Lepton Flavour Universality Violation via Angular Analysis of $\Lambda_b^0 \to pK^-\ell^+\ell^-$

Biplab Dey, Debashis Sahoo

Eötvös Loránd University, Budapest

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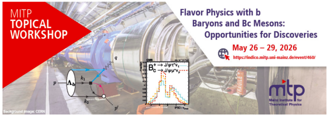

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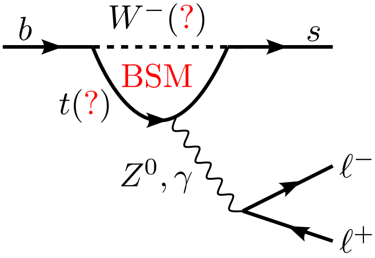
Why study $b \to s\ell^+\ell^-$?

Loop-level FCNC in the Standard Model

- Forbidden at tree level $\Rightarrow$ small SM rates, $\mathcal{B} \sim 10^{-7}$
- Sensitive to heavy new particles entering the loop
- Recent tensions in $b \to s\mu\mu$ angular and branching observables



Lepton Flavour Universality (LFU)

- SM: lepton couplings differ only by their masses
- Above $q^2 \gtrsim 1\,\text{GeV}^2$, e and μ should be identical up to corrections of $\mathcal{O}(m_\ell^2/q^2)$
- Any departure $\Rightarrow$ clean BSM signal

- Moments analysis for all the spins
- For LFUV, take the difference of moments between electron and muon channels.

Why baryonic Λ_b^0 decays?

Mesonic vs baryonic $b \to s\ell\ell$

- In B-mesons: spin information of b-quark lost in hadronisation
- In $\Lambda_b^0 = |bud\rangle$: the $[ud]$ diquark spectator is spin/isospin singlet, so Λ_b^0 "inherits" the b-quark spin $\Rightarrow$ baryonic modes provide complementary information

Why $\Lambda_b^0 \to pK^-\ell\ell$ specifically?

- $\Lambda_b^0 \to \Lambda(\to p\pi)\ell\ell$ has reconstruction issues at LHCb (long Λ lifetime, low-q^2 acceptance)
- $\Lambda_b^0 \to pK^-\ell\ell$ has prompt $\Lambda^* \to pK^-$ decays $\Rightarrow$ stays inside VELO
- Angular Observable very sensitive to NP [S. Descotes-Genon, J. Matias, and J. Virto, Phys. Rev. D88, 074002 (2013)]
- $\Lambda_b^0 \to \Lambda^{(*)}$ FFs, $\Lambda(1520)$ by Lattice, [Phys. Rev. D 105, 054511 (2022)]
- Quark Model for a lot of states [L. Mott and W. Roberts, Int. J. Mod. Phys. A 27, 1250016 (2012)]
- Moments method for Spin $\leq 5/2$ Anja Beck, Thomas Blake, Michal Kreps JHEP 02 (2023) 189

Our goal: set up the formalism for measuring angular observables upto *any* spin.

The pK^- spectrum: multiple Λ^* resonances

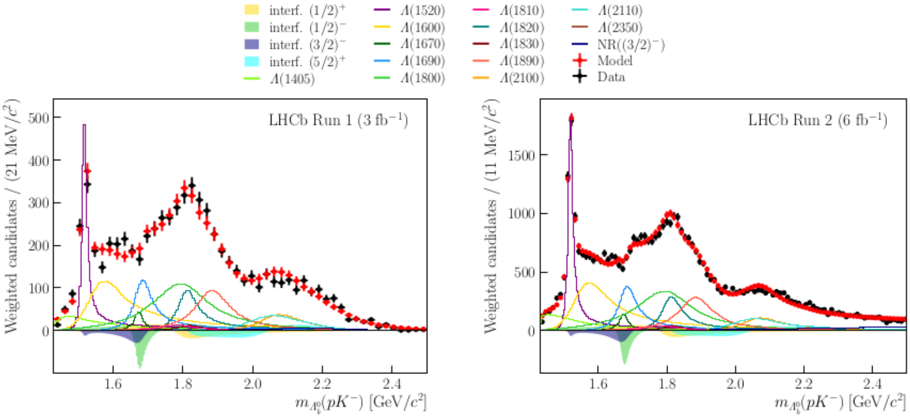
State	J^P	m_0, Γ_0 (MeV)	Dominant modes
$\Lambda(1405)$	$1/2^-$	$1405, 51$	$\Sigma\pi, N\bar{K}$
$\Lambda(1520)$	$3/2^-$	$1519, 16$	$\Sigma\pi, N\bar{K}, \Lambda\pi\pi$
$\Lambda(1600)$	$1/2^+$	$1600, 150$	$\Sigma\pi, N\bar{K}, \Lambda\sigma$
$\Lambda(1670)$	$1/2^-$	$1674, 30$	$\Sigma\pi, N\bar{K}, \Lambda\eta$
$\Lambda(1690)$	$3/2^-$	$1690, 60$	$\Sigma\pi, N\bar{K}, \Lambda\eta$
$\Lambda(1800)$	$1/2^-$	$1800, 200$	$\Sigma\pi, N\bar{K}, \Lambda\sigma$
$\Lambda(1810)$	$1/2^+$	$1790, 110$	$\Sigma\pi, N\bar{K}, \Sigma(1385)\pi$
$\Lambda(1820)$	$5/2^+$	$1820, 80$	$\Sigma\pi, N\bar{K}, \Sigma(1385)\pi$
$\Lambda(1890)$	$3/2^+$	$1890, 120$	$\Sigma\pi, N\bar{K}, \Sigma(1385)\pi$
$\Lambda(2100)$	$7/2^-$	$2100, 200$	$\Sigma\pi, N\bar{K}, N^*\bar{K}$
$\Lambda(2350)$	$9/2^+$	$2350, 150$	$\Sigma\pi, N\bar{K}$

$\Rightarrow$ The analysis m_{pK} range $[1.4, 2.6]$ GeV includes Λ^* of various J^P, all interfering coherently.

$\Rightarrow$ Resonances up to $J = 9/2$ in principle; **full formalism needs all spins.**

The pK^- spectrum from $\Lambda_b^0 \to pK^-\gamma$

- Different resonances contributing to $m(pK^-)$ in JHEP 06 (2024) 098

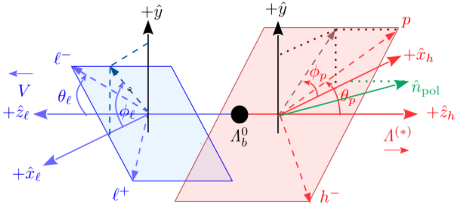


Angle definitions

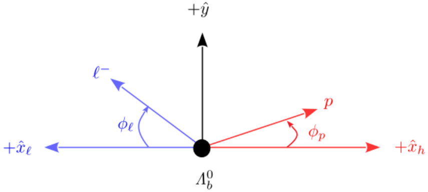
- θ_ℓ – ℓ^- angle in dilepton rest frame
- θ_p – proton angle in pK^- rest frame
- $\chi \equiv \phi_p$ – azimuthal angle between dilepton and hadron frame

Polarisation angle (for polarised Λ_b^0):

- θ — angle of Λ^* flight direction with $\hat{n}_{pol}$
- $\hat{n}_{pol}$: normal to production plane formed by pp beam and Λ_b^0 flight



(b)



Effective Hamiltonian for $b \to s\ell^+\ell^-$ and $b \to s\gamma$

At the $\mu \approx 4.8$ GeV scale, integrate out heavy fields:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=7,9,10} \mathcal{C}_i(\mu)\, \mathcal{O}_i(\mu) + \text{h.c.}$$

The three dominant operators:

$$\mathcal{O}_{7\gamma^{(\prime)}} \sim m_b\, (\bar{s}\sigma_{\mu\nu} P_{R(L)} b)\, F^{\mu\nu} \quad \text{(electromagnetic dipole/magnetic penguin)}$$
$$\mathcal{O}_{9A^{(\prime)}} \sim (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell) \quad \text{(vector)}$$
$$\mathcal{O}_{10V^{(\prime)}} \sim (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma^5 \ell) \quad \text{(axial)}$$

What each piece controls:

- $\mathcal{C}_7$: photon-pole behaviour at low q^2; important for radiative and low-q^2 dilepton decays.
- $\mathcal{C}_9$: vector lepton current; strongly affects rates and angular observables.
- $\mathcal{C}_{10}$: axial lepton current; contributes to parity-odd observables such as A_{FB} through interference with $\mathcal{C}_9$ and $\mathcal{C}_7$.
- Primed Wilson coefficients correspond to right-handed quark currents. In the SM they are suppressed roughly by m_s/m_b, while new physics can enhance them.

Quark-level amplitudes

For $b \to s\ell\ell$ (EWP):

$$\mathcal{M}_{b\to s\ell^+\ell^-} = N_{\text{EWP}} \sum_{L,R} \Big[\mathcal{C}_{9^{(\prime)}} \, (\bar{s}\gamma^\mu P_{L(R)} b)(\bar{\ell}\gamma_\mu \ell) + \mathcal{C}_{10^{(\prime)}} \, (\bar{s}\gamma^\mu P_{L(R)} b)(\bar{\ell}\gamma_\mu\gamma_5 \ell) - \frac{2m_b}{q^2} \mathcal{C}_{7^{(\prime)}} \, (\bar{s} i\sigma^{\mu\nu} q_\nu P_{R(L)} b)(\bar{\ell}\gamma_\mu \ell) \Big]$$

For $b \to s\gamma$ (Rad):

$$\mathcal{M}_{b\to s\gamma} = -N_{\text{Rad}} \sum_{L,R} \mathcal{C}_{7^{(\prime)}} \, (m_b \, \bar{s} i\sigma^{\mu\nu} q_\nu P_{R(L)} b) \, \varepsilon^*_\mu$$

Pre-factors:

$$N_{\text{EWP}} = \frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} \frac{e^2}{4\pi^2}, \qquad N_{\text{Rad}} = \frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} \frac{e}{2\pi^2}$$

Key: The $1/q^2$ factor in $\mathcal{O}_7$ comes from the photon propagator and produces enhanced sensitivity at low dilepton mass.

Strategy: factorise into helicity amplitudes

Idea: Decompose the matrix element using the resolution

$$g_{\mu\nu} = \sum_{\lambda_V,\lambda'_V} g_{\lambda_V \lambda'_V}\, \varepsilon^*_\mu(\lambda_V)\, \varepsilon_\nu(\lambda'_V)$$

This gives:

$$\mathcal{M} = \sum_{\lambda_V,\lambda'_V} g_{\lambda_V \lambda'_V} \left[L^\mu \varepsilon^*_\mu(\lambda_V)\right] \cdot \left[H^\nu \varepsilon_\nu(\lambda'_V)\right]$$

Lorentz-invariant pieces:

- Leptonic helicity amplitudes: $L_\mu(\lambda_1, \lambda_2)\varepsilon^\mu(\lambda_V)$ — computed in V^* rest frame
- Hadronic helicity amplitudes: $H_\nu(\lambda_{\Lambda^*}, \ldots)\varepsilon^\nu(\lambda_V)$ — computed in Λ^* rest frame

Angular dependence:

- Comes from Wigner d-functions $d^J_{m,m'}(\theta)$
- And azimuthal phase factors $e^{i\lambda\phi}$

Leptonic helicity amplitudes

Definition:

$$L_\mu^{(V,A)}(\lambda_1,\lambda_2)\,\varepsilon^{\mu*}(\lambda_V) \equiv h^{(V,A)}_{\lambda_V,\lambda_1,\lambda_2}\, d^{J_\ell}_{\lambda_V,\lambda_1-\lambda_2}(\theta_\ell)\, e^{i\lambda_V\phi_\ell}$$

For massless leptons: only $\eta \equiv \lambda_1 - \lambda_2 = \pm 1$ contribute,

$$L^{L(R)}_{\lambda_V}(\theta_\ell,\phi_\ell) \;=\; \pm\sqrt{2q^2}\, d^J_{\lambda_V,\mp}(\theta_\ell)\, e^{i\lambda_V\phi_\ell}$$

Three available λ_V states:

- $\lambda_V = +1$: right-circular ($e^{+i\phi_\ell}$)
- $\lambda_V = 0$: longitudinal
- $\lambda_V = -1$: left-circular ($e^{-i\phi_\ell}$)

Helicity amplitudes — six per J

For spin J in the pK^- system, six independent helicity amplitudes of the decay $\Lambda_b^0 \to \Lambda^*(\to ph)V(\to \ell^+\ell^-)$:

Amplitude (H_{λ,λ_V})	$\lambda_{\Lambda^*}/2$	λ_V	$\lambda/2$ (Λ_b^0)	λ_p (proton)
C_+^J	$+3/2$	$+1$	$+1/2$	$+1/2$
A_+^J	$+1/2$	0	$+1/2$	$+1/2$
B_+^J	$-1/2$	-1	$+1/2$	$+1/2$
C_-^J	$-3/2$	-1	$-1/2$	$-1/2$
A_-^J	$-1/2$	0	$-1/2$	$-1/2$
B_-^J	$+1/2$	$+1$	$-1/2$	$-1/2$

Conservation rules along Λ^* flight direction:

$$\lambda_{\Lambda^*}/2 = \lambda/2 + \lambda_V \qquad (\Lambda_b^0 \text{ helicity sum})$$

For $J = 1/2$: only $A_\pm$ and $B_\pm$ survive ($\lambda_{\Lambda^*} = \pm 3/2$ forbidden)
For $J \geq 3/2$: all six contribute
For $J > 1/2$: hadronic side adds $\sin\theta_p \cos\theta_p$ structures

The master amplitude formula

For unpolarised Λ_b^0 and massless leptons:

$$\begin{aligned}
\mathcal{M}_{\lambda}^{\lambda_p,\eta} = \sum_J \sqrt{\frac{2J+1}{4\pi}}\, h_\eta\, h^J_{\lambda_p/2} \times \\
\Bigg[\underbrace{d^1_{\lambda,\eta}(\theta_\ell)\, C^J_\lambda\, d^J_{3\lambda/2,\lambda_p/2}(\theta_p)\, e^{+i\lambda\chi}}_{\lambda_V=+\lambda} \\
+ \underbrace{d^1_{0,\eta}(\theta_\ell)\, A^J_\lambda\, d^J_{\lambda/2,\lambda_p/2}(\theta_p)}_{\lambda_V=0\ \text{(longitudinal)}} \\
+ \underbrace{d^1_{-\lambda,\eta}(\theta_\ell)\, B^J_\lambda\, d^J_{-\lambda/2,\lambda_p/2}(\theta_p)\, e^{-i\lambda\chi}}_{\lambda_V=-\lambda} \Bigg]
\end{aligned}$$

What this is:

- Three terms per J, one per dilepton helicity λ_V
- Leptonic Wigner d^1; hadronic Wigner d^J

Squared amplitude and observables

Spin-averaged squared amplitude:

$$|\mathcal{M}|^2_{\text{EWP}} = \frac{1}{2} \sum_{\lambda_p,\eta,\lambda\in\pm1} \left|\mathcal{M}_\lambda^{\lambda_p,\eta}\right|^2$$

Differential rate (4-body phase space, polarised Λ_b^0):

$$\frac{d\Gamma}{dq^2\, d\Omega} = (4\pi\sqrt{2}) \sum_{\lambda_p,\eta,\lambda,\lambda'} H_\lambda^{\lambda_p\eta}\, H_{\lambda'}^{*\lambda_p\eta}\, \rho_{\lambda\lambda'} \equiv (4\pi\sqrt{2}) \sum_i \Gamma_i(q^2)\, f_i(\Omega)$$

Spin density matrix (polarised Λ_b^0):

$$\rho = \frac{1}{2}(I + P_b\, \hat{n}_{\text{pol}} \cdot \vec{\sigma}) = \frac{1}{2} \begin{pmatrix} 1 + P_b \cos\theta & P_b \sin\theta \\ P_b \sin\theta & 1 - P_b \cos\theta \end{pmatrix}$$

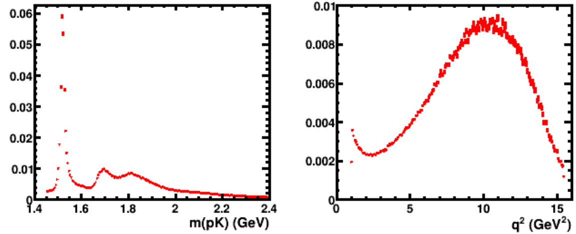
Phase space element:

$$d\Omega \equiv d\cos\theta\, d\cos\theta_p\, d\cos\theta_\ell\, d\phi_p\, d\phi_\ell$$

Goal: extract the moments Γ_i from data $\Rightarrow$ probe BSM physics through Wilson coefficients.

- Events are generated according to Model in Anja Beck et al JHEP 02 (2023) 189.

EWP



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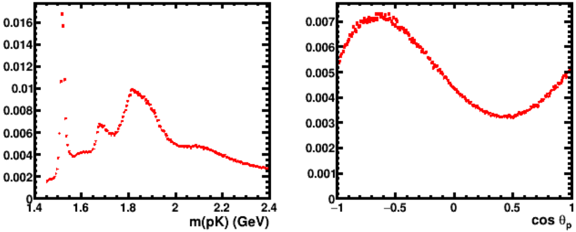


Why moments are the right tool

Maximum likelihood fits vs **method of moments**:

ML fit	**Moments**
Fit convergence at low N unreliable	Always converges (analytic)
Parameter ambiguities can be present	Ambiguity-free
Errors via Hessian	Errors via direct propagation
Hard to combine multiple resonances	Trivial: just compute more moments

At LHCb rare-mode statistics (~ 250 ee events in Run 1+2):

- ML fits suffer from low-statistics pathologies
- Method of moments is provably robust

Bonus: the same moments machinery generalises to higher-J resonances seamlessly.

Angular basis used for the moments

$$\frac{d\Gamma}{d\Omega} = \sum_i \Gamma_i f_i(\Omega), \qquad \Omega = (\theta_\ell, \theta_p, \chi).$$

$$f_i(\Omega) = P_L^m(\cos\theta_p)\, Y_\ell^{m,\mathrm{real}}(\theta_\ell, \chi).$$

- $P_L^m(\cos\theta_p)$ are associated Legendre polynomials for the hadronic angle.
- $Y_\ell^m(\theta_\ell, \chi)$ are spherical harmonics for the leptonic angle and azimuth.
- We use the real basis $\{Y_\ell^0, \sqrt{2}\,\mathrm{Re}\,Y_\ell^m, \sqrt{2}\,\mathrm{Im}\,Y_\ell^m\}$.

$$\begin{aligned}
m = 0:&\quad P_L^0(\cos\theta_p)\{Y_0^0, Y_1^0, Y_2^0\}, && L = 0, \ldots, 2J_{\max},\\
m = 1:&\quad P_L^1(\cos\theta_p)\{\sqrt{2}\,\mathrm{Re}\,Y_1^1, \sqrt{2}\,\mathrm{Im}\,Y_1^1, \sqrt{2}\,\mathrm{Re}\,Y_2^1, \sqrt{2}\,\mathrm{Im}\,Y_2^1\}, && L = 1, \ldots, 2J_{\max},\\
m = 2:&\quad P_L^2(\cos\theta_p)\{\sqrt{2}\,\mathrm{Re}\,Y_2^2, \sqrt{2}\,\mathrm{Im}\,Y_2^2\}, && L = 2, \ldots, 2J_{\max}.
\end{aligned}$$

- Number of Moments:

$$N_{\mathrm{mom}} = 3(2J_{\max} + 1) + 4(2J_{\max}) + 2(2J_{\max} - 1) = 9(2J_{\max}) + 1.$$

Moment extraction and response matrix

Raw moments from the generated model

$$m_i^{\rm raw} = \sum_{k=1}^{N} w_k f_i(\Omega_k), \qquad C_{ij}^{\rm raw} = \sum_{k=1}^{N} (w_k f_i(\Omega_k))(w_k f_j(\Omega_k)).$$

- $\Omega_k = (\theta_{\ell,k}, \theta_{p,k}, \chi_k)$ are generated uniformly.
- The event weight is taken from the full amplitude model:

$$w_k = |\mathcal{M}_{\rm EWP}(q_k^2, m_{pK,k}, \Omega_k)|^2\, \Phi(q_k^2, m_{pK,k}),$$

  where Φ is the four-body phase-space factor.
- Thus the angular moments are computed from the same physics model used for the direct closure test.

Response matrix and unfolding

$$E_{ij} = \langle \epsilon(\Omega) f_i(\Omega) f_j(\Omega) \rangle_{\rm MC}\,, \qquad \Gamma_i^{\rm unfolded} = \sum_j (E^{-1})_{ij} m_j^{\rm raw}.$$

- E_{ij} corrects for the non-orthogonality of the finite basis and for detector acceptance.
- $\epsilon(\Omega)$ is set to one for the ideal closure test; for the e^+e^- toy study an efficiency-weighted response matrix is used.

Angular closure: direct model vs moment expansion

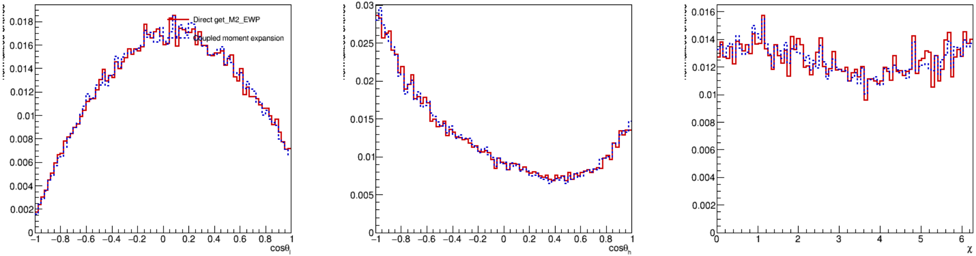
Procedure:

1. Pick $J_{\text{max}} \in \{1/2, 3/2, 5/2, 7/2, 9/2\}$
2. Sample random angles $\Omega^{(k)}$
3. Compute $|\mathcal{M}|^2_{\text{direct}}$
4. Compute Γ_i^{meas} and E_{ij} via MC integration
5. Invert: $\Gamma_i^{\text{true}} = E^{-1}\Gamma^{\text{meas}}$
6. Reconstruct $|\mathcal{M}|^2_{\text{moments}} = \sum_i \Gamma_i^{\text{true}} f_i(\Omega)$
7. Compare $|\mathcal{M}|^2_{\text{direct}}$ vs $|\mathcal{M}|^2_{\text{moments}}$

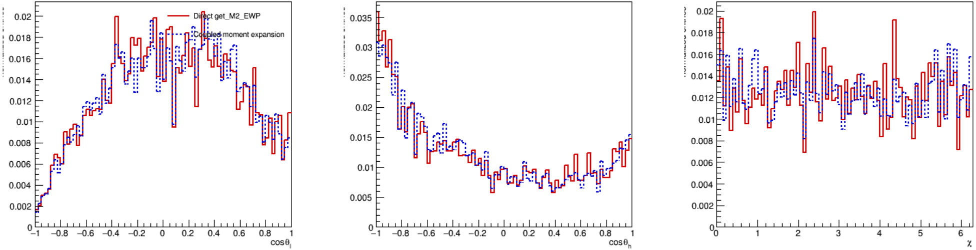
Importance: validates the formalism numerically before applying to real data.

Validation result

2Jmax = 5



2Jmax = 7



Electron-like response used in toys

Efficiency model used for the shown tests:

$$\epsilon_1(\Omega) = 0.82(1 - 0.10\cos^2\theta_\ell)(1 - 0.08\cos^2\theta_p)(1 + 0.04\cos\theta_\ell - 0.03\cos\theta_p)(1 + 0.03\cos\chi).$$

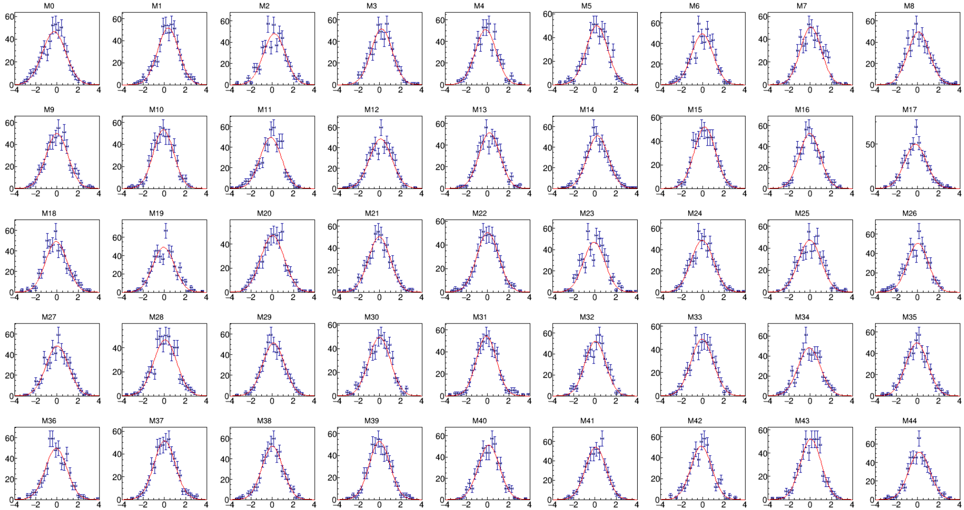
θ_ℓ resolution fudge factor:

$$\theta_\ell^{\mathrm{reco}} = \theta_\ell^{\mathrm{true}}\left[1 + \mathcal{N}(0, \alpha)\right], \qquad \alpha = 0.0431.$$

$$E_{ij}^{ee} = \langle \epsilon_1(\Omega_{\mathrm{reco}}) f_i(\Omega_{\mathrm{reco}}) f_j(\Omega_{\mathrm{reco}}) \rangle_{\mathrm{MC}}\,.$$

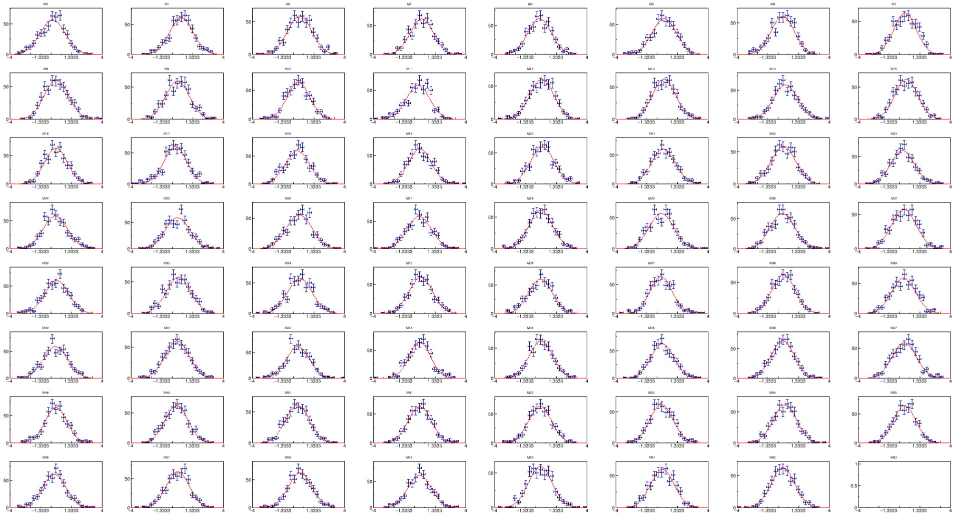
Shown tests: 500 toys, 250 events/toy.

Pulls with real stats- 2Jmax =5



- Pulls for 46 Moments

Pulls with real stats-2Jmax =7



- Pulls for 64 Moments

Summary

- Built a moment-based angular analysis framework including multiple interfering Λ^* resonances.
- The angular expansion uses product functions

$$f_i(\Omega) = P_L^m(\cos\theta_p)\, Y_\ell^{m,\mathrm{real}}(\theta_\ell, \chi),$$

  with the same angular order m in the hadronic and leptonic parts.
- The basis overlap is handled through the MC response matrix

$$E_{ij} = \langle \epsilon(\Omega) f_i(\Omega) f_j(\Omega) \rangle_{\mathrm{MC}},$$

- Closure tests show that the moment expansion reproduces the direct model angular projections for

$$2J_{\max} = 5 \quad \text{and} \quad 2J_{\max} = 7.$$

- Electron-like toy studies with low statistics, θ_ℓ smearing, and an efficiency-weighted response matrix give stable pull widths close to one.

Conclusion: The moment framework is numerically stable for the tested rare-decay statistics and can be extended to higher Λ^* spin hypotheses.

Thank You!