State of the art in self-force theory

Adam Pound

Lost in Translation

20 January 2026

UK Research and Innovation

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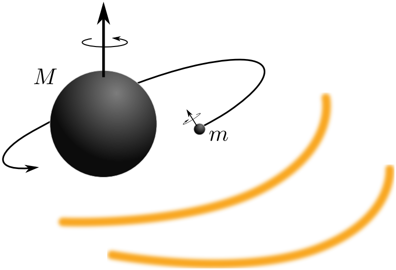
Gravitational self-force theory



- $\varepsilon = 1/q = m/M \ll 1$
- small body perturbs spacetime:

$$\mathsf{g}_{\mu\nu} = g^{\text{Kerr}}_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \ldots$$

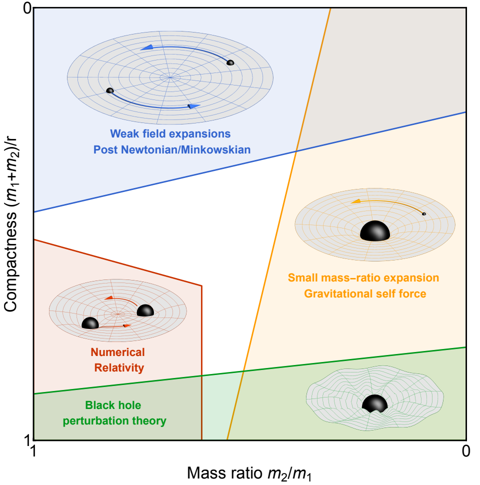
- perturbation affects m's motion:

$$\frac{D^2 x_p^\mu}{d\tau^2} = \varepsilon f^\mu_{(1)} + \varepsilon^2 f^\mu_{(2)} + \ldots$$

(other notation: $M = m_1$, $m = m_2$)

Self-force in the binary landscape

- NR: "exact"
- PN: $v^2 \sim (m_1 + m_2)/r \ll 1$
- PM: $(m_1 + m_2)/r \ll 1$
- GSF: $m_2/m_1 \ll 1$
- ringdown: $|g_{\alpha\beta} - g^{\text{Kerr}}_{\alpha\beta}| \ll 1$



[Credit: LISA Waveforms White Paper]

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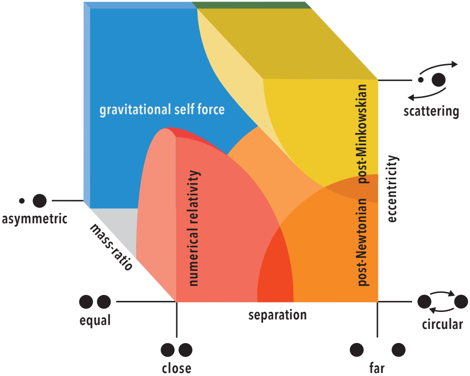

[Credit: Carvalho & GWSky]

Attributes of self-force theory

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Physically simple: waveform determined by secondary's orbit

Limited: currently limited to leading order over most of parameter space

Outline

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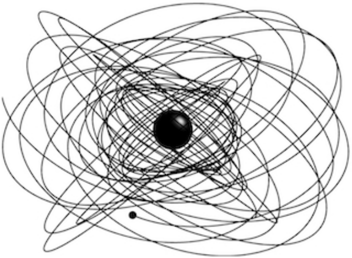
Zeroth order: test mass on a geodesic in Kerr



[image credit: Steve Drasco]

- constants $M_I = (M, a)$ & $P_A = (E, L_z, Q)$:
  1. primary's mass and spin (M, a)
  2. orbital energy E
  3. orbital angular momentum L_z
  4. Carter constant Q, related to orbital inclination
- phases $\varphi_A = (\varphi_r, \varphi_\theta, \varphi_\phi)$ with frequencies $\Omega_A^{(0)}(M_I, P_B)$

- simple ODEs:

$$\frac{d\varphi_A}{dt} = \Omega_A^{(0)}(M_I, P_B)$$
$$\frac{dP_A}{dt} = 0$$
$$\frac{dM_I}{dt} = 0$$

Multiscale expansion

[Mathews, AP; Flanagan, Hinderer, Lynch, Miller, Moxon, van de Meent, Warburton, ...]

- metric perturbations as functions on binary phase space:

$$h^{(n)}_{\alpha\beta} = \sum_{k_A \in \mathbb{Z}^3} h^{(n,k_A)}_{\alpha\beta}(M_I, P_B, r, \theta, \phi) e^{-ik_A \varphi_A}$$

- nice (near-identity transformed) phase-space variables:

$$\frac{d\varphi_A}{dt} = \Omega^{(0)}_A(M_I, P_B) + \varepsilon \Omega^{(1)}_A(M_I, P_B) + \mathcal{O}(\varepsilon^2)$$
$$\frac{dP_A}{dt} = \varepsilon \left[F^{(0)}_A(M_I, P_B) + \varepsilon F^{(1)}_A(M_I, P_B) + \mathcal{O}(\varepsilon^2)\right]$$
$$\frac{dM_I}{dt} = \varepsilon^2 \mathcal{F}^{(1)\mathcal{H}}_I(M_J, P_B) + \mathcal{O}(\varepsilon^3)$$

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Adiabatic order (0PA)

determined by dissipative modes of $h_{\alpha\beta}^{(1)}$ (1SF)

Multiscale expansion

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- metric perturbations as functi...

$$h_{\alpha\beta}^{(n)} = \sum_{k_A \in \mathbb{Z}^3} h_{\alpha\beta}^{(n,k_A)}(M_I, P_B, r, \theta, \phi)e^{-ik_A\varphi_A}$$

- nice (near-identity transformed) phase-space variables:

Adiabatic order (0PA)

determined by dissipative modes of $h^{(1)}_{\alpha\beta}$ (1SF)

First post-adiabatic order (1PA)

determined by

- dissipative modes of $h^{(2)}_{\alpha\beta}$ (2SF)
- all of $h^{(1)}_{\alpha\beta}$ (1SF)

$$\frac{d\varphi_A}{dt} = \Omega_A^{(0)}(M_I, P_B) + \varepsilon\,\Omega_A^{(1)}(M_I, P_B) + \mathcal{O}(\varepsilon^2)$$

$$\frac{dP_A}{dt} = \varepsilon\left[F_A^{(0)}(M_I, P_B) + \varepsilon F_A^{(1)}(M_I, P_B) + \mathcal{O}(\varepsilon^2)\right]$$

$$\frac{dM_I}{dt} = \varepsilon^2 \mathcal{F}_I^{(1)\mathcal{H}}(M_J, P_B) + \mathcal{O}(\varepsilon^3)$$

Required accuracy

- phases have an expansion

$$\varphi_A = \varepsilon^{-1} \underbrace{\varphi_A^{(0)}(\varepsilon t)}_{\text{0PA}} + \varepsilon^0 \underbrace{\varphi_A^{(1)}(\varepsilon t)}_{\text{1PA}} + \mathcal{O}(\varepsilon)$$

- a model that gets $\varphi_A^{(0)}$ and $\varphi_A^{(1)}$ right should be enough for precise parameter extraction [Burke et al.]
- also need to account for transient orbital resonances (0.5PA)
- secondary spin χ_2 also enters at 1PA

Multiscale field equations and rapid waveforms

Offline step

- field equations on phase space:

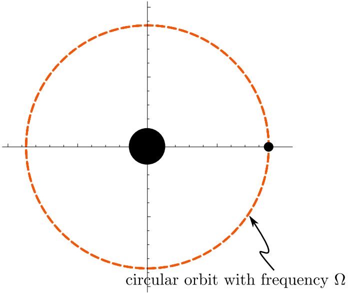
$$\partial_t = \underbrace{\frac{d\varphi_A}{dt}}_{\Omega_A^{(0)}+\varepsilon\Omega_A^{(1)}+\ldots} \partial_{\varphi^A} + \underbrace{\frac{dP_A}{dt}}_{\varepsilon F_A^{(0)}+\varepsilon^2 F_A^{(1)}+\ldots} \partial_{P_A} + \underbrace{\frac{dM_I}{dt}}_{\varepsilon^2 \mathcal{F}_I^{(1)\mathcal{H}}+\ldots} \partial_{M_I}$$

- compute waveform amplitudes $\lim_{r\to\infty} h_{\alpha\beta}^{(n,k_A)}$ and forcing functions $F_A^{(n-1)}$ on grid of (M_I, P_A) values.

Online step

- solve ODEs for φ_A, P_A, and M_I, add up mode $h_{\alpha\beta}^{(n,k_A)} e^{-imK_A\varphi_A}$
- “slow fast”: WaSABI: seconds [Honet, Mathews, Wardell,...]
- “fast fast”: FEW: milliseconds [Chapman-Bird, Chua, Hughes, Katz, Nasipak, Speri, Warburton, ...]

Example: quasicircular inspiral



$$\frac{d\phi_p}{dt} = \Omega$$

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$$h_{\ell m} = \sum_{n\geq 1} \varepsilon^n h^{(n)}_{\ell m}(\Omega) e^{-im\Omega t}$$

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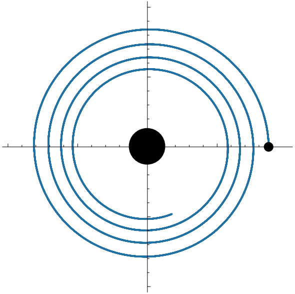


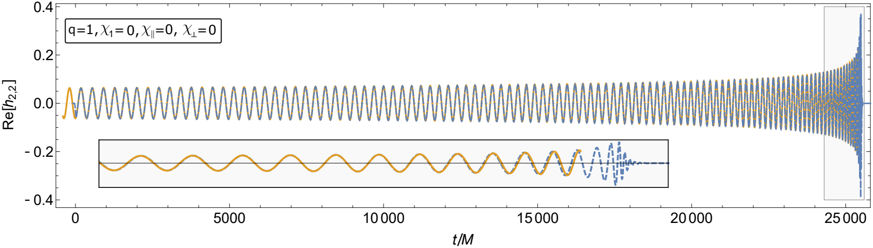
$$\frac{d\phi_p}{dt} = \Omega$$
$$\frac{d\Omega}{dt} = \varepsilon[F_{(0)}(\Omega) + \varepsilon F_{(1)}(\Omega) + \ldots]$$
$$h_{\ell m} = \sum_{n\geq 1} \varepsilon^n h^{(n)}_{\ell m}(\Omega) e^{-im\phi_p}$$

(note: field equations for $h^{(1)}_{\ell m}$ are the same with $e^{-im\Omega t}$ or $e^{-im\phi_p}$)

Precompute and store $h^{(n)}_{\ell m}(\Omega)$ and $F_{(n)}(\Omega)$
$\Rightarrow$ fast waveforms

State of the art in accuracy: 1PA [AP, Warburton, Wardell, &co]

- quasicircular, generic secondary spin, linear primary spin
- comparison with NR:



[Mathews, AP, Warburton, Wardell]

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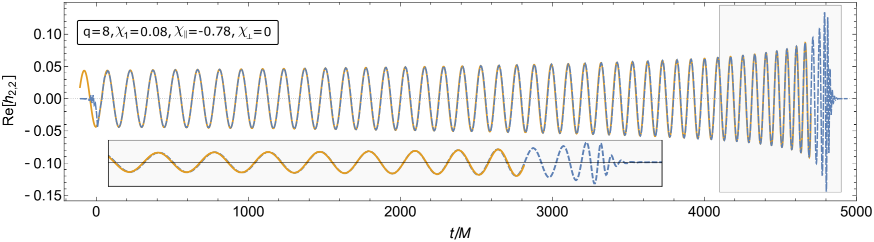
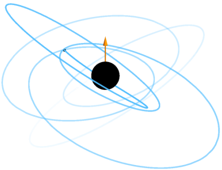


[Mathews, AP, Warburton, Wardell]

State of the art in parameter-space coverage: 0PA

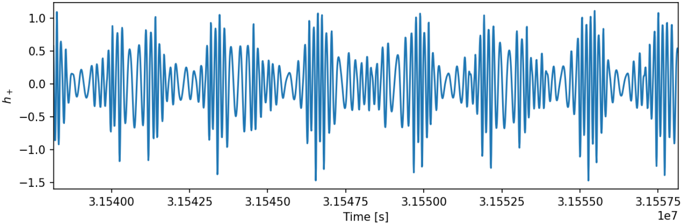
[Isoyama et al.,Hughes et al., Chapman-Bird et al., Warburton, Nasipak, ...]

- LISA-length 0PA waveforms for complex orbits
- FEW implementation: currently used in LISA pipeline development



[Credit: P. Lynch]

[Credit: Chapman-Bird et al.]



Summary: state of play

Goal: 1PA waveforms over full parameter space

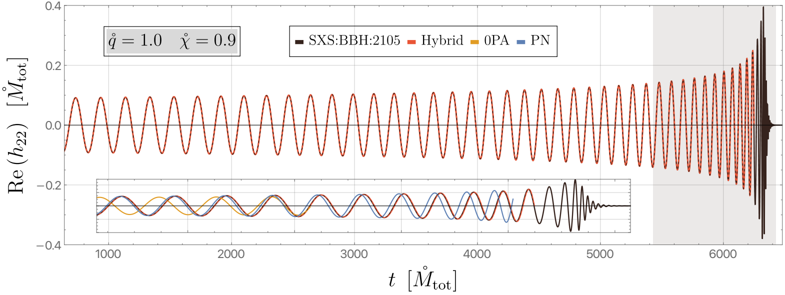
Background Spacetime	Orbital Configuration	Adiabatic	Post-1-adiabatic			
		1SF (Dissipative)	1SF (Conservative)	2SF (Dissipative)	Spin Effects (Conservative)	Spin Effects (Dissipative)
Schwarzschild	Circular	✓✓✓	✓✓✓	✓✓✓	✓✓✓	✓✓✓
	Eccentric	✓✓✓	✓✓✓	✗	✓✓ , ✓✓✓*	✓, ✓✓*
Kerr	Circular	✓✓✓	✓✓	✗	✓,✓✓*	✓✓✓*
	Eccentric Equatorial	✓✓✓	✓✓	✗	✓,✓✓*	✓✓*
	Generic	✓✓✓	✓	✗	✓	✓*
	Resonances	✓✓✓	✓	✗	✗	✗

✓✓✓ Evolving Waveform ✓✓ Driven Inspiral ✓ Snapshot Calculation *(Anti-)Aligned Spin Only

[table courtesy of Josh Mathews]

But! SF-PN-(PM?) hybridization

- modular construction $\Rightarrow$ we can use PN (or PM) approximations for missing inputs
- e.g.: quasicircular, spinning binaries: 0PA, partial 1PA, & 4PN:



[Honet et al.]

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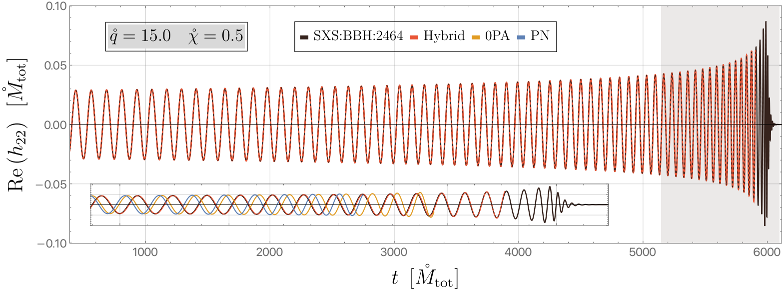


[Honet et al.]

Outline

Transition to plunge

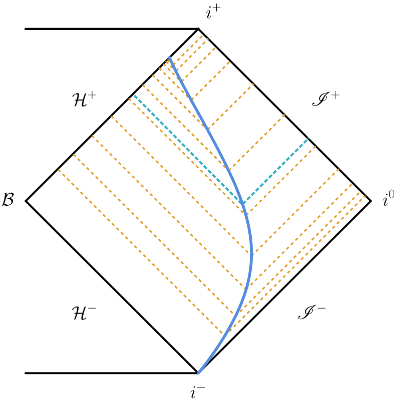
Near innermost stable circular orbit: define $\Delta\Omega := \frac{\Omega - \Omega_{isco}}{\varepsilon^{2/5}}$

$$\frac{d\phi_p}{dt} = \Omega$$

$$\frac{d\Delta\Omega}{dt} = \varepsilon^{1/5}\left[F^{(0)}_{\Delta\Omega}(\Delta\Omega) + \varepsilon^{1/5}F^{(1)}_{\Delta\Omega}(\Delta\Omega) + O(\varepsilon^{2/5})\right]$$

$$h_{\ell m} = \varepsilon \sum_{n\geq 0} \varepsilon^{n/5} j^{(n)}_{\ell m}(\Delta\Omega) e^{-im\phi_p}$$

Merger & ringdown: the final plunge



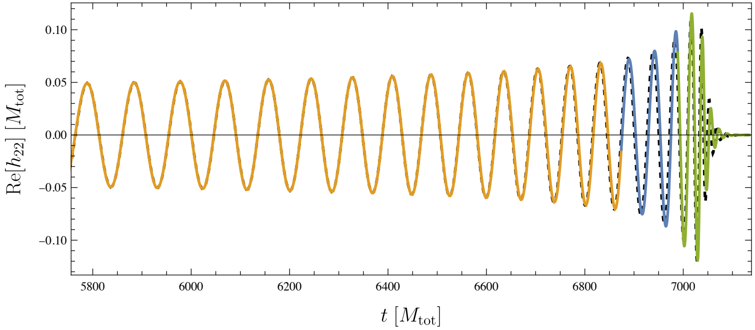
$$\frac{d\phi_p}{dt} = \Omega$$

$$\frac{d\Omega}{dt} = F_\Omega^{(0)}(\Omega) + \varepsilon F_\Omega^{(1)}(\Omega) + O(\varepsilon^2)$$

$$h_{\ell m} = \sum_{n\geq 1} \varepsilon^n h_{\ell m}^{(n)}(\Omega) e^{-im\phi_p}$$

Inspiral-merger-ringdown waveform [Kuchler, Compere, AP]

Mass ratio $\varepsilon = 1/10$



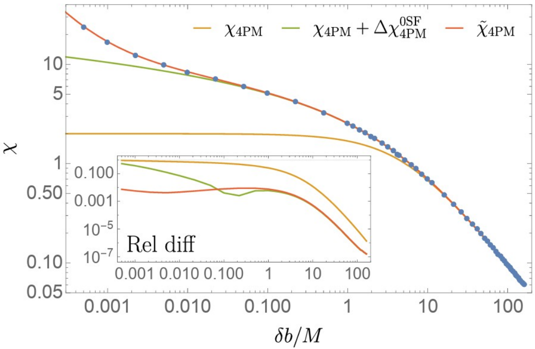
(see also: Dartmouth surrogate model [Islam et al.])

Scattering

2016– scattering can inform bound-orbit models [Damour, Kalin et al., Gonzo et al.]

2025 5PM (2SF) [Driesse et al., Dlapa et al., Bern et al.]

recent direct SF scattering calculations [Barack, Long, Warburton, Whittall]



[Credit: Long, Whittall, Barack]

Outline

EMRI science

- probe of massive BHs:
  - multipole structure
  - presence or absence of horizon
  - deformability, etc.
- probe of stellar-mass objects [Maselli et al.]

  $\Rightarrow$ test large classes of theories
- probe of galactic nuclei:
  - population of nearby bodies
  - properties of accretion disk
  - nature of dark matter

Self-force beyond GR [Barsanti, Maselli, Sotiriou, Spiers &co]

- local EFT extensions of GR: corrections suppressed by curvature
  $\Rightarrow$ effect on large BH is small; effect on small body is large
  $\Rightarrow$ binary system = Kerr plus corrections
  $\Rightarrow$ semi-agnostic tests
- e.g., in class of theories with additional scalar field φ:

$$G^{(1)}_{\alpha\beta}[h^{(1)}] = 8\pi T^{(1)}_{\alpha\beta} \quad \text{(particle mass sources } h^{(1)}_{\alpha\beta}\text{)},$$

$$\Box\varphi^{(1)} = -4\pi\rho^{(1)}_{\alpha\beta} \quad \text{(particle charge sources } \varphi^{(1)}\text{)},$$

$$\vdots$$

Inspiral waveforms beyond GR [Barsanti et al.]

new parameter: $\lambda = q/m$

$$\frac{d\phi_p}{dt} = \Omega$$

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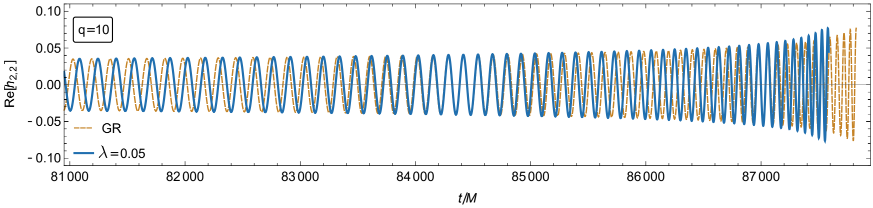

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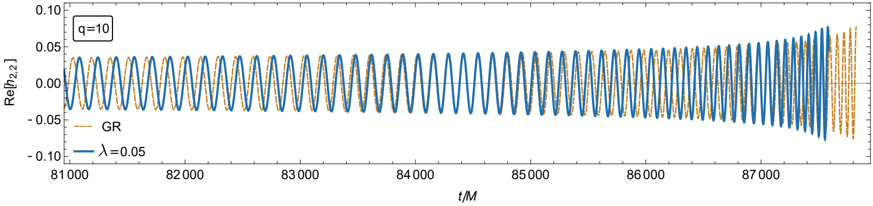

Merger-ringdown beyond GR [Roy, Kuchler, Panosso Macedo, AP]

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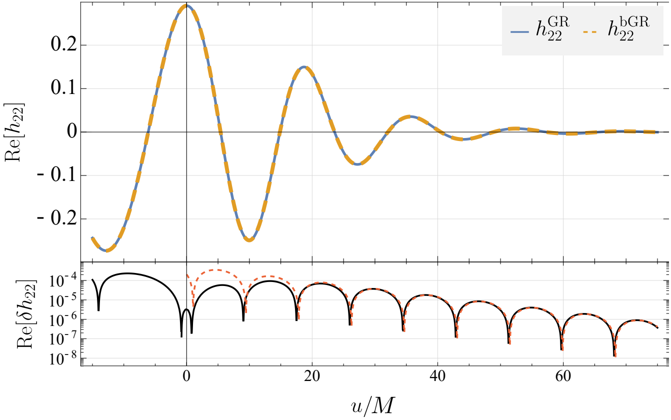
Effect of scalar self-force on ringdown amplitudes



Conclusion

Self-force theory:

- highly accurate for IMRIs and EMRIs
  $\Rightarrow$ will enable high-precisions tests of GR, probes of environment
- fast waveform generation
- unique inroad to merger-ringdown calculations
- modular: readily include beyond-vacuum-GR effects
- poor coverage: 1PA limited to quasicircular orbits around a slowly spinning primary
- can provide inputs to, and take inputs from, other methods