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centra



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TÉCNICO LISBOA

Tachyonic gravitational dark matter production after inflation

Tomás Mendes

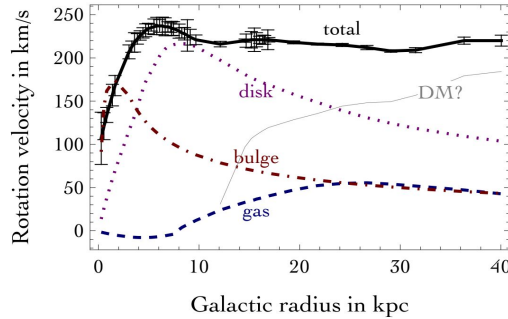
IST-Lisboa

Based on [2601.07670](#), in collaboration with Javier Rubio and Giorgio Laverda

A contribution to the workshop:
YOUNGST@RS - Shaping the Universe: Framework and Footprints of Cosmological Phase Transitions

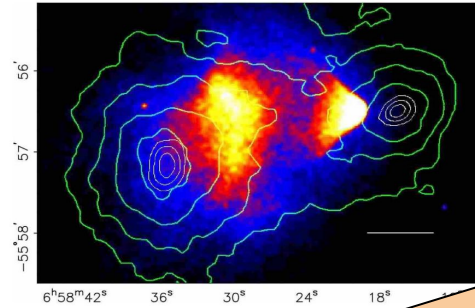
Introduction: The need for dark matter

Galaxy rotation curves



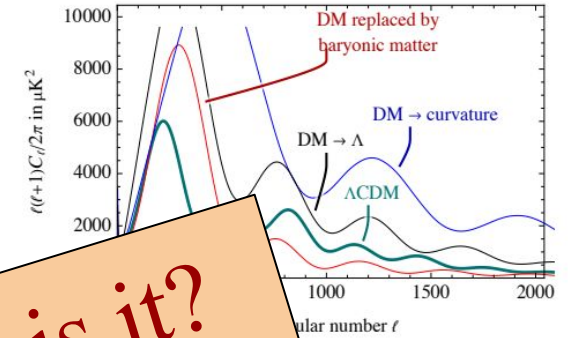
Total rotation velocity (black) for the UGC03205 galaxy, with contributions from the different galactic components [1]

Galaxy clusters



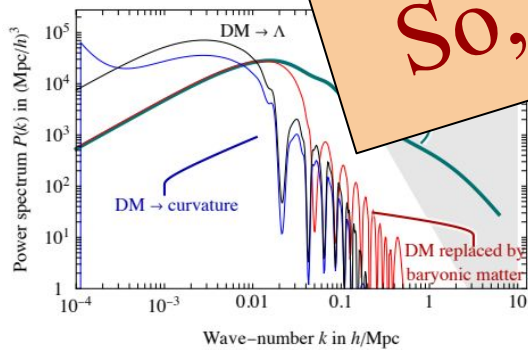
Bullet Cluster. The mass concentrations (DM) are separated from the gas concentration. The DM trace the path of the galaxies, while the gas is concentrated in the center. The CMB acoustic peaks. Alternatives to Λ CDM do not fit data correctly [2]

CMB power spectrum

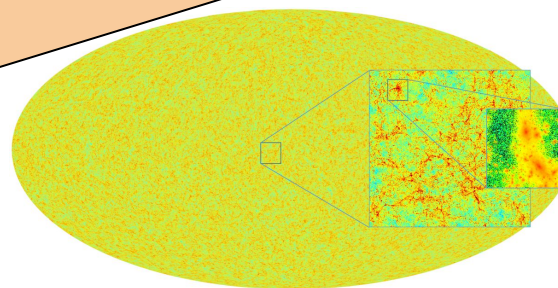


So, it exists! But what is it?

Matter power spectrum

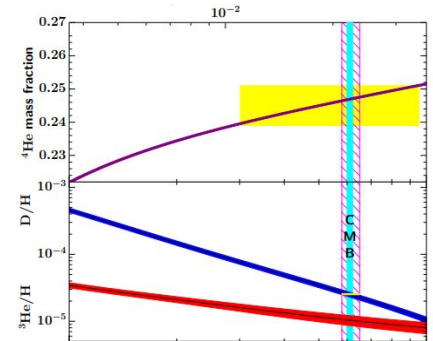


Matter power spectrum, extracted from extensive galaxy surveys. Alternatives to Λ CDM do not fit data correctly [1]



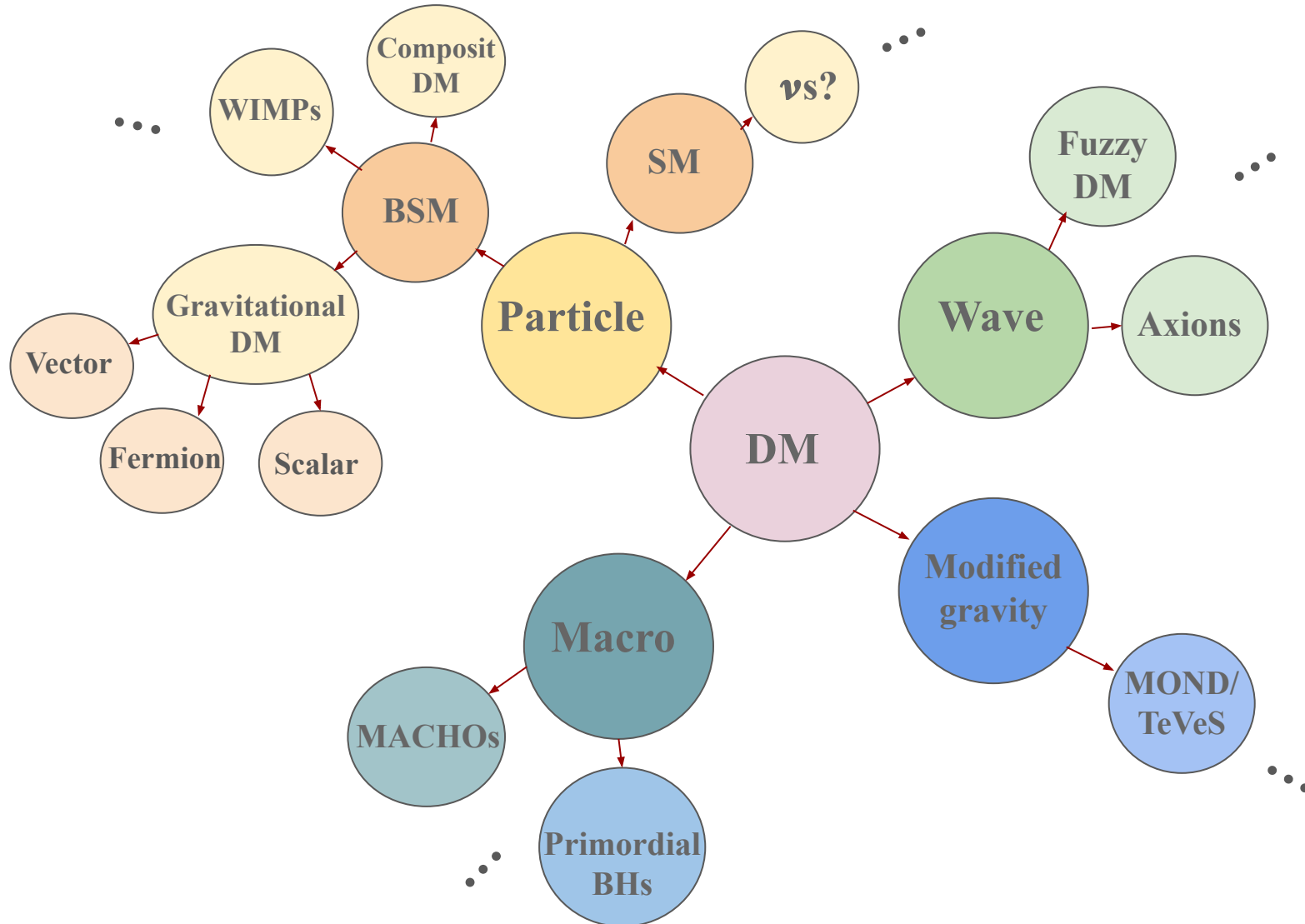
Simulated full-sky matter distribution from a 2 trillion particles simulation. The zoom-in quadrant shows the nonlinear structure of the universe on small scale. [4]

BBN

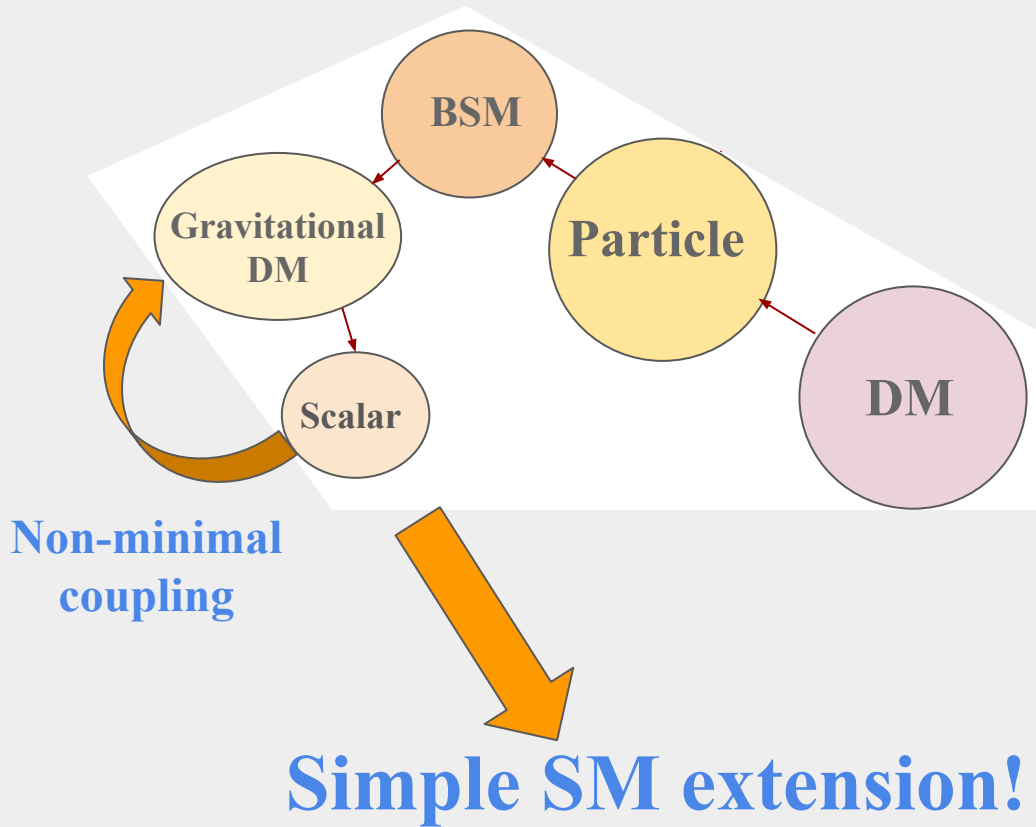


Primordial abundances of ^4He , D and ^3He agree with CMB at low relative abundances [5]

Introduction: The nature of dark matter



Introduction: The nature of dark matter

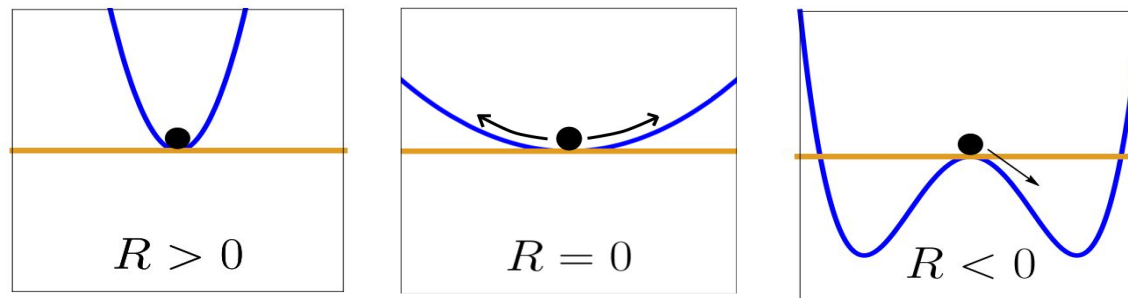


Introduction: Hubble-induced phase transitions

χ \longrightarrow Scalar DM field

$$V(\chi) = \dots + \xi R \chi^2 \longrightarrow \text{Non-minimal coupling to gravity}$$

Changing the sign of the Ricci scalar can change the symmetry of the potential



Breaking of the symmetry of the potential for a model with a quartic self-interaction [6]

In FLRW $R \propto H^2$ \longrightarrow

The phase transition can be induced by the expansion of space!

Other non-minimal couplings can also lead to interesting phenomenology \longrightarrow

Introduction: Hubble-induced phase transitions

Other non-minimal couplings can lead to more interesting phenomenology

Gravitational lagrangian

In RD this term is zero on average!

$$\mathcal{L}_{\text{eff}} = \sqrt{-g} \left[\frac{M_P^2}{2} R + \frac{1}{\Lambda^2} (\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) + \frac{1}{\Lambda^3} \mathcal{O}(R^3) + \dots \right]$$

Einstein-Hilbert action
yields GR

Account for quantum
corrections

Couple to the DM field

$$\mathcal{L}_\chi = \dots + f(R, R^2, \dots) \chi^2$$

Toy Model

To be predictive one needs to choose a combination of (α, β, γ) coefficients

$$\begin{pmatrix} \alpha = 1 \\ \beta = -4 \\ \gamma = 1 \end{pmatrix}$$

Gauss-Bonnet: $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

No additional degrees of freedom

Change in sign occurs for any expansion history after inflation

We show that one could use a wide range of (α, β, γ) coefficients, while always inducing a SSB

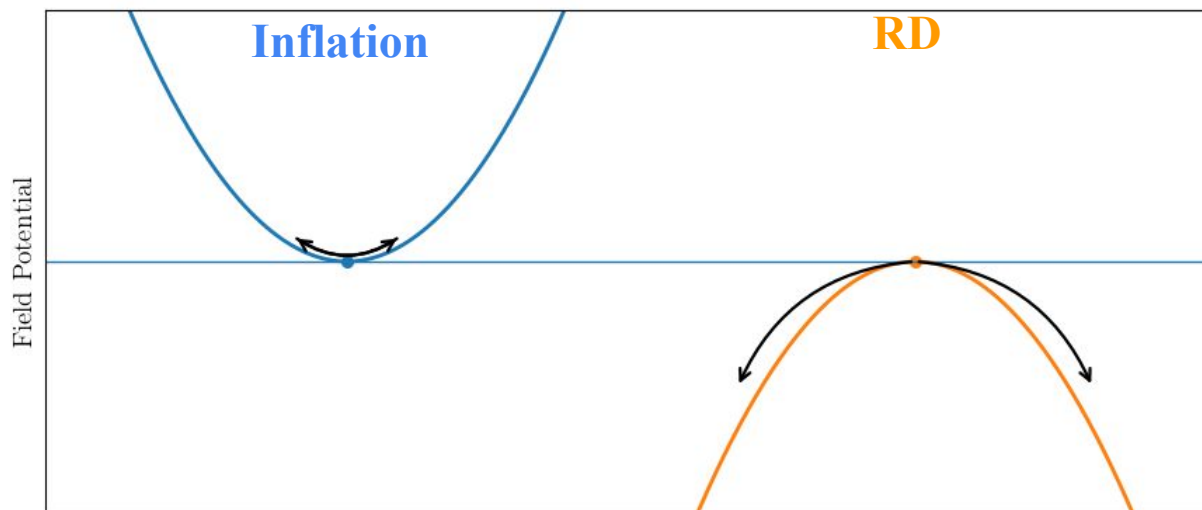
Dynamics of the system in an expanding universe

In FLRW, $\mathcal{G} = -12(1 + 3w)H^4$

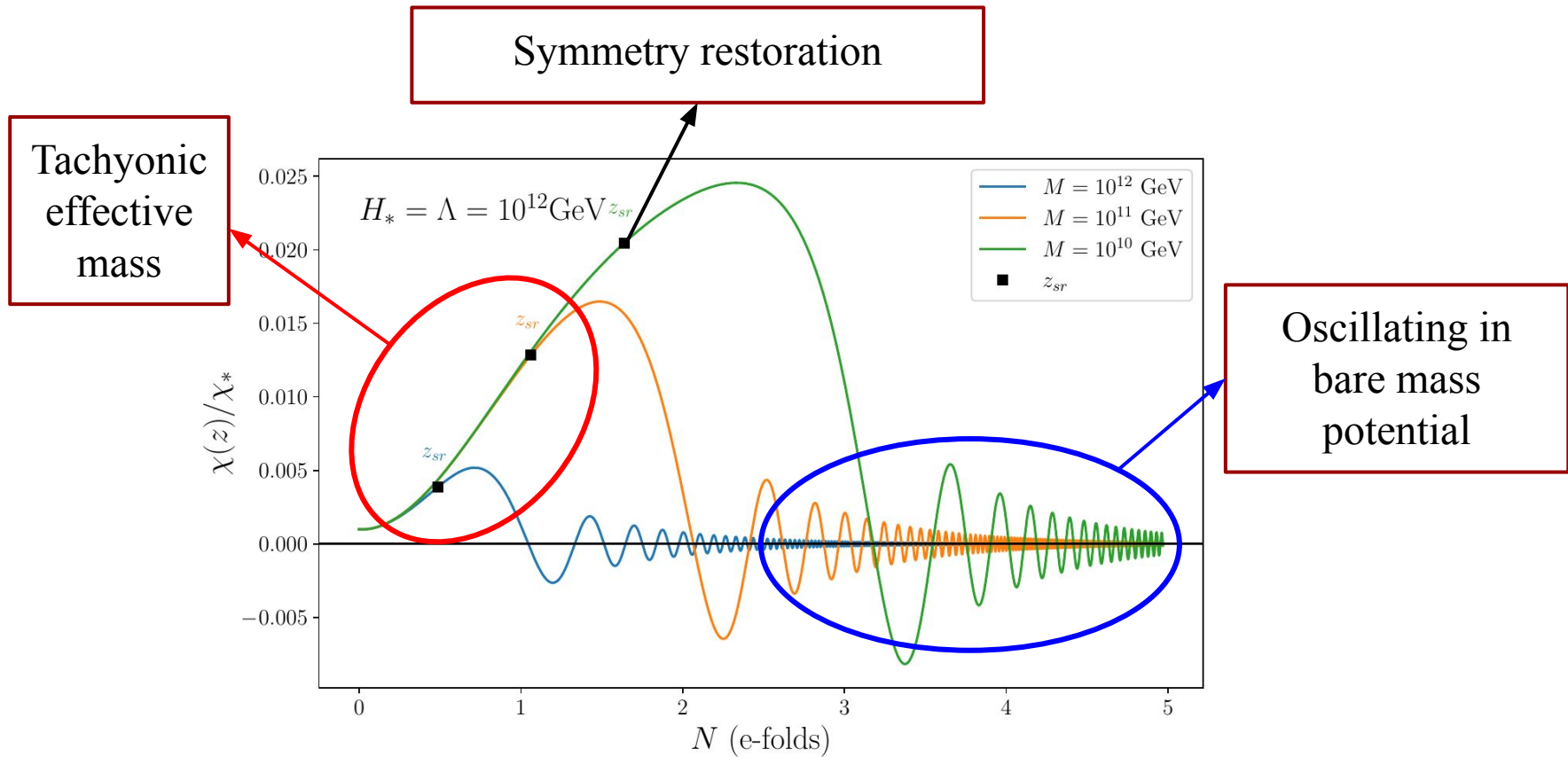
EoS parameter
Hubble rate

Klein-Gordon equation: $\ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\nabla^2\chi + \left(M^2 + \frac{2\mathcal{G}}{\Lambda^2}\right)\chi = 0$

$$M_{\text{eff}}^2 > 0 \qquad M_{\text{eff}}^2 = M^2 - 48\frac{H^4}{\Lambda^2} < 0$$



Homogeneous approximation



Evolution of the energy density of the field, for three masses

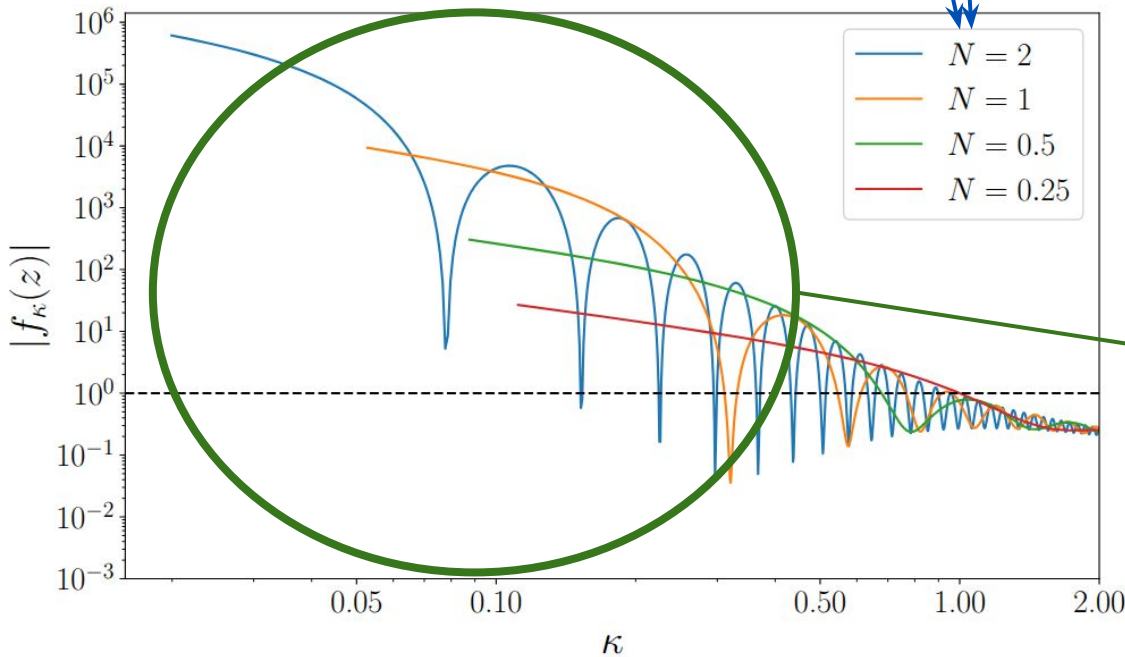
Quantum treatment

Important steps:

- Quantize the field
- Find mode equation and solution
- Find classicalization condition

$$f''_{\kappa} + \omega_{\kappa}^2(z) f_{\kappa} = 0$$

$$\omega_{\kappa}^2 = \kappa^2 - M^2(z)$$



Mode function vs. momenta

Amplification window

$$\kappa_{\min}(z) < \kappa < \kappa_{\max}(z),$$

$$\kappa_{\min}(z) = \mathcal{H}(z), \quad \kappa_{\max}(z) = \nu^3 \mathcal{H}^3(z)$$

- **IR modes** will be **amplified** and will dominate the dynamics
- Large amplification leads to **classicalization**
- Obtained clear prediction of which momentum band will be amplified!

DM abundance estimation

Assumptions:

- Instantaneous Inflation-RD transition
- DM field is a spectator
- Bare mass initially negligible
- Initial radiation-like scaling of DM field

$$\Omega_{\text{DM}}^0 \sim \mathcal{C}(\nu, \kappa_*) \left(\frac{H_*}{M_P} \right)^{5/2} \left(\frac{M}{H_*} \right) \nu^{5/2} \exp[\alpha(\kappa_*) \nu], \quad \nu \sim \frac{H_*}{\Lambda}$$

Outputs the produced DM abundance today, in terms of the parameters of the theory and small numerical factors

$$\kappa_* \sim 10^{-1}$$

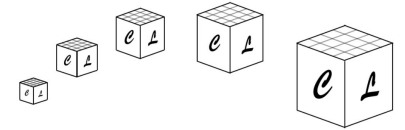
Typical momentum scale

$$(H_*, M, \Lambda)$$

Lattice Simulations

Verify our analytical estimates!

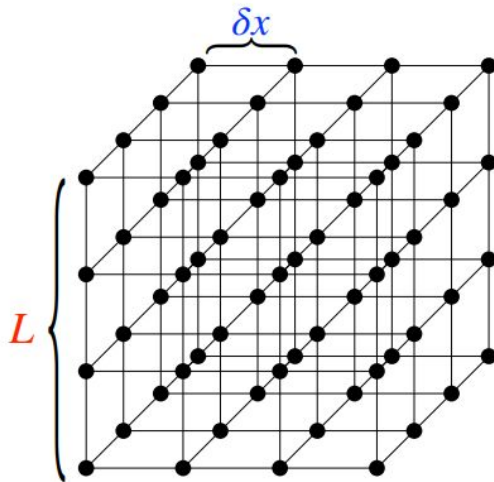
Classicalization seen before \longrightarrow Classical lattice simulations



CosmoLattice [8]

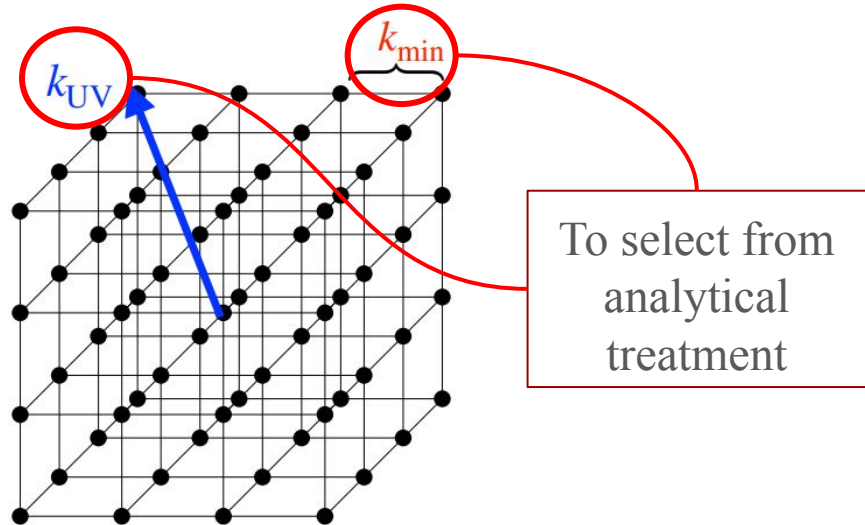
Discretize equations of motion on a 3D grid

Real Lattice



Pictorial lattice [7]

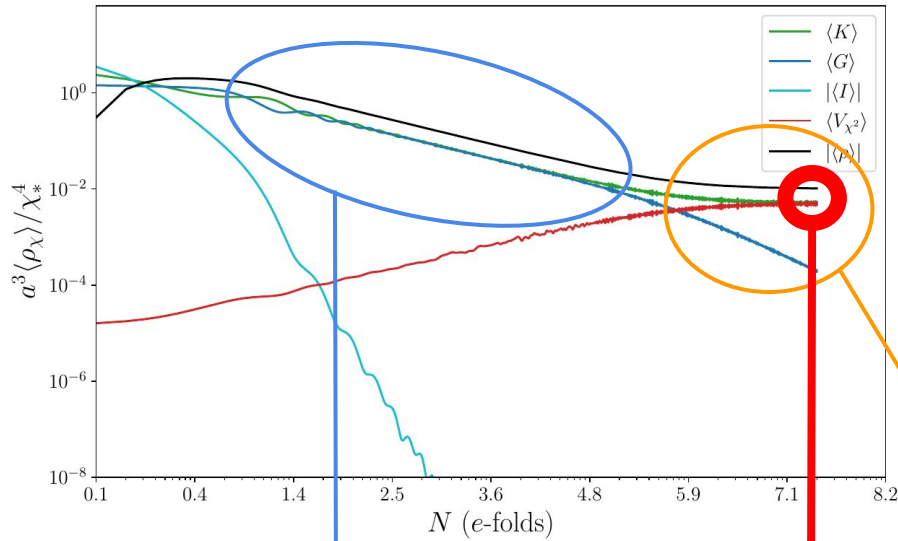
Fourier Lattice



Initialize the lattice points with gaussian random fluctuations over a background

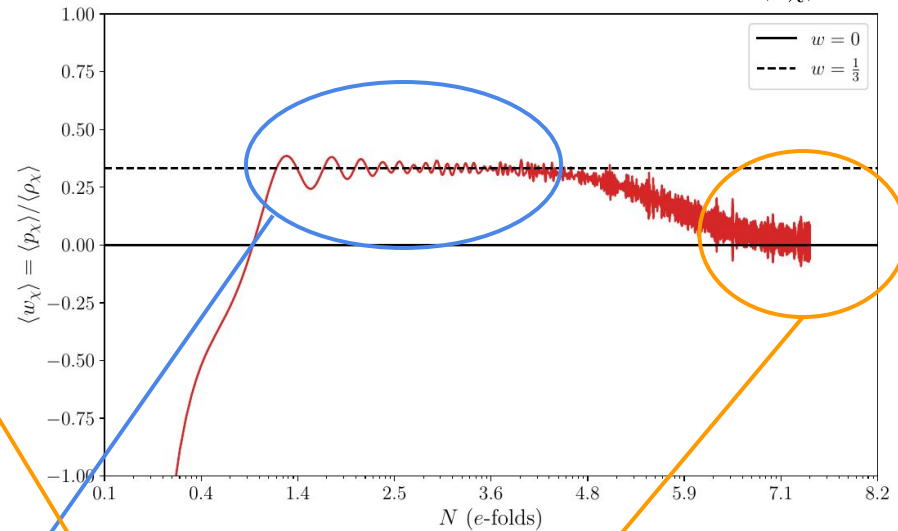
Observables : Energy density and EoS

Energy density



Intermediate period of radiation-like scaling

Equation of state: $\langle w \rangle = \frac{\langle p_X \rangle}{\langle \rho_X \rangle}$



Asymptotic approach to matter-like scaling

Record energy density once it acquires matter-like behaviour and propagate to today

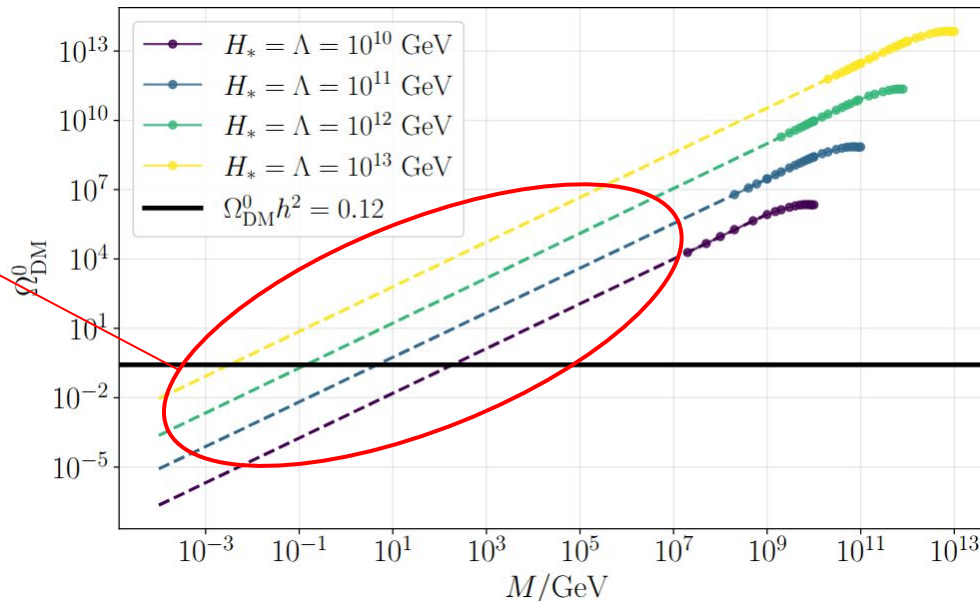
Calculation of DM abundance

Taken from the previous result

$$\Omega_{\text{DM}}^0 = \left(\frac{(\Omega_R^0)^{3/2}}{H_0 H_*^3} \right)^{1/2} \frac{\rho_M(t_{\text{mat}})}{3M_P^2} \left(\frac{a(t_{\text{mat}})}{a_*} \right)^3$$

Calculate this quantity for different values of the model's parameters: $M \ \Lambda \ H_*$

Interpolated in order to cross current observed value



Latest observed abundance from Planck satellite [8]

Today's DM abundance for the first set of simulations (dots)

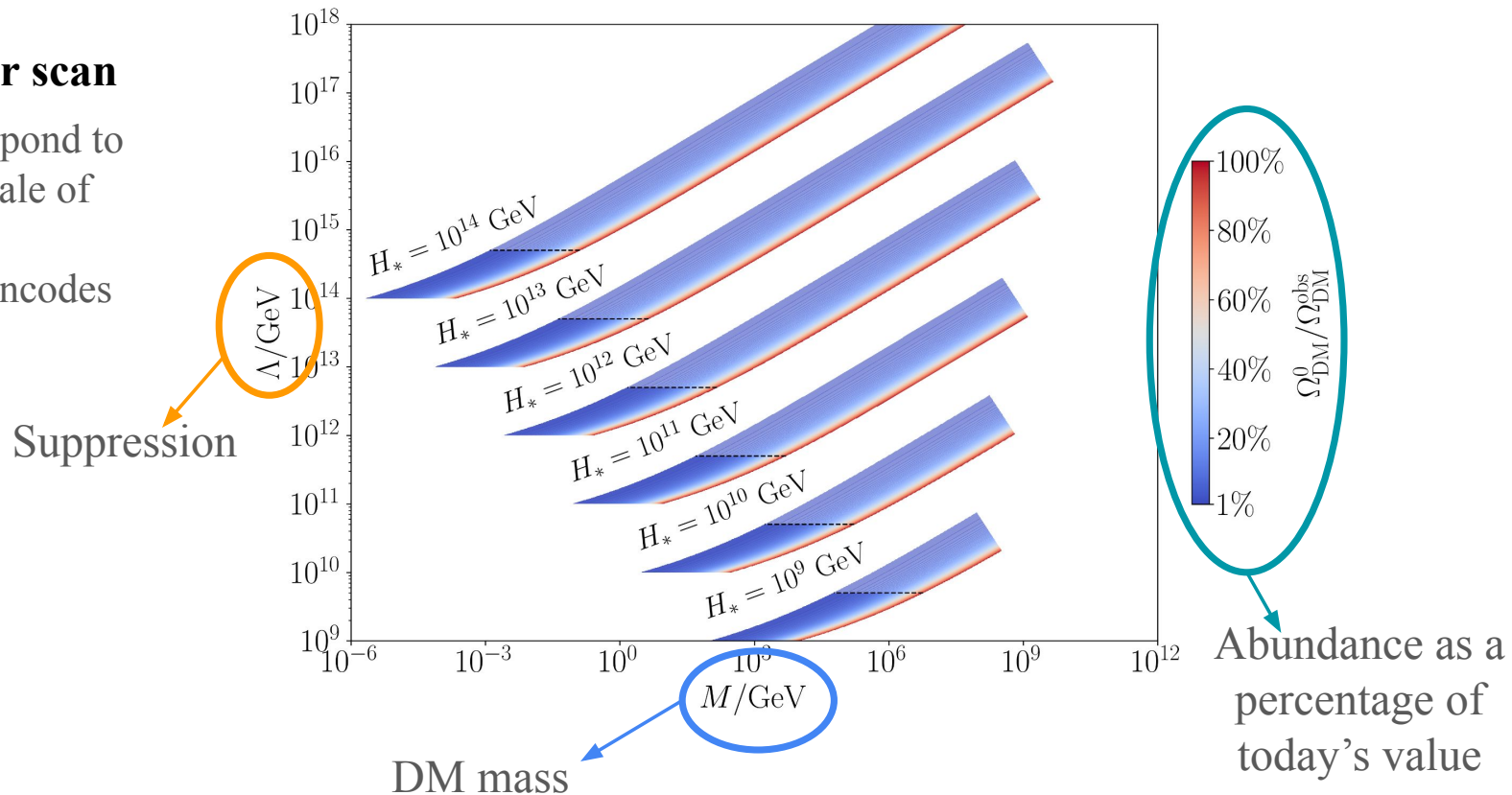
Full parameter scan

$$\Omega_{\text{DM}}^0(H_*, M, \Lambda) \sim 10^a \left(\frac{H_*}{M_P}\right)^b \left(\frac{M}{H_*}\right)^c \nu^d \exp[\alpha_n \nu]$$

$$a = 24.476, \quad b = 2.499, \quad c = 0.987, \quad d = 2.505, \quad \alpha_n = 0.403$$

Full parameter scan

- Bars correspond to a certain scale of inflation
- Color bar encodes abundance



Analytical Vs. Numerical estimates

Analytical

$$\Omega_{\text{DM}}^0 \sim \mathcal{C}(\nu, \kappa_*) \left(\frac{H_*}{M_P}\right)^{5/2} \left(\frac{M}{H_*}\right) \nu^{5/2} \exp[\alpha(\kappa_*) \nu]$$

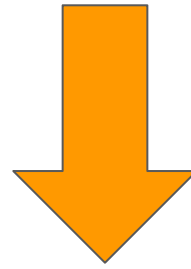
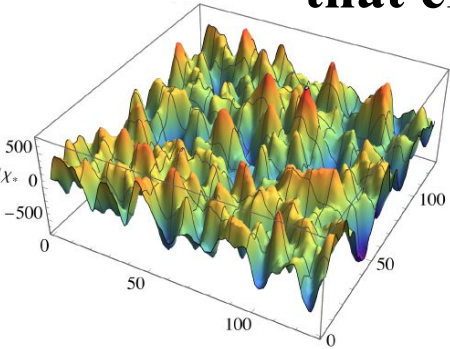
Numerical

$$\Omega_{\text{DM}}^0(H_*, M, \Lambda) \sim 10^{24.476} \left(\frac{H_*}{M_P}\right)^{2.499} \left(\frac{M}{H_*}\right)^{0.987} \nu^{2.505} \exp[0.403\nu]$$

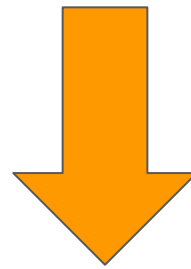
Impressive agreement, given the analytical approximations!

Conclusion

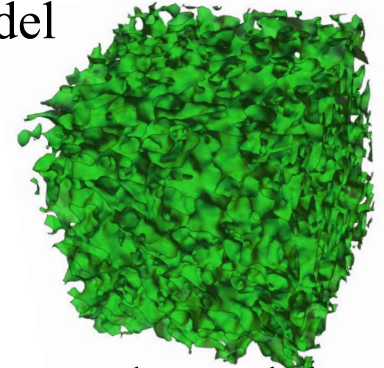
Spacetime curvature alone can trigger a tachyonic instability that efficiently produces dark matter after inflation.



The instability is driven by the background expansion (inflation \rightarrow RD), not by tuned couplings or interactions with the Standard Model



The mechanism works over a wide range of masses and inflationary scales and does not rely on thermal equilibrium or reheating details



Bibliography

- [1] M. Cirelli, A. Strumia and J. Zupan, Dark Matter, [2406.01705]
- [2] D. Clowe, et. al, A direct empirical proof of the existence of dark matter, *Astrophys. J. Lett.*, [astro-ph/0608407]
- [3] PLANCK collaboration, Planck 2018 results. I. Overview and the cosmological legacy of Planck, *Astron. Astrophys.* 641 (2020) A1, [1807.06205]
- [4] D. Potter, J. Stadel, and R. Teyssier, "PKDGRAV3: beyond trillion particle cosmological simulations for the next era of galaxy surveys," *Comput. Astrophys. Cosmol.*, 2017. [1609.08621]
- [5] C. Patrignani et al. (Particle Data Group), "Big-Bang Nucleosynthesis," *Prog. Theor. Exp. Phys.* 2024, 083C01 (2024), [hep-ph/9701286]
- [6] D. Bettoni, G. Domènech, and J. Rubio, "Gravitational waves from global cosmic strings in quintessential inflation," *JCAP* **02**, 034 (2019), [1810.11117]
- [7] Figueroa, Daniel G., Florio, Adrien, Torrenti, Francisco, and Valkenburg, Wessel. "The art of simulating the early universe. Part I. Integration techniques and canonical cases." *Journal of Cosmology and Astroparticle Physics*, [2011.13957]
- [8] D.G. Figueroa, A. Florio, F. Torrenti and W. Valkenburg, CosmoLattice: A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe, *Comput. Phys. Commun.* 283 (2023) 108586 [2102.01031].
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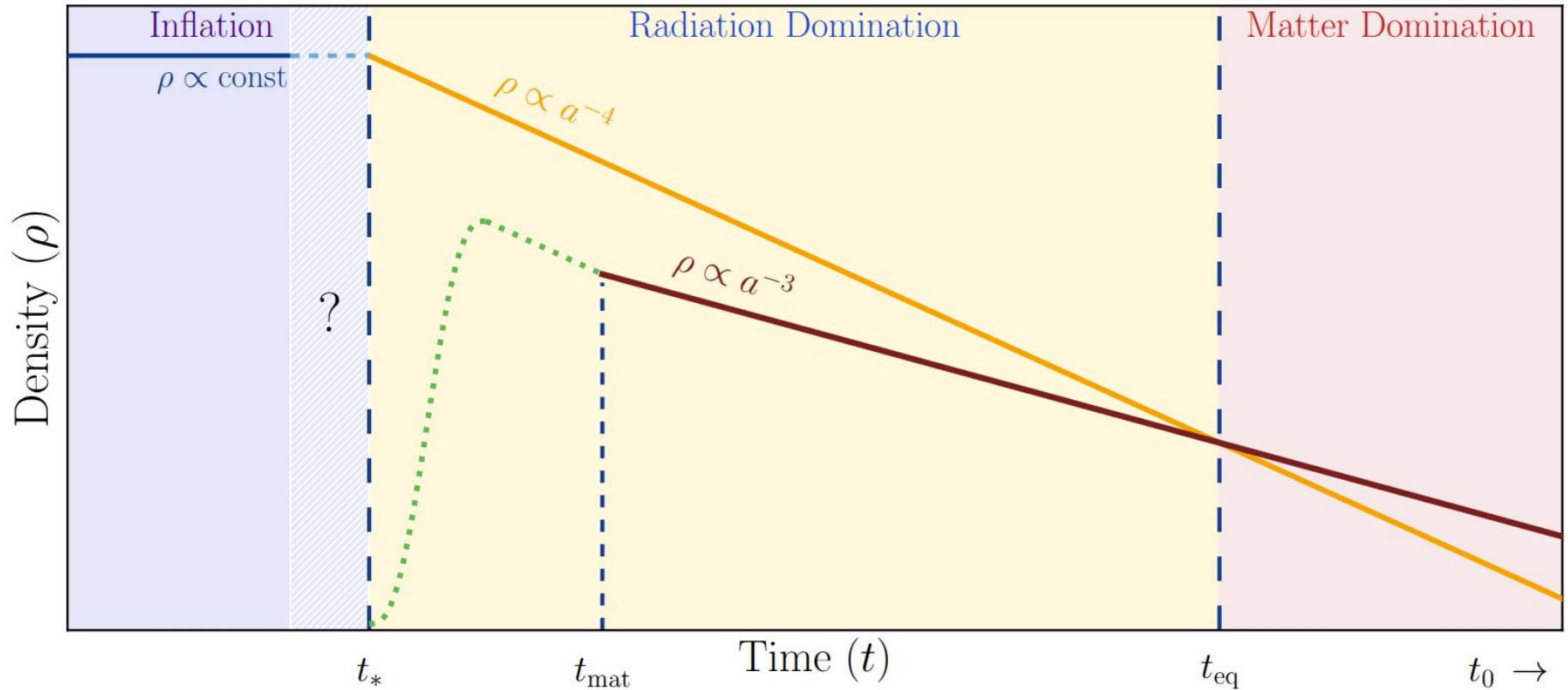
Thank you!
Obrigado!
Gracias!
Danke!



<https://arxiv.org/abs/2601.07670>

Supplementary slides

Additional slide 1



Additional slide 2

From t_{eq} to t_0 , the energy density of the matter and radiation fluids has scaled as dictated in (2.16), giving:

$$\rho_m^0 = \rho_m(t_{\text{eq}}) \left(\frac{a_{\text{eq}}}{a_0} \right)^3 = \rho_r^0 \left(\frac{a_0}{a_{\text{eq}}} \right). \quad (2.17)$$

Moreover, noting that $\rho_r(t_{\text{eq}}) = \rho_m(t_{\text{eq}}) = \rho_{\text{eq}}/2$ and $\Omega_r^0 = \rho_r(t_0)/(3M_p^2 H_0^2)$,

$$\frac{\rho_{\text{eq}}}{2} = \frac{3}{2} M_p^2 H_{\text{eq}}^2 = \rho_r(t_{\text{eq}}) = \rho_r(t_0) \left(\frac{a_0}{a_{\text{eq}}} \right)^4 \implies \frac{a_0}{a_{\text{eq}}} = \left(\frac{H_{\text{eq}}}{H_0} \right)^{1/2} \frac{1}{(2\Omega_r^0)^{1/4}}, \quad (2.18)$$

we obtain a relation between the scale factors at the time of equality (a_{eq}) and today (a_0). The Hubble rate at t_{eq} can be calculated by extrapolating the radiation evolution from t_* ,

$$\frac{\rho_r^*}{\rho_r^{\text{eq}}} = \left(\frac{a_{\text{eq}}}{a_*} \right)^4 = \frac{3M_p^2 H_*^2}{3M_p^2 H_{\text{eq}}^2/2} \implies H_{\text{eq}} = H_* \sqrt{2} \left(\frac{a_*}{a_{\text{eq}}} \right)^2. \quad (2.19)$$

Finally, the scale factor at equality a_{eq} is calculated by evolving radiation from t_* and matter from t_{mat} and setting $\rho_r(t_{\text{eq}}) = \rho_m(t_{\text{eq}})$:

$$\rho_r(t_{\text{eq}}) = \rho_m(t_{\text{eq}}) \implies \rho_r(t_*) \left(\frac{a_*}{a_{\text{eq}}} \right)^4 = \rho_m(t_{\text{mat}}) \left(\frac{a(t_{\text{mat}})}{a_{\text{eq}}} \right)^3 \implies a_{\text{eq}} = \frac{3M_p^2 H_*^2}{\rho_m(t_{\text{mat}})} \frac{a_*^4}{a(t_{\text{mat}})^3}. \quad (2.20)$$

Additional slide 3

$$\ddot{\chi} + 3H\dot{\chi} - a^{-2}\nabla^2\chi + 2\frac{\mathcal{R}_{\text{GB}}^2}{\Lambda^2}\chi + M^2\chi = 0 ,$$

$$\frac{G_{\mu\nu}}{\kappa} = \nabla_{\mu}\chi\nabla_{\nu}\chi - \frac{1}{2}g_{\mu\nu} [\nabla^{\rho}\chi\nabla_{\rho}\chi + M^2\chi^2] + \frac{2}{\Lambda^2}H_{\mu\nu} ,$$

$$H_{\mu\nu} = (-2\nabla_{\mu}\nabla_{\nu}R + 2g_{\mu\nu}\square R + 4\nabla_{\rho}\nabla_{\mu}R_{\nu}{}^{\rho} + 4\nabla_{\rho}\nabla_{\nu}R_{\mu}{}^{\rho} - 4\square R_{\mu\nu} - 4g_{\mu\nu}\nabla_{\rho}\nabla_{\sigma}R^{\rho\sigma} + 4\nabla^{\rho}\nabla^{\sigma}R_{\mu\rho\nu\sigma})\chi^2$$

$$\rho_{\chi} = T_{00} = \frac{\dot{\chi}^2}{2} + \frac{(\nabla\chi)^2}{2a^2} + \frac{1}{2}M^2\chi^2 + 8\frac{H^2}{\Lambda^2} \left(3H\partial_t - \frac{\nabla^2}{a^2} \right) \chi^2 ,$$

$$p_{\chi} = \frac{1}{3a^2} \sum_i T_{ii} = \frac{\dot{\chi}^2}{2} - \frac{(\nabla\chi)^2}{6a^2} - \frac{1}{2}M^2\chi^2 + \frac{8}{3\Lambda^2} \left(2 \left(H^2 + \dot{H} \right) \left(\frac{\nabla^2}{a^2} - 3H\partial_t \right) - 3H^2\partial_t^2 \right) \chi^2$$

$$N = \ln \left(\frac{a}{a_*} \right) = \ln \left(1 + \frac{z}{\nu} \right)$$

Additional slide 4

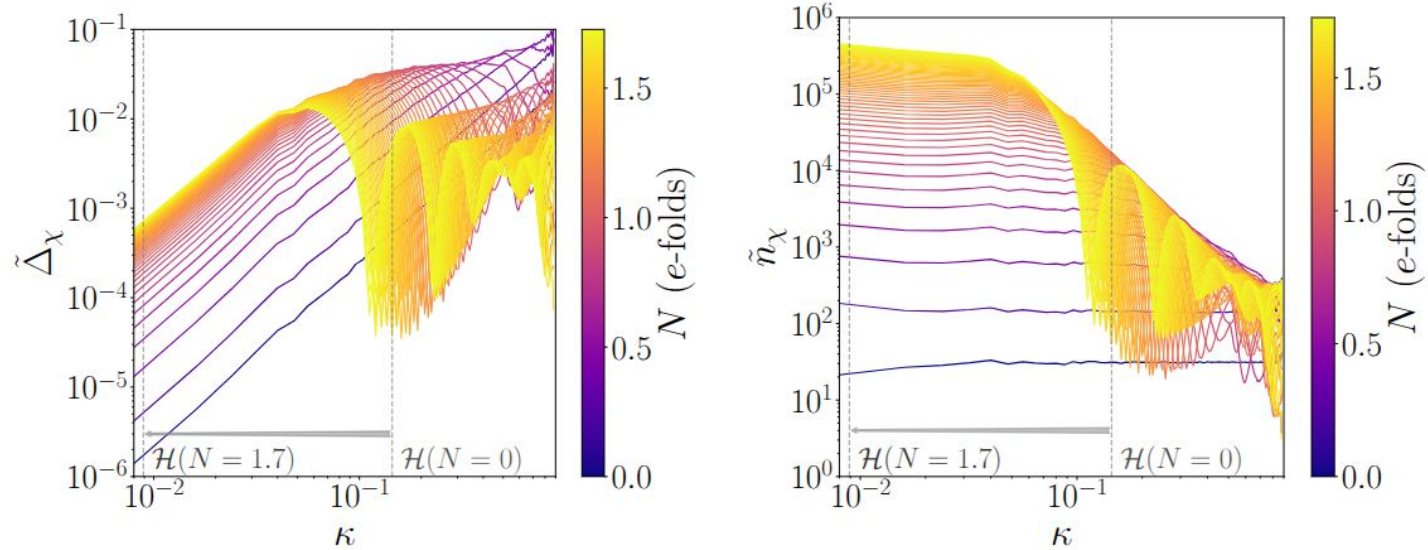


Figure 10: Evolution of the power spectrum of field fluctuations (left panel) and the corresponding lattice-defined occupation numbers (right panel), from $N = 0$ e -folds until the symmetry restoration time z_{sr} . These were obtained from lattice simulations with $N = 128$ lattice points, $H_* = \Lambda = 10^{12}$ GeV and $M = 10^8$ GeV. The spectra are normalized to the scale χ_* , and the color coding indicates different times during the evolution, expressed in e -folds since the onset of RD, with blue (yellow) curves corresponding to earlier (later) times. The grey dashed lines denote the comoving Hubble scale, from the beginning of tachyonic amplification up to z_{sr} . A rapid and strong amplification of IR modes is observed, which dominates the dynamics and drives the subsequent evolution of the system.

Additional slide 5

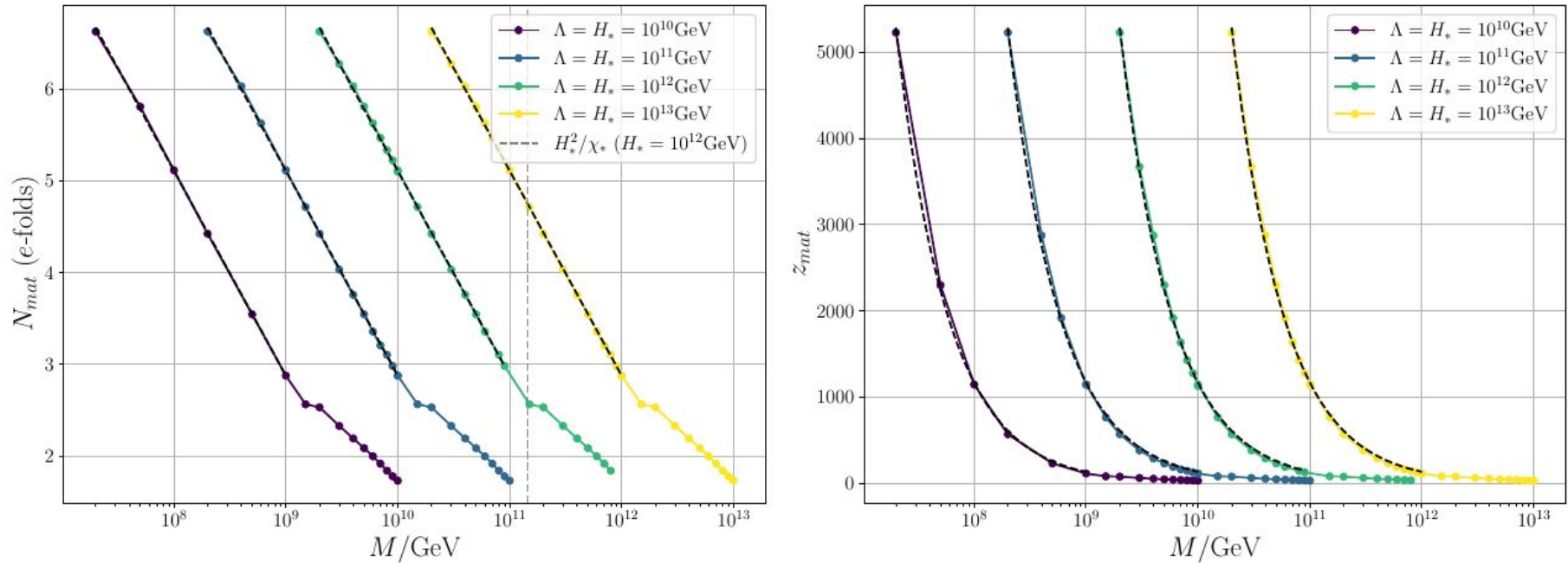
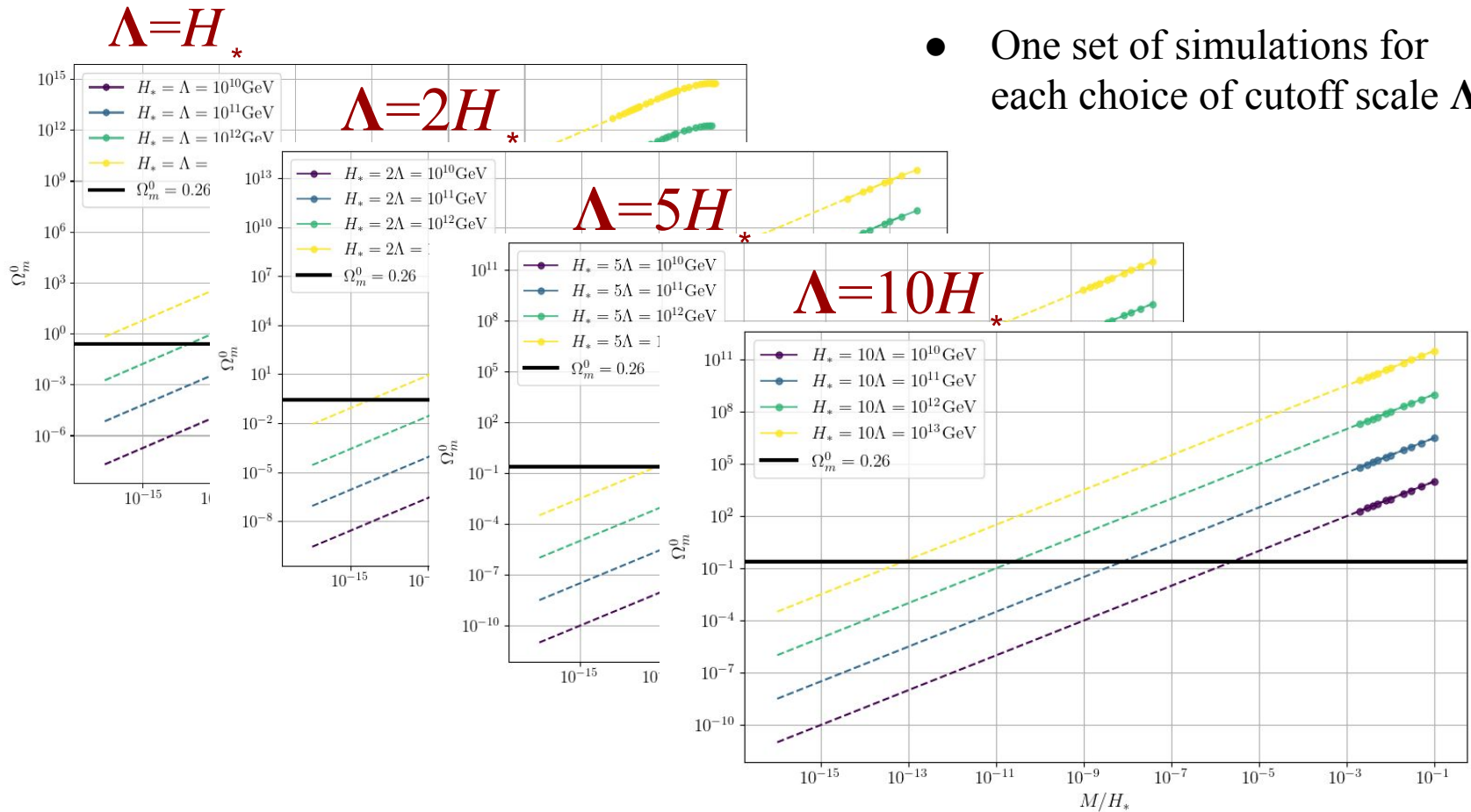


Figure 5.2: Plot of the calculated z_{mat} and the number of e -folds after inflation corresponding to the time of matter-like scaling of χ (N_{mat}), as a function of the field's mass M and for the various values of the scale $H_* = \{10^{14}, 10^{12}, 10^{11}, 10^{10}\}$ GeV. Black dashed lines are the derived parametric equations (5.1). Simulated for $N = 64$ and $\Lambda = H_*$. The gray dashed line in the left panel indicates the position where the slope of the function changes ($M = H_*/\nu$), in the $H_* = 10^{12}$ GeV case.

Parameter scan

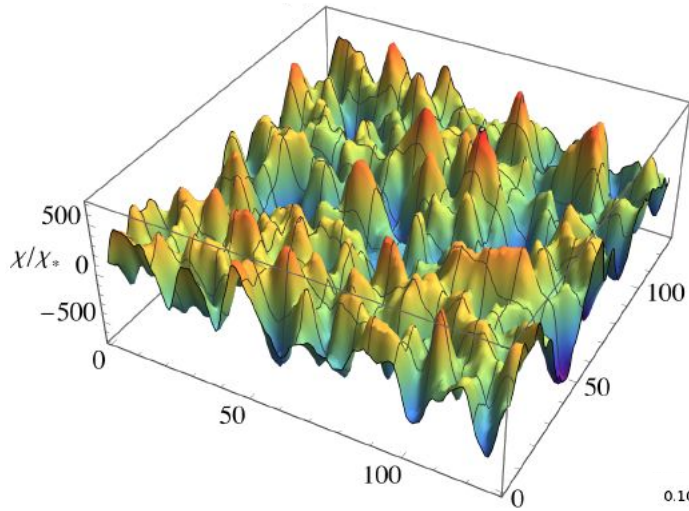
- One set of simulations for each choice of cutoff scale Λ



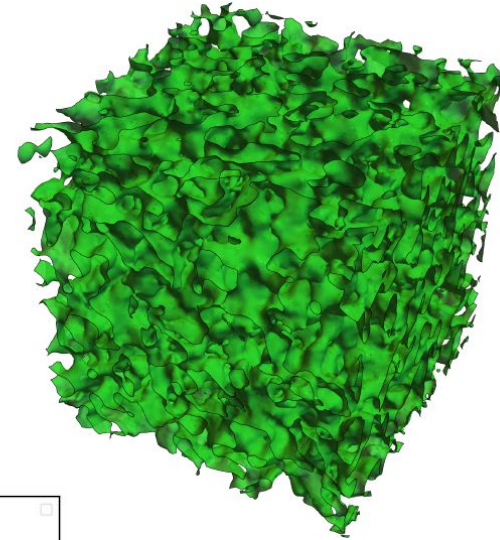
Interpolate each $\Omega_{\text{DM}}^0 = \Omega_{\text{DM}}^0(M, H_*)$ to find $\Omega_{\text{DM}}^0 = \Omega_{\text{DM}}^0(M, H_*, \Lambda)$

Observables

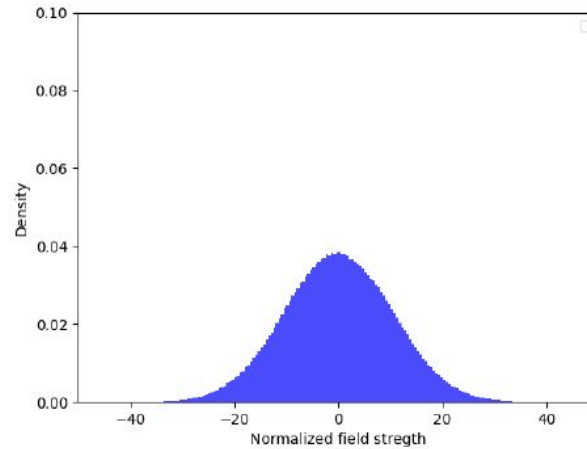
Several observables can be obtained from the simulations



2D slices of the lattice



3D domains of the lattice



Probability distribution

Future directions

- Perform a larger number of better resolved simulations, using more powerful resources
- Specify inflationary potential
- Include a quartic self-interaction term
- Include all the couplings allowed by the symmetry (eg. to the Higgs)
- Consider other symmetry group (eg. $U(1)$)
- Consider different nature for the DM field (fermionic, vectorial,...)