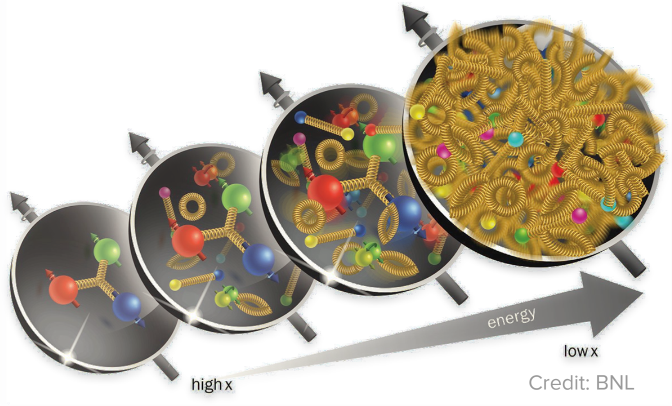
Status and Prospects of Lattice QCD for Baryon Semileptonic Transitions

High-Performance Computing
Theoretical Physics
Applied Mathematics

Simone Bacchio

CaSToRC, The Cyprus Institute
Extended Twisted Mass Collaboration (ETMC)



25/05/26 – FPBM2026 – MITP Workshop – Germany

Overview

- **Lattice QCD:** methodology
- ***Λ → p*:** V_{US}, non-standard interactions, ν-scattering
- ***Σ → Λ system*:** unique opportunities
- **Outlook and other prospects**

Represented experiments: LHCb, BESIII, FAIR, ALADDIN

Workshop on
Baryon Weak Decays
From experiments to Lattice QCD

Cyprus
25-27 March 2026

https://indico.global/e/BWDCy

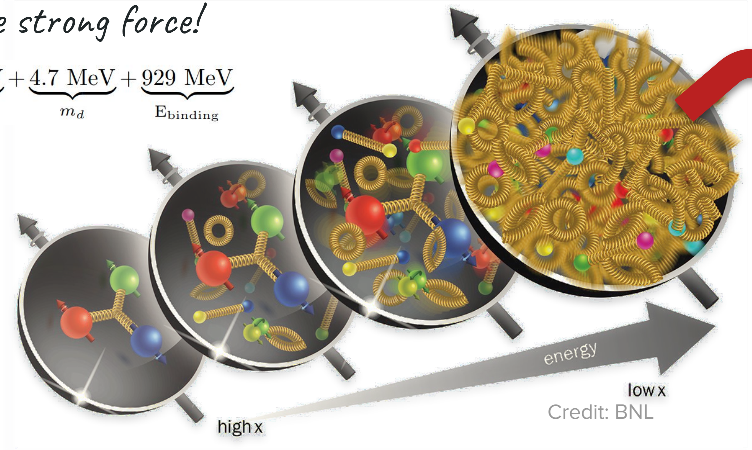
Lattice QCD: Why? How?

Why? **The Lattice formulation** is a non-perturbative, systematically-improvable approach to regularize a quantum field theory; it provides a fundamental framework for the study of QCD at low energies

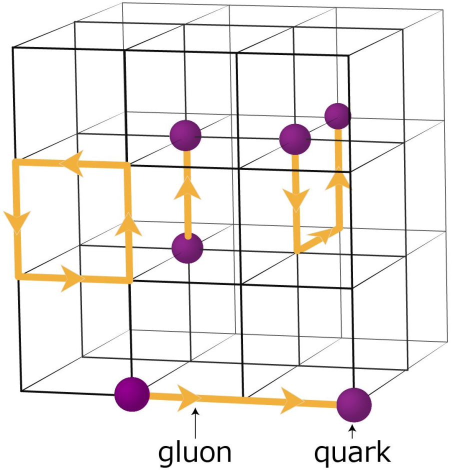
$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

99% of our mass comes from the strong force!

$$m_p = \underbrace{2.3\ \mathrm{MeV}}_{2\times m_u} + \underbrace{4.7\ \mathrm{MeV}}_{m_d} + \underbrace{929\ \mathrm{MeV}}_{\mathrm{E_{binding}}}$$



Numerical approaches required!



Lattice QCD: Why? How?

Why? **The Lattice formulation** is a non-perturbative, systematically-improvable approach to regularize a quantum field theory; it provides a fundamental framework for the study of QCD at low energies

$$\mathcal{L}_{\text{QCD}} = -\tfrac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

How?

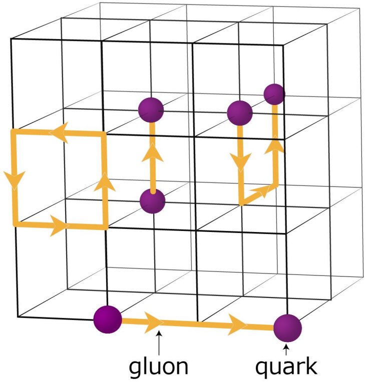
K. G. Wilson
1974

From QCD

- Continuous space-time
- Minkowski space-time
- Lie algebra $A_\mu(x)$
- Lagrangian $\mathcal{L}_{\text{QCD}}(g, m_f)$

To Lattice QCD

- 4D hypercubic lattice a, V
- Euclidean space-time
- Lie group $U_\mu(x) = e^{igaA_\mu(x)}$
- Lattice action $S(a, \mu_f)$



$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U]\mathcal{D}[\psi\bar{\psi}]\, O(U, \psi, \bar{\psi})\, e^{-S(U,\psi,\bar{\psi})}$$

Lattice QCD: Why? How?

Why?

The Lattice formulation is a non-perturbative, systematically-improvable approach to regularize a quantum field theory; it provides a fundamental framework for the study of QCD at low energies

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

$$\langle \mathcal{O} \rangle = \lim_{\substack{\mu_f \to m_f \\ V \to \infty \\ a \to 0}} \langle \mathcal{O} \rangle_{a,V,\mu_f}$$

How?

K. G. Wilson
1974

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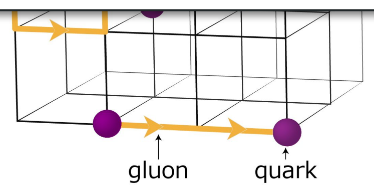
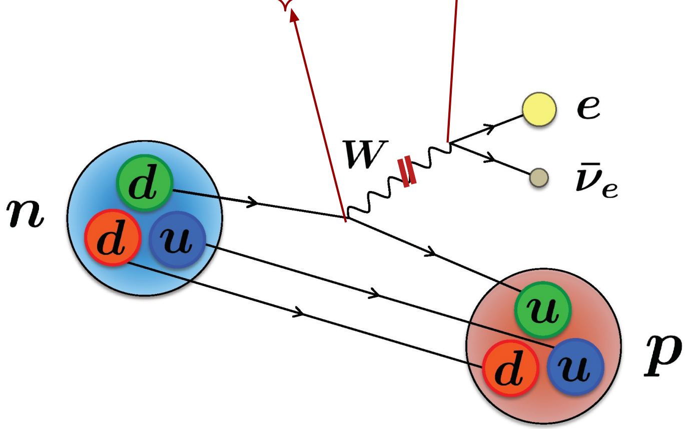


$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{D}[\psi \bar{\psi}] O(U, \psi, \bar{\psi}) \, e^{-S(U,\psi,\bar{\psi})}$$

Semileptonic decays in Lattice QCD

The focus of this talk are baryon semileptonic decays, e.g. neutron beta decay.

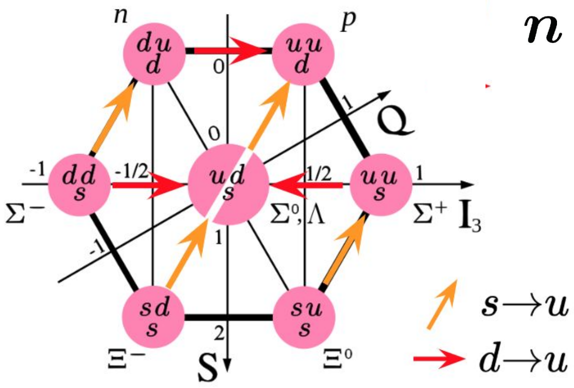
$$\mathcal{M}(n \to p\, e^- \bar{\nu}_e) = \frac{G_F}{\sqrt{2}}\, V_{ud}\, \sum_\mu \langle p(p')|W_\mu|n(p)\rangle\, L_\mu$$



Key feature: *in absence of radiative corrections*, the heavy W-boson enables exact factorization of the transition amplitude at tree-level.

Key ingredients:

- Hadronic matrix elements (QCD)
- Leptonic matrix elements (QED)
- CKM matrix elements (Weak)



Key tool: Euclidean correlation functions

The correlation function between two states ψ allows for a spectral decomposition

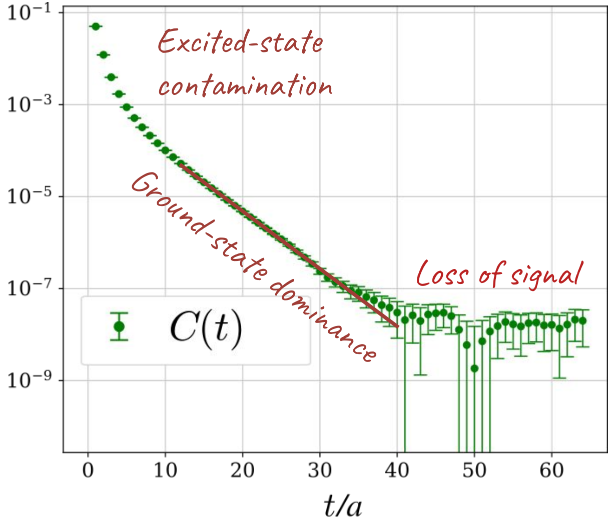
$$C_{2pt}(t) \equiv \langle \psi_N(t)\psi_N(0)\rangle = \langle \psi_N | T^t | \psi_N \rangle = \sum_{n=N} \langle \psi_N | n \rangle \langle n | \psi_N \rangle e^{-E_n t}$$

$T \to e^{-H}$

→ *Exponential suppression!*

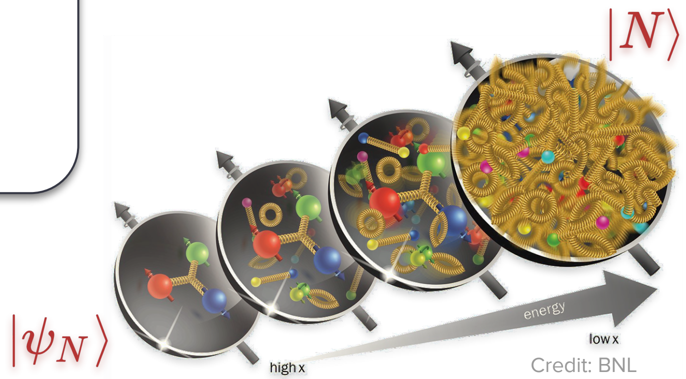
✔ *Great for ground-states*

✘ *Awful for anything else!*



Key Challenges:

- Exponential growth of the noise with distance
- Reliably assessing ground-state dominance



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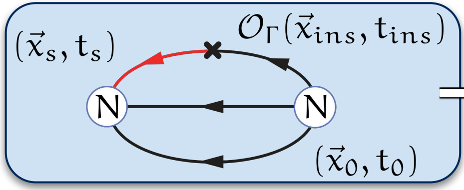
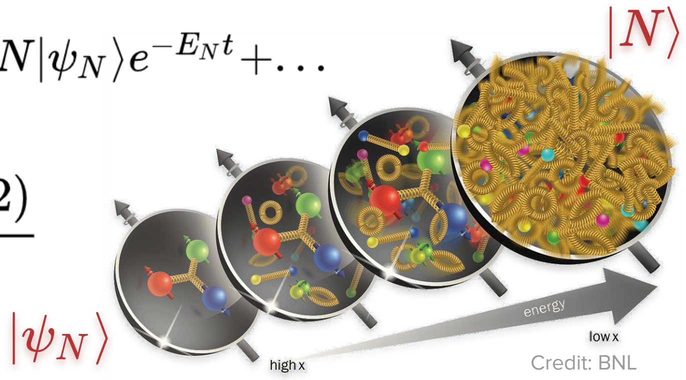
Matrix elements are extracted from three-point correlation functions

$$C_{3pt}(t, t_{ins}) \equiv \langle \psi_N(t) O(t_{ins}) \psi_N(0)\rangle \approx \langle \psi_N|N\rangle\langle N|O|N\rangle\langle N|\psi_N\rangle e^{-E_N t} + \ldots$$



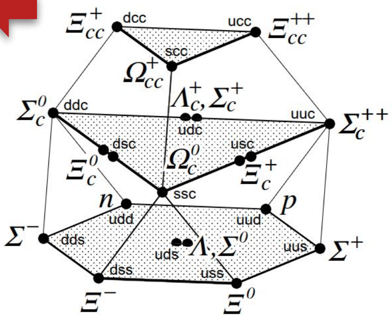
$$\Longrightarrow \quad \langle N|O|N\rangle = \lim_{t \gg \Delta E^{-1}} \frac{C_{3pt}(t, t/2)}{C_{2pt}(t)}$$



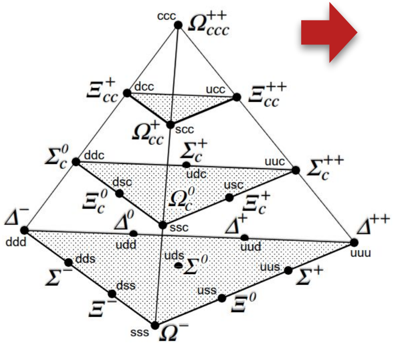
Accessible low-lying baryons with u,d,s,c

Charm	Strange	Baryon	Quark content	I	I_z
$c=2$	$s=0$	Ξ_{cc}^{++}	ucc	1/2	+1/2
		Ξ_{cc}^{+}	dcc	1/2	-1/2
	$s=1$	Ω_{cc}^{+}	scc	0	0
$c=1$	$s=0$	Σ_{c}^{++}	uuc	1	+1
		Σ_{c}^{+}	udc	1	0
		Σ_{c}^{0}	ddc	1	-1
	$s=1$	$\Xi_{c}^{\prime +}$	usc	1/2	+1/2
		$\Xi_{c}^{\prime 0}$	dsc	1/2	-1/2
	$s=2$	Ω_{c}^{0}	ssc	0	0
	$s=0$	Λ_{c}^{+}	udc	0	0
	$s=1$	Ξ_{c}^{+}	usc	1/2	+1/2
		Ξ_{c}^{0}	dsc	1/2	-1/2
$c=0$	$s=0$	p	uud	1/2	+1/2
		n	udd	1/2	-1/2
	$s=1$	Λ	uds	0	0
		Σ^{+}	uus	1	+1
		Σ^{0}	uds	1	0
		Σ^{-}	dds	1	-1
	$s=2$	Ξ^{0}	uss	1/2	+1/2
		Ξ^{-}	dss	1/2	-1/2

Spin 1/2



Spin 3/2



Charm	Strange	Baryon	Quark content	I	I_z
$c=3$	$s=0$	Ω_{ccc}^{++}	ccc	0	0
$c=2$	$s=0$	$\Xi_{cc}^{\star ++}$	ucc	1/2	+1/2
		$\Xi_{cc}^{\star +}$	dcc	1/2	-1/2
	$s=1$	$\Omega_{cc}^{\star +}$	scc	0	0
$c=1$	$s=0$	$\Sigma_{c}^{\star ++}$	uuc	1	+1
		$\Sigma_{c}^{\star +}$	udc	1	0
		$\Sigma_{c}^{\star 0}$	ddc	1	-1
	$s=1$	$\Xi_{c}^{\star +}$	usc	1/2	+1/2
		$\Xi_{c}^{\star 0}$	dsc	1/2	-1/2
	$s=2$	$\Omega_{c}^{\star 0}$	ssc	0	0
$c=0$	$s=0$	Δ^{++}	uuu	3/2	+3/2
		Δ^{+}	uud	3/2	+1/2
		Δ^{0}	udd	3/2	-1/2
		Δ^{-}	ddd	3/2	-3/2
	$s=1$	$\Sigma^{\star +}$	uus	1	+1
		$\Sigma^{\star 0}$	uds	1	0
		$\Sigma^{\star -}$	dds	1	-1
	$s=2$	$\Xi^{\star 0}$	uss	1/2	+1/2
		$\Xi^{\star -}$	dss	1/2	-1/2
	$s=3$	Ω^{-}	sss	0	0

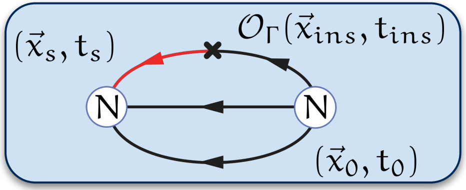
From Three-point Functions to Nucleon Matrix Elements

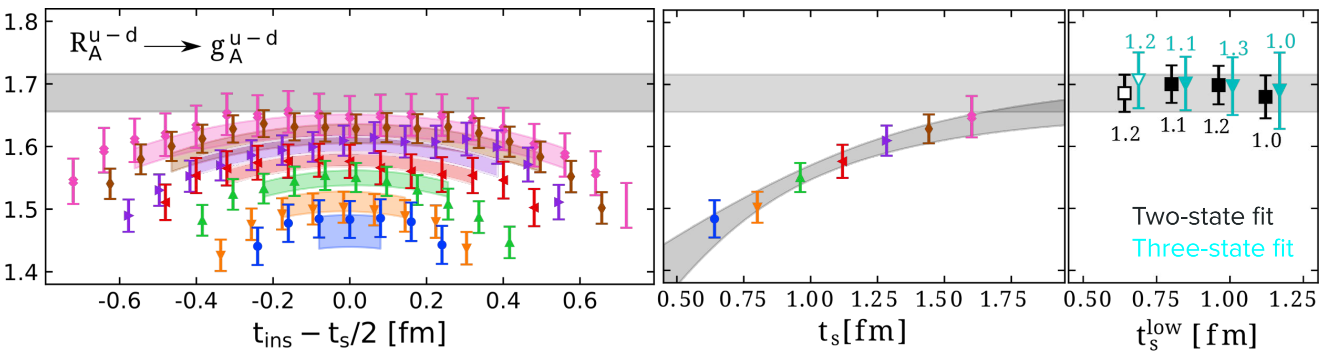
$$C_{3pt}(t, t_{ins}) \equiv \langle \psi_N(t) O(t_{ins}) \psi_N(0) \rangle \approx \langle \psi_N | N \rangle \langle N | O | N \rangle \langle N | \psi_N \rangle e^{-E_N t} + \ldots$$

$$C_{3pt}(t, t_{\rm ins}) \simeq A_{00} e^{-m_N t} + A_{01}\left(e^{-E_1 t_{\rm ins}} + e^{-E_1 t + (E_1 - m_N) t_{\rm ins}}\right) + A_{11} e^{-E_1 t}$$

$$C_{2pt}(t) \simeq c_0 e^{-m_N t_s} + c_1 e^{-E_1 t_s}$$

Desired matrix element: $\mathcal{M} = \frac{A_{00}}{c_0}$





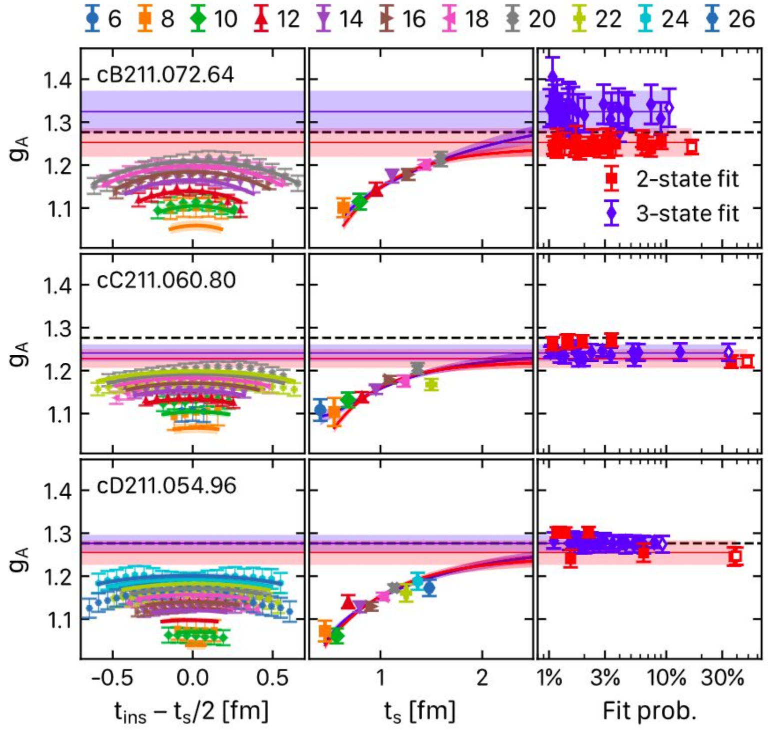
cB211.072.64		
750 configurations		
t_s/a	t_s[fm]	n_{src}
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128
Nucleon 2pt		477

~30M inversions!

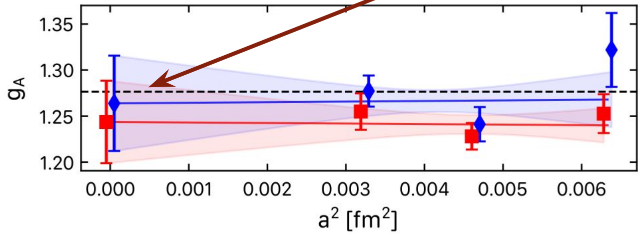
Determination of Nucleon Matrix Elements

$$\langle \mathcal{O} \rangle = \lim_{a \to 0} \langle \mathcal{O} \rangle_{a,V,\vec{\mu}_f}$$

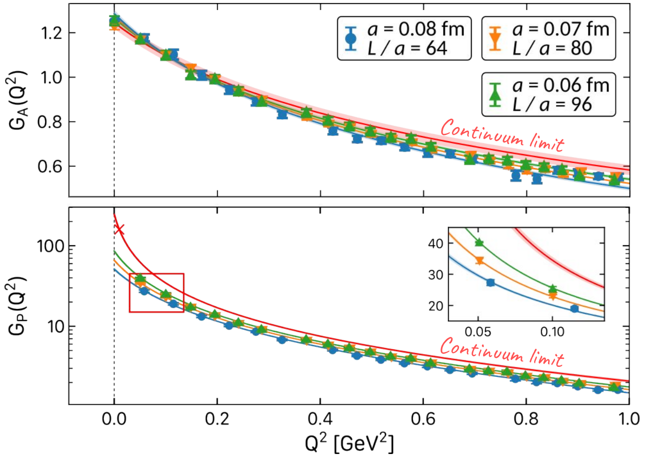
Analysis repeated on multiple ensembles



Continuum limit



And at finite momentum transfer



Determination of Nucleon Matrix Elements

Analysis repeated on multiple ensembles

Extrapolation to the continuum limit

FLAG: Biyearly review of lattice QCD results for well-established observables (QCD params., mesonic & baryonic)

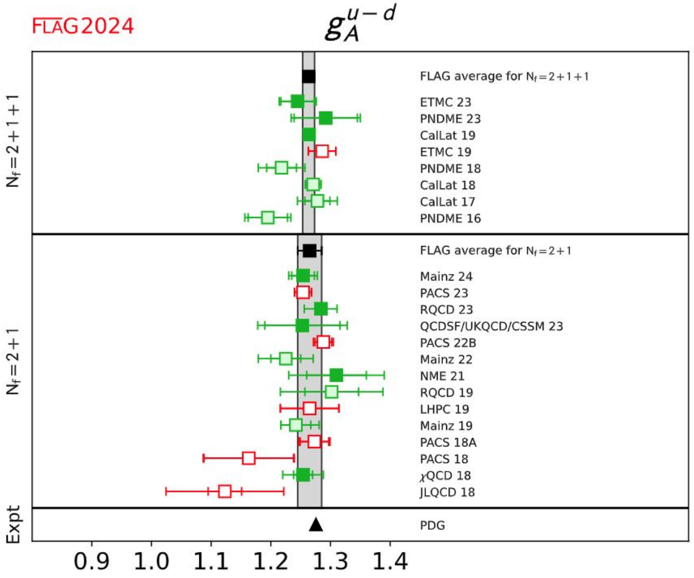
"The PDG of Lattice QCD"

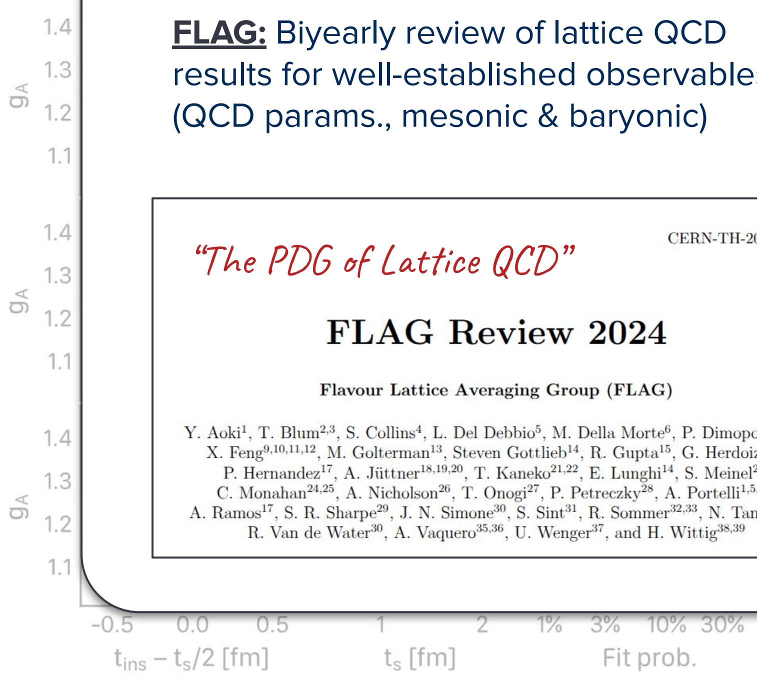
CERN-TH-2024-192

FLAG Review 2024

Flavour Lattice Averaging Group (FLAG)

Y. Aoki[1], T. Blum[2,3], S. Collins[4], L. Del Debbio[5], M. Della Morte[6], P. Dimopoulos[7,8], X. Feng[9,10,11,12], M. Golterman[13], Steven Gottlieb[14], R. Gupta[15], G. Herdoiza[16], P. Hernandez[17], A. Jüttner[18,19,20], T. Kaneko[21,22], E. Lunghi[14], S. Meinel[23], C. Monahan[24,25], A. Nicholson[26], T. Onogi[27], P. Petreczky[28], A. Portelli[1,5,20], A. Ramos[17], S. R. Sharpe[29], J. N. Simone[30], S. Sint[31], R. Sommer[32,33], N. Tantalo[34], R. Van de Water[30], A. Vaquero[35,36], U. Wenger[37], and H. Wittig[38,39]

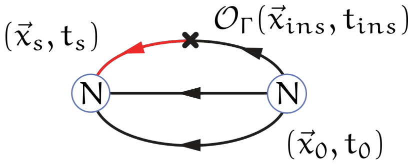




[C. Alexandrou, **S. B.**, et al. "Nucleon axial and pseudoscalar form factors using twisted-mass fermion ensembles at the physical point". Phys. Rev., D109(3), 2024]

And moments of PDFs and GPDs

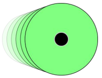
By changing the insertion we can study moments of PDFs and GPDs



Unpolarized = Vector struct.

$$\mathcal{O}_V^{\mu\mu_1\mu_2\dots\mu_n} = \bar{\psi}\gamma^{\{\mu} iD^{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}}\psi$$

$$\langle 1 \rangle_{u-d} = g_V\,, \quad \langle x \rangle_{u-d}, \quad \dots$$



Helicity = Axial struct.

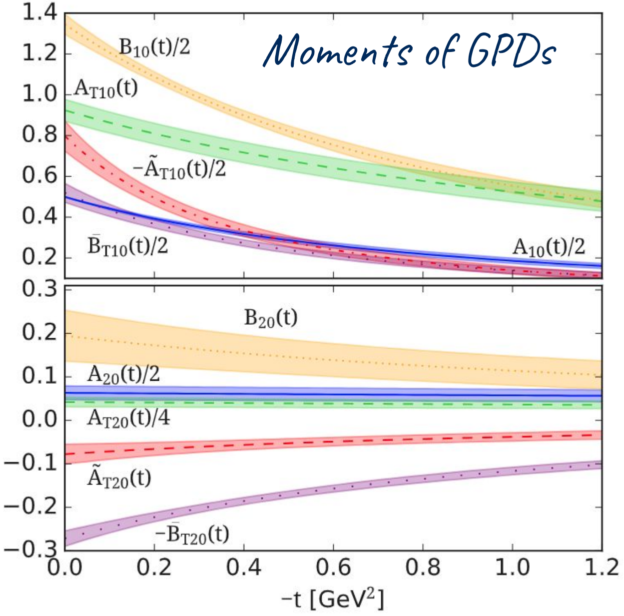
$$\mathcal{O}_A^{\mu\mu_1\mu_2\dots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu} iD^{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}}\psi$$

$$\langle 1 \rangle_{\Delta u-\Delta d} = g_A, \quad \langle x \rangle_{\Delta u-\Delta d}, \quad \dots$$

Transverse = Tensor struct.

$$\mathcal{O}_T^{\nu\mu\mu_1\mu_2\dots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu} iD^{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}}\psi$$

$$\langle 1 \rangle_{\delta u-\delta d} = g_T, \quad \langle x \rangle_{\delta u-\delta d}, \quad \dots$$



[C. Alexandrou, **S. B.**, et al. "Moments of the nucleon transverse quark spin densities using lattice QCD". Phys.Rev.D 107(5), 2023]

And moments of PDFs and GPDs

In summary: Ground-state matrix elements up to fourth Mellin moment for any Dirac structure are computable with standard three-point functions; and it is a very well-established approach.

Recent results in Nucleon Structure (arXiv #):

- **Charges and σ-terms:** 2412.01535 (ETMC), 2305.04717 2412.13138 (RQCD), 2303.08741 (Mainz), 2105.12095 2305.11330 (PNDME), 1805.12130 (CalLat)
- **Electromagnetic Form Factors:** 2309.06590 (Mainz), 2507.20910 (ETMC), 2311.10345 (PACS), 1906.07217 (MILC)
- **Axial Form Factors:** 2305.11330 (PNDME), 2106.13468 2309.05774 (ETMC), 1810.05569 (RQCD), 2207.03440 2503.18848 (Mainz)
- **Second Mellin Moments:** 1908.10706 2003.08486 2202.09871 (ETMC), 2011.12787 (NME), 2005.13779 (PNDME), 2401.05360 (DE+US on BMW)
- **Gravitational Form Factors:** 2107.10368 2310.08484 (MIT), 2401.05496 (χQCD)

[C. Alexandrou, **S. B.**, et al. "Moments of the nucleon transverse quark spin densities using lattice QCD". Phys.Rev.D 107(5), 2023]

What about other baryons?

Heavy baryons are easier, but there is also less motivation: they are harder to access experimentally.

Recent results for heavy baryons (arXiv #):

- **Spectroscopy:** 2411.18402 (CLQCD), 2309.04401 (ETMC), 2211.03744 (RQCD), 0906.3599 (BMW)

- **Transition FF:** 2507.09970 ($\Lambda \to N$), 2504.07302 ($\Xi_C \to \Xi$), 2107.13140 ($\Lambda_C \to \Lambda^*$), 2103.07064 ($\Xi_C \to \Xi$), 2107.13909 ($\Lambda_C \to \Lambda$), 2103.08775 ($\Lambda_B \to \Lambda^*_C$), 2009.09313 ($\Lambda_B \to \Lambda^*$), 1712.05783 ($\Lambda_C \to N$), 1708.04008 ($\Xi \to \Sigma$, $\Sigma \to N$), 1702.02243 ($\Lambda_B \to \Lambda_C$), 1611.09696 ($\Lambda_C \to \Lambda$), 1602.01399 ($\Lambda_B \to \Lambda$), 1503.01421 ($\Lambda_B \to \Lambda_C$), 1306.0446 ($\Lambda_B \to N$).

- **Structure:** 2305.04717 (octet isovector charges), 1901.00018 ($g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$), 1606.01650 (octet & decuplet charges), 1609.0398 (octet magnetic moment), 1310.5915 (charmed baryon EM)

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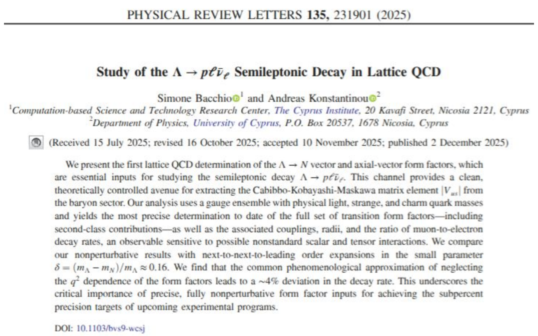
  2103.08775 ($\Lambda_B \to \Lambda^*_C$), 2009.09313 ($\Lambda_B \to \Lambda^*$), 1712.... Λ_C),
  1611.09696 ($\Lambda_C \to \Lambda$), 1602.01399 ($\Lambda_B \to \Lambda$), 1503.01...



PHYSICAL REVIEW LETTERS **135,** 231901 (2025)

Study of the $\Lambda \to p\ell\bar{\nu}_\ell$ Semileptonic Decay in Lattice QCD

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[1]*Computation-based Science and Technology Research Center, The Cyprus Institute, 20 Kavafi Street, Nicosia 2121, Cyprus*
[2]*Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus*

(Received 15 July 2025; revised 16 October 2025; accepted 10 November 2025; published 2 December 2025)

We present the first lattice QCD determination of the $\Lambda \to N$ vector and axial-vector form factors, which are essential inputs for studying the semileptonic decay $\Lambda \to p\ell\bar{\nu}_\ell$. This channel provides a clean, theoretically controlled avenue for extracting the Cabibbo-Kobayashi-Maskawa matrix element $|V_{us}|$ from the baryon sector. Our analysis uses a gauge ensemble with physical light, strange, and charm quark masses and yields the most precise determination to date of the full set of transition form factors—including second-class contributions—as well as the associated couplings, radii, and the ratio of muon-to-electron decay rates, an observable sensitive to possible nonstandard scalar and tensor interactions. We compare our nonperturbative results with next-to-next-to-leading order expansions in the small parameter $\delta = (m_\Lambda - m_N)/m_\Lambda \approx 0.16$. We find that the common phenomenological approximation of neglecting the q^2 dependence of the form factors leads to a ~4% deviation in the decay rate. This underscores the critical importance of precise, fully nonperturbative form factor inputs for achieving the subpercent precision targets of upcoming experimental programs.

DOI: 10.1103/bvs9-wcsj

- **Structure:** 2305.04717 (octet isovector charges), 1901.00018 ($g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$), 1606.01650 (octet & decuplet charges), 1609.0398 (octet magnetic moment), 1310.5915 (charmed baryon EM)

Test of CKM matrix unitarity

The CKM matrix is expected to be unitary in the SM. Currently the most stringent test is the first-row unitary relation:

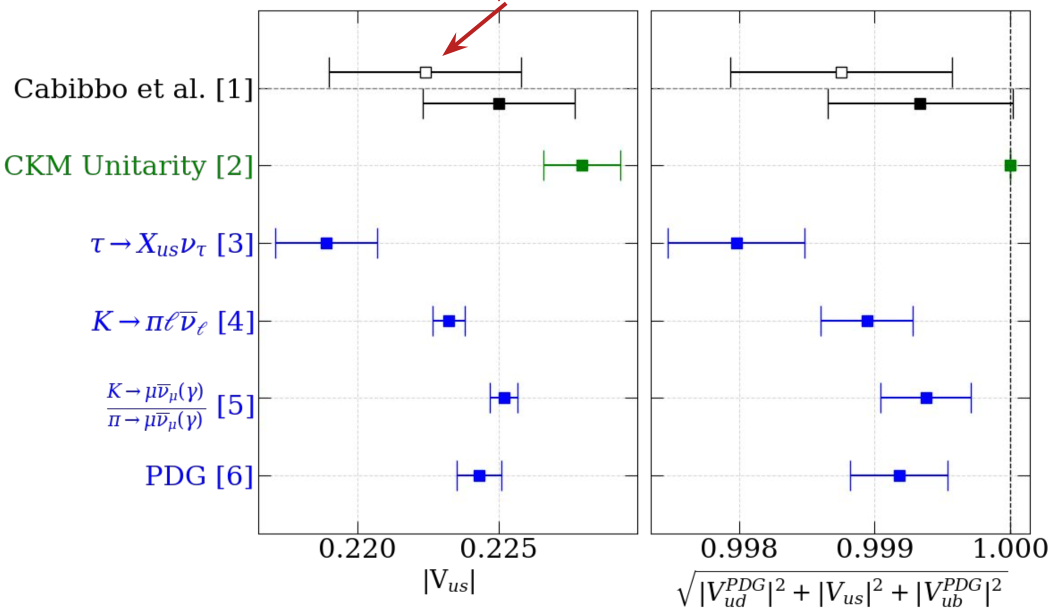
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$|V_{ud}^{\mathrm{PDG}}| = 0.97367(32)$$

$$|V_{ub}^{\mathrm{PDG}}| = 3.82(20) \cdot 10^{-3}$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

We aim to improve this determination!



CKM matrix elements from decay rates

CKM matrix elements cannot be determined from first principles and require combining theoretical (lattice QCD) results to experimental inputs:

$$\Gamma = \frac{1}{2m_\Lambda}\int \frac{d^3\vec{p}_p}{(2\pi)^3 2E_p}\frac{d^3\vec{p}_\ell}{(2\pi)^3 2E_\ell}\frac{d^3\vec{p}_\nu}{(2\pi)^3 2E_\nu} \times (2\pi)^4\delta^{(4)}(p_\Lambda - p_p - p_\ell - p_\nu)\,|\mathcal{M}|^2.$$

E.g. for lambda semileptonic decay

$$\mathcal{M}(\Lambda \to p e^- \bar{\nu}_e) = \frac{G_F}{\sqrt{2}} V_{us} \langle \Lambda | W_\mu | p \rangle L^\mu$$

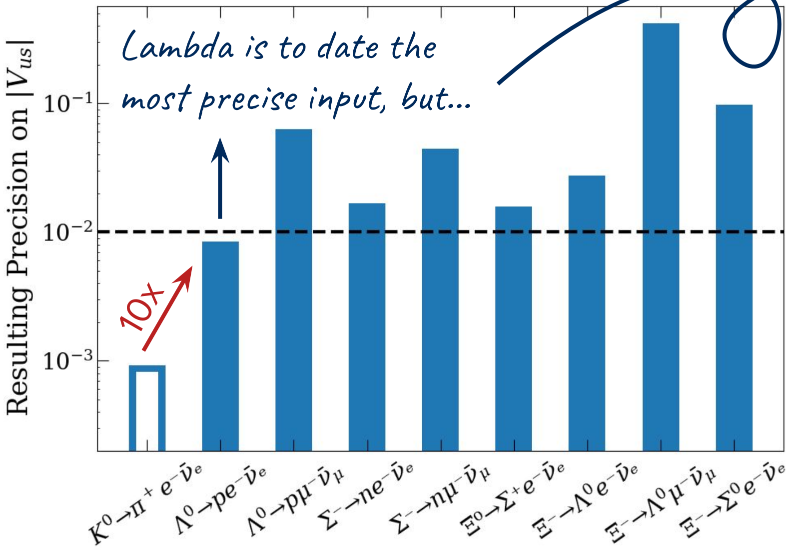
After integrating over the angular degrees of freedom, we obtain the expression

$$\Gamma = \frac{{G_F}^2 |V_{us}|^2}{192\pi^3 {m_\Lambda}^3} \times \int_{{m_\ell}^2}^{{q_{\max}}^2} dq^2 \sqrt{\lambda({m_\Lambda}^2, {m_p}^2, q^2)}\, L^{\mu\nu} H_{\mu\nu}$$

- Precise **experimental** inputs
- Necessary **lattice QCD** results

Baryons vs mesons inputs

Meson *(K→π)* provide the most precise inputs, but are also the source of discrepancy.
Baryons provide valuable alternative approaches with fully independent systematics.



Results included in the PDG average

VALUE (10^{-3})	EVTS	DOCUMENT ID	
1.301 ± 0.019	OUR AVERAGE		
1.335 ±0.056	7111	BOURQUIN	1983
1.313 ±0.024	10k	WISE	1980
1.23 ±0.11	544	LINDQUIST	1977
1.27 ±0.07	1089	KATZ	1973
1.31 ±0.06	1078	ALTHOFF	1971
1.17 ±0.13	86	[1] CANTER	1971
1.20 ±0.12	143	[2] MALONEY	1969
1.17 ±0.18	120	[2] BAGLIN	1964
1.23 ±0.20	150	[2] ELY	1963

Until PDG25, all results dated before 1983.

Now there are new results by BESIII, LHCb, and more coming!

Lattice QCD results on Lambda semileptonic decay

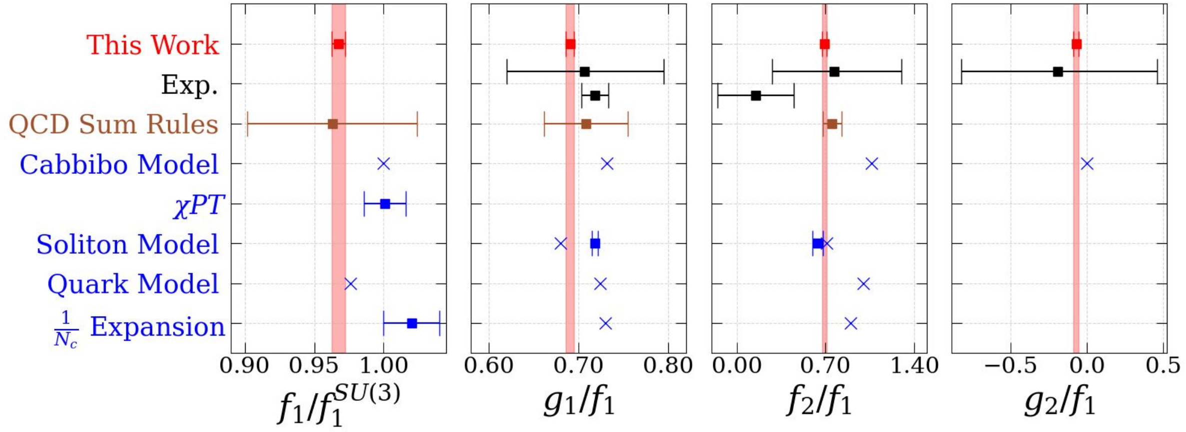
Ours was the first LQCD calculation of $\Lambda \to p$ matrix elements and form factors (FFs)

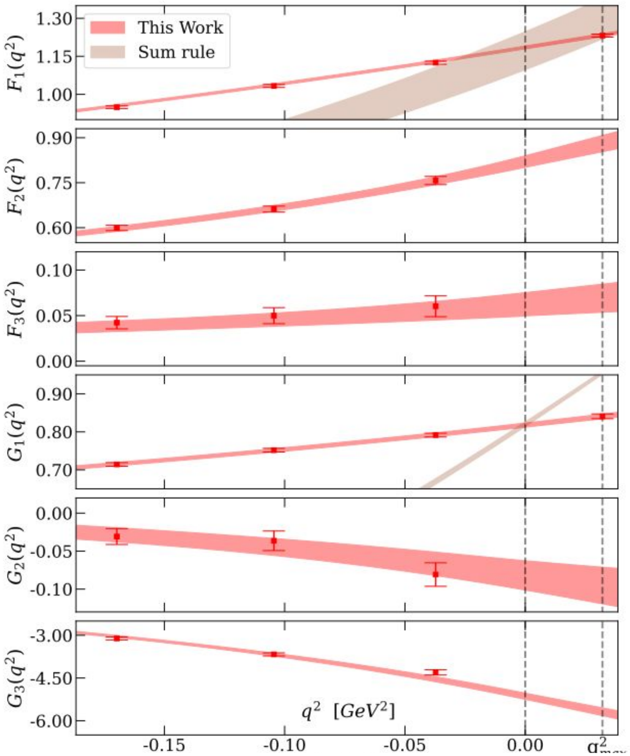
2nd class, m_ℓ suppressed

$$M_\mu^{\mathrm{V}} = \bar{u}_p \left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\nu} q^\nu \frac{F_2(q^2)}{m_\Lambda} + q_\mu \frac{F_3(q^2)}{m_\Lambda} \right] u_\Lambda$$

$$M_\mu^{\mathrm{A}} = \bar{u}_p \left[\gamma_\mu G_1(q^2) - i\sigma_{\mu\nu} q^\nu \frac{G_2(q^2)}{m_\Lambda} + q_\mu \frac{G_3(q^2)}{m_\Lambda} \right] \gamma_5 u_\Lambda$$

2nd class | m_ℓ suppressed

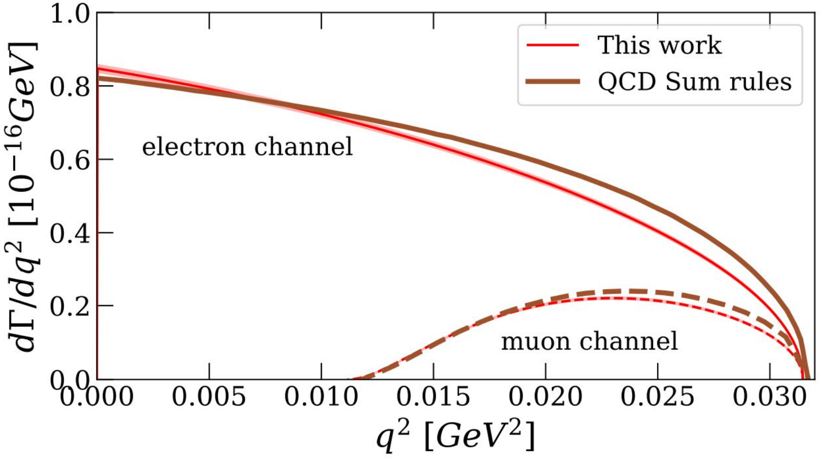


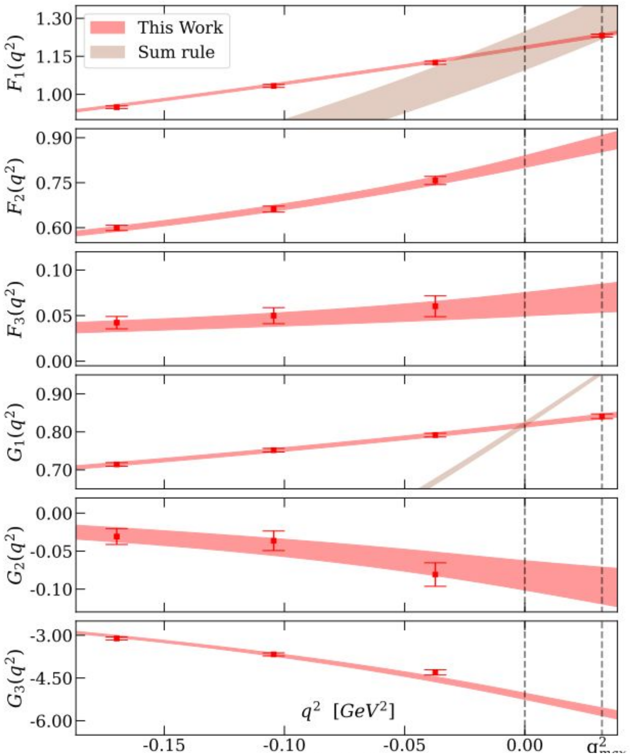


Lattice QCD results on Lambda semileptonic decay

Ours was the first LQCD calculation of $\Lambda \to p$ matrix elements and form factors (FFs)

- G_1 and F_1 dominate the decay rate $\Gamma_{\mathrm{NLO}} = \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^5}{60\pi^3}\left(1 - \frac{3}{2}\delta\right)\left(f_1^2 + 3g_1^2\right)$
- G_2 and F_3 are second-class FFs (baryon-masses-diff. suppressed)
- G_3 and F_3 are lepton-mass suppressed in the decay rate



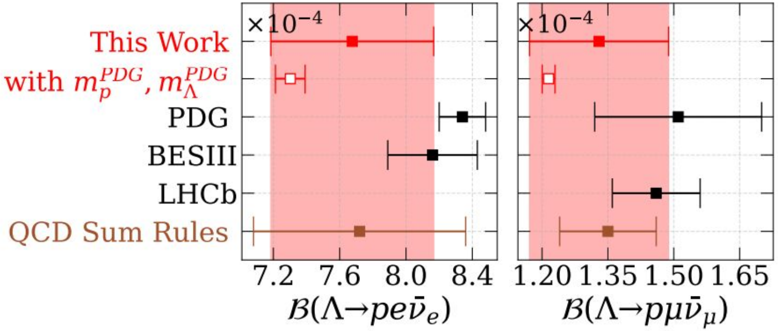


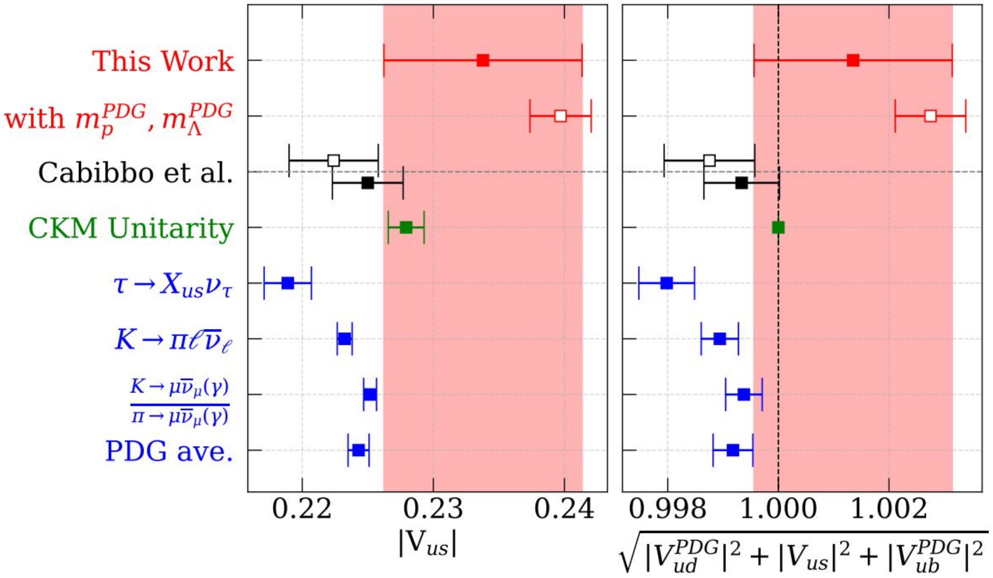
... and combined with experiments to get Vus

We are not ready yet to provide final results, control of systematics is critical...
But we learnt useful things:

- (Sub-)percent level precision is within reach
- Baryon mass errors plays a critical role
- Non-perturbative approach is critical!

$$\Delta_{\text{NLO}} = 5.5(2)\% \quad \text{and} \quad \Delta_{\text{NNLO}} = -1.14(8)\%$$





Ratio of muon-to-electron mode

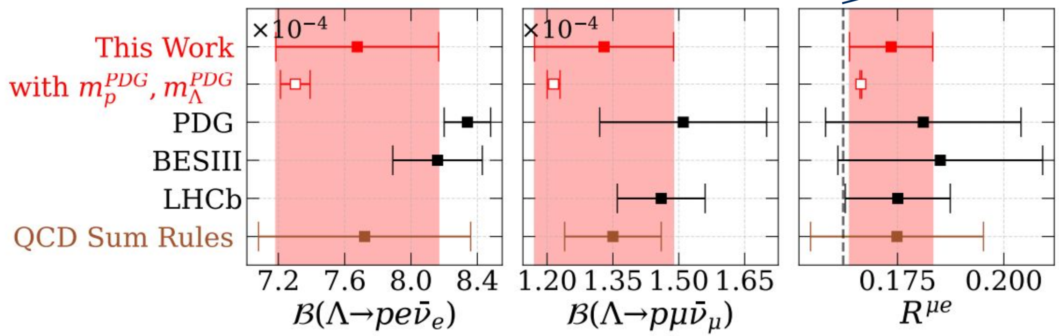
A very interesting quantity is the ratio of muon-to-electron mode $R^{\mu e} \equiv \dfrac{\Gamma(\Lambda \to p\,\mu\,\bar{\nu}_\mu)}{\Gamma(\Lambda \to p\,e\,\bar{\nu}_e)}$

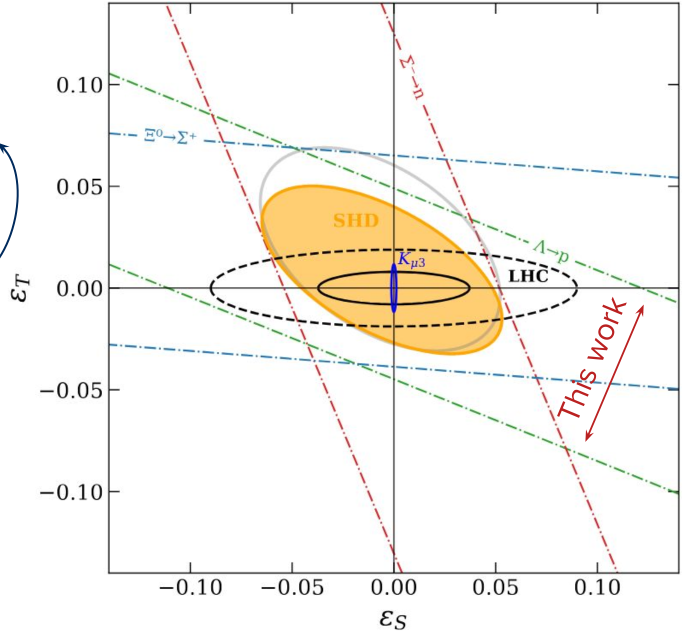
1. It is a pure first-principle prediction independent of external inputs
2. At NLO, it does not depend on hadronic matrix elements

$$R^{\mu e}_{\rm NLO} = \sqrt{1-\frac{m_\mu^2}{\Delta^2}}\left(1-\frac{9}{2}\frac{m_\mu^2}{\Delta^2}-4\frac{m_\mu^4}{\Delta^4}\right)+\frac{15}{2}\frac{m_\mu^4}{\Delta^4}\,\mathrm{artanh}\left(\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)$$

3. In presence of non-standard interaction it is sensitive to scalar and tensor interactions only at LO in ε

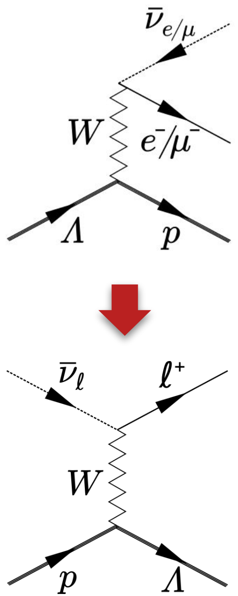
$$\frac{R^{\mu e}_{\rm NP}}{R^{\mu e}_{\rm SM}} \simeq 1 + r_S\,\epsilon_S + r_T\,\epsilon_T$$

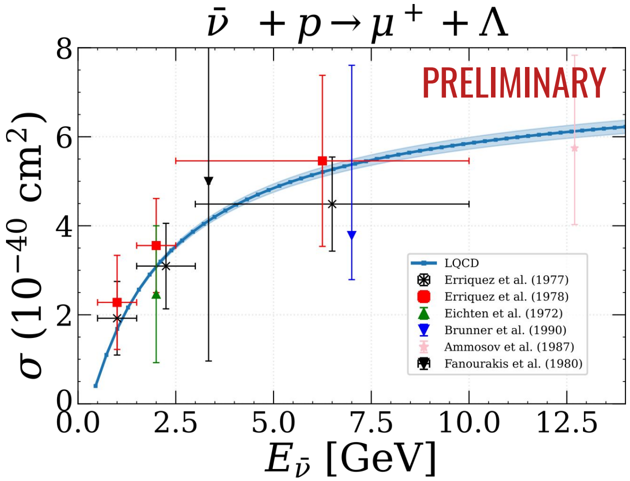


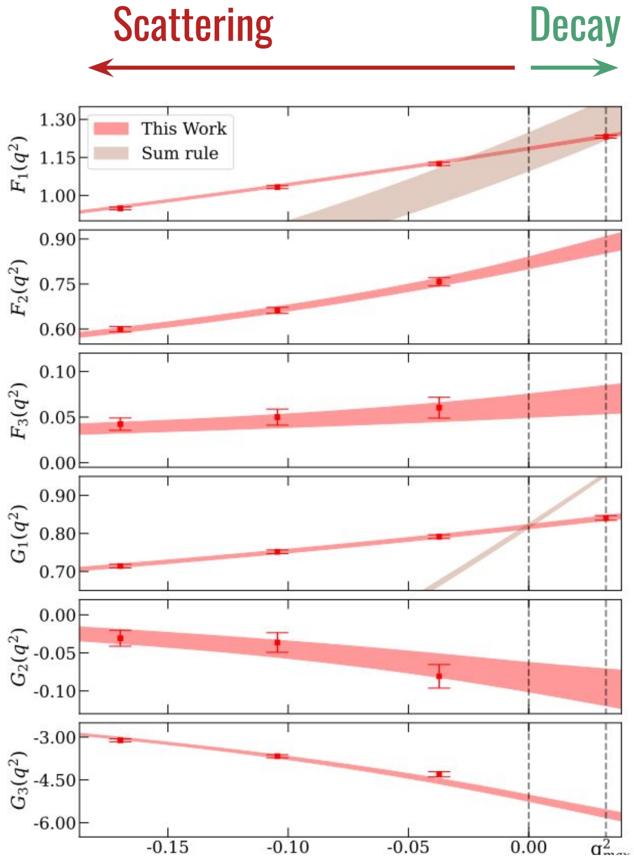


Lambda production in neutrino-nucleon scattering

The same FF can be used for a first-principle prediction of Lambda production in neutrino-nucleon scattering!

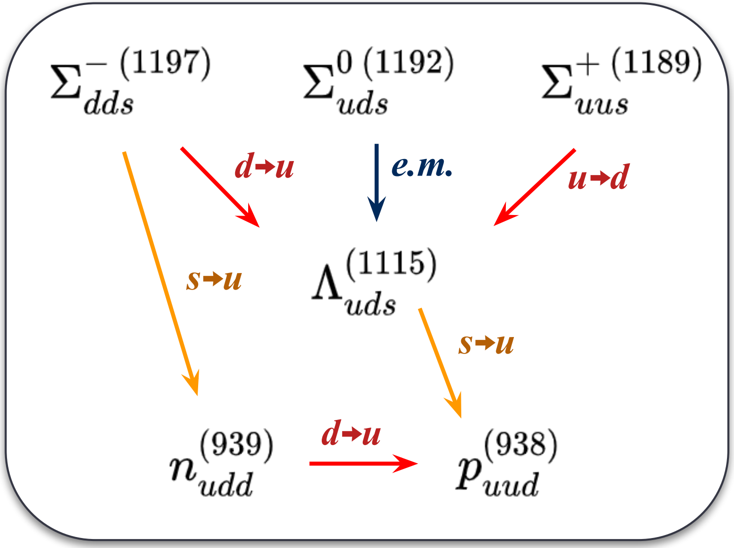


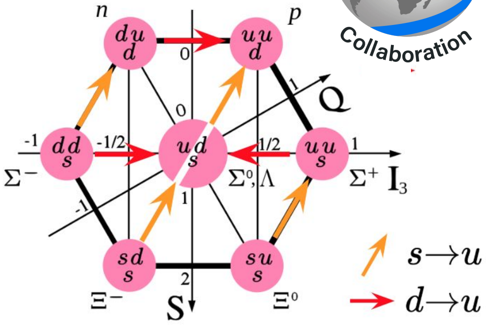




Beyond the Lambda hyperon: the Sigma!

Still looking at the baryon octet, of main interest is the Sigma!





Same quark content but Lambda is odd under exchange of $u \leftrightarrow d$, while Sigmas are even under the exchange of $u \leftrightarrow d$.

Λ	$\frac{1}{\sqrt{6}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_5 d_b\right)s_c + \left(u_a^T C\gamma_5 s_b\right)d_c - \left(d_a^T C\gamma_5 s_b\right)u_c\right]$
Σ^+	$\epsilon_{abc}\left(u_a^T C\gamma_5 s_b\right)u_c$
Σ^0	$\frac{1}{\sqrt{2}}\epsilon_{abc}\left[\left(u_a^T C\gamma_5 s_b\right)d_c + \left(d_a^T C\gamma_5 s_b\right)u_c\right]$
Σ^-	$\epsilon_{abc}\left(d_a^T C\gamma_5 s_b\right)d_c$

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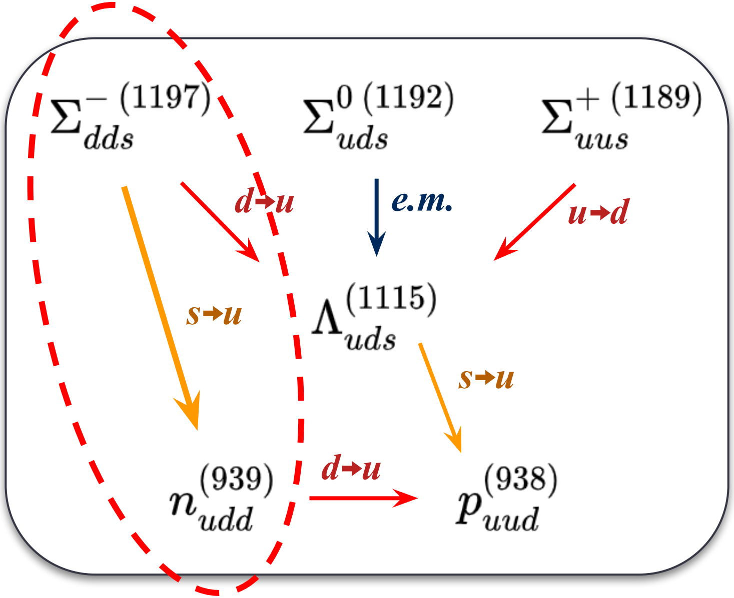

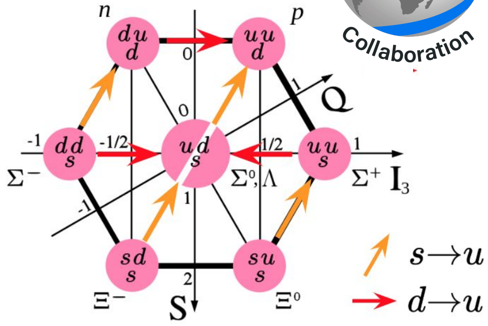

- $\Sigma \to n$: is the second most precise s→u decay

Beyond the Lambda hyperon: the Sigma!

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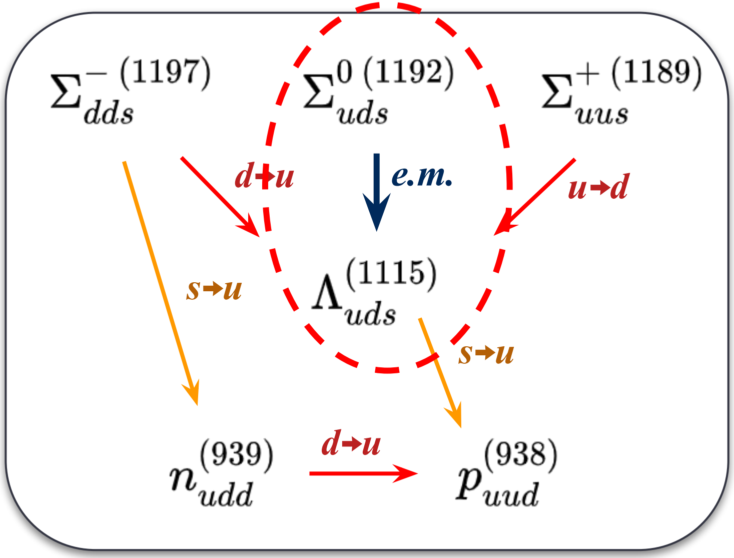
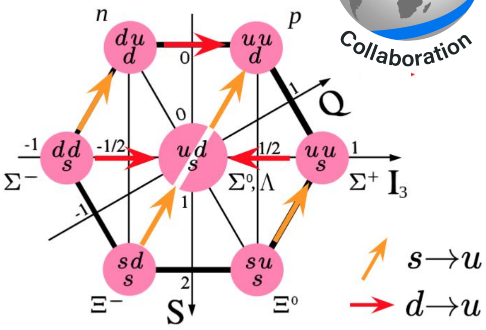
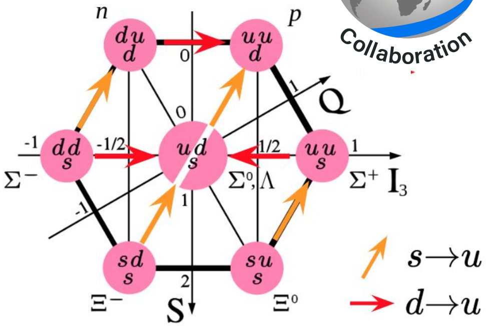


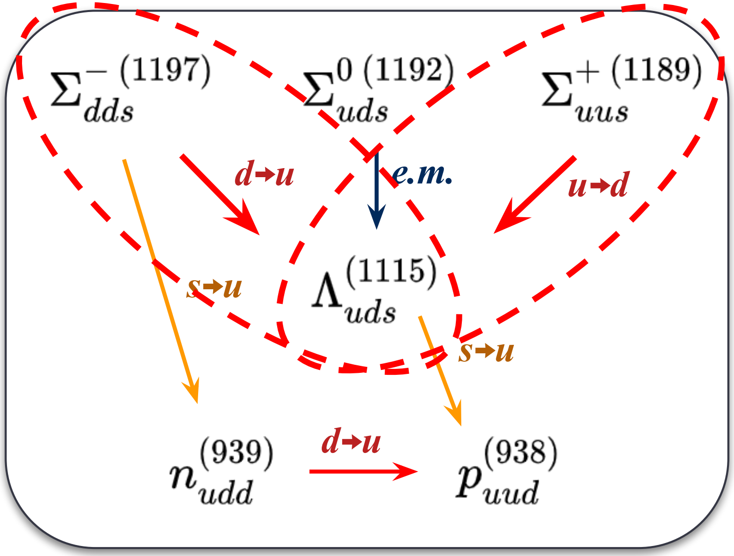


- $\Sigma \rightarrow n$**:** is the second most precise s→u decay
- $\Sigma^0 \rightarrow \Lambda$**:** is a Dalitz (EM) decay. Experimentally, allows to access vector couplings and form factors!

Beyond the Lambda hyperon: the Sigma!

Still looking at the baryon octet, of main interest is the Sigma!





- $\Sigma \rightarrow n$**:** is the second most precise s→u decay
- $\Sigma^0 \rightarrow \Lambda$**:** is a Dalitz (EM) decay. Experimentally, allows to access vector couplings and form factors!
- $\Sigma^\pm \rightarrow \Lambda$**:** are charge-conjugated semileptonic decays, very sensitive to isospin breaking corrections!

The $\Sigma^{\pm} \to \Lambda$ decay rates

What are the differences between the two decay rates in $\Sigma^{\pm} \longrightarrow \Lambda$?

$$\Gamma_{\Sigma^{\pm}\to\Lambda e^{\pm}\nu} \simeq \frac{G_F^2|V_{ud}|^2 m_{\Sigma^{\pm}}^5 \delta_{\pm}^5}{60\pi^3}\left(1+\Delta_{\mathrm{RC}}\right)\left(1-\frac{3}{2}\delta_{\pm}\right)\left(f_{1,\pm}^2+3g_{1,\pm}^2\right)\left(1+\mathcal{O}(\delta_{\pm}^2)\right)$$

$$\delta_{\pm} = (m_{\Sigma^{\pm}} - m_{\Lambda})/m_{\Sigma^{\pm}}$$

$$\mathcal{O}(\delta_{\pm}^2) \approx 0.4\%$$

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$$\delta_\pm = (m_{\Sigma^\pm} - m_\Lambda)/m_{\Sigma^\pm}$$

$$\mathcal{O}(\delta_\pm^2) \approx 0.4\%$$

And many cancellations happen once we take the ratio:

$$\frac{\Gamma_{\Sigma^- \to \Lambda e^- \bar{\nu}_e}}{\Gamma_{\Sigma^+ \to \Lambda e^+ \nu_e}} = \left(\frac{m_{\Sigma^-} - m_\Lambda}{m_{\Sigma^+} - m_\Lambda}\right)^5 \left(\frac{2 - 3\delta_-}{2 - 3\delta_+}\right)\left(\frac{f_{1,-}^2 + 3g_{1,-}^2}{f_{1,+}^2 + 3g_{1,+}^2}\right)\Big(1 + \mathcal{O}(\delta_-^2 - \delta_+^2,\, \delta_\pm^2 \alpha)\Big)$$

Higher order corrections cancel up to isospin-breaking effects because hadronic matrix elements are the same in isoQCD

$$\mathcal{O}(\delta_-^2 - \delta_+^2) \approx 0.08\%$$
$$\mathcal{O}(\delta_\pm^2 \alpha) \approx 0.05\%$$

The $\Sigma^\pm \to \Lambda$ decay rates

$$\frac{\Gamma_{\Sigma^- \to \Lambda e^- \bar{\nu}_e}}{\Gamma_{\Sigma^+ \to \Lambda e^+ \nu_e}} = \left(\frac{m_{\Sigma^-} - m_\Lambda}{m_{\Sigma^+} - m_\Lambda}\right)^5 \left(\frac{2 - 3\delta_-}{2 - 3\delta_+}\right) \left(\frac{f_{1,-}^2 + 3g_{1,-}^2}{f_{1,+}^2 + 3g_{1,+}^2}\right) \left(1 + \mathcal{O}(\delta_-^2 - \delta_+^2,\, \delta_\pm^2 \alpha)\right)$$

Using the Ademollo-Gatto theorem (based on CVC) we can expand also the vector coupling:

zero for this process!

$$\frac{f_{1,-}^2 + 3g_{1,-}^2}{f_{1,+}^2 + 3g_{1,+}^2} = \frac{f_1^2|_{\mathrm{SU}(3)} + 3g_{1,-}^2 + \mathcal{O}(2\delta_-^2)}{f_1^2|_{\mathrm{SU}(3)} + 3g_{1,+}^2 + \mathcal{O}(2\delta_+^2)} = \frac{g_{1,-}^2}{g_{1,+}^2} \left(1 + \mathcal{O}(2(\delta_-^2 - \delta_+^2),\, 2\delta_\pm^2 \alpha^2)\right)$$

The $\Sigma^{\pm} \to \Lambda$ decay rates

$$\frac{\Gamma_{\Sigma^- \to \Lambda e^- \bar{\nu}_e}}{\Gamma_{\Sigma^+ \to \Lambda e^+ \nu_e}} = \left(\frac{m_{\Sigma^-} - m_\Lambda}{m_{\Sigma^+} - m_\Lambda}\right)^5 \left(\frac{2 - 3\delta_-}{2 - 3\delta_+}\right) \left(\frac{f_{1,-}^2 + 3g_{1,-}^2}{f_{1,+}^2 + 3g_{1,+}^2}\right) \left(1 + \mathcal{O}(\delta_-^2 - \delta_+^2,\, \delta_\pm^2 \alpha)\right)$$

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In conclusions, we obtain:

NNLO effects estimated to contribute at 0.2%

$$\left.\frac{\Gamma_{\Sigma^- \to \Lambda e^- \bar{\nu}_e}}{\Gamma_{\Sigma^+ \to \Lambda e^+ \nu_e}}\right|_{\mathrm{SM}} = (1 - \Delta_g)^2 \times 1.6647(72)_{\mathrm{mPDG}}(40)_{\mathrm{NNLO}} \quad \text{with} \quad \Delta_g \equiv \frac{g_{1,+} - g_{1,-}}{g_{1,+}}$$

Leading corrections are percent-level isospin-breaking effects in g_1

The $\Sigma^{\pm} \to \Lambda$ decay rates

$$\left.\frac{\Gamma_{\Sigma^-\to\Lambda e^-\bar{\nu}_e}}{\Gamma_{\Sigma^+\to\Lambda e^+\nu_e}}\right|_{\rm SM} = (1-\Delta_g)^2 \times 1.6647(72)_{\rm m_{PDG}}(40)_{\rm NNLO} \quad \text{with} \quad \Delta_g \equiv \frac{g_{1,+} - g_{1,-}}{g_{1,+}}$$

$$\frac{\Gamma^{\rm PDG\ ave.}_{\Sigma^-\to\Lambda e^-\bar{\nu}_e}}{\Gamma^{\rm PDG\ ave.}_{\Sigma^+\to\Lambda e^+\nu_e}} = 1.35(24)$$

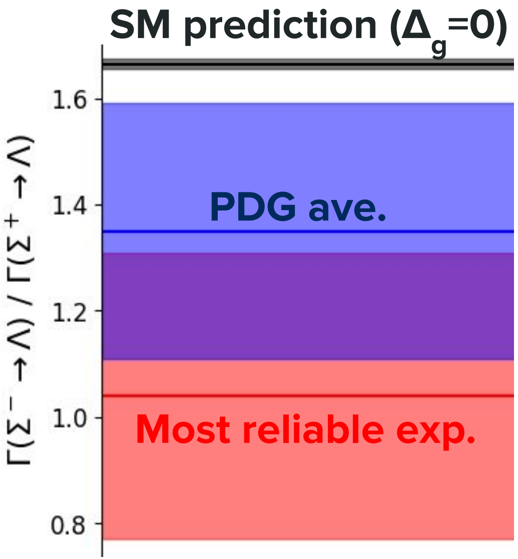
$$\frac{\Gamma^{\rm CERN,\ 1982}_{\Sigma^-\to\Lambda e^-\bar{\nu}_e}}{\Gamma^{\rm BESIII,\ 2023}_{\Sigma^+\to\Lambda e^+\nu_e}} = 1.04(27)$$

$\Gamma_{\Sigma^-\to\Lambda e^-\bar{\nu}_e}$

VALUE (10^{-4})	*EVTS*	*DOCUMENT ID*	
0.561 ± 0.031	1620	[1] BOURQUIN	1982
0.63 ± 0.11	114	THOMPSON	1980
0.52 ± 0.09	31	BALTAY	1969
0.69 ± 0.12	31	EISELE	1969
0.64 ± 0.12	35	BARASH	1967
0.75 ± 0.28	11	COURANT	1964

$\Gamma_{\Sigma^+\to\Lambda e^+\nu_e}$

VALUE (10^{-5})	*EVTS*	*DOCUMENT ID*	
$2.93 \pm 0.74 \pm 0.13$	16	ABLIKIM	2023AA
1.6 ± 0.7	5	BALTAY	1969
2.9 ± 1.0	10	EISELE	1969
2.0 ± 0.8	6	BARASH	1967



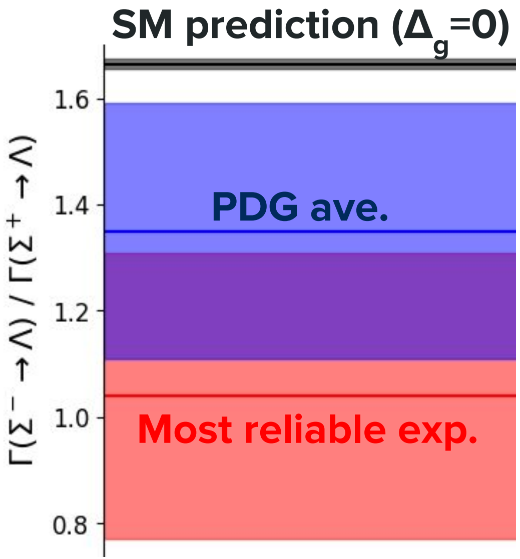
The $\Sigma^{\pm} \to \Lambda$ decay rates

$$\left.\frac{\Gamma_{\Sigma^-\to\Lambda e^-\bar\nu_e}}{\Gamma_{\Sigma^+\to\Lambda e^+\nu_e}}\right|_{\rm SM} = (1-\Delta_g)^2\times 1.6647(72)_{\rm mPDG}(40)_{\rm NNLO} \quad \text{with} \quad \Delta_g \equiv \frac{g_{1,+}-g_{1,-}}{g_{1,+}}$$

$$\frac{\Gamma^{\rm PDG\,ave.}_{\Sigma^-\to\Lambda e^-\bar\nu_e}}{\Gamma^{\rm PDG\,ave.}_{\Sigma^+\to\Lambda e^+\nu_e}} = 1.35(24) \;\Rightarrow\; \Delta_{g,\rm ave.} = 9.9(8.1)\%$$

$$\frac{\Gamma^{\rm CERN,\,1982}_{\Sigma^-\to\Lambda e^-\bar\nu_e}}{\Gamma^{\rm BESIII,\,2023}_{\Sigma^+\to\Lambda e^+\nu_e}} = 1.04(27) \;\Rightarrow\; \Delta_{g,\rm sel.} = 21(10)\%$$

Predicted huge isospin breaking corrections on g_1!



Possible explanations of the tension:

- **LIBE:** a very few diagrams survive in the g_1 splitting
- **Exp. systematics:** most probable? Needs new results!
- **New physics:** would be interesting to compute which non-standard interactions survive at leading order in the ratio.

Candidate: pion-cloud effects

arXiv:2202.10439

Pion-induced radiative corrections to neutron beta-decay

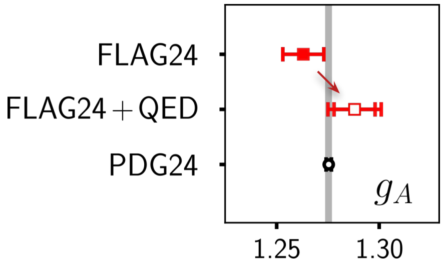
Vincenzo Cirigliano,[1,2,*] Jordy de Vries,[3,4,†] Leendert Hayen,[5,6,‡]
Emanuele Mereghetti,[1,§] and André Walker-Loud[7,¶]

[1]*Los Alamos National Laboratory, Theoretical Division T-2, Los Alamos, NM 87545, USA*
[2]*Institute for Nuclear Theory, University of Washington, Seattle WA 98195-1550*
[3]*Institute for Theoretical Physics Amsterdam and Delta Institute for Theoretical Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands*
[4]*Nikhef, Theory Group, Science Park 105, 1098 XG, Amsterdam, The Netherlands*
[5]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*
[6]*Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708, USA*
[7]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

We compute the electromagnetic corrections to neutron beta decay using a low-energy hadronic effective field theory. We identify and compute new radiative corrections arising from virtual pions that were missed in previous studies. The largest correction is a percent-level shift in the axial charge of the nucleon proportional to the electromagnetic part of the pion-mass splitting. Smaller corrections, comparable to anticipated experimental precision, impact the β-ν angular correlations and the β-asymmetry. We comment on implications of our results for the comparison of the experimentally measured axial charge with first-principle computations using lattice QCD and on the potential of β-decay experiments to constrain beyond-the-Standard-Model interactions.

In the case of the nucleon, pion-cloud effects are expected to contribute between 1.4% to 2.6% on g_A

[arXiv:2503.09891]



In **Σ± → Λ ratio**, pion-cloud effects are expected to be about **4x larger (5.6% to 10.4%)**:

Σ± → Λ: $(m_\Lambda + m_\pi) - m_\Sigma \approx 60\,\text{MeV}$

vs

n → p: $(m_N + m_\pi) - m_N \approx 135\,\text{MeV}$

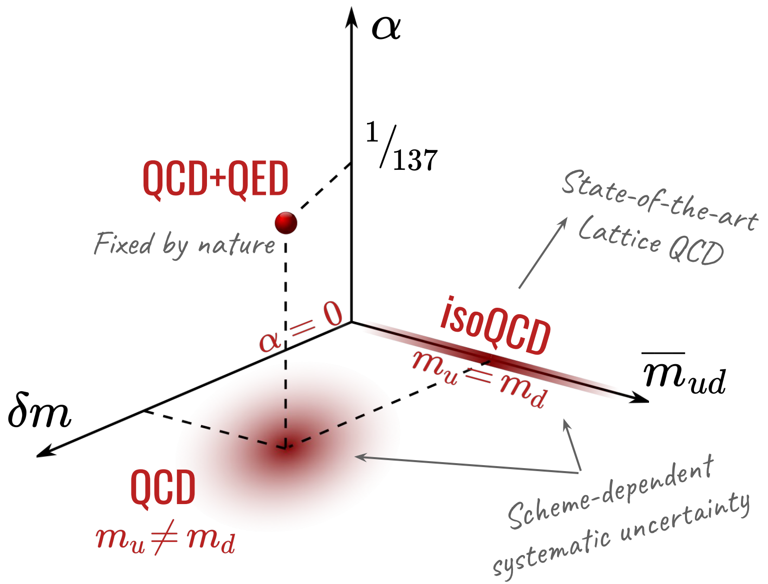
} 2x

$$\delta g^{\pm} = \delta g^{+} - \delta g^{-} = 2\delta_{\text{P.C.}}$$

$$\delta\lambda = \delta g_A - \delta g_V = \delta_{\text{P.C.}}$$

} 2x

Why QCD+QED?

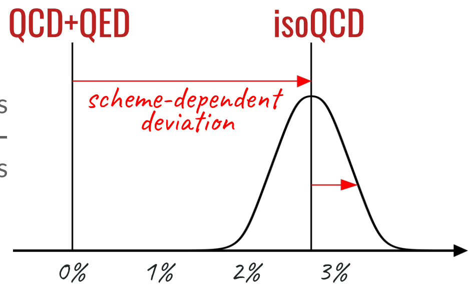


QCD+QED is the first-principles framework with the necessary parameters for a sub-percent matching to nature.

Theory parameters: $\alpha, m_u, m_d, m_s, m_c$ set matching

E.g. external inputs: $M_{\pi^\pm}, M_{K^\pm}, M_{D_s}, \Delta M_K, \Delta M_D$

At percent level, isoQCD results *might* have sizeable scheme-dependent systematic effects



Towards lattice QCD + QED

Lattice QCD has reached a mature stage:

- **Physical point ensembles & high-precision:**

$$\langle \mathcal{O} \rangle_{iso} \equiv \lim_{\substack{V\to\infty \\ a\to 0}} \langle \mathcal{O} \rangle^{\mathrm{isoQCD}}_{a,V}$$

- **Leading order isospin-breaking effects (LIBE):**

$$\langle \mathcal{O} \rangle \equiv \langle \mathcal{O} \rangle_{iso} + 4\pi\alpha \langle \frac{\partial^2 \mathcal{O}}{\partial e^2} \rangle_{iso} + \delta m \langle \frac{\partial \mathcal{O}}{\partial m} \rangle_{iso}$$

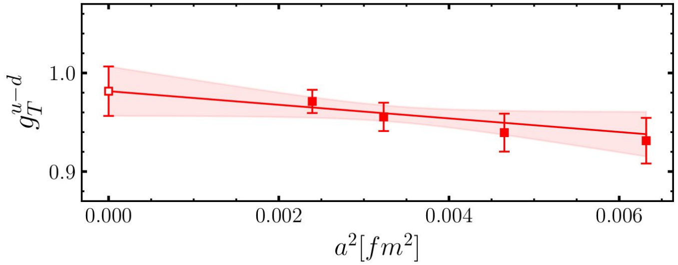
Radiative — *SU(2)-breaking*

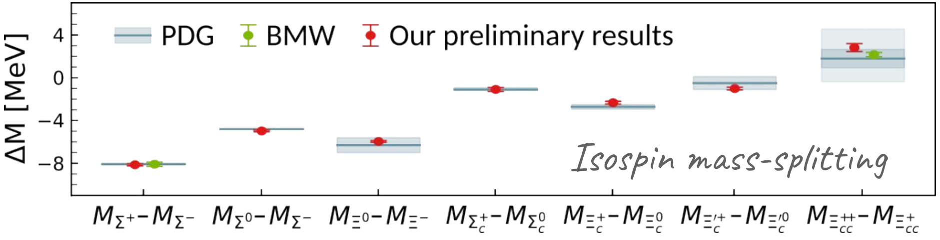
- **Benchmark calculations & scale-setting:**

$$\langle \mathcal{O} \rangle \in \{ \Delta M, \quad a_\mu^{\mathrm{HVP}}, \quad \Gamma_{P\to\ell\nu(\gamma)}, \quad \dots \}$$

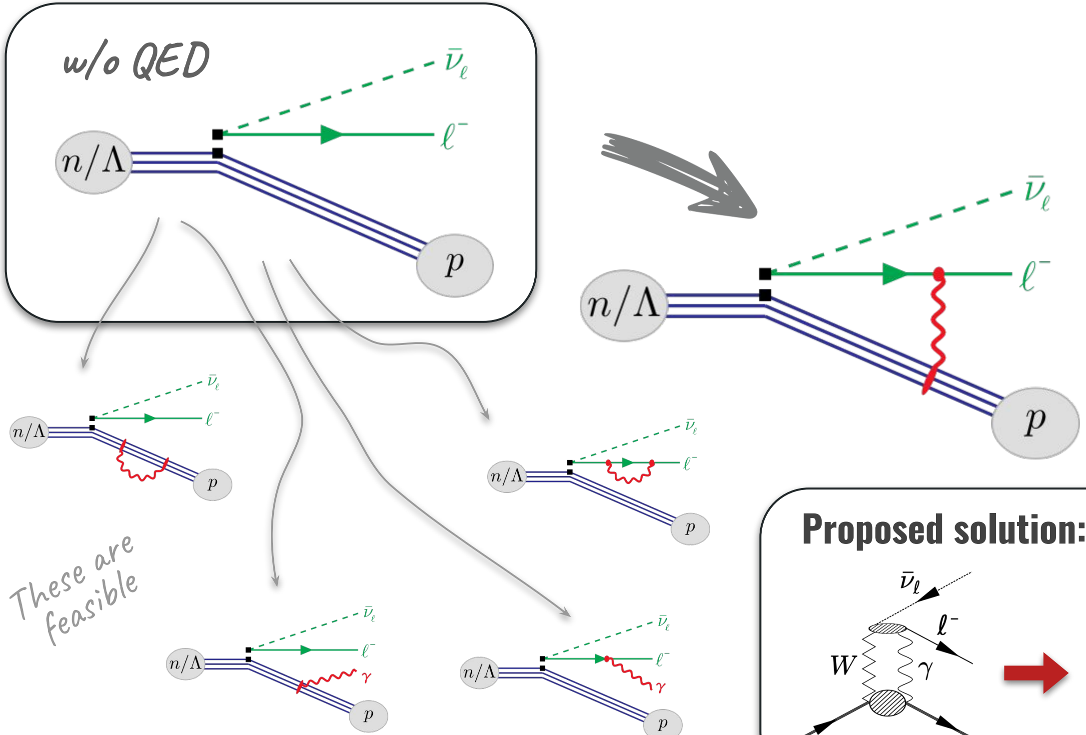
Mass split. — *Muon g-2* — *Leptonic decays*

Today, computing **LIBE** is not only becoming feasible, but a necessary next step!



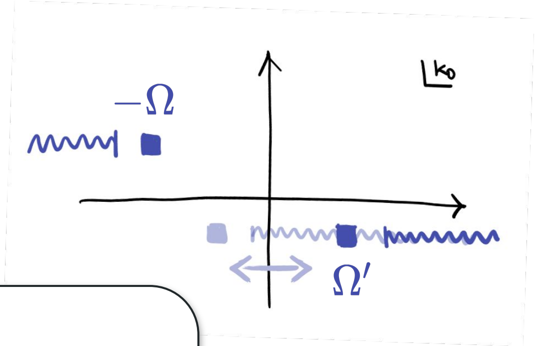


The challenge in radiative corrections



The **exchange** is the most challenging:

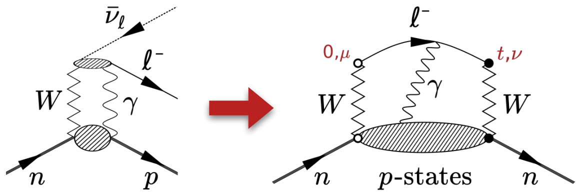
- ➢ Breaks lepton-hadron factorization
- ➢ Breaks Minkowski ➔ Euclidean map



$$\Omega = \sqrt{m_B^2 + \vec{k}_\gamma^2} - m_B$$

$$\Omega' = \sqrt{\omega_{B'}^2 + 2\,\vec{p}_{B'}\cdot\vec{k}_\gamma + \vec{k}_\gamma^2} - \omega_{B'}$$

Proposed solution:



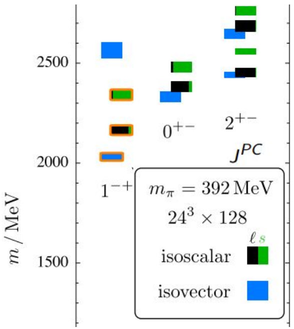
[Matteo Di Carlo, https://indico.global/event/16051/contributions/149080/]

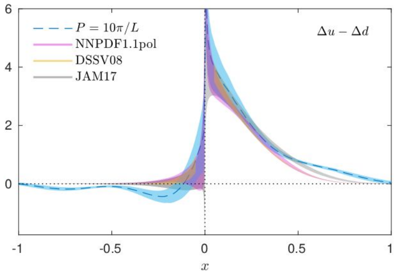
Conclusions

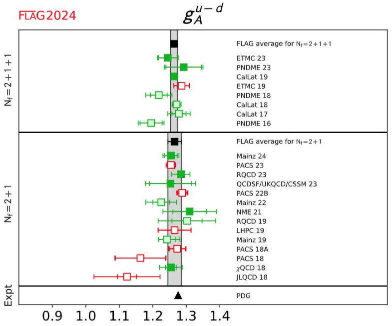
Lattice QCD allows for first-principles, non-perturbative, and systematically-improvable results.

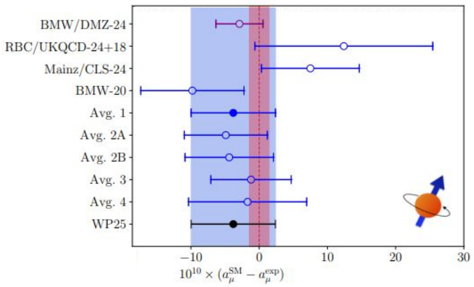
Increasing Quality / Time / Effort / Computational Resources / Motivation

Qualitative Results	**< 10% accuracy**	**≃ 1% accuracy**	**< 1% accuracy**
1 ensemble	*Few ensembles*	*Many ensembles*	*Inclusion of QED and isospin*
Heavy pion mass	*Good understanding*	*Full control of systematics*	*breaking corrections!*
No control of systematics	*of systematics*	*in isosymmetric QCD*	*(so far not always possible)*









Conclusions

Lattice QCD allows for first-principles, non-perturbative, and systematically-improvable results.

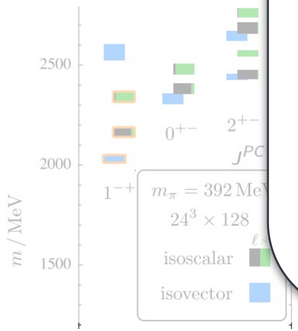
Increasing Quality / Time / Effort / Computational Resources / Motivation

Qualitative Resu

1 ensemble

Heavy pion mass

No control of system



Constructive feedback loop with experiments is vital for our field!

provides theoretical inputs

Lattice QCD **Experiments**

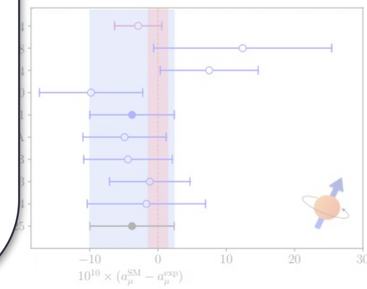
provides precise result to validate the SM / direct searches of BSM

< 1% accuracy

sion of QED and isospin

reaking corrections!

far not always possible)



Conclusions

Lattice QCD allows for first-principles, non-perturbative, and systematically-improvable results.

Qualitativ…

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Heavy pio…

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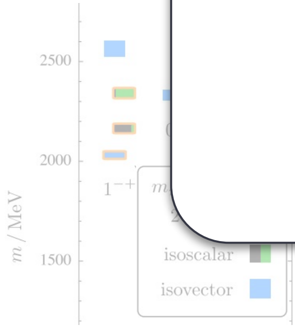
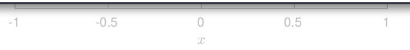
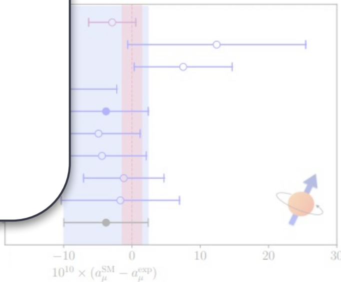
…f QED and isospin

…g corrections!

…t always possible)







Thank you for your attention!

Extended Twisted Mass Collaboration

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Republic of Cyprus

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