Non-factorizable contributions in rare Λ_b decays

"Flavor Physics with b-Baryons and B_c Mesons"
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color meets flavor

P3H CRC TRR 257 Particle Physics Phenomenology after the Higgs Discovery

Outline

1. Motivation
2. Theoretical Framework
3. Calculation
4. Summary & Outlook

Rare $b \to s\ell^+\ell^-$ Transitions

- Rare $b \to s$ transitions: sensitive probes of SM flavour sector and possible New Physics
- Phenomenological studies for various exclusive processes

$$B \to K\ell^+\ell^- \,, \qquad B \to K^*\ell^+\ell^- \,, \qquad \Lambda_b \to \Lambda\ell^+\ell^- \,, \qquad \ldots$$

- Theoretical frameworks based on effective field theories and OPE/factorization (SMEFT, WET, HQET, SCET, QCDF, LCSRs, ...)
- Some tensions between theory expectations and experimental measurements

$\to$ “flavour anomalies”

Main theory challenge:

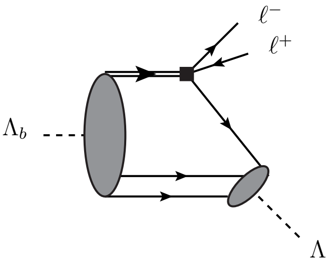
Reliable treatment of hadronic uncertainties

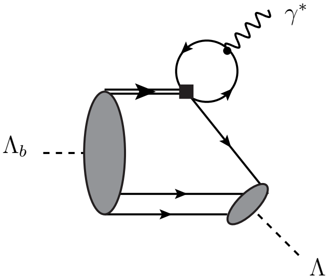
Why Baryonic Decays?

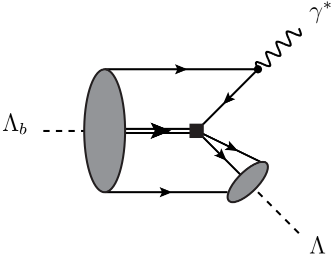
- $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$ provides complementary information to mesonic decays:
  - independent set of angular observables
    $\hookrightarrow$ sensitive to different linear combinations of Wilson coefficients
  - independent hadronic parameters
- $\Lambda_b \to \Lambda$ form factors for semi-leptonic operators (fairly well) known from:
  - lattice QCD at low hadronic recoil energy
  - light-cone sum rules (LCSR) at large recoil
- But: Also long-distance contributions from $T\{O_i^{\text{had}} J_\mu^{\text{em}}\}$
  (i.e. photon radiation off internal quark lines)

$\hookrightarrow$ **Non-factorizable contributions are less understood for baryons**

Examples for different Decay Topologies







semi-leptonic

operators O_7^γ, O_9, O_{10}
(naively) factorizable in QCD
only needs (local) form factors

quark-loop

e.g. charm-loop from O_1^c, O_2^c
factorizable and non-factorizable
corrections $\rightarrow C_9^{\text{eff}}(q^2, m_c^2) + \ldots$

“annihilation-like”

strong penguin operators O_{3-6}
non-factorizable
?

$\hookrightarrow$ **Our paper studies the third topology for the first time**

Use analogous Lorentz decomposition as for local form factors: $\mathcal{O}_i = \mathcal{O}_{1-6,8g}$

$$\mathcal{H}_\mu^{(i)}(p,q) = i \int d^4x\, e^{iq\cdot x} \langle \Lambda(p') | \mathcal{T}\{\mathcal{O}_i(0)\, J_\mu^{\rm em}(x)\} | \Lambda_b(p) \rangle$$

with $s\pm = (M_{\Lambda_b} \pm m_\Lambda)^2 - q^2$

$$p^\mu\, \mathcal{H}_\mu^{(i)} = -\frac{M_{\Lambda_b}^2}{2q^2}\, \bar{u}_\Lambda(p') \left[s_-(M_{\Lambda_b} + m_\Lambda)\, \mathcal{H}_+^{(i)}(q^2) - s_+(M_{\Lambda_b} - m_\Lambda)\, \mathcal{H}_{+5}(q^2)\gamma_5 \right] u_{\Lambda_b}(p)\,,$$

$$g_\perp^{\mu\nu}\, \mathcal{H}_\mu^{(i)} = M_{\Lambda_b}^2 \bar{u}_\Lambda(p') \left[\mathcal{H}_\perp^{(i)}(q^2)\, \gamma_\perp^\nu - \mathcal{H}_{\perp 5}^{(i)}(q^2) \gamma_\perp^\nu \gamma_5 \right] u_{\Lambda_b}(p)\,.$$

$$p^\mu g_{\mu\nu}^\perp = q^\mu g_{\mu\nu}^\perp = 0$$

$\hookrightarrow$ **accounted for by replacing**

$$C_9\, f_+(q^2) \to C_9\, f_+(q^2) - 16\pi^2 \frac{2M_{\Lambda_b}^2}{q^2} \sum_i C_i\, \mathcal{H}_+^{(i)}(q^2) \qquad \text{etc.}$$

- concentrate on the large-recoil region

$$0 < q^2 = (p_{\Lambda_b} - p_\Lambda)^2 \ll m_b^2 \qquad \leftrightarrow \qquad E_\Lambda \sim \frac{m_b}{2} \gg m_\Lambda$$

- two light-like directions w.r.t. Λ_b rest frame:
  - partons ending up in Λ baryon move aligned to n^μ ("collinear")
  - partons ending up in virtual photon move aligned to $\bar{n}^\mu$ ("anti-collinear")

$\Rightarrow$ energy-transfer from soft constituents in Λ_b to collinear or anticollinear direction:

- hard-scattering picture:
  ...by *a few* far off-shell gluons (or quarks) $\rightarrow$ perturbative
- soft mechanism:
  ...by *many* low-virtuality exchanges $\rightarrow$ non-perturbative

Here: Use light-cone sum rule approach to account for non-perturbative effects

Starting point: (analogous setup as in [Feldmann/Yip 2011])

- replace the light Λ baryon state by an appropriate interpolating current

$$J_\Lambda(y) \equiv \epsilon_{ijk} \left[u^{T,i}(y)\, C\gamma_5 \not{\bar{n}}\, d^j(y)\right] \frac{\not{n}\not{\bar{n}}}{4} s^k(y)$$

- Define the correlation function as a function of the external momentum p'

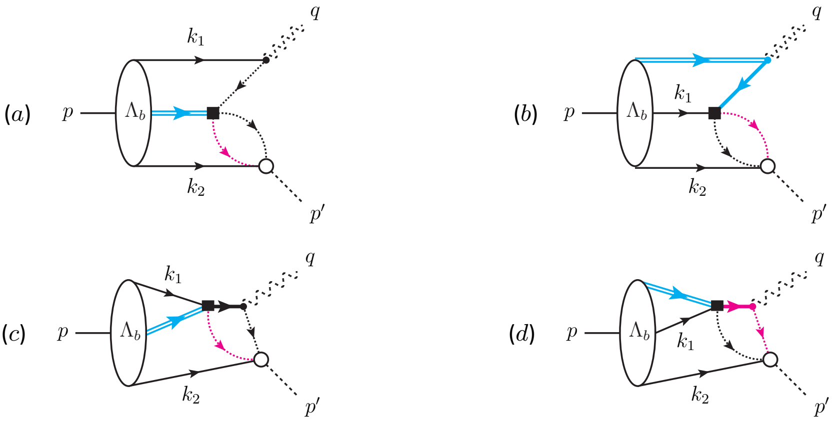
$$\Pi_\mu(p') \sim \int d^4x\, e^{iq\cdot x} \int d^4y\, e^{ip'\cdot y} \langle 0| T\left\{J_\Lambda(y)\, \mathcal{O}_{3-6}(0)\, J_\mu^{\text{em}}(x)\right\} |\Lambda_b(p)\rangle$$

- consider a frame where $(\bar{n}\cdot p') \simeq 2E \simeq \frac{M_{\Lambda_b}^2 - q^2}{M_{\Lambda_b}}$ is fixed and $p'_\perp = 0$

$\hookrightarrow$ **Correlator can be calculated perturbatively**, for

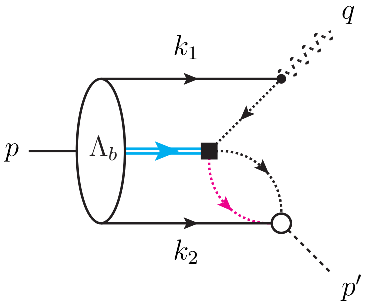
$n\cdot p' \sim \Lambda_{\text{had}} < 0$ and $\Rightarrow$ $|p'|^2 \sim m_b\,\Lambda_{\text{had}}$ (Euclidean, "hard-collinear")

Leading-order diagrams for “annihilation-like topologies”



- lines labelled with k_1 and k_2 represent up- and down-quark, or vice versa
- b-quark indicated by a light blue line ; the strange quark by a magenta line
- hard-collinear propagators indicated by dotted lines ; hard propagators by thick lines
- double line denotes the (static) b-quark in HQET

Leading-order diagrams for “annihilation-like topologies”

(a) 

$\leftrightarrow$ isospin

- diagrams with hard propagators are power-suppressed by $1/2E$
- remaining diagram has two hard-collinear propagators in p' direction and one hard-collinear propagator in q direction

$\hookrightarrow$ light-quark fields in Λ_b state are probed at ***different*** **light-cone separations !**

- start from Lorentz decomposition of a generic tri-local operator, [Bell/Feldmann/Wang/Yip 2013]

$$\epsilon_{ijk}\,\langle 0|\,\Big(u^i_\alpha(z_1)\,d^j_\beta(z_2)\Big)\,b^k_\delta(0)|\Lambda_b(p)\rangle \equiv$$
$$\frac{1}{4}\left\{f^{(1)}_{\Lambda_b}\left[\tilde{M}^{(1)}(v,z_1,z_2)\gamma_5 C^{-1}\right]_{\beta\alpha} + f^{(2)}_{\Lambda_b}\left[\tilde{M}^{(2)}(v,z_1,z_2)\gamma_5 C^{-1}\right]_{\beta\alpha}\right\} u_{\Lambda_b,\delta}(v)$$

- odd number of Dirac matrices are contained in $t_i = v\cdot z_i$

$$\tilde{M}^{(2)}(v,z_1,z_2) = \tilde{\Phi}_2\,\not{v} + \tilde{\Phi}_X\,\frac{\not{z}_2\not{v}\not{z}_1 - \not{z}_1\not{v}\not{z}_2}{4t_1t_2} + \tilde{\Phi}^{(i)}_{42}\,\frac{\not{z}_1}{2t_1} + \tilde{\Phi}^{(ii)}_{42}\,\frac{\not{z}_2}{2t_2}$$

- similarly for even numbers of Dirac matrices encoded in $\tilde{M}^{(1)}$.

$f^{(1,2)}_{\Lambda_b}$ from normalization of *local* matrix elements

---

↪ each function depends on **five Lorentz invariants**,

$$\tilde{\Phi}_2 = \tilde{\Phi}_2(t_1,t_2,z_1^2,z_2^2,z_1\cdot z_2)$$

- expand around light-like limit:

$$\tilde{M}^{(2)}(v, z_1, z_2) \longrightarrow \left(\tilde{\chi}_2(\bar{\tau}_1, \tau_2) + \tilde{\chi}_{42}^{(i)}(\bar{\tau}_1, \tau_2)\right) \frac{\not{\bar{n}}}{2} + \left(\tilde{\chi}_2(\bar{\tau}_1, \tau_2) + \tilde{\chi}_{42}^{(ii)}(\bar{\tau}_1, \tau_2)\right) \frac{\not{n}}{2}$$
$$+ \tilde{\chi}_{42}^{(i)}(\bar{\tau}_1, \tau_2) \frac{\not{z}_1^{\perp}}{2\bar{\tau}_1} + \tilde{\chi}_{42}^{(ii)}(\bar{\tau}_1, \tau_2) \frac{\not{z}_2^{\perp}}{2\tau_2}$$
$$+ \tilde{\chi}_X(\bar{\tau}_1, \tau_2) \left(\frac{\not{z}_1^{\perp}}{2\bar{\tau}_1} + \frac{\not{z}_2^{\perp}}{2\tau_2}\right)\left(\frac{\not{n}\not{\bar{n}}}{4} - \frac{\not{\bar{n}}\not{n}}{4}\right) + \mathcal{O}(z_{i\perp}^2, n\cdot z_2, \bar{n}\cdot z_1)\,,$$

where now $\bar{\tau}_1 = \frac{n\cdot z_1}{2}$ and $\tau_2 = \frac{\bar{n}\cdot z_2}{2}$

↪ **Comparison:**

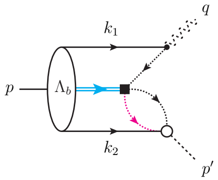
DLCDA:	$\tilde{\chi}_2(\bar{\tau}_1, \tau_2) = \tilde{\Phi}_2(\bar{\tau}_1, \tau_2, 0, 0, 2\bar{\tau}_1\tau_2)$	etc.	[this work]
LCDA:	$\tilde{\phi}_2(\tau_1, \tau_2) = \tilde{\Phi}_2(\tau_1, \tau_2, 0, 0, 0)$	etc.	[Ball, Braun, Gardi 2008]

Discontinuity of the correlator

- LCSRs are derived from a dispersion relation

$$\Pi_\mu^{\mathrm{OPE}}(n\cdot p') = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\,\Pi_\mu^{\mathrm{OPE}}(s/(\bar{n}\cdot p'))}{s-(\bar{n}\cdot p')(n\cdot p') - i\epsilon}\,, \qquad \bar{n}\cdot p' > 0$$

- external momentum enters the hard-collinear quark-loop
  $\hookrightarrow$ perform the hard-collinear quark loop integration
  $\hookrightarrow$ read off the discontinuity in $(n\cdot p')$



$\hookrightarrow$ leading contribution to non-local form factors from $\mathcal{O}_5$ and $\mathcal{O}_6$ only !

(complementary to annihilation in $B\to K^*$)

- Insert complete set of states:

$$\Pi_\mu^{\rm had}(n\cdot p') = \sum_{s'} \frac{\langle 0|J_\Lambda(0)|\Lambda(p',s')\rangle}{m_\Lambda^2-(p')^2}\,\mathcal{H}_\mu^{(i)} + \text{excited states and continuum},$$

with "decay constant" of the Λ baryon

$$\langle 0|J_\Lambda(0)|\Lambda(p',s')\rangle = f_\Lambda\,(\bar n\cdot p')\,\frac{\not n \not{\bar n}}{4}\,u_\Lambda(p',s')\,,$$

- equate OPE and hadronic representation of spectral densitiy above some threshold

$$s > s_0 \approx 2.55\ \mathrm{GeV}^2 \qquad \text{(global) quark-hadron duality}$$

- improve convergence by Borel transform

$$\frac{1}{A-n\cdot p'} \longrightarrow \exp\left(-A/\omega_M\right) \qquad \text{varying } \omega_M\,(\bar n\cdot p') = 2.5\pm 0.5\ \mathrm{GeV}^2$$

LCSR after Borel transform and continuum subtraction

After projecting onto non-local FFs, the LCSR takes the form

$$\mathcal{H}_\perp^{(5)}(q^2) = -\mathcal{H}_\perp^{(6)}(q^2) = \frac{f_{\Lambda_b}^{(2)}}{f_\Lambda} \frac{(Q_u + Q_d)}{192\pi^2} \frac{\bar{n}\cdot p'}{M_{\Lambda_b}^2 (\bar{n}\cdot p' - m_\Lambda)}$$

$$\times \int_0^{s_0} ds \int_0^{s/\bar{n}\cdot p'} d\omega_2 \int_0^\infty d\bar{\omega}_1 \left(\chi_2(\bar{\omega}_1, \omega_2) + \chi_{42}^{(ii)}(\bar{\omega}_1, \omega_2)\right) \frac{\exp\left(\frac{m_\Lambda^2 - s}{\omega_M\, \bar{n}\cdot p'}\right)}{\bar{n}\cdot q - \bar{\omega}_1 + i\epsilon}$$

and similarly for

$\mathcal{H}_{+(5)}^{(5,6)}(q^2)$ in terms of $\left(\hat{\chi}_{42}^{(ii)}(\bar{\omega}_1, \omega_2) + \hat{\chi}_X(\bar{\omega}_1, \omega_2)\right)$

- $\hookrightarrow$ complex-valued (branch cut for $q^2 > 0$ from intermediate hadronic states in $q\bar{q} \to \gamma^*$)
- $\hookrightarrow$ dependence on $f_{\Lambda_b}^{(2)}/f_\Lambda$ drops out in ratio with LCSR for local form factors [Feldmann/Yip 2011]
- $\hookrightarrow$ dependence on continuum threshold s_0 and Borel parameter ω_M

- requires model for the DLCDAs: we use parametrization with exponential fall-off, e.g.

$$\chi_2(\bar{\omega}_1, \omega_2) + \chi_{42}^{(ii)}(\bar{\omega}_1, \omega_2) = \frac{\bar{\omega}_1}{\omega_0^3}\, e^{-(\bar{\omega}_1+\omega_2)/\omega_0}$$

$\hookrightarrow$ we take $1/\omega_0 = 3.4 \pm 1.6$ GeV^{-1} $\rightarrow$ largest source of parametric uncertainty

For the q^2-dependent shift in the Wilson coefficients we find:

- effects of order 1% in

$$10^2 \cdot \Delta C_{9,\perp}(2\text{GeV}^2) = (0.6 \pm 2.5) + i(6.9 \pm 1.3),$$
$$10^2 \cdot \Delta C_{9,\perp}(4\text{GeV}^2) = -(0.97 \pm 0.98) + i(1.89 \pm 0.62),$$
$$10^2 \cdot \Delta C_{9,\perp}(6\text{GeV}^2) = -(0.64 \pm 0.39) + i(0.63 \pm 0.42),$$

- and power-suppressed effects in

$$10^5 \cdot \Delta C_{9,+}(2\text{GeV}^2) = (5.2 \pm 3.1) - i(8.7 \pm 3.8),$$
$$10^5 \cdot \Delta C_{9,+}(4\text{GeV}^2) = (5.1 \pm 2.2) - i(4.1 \pm 4.0),$$
$$10^5 \cdot \Delta C_{9,+}(6\text{GeV}^2) = (4.3 \pm 2.1) - i(2.3 \pm 3.7).$$

Summary & Outlook

“Annihilation-like” topologies in $\Lambda_b \to \Lambda \ell^+ \ell^-$ (at large hadronic recoil $q^2 \in [2, 6]\ \mathrm{GeV}^2$):

- non-local, non-factorizable effects treated in LCSR setup
- requires di-light-cone distribution amplitudes (DLCDAs) of Λ_b as hadronic input
- strong penguin operators $\mathcal{O}_5$ and $\mathcal{O}_6$ contribute at leading order

$\hookrightarrow$ numerical effects of order 1% (q^2-dependent, complex-valued)

$\hookrightarrow$ dominant parametric uncertainty from modelling of DLCDAs

Future directions:

- other (sub-leading) non-factorizable topologies
- α_s corrections
- better understanding of b-hadron DLCDAs

Thank You!