The Pion-Nucleon-Nucleon Coupling Constants

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Determination of the Fundamental Parameters in QCD
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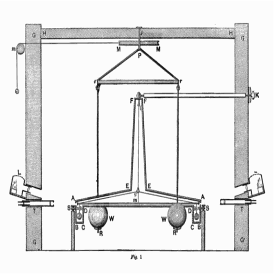
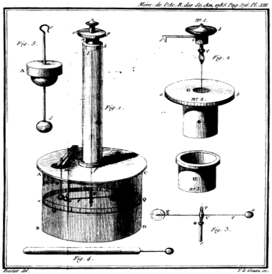
Fundamental Forces





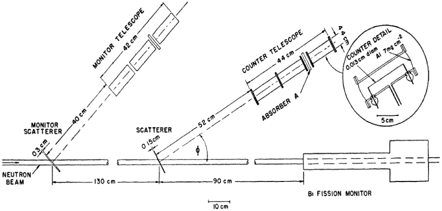
Cavendish (1798) "Experiments to determine the Density of the Earth"
Coulomb (1785) "Premier mmoire sur llectricit et le magntisme"

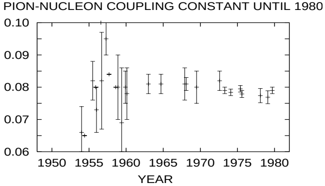
Strong Forces

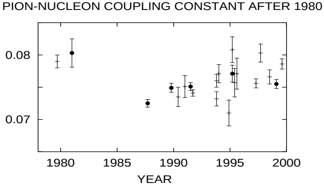
- Yukawa (1935)

$$V(r) = -f^2 \frac{e^{-mr}}{r}$$

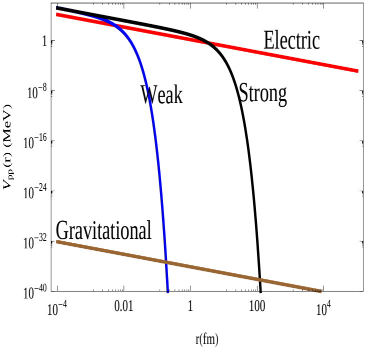
- Kemmer (1938) Isospin $\to \pi^0$
- Bethe (1940) $f^2 = 0.077 - 0.080$ (Deuteron)







Fundamental Forces beween protons



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- [4] **Nuclear Binding Energies and NN uncertainties** PoS QNP **2012** (2012) 145
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- [7] **Partial Wave Analysis of Nucleon-Nucleon Scattering below pion production** Phys.Rev. C88 (2013) 024002, Phys.Rev. C88 (2013) 6, 069902.
- [8] **Coarse-grained potential analysis of neutron-proton and proton-proton scattering below the pion production threshold** Phys.Rev. C88 (2013) 6, 064002, Phys.Rev. C91 (2015) 2, 029901.
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- [22] **Validation of NN forces in light nuclei** (in preparation) (with A. Nogga)

Bottomline

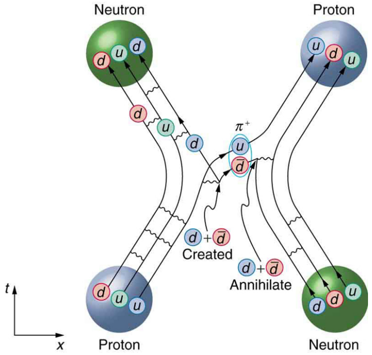
THE PROBLEM IN NUCLEAR PHYSICS

- GOAL: Estimate uncertainties in Nuclear Physics from IGNORANCE of NN,3N,4N interaction
  Reduce computational cost
- Statistical Uncertainties: NN,3N,4N Data
  Data abundance bias
- Systematic Uncertainties: NN,3N,4N potential
  Many forms of potentials possible
- Confidence level of Imperfect theories vs Perfect experiments

OUR APPROACH

- Start with NN
- Fit data WITH ERRORS with a simple interaction
- Estimate uncertainties of Effective Interactions and Matrix elements

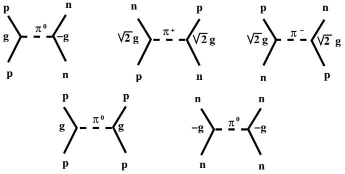
Fundamental approach: QCD



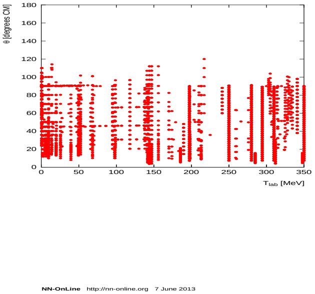
- Lattice form factor $g_{\pi NN} \sim 10 - 12$
- Lattice NN potential $g^2_{\pi NN}/(4\pi) = 12.1 \pm 2.7$
- QCD sum rules $g_{\pi NN} \sim 13(1)$

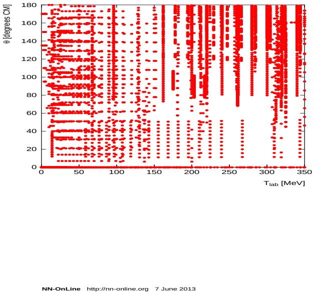
Long distances

- Nucleons exchange JUST one pion



- Low energies (about pion production) 8000 pp + np scattering data (polarizations etc.)





ANATOMY OF NUCLEAR FORCES

Nucleon-Nucleon Scattering

- Scattering amplitude

$$
\begin{aligned}
M &= a + m(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + (g-h)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}) \\
&+ (g+h)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + c(\sigma_1 + \sigma_2) \cdot \mathbf{n} \\
\mathbf{l} &= \frac{\mathbf{k}_f + \mathbf{k}_i}{|\mathbf{k}_f + \mathbf{k}_i|} \qquad \mathbf{m} = \frac{\mathbf{k}_f - \mathbf{k}_i}{|\mathbf{k}_f - \mathbf{k}_i|} \qquad \mathbf{n} = \frac{\mathbf{k}_f \wedge \mathbf{k}_i}{|\mathbf{k}_f \wedge \mathbf{k}_i|}
\end{aligned}
$$

- 5 complex amplitudes $\to$ 24 measurable cross-sections and polarization asymmetries
- Partial Wave Expansion

$$
\begin{aligned}
M^s_{m'_s, m_s}(\theta) &= \frac{1}{2ik} \sum_{J,l',l} \sqrt{4\pi(2l+1)} Y^{l'}_{m'_s - m_s}(\theta, 0) \\
&\times C^{l',s,J}_{m_s - m'_s, m'_s, m_s} i^{l-l'} (S^{J,s}_{l,l'} - \delta_{l',l}) C^{l,s,J}_{0,m_s,m_s},
\end{aligned} \tag{1}
$$

- S-matrix

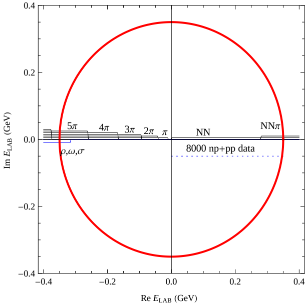
$$
S^J = \begin{pmatrix} e^{2i\delta^{J,1}_{J-1}} \cos 2\epsilon_J & ie^{i(\delta^{J,1}_{J-1} + \delta^{J,1}_{J+1})} \sin 2\epsilon_J \\ ie^{i(\delta^{J,1}_{J-1} + \delta^{J,1}_{J+1})} \sin 2\epsilon_J & e^{2i\delta^{J,1}_{J+1}} \cos 2\epsilon_J \end{pmatrix}, \tag{2}
$$

Analytical Structure

- $s = 4(M_N^2 + p^2) \to E_{\rm LAB} = 2p^2/M_N$
- Partial Wave Scattering Amplitude analytical for $|p| \le m_\pi/2$

$$T^J_{ll'}(p) \equiv S^J_{ll'}(p) - \delta_{l,l'} = p^{l+l'} \sum_n C_{n,l,l'} p^{2n}$$

- Nucleons behave as elementary (AT WHAT SCALE ?)



- Nucleons are heavy → Local Potentials

$$V_{n\pi}(r) \sim \frac{g^{2n}}{r} e^{-n m_\pi r}$$

Charge dependent One Pion Exchange

$$
\begin{aligned}
V_{\mathrm{OPE},pp}(r) &= f_{pp}^2 V_{m_{\pi^0},\mathrm{OPE}}(r), \\
V_{\mathrm{OPE},np}(r) &= -f_{nn} f_{pp} V_{m_{\pi^0},\mathrm{OPE}}(r) + (-)^{(T+1)} 2 f_c^2 V_{m_{\pi^\pm},\mathrm{OPE}}(r),
\end{aligned}
$$

where $V_{m,\mathrm{OPE}}$ is given by

$$
\begin{aligned}
V_{m,\mathrm{OPE}}(r) &= \left(\frac{m}{m_{\pi^\pm}}\right)^2 \frac{1}{3} m \left[Y_m(r) \sigma_1 \cdot \sigma_2 + T_m(r) S_{1,2}\right], \\
S_{1,2} &= 3 \sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2 \\
Y_m(r) &= \frac{e^{-mr}}{mr} \\
T_m(r) &= \frac{e^{-mr}}{mr} \left[1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right]
\end{aligned}
$$

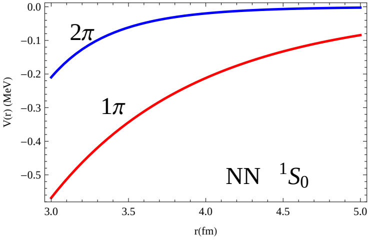
Small short range

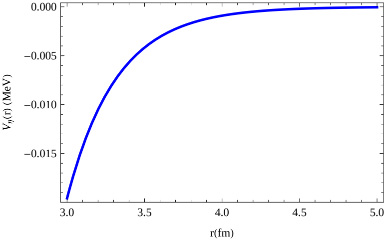
- OPE exchange

$$V_{1\pi}(r) = -f_{\pi NN}^2 \frac{e^{-m_\eta r}}{r}$$

- TPE exchange
- η-exchange

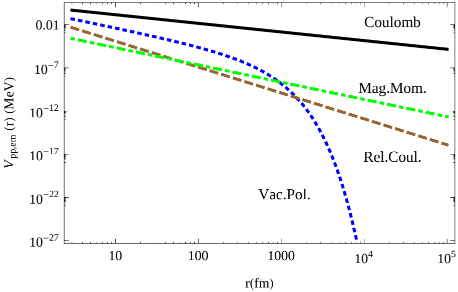
$$V_{\eta}(r) = -f_{\eta NN}^2 \frac{e^{-m_\eta r}}{r}$$





Small but crucial long range

- Coulomb interaction (pp) e/r
- Magnetic moments $\sim \mu_p \mu_n / r^3, \mu_p \mu_p / r^3, \mu_n \mu_n / r^3$
  Lowered $\chi^2/\nu \sim 2 \to \chi^2/\nu \sim 2 \to 1$
  Summing 1000-2000 partial waves
- Vacuum polarization (Uehling potential,Lamb-shift)
- Relativistic corrections $1/r^2$



Effective Elementary

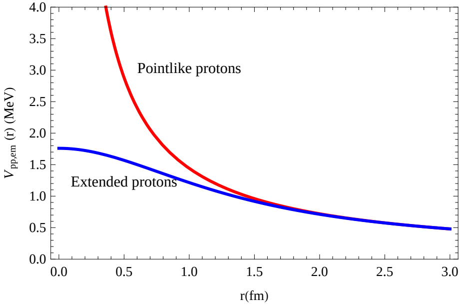
When are two protons interacting as point-like particles ?

- Electromagnetic Form factor

$$F_i(q) = \int d^3 r e^{iq\cdot r} \rho_i(r)$$

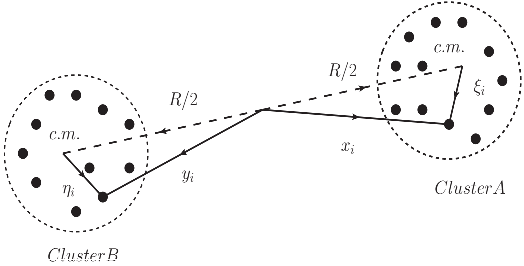
- Electrostatic interaction

$$V_{pp}^{\rm el}(r) = e^2 \int d^3 r_1 d^3 r_2 \frac{\rho_p(r_1)\rho_p(r_2)}{|\vec{r}_1 - \vec{r}_2 - \vec{r}|} \to \frac{e^2}{r} \qquad r > r_e \sim 2{\rm fm}$$



Quark Cluster Dynamics (qcd)

- Atomic analogue. Neutral atoms
- Non-overlapping atoms exchange TWO photons (Van der Waals force)
- Overlapping atoms are not locally neutral; ONE photon exchange is possible (Chemical bonding)

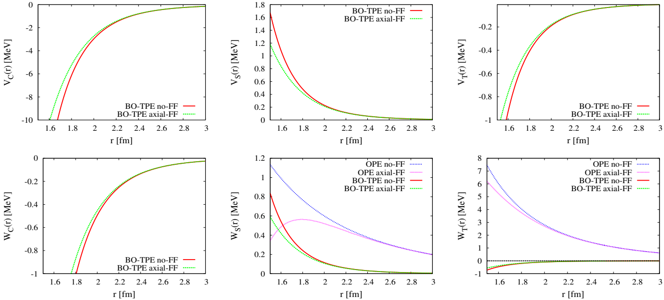


Finite size effects

- NN potential in the Born-Oppenheimer approximation — Calle Cordon, RA, '12

$$\bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\boldsymbol{r}) = V^{1\pi}_{NN,NN}(\boldsymbol{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,,$$

- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Finite size effects set in at 2fm $\rightarrow$ exchange quark effects become explicit
- High quality potentials confirm these trends.



COARSE GRAINING

The number of parameters (for $E_{\rm LAB} \le 350$ MeV)

- At what distance look nucleons point-like ?

$$r > 2\text{fm}$$

- When is OPE the ONLY contribution ?

$$r_c > 3\text{fm}$$

- What is the minimal resolution where interaction is elastic ?

$$p_{\rm max} \sim \sqrt{M_N m_\pi} \to \Delta r = 1/p_{\rm max} = 0.6\text{fm}$$

- How many partial waves must be fitted ?

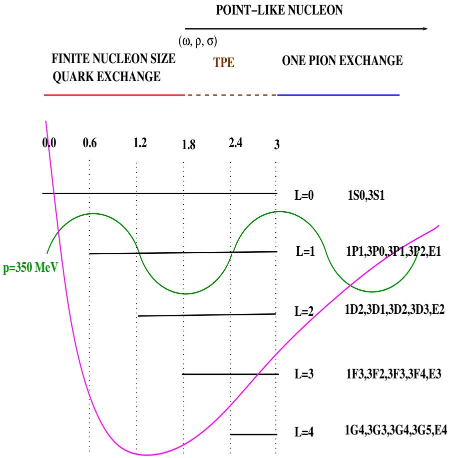
$$l_{\rm max} = p_{\rm max} r_c = r_c/\Delta r = 5$$

- Minimal distance where centrifugal barrier dominates

$$\frac{l(l+1)}{r_{\rm min}^2} \le p^2$$

- How many parameters ?
$({}^1S_0, {}^3S_1)$, $({}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2)$, $({}^1D_2, {}^3D_1, {}^3D_2, {}^3D_3)$, $({}^1F_3, {}^3F_2, {}^3F_3, {}^3F_4)$

$$2 \times 5 + 4 \times 4 + 4 \times 3 + 4 \times 2 + 4 \times 1 = 50$$


POINT–LIKE NUCLEON
(ω, ρ, σ)
FINITE NUCLEON SIZE
QUARK EXCHANGE
TPE
ONE PION EXCHANGE
0.0
0.6
1.2
1.8
2.4
3
L=0
1S0,3S1
p=350 MeV
L=1
1P1,3P0,3P1,3P2,E1
L=2
1D2,3D1,3D2,3D3,E2
L=3
1F3,3F2,3F3,3F4,E3
L=4
1G4,3G3,3G4,3G5,E4

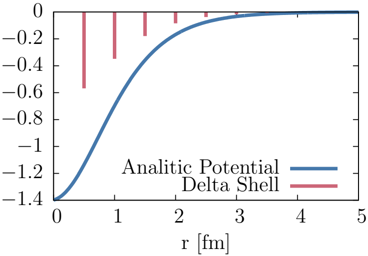
Delta Shell Potential

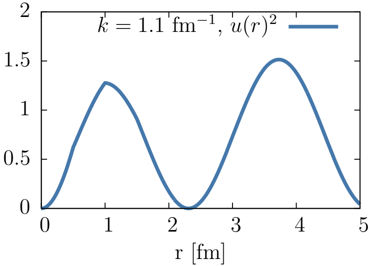
- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu}\delta(r - r_i)$$

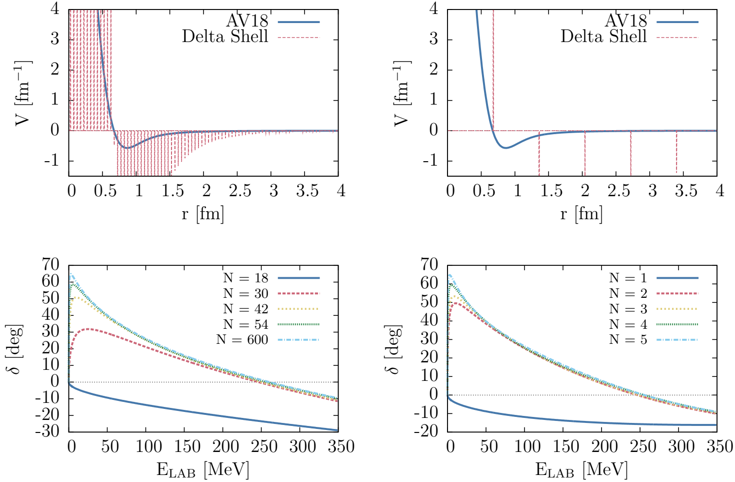
[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k \sim 2$ fm^{-1}
- Optimal sampling, $\Delta r \sim 0.5$fm





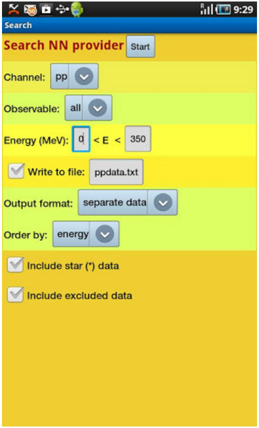
Coarse Graining the AV18 potential



Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq 3.0$ fm
  - Unknown interaction
  - λ_i parameters fitted to scattering data
- Outermost region $r \geq 3.0$ fm
  - Long range interaction
  - Described by OPE and EM effects
    - Coulomb interaction V_{C1} and relativistic correction V_{C2} (pp)
    - Vacuum polarization V_{VP} (pp)
    - Magnetic moment V_{MM} (pp and np)

Fitting NN observables



- Database of NN scattering data obtained till 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN provider for Android
    - Google Play Store

[J.E. Amaro, R. Navarro-Perez, and E. Ruiz-Arriola]

- 2868 pp data and 4991 np data
- 3σ criterion by Nijmegen to remove possible outliers

Fitting NN observables

- Delta shell potential in every partial wave

$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{n=1}^{N} (\lambda_n)_{l,l'}^{JS} \delta(r - r_n) \qquad r \leq r_c = 3.0\text{fm}$$

- Strength coefficients λ_n as fit parameters
- Fixed and equidistant concentration radii $\Delta r = 0.6$ fm
- EM interaction is crucial for pp scattering amplitude

$$\begin{aligned} V_{C1}(r) &= \frac{\alpha'}{r} \,, \\ V_{C2}(r) &\approx -\frac{\alpha\alpha'}{M_p r^2} \,, \\ V_{VP}(r) &= \frac{2\alpha\alpha'}{3\pi r} \int_1^\infty dx\, e^{-2m_e r x} \left[1 + \frac{1}{2x^2}\right] \frac{(x^2-1)^{1/2}}{x^2} \,, \\ V_{MM}(r) &= -\frac{\alpha}{4M_p^2 r^3} \left[\mu_p^2 S_{ij} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}\right] \end{aligned}$$

STATISTICS

Self-consistent fits

- We test the assumption

$$O_i^{\rm exp} = O_i^{\rm th} + \xi_i \Delta O_i \qquad i = 1, \dots, N_{\rm Data} \qquad \xi_i \in N[0,1]$$

- Least squares minimization $\mathbf{p} = (p_1, \dots,)$

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{N} \left(\frac{O_i^{\rm exp} - F_i(\mathbf{p})}{\Delta O_i^{\rm exp}} \right)^2 \to \min_{\lambda_i} \chi^2(\mathbf{p}\chi^2(\mathbf{p}_0) \qquad (3)$$

- Are residuals Gaussian ?

$$R_i = \frac{O_i^{\rm exp} - O_i^{\rm th}}{\Delta O_i} \qquad O_i^{\rm th} = F_i(\mathbf{p}_0) \qquad i = 1, \dots, N \qquad (4)$$

If $R_i \in N[0,1]$ self-consistent fit.

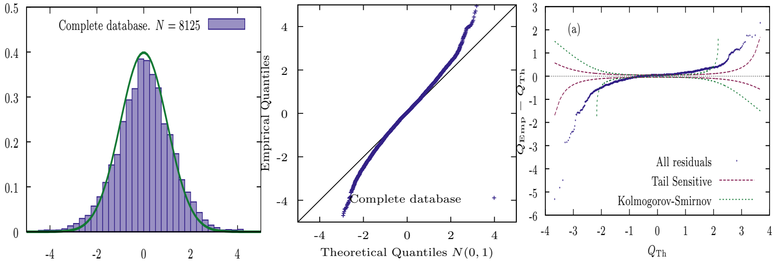
- Normality test for a finite sample with N elements $\to$ Probability (Confidence level) p-value

$$\chi^2_{\rm min} = 1 \pm \sigma \sqrt{\frac{2}{\nu}} \qquad \nu = N_{\rm Dat} - N_{\rm Par} \qquad p = 1 - \int_\sigma^\sigma dt \frac{e^{-t^2}}{\sqrt{2\pi}}$$

Histograms, Moments, Kolmogorov-Smirnov, **Tail Sentitive** QQ-plots

Normality tests

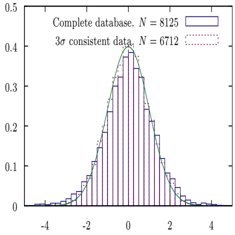
- Does the sequence
$$x_1^{\rm exp} \le x_2^{\rm exp} \le \cdots \le x_N^{\rm exp} \in N[0,1]$$
- We compute the theoretical points
$$\frac{n}{N+1} = \int_{-\infty}^{x_n^{\rm th}} dt \frac{e^{-t^2/2}}{\sqrt{2\pi}}$$
- The Q-Q plot is $x_n^{\rm th}$ vs $x_n^{\rm exp}$
- For large N
$$x_n^{\rm th} - x_n^{\rm exp} = \mathcal{O}(1/\sqrt{N})$$

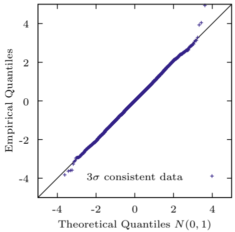


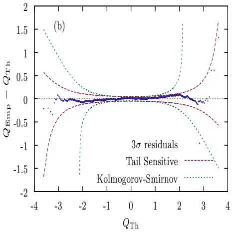
Granada-2013 np+pp database

Selection criterium

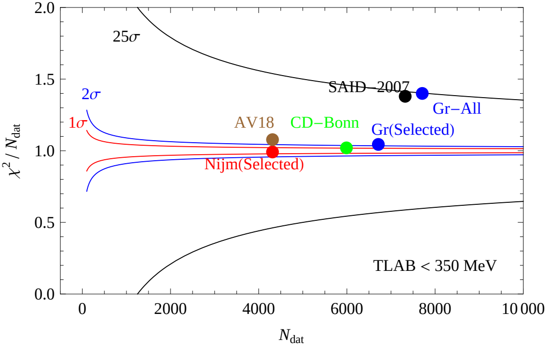
- Mutually incompatible data. Which experiment is correct? Is any of the two correct?
- Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  1. Fit to all data $(\chi^2/\nu > 1)$
  2. Remove data sets with improbably high or low χ^2 (3σ criterion)
  3. Refit parameters
  4. Re-apply 3σ criterion to all data
  5. Repeat until no more data is excluded or recovered







To believe or not to believe

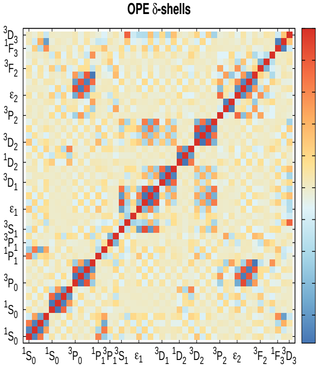


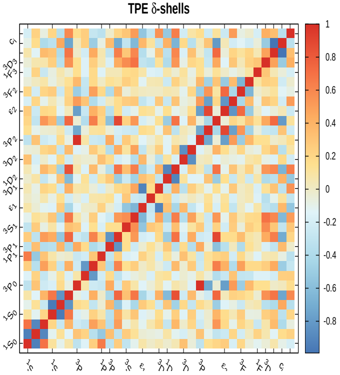
$$\chi^2_{\min}/\nu = 1 \pm \sqrt{2/\nu}$$

- Charge dependence in OPE
- Magnetic-Moments, Vacuum polarization, ...

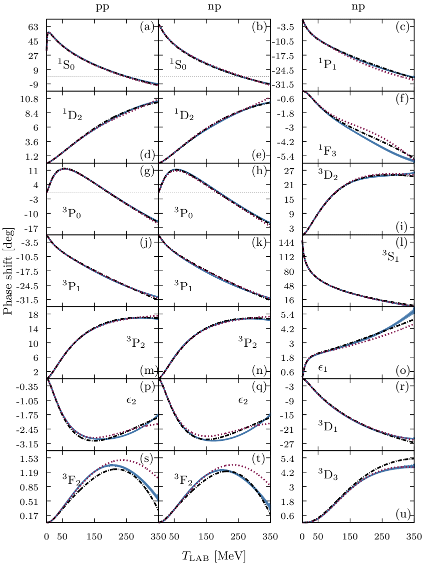
Correlations

The strengths of the coarse grained potential are largely independent !!
$\rightarrow$ Good Fitting Parameters

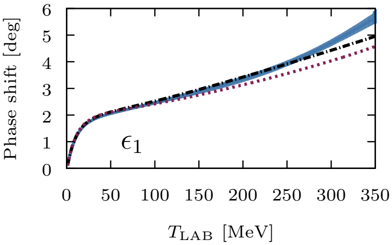




Phase shifts



- Phase shifts for every partial
- Statistical uncertainty propagated directly from covariance matrix



Wolfenstein Parameters

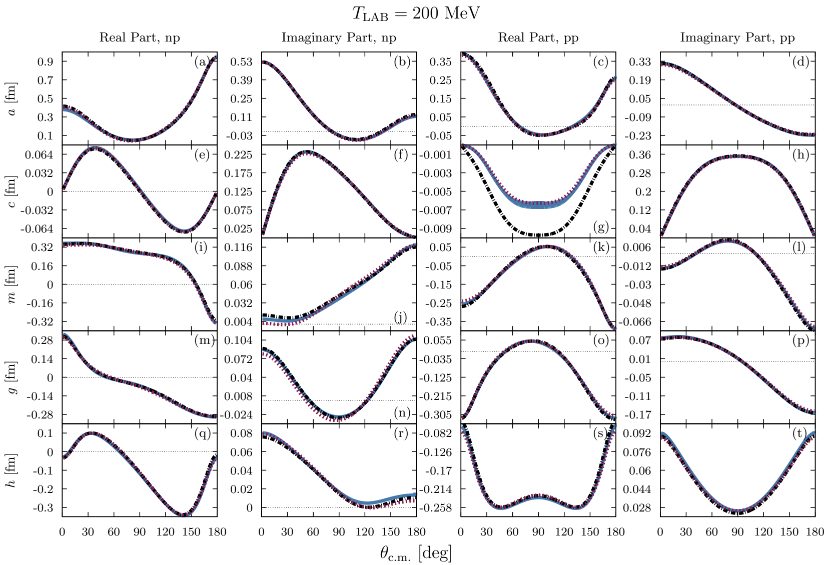
- A complete parametrization of the on-shell scattering amplitudes
- Five independent complex quantities
- Function of Energy and Angle

$$
\begin{aligned}
M(\mathbf{k}_f, \mathbf{k}_i) &= a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g - h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) \\
&\quad + (g + h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, \mathbf{n})
\end{aligned}
$$

- Scattering observables can be calculated from M

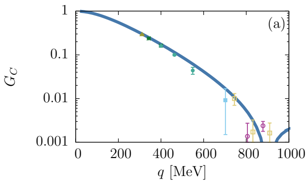
[Bystricky, J. et al, Jour. de Phys. 39.1 (1978) 1]

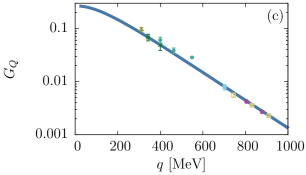
Wolfenstein Parameters

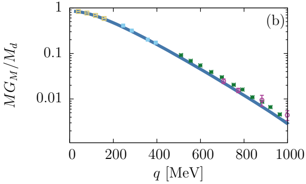


Deuteron Properties

	Delta Shell	Empirical	Nijm I	Nijm II	Reid93	AV18	CD-Bonn
E_d(MeV)	Input	2.224575(9)	Input	Input	Input	Input	Input
η	0.02493(8)	0.0256(5)	0.0253	0.0252	0.0251	0.0250	0.0256
$A_S(\mathrm{fm}^{1/2})$	0.8829(4)	0.8781(44)	0.8841	0.8845	0.8853	0.8850	0.8846
$r_m(\mathrm{fm})$	1.9645(9)	1.953(3)	1.9666	1.9675	1.9686	1.967	1.966
$Q_D(\mathrm{fm}^2)$	0.2679(9)	0.2859(3)	0.2719	0.2707	0.2703	0.270	0.270
P_D	5.62(5)	5.67(4)	5.664	5.635	5.699	5.76	4.85
$\langle r^{-1}\rangle(\mathrm{fm}^{-1})$	0.4540(5)			0.4502	0.4515		

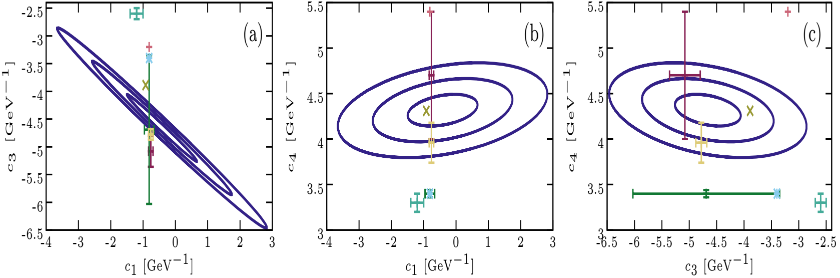






Chiral Two Pion Exchange

Fit to from Granada-2013 np+pp database



To count or not to count

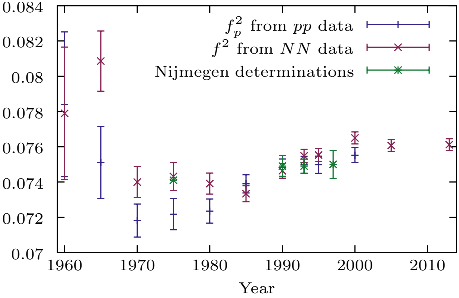
- We can fit CHIRAL forces to ANY energy and look if counterterms are compatible with zero within errors
- We find that if $E_{\rm LAB} \le 125 {\rm MeV}$ Weinberg counting is INCOMPATIBLE with data.
- You have to promote D-wave counterterms.
  N2LO-Chiral TPE + N3LO-Counterterms $\to$ Residuals are normal
  Piarulli,Girlanda,Schiavilla,Navarro Pérez,Amaro,RA, PRC
- We find that if $E_{\rm LAB} \le 40 {\rm MeV}$ TPE is INVISIBLE
- We find that peripheral waves predicted by 5th-order chiral perturbation theory ARE NOT consistent with data within uncertainties

$$|\delta^{\rm Ch,N4LO} - \delta^{\rm PWA}| > \Delta\delta^{\rm PWA,stat}$$

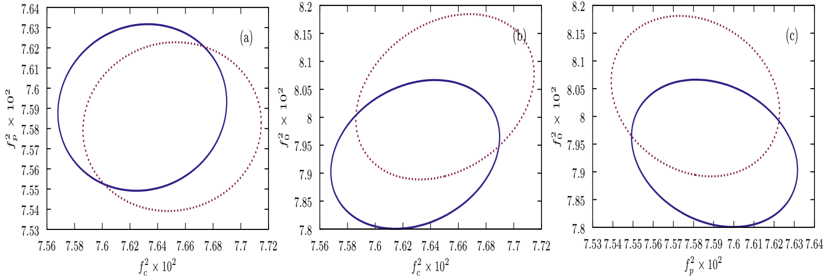
COUPLING CONSTANTS

Arqueological Flashback

Chronological recreation of pion-nucleon coupling constants



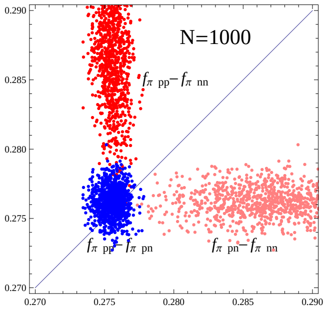
The pion-nucleon coupling constants f_p^2, f_0^2 and f_c^2

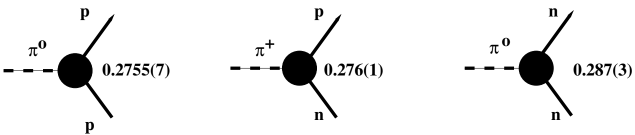


Fits to the Granada-2013 database.

f^2	f_0^2	f_c^2	CD-waves	χ^2_{pp}	χ^2_{np}	$N_{\rm Dat}$	$N_{\rm Par}$	χ^2/ν
0.075	idem	idem	$^1S_0$	3051	3951	6713	46	1.051
0.0761(3)	idem	idem	$^1S_0$	3051	3951	6713	46+1	1.051
-	-	-	$^1S_0, P$	2999	3951.40	6713	46+3	1.043
0.0759(4)	0.079(1)	0.0763(6)	$^1S_0, P$	3045	3870	6713	46+3+9	1.039

The πNN vertices





CONCLUSIONS

Neutron-Neutron vs Proton-Proton (Polarized)

nn interaction is more intense than pp interaction