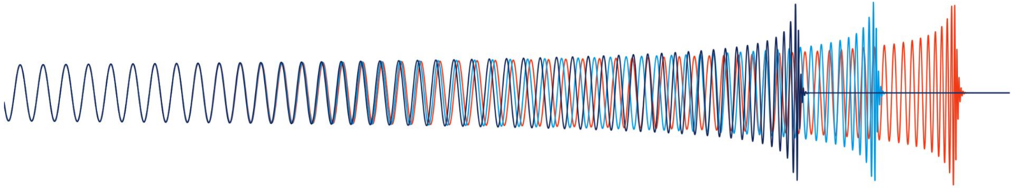
Effective-one-body waveforms and fluxes in Einstein-scalar-Gauss-Bonnet gravity

Lorenzo Pompili
University of Nottingham

lorenzo.pompili@nottingham.ac.uk



Based on Phys.Rev. D 111 (2025) with Félix-Louis Julié and Alessandra Buonanno

University of Nottingham
UK | CHINA | MALAYSIA

YOUNGST@RS – Lost in Translation:
The languages of Gravitational Waves

20.01.2026

Motivation: tests of GR with gravitational waves

- Gravitational waves (GWs) from coalescing compact binaries allow us to test Einstein's theory of general relativity (GR) in the highly nonlinear and dynamical regime.

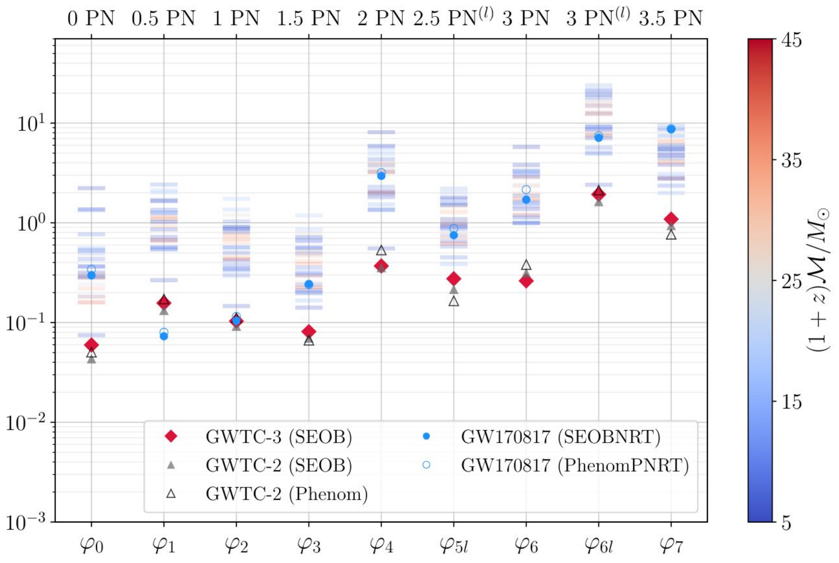
- Theory-independent tests: modify GR waveform models by introducing generic, parameterized deviations or look for consistency between the signal and the data.

  ✓ Can, in principle, constrain a wide range of alternative theories.

  ✗ No guarantee that parameterized deviations represent waveforms in actual beyond-GR theories.

  ✗ Degeneracies complicate constraining several deviation parameters simultaneously.

- Theory-specific tests: predict waveforms in a particular alternative theory of gravity and estimate the underlying physical parameters of the theory from GW data.



LIGO, Virgo, KAGRA; PRD (2025)

Motivation: Einstein-scalar-Gauss-Bonnet gravity

In this work, we present a first example of an inspiral-merger-ringdown waveform model in a beyond-GR theory, focusing on Einstein-scalar-Gauss-Bonnet (ESGB) gravity.

$$\mathrm{S}_{\mathrm{EsGB}} = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi}R - \frac{1}{2}(\nabla\varphi)^2 + \alpha_{\mathrm{GB}} f(\varphi) R^2_{\mathrm{GB}}\right)$$

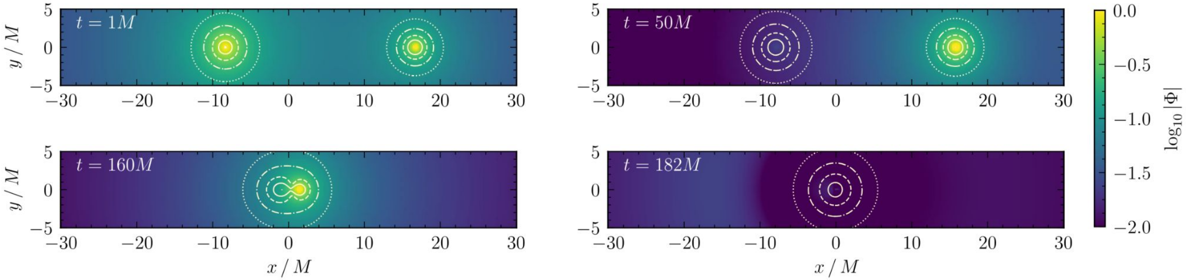
- Massless scalar field φ
- Gauss-Bonnet scalar $R^2_{\mathrm{GB}} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
- Fundamental length $\sqrt{\alpha_{\mathrm{GB}}}$ and dimensionless function $f(\varphi)$ that define the theory

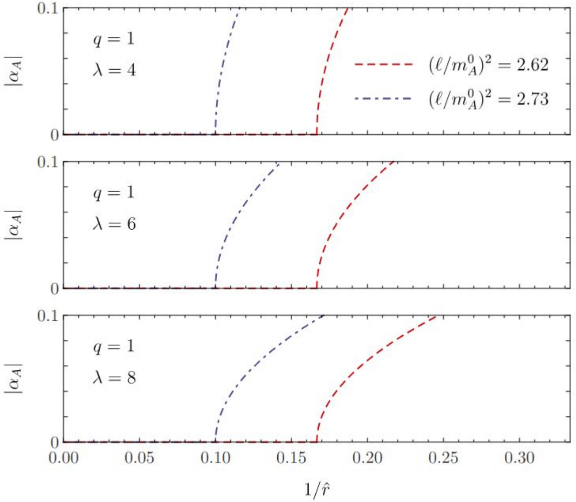
Among the simplest modifications of GR, arising from an effective field theory perspective and in the low-energy limit of string theory, subclass of Horndeski theory.

Gross and Sloan, Nucl. Phys. B (1987); Kobayashi, Rept. Prog. Phys. (2019)

Motivation: Einstein-scalar-Gauss-Bonnet gravity

Black-hole solutions with scalar “hair” and a rich phenomenology (e.g. “spontaneous” or “dynamical” “(de)scalarization” of BH systems).





Credit: Silva+, PRL (2021); Julié, arXiv:2312.16764 (2023)

Doneva and Yazadjiev, PRL (2019); Silva+, PRL (2018); Silva+, PRL (2021); Julié, (2023); Lara+, (2025)

Effective-one-body waveforms in GR

The effective-one-body (EOB) formalism maps the dynamics of a compact binary to that of a test particle in a deformed BH background, with the deformation parameter being the symmetric mass ratio ν. Buonanno and Damour, PRD (1999; 2000),

We extend the EOB waveform model for BBHs SEOBNRv5PHM, which includes effects of spin precession (P) and higher modes (HM).

Pompili+, PRD (2023); Ramos-Buades+, PRD (2023)

- Inspiral-plunge: combine and resum information from analytical approximation methods (PN/PM theory, gravitational self-force).

$$H^{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu}-1\right)}$$

$$\mathcal{F} = \frac{\Omega^2}{8\pi}\sum_{\ell,m} m^2 \left|h_{\ell m}\right|^2$$

$$\dot{\vec{r}} = \frac{\partial H^{\rm EOB}}{\partial \vec{p}}$$

$$\dot{\vec{p}} = -\frac{\partial H^{\rm EOB}}{\partial \vec{r}} + \vec{\mathcal{F}}$$

$$h_{\ell m}^{\text{insp-plunge}} = h_{\ell m}^{\rm Newt} S_{\ell m} T_{\ell m} e^{i\delta_{\ell m}} (\rho_{\ell m})^{\ell} h_{\ell m}^{\rm NQC}$$

Notation

$M = m_A + m_B$

$q = m_A/m_B \geq 1$

$\mu = m_A m_B/M$

$\nu = \mu/M$

- Merger-ringdown: physically motivated ansatz calibrated to NR simulations / BH perturbation theory.

Damour and Nagar, PRD (2014)

$$h_{\ell m}^{\text{merger-RD}}(t) = \tilde{A}_{\ell m}(t) e^{i\tilde{\phi}_{\ell m}(t)} e^{-i\sigma_{\ell m0}(t-t_{\rm match})}$$

EsGB corrections: EOB Hamiltonian

1. Supplement the (Taylor-expanded) EOB potentials with 3PN non-spinning ESGB corrections.

   Damour and Esposito-Farese (1992); Mirshekari and Will (2013); Bernard+ (2019) Julié+ (2018, 2023); Jain+ (2023)

2. Formally replace u with its ESGB counterpart at *all* PN orders.
3. Pade'-resum using same the choices as in GR.

$$H_{\rm eff} = \sqrt{p_{r_*}^2 + A(r)\left[\mu^2 + \frac{p_\phi^2}{r^2} + Q(r, p_{r_*})\right]},$$

$$p_{r_*} = p_r \xi(r) \qquad \xi(r) \equiv A(r)\sqrt{\bar{D}(r)}$$

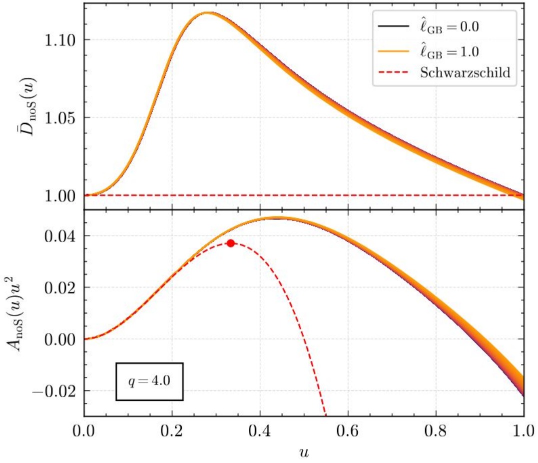
$$\delta A\left(r, \hat{\ell}\right),\ \delta D\left(r, \hat{\ell}\right),\ \delta Q\left(r, p_{r_*}, \hat{\ell}\right)$$

$$u = \frac{1}{r} \rightarrow \frac{G_{AB}}{r} = \frac{1}{r}\left(1 + \alpha_A^0 \alpha_B^0\right) \sim \frac{1}{r}\left(1 + \frac{\ell^4}{m_{A,0}^2 m_{B,0}^2}\right)$$

$$\hat{\ell} = \ell/\mu \qquad \ell = \sqrt{\alpha_{\rm GB}}\ 4\pi^{1/4}$$

Example

Binary with q=4 and $f(\varphi) = \exp(2\varphi)/4$



ESGB corrections: scalar flux

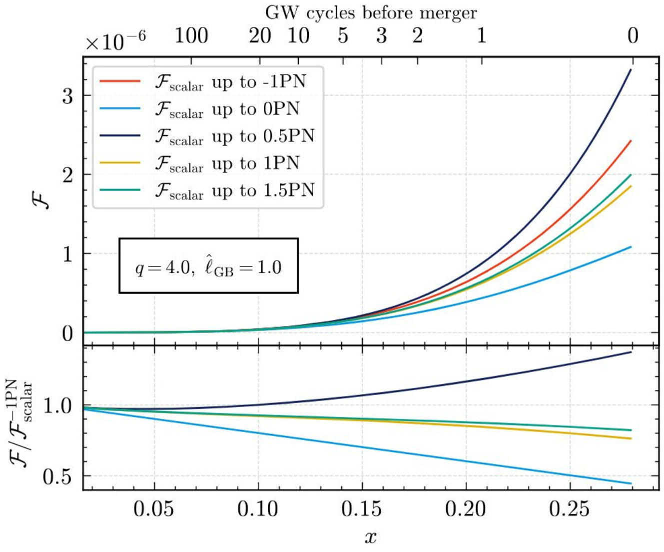
In ESGB theories, we have to account for an additional contribution to the flux from the scalar field.

$$\mathcal{F} = \mathcal{F}^{\text{metric}} + \mathcal{F}^{\text{scalar}}$$

- PN-expanded scalar flux computed up to 1.5PN (relative to quadrupole). Lang, PRD (2015); Bernard, PRD (2022)

$$\mathcal{F}^{\text{scalar}} = \frac{\nu^2 x^5}{\left(1+\alpha_A^0\alpha_B^0\right)^2}\Big[x^{-1} f_{-1\text{PN}} + f_{0\text{PN}} + x^{1/2} f_{0.5\text{PN}} + x f_{1\text{PN}} + x^{3/2} f_{1.5\text{PN}} + \cdots\Big],$$

$$x = (M\Omega)^{2/3} \to (G_{AB} M\Omega)^{2/3}$$



- Dipolar radiation (-1PN) dominant at low frequencies: larger effect for long inspirals.
- $f_{\text{-1PN}}$ proportional to $(\alpha_A - \alpha_B)^2$: larger effect for asymmetric binaries and NSBHs (α_i=0 for NSs).

An aside: resumming spin effects (in GR)

The factorized metric modes in GR are defined as

$$h^{\rm F}_{\ell m} = h^{\rm N}_{\ell m} \hat{S}_{\rm eff} T_{\ell m} f_{\ell m} e^{i\delta_{\ell m}}$$

Damour and Nagar, PRD (2007); Damour+, PRD (2008)

$$\hat{S}^{\rm GR}_{\rm eff} = \begin{cases} \hat{H}^{\rm GR}_{\rm eff}, & \ell+m \text{ even}, \\ \sqrt{x_{\rm GR}}\,\hat{p}_\phi, & \ell+m \text{ odd}. \end{cases}$$

$$T^{\rm GR}_{\ell m} = \frac{\Gamma\left(\ell+1-2i\hat{k}_{\rm GR}\right)}{\Gamma(\ell+1)} e^{\pi \hat{k}_{\rm GR}} e^{2i\hat{k}_{\rm GR}\ln(2m\Omega R_0)}$$

- For even m or non-spinning binaries $f_{\ell m}$ is further resummed as $f^{\rm GR}_{\ell m} = (\rho^{\rm GR}_{\ell m})^\ell$
- However, for odd-m modes, the spins require a particular treatment.
- Spin contributions to $f_{\ell m}$ and $\delta_{\ell m}$ inversely proportional to $\delta = (m_1 - m_2) / M$, making $h_{\ell m}{}^{\rm F}$ ill-defined in the equal-mass case.
    - Traces back to $h_{\ell m}$ being zero for $\delta = 0$ at Newtonian order, while some of its spin contributions entering at sub-leading PN orders are not.
- Resummed as:

Pan+ PRD (2011)
Taracchini+ PRD (2012)
Cotesta+ PRD (2018)

$$\bar{f}^{\rm GR}_{\ell m} = \begin{cases} (\rho^{\rm GR}_{\ell m})^\ell, & m \text{ even}, \\ (\rho^{\rm noS,GR}_{\ell m})^\ell + \bar{f}^{\rm S,GR}_{\ell m}, & m \text{ odd}, \end{cases}$$

$$\bar{\delta}^{\rm GR}_{\ell m} = \begin{cases} \delta^{\rm GR}_{\ell m}, & m \text{ even}, \\ \delta^{\rm noS,GR}_{\ell m}, & m \text{ odd}, \end{cases}$$

$f_{\ell m}{}^{\rm S,GR}$ reabsorbs the spin contributions by expanding the (complex) exponential.
Non-ambiguous equal-mass limit because the terms inversely proportional to δ only enter $f_{\ell m}{}^{\rm S,GR}$ which is multiplied by $h_{\ell m}{}^{\rm N} \propto \delta$.

ESGB corrections: metric modes

We generalize the modes' factorization to ESGB gravity, starting from the PN-expanded metric modes up to 2PN.

Damour & Esposito-Farese, CQG (1992); Lang, PRD (2014); Sennet+, PRD (2016)

1. Replace the effective and EOB energies by their ESGB expression in $h_{\ell m}^{\mathrm{F,\,GR}}$, $S_{\mathrm{eff}}^{\mathrm{GR}}$, $T_{\ell m}^{\mathrm{GR}}$
2. For $\ell + m$ odd, normalize $S_{\mathrm{eff}}^{\mathrm{GR}}$ by its Newtonian value on circular orbits in EsGB gravity, $\hat{S}_{\mathrm{eff}} = \dfrac{\sqrt{x}\hat{p}_\phi}{1+\alpha_A^0\alpha_B^0}, \qquad \ell + m \text{ odd.}$
3. Replace $x = (M\Omega)^{2/3} \rightarrow (G_{AB}M\Omega)^{2/3}$
4. Deform the resulting amplitude and phase by ESGB corrections. When m is odd, the modes $h_{\ell m}$ are imaginary and proportional to δ at Newtonian order. However, some real ESGB corrections, entering at subleading PN orders, are not. The situation is similar to that with spins, and thus resum ESGB corrections in a similar fashion.

$$\bar{f}_{\ell m} = \begin{cases} (\rho_{\ell m}^{\mathrm{GR}} + \delta\rho_{\ell m})^{\ell}, & m \text{ even}, \\ (\rho_{\ell m}^{\mathrm{noS,GR}})^{\ell} + \bar{f}_{\ell m}^{\mathrm{S,GR}} + \delta \bar{f}_{\ell m}, & m \text{ odd}, \end{cases}$$

$$\bar{\delta}_{\ell m} = \begin{cases} \delta_{\ell m}^{\mathrm{GR}} + \delta\delta_{\ell m}, & m \text{ even}, \\ \delta_{\ell m}^{\mathrm{noS,GR}}, & m \text{ odd}. \end{cases}$$

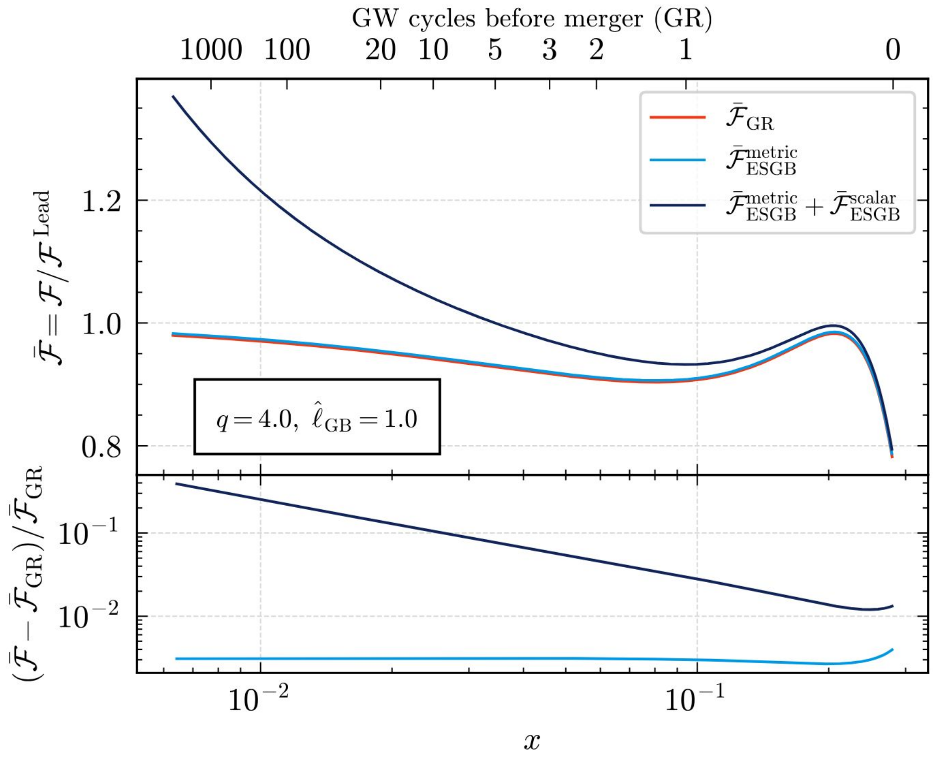
ESGB corrections: metric modes

Example: fluxes in GR and ESGB gravity normalized by the leading order GR contribution.

- Difference in the metric flux at low frequencies is constant with x (e.g., at leading PN order) and is due to the redefinition

$$x = (M\Omega)^{2/3} \rightarrow (G_{AB} M\Omega)^{2/3}$$

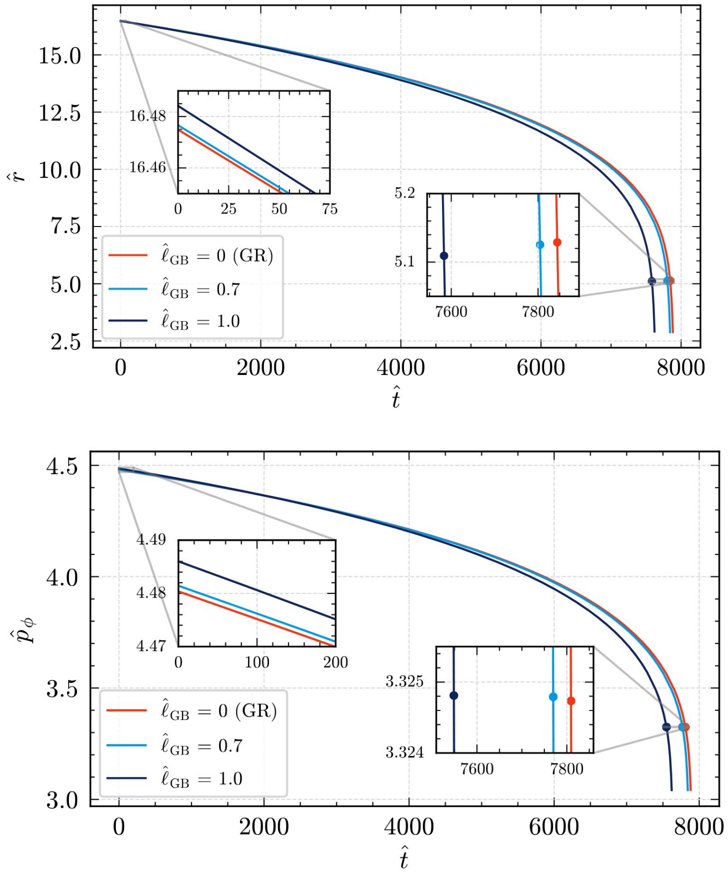
- Difference including the scalar flux increases at low frequencies (as x^{-1}) due to the dipolar -1PN term.



ESGB corrections: orbital dynamics

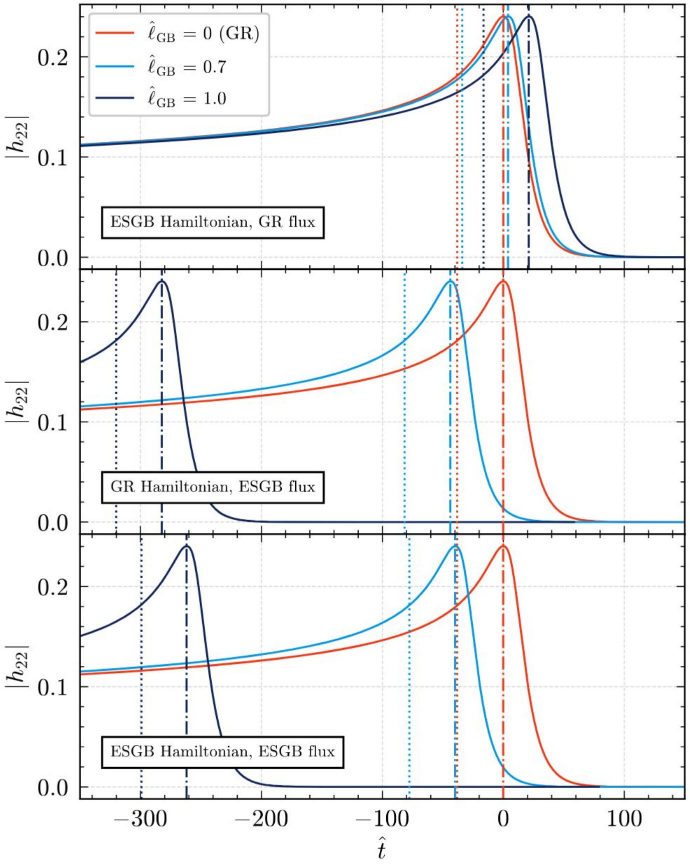
Example: orbital dynamics (q=4 binary)

- The binary merges at earlier times for larger values of the ESGB coupling (more energy is radiated away through the additional scalar field).
- ESGB binaries at the same initial dimensionless frequency start at a larger radius and angular momentum. Due to the leading-order correction $1/r \to G_{AB}/r$ in the EOB potentials with $G_{AB} \geq 1$.
- The ISCO (dots) is at a smaller separation and slightly larger angular momentum compared to GR. Non-trivial effect due to higher-order PN corrections in the EOB Hamiltonian.



ESGB corrections: inspiral

- Amplitude of the (2,2) mode of a nonspinning BBH with $q = 4$, starting from a fixed dimensionless frequency, for the coupling $f(\varphi) = \exp(2\varphi)/4$
- Corrections to the conservative dynamics tend to delay the merger, while corrections to the dissipative sector accelerate the inspiral.
- The trend is consistent for other parameters. When combining all contributions, the changes to the dissipative dynamics are dominant, and the binary merges earlier than in GR.



ESGB corrections: inspiral

- Corrections to the QNM frequencies $\sigma_{\ell m 0}$ (accurate up to $\chi_f \simeq 0.8$)

  Chung and Yunes, PRL (2024)

- Corrections to the final mass and spin from EOB Hamiltonian and angular momentum at merger.

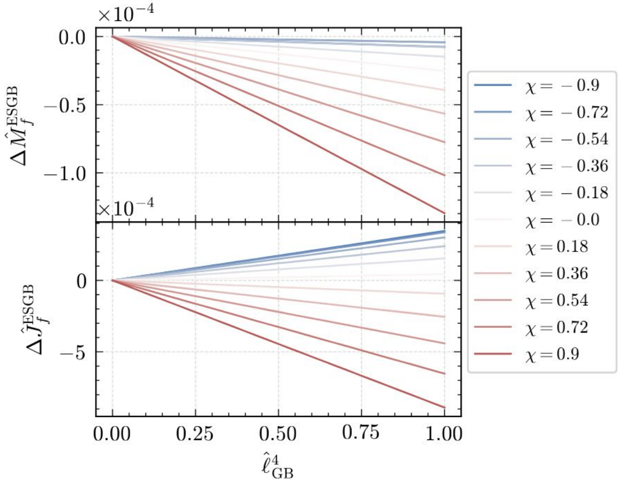
  Buonanno and Damour, PRD (2000); Damour and Nagar, PRD (2007)

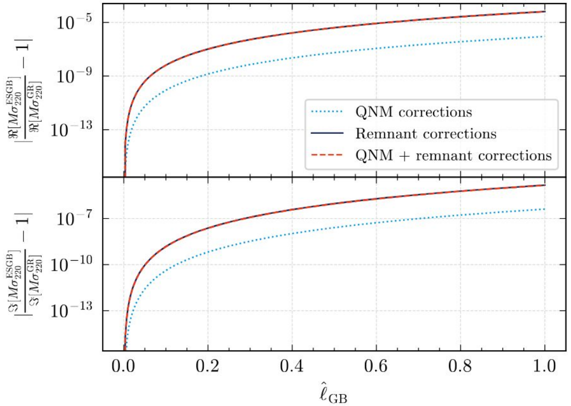
$$M_f^{\rm EsGB} \simeq M_f^{\rm GR} + \left[H_{\rm EOB}^{\rm EsGB}\left(t_{\rm match}^{\rm EsGB}\right) - H_{\rm EOB}^{\rm GR}\left(t_{\rm match}^{\rm GR}\right)\right]$$

$$J_f^{\rm EsGB} \simeq J_f^{\rm GR} + \left[p_\phi\left(t_{\rm match}^{\rm EsGB}\right) - p_\phi\left(t_{\rm match}^{\rm GR}\right)\right]$$

- Waveform amplitude and frequency at merger should be calibrated to NR simulations beyond-GR, not yet available in sufficient number.
  - We parameterize our ignorance of the merger morphology with extra nuisance parameters (inspired by parameterized tests of GR), to be marginalized in parameter estimation.

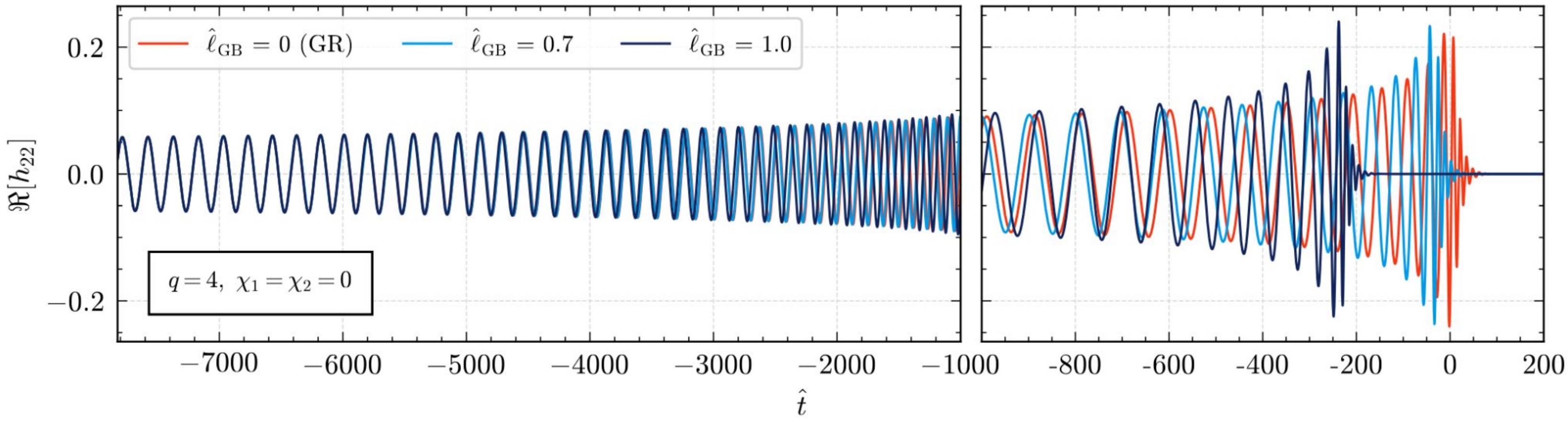
Maggio+, PRD (2023); Pompili+, PRD (2025)





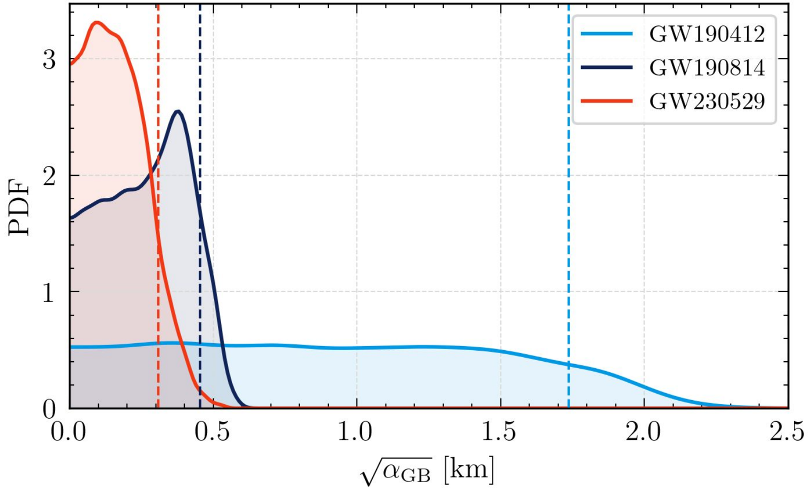
Waveform morphology

- ESGB corrections accelerate the inspiral, mostly due to the additional energy dissipation via the scalar field, and the binary merges earlier in time.
- Corrections to the conservative dynamics are smaller, but not negligible ($\Delta\ \Omega_{\rm ISCO}$ / $\Omega_{\rm ISCO}$ ~ 10^{-2}).
- The effect of QNM corrections is very small ($\Delta\sigma_{\ell m0}/\sigma_{\ell m0}$ ~ 10^{-4}).
- Systems with $\ell_{\rm GB}$~ 0.7 (depending on q and on the SNR) could be distinguished from GR waveforms with LIGO.



Constraints on the ESGB coupling

- We perform parameter estimation on the events GW190412, GW190814 and GW230529 with parallel bilby, including in the analysis the ESGB coupling $\sqrt{\alpha_{GB}}$ with uniform priors.
- We can place constraints on the coupling of the theory $\sqrt{\alpha_{GB}}$ and perform Bayesian model selection by computing the natural log Bayes factor between the ESGB and GR hypotheses.
- GW230529 poses the current best constraint on ESGB gravity with $\sqrt{\alpha_{GB}} \leq 0.31$ km (90% CI).



Event	$\sqrt{\alpha_{GB}}$ (90% CI)	$\ln BF^{\rm EsGB}_{\rm GR}$
GW190412	1.77 km	-0.52
GW190814	0.48 km	-0.02
GW230529	0.31 km	-0.94

See also Lyu+, PRD (2022); Perkins+, PRD (2021); Sänger+ (2024)

Conclusions

- We presented the first example of an IMR waveform model in a beyond-GR theory, focusing on ESGB gravity.
- We used our model to place constraints on the coupling of the theory and to perform Bayesian model selection between ESGB and GR. GW230529 currently poses the best constraint with $\sqrt{\alpha_{GB}} \leq 0.31$ km (90% CI).
- Illustrates the flexibility of the EOB framework for including beyond vacuum-GR effects: similar strategy to include astrophysical environmental effects in EOB models?

Ongoing and future work

- Validation/calibration against NR simulations in EsGB.

  Corman+, PRD (2023, 2025); Aresté Saló+, PRD (2023, 2025); Doneva+ PRD (2023); Lara+, PRD (2025) ...
- Effective modeling of dynamical scalarization. Khalil+ PRD (2019, 2022)
- BNS and NSBH models in EsGB and scalar tensor gravity, extending SEOBNRv5THM.

  Haberland+ PRD (2025) ; Bernard+ PRD (2023); Dones+ (2025)
- Inclusion of spin and eccentricity corrections.

  Almeida+ PRD (2024), Jain and Rettegno, PRD (2024, 2025); Trestini, PRD (2024); Usseglio+ PRD (2025)