

Flavor physics with  $b$  baryons and  $B_c$  mesons: Opportunities for discoveries  
Mainz Institute for Theoretical Physics, May 26-29, 2026

## Heavy-baryon decay form factors from lattice QCD

Stefan Meinel



# Overview: $b$ and $c$ baryon semileptonic form factors from lattice QCD

Early lattice studies of  $\Lambda_b \rightarrow \Lambda_c$  (quenched, focused on Isgur-Wise function):

K. C. Bowler *et al.* (UKQCD Collaboration), hep-lat/9709028/PRD 1998 ; S. Gottlieb and S. Tamhankar, hep-lat/0301022/Lattice 2002

Our lattice calculations, using RBC/UKQCD gauge-field configurations,  $N_f = 2 + 1$

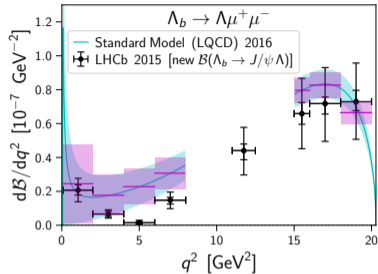
Transition	$m_Q$	$a$ [fm]	$m_\pi$ [MeV]	Reference
$\Lambda_b \rightarrow \Lambda$	$\infty$	0.08, 0.11	230–360	WD, DL, SM, MW, 1212.4827/PRD 2013
$\Lambda_b \rightarrow p$	$\infty$	0.08, 0.11	230–360	WD, DL, SM, MW, 1306.0446/PRD 2013
$\Lambda_b \rightarrow p$	phys.	0.08, 0.11	230–360	WD, CL, SM, 1503.01421/PRD 2015
$\Lambda_b \rightarrow \Lambda_c$	phys.	0.08, 0.11	230–360	WD, CL, SM, 1503.01421/PRD 2015; AD, SK, SM, AR, 1702.02243/JHEP 2017
$\Lambda_b \rightarrow \Lambda$	phys.	0.08, 0.11	230–360	WD, SM, 1602.01399/PRD 2016
$\Lambda_b \rightarrow \Lambda^*(1520)$	phys.	0.08, 0.11	300–430	SM, GR, 2009.09313/PRD 2021; 2107.13140/PRD 2022
$\Lambda_b \rightarrow \Lambda_c^*(2595)$	phys.	0.08, 0.11	300–430	SM, GR, 2103.08775/PRD 2021; 2107.13140/PRD 2022
$\Lambda_b \rightarrow \Lambda_c^*(2625)$	phys.	0.08, 0.11	300–430	SM, GR, 2103.08775/PRD 2021; 2107.13140/PRD 2022
$\Xi_b \rightarrow \Xi$	phys.	0.07, 0.08, 0.11	230–430	CF, SM, 2603.18438/accepted by PRD
$\Lambda_b \rightarrow p, \Lambda, \Lambda_c$	phys.	0.07, 0.08, 0.11	140–360	SM, in preparation
$\Lambda_c \rightarrow \Lambda$	phys.	0.08, 0.11	140–360	SM, 1611.09696/PRL 2017
$\Lambda_c \rightarrow n$	phys.	0.08, 0.11	230–360	SM, 1712.05783/PRD 2018
$\Lambda_c \rightarrow \Lambda^*(1520)$	phys.	0.08, 0.11	300–430	SM, GR, 2107.13140/PRD 2022; 2107.13084/PRD 2022
$\Xi_c \rightarrow \Xi$	phys.	0.07, 0.08, 0.11	230–430	CF, SM, 2504.07302/PRD 2025

Recent lattice calculations by other collaborations:

$\Xi_c \rightarrow \Xi$ : Q.-A. Zhang, J. Hua, F. Huang, R. Li, Y. Li, C.-D. Lu, P. Sun, W. Sun, W. Wang, Y.-B. Yang, 2103.07064/CPC 2022

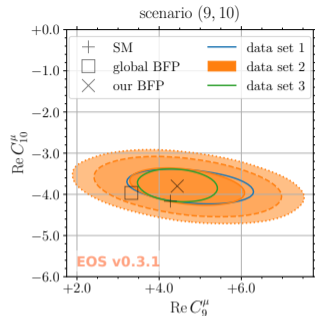
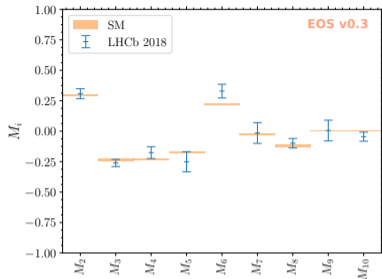
$\Lambda_c \rightarrow \Lambda$ : H. Bahtiyar, 2107.13909/Turk.J.Phys. 2021

$$\Lambda_b \rightarrow \Lambda^{(*)} \mu^+ \mu^-$$

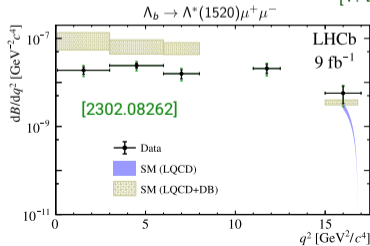


[W. Detmold, S. Meinel, 1602.01399/PRD 2016]

Next-gen  $\Lambda_b \rightarrow \Lambda$  LQCD calculation underway.



[T. Blake, S. Meinel, D. van Dyk, 1912.05811/PRD 2020]

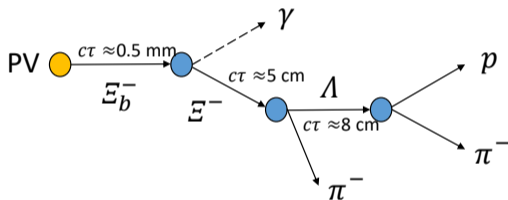


[LQCD: S. Meinel, G. Rendon, 2107.13140/PRD 2022]

# What about rare decays of other $b$ baryons?

LHCb is also analyzing  $\Xi_b^-$  decays. Low production fraction but distinctive final states.

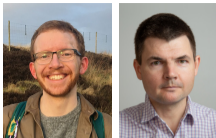
- $\Xi_b^- \rightarrow \Xi^- \gamma$



First result:  $\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \gamma) < 1.3 \times 10^{-4}$  at 95% CL [LHCb, 2108.07678/JHEP 2022]

- $\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-$

Analysis in progress [Janina Nicolini, PhD Thesis, 2024, DOI:10.17877/DE290R-24786]



## $\Xi_b \rightarrow \Xi$ form factors from lattice QCD and Standard-Model predictions for $\Xi_b \rightarrow \Xi \mu^+ \mu^-$ and $\Xi_b \rightarrow \Xi \gamma$ decays

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(Dated: March 18, 2026)

We present the first lattice QCD determination of the  $\Xi_b \rightarrow \Xi$  vector, axial-vector, and tensor form factors, which are relevant for the theory of rare decays including  $\Xi_b \rightarrow \Xi \ell^+ \ell^-$  and  $\Xi_b \rightarrow \Xi \gamma$ . The calculation is performed with 2+1 flavors of domain-wall fermions at three different lattice spacings and pion masses in the range from approximately 430 to 230 MeV. The bottom quark is implemented using an anisotropic clover action. Three-point functions with a wide range of source-sink separations and model averaging are used to extract the ground-state contributions. We fit the dependence of the form factors on the momentum transfer, the pion mass, and the lattice spacing using modified  $z$  expansions that account for subthreshold branch cuts, and apply dispersive bounds and asymptotic-behavior constraints to achieve controlled uncertainties in the full semileptonic kinematic region. Using our form factor results, we present Standard-Model predictions for the  $\Xi_b^- \rightarrow \Xi^- \gamma$  and  $\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-$  branching fractions and two angular observables.

[2603.18438]

# Form factor definitions

Current	Form factors
Vector	$f_0, f_+, f_\perp$
Axial vector	$g_0, g_+, g_\perp$
Tensor	$h_+, h_\perp, \tilde{h}_+, \tilde{h}_\perp$

Example:

$$\begin{aligned}
 \langle \Xi(p', s') | \bar{s} \gamma^\mu \gamma_5 b | \Xi_b(p, s) \rangle &= -\bar{u}_\Xi(p', s') \gamma_5 \left[ (m_{\Xi_b} + m_\Xi) \frac{q^\mu}{q^2} g_0(q^2) \right. \\
 &\quad + \frac{m_{\Xi_b} - m_\Xi}{s_-} \left( p^\mu + p'^\mu - (m_{\Xi_b}^2 - m_\Xi^2) \frac{q^\mu}{q^2} \right) g_+(q^2) \\
 &\quad \left. + \left( \gamma^\mu + \frac{2m_\Xi}{s_-} p^\mu - \frac{2m_{\Xi_b}}{s_-} p'^\mu \right) g_\perp(q^2) \right] u_{\Xi_b}(p, s)
 \end{aligned}$$

where  $q = p - p'$  and  $s_\pm = (m_{\Xi_b} \pm m_\Xi)^2 - q^2$

[T. Feldmann, M. W. Y. Yip, 1111.1844/PRD 2012]

# Lattice actions and ensembles

- Gluons: Iwasaki action
- $u, d, s$  quarks: domain-wall action  $\rightarrow$  nearly exact chiral symmetry and no  $O(a)$  errors
- Gauge configurations generated by RBC/UKQCD

Label	$N_s^3 \times N_t$	$\beta$	$am_{u,d}^{(\text{sea, val})}$	$am_s^{(\text{sea})}$	$am_s^{(\text{val})}$	$a$ [fm]	$m_\pi^{(\text{sea, val})}$ [MeV]	Samples
C01	$24^3 \times 64$	2.13	0.01	0.04	0.0323	$\approx 0.111$	$\approx 430$	283 ex, 2264 sl
C005	$24^3 \times 64$	2.13	0.005	0.04	0.0323	$\approx 0.111$	$\approx 340$	311 ex, 2488 sl
F004	$32^3 \times 64$	2.25	0.004	0.03	0.0248	$\approx 0.083$	$\approx 300$	251 ex, 2008 sl
F1M	$48^3 \times 96$	2.31	0.002144	0.02144	0.02217	$\approx 0.073$	$\approx 230$	113 ex, 1808 sl

# Lattice actions and ensembles

- $b$  quarks: anisotropic clover action

$$S_Q = a^4 \sum_x \bar{Q} \left[ m_Q + \gamma_0 \nabla_0 - \frac{a}{2} \nabla_0^{(2)} + \nu \sum_{i=1}^3 \left( \gamma_i \nabla_i - \frac{a}{2} \nabla_i^{(2)} \right) - c_E \frac{a}{2} \sum_{i=1}^3 \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,j=1}^3 \sigma_{ij} F_{ij} \right] Q$$

with  $m_Q$ ,  $\nu$ ,  $c_E = c_B$  tuned nonperturbatively using  $B_s$  energy, “speed of light”  $c^2 = M_{\text{rest}}/M_{\text{kin}}$ , and hyperfine splitting. This removes all  $(am_Q)^n$  discretization errors in on-shell observables.

Ensemble	$am_Q$	$\nu$	$c_{E,B}$	$aE_{B_s}$	$E_{B_s}$ [GeV]	$c_{B_s}^2$	$aE_{B_s^*} - aE_{B_s}$	$E_{B_s^*} - E_{B_s}$ [MeV]
C005	7.42	2.92	4.86	3.00324(81)	5.360(15)	0.9184(89)	0.02602(52)	46.44(94)
C005	7.471	2.929	4.92	3.00801(81)	5.369(15)	0.9184(89)	0.02616(53)	46.69(95)
C005	7.9	2.929	4.92	3.05463(85)	5.452(15)	0.8895(91)	0.02482(53)	44.29(96)
C005	7.471	2.929	6	2.97872(77)	5.316(15)	0.9399(85)	0.03165(58)	56.5(1.1)
C005	7.471	3.3	4.92	3.03049(75)	5.409(15)	1.0186(89)	0.02665(48)	47.56(86)
C005	7.3258	3.1918	4.9625	3.00695(75)	5.367(15)	1.0013(88)	0.02719(49)	48.52(88)
F004	3.485	1.76	3.06	2.25010(82)	5.363(19)	0.855(12)	0.02053(62)	48.9(1.5)
F004	3.3	1.76	3.06	2.20958(79)	5.266(19)	0.877(12)	0.02148(61)	51.2(1.5)
F004	3.485	1.76	3.7	2.21551(78)	5.280(19)	0.877(12)	0.02468(68)	58.8(1.6)
F004	3.485	2.2	3.06	2.29690(70)	5.474(20)	1.047(12)	0.02126(51)	50.7(1.2)
F004	3.2823	2.0600	2.7960	2.25175(71)	5.366(19)	1.004(12)	0.02046(51)	48.8(1.2)
F1M	2.4538	1.7563	2.6522	1.97455(85)	5.347(20)	0.960(16)	0.01876(79)	50.8(2.2)
F1M	2.55	1.7563	2.6522	2.00012(88)	5.416(20)	0.943(17)	0.01821(81)	49.3(2.2)
F1M	2.4538	2	2.6522	2.00698(75)	5.435(20)	1.089(16)	0.01926(70)	52.2(1.9)
F1M	2.4538	1.7563	3.1	1.94472(81)	5.266(20)	0.983(16)	0.02151(83)	58.3(2.3)
F1M	2.3867	1.8323	2.4262	1.98009(81)	5.362(20)	1.003(16)	0.01799(73)	48.7(2.0)

# Extraction of the form factors from correlation functions

Each form factor  $f$  is obtained from the large- $t$  limit ( $t =$  Euclidean-time source-sink separation) of a combination of three-point and two-point correlation functions,

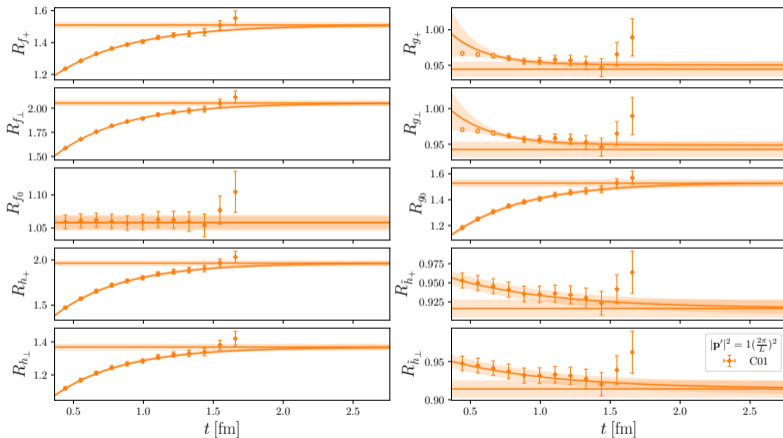
$$R_f = \text{(kinematic factors)} \times \frac{\langle \text{Diagram 1} \rangle \times \langle \text{Diagram 2} \rangle}{\langle \text{Diagram 3} \rangle \times \langle \text{Diagram 4} \rangle}$$

The diagram illustrates the extraction of form factors  $R_f$  from correlation functions. It shows a ratio of four diagrams, with the top row diagrams multiplied together and the bottom row diagrams multiplied together. The diagrams are labeled with  $s$ ,  $d$ , and  $b$  for different paths or states. The top row diagrams include time intervals  $t'$  and  $t-t'$ , while the bottom row diagrams include the total time interval  $t$ .

where we are setting  $t' = t/2$ .

# Extraction of the form factors from correlation functions

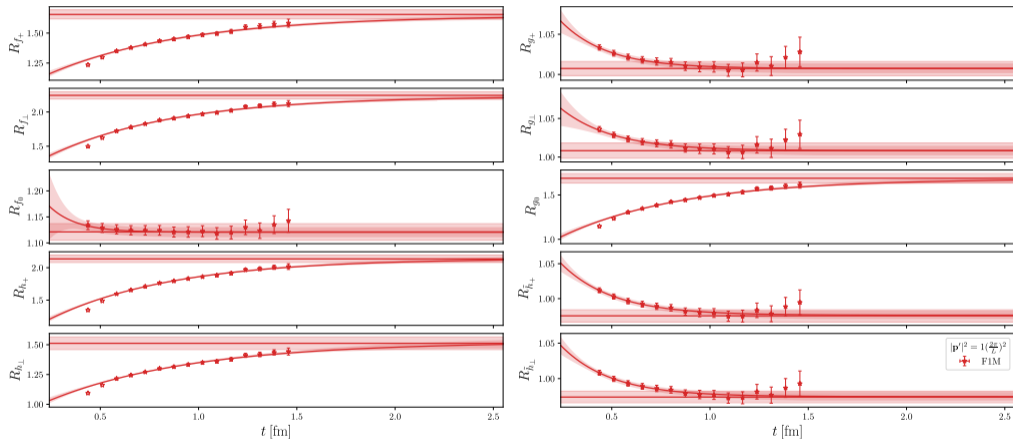
$R_f(t) \rightarrow f$  for large  $t$ . We use fits of the form  $R_f(t) = f + A e^{-\Delta E t}$ , where  $f$ ,  $A$ ,  $\Delta E$  are fit parameters.



We use a generalized version of the Akaike information criterion to average over fits with different  $t_{\min}$  [W. Jay and E. Neil, 2008.01069/PRD 2021]. Horizontal bands = AIC averages of  $f$

# Extraction of the form factors from correlation functions

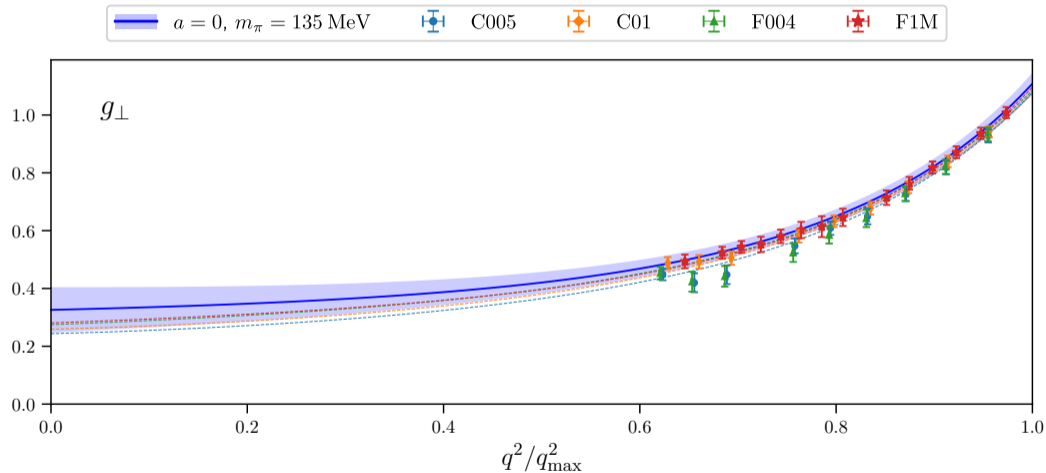
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# Chiral-continuum-kinematic extrapolations

Example:



The fit is done using a modified  $z$  expansion of order  $z^8$  with a dispersive bound on the coefficients and additional constraints ensuring reasonable asymptotic behavior for  $q^2 \rightarrow -\infty$ .

# Dispersive bounds

[T. Blake, S. Meinel, M. Rahimi, D. van Dyk, 2205.06041/PRD 2023]

An example of a dispersive bound is

$$\underbrace{\chi_V^{J=1}(Q^2)}_{\text{perturbation theory}} \Big|_{\text{OPE}} \geq \int_{t_{\text{th}}}^{\infty} dt \frac{1}{24\pi^2} \frac{\sqrt{\lambda(m_{\Xi_b}^2, m_{\Xi}^2, t)}}{t^2(t-Q^2)^3} s_-(t) \left( (m_{\Xi_b} + m_{\Xi})^2 |f_+(t)|^2 + 2t |f_{\perp}(t)|^2 \right),$$

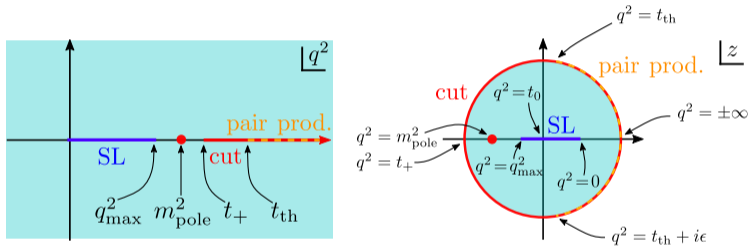
where  $t_{\text{th}} = (m_{\Xi_b} + m_{\Xi})^2$ ,  $\chi_V^{J=1}(Q^2) = \frac{1}{2!} \left( \frac{d}{dQ^2} \right)^2 \Pi_V^{J=1}(Q^2)$ , and we use  $Q^2 = 0$ .

NB: 2205.06041 uses different names for the form factors, such as  $(f_0^V, f_{\perp}^V, f_t^V)$  instead of  $(f_+, f_{\perp}, f_0)$ , but the definitions are identical.

# Dispersive bounds

The vector form factors have a branch cut starting at  $t_+ = (m_B + m_K)^2$ . We define the variable

$$z(t; t_0, t_+) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}.$$



Then the dispersive bound can be written as

$$1 \geq \int_{-\alpha_{\Xi_b}}^{+\alpha_{\Xi_b}} d\alpha \left( |\phi_{f_+}(z)|^2 |f_+(z)|^2 + |\phi_{f_\perp}(z)|^2 |f_\perp(z)|^2 \right)_{z=e^{i\alpha}}$$

where the  $\phi$ 's are the *outer functions* and

$$\alpha_{\Xi_b} = \arg z((m_{\Xi_b} + m_{\Xi})^2, t_0, t_+) \approx 1.43829 \quad [\text{for } t_0 = q_{\max}^2 = (m_{\Xi_b} - m_{\Xi})^2].$$

# Dispersive bounds

We use BGL-type  $z$ -expansions to describe the continuum form factors,

$$f(q^2) = \frac{1}{\mathcal{P}_f(q^2) \phi_f(z)} \sum_n \mathbf{a}_{f,n} z^n,$$

where  $\mathcal{P}_f(q^2) = z(q^2; m_{\text{pole},f}^2, t_+)$  is the Blaschke factor. If we had  $t_{\text{th}} = t_+$ , so that the dispersive integral would go over the entire unit circle, we would have  $1 \geq \sum_n (\mathbf{a}_{f_+,n}^2 + \mathbf{a}_{f_\perp,n}^2)$ . Instead, because the integral only goes over a smaller arc, we have

$$1 \geq \sum_{n,n'} \mathbf{a}_{f_+,n} \langle z^n | z^{n'} \rangle \mathbf{a}_{f_+,n'} + \sum_{n,n'} \mathbf{a}_{f_\perp,n} \langle z^n | z^{n'} \rangle \mathbf{a}_{f_\perp,n'},$$

where [J.M. Flynn, A. Jüttner, J.T. Tsang, 2303.11285/JHEP 2023]

$$\langle z^n | z^{n'} \rangle = \begin{cases} \frac{\sin(\alpha_{\Xi_b \Xi}(n - n'))}{\pi(n - n')}, & n \neq n', \\ \frac{\pi}{\alpha_{\Xi_b \Xi}}, & n = n'. \end{cases}$$

This is equivalent to the orthogonal-polynomial expansion in [T. Blake, S. Meinel, M. Rahimi, D. van Dyk, 2205.06041/PRD 2023]

# Asymptotic behavior

A possible problem with BGL  $z$  expansion is that  $\frac{1}{\mathcal{P}_f(q^2)\phi_f(z)}$  diverges for  $q^2 \rightarrow -\infty$  ( $z \rightarrow 1$ ), while  $\sum_n a_{f,n} z^n$  remains finite. This allows the form factor to diverge, which contradicts the expectation from perturbative QCD.

One possible solution is to use a BCL  $z$  expansion with  $\frac{1}{1 - q^2/m_{\text{pole},f}^2}$  instead of  $\frac{1}{\mathcal{P}_f(q^2)\phi_f(z)}$ , but then the dispersive bounds become more complicated.

We instead solve the problem by imposing the sum rules  
[J.M. Flynn, A. Jüttner, J.T. Tsang, 2303.11285/JHEP 2023]

$$\left(\frac{d}{dz}\right)^m \sum_{n=0}^{N+M} a_{f,n} z^n \Big|_{z=1} = 0 \quad \forall m \in \{0, 1, \dots, M-1\}$$

with  $M = 4$  for  $f_+$ ,  $g_+$ ,  $h_\perp$ , and  $\tilde{h}_\perp$  and  $M = 3$  for the other form factors, which, given our outer functions, ensures fall-off at least as fast as  $1/(-q^2)$  for  $q^2 \rightarrow -\infty$ . We solve for the highest  $M$  coefficients in terms of the lower coefficients.

# Lattice-spacing and pion-mass dependence

We fit the lattice data using

$$f(q^2) = \frac{1}{\mathcal{P}_f(q^2)\phi_f(q^2)} \left[ a_{f,0} \left( 1 + c_0^f \frac{m_\pi^2 - m_{\pi,\text{phys}}^2}{\Lambda_\chi^2} + \tilde{c}_0^f \frac{m_\pi^3 - m_{\pi,\text{phys}}^3}{\Lambda_\chi^3} \right) + a_{f,1} \left( 1 + c_1^f \frac{m_\pi^2 - m_{\pi,\text{phys}}^2}{\Lambda_\chi^2} + \tilde{c}_1^f \frac{m_\pi^3 - m_{\pi,\text{phys}}^3}{\Lambda_\chi^3} \right) z(q^2, q_{\text{max}}^2, t_+^f) + \sum_{n=2}^{N+M} a_{f,n} [z(q^2, q_{\text{max}}^2, t_+^f)]^n \right] \times \left[ 1 + b^f a^2 |\mathbf{p}'|^2 + d^f a^2 \Lambda_{\text{had}}^2 + \tilde{b}^f a^4 |\mathbf{p}'|^4 + \hat{d}^f a^3 \Lambda_{\text{had}}^3 + \tilde{d}^f a^4 \Lambda_{\text{had}}^4 + j^f a^4 |\mathbf{p}'|^2 \Lambda_{\text{had}}^2 \right],$$

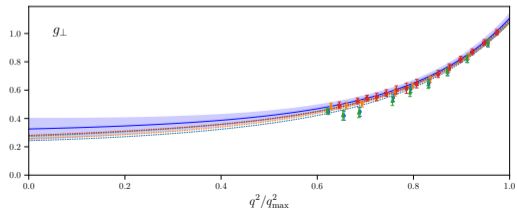
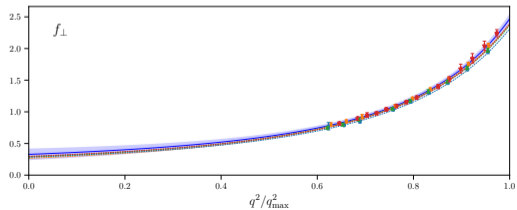
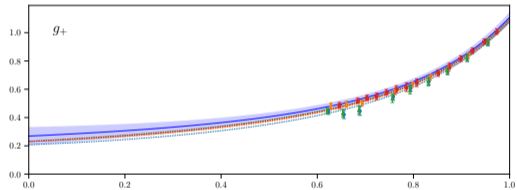
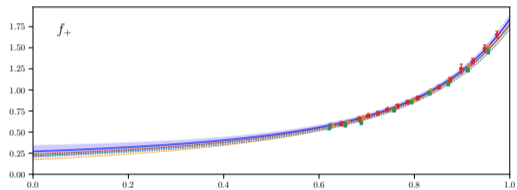
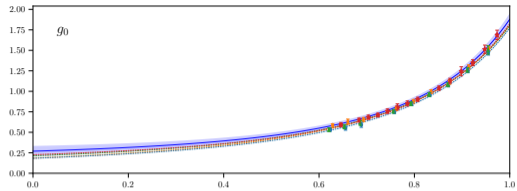
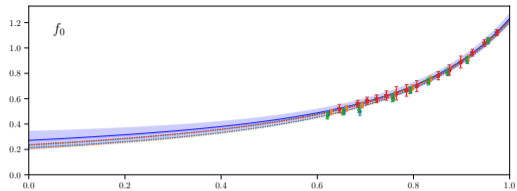
where the higher-order parameters  $\tilde{c}_0^f$ ,  $\tilde{c}_1^f$ ,  $\tilde{b}^f$ ,  $\hat{d}^f$ ,  $\tilde{d}^f$ , and  $j^f$ , which are not needed to describe the data, were constrained with Gaussian priors to be not unnaturally large.

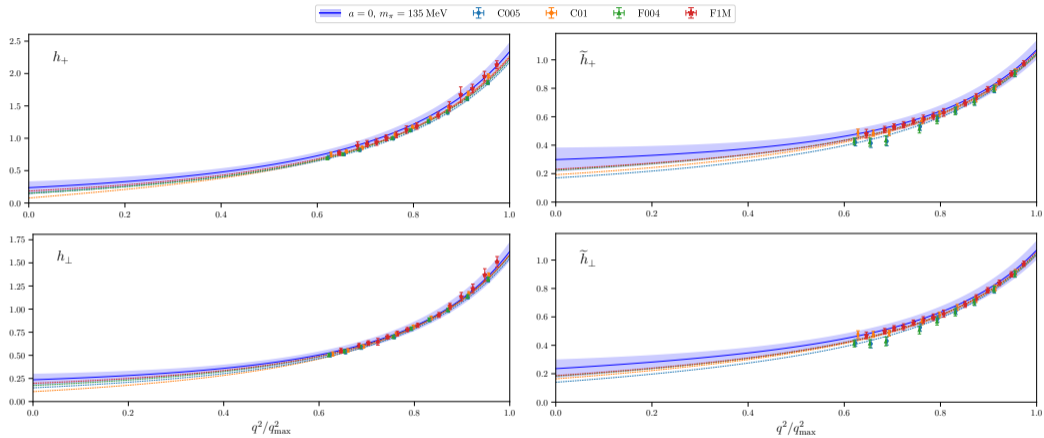
# Statistical approach

We use the algorithm of [J.M. Flynn, A. Jüttner, J.T. Tsang, 2303.11285/JHEP 2023], which involves an initial fit with a Gaussian prior on the dispersive-bound function, followed by random sampling and an accept-reject step to apply the dispersive bound and reweight to remove the initial prior.

We increase the order of the  $z$  expansion until the central values and uncertainties of the form factors stabilize in the full kinematic range. We found that we need  $N = 5$  (and recall that there are an additional  $M = 3$  or  $M = 4$  higher powers, so we go up to  $z^8$  or  $z^9$ ).

$a = 0, m_\pi = 135 \text{ MeV}$    C005   C01   F004   F1M





Weak effective Hamiltonian for  $b \rightarrow sl^+l^-$ ,  $b \rightarrow s\gamma$

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$O_1 = \bar{c}^b \gamma^\mu b_L^a \bar{s}^a \gamma_\mu c_L^b,$$

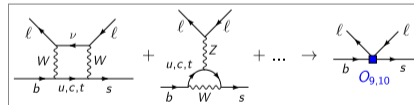
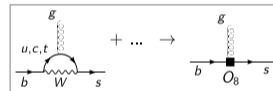
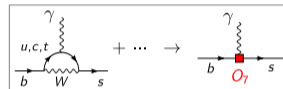
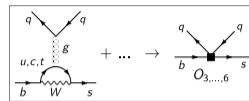
$$O_2 = \bar{c}^a \gamma^\mu b_L^a \bar{s}^b \gamma_\mu c_L^b,$$

$$O_7 = (e m_b)/(16\pi^2) \bar{s} \sigma^{\mu\nu} b_R F_{\mu\nu}^{(e.m.)},$$

$$O_9 = e^2/(16\pi^2) \bar{s} \gamma^\mu b_L \bar{l} \gamma_\mu l,$$

$$O_{10} = e^2/(16\pi^2) \bar{s} \gamma^\mu b_L \bar{l} \gamma_\mu \gamma_5 l,$$

...



In the Standard Model,  $\overline{\text{MS}}$  scheme, at  $\mu = 4.2$  GeV,

$C_1$	$C_2$	$C_7$	$C_9$	$C_{10}$	...
-0.289	1.010	-0.336	4.267	-4.113	...

[Computed using EOS, <https://eoshep.org/>]

# Hadronic matrix elements for exclusive $b \rightarrow s\ell^+\ell^-$ , $b \rightarrow s\gamma$ decays

For a generic decay  $H_b \rightarrow H_s\ell^+\ell^-$  or  $H_b \rightarrow H_s\gamma$ :

Contributions from  $O_7$ ,  $[O_9, O_{10}]$ :  $\langle H_s(p') | \bar{s}\Gamma b | H_b(p) \rangle \rightarrow$  local form factors

Contributions from  $O_{1,\dots,6}$ ,  $O_8$ :  $\int d^4x e^{iq\cdot x} \langle H_s(p') | T O_i(0) J_{\text{e.m.}}^\mu(x) | H_b(p) \rangle \rightarrow$  nonlocal form factors

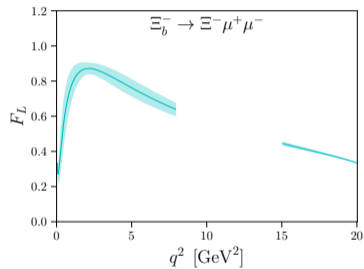
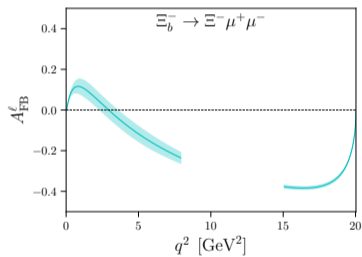
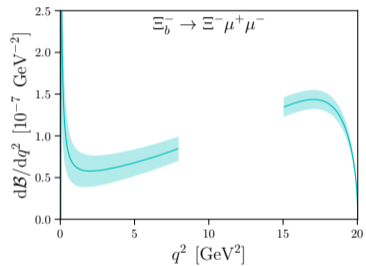
## Calculation of $\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-$ and $\Xi_b^- \rightarrow \Xi^- \gamma$ observables

We limit our predictions to the kinematic regions  $q^2 < 8 \text{ GeV}^2$  and  $q^2 > 15 \text{ GeV}^2$  to avoid the effects of narrow charmonium resonances, and use the approximation in which all nonlocal hadronic matrix elements are reduced to local matrix elements through the effective Wilson coefficients

$$\begin{aligned} C_7^{\text{eff}}(q^2) &= C_7 - \frac{1}{3} \left[ C_3 + \frac{4}{3} C_4 + 20 C_5 + \frac{80}{3} C_6 \right] - \frac{\alpha_s}{4\pi} \left[ (C_1 - 6 C_2) F_{1,c}^{(7)}(q^2) + C_8 F_8^{(7)}(q^2) \right], \\ C_9^{\text{eff}}(q^2) &= C_9 + \frac{4}{3} C_3 + \frac{64}{9} C_5 + \frac{64}{27} C_6 + h(0, q^2) \left( -\frac{1}{2} C_3 - \frac{2}{3} C_4 - 8 C_5 - \frac{32}{3} C_6 \right) \\ &\quad + h(m_b, q^2) \left( -\frac{7}{2} C_3 - \frac{2}{3} C_4 - 38 C_5 - \frac{32}{3} C_6 \right) + h(m_c, q^2) \left( \frac{4}{3} C_1 + C_2 + 6 C_3 + 60 C_5 \right) \\ &\quad - \frac{\alpha_s}{4\pi} \left[ C_1 F_{1,c}^{(9)}(q^2) + C_2 F_{2,c}^{(9)}(q^2) + C_8 F_8^{(9)}(q^2) \right]. \end{aligned}$$

See [D. Du *et al.*, 1510.02349/PRD 2016] and references therein

$\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-$  Standard-Model predictions



$\Xi_b^- \rightarrow \Xi^- \gamma$  Standard-Model predictions

	Method	$h_{\perp}(0) = \tilde{h}_{\perp}(0)$	$\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \gamma)$
LHCb, 2021 [87]	Experiment		$< 1.3 \times 10^{-4}$
This work	Lattice QCD	$0.235 \pm 0.065$	$(2.9 \pm 1.6) \times 10^{-5}$
Rui <i>et al.</i> , 2025 [85]	pQCD approach	$0.338_{-0.000-0.000-0.015}^{+0.015+0.009+0.024}$	
Aliev <i>et al.</i> , 2023 [92]	Light-cone sum rules	$0.31 \pm 0.04$	$(4.8 \pm 1.3) \times 10^{-5}$
Davydov <i>et al.</i> , 2022 [91]	Relativistic quark model	$0.144 \pm 0.007$	$(0.95 \pm 0.15) \times 10^{-5}$
Geng <i>et al.</i> , 2022 [90]	Light-front quark model	0.143	$(1.1 \pm 0.1) \times 10^{-5}$
Olamaei <i>et al.</i> , 2021 [89]	Light-cone sum rules [81]		$(1.08_{-0.49}^{+0.63}) \times 10^{-5}$
Wang <i>et al.</i> , 2021 [88]	Flavor- $SU(3)$ symmetry		$(1.23 \pm 0.64) \times 10^{-5}$
Liu <i>et al.</i> , 2011 [93]	Light-cone sum rules	$\approx 0.6$	$(3.03 \pm 0.10) \times 10^{-4}$

Outlook: next-generation  $\Lambda_b \rightarrow p, \Lambda, \Lambda_c$  form-factor calculations

Goals of the next-generation calculations:

- Higher precision
- Controlled uncertainties in full kinematic range
- Satisfy FLAG quality criteria

# Parameters of the 2015/2016 calculations

Label	$N_s^3 \times N_t$	$\beta$	$am_{u,d}^{(\text{sea})}$	$am_{u,d}^{(\text{val})}$	$am_s^{(\text{sea})}$	$am_s^{(\text{val})}$	$a$ [fm]	$m_\pi^{(\text{sea})}$ [MeV]	$m_\pi^{(\text{val})}$ [MeV]	Samples
C14	$24^3 \times 64$	2.13	0.005	0.001	0.04	0.04	$\approx 0.111$	$\approx 340$	$\approx 250$	2672
C24	$24^3 \times 64$	2.13	0.005	0.002	0.04	0.04	$\approx 0.111$	$\approx 340$	$\approx 270$	2676
C54	$24^3 \times 64$	2.13	0.005	0.005	0.04	0.04	$\approx 0.111$	$\approx 340$	$\approx 340$	2782
C53	$24^3 \times 64$	2.13	0.005	0.005	0.04	0.03	$\approx 0.111$	$\approx 340$	$\approx 340$	1205
F23	$32^3 \times 64$	2.25	0.004	0.002	0.03	0.03	$\approx 0.083$	$\approx 304$	$\approx 230$	1907
F43	$32^3 \times 64$	2.25	0.004	0.004	0.03	0.03	$\approx 0.083$	$\approx 304$	$\approx 304$	1917
F63	$32^3 \times 64$	2.25	0.006	0.006	0.03	0.03	$\approx 0.083$	$\approx 361$	$\approx 361$	2782

These data sets come from three ensembles only. The red color indicates  $m_{u,d}^{(\text{val})} < m_{u,d}^{(\text{sea})}$ , leading to small  $m_\pi^{(\text{val})} L$  and making finite-volume errors the dominant systematic uncertainty.

The bottom-quark parameters (taken from RBC/UKQCD) violate the kinetic-mass tuning condition by  $O(10\%)$  [S. Meinel, 2309.01821]

The extrapolations to  $q^2 = 0$  use first-order  $z$  expansions only, with ad-hoc estimates of missing higher-order terms.

# Parameters of the next-generation calculations

Label	$N_s^3 \times N_t$	$\beta$	$am_{u,d}^{(\text{sea, val})}$	$am_s^{(\text{sea})}$	$am_s^{(\text{val})}$	$a$ [fm]	$m_\pi^{(\text{sea, val})}$ [MeV]	Samples
CP	$48^3 \times 96$	2.13	0.00078	0.0362	0.0362	$\approx 0.114$	$\approx 139$	80 ex, 2560 sl
CPB*	$48^3 \times 96$	2.13	0.00078	0.0362	0.0362	$\approx 0.114$	$\approx 139$	78 ex, 2496 sl
C005LV	$32^3 \times 64$	2.13	0.005	0.04	0.0323	$\approx 0.111$	$\approx 340$	186 ex, 5022 sl
C005	$24^3 \times 64$	2.13	0.005	0.04	0.0323	$\approx 0.111$	$\approx 340$	311 ex, 4976 sl
F004	$32^3 \times 64$	2.25	0.004	0.03	0.0248	$\approx 0.083$	$\approx 304$	251 ex, 4016 sl
F006	$32^3 \times 64$	2.25	0.006	0.03	0.0248	$\approx 0.083$	$\approx 361$	223 ex, 3568 sl
F1M	$48^3 \times 96$	2.31	0.002144	0.02144	0.02217	$\approx 0.073$	$\approx 232$	226 ex, 7232 sl

\* same ensemble, but half the source/sink smearing width for propagators; different subset of configs

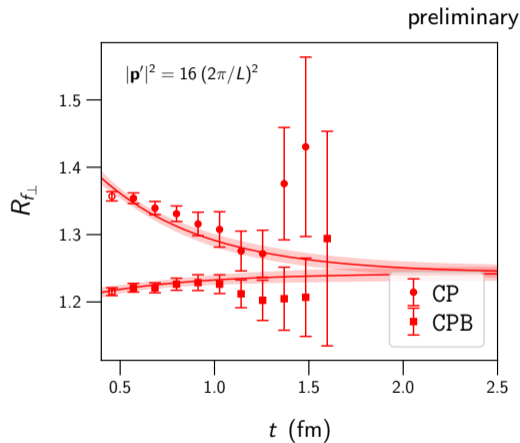
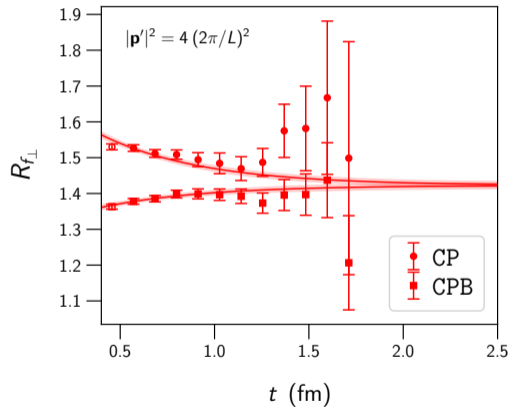
Six different ensembles, all data sets use  $m_{u,d}^{(\text{val})} = m_{u,d}^{(\text{sea})}$ . Now using physical  $am_s^{(\text{val})}$ .

I have newly computed all propagators with improved source smearing, and using all-mode averaging [E. Shintani *et al.*, 1402.0244/PRD 2015].

I have re-tuned the bottom and charm action parameters [S. Meinel, 2309.01821].

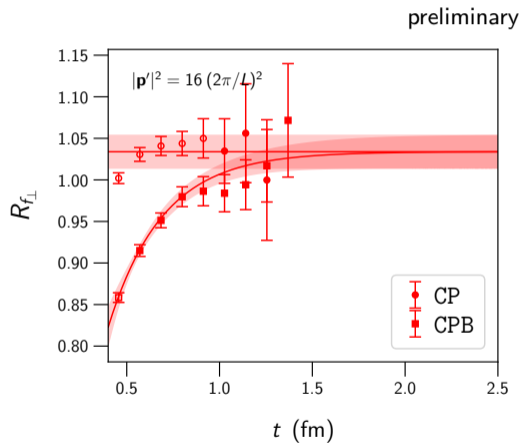
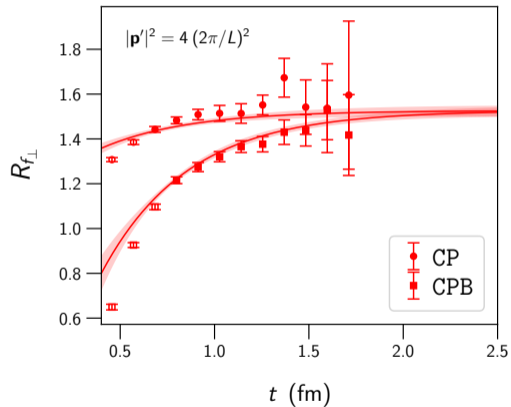
# Simultaneous fits of CP and CPB data

$$\Lambda_b \rightarrow \Lambda_c$$



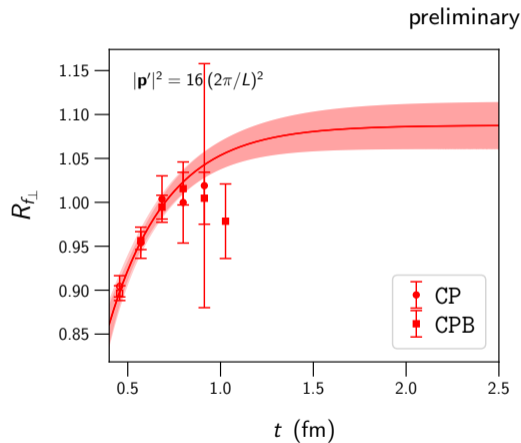
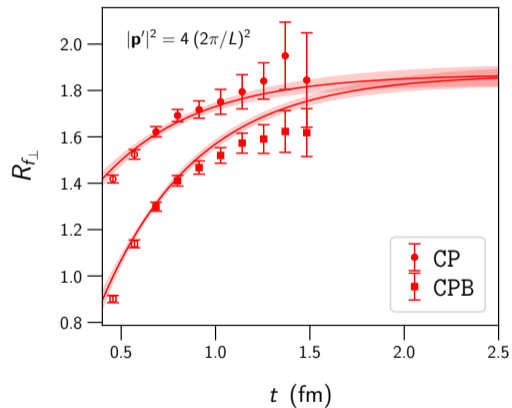
# Simultaneous fits of CP and CPB data

$$\Lambda_b \rightarrow \Lambda$$



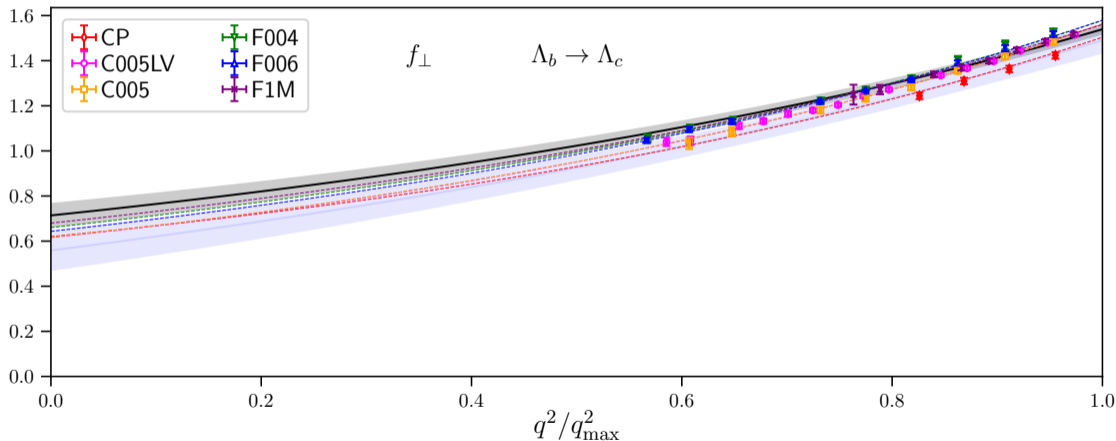
# Simultaneous fits of CP and CPB data

$$\Lambda_b \rightarrow p$$



# Preliminary chiral-continuum-kinematic extrapolations

I am still working on finalizing the form-factor extractions from the ratios. The data points used here may still change.

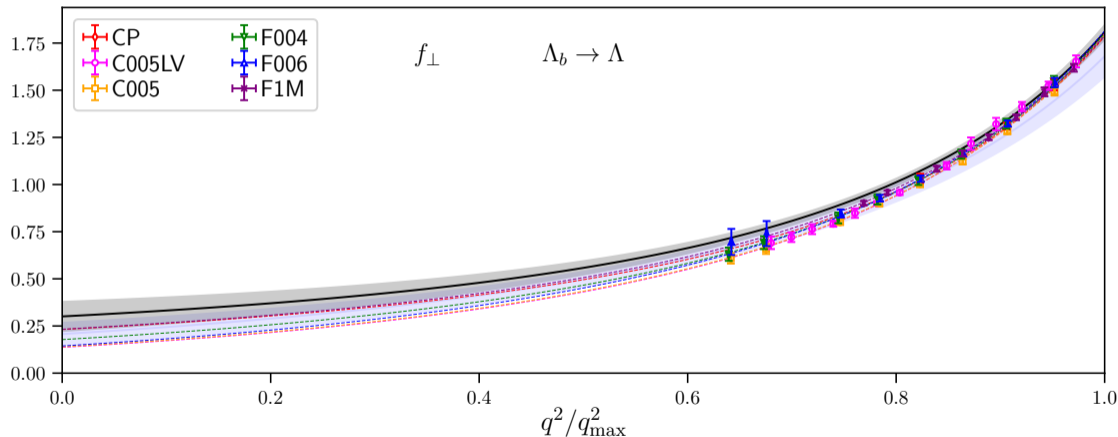


Black curve = preliminary new continuum form factor (fit with  $N = 5$  and dispersive bound).

Light-blue curve = 2015 form factor

# Preliminary chiral-continuum-kinematic extrapolations

I am still working on finalizing the form-factor extractions from the ratios. The data points used here may still change.



Black curve = preliminary new continuum form factor (fit with  $N = 5$  and dispersive bound).

Light-blue curve = 2016 form factor

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