Conservative Black Hole Scattering @ 5PM-2SF

Based on [2601.xxxxx]
in collaboration with:
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Mathias Driesse, Lost in Translation 19.01.2026

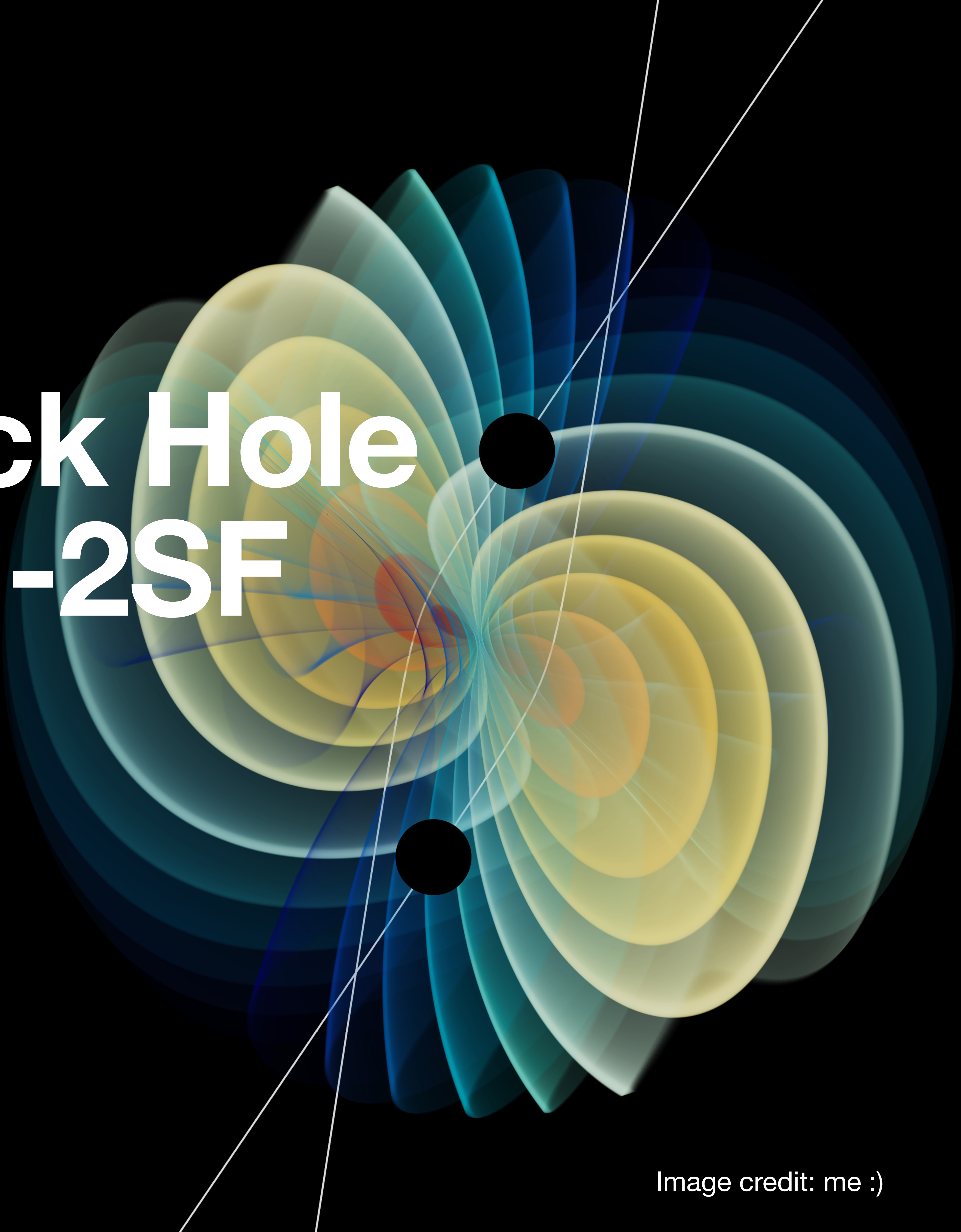


Image credit: me :)

By the end of this talk, you will know:

Motivation

WQFT formalism for gravity

Comparisons of 5PM-1SF to SF and NR

Preliminary 2SF results

The Big Picture

Building Analytical Models

- Effective-One-Body [Buonanno, Damour 1999]
- Self-force/PN/PM hybridization [Honet, Pound, Compère 2025]
- Phenomenological models [many people, ~2007]
- That's all I know of

Straight line trajectory $b + v\tau$
Impact parameter b
Scattering angle θ

[Mogull, Plefka, Steinhoff 2021]

Worldline Quantum Field Theory

"Tree-level one-point functions solve the classical equations of motion"

[Boulware, Brown 1967]

↓

$$S = \int d\tau \, \text{point-particle} + \int d^D x \, \text{gravity}$$

$\Delta p = \lim_{\omega \to 0} \omega^2 \langle z(\omega) \rangle$
Quantized deflection $z(\tau)$
Point particles
Scattering angle θ
Quantized metric perturbation $h_{\mu\nu}(x)$

The post-Minkowskian expansion

Basic idea: Post-Minkowskian expansion in $\frac{Gm}{b}$

$$\Delta p = G^1 \Delta p^{(1)} + G^2 \Delta p^{(2)} + \ldots$$

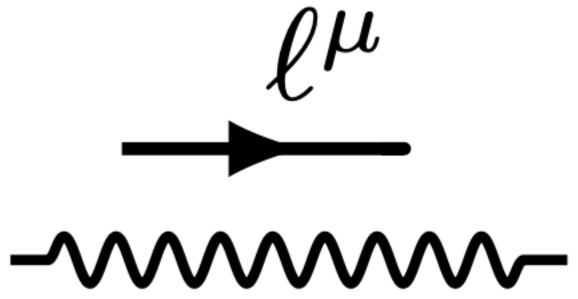
ONLY function of $\frac{1}{\sqrt{1-v^2}} = \gamma = \frac{\left(x + \frac{1}{x}\right)}{2}$

Features of the WQFT formalism

- Tree-level integrand → **recursion**
- Schwinger-Keldysh in-in formalism → **causality, retarded propagators**
- Energy is conserved at worldline vertices → **loop integration**

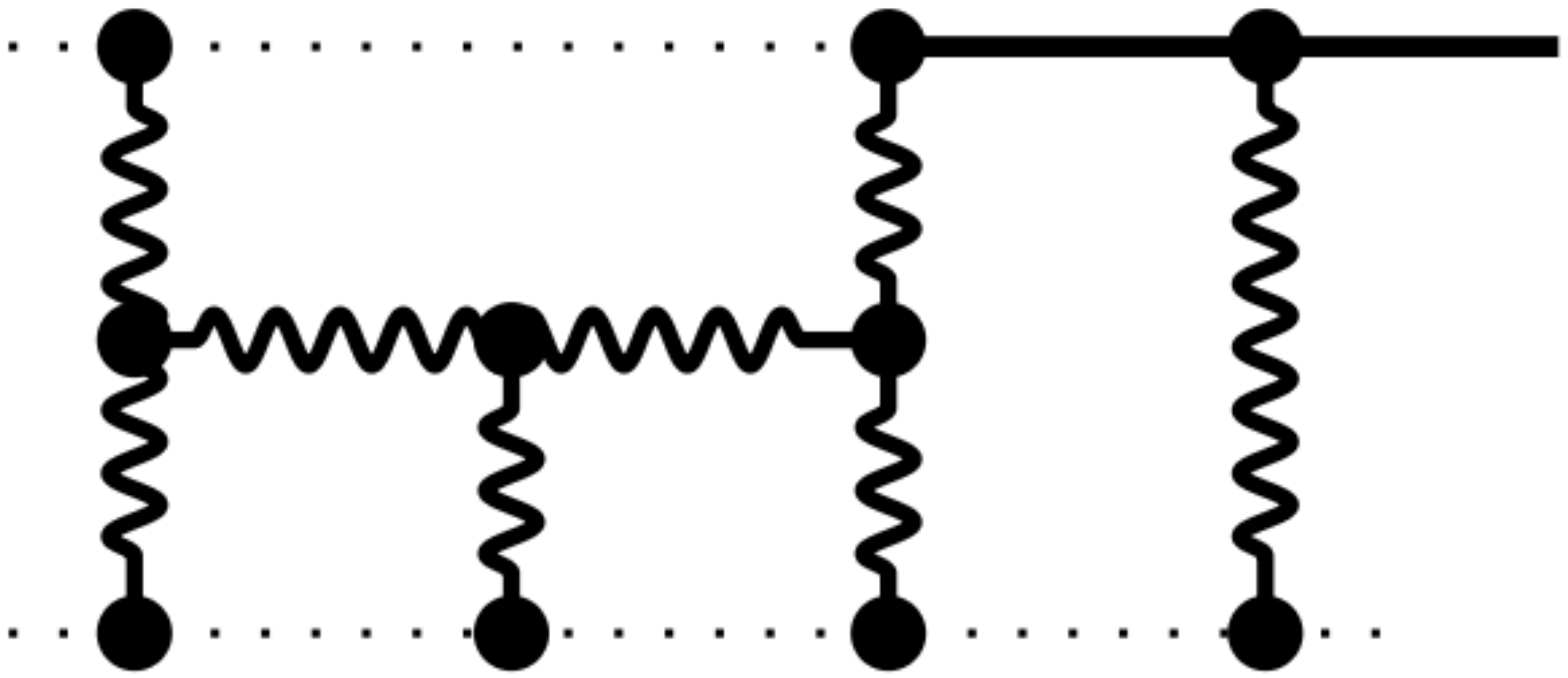
Example graph:

$$= \frac{1}{\ell \cdot v_j + \mathrm{i}0^+}$$

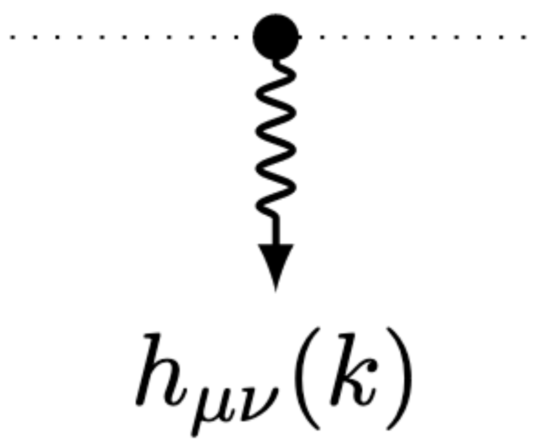


$$= \frac{1}{(\ell^0 + \mathrm{i}0^+)^2 - \boldsymbol{\ell}^2}$$

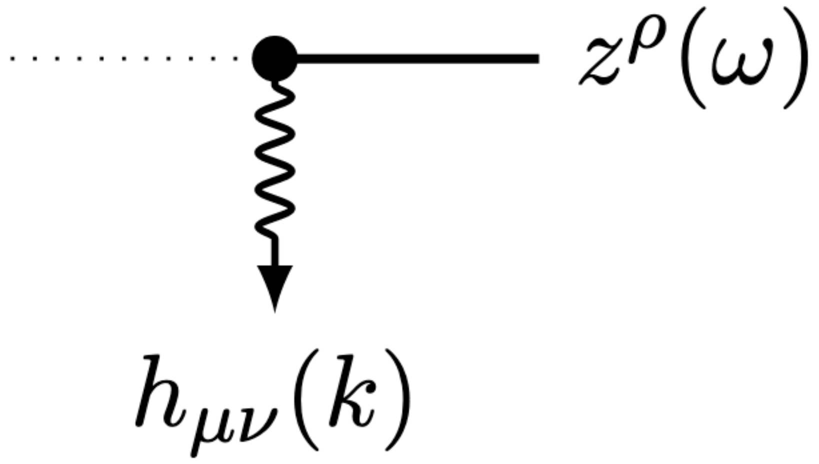
$$= \delta(\ell \cdot v_j)$$



The self-force expansion from WQFT

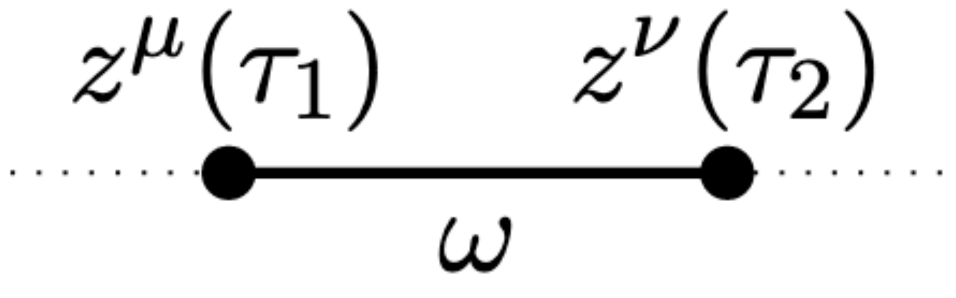


$\sim m$



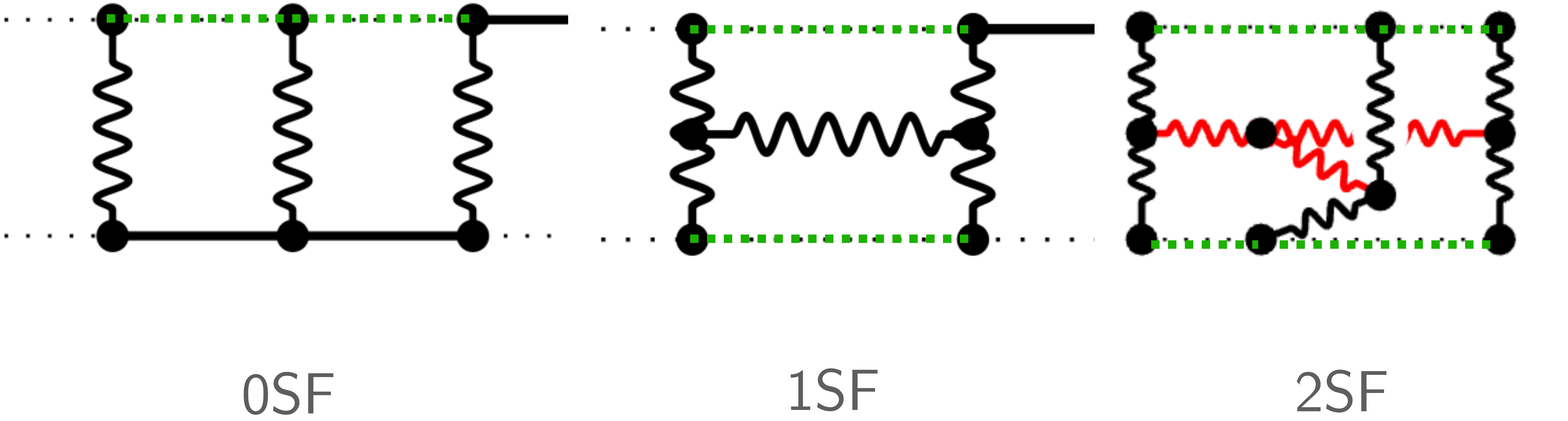
$\sim m$

Polynomial mass dependence!



$$\sim \frac{1}{m}$$

The self-force expansion



The self-force expansion of the impulse

	0SF	1SF	2SF
1PM	✓	–	–
2PM	✓	–	–
3PM	✓	✓	–
4PM	✓	✓	–
5PM	✓	✓	✓ / ✗

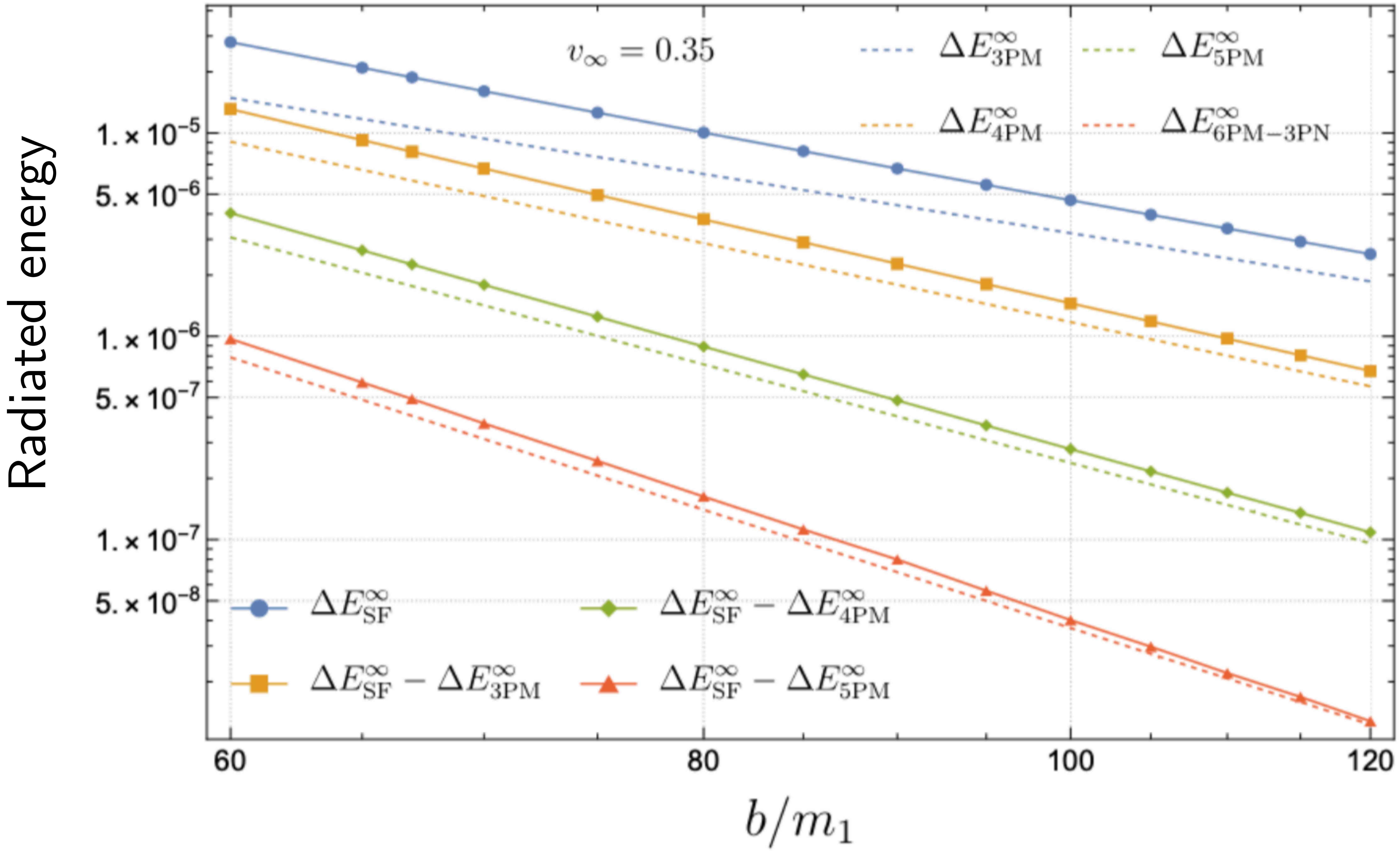
[Westpfahl, 1985]

[Bern, Cheung, Roiban, Shen, Solon, Zeng]
[Källin, Liu, Porto][Di Vecchia, Heissenberg, Russo,Veneziano]
[Bjerrum-Bohr, Vanhove, Damgaard]
[Brandhuber, Chen, Travaglini, Wen]
[Jakobsen, Mogull, Plefka, Sauer]

[Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng]
[Dlapa, Källin, Liu, Porto]
[Vanhove, Damgaard, Plante]
[Jakobsen, Mogull, Plefka, Sauer]

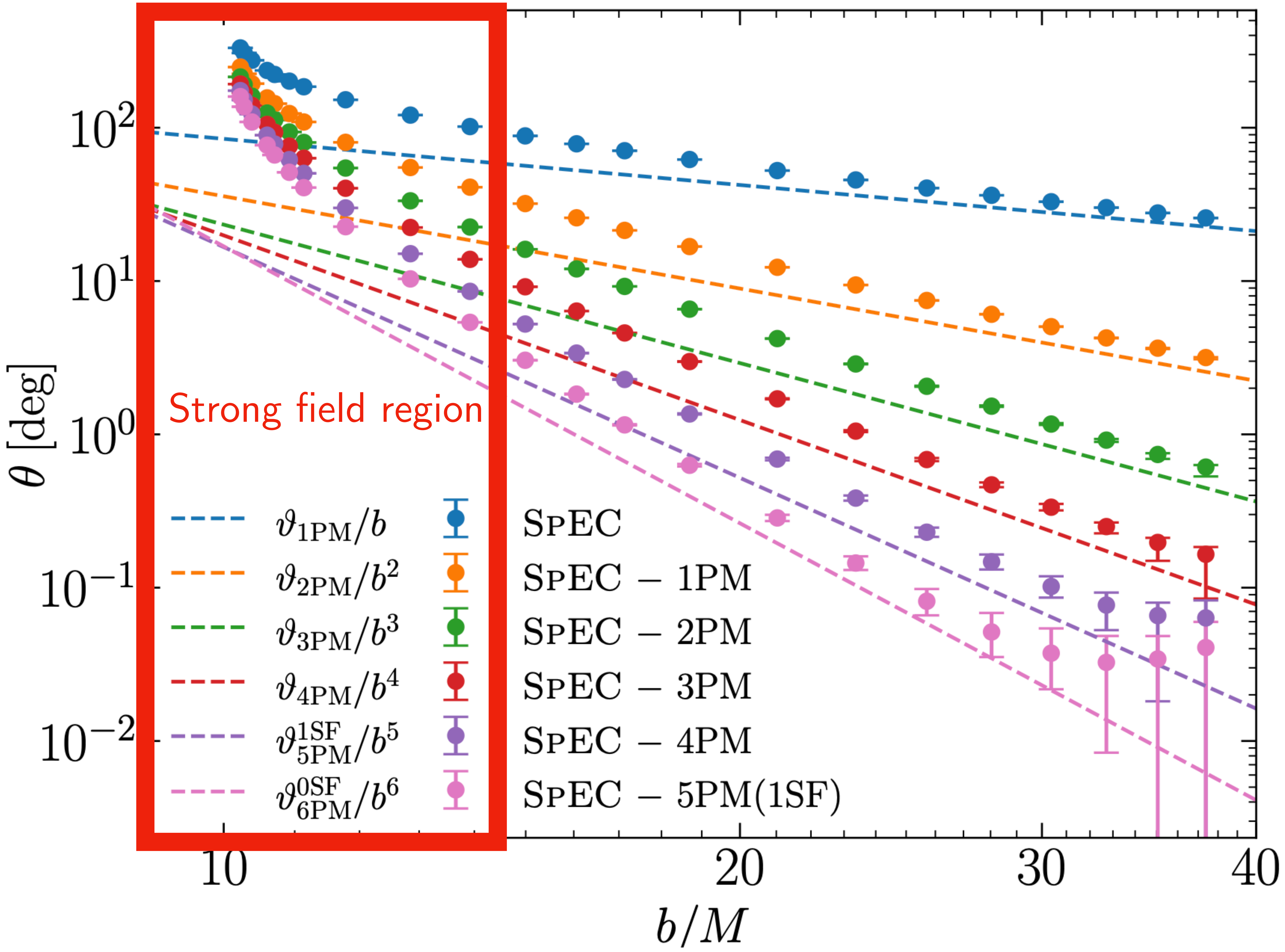
[**MD**, Jakobsen, Mogull, Plefka, Sauer, Usovitsch]
[**MD**, Jakobsen, Klemm, Mogull, Nega, Plefka, Sauer, Usovitsch]
[Bern, Hermann, Roiban, Ruf, Smirnov, Smith, Zeng]

Matching PM to Self-Force



[Warburton 2025]

Matching PM to Numerical Relativity



[Long, Pfeiffer, Kidder, Scheel 2025]

Method of differential equations

Idea: relate derivatives of integrals to themselves

$$\frac{d}{dx}\vec{I} = M(x,\epsilon)\,\vec{I}$$

[Gehrmann, Remiddi 1999]

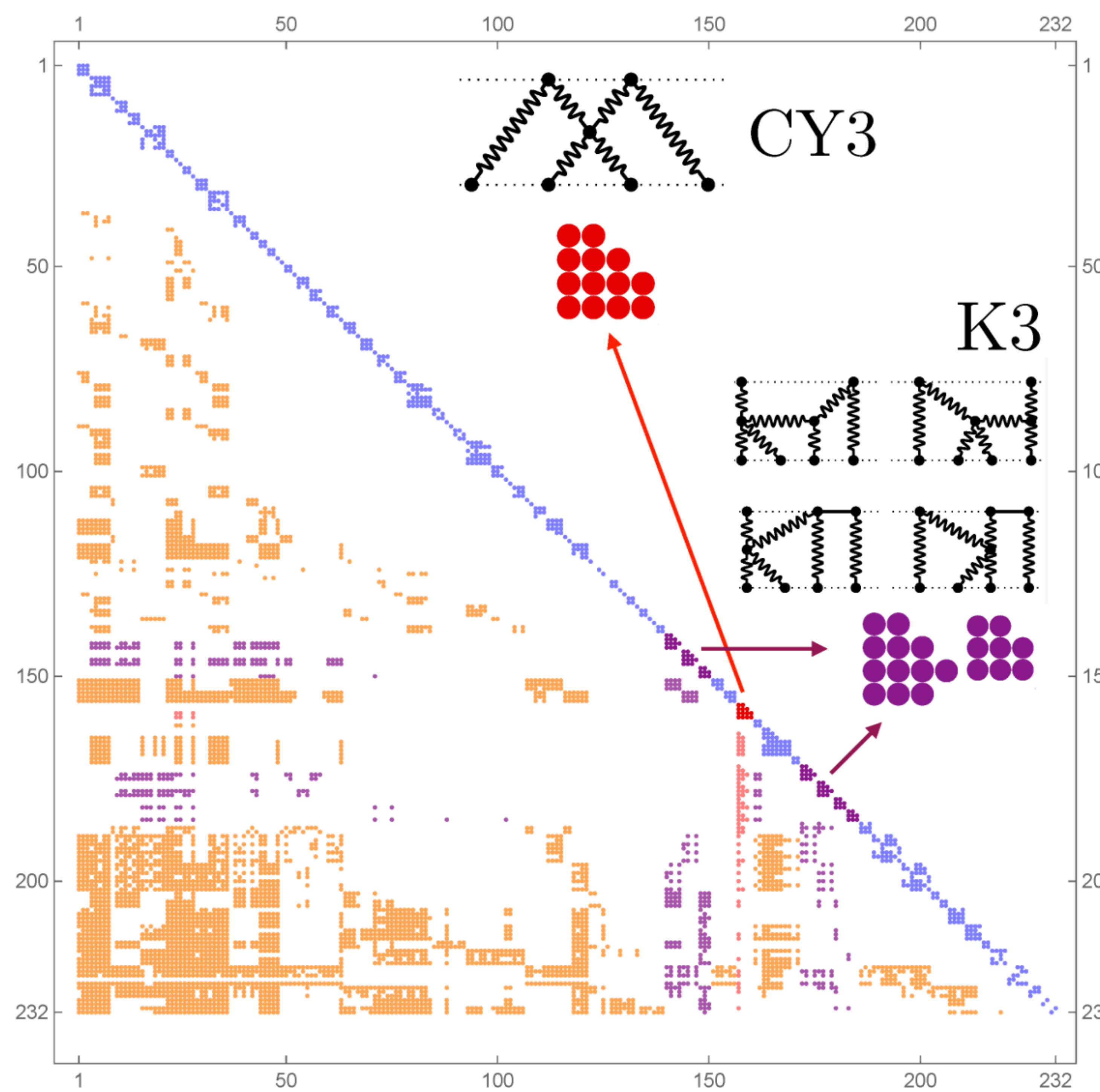
Problem: non-trivial x, ϵ dependence

Solution: find basis transformation s.t.

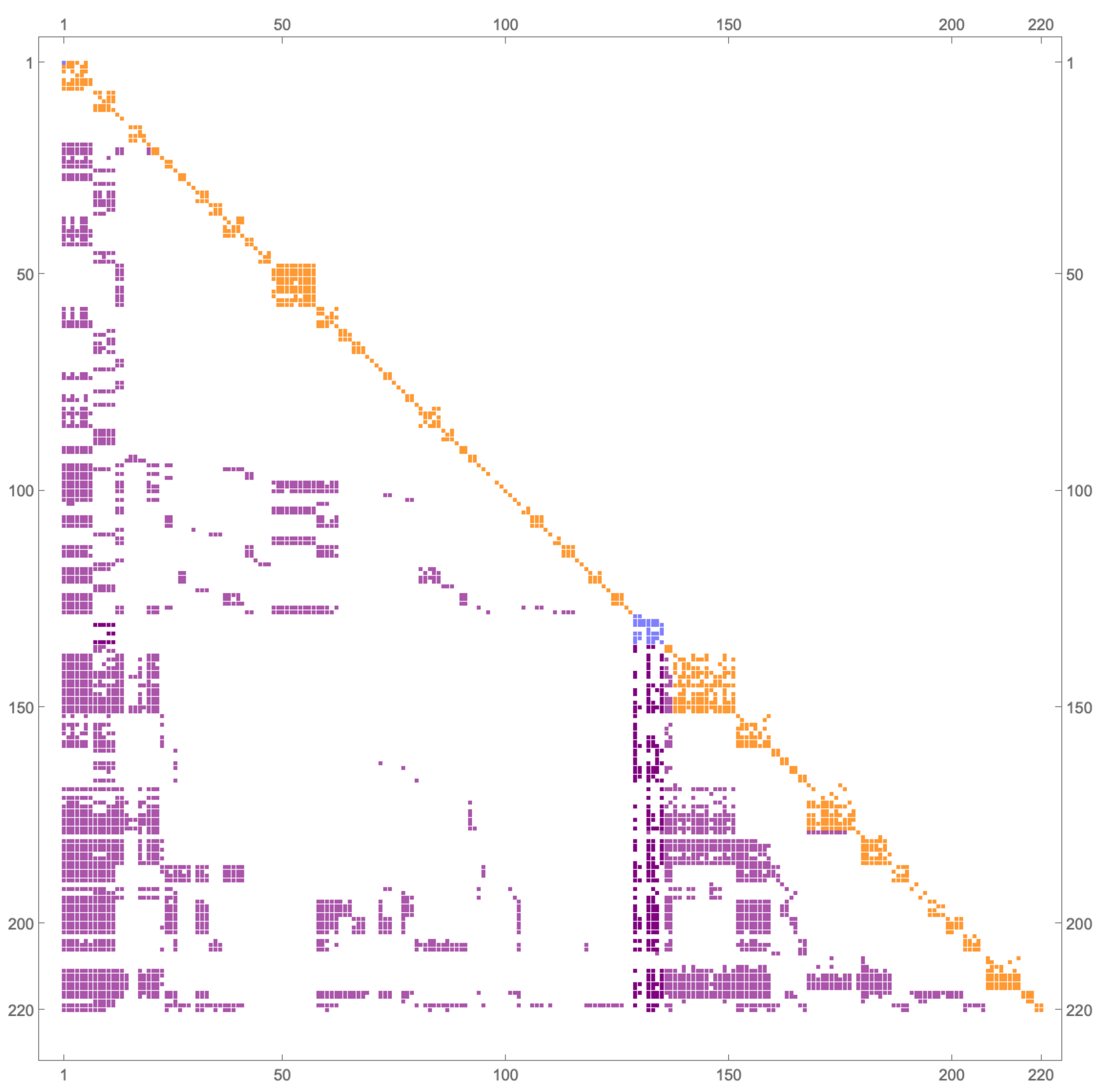
$$\frac{d}{dx}\vec{I} = \epsilon\,M'(x)\,\vec{I}$$

[Henn 2013]

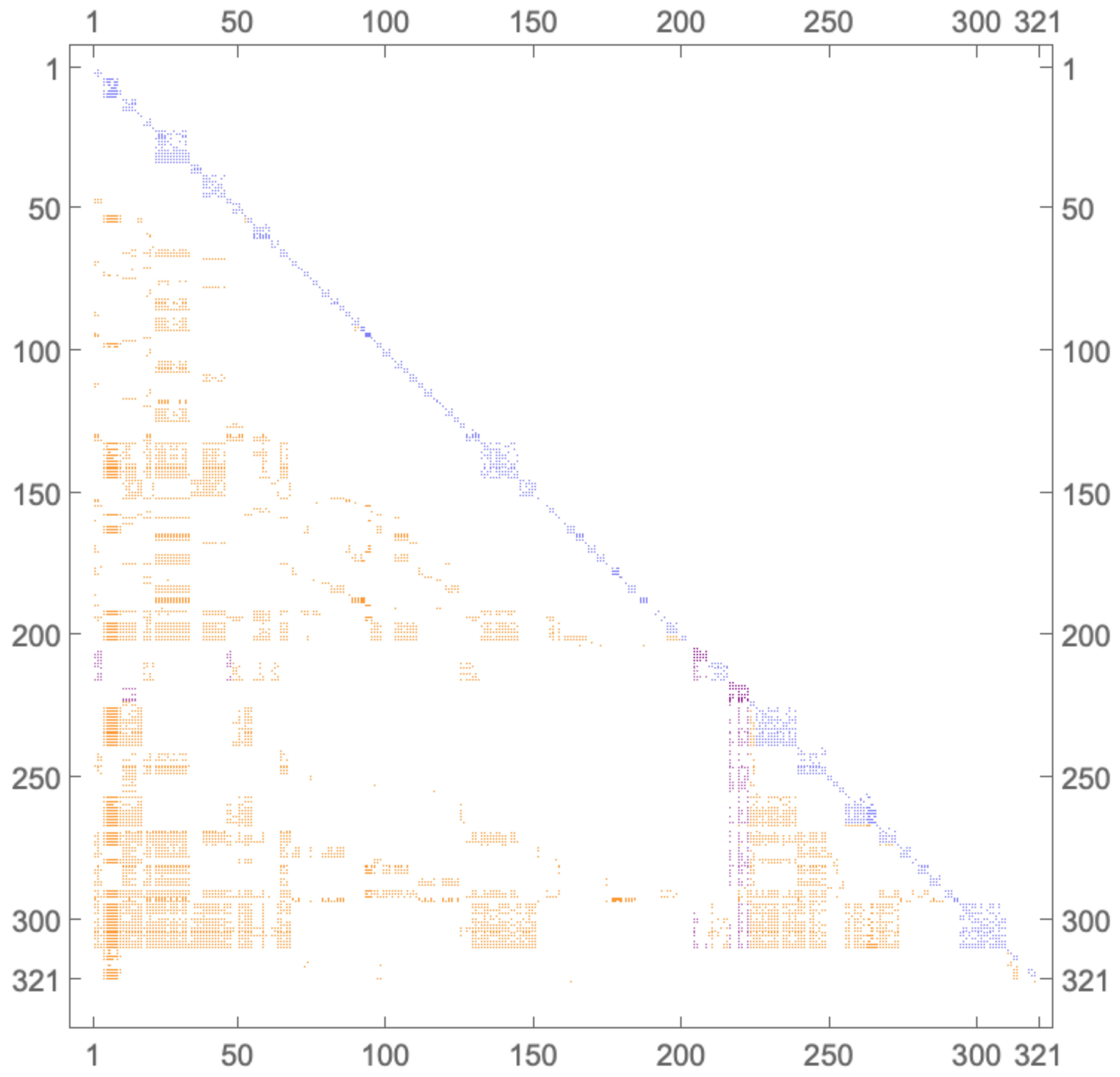
Differential equations



Odd, 1SF



Even, non-planar, 2SF

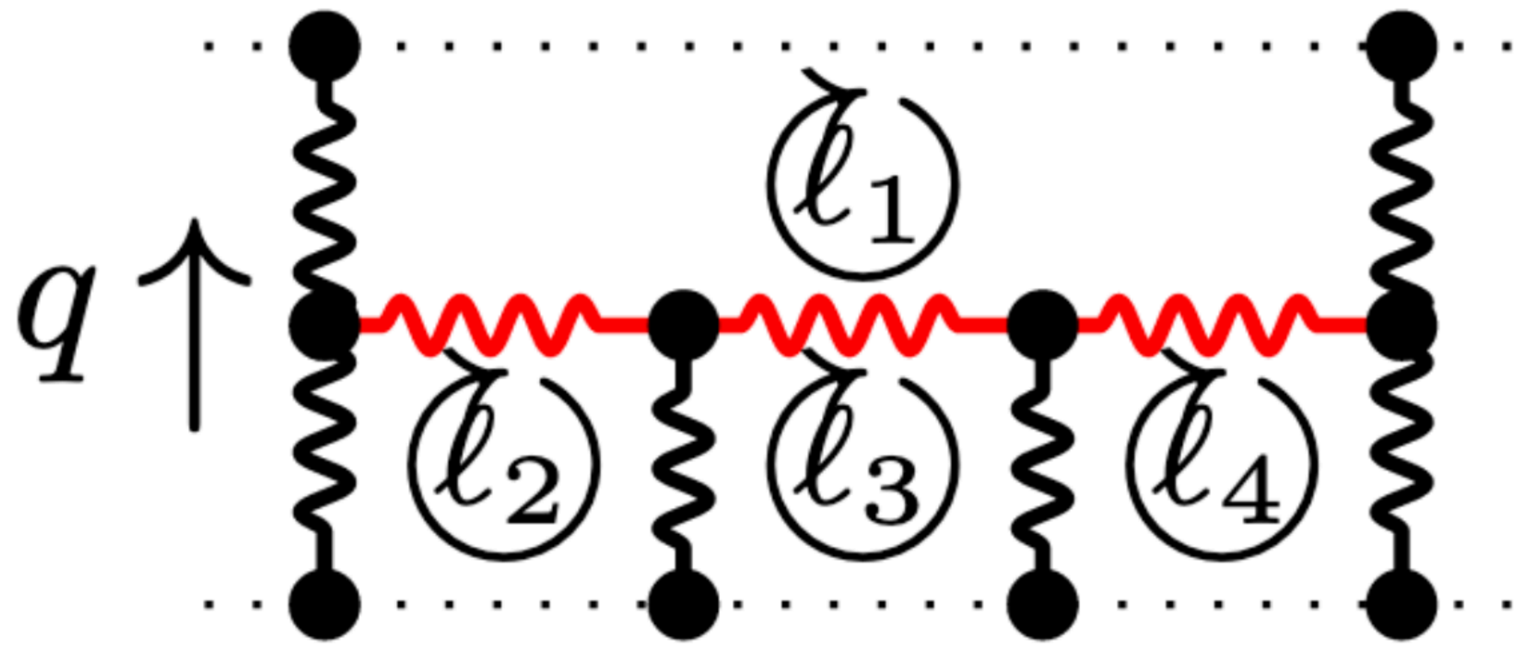


Even, planar, 2SF

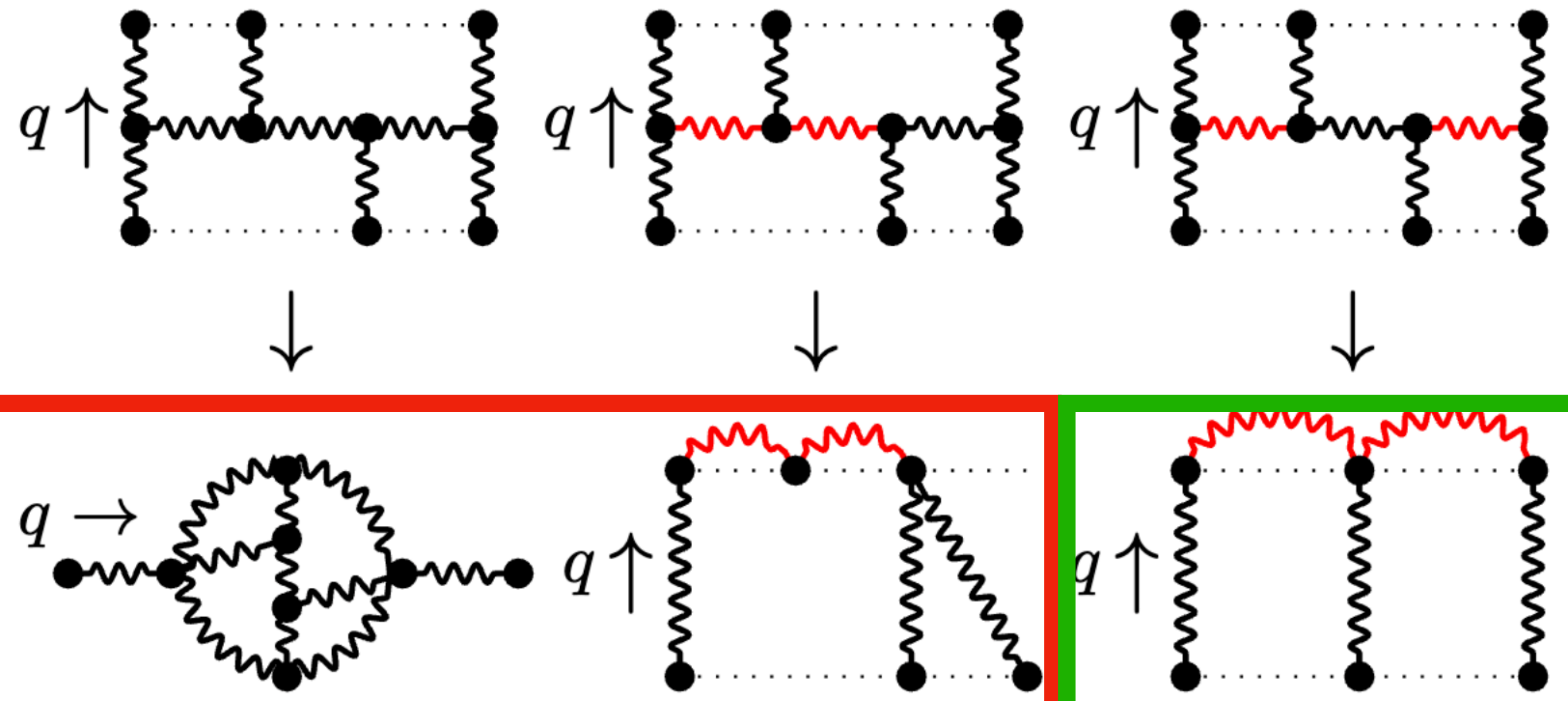
Boundary conditions: Expansion by regions

$$\ell_P \sim (l_0, \vec{\ell}\,) \sim (1 - x, 1)$$

$$\ell_R \sim (l_0, \vec{\ell}\,) \sim (1 - x, 1 - x)$$



Expansion by regions



(a) Potential region

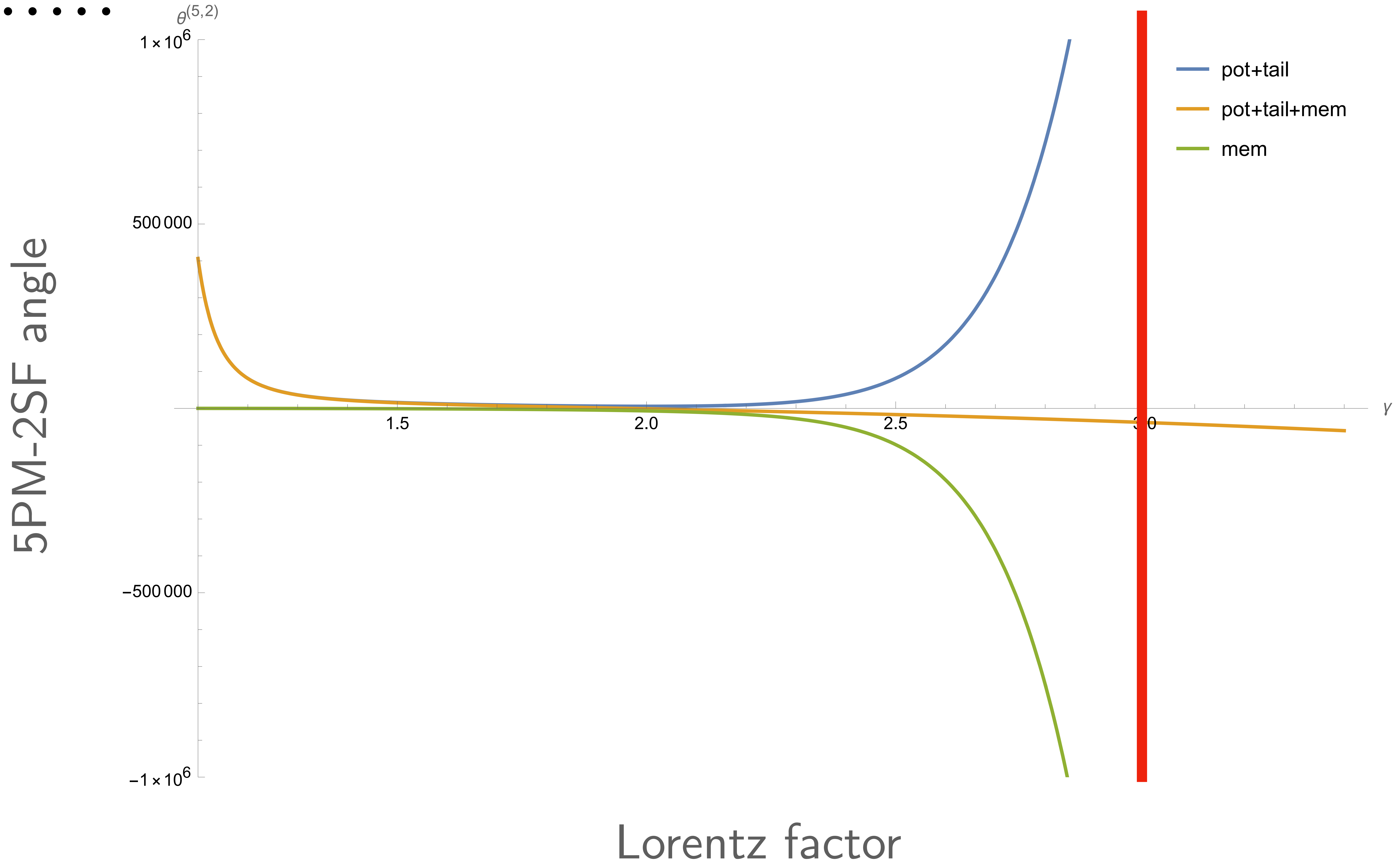
(b) Tail region

(c) Memory region

$$\propto \frac{1}{\epsilon} + \frac{1}{\sqrt{\gamma - 3}}$$

$$\propto -\frac{1}{\epsilon} - \frac{1}{\sqrt{\gamma - 3}}$$

Cancelations



Functions

- Iterated integrals over the following:

$$\left\{ \frac{1}{x}, \frac{1}{1-x^2}, \frac{x}{1-x^2}, \frac{1}{1+x^2}, \frac{x}{1+x^2}, \frac{1-x^2}{1+x^2+x^4}, \frac{x+2x^3}{1+x^2+x^4}, \frac{(1+x^2)\,\varpi_{\mathsf{K3}}(x)}{x}, \frac{1}{x r(x)\,\varpi_{\mathsf{K3}}(x)} \right\}$$

Where r(x) is $\sqrt{-1+34x^2-x^4}$

- Calabi-Yau 3-folds (even more complicated) drop out

Contributions

- Agreement with SF/NR
- First analytic 2SF results
- Cancellation of Calabi-Yaus
- Double cancellation of poles

What now?

- 2SF dissipative — the full answer coming soon™
- Comparisons!
- Analytic continuation to bound states!
- Building waveform models!

Conclusion

"Lost in Translation is a great conference that brings together researchers from different fields. I am very hopeful for the future of analytical gravitational-wave physics in the LISA/Einstein Telescope era."

- Albert Einstein