The case for ***Duality Violations*** *in the analysis of hadronic τ decays*

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Workshop on the Determination of the Fundamental Parameters in QCD

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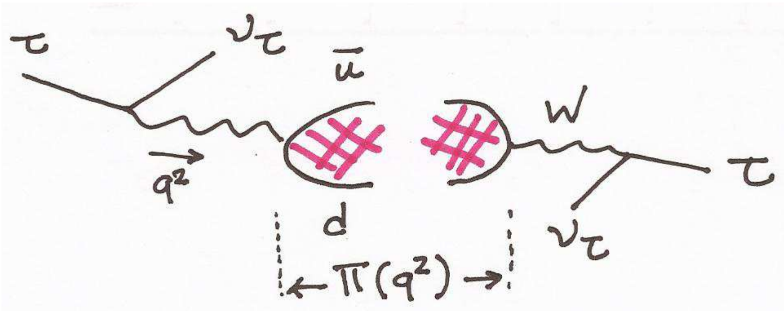
QCD in τ decay

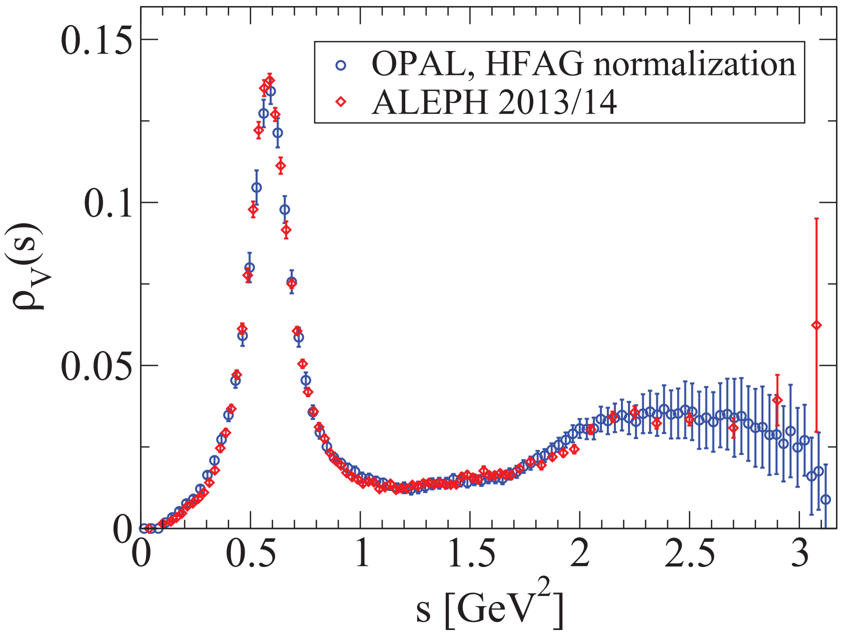


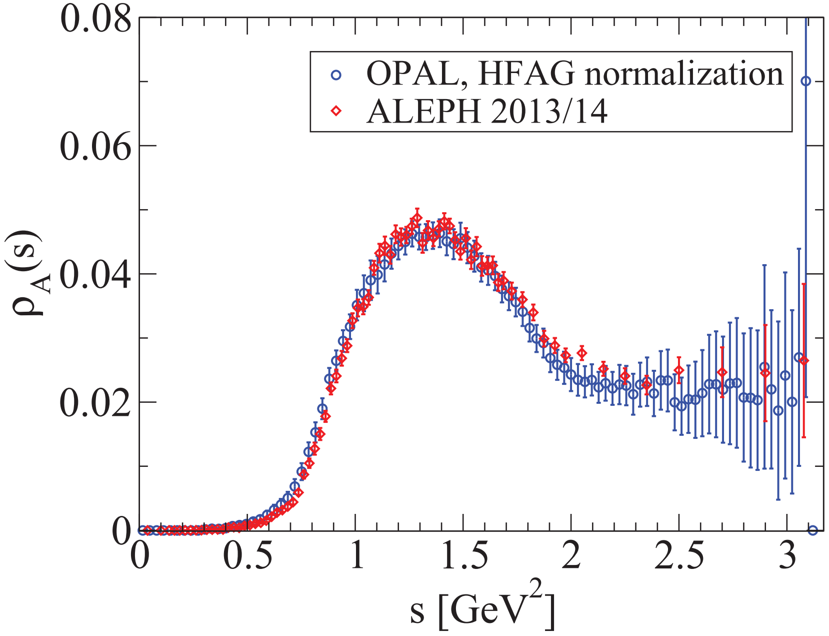
$$w_T(s;s_0) = \left(1 + 2\frac{s}{s_0}\right)\underbrace{\left(1 - \frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$$w_L(s;s_0) = 2\left(\frac{s}{s_0}\right)\underbrace{\left(1 - \frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$s_0 = m_\tau^2$ $\qquad \rho_{V,A} = \frac{1}{\pi}\mathrm{Im}\Pi_{V,A}$

$$\frac{\Gamma(\tau\to\nu_\tau \mathrm{H}_{ud}(\gamma))}{\Gamma[\tau\to\nu_\tau e\bar{\nu}_e(\gamma)]} = 12\pi^2|V_{ud}|^2 S_{EW}\int_0^{s_0}\frac{ds}{s_0}\left[w_T(s;s_0)\rho^{(1+0)}_{V+A}(s) - w_L(s;s_0)\rho^{(0)}_A(s)\right]$$

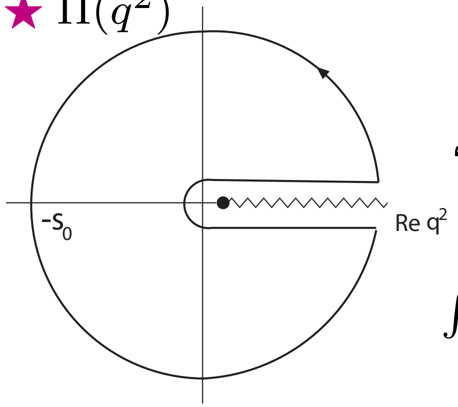




Theoretical Foundations (I)

Shankar '77; Braaten-Narison-Pich '92

★ $\Pi(q^2)$



"Cauchy's Theorem" $(z = q^2\ ;\ \rho(t) = \frac{1}{\pi}\mathrm{Im}\Pi\ ;\ w_n = \text{polynomial})$:

$$\int_0^{s_0} dt\ w_n(t)\ \underbrace{\rho(t)}_{exp.} = \frac{1}{2i\pi}\oint_{|z|=s_0} dz\ w_n(z)\ \Pi(z)$$

$$= \frac{1}{2i\pi}\oint_{|z|=s_0} dz\ w_n(z)\ \Big[\underbrace{\Pi_{\mathrm{OPE}}(z)}_{\mathcal{O}(\alpha_s^4)} + \underbrace{\Pi(z) - \Pi_{\mathrm{OPE}}(z)}_{\Pi_{DV}(z)}\Big]$$

★ $\Pi_{DV} \to 0 \Longleftrightarrow \Pi_{\mathrm{OPE}} \to \Pi$.

(Cata-Golterman-S.P. '05)

However,

- Π_{OPE} expected asymptotic (at best) : $\Pi_{DV}(z) \to 0,\ z \to \infty$.
- OPE no good on the Minkowski axis (spect. fnct. shows oscillations)

$\Longrightarrow$ pinching

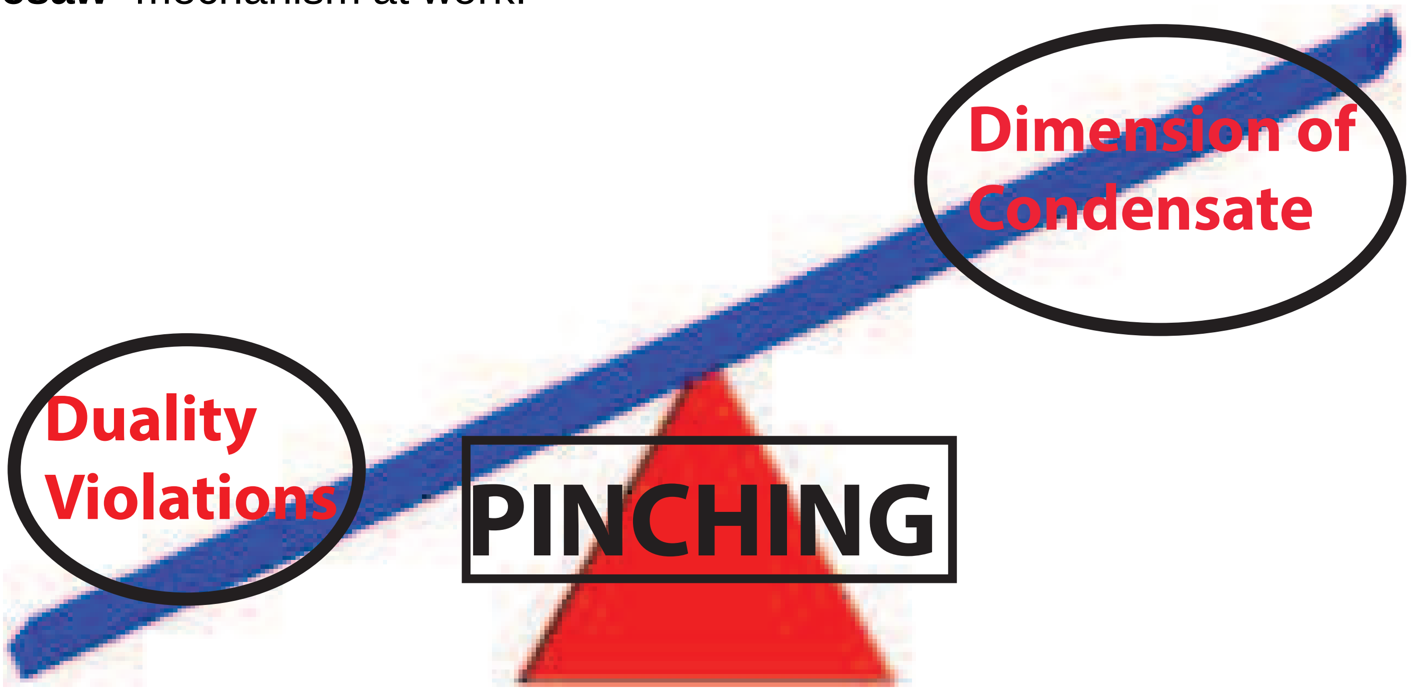
Main Theoretical Message:

(Maltman-Yavin ’08, Boito et al. ’11)

★ No free lunch: with pinching one has a **price to pay**:

> It is not possible to simultaneously **suppress DVs** and **condensates**.

★ “**Seesaw**” mechanism at work:

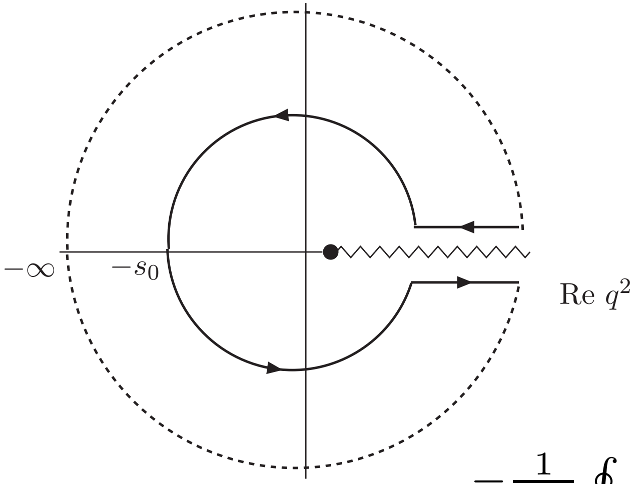


Theoretical Foundations (II)

- Need a better control of systematic error

$\Longrightarrow$ need **quantitative** knowledge of DVs.

- $\Pi_{DV}(s) \to 0$, as $s \to \infty$. Then:



(Cata-Golterman-S.P. '05)

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi_{DV}(z) = -\underbrace{\int_{s_0}^{\infty} ds}_{\text{extrapolation!}}\; w(s)\; \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s)$$

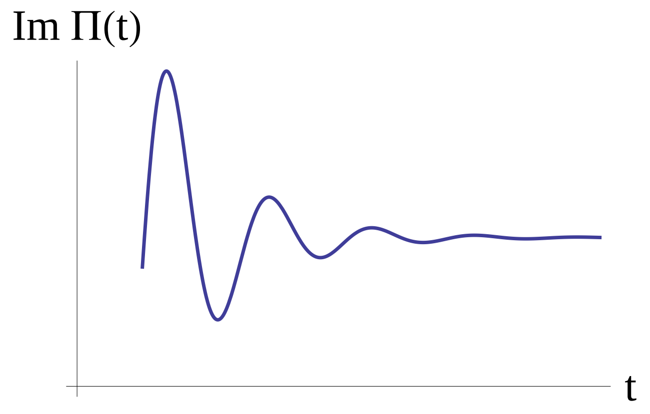
Theoretical Foundations (III)

Cata,Golterman, S.P. '05, '08

★ We make an **educated guess:**

For s_0 large enough:

$$\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s) \simeq e^{-\delta}\ \underbrace{e^{-\gamma\, s}}_{\text{asy. exp.}}\ \underbrace{\sin(\alpha+\beta s)}_{\text{Regge Th.}}$$



independently for V and A (i.e. 8 DV parameters in total).

- Assuming no DVs $\equiv$ assuming $e^{-\delta}=0$ (**not** favored by data$\rightarrow$ Boito's talk).

Theoretical Foundations (and IV)

- $e^{-\gamma s}$ natural in asymptotic analysis (e.g. Renormalons, etc...).

- Regge Theory (i.e. equally-spaced spectrum) works rather well phenomenologically.

- Ansatz is reproduced in a specific model (Blok, Shifman, Zhang '97) .

- It has been applied to determine LECs and condensates from $\langle VV - AA \rangle$.

(Glez.-Alonso, Pich, Prades '10; Rguez.-Sanchez, Glez.-Alonso, Pich '15, '16 ;

(Boito et al. '13, Golterman et al. '14, Boito et al. '15)

(Critical) Review of the "Old Strategy"

(LeDiberder-Pich '92)

- Use 5 pinched weights

$$w_{kl}(y) = (1-y)^2(1+2y)(1-y)^k y^l \quad , \quad y = s/s_0$$

with $(k,l) = \{(0,0),(1,0),(1,1),(1,2),(1,3)\}$.

- Set OPE condensates $C_{10,12,14,16} = 0$. (This assumption $\sim$ OPE is convergent.)
- Set Duality Violations = 0.
- Fit to 5 data points to extract 4 param. (1 dof) : α_s and $C_{4,6,8}$ only at $s = m_\tau^2$.
- May use V and A, but assume $V+A$ more reliable.

(Davier et al. '14)

$$\begin{aligned} \langle \frac{\alpha_s}{\pi} GG \rangle \quad &= \quad (-0.5 \pm 0.3) \times 10^{-2}\ \text{GeV}^4 \ , \qquad \chi^2 = 0.43,\ p = 51\% \qquad V\ , \\ &\quad\ \ (-3.4 \pm 0.4) \times 10^{-2}\ \text{GeV}^4 \ , \qquad \chi^2 = 3.4,\ p = 7\% \qquad A\ , \\ &\quad\ \ (-2.0 \pm 0.3) \times 10^{-2}\ \text{GeV}^4 \ , \qquad \chi^2 = 1.1,\ p = 29\% \qquad V + A\ . \end{aligned}$$

- Check Weinberg sum rules.

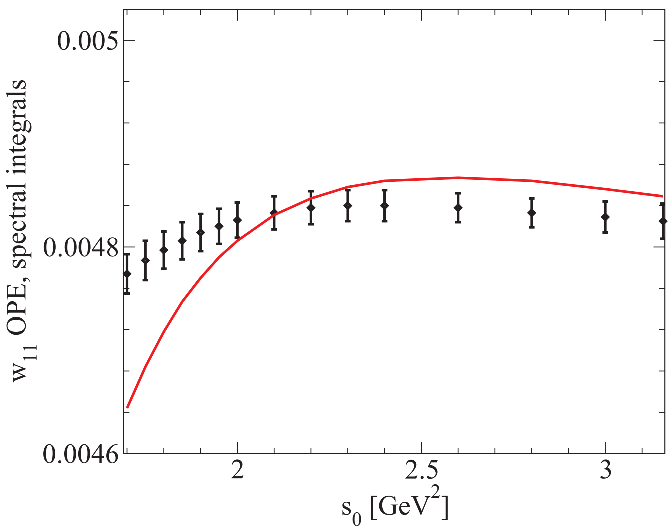
Tests: w_{11}, w_{12}, w_{13}

Looking only at $s = m_\tau^2$ potentially misleading. (Maltman-Yavin '08).

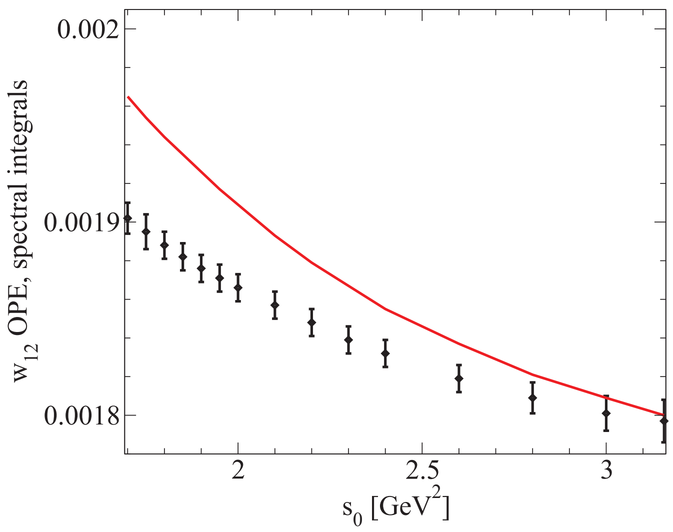
(Davier et al. '14)

CIPT, V+A:

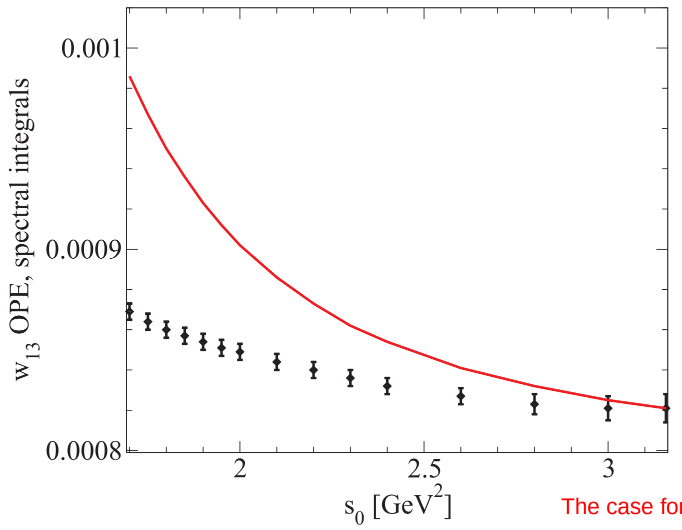
w_{11}



w_{12}



w_{13}



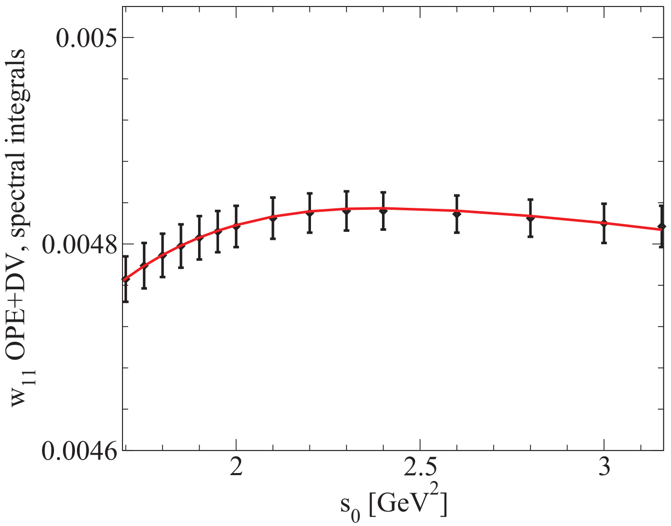
Tests: w_{11}, w_{12}, w_{13}

$D > 8$ condensates vital !

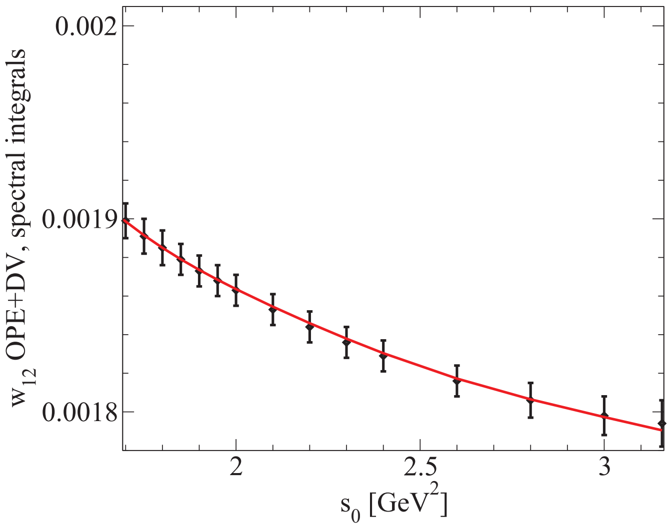
(Boito et al. '15)

CIPT, V+A:

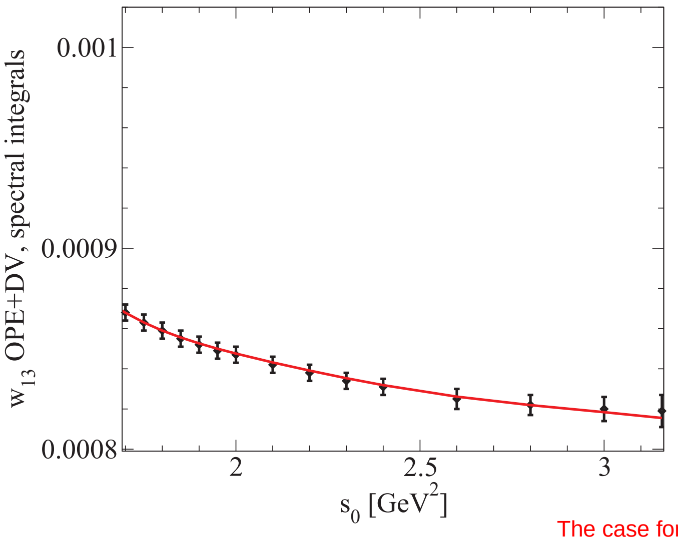
w_{11}



w_{12}



w_{13}



Not part of our fit !

Used $s_{min} = 1.55\ \text{GeV}^2$
$w = 1, 1 - y^2, w_\tau$

Results

(Boito et al. '15)$\longrightarrow$ Boito's talk.

Analysis based on $w = 1, 1 - y^2, (1-y)^2(1+2y)$; V and/or A (ALEPH).

(FOPT) $\alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$

(CIPT) $\alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$

N.B. "Old Strategy" produces a shift: $\alpha_s(m_\tau) \sim +0.03$ higher, (and $\sim$ half errors)
(Davier et al. '14)

$$R_{V+A} = N_c\, S_{EW} |V_{ud}|^2 \Big(1 + \delta_P + \underbrace{\delta_6 + \delta_8 + \delta_{DV}}_{\delta_{NP}}\Big)$$

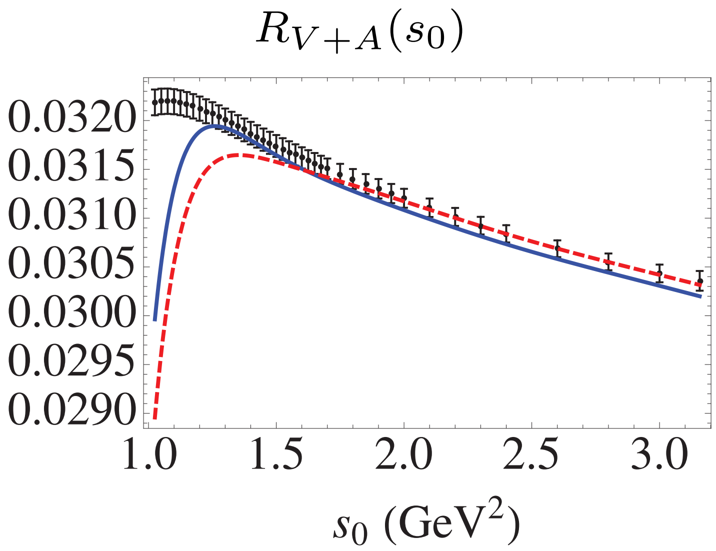
(FOPT) $\delta_{NP} = 0.020 \pm \mathbf{0.009}$

(CIPT) $\delta_{NP} = 0.016 \pm \mathbf{0.010} \longleftrightarrow \delta_{NP}^{\text{"Old Strategy"}} = -0.0064 \pm \mathbf{0.0013}$

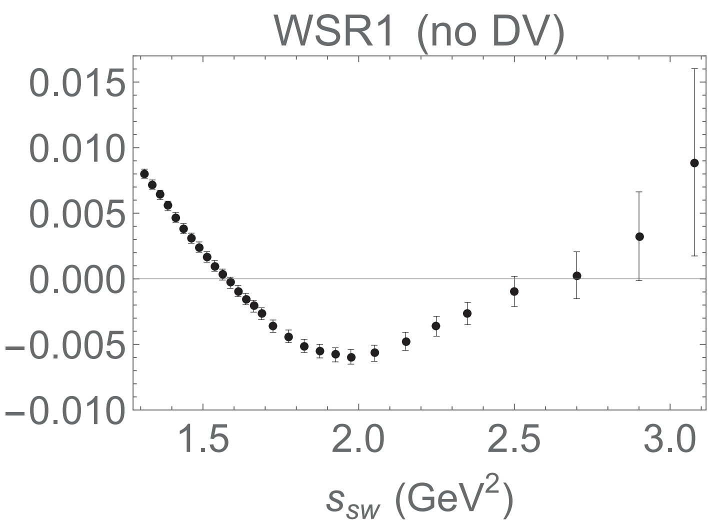
(Davier et al. '14)

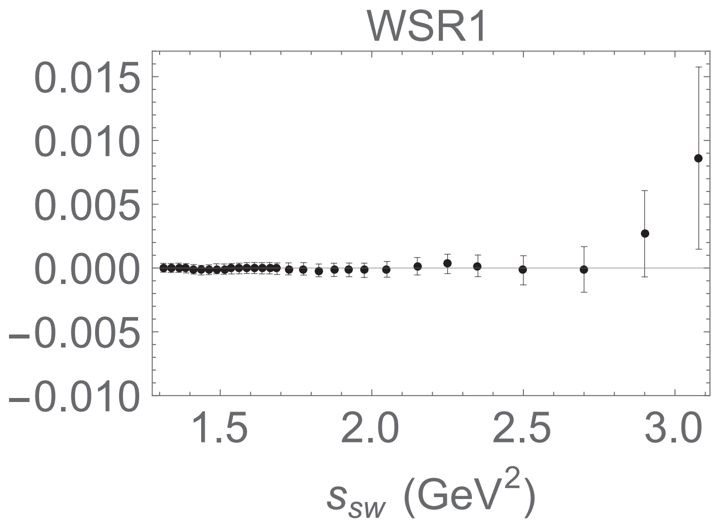
Classic Tests

red=CIPT blue =FOPT (Boito et al. ’15).



Weinberg sum rule: $\int_0^\infty ds\, \left(\rho_V^{(1)}(s) - \rho_A^{(1)}(s)\right) - 2f_\pi^2 = 0$

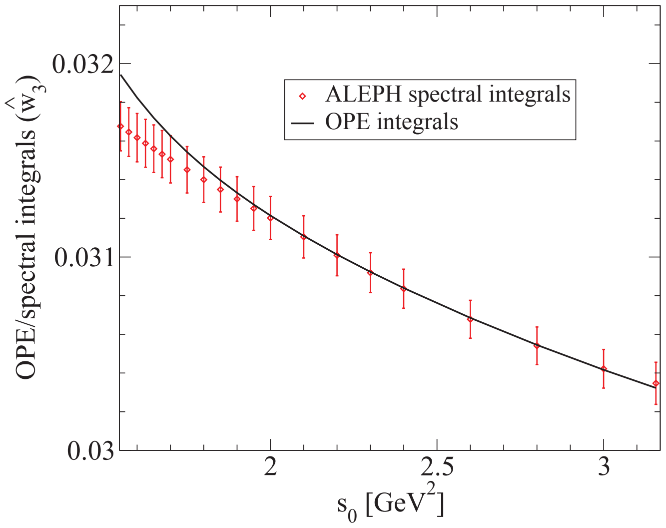




An illustrative exercise

- **V + A , FOPT** , fit to w_τ for $1.95\ \mathrm{GeV}^2 \leq s \leq m_\tau^2$. ($s_{min} = 2.2\ \mathrm{GeV}^2$).

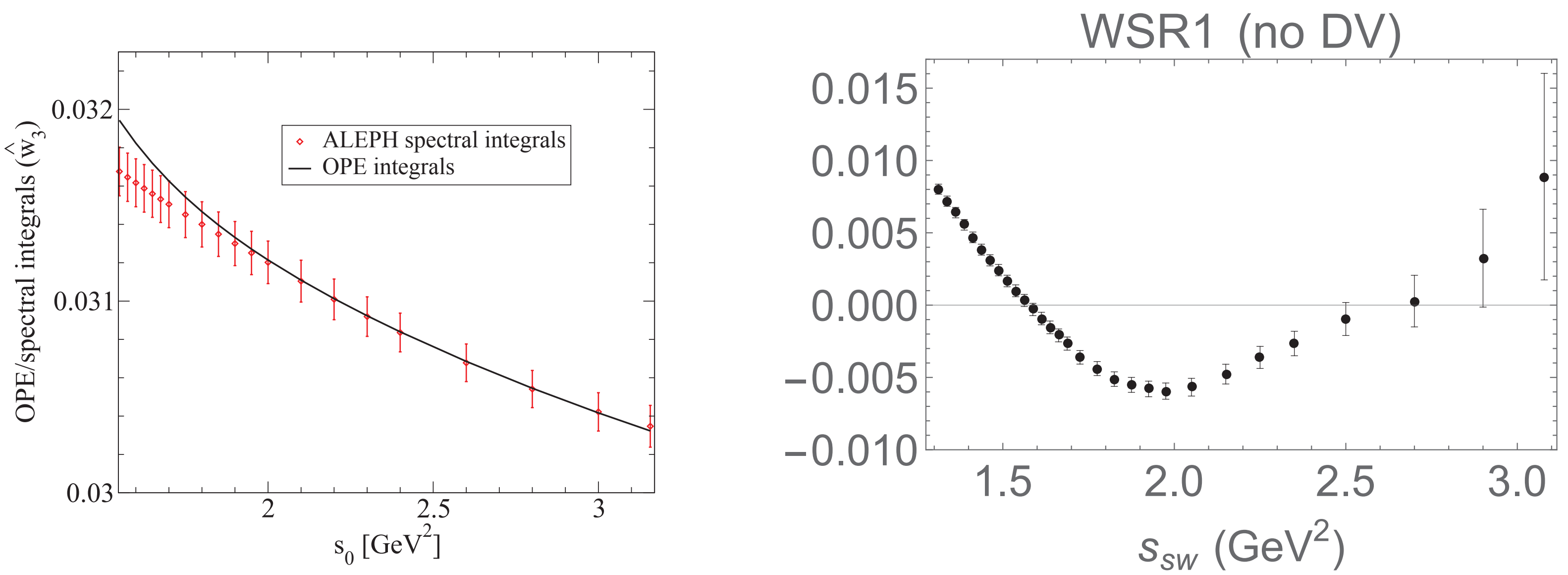
No DVs included !



$$\alpha_s(m_\tau^2) = 0.330 \pm 0.006\ ,$$
$$C_{6,V+A} = 0.0070 \pm 0.0022\ \mathrm{GeV}^6\ ,$$
$$C_{8,V+A} = -0.0088 \pm 0.0042\ \mathrm{GeV}^8\ .$$

An illustrative exercise

- $\mathbf{V+A}$, **FOPT** , fit to w_3 for $1.95\ \mathrm{GeV}^2 \le s \le m_\tau^2$. ($s_{min} = 2.2\ \mathrm{GeV}^2$).



$$\alpha_s(m_\tau^2) = 0.330 \pm 0.006 \xrightarrow{DV} 0.301 \pm 0.006 \pm 0.009$$

$$C_{6,V+A} = 0.0070 \pm 0.0022\ \mathrm{GeV}^6 \xrightarrow{DV} -0.0127 \pm 0.0020 \pm 0.0066\ \mathrm{GeV}^6$$

$$C_{8,V+A} = -0.0088 \pm 0.0042\ \mathrm{GeV}^8 \xrightarrow{DV} 0.0399 \pm 0.0040 \pm 0.021\ \mathrm{GeV}^8$$

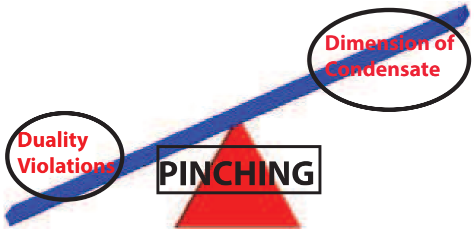
Conclusions and Outlook

- DVs are clearly visible in the data.

(DVs are not a question of principle, they exist in practice.)

- Pinching does not allow a simultaneous reduction of DVs **and** higher-dim condensates

(unlike what has been assumed so far in the “Old Strategy" Method).



This introduced an unquantified systematic error.

- I see no way to make progress without a better understanding of DVs and/or the OPE as a series expansion.

Resurgence ? (Shifman ’14)

Functional Analysis Methods ? (Caprini, Golterman, S.P. ’14)

Conclusions and Outlook (II)

- We have introduced a new strategy based on an educated guess for DVs which avoids this flaw and allows the data to determine both the contribution from DVs and condensates.($\rightarrow$ Boito's talk)

- The new strategy passes all known tests, experimental and theoretical, performing better than the "Old Strategy".

N.B. The "Old Strategy" also uses **a model**:

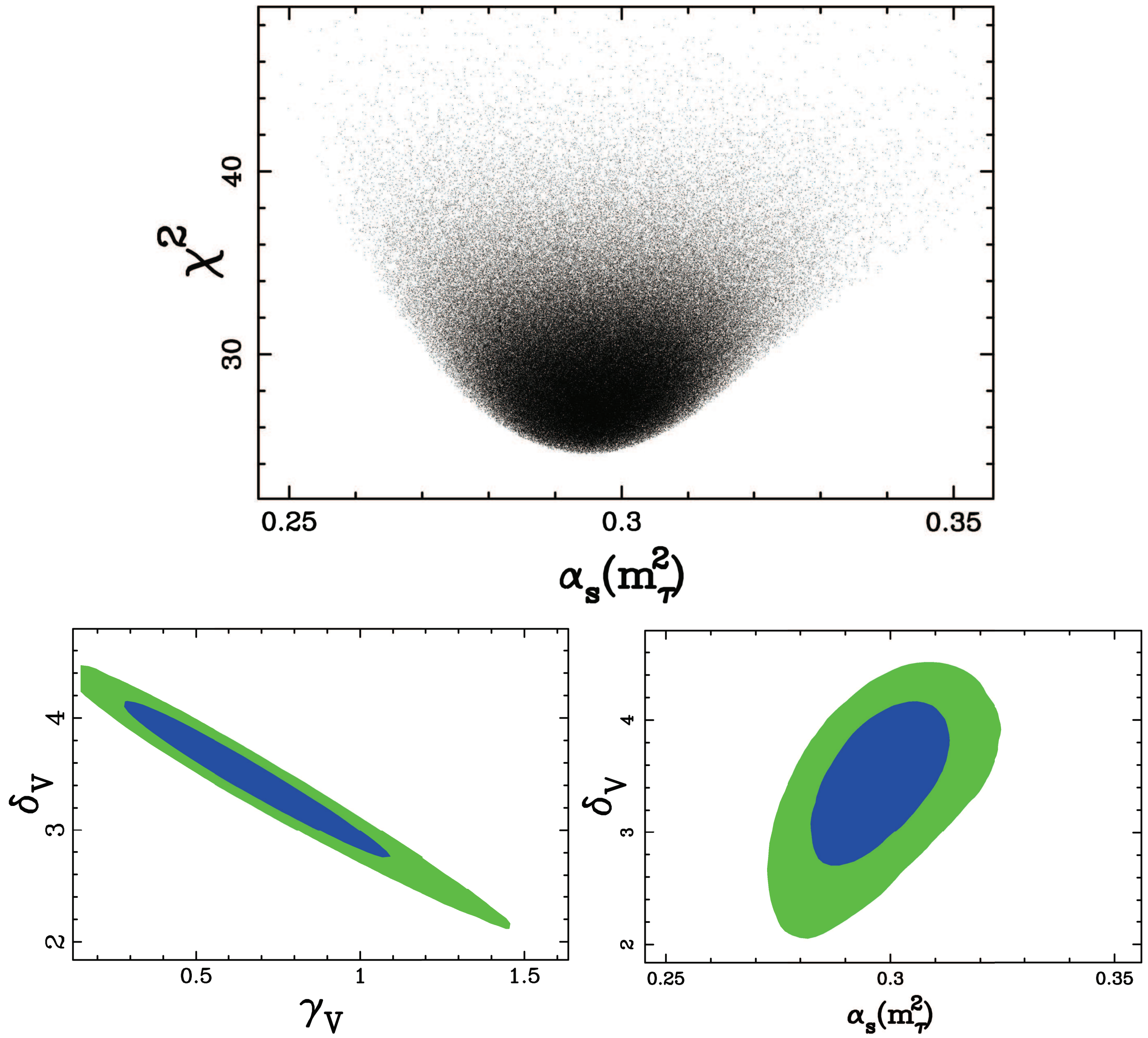
$$\mathrm{e}^{-\delta} = 0 \quad \text{and} \quad \langle O_{10,12,14,16} \rangle = 0.$$

Not favored by data/present theoretical knowledge.($\rightarrow$ Boito's talk)

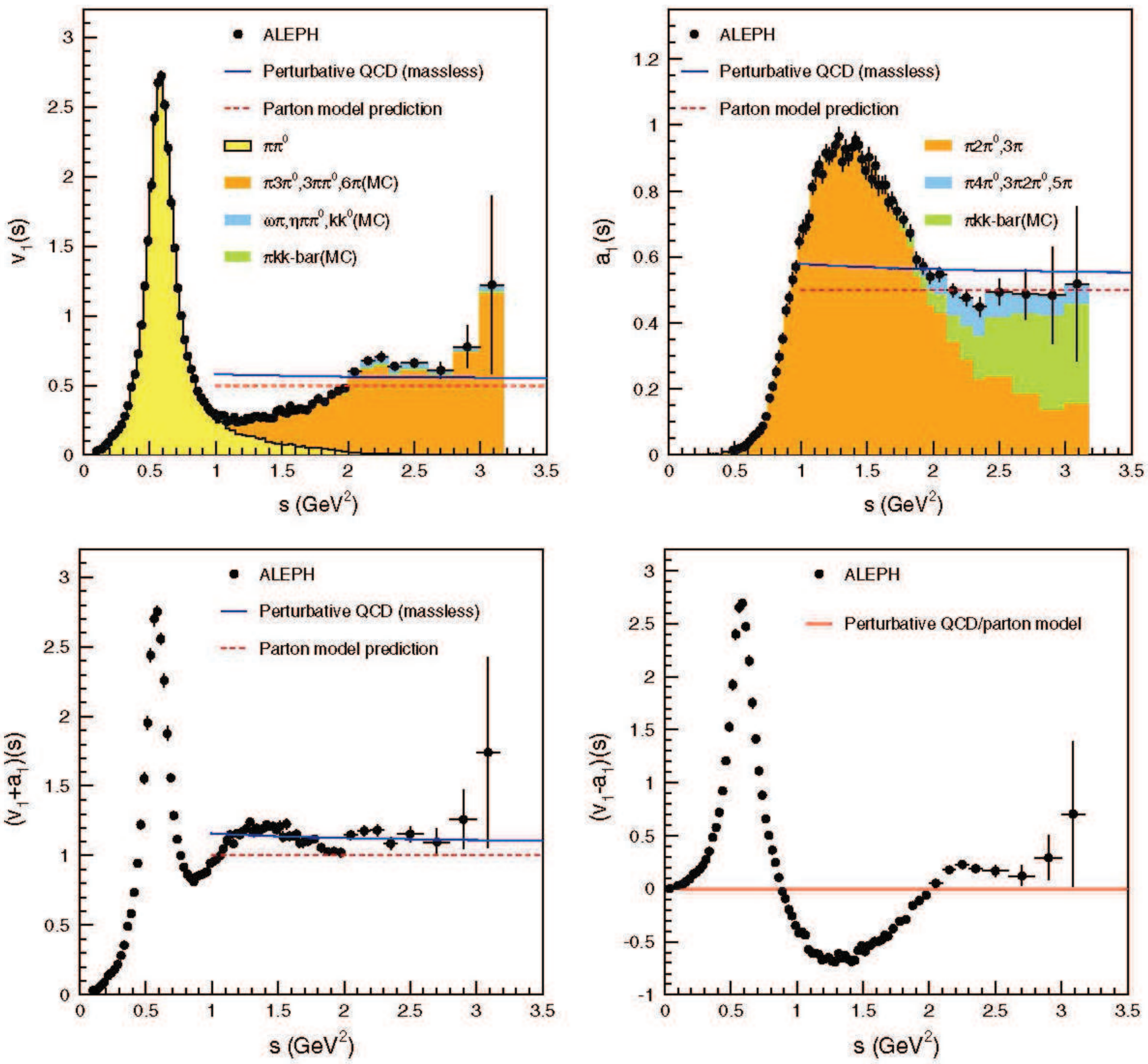
- Better data (Babar and Belle ?) would help significantly.

BACK-UP SLIDES

Example: FOPT χ^2 Fit Results (ALEPH)



2014 Aleph's Spectral Functions



Comments on $V + A$

- DV oscillations are still present in $V + A$ (although of a smaller size than in V and A).

- Since we have a good representation of V and A, we also have it of their sum $V + A$.

- Fit results show that DV's exponent in A is $\ll$ than in V, so the reduction for $s \sim 2 - 3\, GeV^2$ is accidental. At still larger s, the DVs in V will dominate.

- The expectation of strong cancellation of $D = 6$ terms in OPE in $V + A$ is based on the vacuum saturation ansatz. However, the data shows that this ansatz is a very poor approximation.

A Toy Model

Blok-Shifman-Zhang '98; Cata-Golterman-SP '05, '08; Jamin '11

Take $\Lambda_{QCD} = 1$; $F = 1$, decay constant.

- 1 resonance ($M \to M + i\Gamma/2$):

$$\frac{1}{q^2 - n} \longrightarrow \frac{1}{q^2 - n - i\sqrt{n}\,\Gamma}$$

- Regge-like tower: $n = 1, 2, 3, \ldots$

$$\Pi(q^2) \sim \sum_n^\infty \frac{1}{z+n} \sim \psi(z) = \frac{d \log \Gamma(z)}{dz} \,, \quad z = \underbrace{(-q^2)^\zeta}_{\text{cut, } q^2>0} \,, \quad \zeta \simeq 1 - \frac{1}{N_c}$$

- For $q^2 < 0 \longrightarrow \Pi(q^2) \sim \log z + \sum \frac{c_n}{z^n}$, $c_n \sim n!$

- For $q^2 > 0 \longrightarrow \psi(z) = \psi(-z) - \frac{1}{z} - \pi \cot(\pi z)$,

$$\mathrm{Im}\Pi(q^2) \sim \mathrm{Im}(\log z) + \mathrm{Im} \sum \frac{c_n}{z^n} + \quad \mathrm{e}^{-\frac{2\pi}{N_c} q^2} \sin(\alpha + \beta\, q^2) \quad ; \quad \alpha, \beta \sim 1$$

Relative weight of $D = 0, 4, 6, 8$ *to* w_{kl}

TABLE VI. The $D = 4$, 6 and 8 and α_s-dependent $D = 0$ contributions to the $s_0 = m_\tau^2$, $V + A$, $w_{k\ell}$ moments corresponding to the $V + A$ OPE fit parameter results of Table 4 of Ref. [1].

(k, ℓ)	α_s-dependent $D = 0$	$D = 4$	$D = 6$	$D = 8$
(0,0)	0.005173	−0.000008	−0.000117	0.000033
(1,0)	0.004399	−0.000361	−0.000117	0.000082
(1,1)	0.000365	0.000350	−0.000039	−0.000049
(1,2)	0.000208	0.000002	0.000039	−0.000016
(1,3)	0.000081	0.000000	0.000000	0.000016

$C_{4,6,8}$ determined by $w_{11,12,13}$.