Charm quark mass in 2+1 flavor QCD from the lattice

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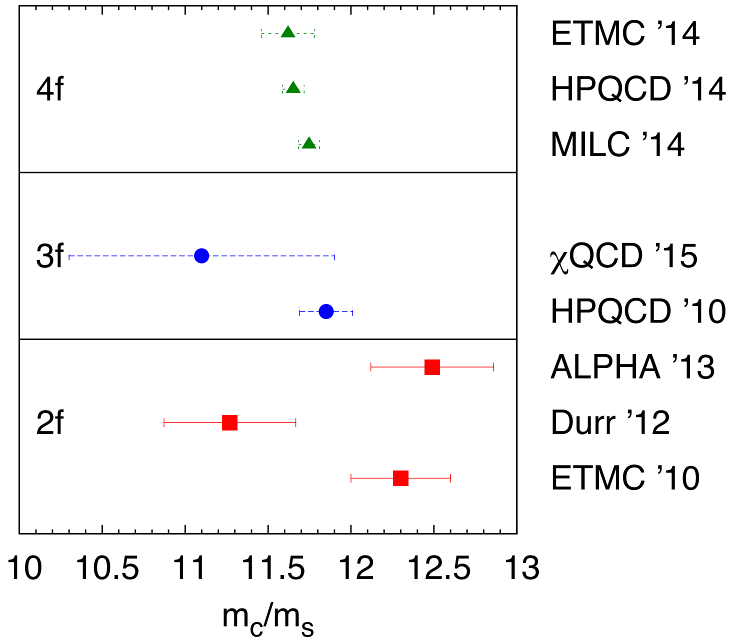
Motivation: large scattering in lattice determinations of the charm quark masses and QCD coupling constant, e.g.

$m_c=1.2715(95)$ GeV (HPQCD 2014) vs. $m_c=1.348(42)$ GeV (ETMC 2014)



Additional lattice calculations for cross-check ?

In this talk:

1) m_c from moments of charm correlators,

2) m_c/m_s from meson masses

3) α_s from moments of charm correlators

Based on work in progress with Yu Maezawa, Yukawa Institute, Kyoto

MITP workshop : Determination of fundamenta parameters in QCD, Mainz, March 7-12,2016

Some lattice details

Highly improved Staggered Quark (HISQ) action and tree-level improved gauge action

HotQCD gauge configurations : 2+1 flavor QCD
physical m_s , $m_l = m_s/20$: $m_K = 504$ MeV, $m_\pi = 161$ MeV
Bazavov et al, PRD90 (2014) 094503

Lattice spacing set by the r_1 scale

$$\left(r^2 \frac{dV_{q\bar{q}}(r)}{dr} \right)_{r=r_1} = 1.0$$

$r_1 = 0.3106(14)(8)(4)$ fm (pion decay constant)

Temperature is varied by the lattice spacing a

$T = (1/N_\tau a)$

Many lattice spacings available, $a_{min} = 0.041$ fm

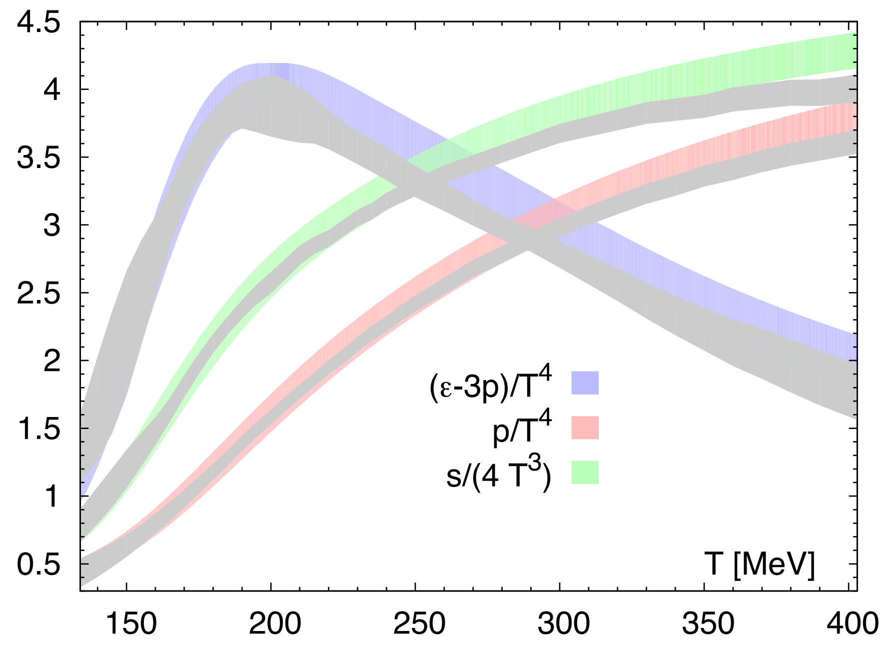
Key differences compared to HPQCD and MILC:

1) 2+1 flavor vs. 2+1+1 flavor
2) Tree level improved gauge action vs. 1-loop tadpole improved gauge action
3) r_1 scale vs. w_0 scale

statistics for the $T=0$ runs:

24^3x32:	4-8K TU
32^4, 32^3x64:	7-40K TU
48^4:	8-16K TU
48^3x64:	8-9K TU
64^4 :	9K TU

in molecular dynamic time units (TU)



Calculations of meson propagators on the lattice

We calculated meson propagators with point and corner wall sources for various valence charm and strange quark masses around their physical values and extracted ground state masses from exponential fits

Ground state charmonia

$$\overline{M} = \frac{1}{4}(3M_{J/\psi} + M_{\eta_c})$$

Unmixed ss meson

$$M_{\eta_{ss}} = 2M_K^2 - M_\pi^2$$

Tune quark masse to reach the "physical" values

$$\overline{M} = d + bm_c,$$

$$M_{\eta_{ss}}^2 = Bm_s$$

Use PDG value and assign errors for EM effects and absence of disconnected diagrams

$$\overline{M} = 3.067(3)\ \mathrm{GeV}$$

$$M_\pi^2 = M_{\pi^0}^2,$$

$$M_K^2 = \frac{1}{2}(M_{K^0}^2 + M_{K^+}^2 - (1+\Delta_E)(M_{\pi^+}^2 - M_{\pi^0}^2))$$

$$M_{\eta_{ss}} = 686.00(92)\ \mathrm{MeV}\ (\Delta_E = 0-2)$$

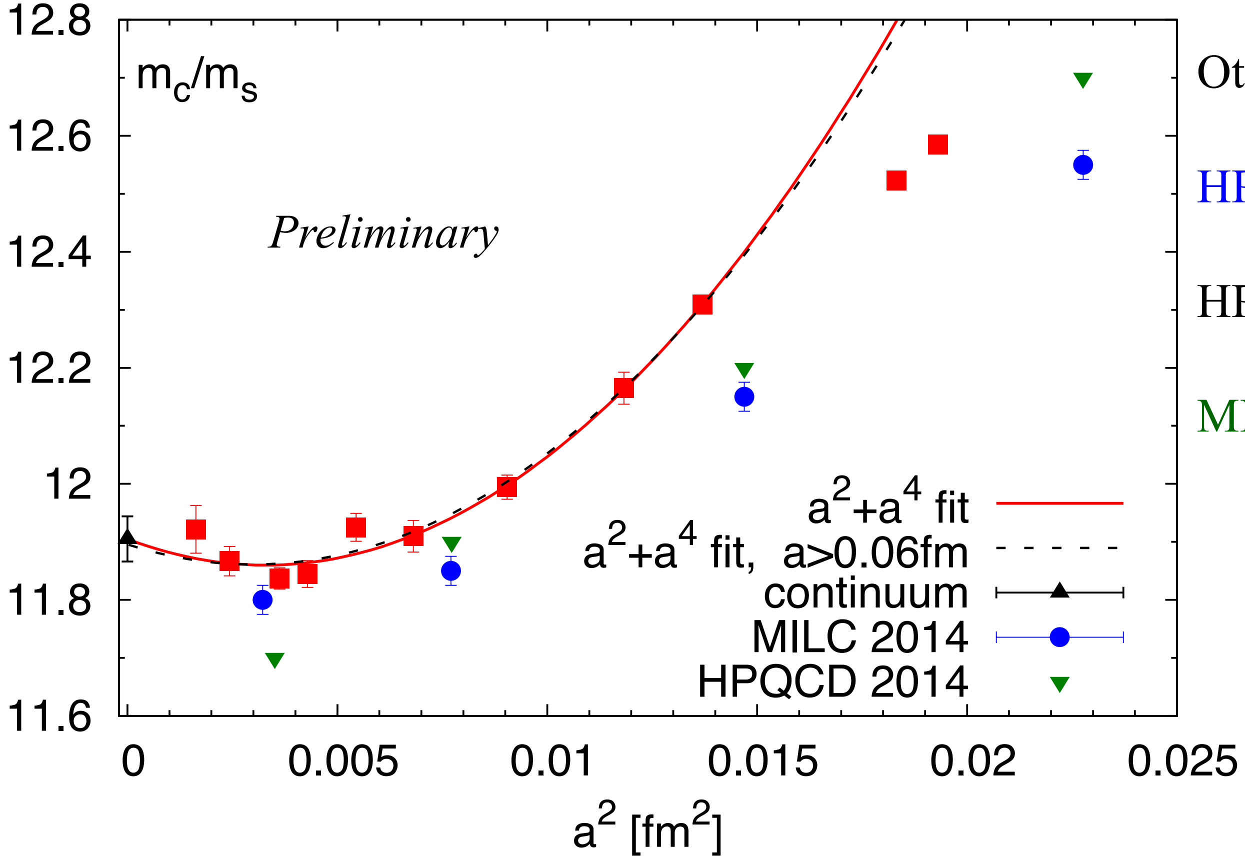
Levkova, Detar, PRD 83 (2011) 074504

Aubin et al (MILC), PRD 70 (2004) 114501

HPQCD: 0.6858(38)(12), 0.6885(22) GeV

Physical values of m_c and m_s in our lattice scheme instead of Msbar scheme, but the ratio is scheme independent

Charm to strange mass ratio



Other determinations:

HPQCD 2009: 11.85(16)

HPQCD 2014: 11.65(65)

MILC 2014: 11.747(19)(+59)(-43)

$$\frac{m_c}{m_s} = 11.905 \pm 0.039(stat.) \pm 0.049(scale) \pm 0.034(\eta_{ss}\ mass) \pm 0.018(\eta_c\ mass)$$

$$\frac{m_c}{m_s} = 11.905(73)$$ Preliminary

Moments of charm current correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G(t) = a^6 m_{c0}^2 \sum_{\mathbf{x}} \langle j_5(\mathbf{x},t) j_5(0,0) \rangle, \;\; j_5 = \bar{\psi}_c \gamma_5 \psi_c$$

$$G_n = \sum_t (t/a)^n \, G(t)$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), m_c(\mu))}{m_c^{n-4}(\mu)}, \quad g_n = \sum_j g_{nj}(m_c,\mu)\alpha_s^j(\mu)$$

To cancel lattice effects consider the reduced moments $$R_n = \left(\frac{G_n}{G_n^0}\right)^{1/(n-4)}$$

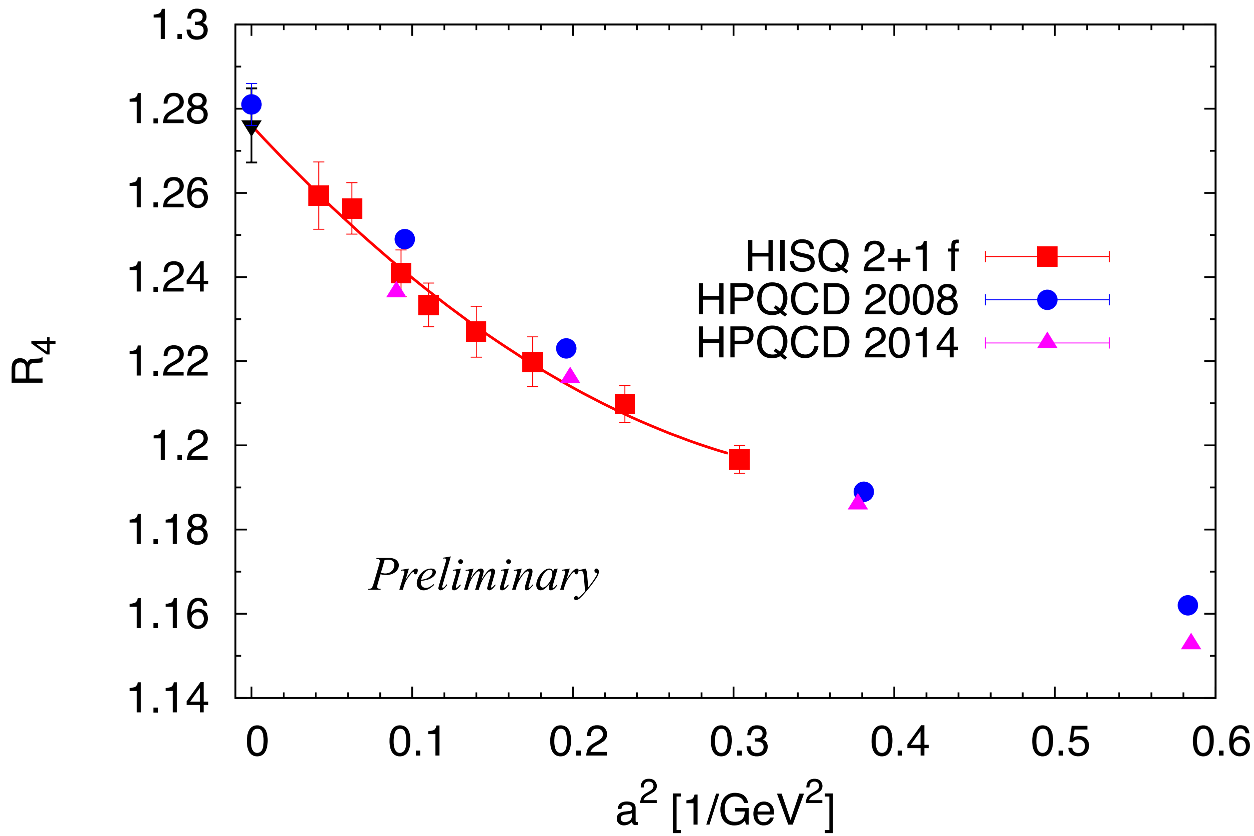
and similarly on the weak coupling side:

$$r_n = \sum_j r_{nj}(m_c,\mu)\alpha_s^j(\mu)$$

Effects of charm loop are 0.7% for R_4 and 0.1% for R_6 in perturbation theory
This information can be used to correct for charm loops in for in 2+1 flavor simulations

Allison et al, PRD78 (2008) 054513

Moments of charm current correlators (cont'd)



In the continuum limit the results for the 4th moment agree; our central value is slightly smaller

HPQCD 2008: Allison et al, PRD78 (2008) 054513

HPQCD 2014: Chakraborty et al, PRD 91 (2015) 054508

$\mu = m_c$ → No large logs in r_4 → $\alpha_s(m_c) = 0.372(12)$

Determination at the lowest scale !

Preliminary

$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.334(10)$

$\alpha_s(M_Z, n_f = 5) = 0.1164(10)$

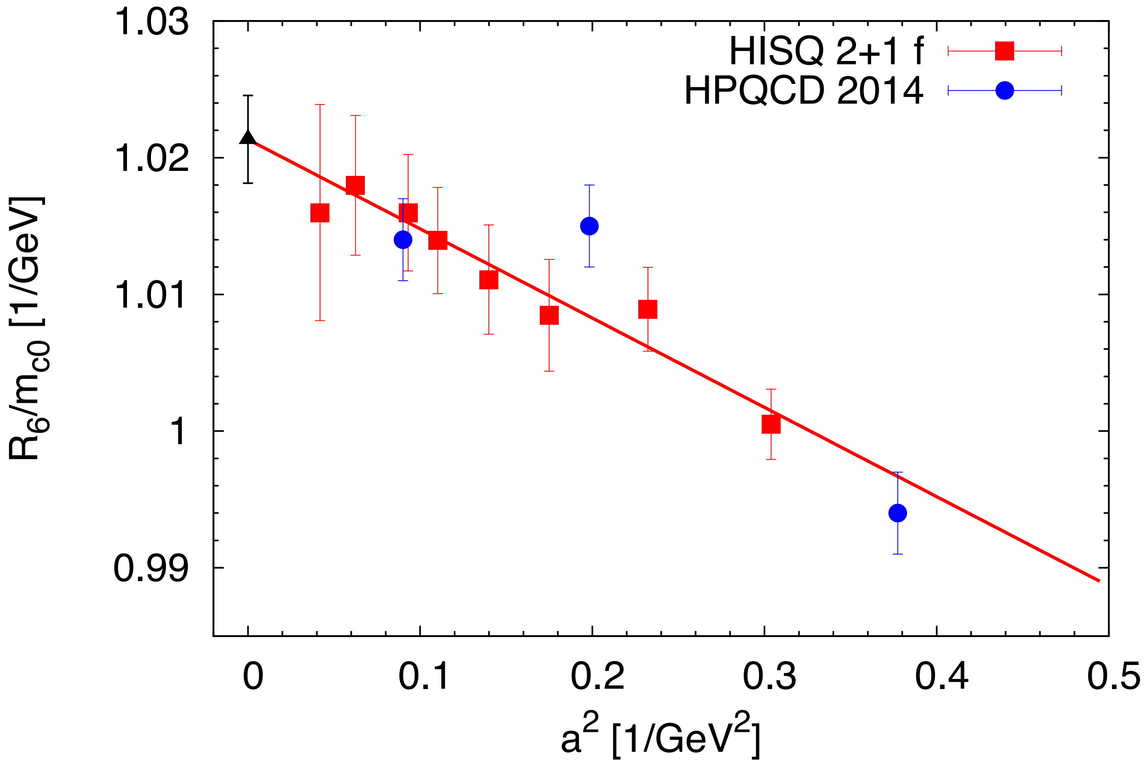
Static energy, Brambilla et al, PRD90 ('14)074038

$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.336(+12)(-08)$

$\alpha_s(M_Z, n_f = 5) = 0.1166(+12)(-08)$

also agrees with 2008 HPQCD value: $\alpha_s(M_Z, n_f = 5) = 0.1174(12)$

Moments of charm current correlators (cont'd)



Use 6th moment to determine the charm quark mass

$$\mu = m_c$$

$$m_c(m_c) = r_6(\alpha_s(m_c))\frac{m_{c0}}{R_6}$$

$$\left(\frac{m_{c0}}{R_6}\right)^{-1}_{cont} = (1.0213 \pm 0.0032 \pm 0.0058(scale))\,\mathrm{GeV}^{-1} = 1.0213(66)\mathrm{GeV}^{-1}$$

$$m_c(m_c) = 1.266(+0.008)(-0.011)(0.008)\ \mathrm{GeV}$$

HPQCD 2014: Chakraborty et al, PRD 91 (2015) 054508 $\quad m_c(m_c) = 1.2733(76)\mathrm{GeV}$

Summary

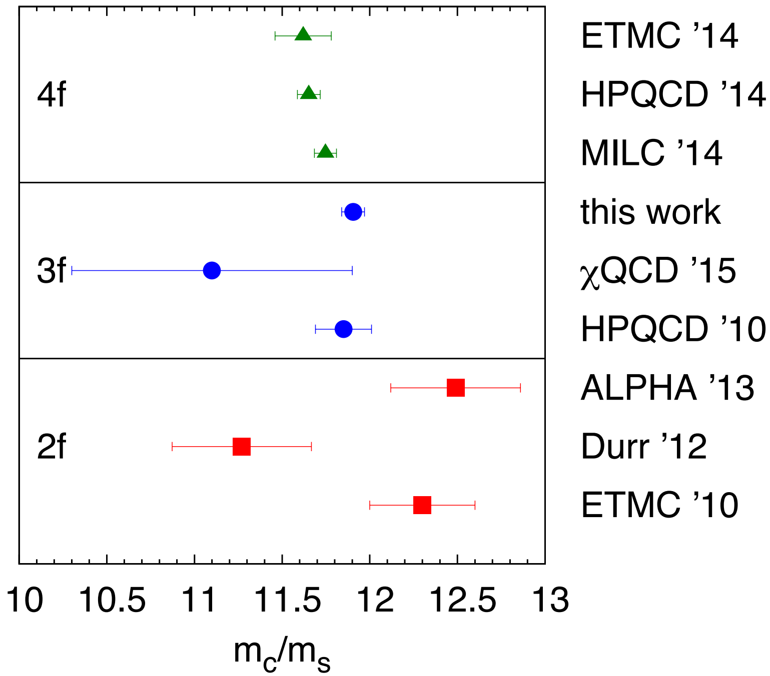
Using 2+1 flavor LQCD and moments methods we determined:

$$\alpha_s(M_Z, n_f = 5) = 0.1164(10)$$

in excellent agreement with the results obtained from static energy, Brambilla et al, 2014

$$m_c(m_c) = 1.266(+0.008)(-0.011)(0.008)\ \mathrm{GeV}$$

in agreement with HPQCD 2014 result but with x2 error



Somewhat larger value of charm to strange quark mass ratio than HPQCD 2014 and MILC 2014 but similar errors

$$\frac{m_c}{m_s} = 11.905(73)$$

$$m_s(\mu = 2\mathrm{GeV}, n_f = 3) = 91.0(1.6)\mathrm{MeV}$$