

Hadronic Cross Section Measurements with ISR and the Implications on $g_\mu - 2$

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Determination of Fundamental QCD Parameters,
09 March 2016



Outline

1 Introduction

- Theoretical relation $a_\mu \leftrightarrow \sigma_{\text{had}}$
- Experimental setup at *BABAR*

2 Recent Results

- $e^+e^- \rightarrow K^+K^-$
- $e^+e^- \rightarrow K_S^0 K_L^0$
- $e^+e^- \rightarrow K^0 K^0 \pi^+ \pi^-$ and $e^+e^- \rightarrow K_S^0 K_S^0 K^+ K^-$

3 Summary

The contributions to a_μ and its uncertainty

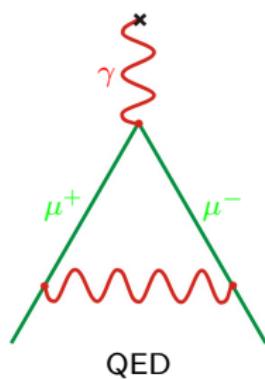
$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$
$$(g_\mu - 2)/2 =: a_\mu = 0 \quad (\text{Dirac})$$

The contributions to a_μ and its uncertainty

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$
$$(g_\mu - 2)/2 =: a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadronic}}$$

The contributions to a_μ and its uncertainty

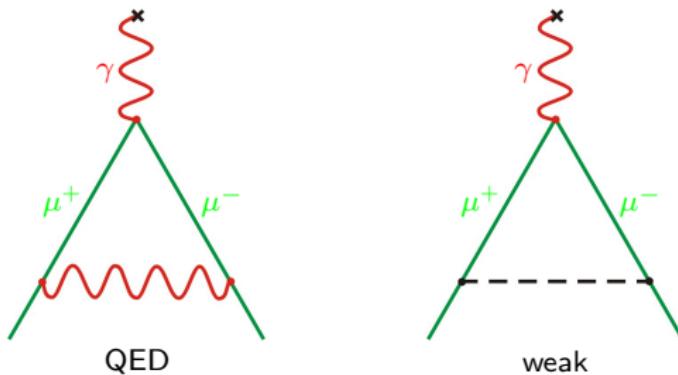
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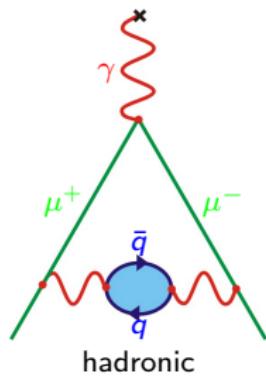
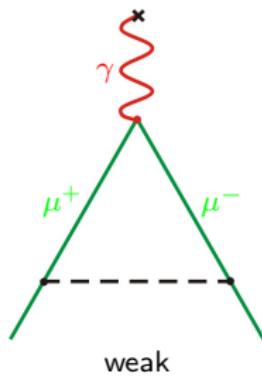
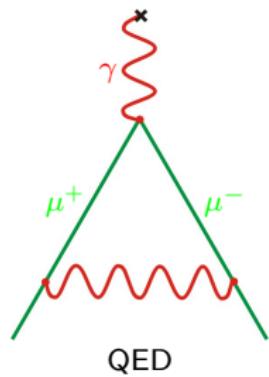
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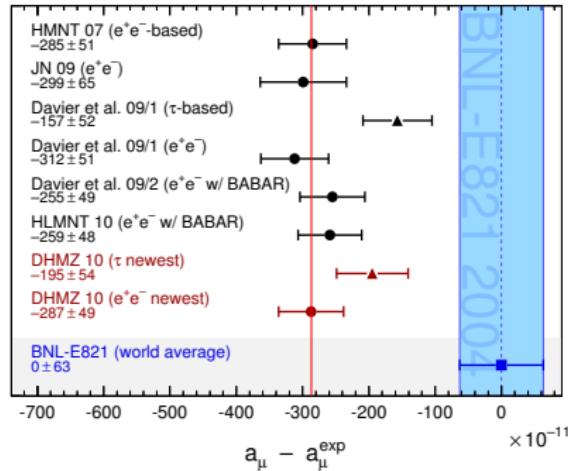
The contributions to a_μ and its uncertainty

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

$$(g_\mu - 2)/2 =: a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadronic}}$$

Interaction	Contribution [$\cdot 10^{-11}$]	Uncertainty [$\cdot 10^{-11}$]
QED [1]	116 584 718.951	0.080
EW [5]	153.6	1
hadronic VP [4, 9]	6837	43
hadronic LbL [8, 3]	119	41
total theory	116 591 828	60
E821 experiment [13]	116 592 089	63
deviation exp-theo	261	87

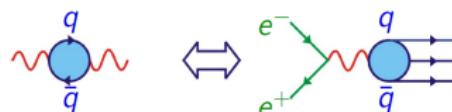
Discrepancy between SM prediction and direct measurement from Eur.Phys.J., C71:1515, 2011 [4].



Just a fluctuation?

3σ effect, thus reduction of uncertainties necessary!

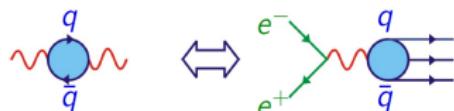
Connection between a_μ and σ_{had}



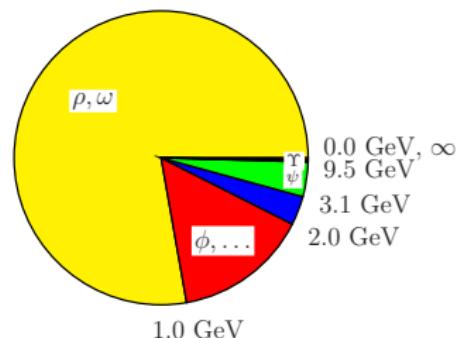
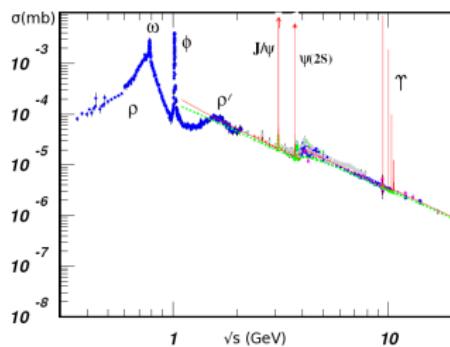
$$a_\mu^{\text{had}} \approx \frac{1}{4\pi^3} \int_{m_\pi^2}^{\infty} K_\mu(s) \cdot \sigma_{e^+ e^- \rightarrow \text{had}}(s) ds$$

Kernel function cross section

Connection between a_μ and σ_{had}

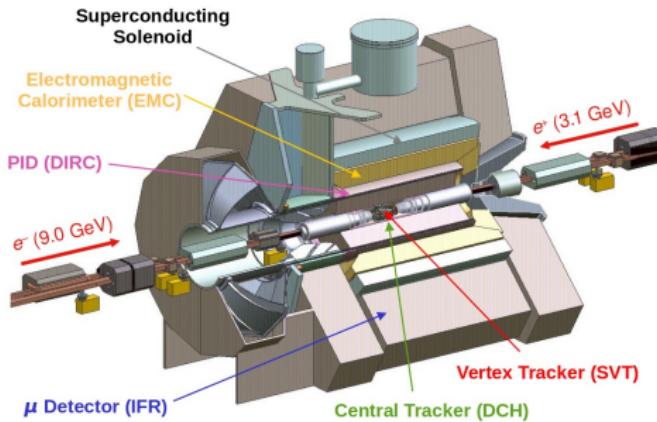


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σ_{had} (left) from Nuovo Cim., C034S1:31-40, 2011 [7] and relative contributions to a_μ^{had} (right).

The *BABAR* Experiment

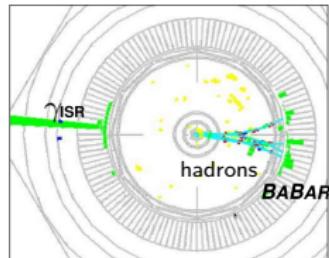
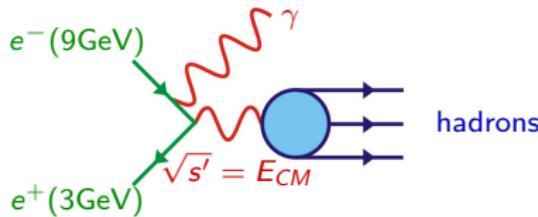


Experimental specifications

Energy: $\sqrt{s} \approx 10.58 \text{ GeV}$ ($E_{e^-} \approx 9.0 \text{ GeV}$, $E_{e^+} \approx 3.1 \text{ GeV}$),

Luminosity: $\mathcal{L} \approx 454 \text{ fb}^{-1}$ ($\Upsilon(4S)$)

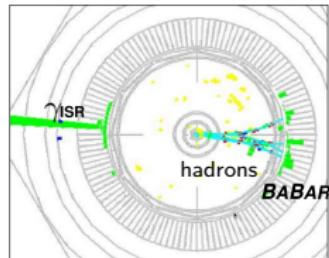
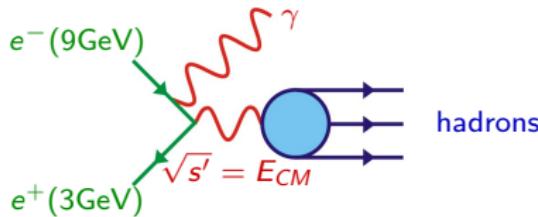
Initial State Radiation (ISR) events at *BABAR*



ISR selection: large angle analyses

- Detected high energy photon: $E_\gamma > 3\text{GeV}$
→ defines E_{CM} & provides strong background rejection
- Event topology: γ_{ISR} back-to-back to hadrons
→ high acceptance
- Kinematic fit including γ_{ISR}
→ very good energy resolution (4 – 15MeV)
- e^+e^- -boost into the laboratory reference frame
→ high efficiency at production threshold of hadronic system
- Continuous measurement from threshold to $\sim 5\text{GeV}$
→ provides common, consistent systematic uncertainties

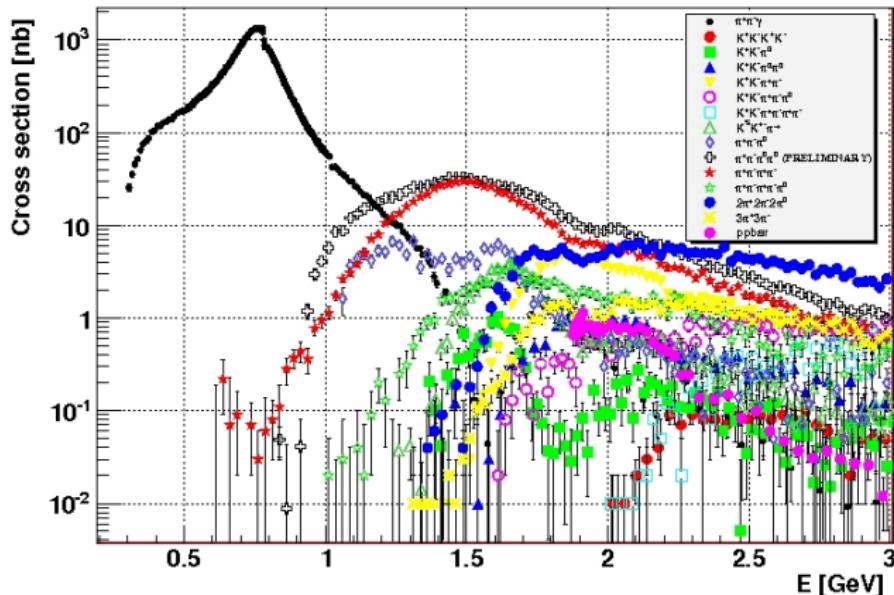
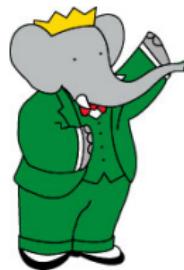
Initial State Radiation (ISR) events at *BABAR*



ISR selection: small angle analyses

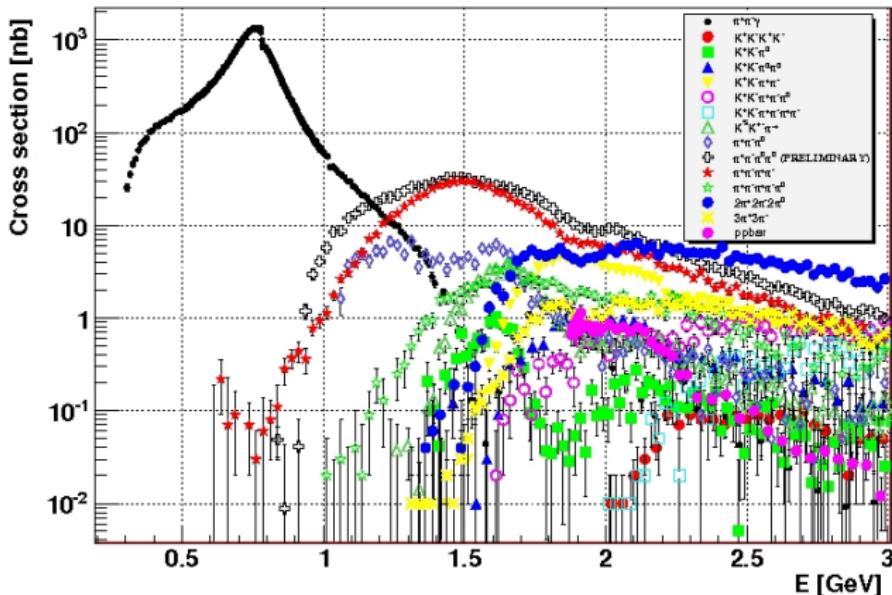
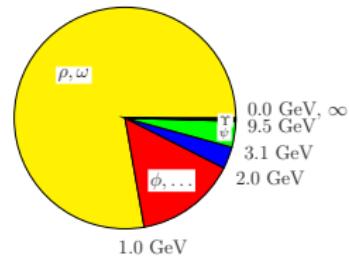
- ISR photon γ_{ISR} not detected
→ more statistics at high energies
- Event topology: γ_{ISR} back-to-back to hadrons
→ high acceptance
- Kinematic fit not including γ_{ISR}
→ energy resolution $\sim 10 - 15\text{MeV}$
- e^+e^- -boost into the laboratory reference frame
→ high efficiency at production threshold of hadronic system
- Continuous measurement from threshold to $\sim 8\text{GeV}$
→ provides common, consistent systematic uncertainties

Most important channels



Cross Sections of the single channels measured at *BABAR* (from Nucl.Phys.Proc.Suppl., 207-208:133-136, 2010 [6]).

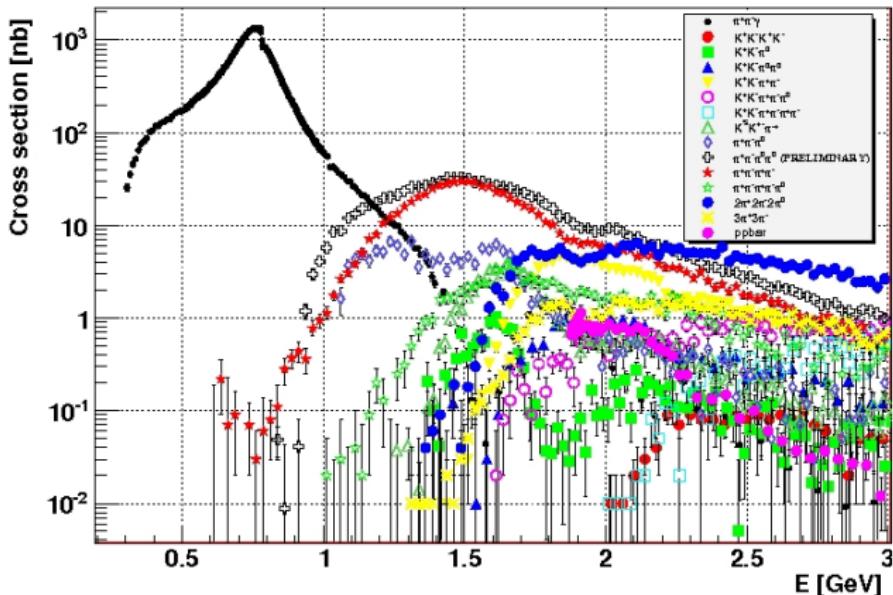
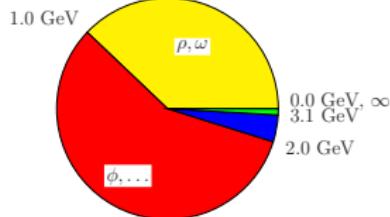
Most important channels



Right panel: Cross Sect. of single channels (from Nucl.Phys.Proc.Supp., 207-208:133-136, 2010 [6]).

Left panel: Relative contributions to a_μ^{had} (from Nuovo Cim., C034S1:31-40, 2011 [7]).

Most important channels

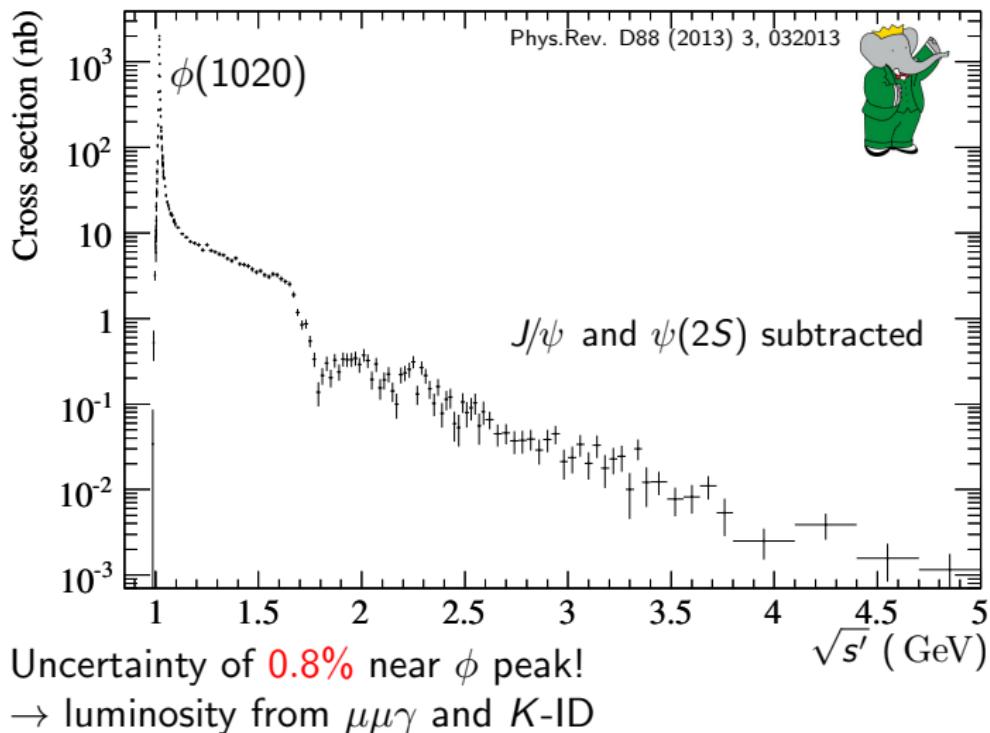


Right panel: Cross Sect. of single channels (from Nucl.Phys.Proc.Supp., 207-208:133-136, 2010 [6]).

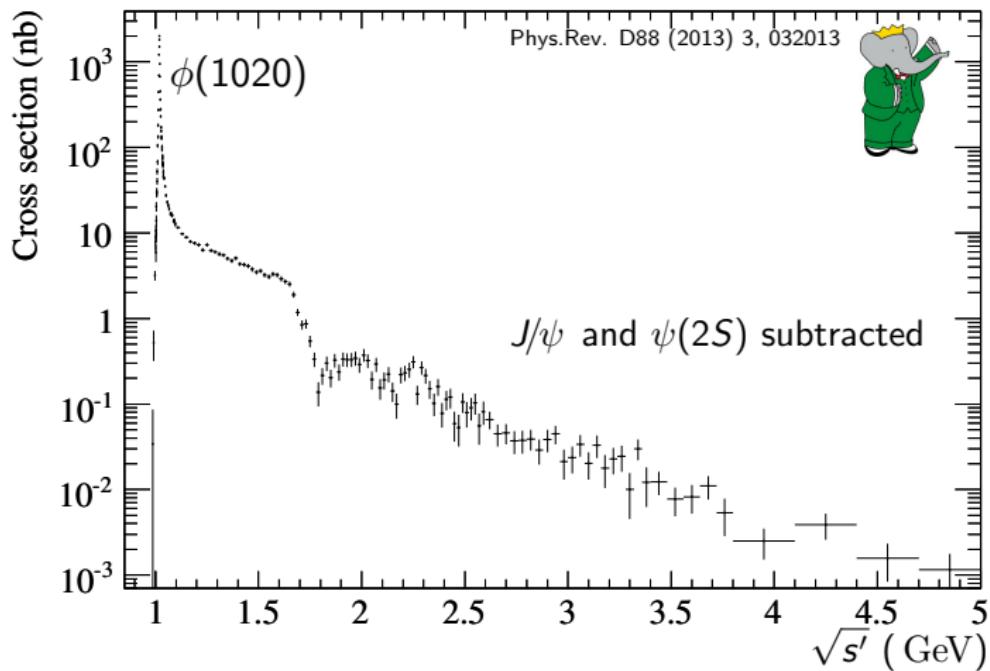
Left panel: Relative contributions to δa_μ^{had} (from Nuovo Cim., C034S1:31-40, 2011 [7]).

$$e^+ e^- \rightarrow K^+ K^-$$

Phys.Rev. D88 (2013) 3, 032013 [10]
Phys.Rev. D92 (2015) 7, 072008 [12]

Cross section $\sigma(e^+e^- \rightarrow K^+K^-)$ 

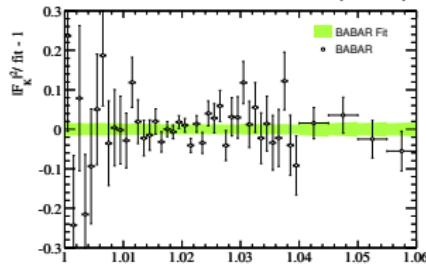
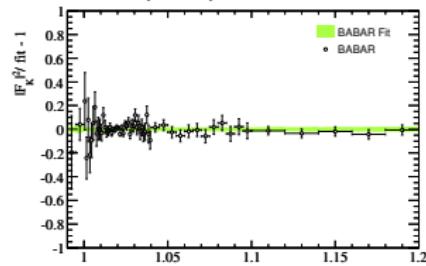
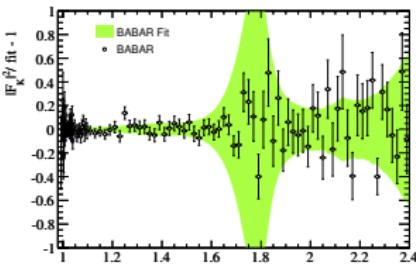
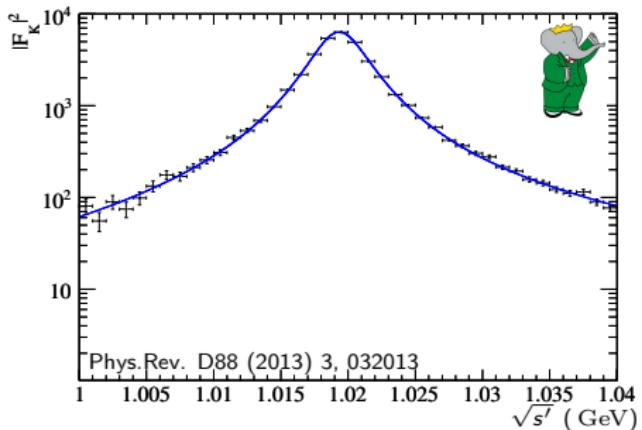
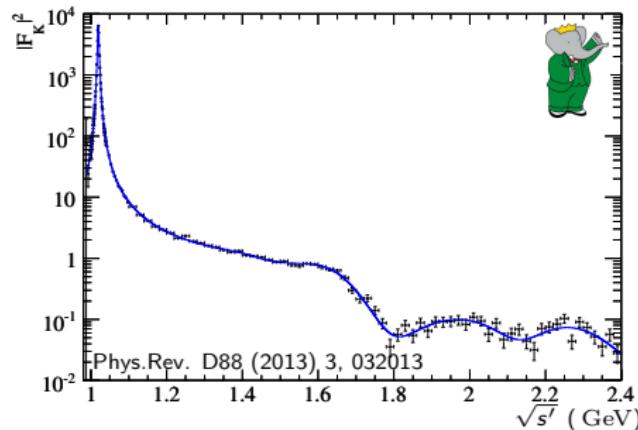
Cross section $\sigma(e^+ e^- \rightarrow K^+ K^-)$



$$\Rightarrow a_\mu^{had}(K^+ K^-) = (229.5 \pm 1.4 \pm 2.2) \times 10^{-11}$$

→ improvement by factor 2.8 over previous world data

A phenomenological fit to the form factor



The Φ parameters

m_Φ and Γ_Φ obtained from the fit of the form factor

BABAR

$$m_\Phi = 1019.51 \pm 0.02 (\pm 0.11) \text{ MeV}$$

$$\Gamma_\Phi = 4.29 \pm 0.04 (\pm 0.07) \text{ MeV}$$

PDG

$$m_\Phi = 1019.455 \pm 0.020 \text{ MeV}$$

$$\Gamma_\Phi = 4.26 \pm 0.04 \text{ MeV}$$

→ good agreement

From integrated Φ peak: $\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \rightarrow K^+ K^-) = \frac{\alpha^2 \beta^3(s, m_K)}{324} \frac{m_\Phi^2}{\Gamma_\Phi} a_\Phi^2 C_{FS}$

BABAR:

$$\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \rightarrow K^+ K^-) = 0.6344 \pm 0.0059_{exp} \pm 0.0028_{fit} \pm 0.0015_{cal} \text{ keV (1.1%)}$$

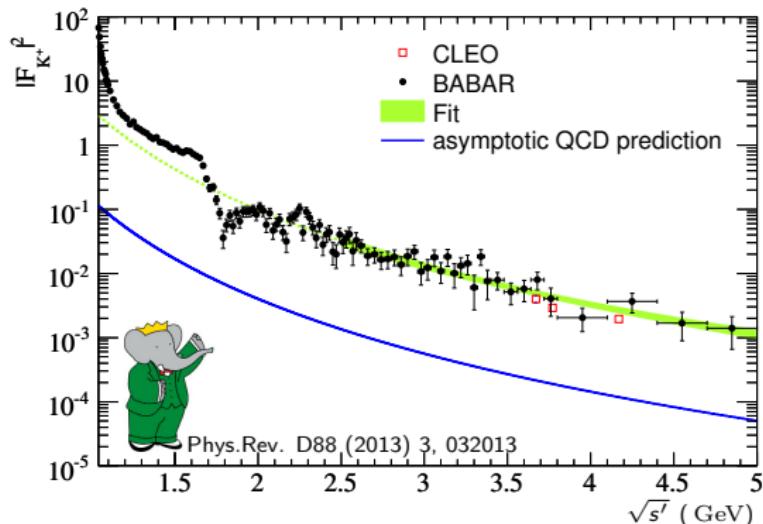
CMD2:

$$\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \rightarrow K^+ K^-) = 0.605 \pm 0.002 \pm 0.013 \text{ keV (2.1%)}$$

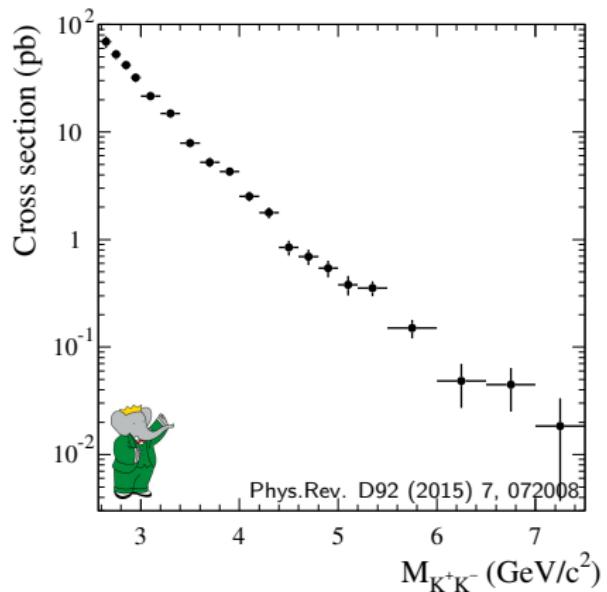
Charged kaon form factor at large Q^2

Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)

- Power law: $F_K \sim \alpha_S(Q^2)Q^{-n}$ with $n=2$
→ in good agreement with the data (2.5-5 GeV $n = 2.10 \pm 0.23$)
- HOWEVER: data on $|F_K|^2$ factor ~ 20 above prediction!
- No trend in data up to 5 GeV for approaching the asympt. QCD prediction

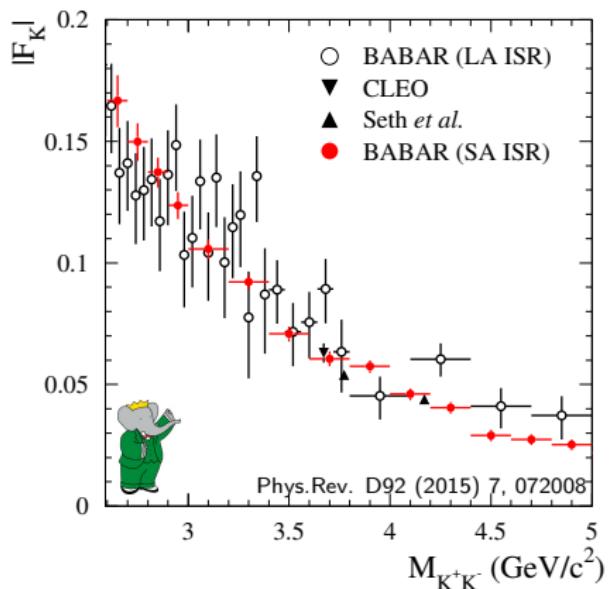


Small angle analysis \rightarrow even larger Q^2



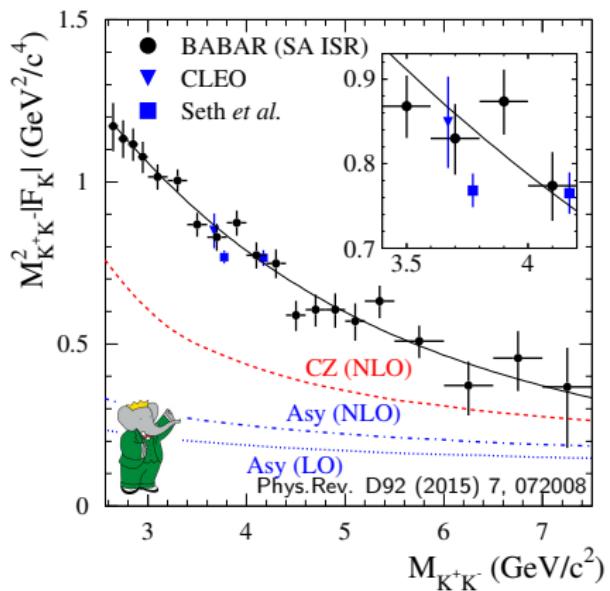
- Small angle measurement reaches energies above 5 GeV
- Smooth decrease over full energy range

Comparison between small and large angle analysis



Good agreement between measurements in overlapping region

Asymptotic behavior of the form factor at largest Q^2 ?



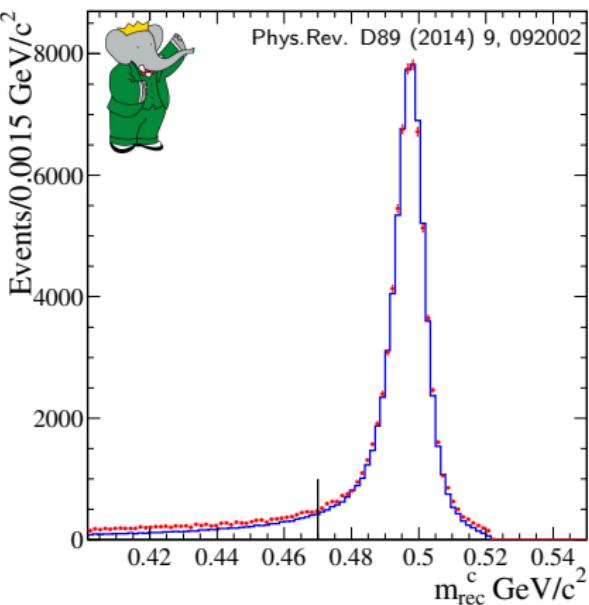
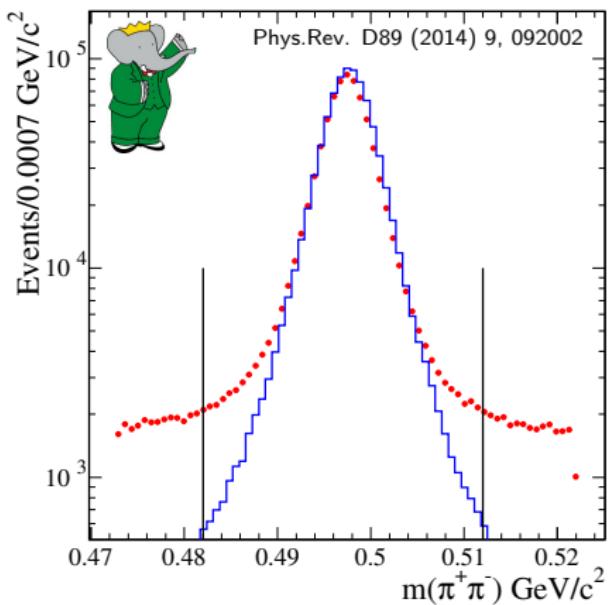
At large energies, data tends to agree better with predictions

$$e^+ e^- \rightarrow K_S^0 K_L^0$$

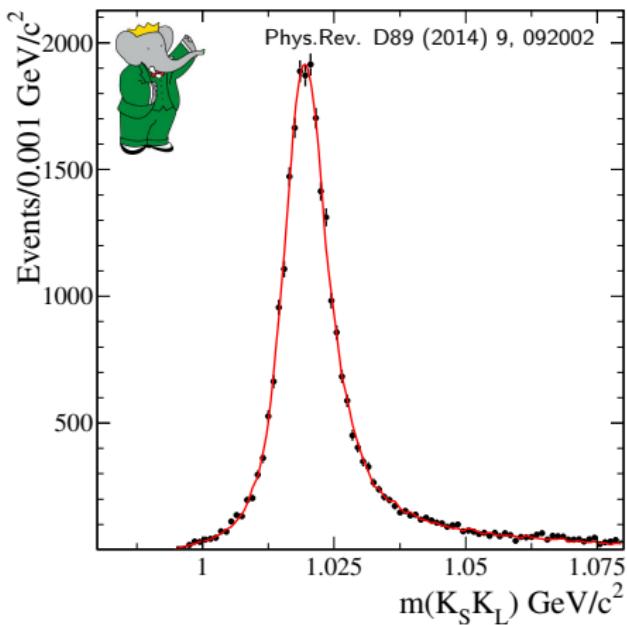
Phys.Rev. D89 (2014) 9, 092002 [11]

Detection of $K_S^0 K_L^0$

$K_S^0 \rightarrow \pi^+ \pi^-$ and K_L^0 selected via recoil mass



$K_S^0 K_L^0$ mass spectrum



Fit results

$$\sigma_\phi = 1409 \pm 33 \pm 42 \pm 15 \text{ nb}$$

$$m_\phi = 1019.462 \pm 0.042 \pm 0.050 \pm 0.025 \text{ MeV}/c^2$$

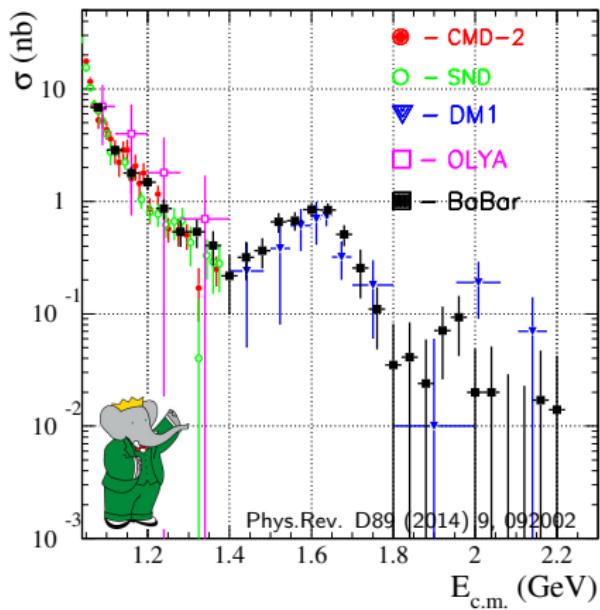
$$\Gamma_\phi =$$

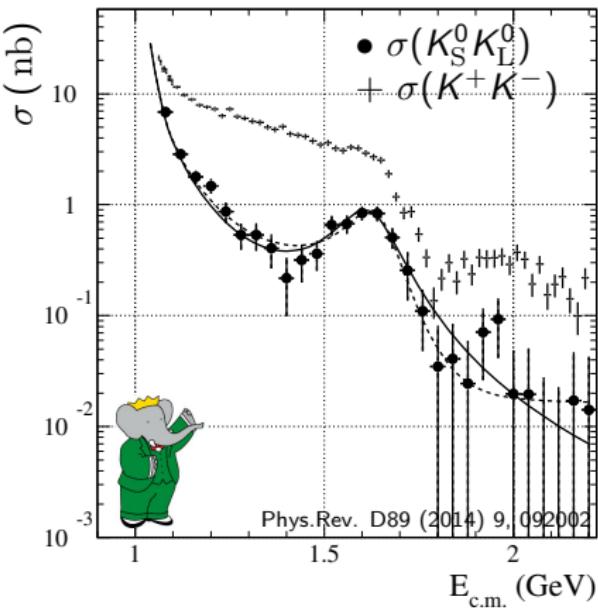
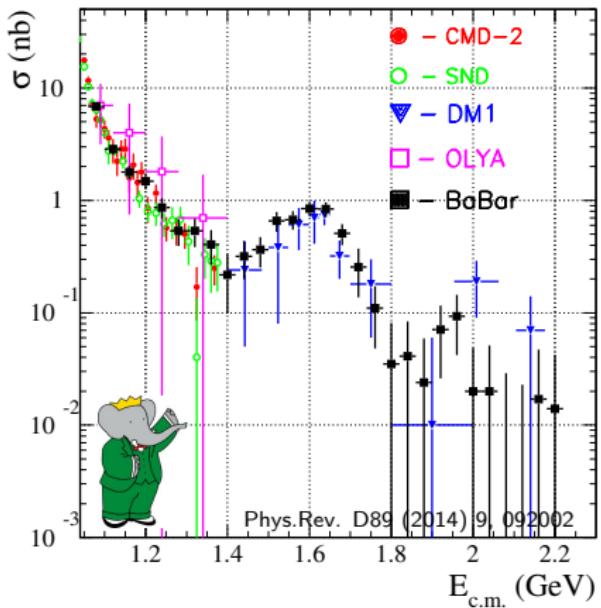
$$4.205 \pm 0.103 \pm 0.050 \pm 0.045 \text{ MeV}$$

$$\Gamma_\phi^{ee} \times \mathcal{B}(\Phi \rightarrow K_S^0 K_L^0) = \\ 0.4200 \pm 0.0033 \pm 0.0122 \pm 0.0013 \text{ keV}$$

→ consistent with world data

Cross section of $e^+e^- \rightarrow K_S^0 K_L^0$



Cross section of $e^+e^- \rightarrow K_S^0 K_L^0$ 

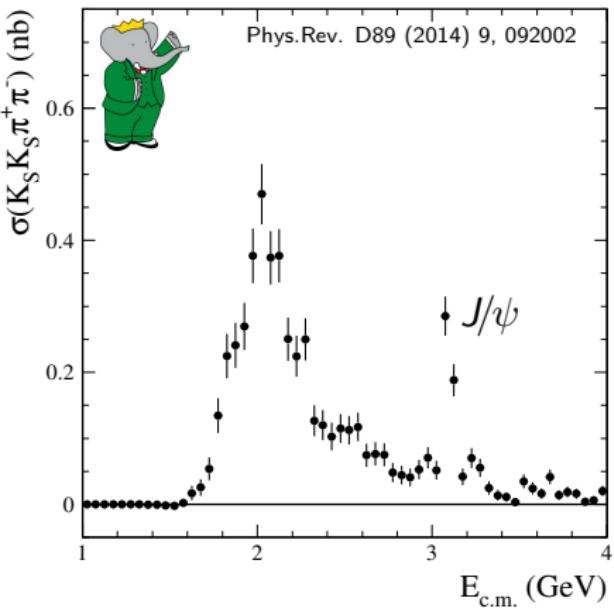
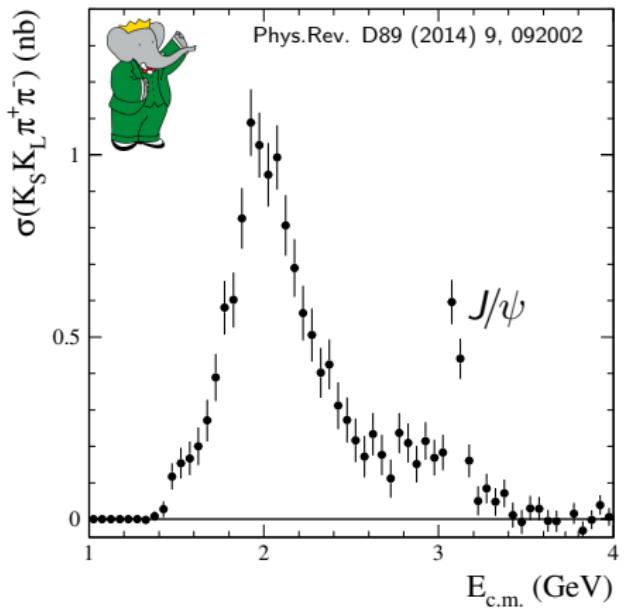
$$e^+e^- \rightarrow K_S^0K_L^0\pi^+\pi^-$$

$$e^+e^- \rightarrow K_S^0K_S^0\pi^+\pi^-$$

$$e^+e^- \rightarrow K_S^0K_S^0K^+K^-$$

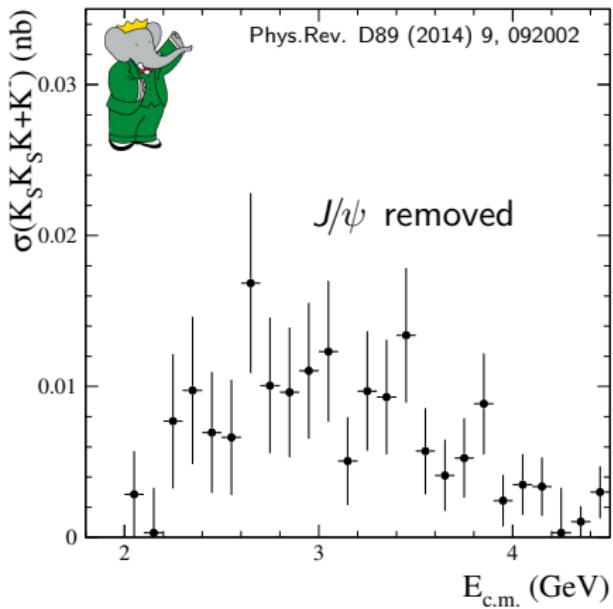
Phys.Rev. D89 (2014) 9, 092002 [11]

Cross sections of $e^+e^- \rightarrow K_S^0\bar{K}_L^0\pi^+\pi^-$ and $K_S^0\bar{K}_S^0\pi^+\pi^-$



First cross section measurements
Clear J/ψ peaks

Cross section of $e^+e^- \rightarrow K_S^0K_S^0K^+K^-$



First cross section measurement
 J/ψ observed in mass distribution

Summary

- ISR physics has proven to be a very productive field even years after the end of data taking at the B-factories
- Precision measurements of hadronic cross sections have greatly improved a_{μ}^{SM} & more hadronic final states in preparation
- $g_{\mu} - 2$ puzzle needs to be solved
 - ★ Data from new experiments (e.g. BES-III)
 - ★ Light-By-Light scattering needs to be studied
 - ★ *E989* at Fermilab and J-PARC $g-2$ /EDM
- QCD predictions on form factors are tested experimentally



Thank you!
Any questions?

Backup slides

References I

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Complete tenth-order qed contribution to the muon $g - 2$.
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Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL.
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The electroweak contributions to $(g - 2)_\mu$ after the Higgs boson
mass measurement.
Phys.Rev., D88(5):053005, 2013.
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Exclusive hadronic cross sections measured via ISR from BaBar.
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Electroweak effective couplings for future precision experiments.
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The Muon g-2.
Phys.Rept., 477:1–110, 2009.
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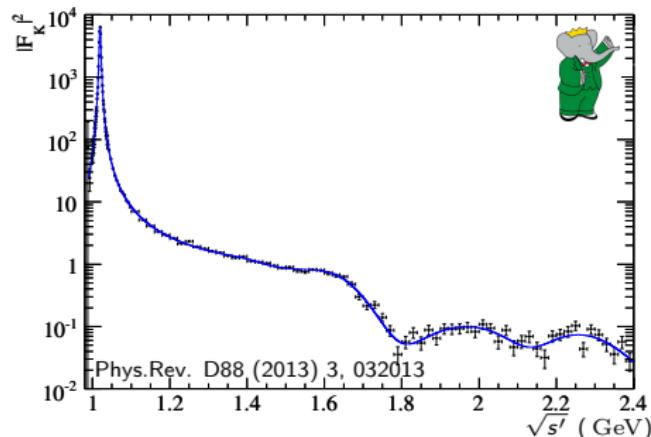
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Precision measurement of the $e^+e^- \rightarrow K^+K^-(\gamma)$ cross section with the initial-state radiation method at BABAR.
Phys. Rev., D88(3):032013, 2013.
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Cross sections for the reactions $e^+e^- \rightarrow K_S^0K_L^0$, $K_S^0K_L^0\pi^+\pi^-$, $K_S^0K_S^0\pi^+\pi^-$, and $K_S^0K_S^0K^+K^-$ from events with initial-state radiation.
Phys. Rev., D89(9):092002, 2014.
- [12] J. P. Lees et al.
Study of the $e^+e^- \rightarrow K^+K^-$ reaction in the energy range from 2.6 to 8.0 GeV.
Phys. Rev., D92(7):072008, 2015.

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- [13] K. Olive et al.
Review of Particle Physics.
Chin.Phys., C38:090001, 2014.
- [14] N. Saito.
A novel precision measurement of muon g-2 and EDM at J-PARC.
AIP Conf.Proc., 1467:45–56, 2012.

Breit-Wigner fit function



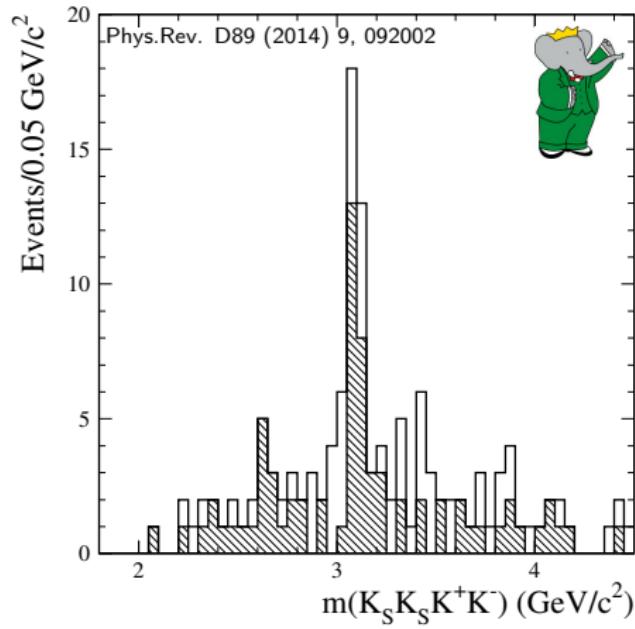
$$\begin{aligned} F_K(s) &= (a_\phi \cdot BW_\phi(s) + a_{\phi'} \cdot BW_{\phi'}(s) + a_{\phi''} \cdot BW_{\phi''}(s))/3 \\ &+ (a_\rho \cdot BW_\rho(s) + a_{\rho'} \cdot BW_{\rho'}(s) + a_{\rho''} \cdot BW_{\rho''}(s) + a_{\rho'''} \cdot BW_{\rho'''}(s))/2 \\ &+ (a_\omega \cdot BW_\omega(s) + a_{\omega'} \cdot BW_{\omega'}(s) + a_{\omega''} \cdot BW_{\omega''}(s) + a_{\omega'''} \cdot BW_{\omega'''}(s))/6 \end{aligned}$$

with $a_\phi + a_{\phi'} + a_{\phi''} = 1$

$$a_\rho + a_{\rho'} + a_{\rho''} + a_{\rho'''} = 1$$

$$a_\omega + a_{\omega'} + a_{\omega''} + a_{\omega'''} = 1$$

$K_S K_S K^+ K^-$ mass spectrum



Basic method

Definition of g :

$$\vec{\mu} = g \frac{e}{2m} \vec{s} .$$

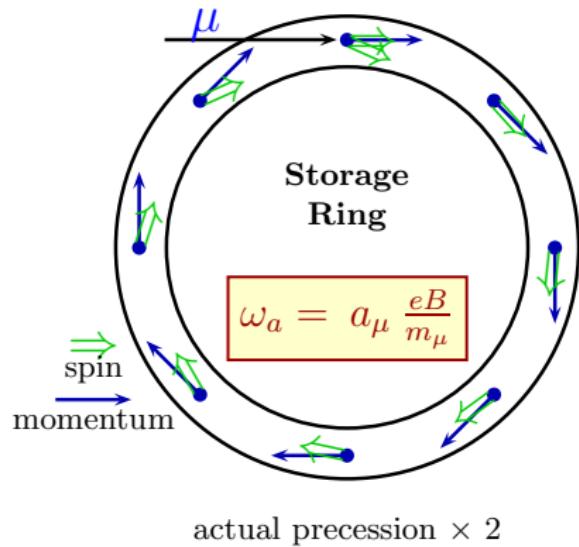
Motion in magnetic field:

$$\vec{\omega}_c = \frac{e\vec{B}}{m\gamma},$$

$$\vec{\omega}_l = \frac{e\vec{B}}{m\gamma} + a \frac{e\vec{B}}{m},$$

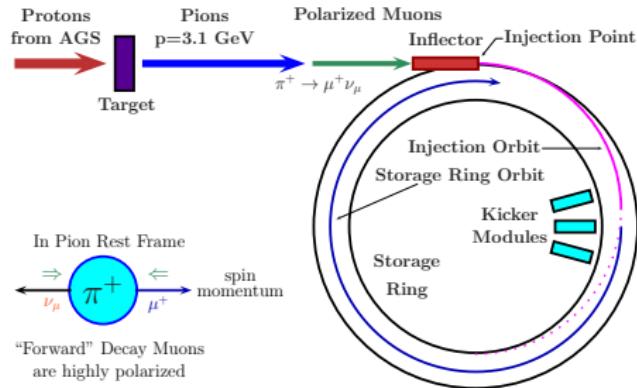
$$\Rightarrow \vec{\omega}_a = a \frac{e\vec{B}}{m}.$$

$$(a = (g - 2)/2)$$



Spin precession. [7]

Realization in detail



From π^+ production to μ^+ decay. [7]

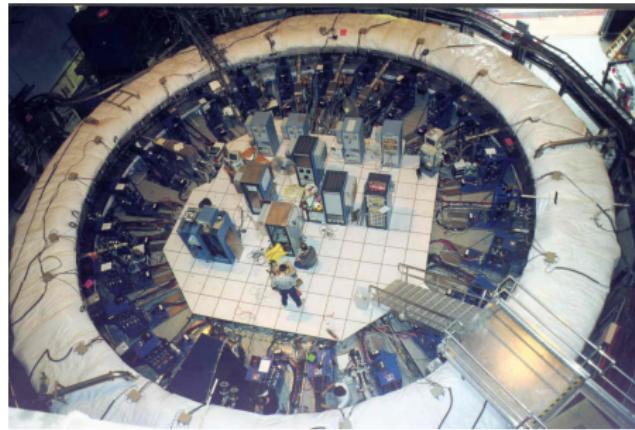
Electric field necessary for focussing: BMT equation

$$\vec{\omega}_a = \frac{e}{m_\mu} \left(a_\mu \vec{B} - \left[a_\mu - \frac{1}{\gamma^2 - 1} \right] \vec{v} \times \vec{E} \right) .$$

$\vec{\omega}_a$ is independent of \vec{E} for $\gamma = 29.3 \Leftrightarrow E_\mu = \gamma m_\mu = 3.1 \text{ GeV}$.

Direct Measurement of $(g_\mu - 2)$

Experiment E821 at Brookhaven National Laboratory



The result [2]:

$$a_\mu = (116\,592\,089 \pm 54_{stat} \pm 33_{syst}) \cdot 10^{-11}$$

New Direct Measurement of $(g_\mu - 2)$

Experiment E989 at Fermilab

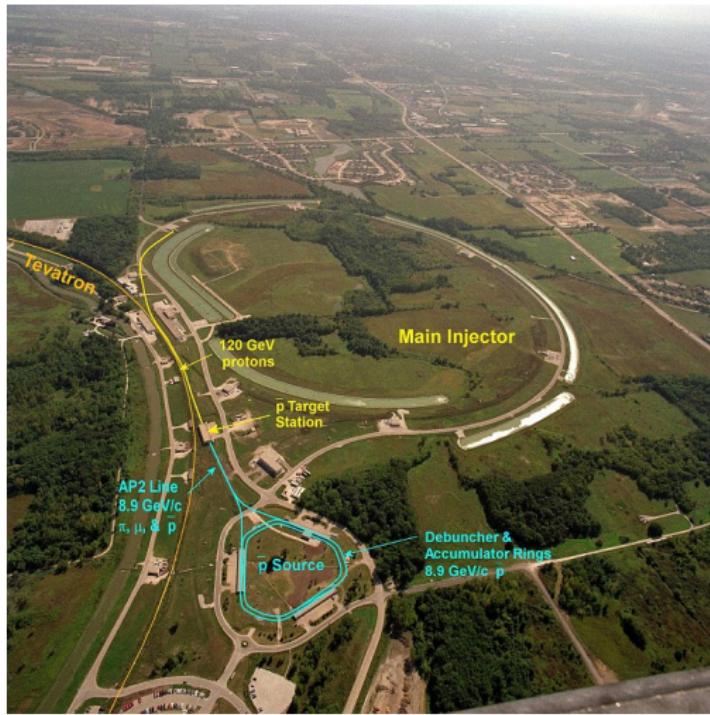


The goal:

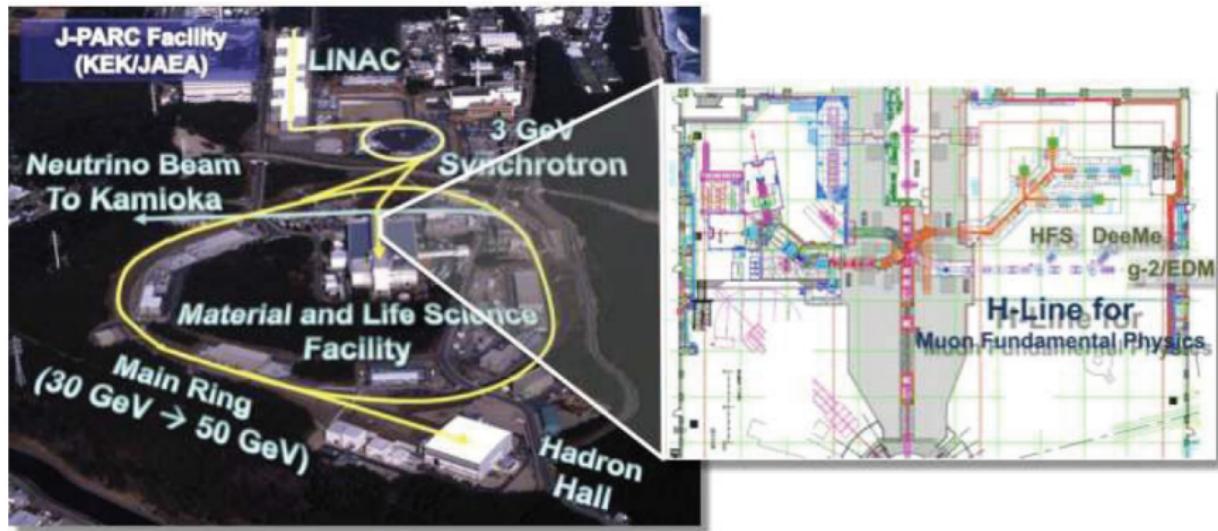
Reduce uncertainty by factor 4 (!)

New Direct Measurement of $(g_\mu - 2)$

Experiment E989 at Fermilab



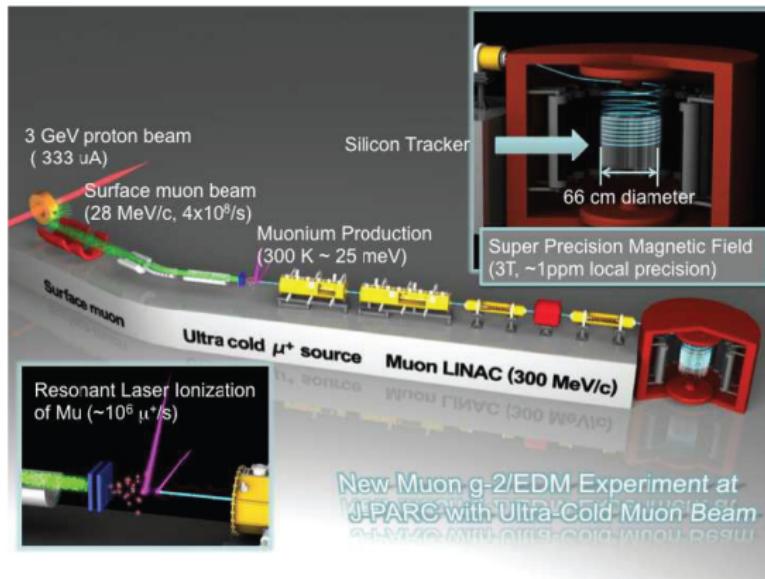
Ultra-cold muon experiment at J-PARC MLF from [14]



The goal:

Uncertainty of $\sim 10 \cdot 10^{-11}$ (!)

Ultra-cold muon experiment at J-PARC MLF from [14]



New Method:

Produce muons from ionization of muonium, store them and track decay.