Testing QCD with τ Decays

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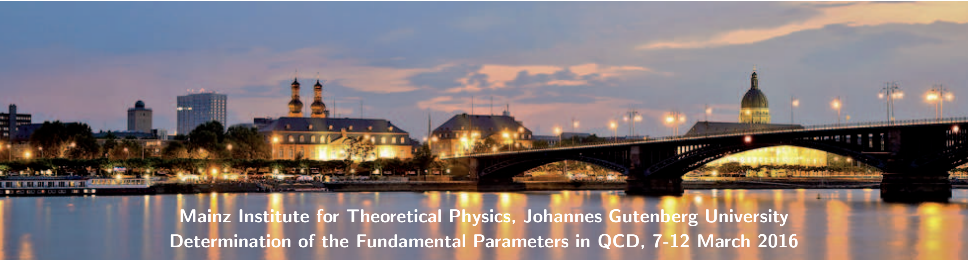


Mainz Institute for Theoretical Physics, Johannes Gutenberg University

Determination of the Fundamental Parameters in QCD, 7-12 March 2016

Theoretical Framework

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \mathrm{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

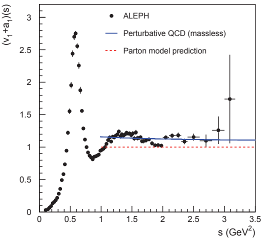
$$= 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

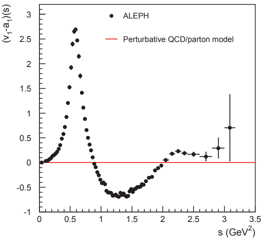
$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2\, \Pi^{(J)}_{us,V+A}(s)$$

Davier et al, 1312.1501

$$v_1 = 2\pi\, \mathrm{Im}\Pi^{(1)}_{ud,V}(s)$$

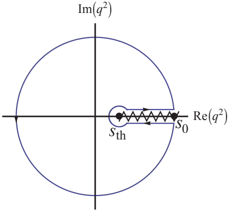
$$a_1 = 2\pi\, \mathrm{Im}\Pi^{(1)}_{ud,A}(s)$$





$$i \int d^4x\, e^{iqx} \langle 0|\, T\left[\mathcal{J}^{\mu}_{ij}(x)\mathcal{J}^{\nu\dagger}_{ij}(0)\right] |0\rangle \;=\; (-g^{\mu\nu} q^2 + q^\mu q^\nu)\, \Pi^{(1)}_{ij,\mathcal{J}}(q^2) + q^\mu q^\nu\, \Pi^{(0)}_{ij,\mathcal{J}}(q^2)$$

$$A^{\omega}_{\mathcal{J}}(s_0) \;\equiv\; \int_{s_{th}}^{s_0} \frac{ds}{s_0}\, \omega(s)\; \mathrm{Im}\, \Pi^{(J)}_{\mathcal{J}}(s) \;=\; -\frac{1}{2i} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi^{(J)}_{\mathcal{J}}(s)$$



$$\Pi^{(J)}_{\mathcal{J}}(s) \;\approx\; \Pi^{(J)}_{\mathcal{J}}(s)^{\mathrm{OPE}} \;=\; \sum_D \frac{\mathcal{O}^{(J)}_{D,\mathcal{J}}}{(-s)^{D/2}}$$

$$\delta_{\mathrm{DV}} \;\equiv\; \frac{1}{2\pi i} \oint_{|s|=s_0} ds\, \omega(s) \left\{\Pi^{(J)}_{\mathcal{J}}(s) - \Pi^{(J)}_{\mathcal{J}}(s)^{\mathrm{OPE}}\right\} \;=\; \frac{1}{\pi}\int_{s_0}^{\infty} ds\, \omega(s)\, \mathrm{Im}\left[\Pi^{(J)}_{\mathcal{J}}(s) - \Pi^{(J)}_{\mathcal{J}}(s)^{\mathrm{OPE}}\right]$$

Add residues whenever there are poles within the integration contour

Left-Right Correlator

$$\Pi(s) \;\equiv\; \Pi^{(0+1)}_{ud,LR}(s) \;\equiv\; \Pi^{(0+1)}_{ud,V}(s) - \Pi^{(0+1)}_{ud,A}(s) \;=\; \frac{2f_\pi^2}{s-m_\pi^2} + \overline{\Pi}(s)$$

- **Pure non-perturbative quantity:** Ideal to test the OPE
- **Known short-distance constraints:** $\Pi(s)^{\mathrm{OPE}}_{m_q=0} \;=\; \sum_{D\geq 6} \frac{\mathcal{O}_D}{(-s)^{D/2}}$

❶ **1st WSR:** $$\int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) \;=\; 2f_\pi^2$$

❷ **2nd WSR:** $$\int_{s_{\mathrm{th}}}^{\infty} ds\; s\, \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) \;=\; 2f_\pi^2 m_\pi^2$$

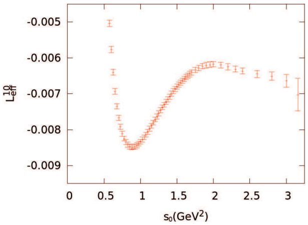
❸ **πSR:** $$\int_{s_{\mathrm{th}}}^{\infty} ds\; s\, \log\left(\frac{s}{\Lambda^2}\right) \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)\Big|_{m_q=0} \;=\; \left(m_{\pi^0}^2 - m_{\pi^+}^2\right)_{\mathrm{em}} \frac{8\pi}{3\alpha}\; f_\pi^2\Big|_{m_q=0}$$

- **χPT:** $\overline{\Pi}(s) \;=\; -8\, L_{10} + \chi\mathrm{logs} + s\,[16\, C_{87} + \chi\mathrm{logs}] + \mathcal{O}(s^2)$

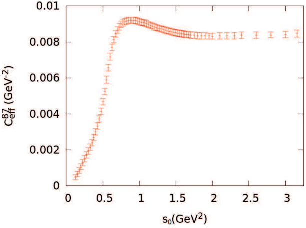
Non-Pinched & Pinched Weights

González-Alonso, Pich, Rodríguez-Sánchez

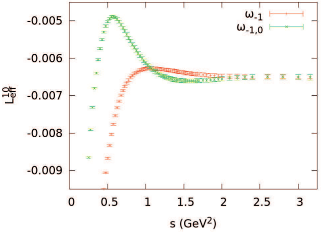
$$L_{10}^{\mathrm{eff}} = -\frac{1}{8}\int_{s_{\mathrm{th}}}^{s_0} ds\; s^{-1}\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$

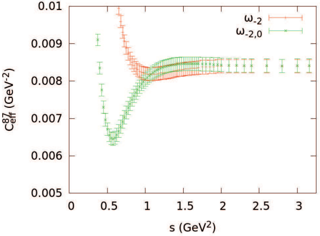


$$C_{87}^{\mathrm{eff}} = \frac{1}{16}\int_{s_{\mathrm{th}}}^{s_0} ds\; s^{-2}\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$



$$\omega_{-1,0} = s^{-1}\,(1 - s/s_0)\quad,\quad \omega_{-1} = s^{-1}\,(1 - s/s_0)^2\quad,\quad \omega_{-2,0} = s^{-2}\,(1 - s^2/s_0^2)\quad,\quad \omega_{-2} = s^{-2}\,(1 - s/s_0)^2(1 + 2s/s_0)$$





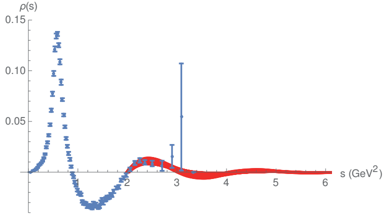
Playing with Duality Violations

González-Alonso, Pich, Rodríguez-Sánchez

- **Ad-hoc Ansatz:** $\rho(s > s_z) \,=\, \kappa\; e^{-\gamma s}\, \sin\{\beta(s - s_z)\}$ (Cata et al)
- **$3 \cdot 10^6$ randomly generated tuples $(\kappa, \gamma, \beta, s_z)$**
- **Select "Acceptable" Spectral Functions, satisfying:**
  1. Consistent with ALEPH at 90% C.L., above $s = 1.7\ \mathrm{GeV}^2$
  2. Fulfill 1st and 2nd WSRs, and πSR

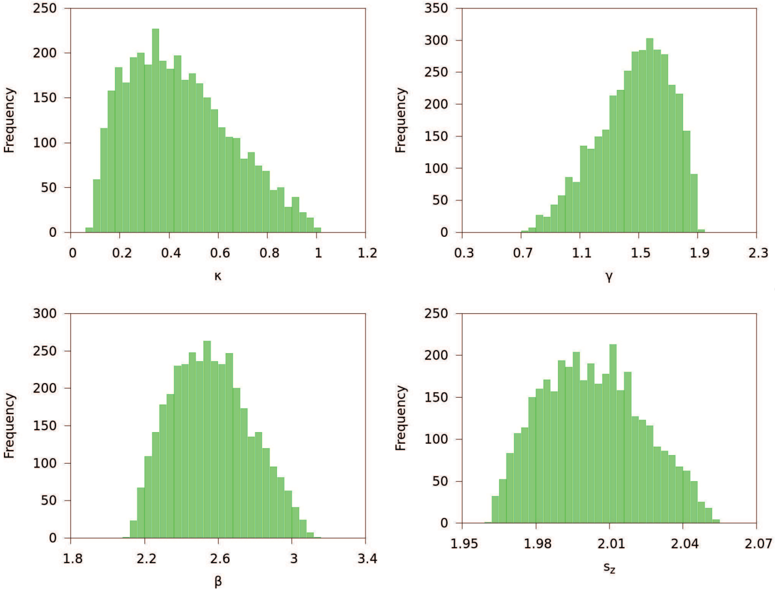
3716 tuples selected

(red band)



Statistical Distributions of Selected Tuples

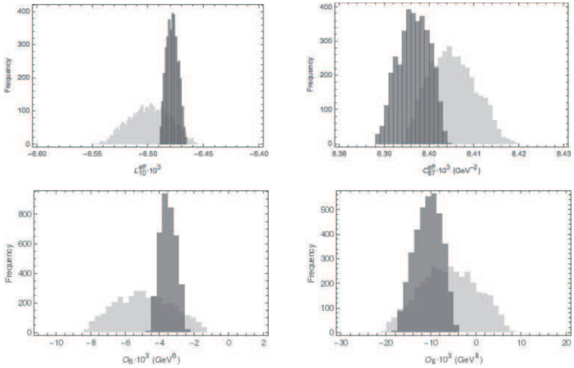
González-Alonso, Pich, Rodríguez-Sánchez



GeV units

Statistical Distributions of $L_{10}^{\rm eff}$, $C_{87}^{\rm eff}$, $\mathcal{O}_6$ and $\mathcal{O}_8$

González-Alonso, Pich, Rodríguez-Sánchez



Non-pinched

& pinched

weights

$$L_{10}^{\rm eff} = (-6.477\,^{+0.004}_{-0.006} \pm 0.05)\cdot 10^{-3} = (-6.48 \pm 0.05)\cdot 10^{-3}$$

$$C_{87}^{\rm eff} = (8.399\,^{+0.002}_{-0.005} \pm 0.18)\cdot 10^{-3}\ \mathrm{GeV}^{-2} = (8.40 \pm 0.18)\cdot 10^{-3}\ \mathrm{GeV}^{-2}$$

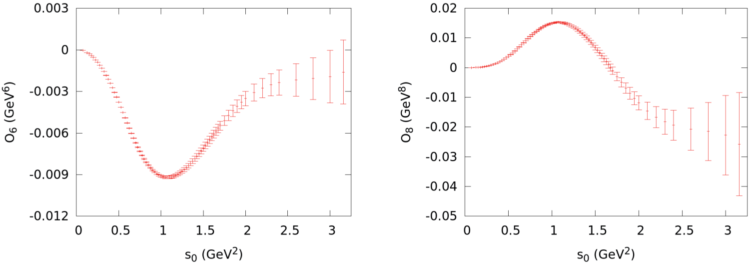
$$\mathcal{O}_6 = (-3.6\,^{+0.5}_{-0.4} \pm 0.5)\cdot 10^{-3}\ \mathrm{GeV}^6 = (-3.6\,^{+0.7}_{-0.6})\cdot 10^{-3}\ \mathrm{GeV}^6$$

$$\mathcal{O}_8 = (-1.0 \pm 0.3 \pm 0.2)\cdot 10^{-2}\ \mathrm{GeV}^8 = (-1.0 \pm 0.4)\cdot 10^{-2}\ \mathrm{GeV}^8$$

$\mathcal{O}_{6,8}$ with Pinched Weights, ignoring DV

González-Alonso, Pich, Rodríguez-Sánchez

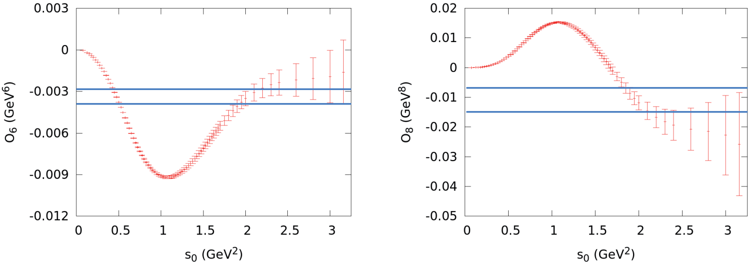
$$\int_{s_{\rm th}}^{s_0} ds\; \frac{1}{\pi}\, \mathrm{Im}\,\Pi(s)\;\; (s-s_0)^2\;\; \{1\,,\; (s+2s_0)\}$$



$\mathcal{O}_{6,8}$ with Pinched Weights, ignoring DV

González-Alonso, Pich, Rodríguez-Sánchez

$$\int_{s_{\rm th}}^{s_0} ds \; \frac{1}{\pi} \, \mathrm{Im}\,\Pi(s) \;\; (s - s_0)^2 \;\; \{1\,,\; (s + 2s_0)\}$$



Result with DV fully taken into account

Pinched weights suppress very efficiently duality violation effects

Big reduction of errors mainly due to short-distance constraints which overcome the large data uncertainties at large s_0 values

Comparison with Previous Works

González-Alonso, Pich, Rodríguez-Sánchez

$10^3 \cdot L_{10}^{\rm eff}$	$10^3 \cdot C_{87}^{\rm eff}$ $(\rm GeV^{-2})$	$10^3 \cdot \mathcal{O}_6$ $(\rm GeV^6)$	$10^2 \cdot \mathcal{O}_8$ $(\rm GeV^8)$	Reference	Comments
-6.45 ± 0.06	–	-2.3 ± 0.6	-5.4 ± 3.3	BPDS'06	ALEPH'05 + DV = 0
–	–	$-6.8\,^{+2.0}_{-0.8}$	$3.2\,^{+2.8}_{-9.2}$	ASS'08	ALEPH'05 + DV = 0
-6.48 ± 0.06	8.18 ± 0.14	–	–	GPP'08	ALEPH'05 + DV = 0
-6.44 ± 0.05	8.17 ± 0.12	-4.4 ± 0.8	-0.7 ± 0.5	GPP'10	ALEPH'05 + $\rm DV_{V-A}$
-6.45 ± 0.09	8.47 ± 0.29	-6.6 ± 1.1	0.5 ± 0.5	Boito'12	OPAL'99 + $\rm DV_{V/A}$
-6.50 ± 0.10	–	-5.0 ± 0.7	-0.9 ± 0.5	DHSS'15	ALEPH'14 + DV = 0
-6.45 ± 0.05	8.38 ± 0.18	-3.2 ± 0.9	-1.3 ± 0.6	Boito'15	ALEPH'14 + $\rm DV_{V/A}$
-6.42 ± 0.10	8.35 ± 0.29	$-5.7\,^{+1.1}_{-1.2}$	$0.0\,^{+0.5}_{-0.6}$	this work	OPAL'99 + $\rm DV_{V-A}$
-6.48 ± 0.05	8.40 ± 0.18	$-3.6\,^{+0.7}_{-0.6}$	-1.0 ± 0.4	this work	ALEPH'14 + $\rm DV_{V-A}$

$V + A$ Correlator: R_τ (ud)

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \mathrm{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

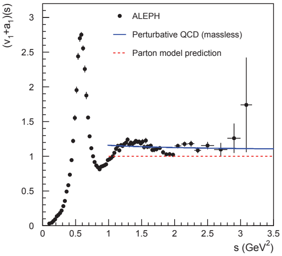
$$= 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(0+1)}(s) - 2\frac{s}{m_\tau^2} \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

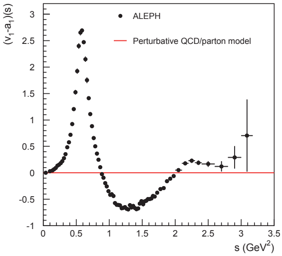
$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2 \, \Pi^{(J)}_{us,V+A}(s)$$

Davier et al, 1312.1501

$$v_1 = 2\pi \, \mathrm{Im}\Pi^{(1)}_{ud,V}(s)$$

$$a_1 = 2\pi \, \mathrm{Im}\Pi^{(1)}_{ud,A}(s)$$





$$R_\tau = N_C\, S_{EW}\, (1 + \delta_P + \delta_{NP})$$

Perturbative ($m_q=0$)

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$K_0 = K_1 = 1$, $K_2 = 1.63982$, $K_3 = 6.37101$, $K_4 = 49.07570$ **Baikov-Chetyrkin-Kühn '08**

$$\delta_P = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

$$R_\tau = N_C\, S_{EW}\, (1 + \delta_P + \delta_{NP})$$

Perturbative (m_q=0)

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$K_0 = K_1 = 1$, $K_2 = 1.63982$, $K_3 = 6.37101$, $K_4 = 49.07570$ **Baikov-Chetyrkin-Kühn '08**

$$\delta_{\mathrm{P}} = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) \left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2} \sum_{n\ge 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4 \langle O_4 \rangle \approx \frac{2\pi}{3} \langle 0 | \alpha_s G^{\mu\nu} G_{\mu\nu} | 0 \rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i} \oint_{|x|=1} dx\, (1 - 3x^2 + 2x^3) \sum_{n\ge 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-x m_\tau^2)^n} = -3\, \frac{C_6 \langle O_6 \rangle}{m_\tau^6} - 2\, \frac{C_8 \langle O_8 \rangle}{m_\tau^8}$$

Suppressed by m_τ^6 **[additional chiral suppression in** $C_6 \langle O_6 \rangle^{V+A}$ **]**

Perturbative Uncertainty in $\alpha_s(m_\tau^2)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\; a(-s)^n$$

$$\delta_{\rm P} = \underbrace{\sum_{n=1} K_n\; A^{(n)}(\alpha_s)}_{\textbf{CIPT}} = \underbrace{\sum_{n=0} r_n\; a_\tau^n}_{\textbf{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a_\tau(-x\,m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

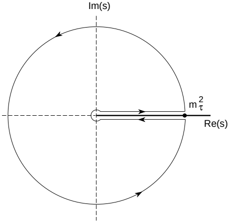
n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

Large running of α_s along the circle $s = m_\tau^2\, e^{i\varphi}$, $\varphi \in [0\,,2\pi]$

R_τ suitable for a precise α_s determination



$$R_\tau = 6\pi i \oint_{|x|=1} (1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

$$\Pi^{(J)}_{\mathcal{J}}(s) \approx \Pi^{(J)}_{\mathcal{J}}(s)^{\rm OPE} = \sum_D \frac{\mathcal{O}^{(J)}_{D,\mathcal{J}}}{(-s)^{D/2}}$$

- **R_τ large enough to safely use the OPE**
- **OPE only valid away from real axis:** $(1-x)^2$ **pinched at** $s = m_\tau^2$
- $m_{u,d} = 0$ ➔ $s\,\Pi^{(0)} = 0$ ➔ $R_{\tau,V+A} = 6\pi i \oint_{|x|=1} (1-3x^2+2x^3)\,\Pi^{(0+1)}_{ud,V+A}(x m_\tau^2)$

  ➔ $\delta_{NP} \sim 1/m_\tau^6$ **Strong suppression of non-perturbative effects**
- **$D = 6$ OPE contributions have opposite sign for V & A. Cancellation**
- **δ_{NP} can be determined from data**

α_s determination with ALEPH-like fit

Rodríguez-Sánchez, A.P.

$$\omega_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^{l} \left(1 + \frac{2s}{m_\tau^2}\right)$$

$(k,l) = (0,0) \quad \rightarrow \quad \alpha_s, \mathcal{O}_{6\,V/A}, \mathcal{O}_{8\,V/A}$

$(k,l) = (1,0) \quad \rightarrow \quad \alpha_s, \langle a_s GG\rangle, \mathcal{O}_{6\,V/A}, \mathcal{O}_{8\,V/A}, \mathcal{O}_{10\,V/A}$

$(k,l) = (1,1) \quad \rightarrow \quad \alpha_s, \langle a_s GG\rangle, \mathcal{O}_{6\,V/A}, \mathcal{O}_{8\,V/A}, \mathcal{O}_{10\,V/A}, \mathcal{O}_{12\,V/A}$

$(k,l) = (1,2) \quad \rightarrow \quad \alpha_s, \mathcal{O}_{6\,V/A}, \mathcal{O}_{8\,V/A}, \mathcal{O}_{10\,V/A}, \mathcal{O}_{12\,V/A}, \mathcal{O}_{14\,V/A}$

$(k,l) = (1,3) \quad \rightarrow \quad \alpha_s, \mathcal{O}_{8\,V/A}, \mathcal{O}_{10\,V/A}, \mathcal{O}_{12\,V/A}, \mathcal{O}_{14\,V/A}, \mathcal{O}_{16\,V/A}$

Channel	$\alpha_s(m_\tau^2)/\pi$	$\langle \frac{\alpha_s}{\pi} GG\rangle$ (10^{-3} GeV^4)	$\mathcal{O}_6$ (10^{-3} GeV^6)	$\mathcal{O}_8$ 10^{-3} GeV^8)
V (FOPT)	$0.104\,^{+0.004}_{-0.002}$	$8\,^{+7}_{-14}$	$-3.2\,^{+0.8}_{-0.5}$	$5.0\,^{+0.4}_{-0.7}$
V (CIPT)	$0.112\,^{+0.004}_{-0.003}$	$-8\,^{+7}_{-7}$	$-3.5\,^{+0.3}_{-0.3}$	$4.9\,^{+0.4}_{-0.5}$
A (FOPT)	$0.097\,^{+0.003}_{-0.002}$	$-15\,^{+5}_{-8}$	$4.4\,^{+0.5}_{-0.4}$	$-5.8\,^{+0.3}_{-0.4}$
A (CIPT)	$0.102\,^{+0.004}_{-0.003}$	$-25\,^{+5}_{-5}$	$4.3\,^{+0.2}_{-0.2}$	$-5.8\,^{+0.3}_{-0.3}$
V+A (FOPT)	$0.102\,^{+0.003}_{-0.002}$	$-3\,^{+6}_{-11}$	$1.3\,^{+1.4}_{-0.8}$	$-0.8\,^{+0.4}_{-0.7}$
V+A (CIPT)	$0.108\,^{+0.003}_{-0.003}$	$-16\,^{+5}_{-5}$	$0.9\,^{+0.3}_{-0.4}$	$-1.0\,^{+0.5}_{-0.7}$

Good agreement with ALEPH

❶ Fit one more condensate to test stability/uncertainties

Channel	$\alpha_s(m_\tau^2)/\pi$	$\langle \frac{\alpha_s}{\pi} GG \rangle$ $(10^{-3}\ \mathrm{GeV}^4)$	$\mathcal{O}_6$ $(10^{-3}\ \mathrm{GeV}^6)$	$\mathcal{O}_8$ $10^{-3}\ \mathrm{GeV}^8)$	$\mathcal{O}_{10}$ $(10^{-3}\ \mathrm{GeV}^{10})$
V (FOPT)	$0.102\,^{+0.005}_{-0.005}$	$10\,^{+9}_{-17}$	$-4\,^{+3}_{-2}$	$6\,^{+2}_{-2}$	$-2\,^{+5}_{-5}$
V (CIPT)	$0.108\,^{+0.007}_{-0.006}$	$-1\,^{+11}_{-10}$	$-5\,^{+2}_{-2}$	$6\,^{+2}_{-2}$	$-3\,^{+4}_{-4}$
A (FOPT)	$0.110\,^{+0.007}_{-0.007}$	$-31\,^{+16}_{-34}$	$11\,^{+5}_{-4}$	$-12\,^{+4}_{-5}$	$15\,^{+10}_{-9}$
A (CIPT)	$0.118\,^{+0.007}_{-0.007}$	$-50\,^{+13}_{-12}$	$10\,^{+2}_{-2}$	$-11\,^{+2}_{-2}$	$13\,^{+5}_{-5}$
V+A (FOPT)	$0.106\,^{+0.004}_{-0.004}$	$-8\,^{+10}_{-24}$	$7\,^{+7}_{-4}$	$-5\,^{+4}_{-6}$	$12\,^{+12}_{-9}$
V+A (CIPT)	$0.113\,^{+0.005}_{-0.005}$	$-23\,^{+10}_{-9}$	$5\,^{+3}_{-3}$	-5 ± 3	$10\,^{+9}_{-9}$

- **Good stability of α_s with respect to previous fit**
- **Larger variation in condensates values and increased errors**

❷ Take central values from first fit, adding differences as errors

α_s determination with ALEPH-like fit

Rodríguez-Sánchez, A.P.

$$\omega_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^l \left(1 + \frac{2s}{m_\tau^2}\right)$$

Channel	$\alpha_s(m_\tau^2)/\pi$	$\langle \frac{\alpha_s}{\pi} GG \rangle$ $(10^{-3}\ \mathrm{GeV}^4)$	$\mathcal{O}_6$ $(10^{-3}\ \mathrm{GeV}^6)$	$\mathcal{O}_8$ $10^{-3}\ \mathrm{GeV}^8)$
V (FOPT)	$0.104\,^{+0.004}_{-0.002}$	$8\,^{+7}_{-14}$	$-3.2\,^{+0.8}_{-0.5}$	$5.0\,^{+0.4}_{-0.7}$
V (CIPT)	$0.112\,^{+0.004}_{-0.003}$	$-8\,^{+7}_{-7}$	$-3.5\,^{+0.3}_{-0.3}$	$4.9\,^{+0.4}_{-0.5}$
A (FOPT)	$0.097\,^{+0.003}_{-0.002}$	$-15\,^{+5}_{-8}$	$4.4\,^{+0.5}_{-0.4}$	$-5.8\,^{+0.3}_{-0.4}$
A (CIPT)	$0.102\,^{+0.004}_{-0.003}$	$-25\,^{+5}_{-5}$	$4.3\,^{+0.2}_{-0.2}$	$-5.8\,^{+0.3}_{-0.3}$
V+A (FOPT)	$0.102\,^{+0.003}_{-0.002}$	$-3\,^{+6}_{-11}$	$1.3\,^{+1.4}_{-0.8}$	$-0.8\,^{+0.4}_{-0.7}$
V+A (CIPT)	$0.108\,^{+0.003}_{-0.003}$	$-16\,^{+5}_{-5}$	$0.9\,^{+0.3}_{-0.4}$	$-1.0\,^{+0.5}_{-0.7}$

$$\alpha_s(m_\tau^2)^{\mathrm{FOPT}} = 0.319\,^{+0.017}_{-0.015}$$

$$\alpha_s(m_\tau^2)^{\mathrm{CIPT}} = 0.339\,^{+0.019}_{-0.017}$$

→

$$\boxed{\alpha_s(m_\tau^2) = 0.329\,^{+0.018}_{-0.015}}$$

Experiment vs. (pinched) Perturbation Theory (only)

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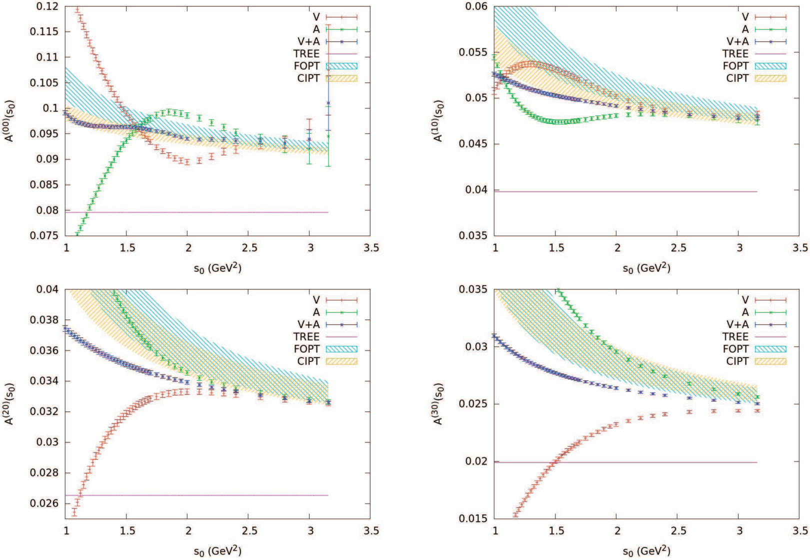
$$\omega^{(n0)}(s = s_0 x) = (1-x)^n \qquad , \qquad \alpha_s(m_\tau^2) = 0.329\,^{+0.018}_{-0.015}$$



Playing with the s_0 dependence

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$$\omega^{(20)}(s = s_0 x) = (1-x)^2 \qquad \mathcal{O}_4\,,\ \mathcal{O}_6$$

$$\omega^{(21)}(s = s_0 x) = (1-x)^2\,(1+2x) \qquad \mathcal{O}_6\,,\ \mathcal{O}_8$$

$$\omega^{(22)}(s = s_0 x) = (1-x)^2\,(1+2x+3x^2) \qquad \mathcal{O}_8\,,\ \mathcal{O}_{10}$$

Fit to the (9) $s_0 > 2\ \mathrm{GeV}^2$ points. One moment only (avoid correlations)

V + A channel

Moment	Method	$\alpha_s(m_\tau^2)/\pi$	Lower-D Condensate (10^{-3} GeV^D)	Higher-D Condensate (10^{-3} GeV^D)
$A^{\omega^{(20)}}(s_0)$	FOPT	$0.105\,^{+\,0.004}_{-\,0.007}$	$-9\,^{+12}_{-4}$	$-4\,^{+3}_{-7}$
$A^{\omega^{(20)}}(s_0)$	CIPT	0.106 ± 0.003	$-11\,^{+7}_{-6}$	0 ± 1
$A^{\omega^{(21)}}(s_0)$	FOPT	0.102 ± 0.003	$3\,^{+1}_{-2}$	0 ± 2
$A^{\omega^{(21)}}(s_0)$	CIPT	0.106 ± 0.003	0 ± 1	1 ± 2
$A^{\omega^{(22)}}(s_0)$	FOPT	0.101 ± 0.003	-2 ± 3	0 ± 2
$A^{\omega^{(22)}}(s_0)$	CIPT	0.106 ± 0.003	1 ± 2	$2\,^{+3}_{-2}$

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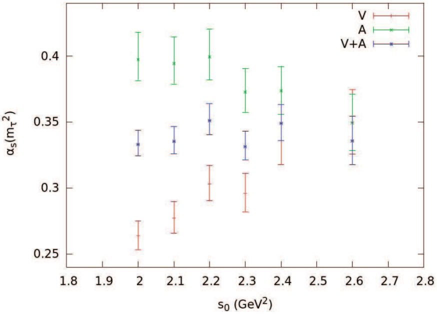
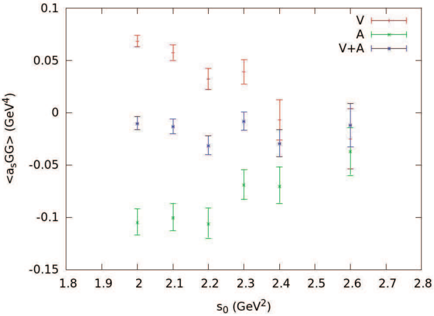
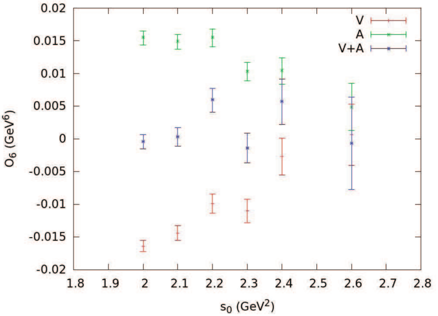
$A^{\omega^{(20)}}(s_0)$ CIPT

Fit to last $n = 4, \cdots, 8$ s_0 points
vs. starting s_0 fitted value







→ $\alpha_s(m_\tau^2) \; = \; 0.326\,{}^{+0.014}_{-0.013}$ from $V+A$

BUT...

- **Bad quality fit** $(\chi^2_{\rm min}/\text{d.o.f.})$
- **Much worse behaviour in separate V & A channels**
- **Fitting the s_0 dependence removes pinching:**

Fitting m points of $A^{(n0)}(s_0)$ is equivalent to a fit of

$$\left\{A^{(n0)}(s_0), A^{(n-1\,0)}(s_0), \ldots, A^{(0\,0)}(s_0), \mathrm{Im}\,\Pi(s_0), \frac{d\,\mathrm{Im}\,\Pi(s_0)}{ds_0}, \ldots, \frac{d^{m-n-1}\,\mathrm{Im}\,\Pi(s_0)}{ds_0}\right\}$$

Local duality assumed! → **Violations of Duality**

Boito et al. approach

- **Ansatz:** $\rho^{DV}_{V/A}(s) = e^{-(\delta_{V/A}+\gamma_{V/A}s)}\,\sin(\alpha_{V/A}+\beta_{V/A}s)$ $\qquad s > s_{th} \sim 1.5\,\mathrm{GeV}^2$

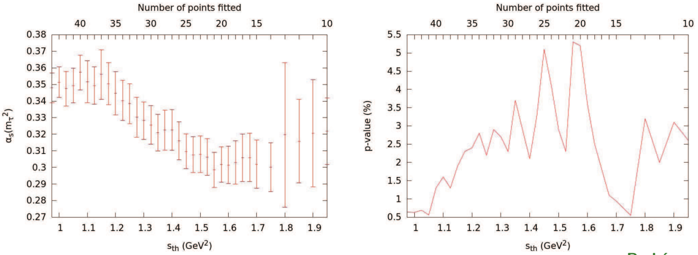
- $\omega(x) = 1$: **Fit s_0 dependence of** $A^{(00)}(s_0) = \int^{s_0} \frac{ds}{s_0}\, \mathrm{Im}\,\Pi(s)$

  n-point fit ➔ $\{A^{(00)}(s_0), \mathrm{Im}\,\Pi(s_0+\Delta s_0), \cdots, \mathrm{Im}\,\Pi(s_0+(n-1)\Delta s_0)\}$

  $n-1$ points dedicated just to fit the spectral function

- **Uncertainties too large in A channel. Only V channel fitted**

FOPT



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Boito et al. approach (cont)

Fit s_0 dependence of $A^{(00)}(s_0) = \int^{s_0} \frac{ds}{s_0}\; \mathrm{Im}\,\Pi_V(s)$

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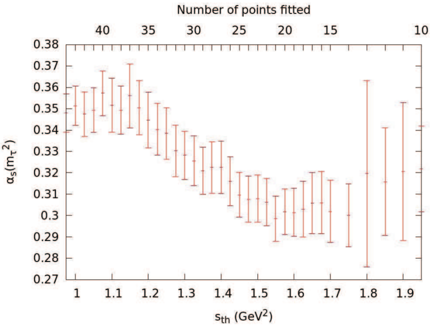
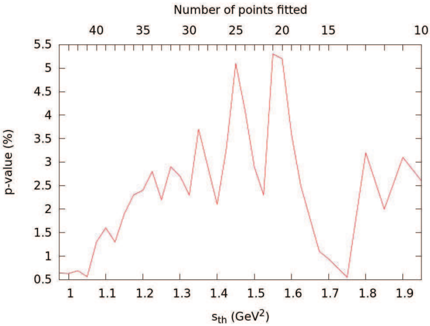
FOPT





Problematic fit. Model dependence. Instabilities. Very low p-value

$\longrightarrow \quad \alpha_s(m_\tau^2) = \quad ?$

Many more fits, tests, plots . . .

Consistent value of $\alpha_s(m_\tau^2)$ obtained **(no surprises)**

Detailed analysis will be released soon

Summary

- Many interesting test of QCD with spectral information
- Current data has limited accuracy & large correlations

  Better experimental data (high statistics & precision) needed

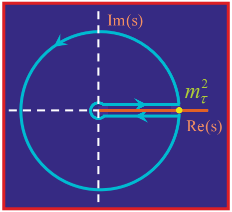
- Π_{V-A} allows clean tests of OPE and χPT
- Precise determination of α_s from τ decays: Π_{V+A} $(\Pi_{V,A})$
  - Main limitation: perturbation theory

    Better understanding of higher orders needed

  - Improved control of δ_{NP} possible with better data

BACKUP

$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a_\tau(-x\,m_\tau^2)^n \;=\; a_\tau^n+\cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$



$$A^{(1)}(a_\tau) = a_\tau - \frac{19}{24}\,\beta_1\,a_\tau^2 + \left[\beta_1^2\left(\frac{265}{288}-\frac{\pi^2}{12}\right)-\frac{19}{24}\,\beta_2\right]a_\tau^3+\cdots$$

$$a(-s) \simeq \frac{a_\tau}{1-\frac{\beta_1}{2}a_\tau\log\left(-s/m_\tau^2\right)} = \frac{a_\tau}{1-i\frac{\beta_1}{2}a_\tau\phi} = a_\tau\sum_n\left(i\frac{\beta_1}{2}a_\tau\phi\right)^n \qquad ; \qquad \phi\in[0,2\pi]$$

FOPT expansion only convergent if $\alpha_\tau < 0.14$ (0.11) [at 1 (3) loops]

Experimentally $\alpha_\tau \approx 0.11$ ➡ **FOPT should not be used** (divergent series)

FOPT suffers a large renormalization-scale dependence **(Le Diberder- Pich , Menke)**

The difference between FOPT and CIPT grows at higher orders

Renormalon Hypothesis: Asymptotics already reached

Modelling a better behaved FOPT

(Beneke – Jamin)

- **Large higher-order K_n corrections could cancel the g_n ones**

  Happens in the "large-β_0" approximation (UV renormalon chain)

- D = 4 corrections very suppressed in R_τ

  ➡ **n = 2 IR renormalons can do the job** **($K_n \approx - g_n$)**

- No sign of renormalon behaviour in known coefficients

  ➡ **n = -1,2,3 renormalons + linear polynomial**

  5 unknown constants fitted to K_n ($2 \leq n \leq 5$). $K_5 = 283$ assumed

- **Borel summation:** large renormalon contributions. Smaller α

Nice model of higher orders. But too many different possibilities …

(Descotes-Genon – Malaescu)