HADRONIC g-2 OF THE MUON: A THEORETICAL QCD DETERMINATION

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S. Bodenstein et al. PRD **85**,014029 (2012); **88**, 014005 (2013)

MUON

$$a_\mu|_{EXP} = 1\ 165\ 920\ 8.9\ (6.3) \times 10^{-10}$$

$$a_\mu = 0.001\ldots\ !!!$$

$$a_\mu|_{THY} = 1\ 165\ 918\ 1.8\ (7.6) \times 10^{-10}$$

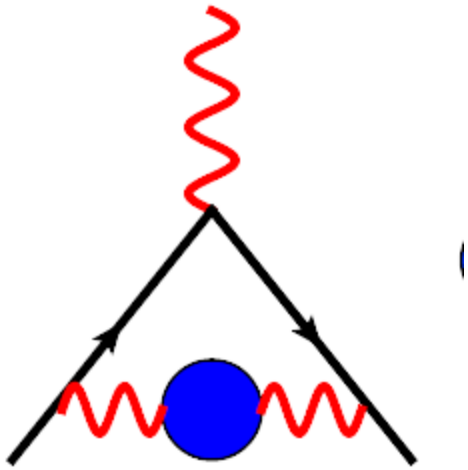
$$a_\mu|_{EXP} - a_\mu|_{THY} = 27.1\ (9.9) \times 10^{-10}$$

$$\Delta a_\mu : 2.7\ \sigma$$

CONTRIBUTIONS TO a = (g-2)/2

$$a_{QED} \quad a_{HAD} \quad a_{EW}$$

$$a_{HAD} = a|_{HAD}(LO) + a|_{HAD}(HO) + a|_{HAD}(LBL)$$



$a_\mu|_{HAD}$ (LO) = 692.7 (6.5)

RELIES ENTIRELY ON

(somewhat unreliable) DATA

$e^+ e^- \rightarrow$ hadrons

(uds) + (charm) + (bottom)

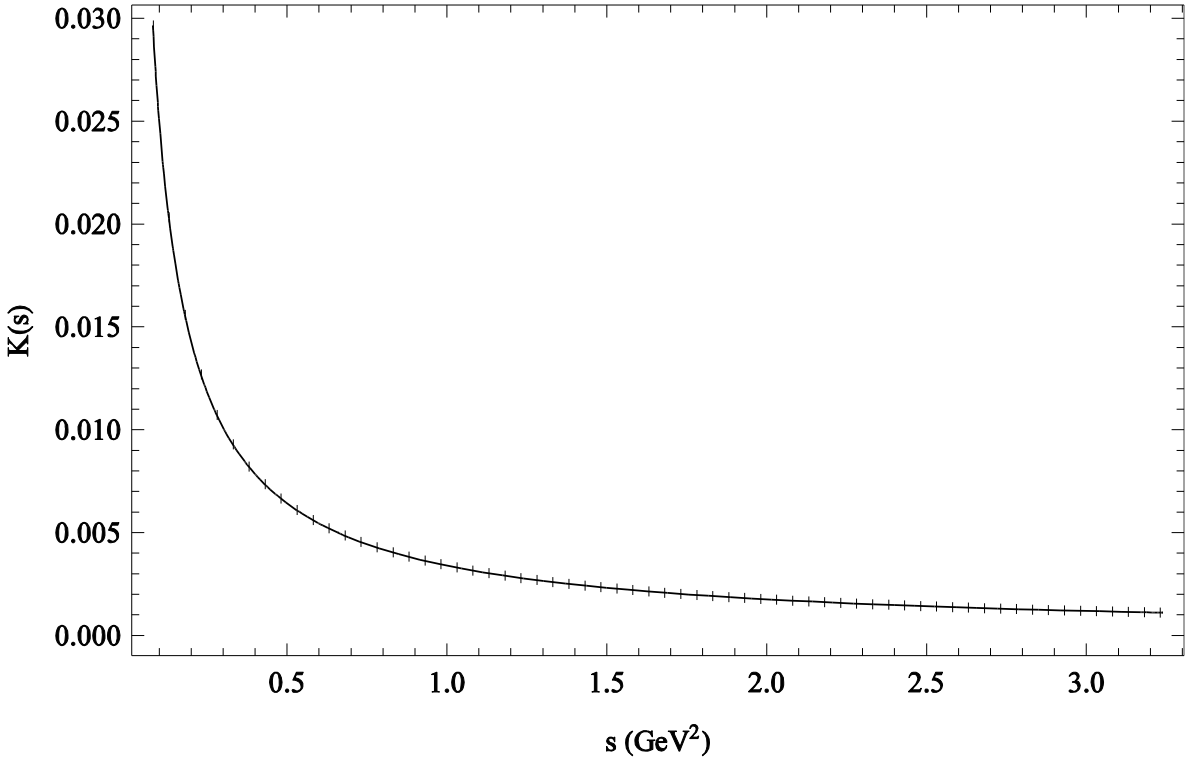
$$a^{HAD} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} K(s)\, R(s) \quad (s=E^2)$$

$$R(s) \equiv \frac{\sigma(e+\, e-\ \rightarrow hadrons}{\sigma(e+\, e-\ \rightarrow leptons} \propto \mathrm{Im}\, \Pi(s)$$

$$\Pi_{\mu\nu}(q^2) = i \int d^4x\, e^{iqx} < 0 | T(J_\mu(x)\, J_\nu^{+}(0)) | 0 >$$

$$= (-g_{\mu\nu}\, q^2 + q_\mu\, q_\nu)\, \Pi_{EM}(q^2)$$

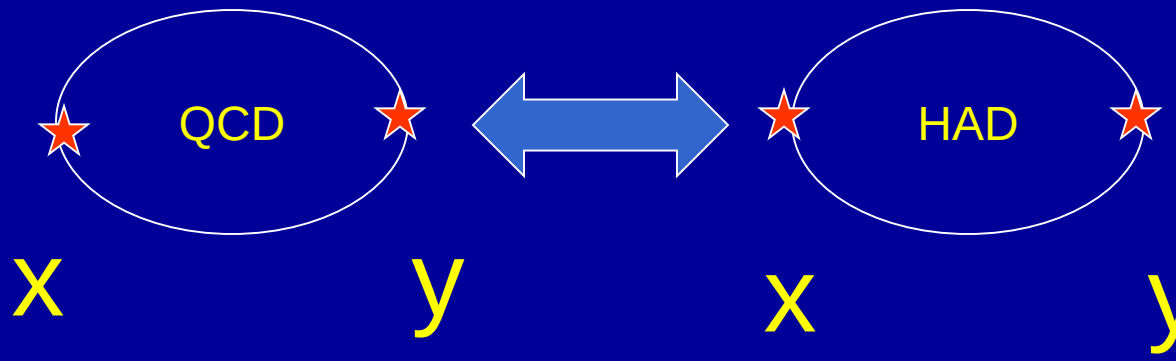
$$\mathrm{Im}\, \Pi_{EM}(q^2) = \frac{1}{8\pi} \left[1 + \frac{\alpha_s(s)}{\pi} + \ldots \right]$$


0.030
0.025
0.020
0.015
0.010
0.005
0.000
K(s)
0.5
1.0
1.5
2.0
2.5
3.0
s (GeV^2)

A THEORETICAL CALCULATION OF

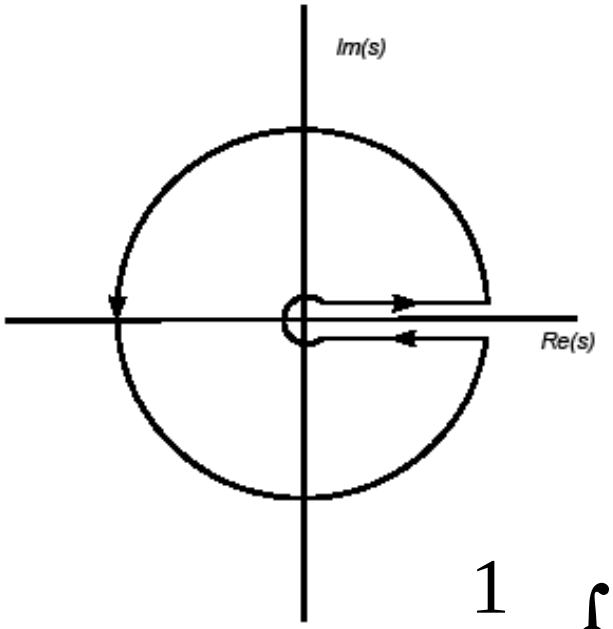
$$a_\mu\, l_{HAD}\ (LO) = a_\mu\, l_{uds} + a_\mu\, l_c + a_\mu\, l_b$$

$$a_\mu\, l_c\ (LQCD)$$



CAUCHY'S THEOREM IN THE COMPLEX ENERGY^2 PLANE

QUARK-HADRON DUALITY

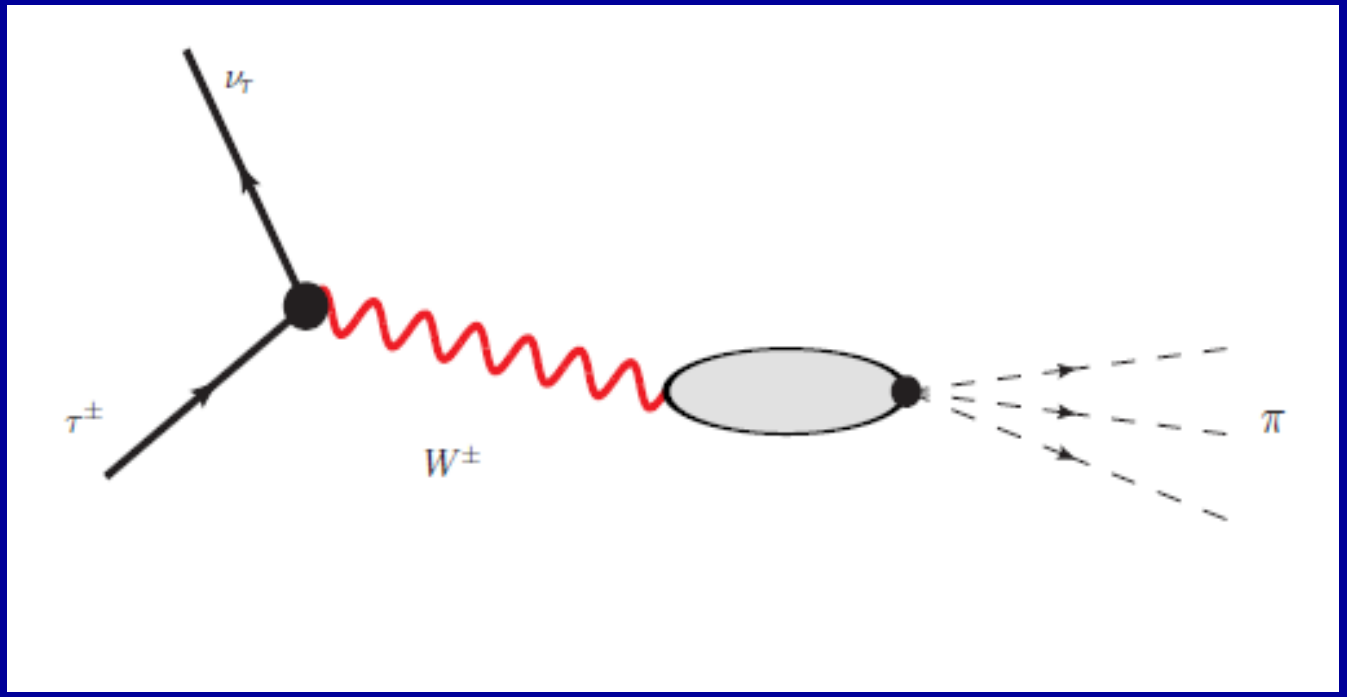


$$\oint_C \Pi(s)\, ds = 0$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi(s) = \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\,\Pi_{QCD}(s) = \int_{s_{th}}^{s_0} ds\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)\,|_{HAD}$$

$\tau \rightarrow$ hadrons (π's)



$$\tau \rightarrow \text{hadrons}$$

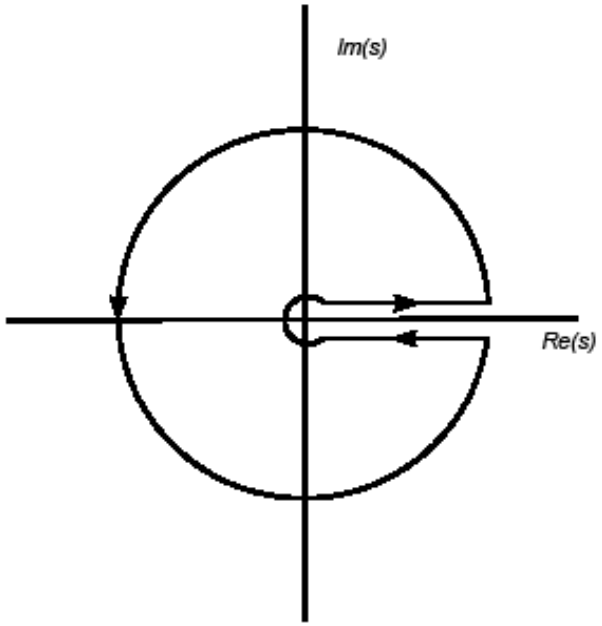
$$R_\tau = \sigma(\tau \rightarrow \text{hadrons}) / \sigma(\tau \rightarrow \text{leptons})$$

$$R_\tau \propto \int_0^{M_\tau^2} ds \; K_\tau(s) \, \mathrm{Im}\,\Pi(s)\,|_{QCD}$$

$$\mathrm{Im}\,\Pi(s)\,|_{QCD} = \sum_n c_n \, \alpha_s^n \qquad (n = 0,1,\ldots 4)$$

CAUCHY'S THEOREM

$$\oint_C \Pi(s)\, ds = 0$$



$$\oint_C \Pi(s)\, ds = \sum_i (\text{Residue Pole})_i$$

$$\frac{1}{2\pi i} \oint_{C(|s_0|)} ds\, \Pi(s) + \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi} \operatorname{Im} \Pi(s) = \sum_i \text{Res}_i$$

$$\mathrm{K(s)} \rightarrow K_1\,(s) \;=\; \frac{a_1}{s} + \frac{a_2}{s^2}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s}\; K_1(\mathrm{s})\, \frac{1}{\pi}\, \mathrm{Im}\; \Pi(\mathrm{s}) = \mathrm{Res}\; [\Pi(\mathrm{s})\, \frac{K_1(s)}{s}\,]_{\mathrm{s}=0}$$

$$- \frac{1}{2\,\pi\, i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(\mathrm{s})\; \Pi(\mathrm{s})$$

PERTURBATIVE QCD EXPANSION (HEAVY QUARKS)

$$z = \frac{s}{4{m_Q}^2}$$

$$\Pi(s)_{\mathrm{PQCD}} \propto \sum_{n\geq 0} C_n \, z^n$$

PQCD up to 4-loop level

Karlsruhe Group (K. Chetyrkin, J. Kuhn, et al.)

$a^{HAD}|_c = 14.4 \pm 0.1 \times 10^{-10}$ $a^{HAD}|_b = 0.29 \pm 0.01 \times 10^{-10}$

(S. Bodenstein et al. 2012)

- LQCD
- (2014)

- $a^{HAD}|_c = 14.1 \pm 0.1 \times 10^{-10}$ (ETM Coll.)

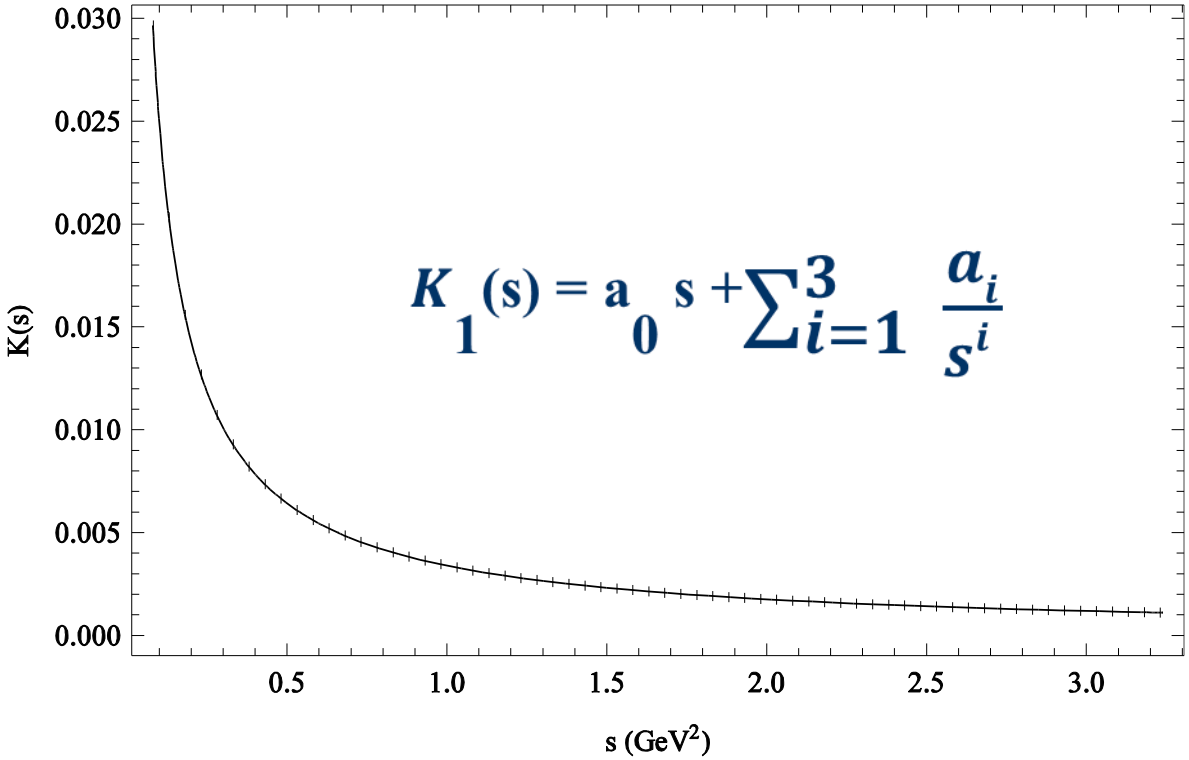
- $a^{HAD}|_c = 14.42 \pm 0.39 \times 10^{-10}$ (HPQCDColl.)

uds

$$\mathbf{K(s) \rightarrow K_1(s) = a_0\, s + \sum_{n=1}^{3} \frac{a_n}{s^n}}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s}\, K_1(s)\, \frac{1}{\pi} \operatorname{Im} \Pi(s) = \operatorname{Res}\left[\Pi(s)\, \frac{K_1(s)}{s}\right]_{s=0}$$

$$- \frac{1}{2\,\pi\, i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(s)\, \Pi(s)$$


0.030
0.025
0.020
0.015
0.010
0.005
0.000
K(s)
0.5
1.0
1.5
2.0
2.5
3.0
s (GeV²)
$K_1(s) = a_0 s + \sum_{i=1}^{3} \frac{a_i}{s^i}$

$$\text{Res}\,[\Pi(s)\,\frac{K_1((s)}{s}]_{s=0} = \lim_{s\to 0}\Sigma\,\frac{a_n}{n!}\,(\frac{d}{ds})^{n}\,\Pi(s)$$

NO (QCD) LOW ENERGY THEOREM FOR

$\Pi(s)|_{uds}$

Need: $d\,\Pi(s) / ds|_{s=0}$ (dominant residue)

$$R(s) \equiv \frac{\sigma(e+e- \rightarrow hadrons}{\sigma(e+e- \rightarrow leptons} \propto \mathrm{Im}\ \Pi(s)$$

$$\mathrm{Im}\ \Pi(s) \propto |F_{\pi}(s)|^2$$

NEED A MODEL OF $F_\pi(s)$ $(s \leq 0)$

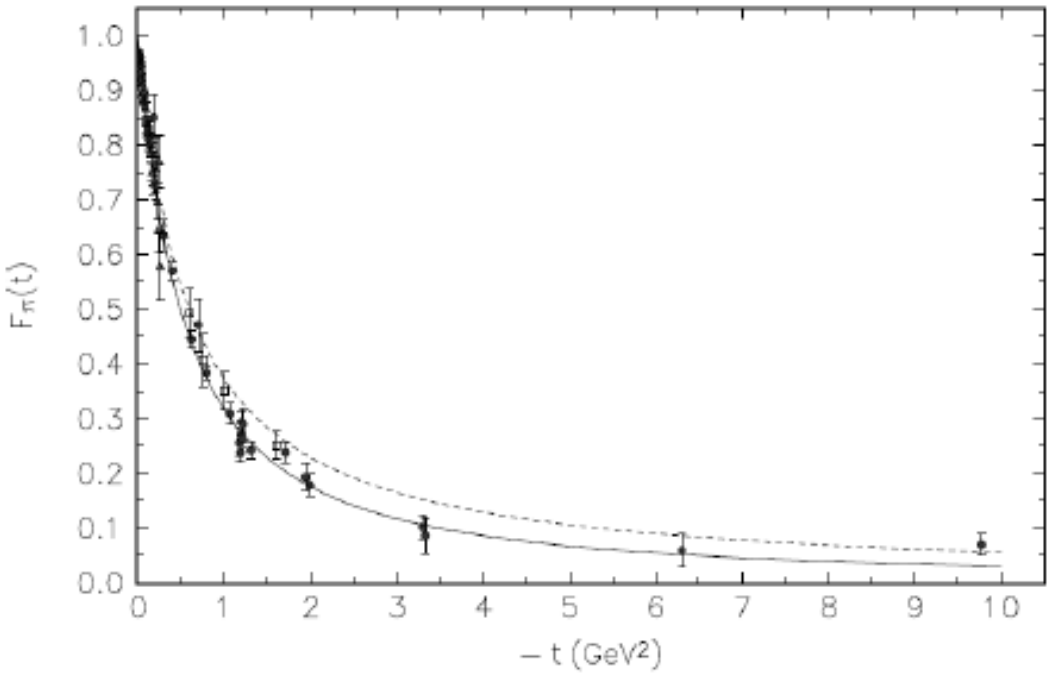
MUST REPRODUCE $\langle r^2_\pi \rangle$

QCD $(N_c \rightarrow \infty)$ (Phys. Lett. B 512, 331 (2001)

$$F_\pi(s) \rightarrow \langle r^2_\pi \rangle = 0.436 \pm 0.004 \text{ fm}^2$$

$$\langle r^2_\pi \rangle_{|EXP} = 0.439 \pm 0.008 \text{ fm}^2$$

FITS DATA (0/ -10 GeV^2): $\chi^2|_{DOF} = 1.5$


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$F_\pi(t)$
0
1
2
3
4
5
6
7
8
9
10
$-t\ (GeV^2)$

$$\Pi(s)|_{uds} : \text{MODEL: } N_c = \infty$$

Phys. Lett. B 512, 331 (2001) CAD

$$a_{HAD} = 722\ (9) \times 10^{-10} \qquad [692\text{-}701(5) \times 10^{-10}]$$

$$a_{QCD\infty} = 116\ \ 592\ \ 10.6 \pm 9.8\ \times 10^{-10}$$

$$a_{EXP} = 116\ \ 592\ \ 08.9 \pm 6.3 \times 10^{-10}$$

Phys. Rev. D **85,** 014029 (2012) [S.Bodenstein, CAD, K. Schilcher]

A model dependent result

CHPT & $d\,\Pi(s) / ds|_{s=0}$

$$\text{Res}\,[\Pi(s)\,\frac{K_1((s)}{s}]_{s=0} = \lim_{s\to 0} \Sigma\ \frac{a_n}{n!}\,(\frac{d}{ds})^n\,\Pi(s)$$

CHPT & $d\,\Pi(s) / ds|_{s=0}$

- $d/ds\ \Pi(0) = 0.0105557 - 4\ C^r_{93} - 0.77725\ L^r_{10} + 1.0346\ L^r_9$
- LQCD: $L^r_9 = 3.08(23)(51) \times 10^{-3}$ (Boyle et al.2008) (Rodriguez, Gonzalez, Pich 2016)
-
- $L^r_{10} = -\ 5.2\ (2)(+5,-3) \times 10^{-3}$ (Shintani et al. 2008)
- $C^r_{93}\ (?)\ |_{VMD} \approx -\ 17 \times 10^{-3}\ GeV^{-2}$ (Bijnens & Talavera 2000)
- $\longrightarrow$ **$a_\mu^{HAD} \approx 815 \times 10^{-10}$**

$a_\mu^{HAD} = 692.3\ (4.2)\ \times 10^{-10} \longrightarrow C^r_{93} = -\ 13.9\ (2)\ GeV^{-2}$

THANK YOU

IN UNITS OF 10^{-10}

$$a_\mu|_{QED} = 11\,658\,471.8853 \pm 0.3650$$

$$a_\mu|_{EW} = 15.4\ (2)$$

$$a_\mu|_{HAD}\ (HO) = -\ 9.84\ (7)$$

$$a_\mu|_{HAD}\ (LBL) = 11.6\ (4.0)$$

$$a_\mu|_{HAD}\ (LO) = 692.7\ (6.5)$$

$$a_\mu|_{THY} = 11\,659\,181.8 \pm 7.6$$

$$a_\mu|_{EXP} - a_\mu|_{THY} = 27.1\ (9.9) \times 10^{-10} \quad [\Delta a_\mu : 2.7\ \sigma]$$