

# *Higgs boson cross section predictions at the LHC and the strong coupling constant*

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MITP workshop *Determination of the Fundamental Parameters in QCD*, Mainz, Mar 07, 2016

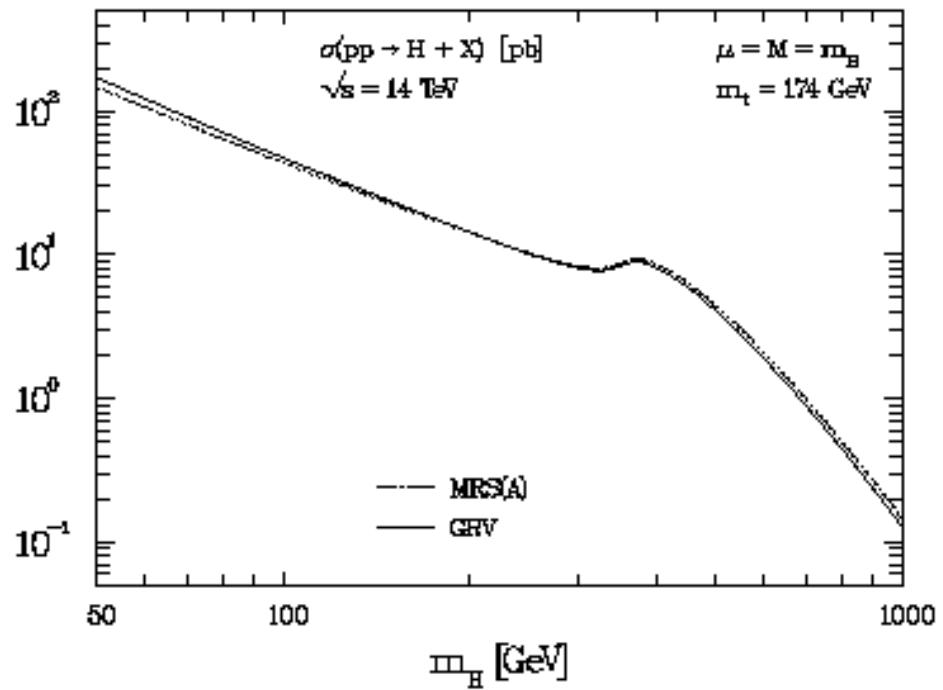
## *Based on work done in collaboration with:*

- *Recommendations for PDF usage in LHC predictions*  
A. Accardi, S. Alekhin, M. Botje, J. Blümlein, M.V. Garzelli, K. Lipka,  
W. Melnitchouk, S. M., R. Plačakytė, J.F. Owens, E. Reya, N. Sato,  
A. Vogt and O. Zenaiev [DESY 16-041](#), to appear
- *Iso-spin asymmetry of quark distributions and implications for single top-quark production at the LHC*  
S. Alekhin, J. Blümlein, S. M. and R. Plačakytė [arXiv:1508.07923](#)
- *Determination of Strange Sea Quark Distributions from Fixed-target and Collider Data*  
S. Alekhin, J. Blümlein, L. Caminada, K. Lipka, K. Lohwasser, S. M.,  
R. Petti, and R. Plačakytė [arXiv:1404.6469](#)
- *The ABM parton distributions tuned to LHC data*  
S. Alekhin, J. Blümlein and S. M. [arXiv:1310.3059](#)
- Many more papers of ABM and friends ...  
[2008](#) – ...

# *Higgs boson production*

# Higgs cross section (1995)

## NLO QCD corrections



MRS(A): Martin, Roberts and Stirling,  
Phys. Rev. D50 (1994) 6734

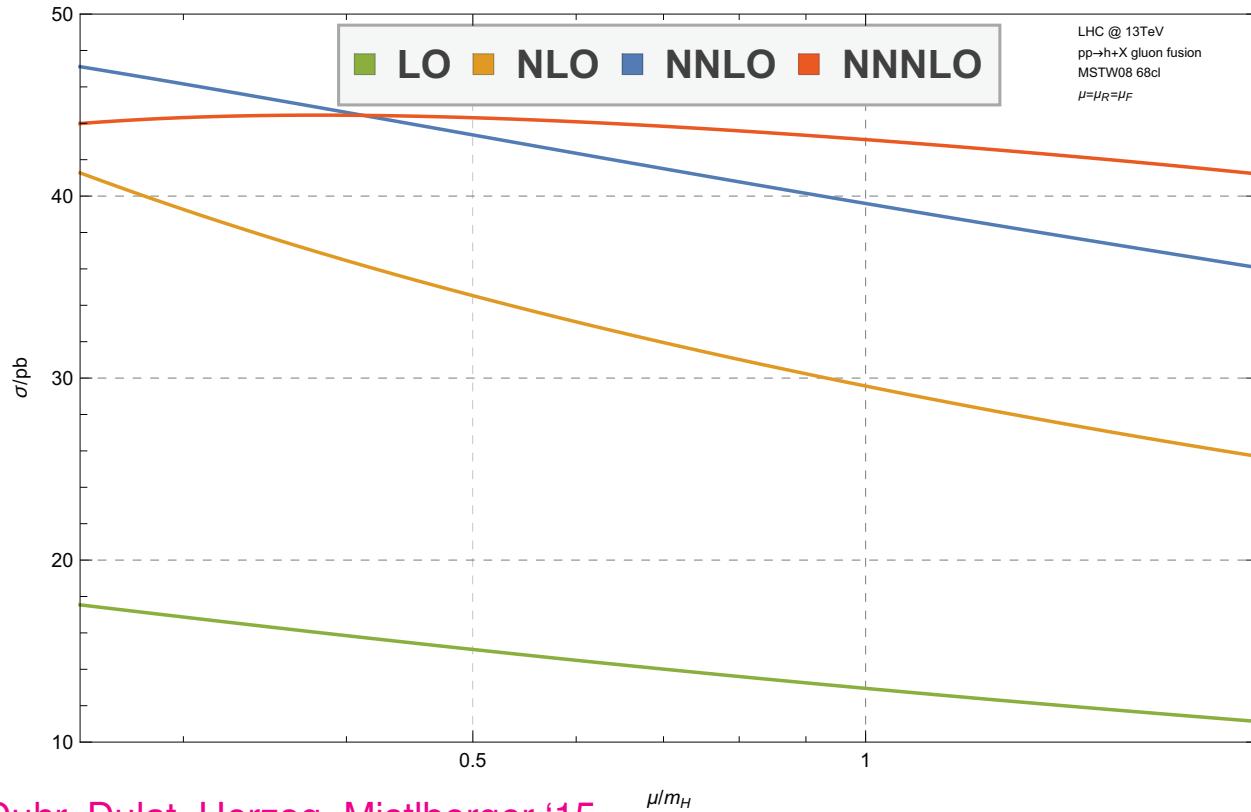
GRV: Glück, Reya and Vogt,  
Z. Phys. C53 (1992) 127

One of the main uncertainties in the prediction of the Higgs production cross section is due to the gluon density. [...] Adopting a set of representative parton distributions [...], we find a variation of about 7% between the maximum and minimum values of the cross section for Higgs masses above  $\sim 100$  GeV.

Spira, Djouadi, Graudenz, Zerwas (1995)  
hep-ph/9504378

# Higgs cross section (2016)

## Exact $N^3LO$ QCD corrections

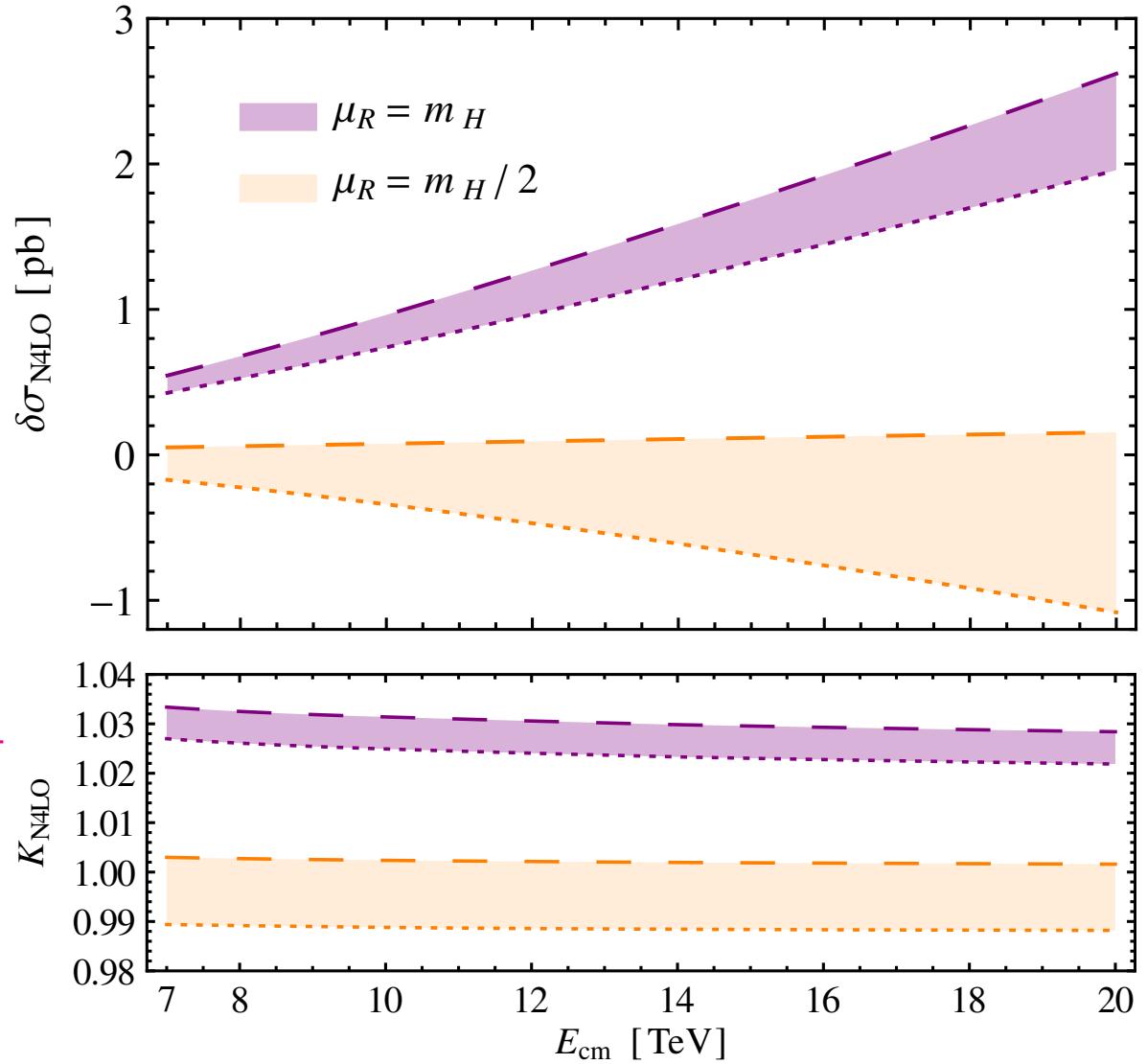


Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15

- Apparent convergence of perturbative expansion
- Scale dependence of exact  $N^3LO$  prediction with residual uncertainty 3%
- Minimal sensitivity at scale  $\mu = m_H/2$

# Approximate $N^4LO$ QCD corrections

- Consistency check with approximate  $N^4LO$  corrections at two scales  $\mu = m_H$  and  $\mu = m_H/2$
- $K$ -factor  $\simeq 1\%$  for  $\mu = m_H/2$  with at  $\sqrt{s} = 13$  TeV  
de Florian, Mazzitelli, S.M., Vogt '14



## Dependence of cross section on parton luminosity

- Cross section  $\sigma(H)$  at NNLO with uncertainties:  $\sigma(H) + \Delta\sigma(\text{PDF} + \alpha_s)$  for  $m_H = 125.0 \text{ GeV}$  at  $\sqrt{s} = 13 \text{ TeV}$  with  $\mu_R = \mu_F = m_H$  and nominal  $\alpha_s$

ABM12 Alekhin, Blümlein, S.M. '13	$39.80 \pm 0.84 \text{ pb}$
CJ15 (NLO) Accardi, Brady, Melnitchouk et al. '16	$45.45 \pm 0.17 \text{ pb}$
CT14 Dulat et al. '15	$42.33 \pm 1.43 \text{ pb}$
HERAPDF2.0 H1+Zeus Coll.	$42.62 \pm 0.35 \text{ pb}$
JR14 (dyn) Jimenez-Delgado, Reya '14	$38.01 \pm 0.34 \text{ pb}$
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	$42.36 \pm 0.56 \text{ pb}$
NNPDF3.0 Ball et al. '14	$42.59 \pm 0.80 \text{ pb}$
PDF4LHC15 Butterworth et al. '15	$42.42 \pm 0.78 \text{ pb}$

- Large spread for predictions from different PDFs  $\sigma(H) = 38.0 \dots 42.6 \text{ pb}$
- PDF and  $\alpha_s$  differences between sets amount to up to 11%
  - significantly larger than residual theory uncertainty due to N<sup>3</sup>LO QCD corrections

*How to explain the differences ?*

## *Strong coupling constant*

# Strong coupling constant (1992)

	$\alpha_s(M_Z^2)$
$R_\tau$	$0.117^{+0.010}_{-0.016}$
DIS	$0.112 \pm 0.007$
$\Upsilon$ Decays	$0.110 \pm 0.010$
$R_{e^+e^-}(s < 62\text{GeV})$	$0.140 \pm 0.020$
$p\bar{p} \rightarrow W + jets$	$0.121 \pm 0.024$
$\Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow l\bar{l})$	$0.132 \pm 0.012$
Jets at LEP	$0.122 \pm 0.009$
Average	$0.118 \pm 0.007$

G. Altarelli (1992)  
in QCD - 20 Years Later,  
CERN-TH-6623-92

## Essential facts

- World average 1992  $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to  $\sim 0.0050 - 0.0040$  (theory scale uncertainty)

# Strong coupling constant (2016)

## Measurements at NNLO

- Values of  $\alpha_s(M_Z)$  at NNLO from PDF fits

SY	$0.1166 \pm 0.013$	$F_2^{ep}$	Santiago, Yndurain '01
	$0.1153 \pm 0.063$	$xF_3^{\nu N}$ (heavy nucl.)	
A02	$0.1143 \pm 0.013$	DIS	Alekhin '01
MRST03	$0.1153 \pm 0.0020$		Martin, Roberts, Stirling, Thorne '03
BBG	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck '06
A06	$0.1128 \pm 0.015$		Alekhin '06
JR08	$0.1128 \pm 0.0010$	dynamical approach	Jimenez-Delgado, Reya '08
	$0.1162 \pm 0.0006$	including NLO jets	
ABKM09	$0.1135 \pm 0.0014$	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
	$0.1129 \pm 0.0014$	HQ: BSMN	
MSTW	$0.1171 \pm 0.0014$		Martin, Stirling, Thorne, Watt '09
Thorne	0.1136	[DIS+DY, HT*] (2013)	Thorne '13
ABM11 <sub>J</sub>	$0.1134 \dots 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. '11
NN21	$0.1173 \pm 0.0007$	(+ heavy nucl.)	NNPDF '11
ABM12	$0.1133 \pm 0.0011$		Alekhin, Blümlein, S.M. '13
	$0.1132 \pm 0.0011$	(without jets)	
CT10	0.1140	(without jets)	Gao et al. '13
CT14	$0.1150^{+0.0060}_{-0.0040}$	$\Delta\chi^2 > 1$ (+ heavy nucl.)	Dulat et al. '15
MMHT	$0.1172 \pm 0.0013$	(+ heavy nucl.)	Martin, Motylinski, Harland-Lang, Thorne '15

# Strong coupling constant (2016)

## Other measurements of $\alpha_s$ at NNLO

- Values of  $\alpha_s(M_Z)$  at NNLO from measurements at colliders

3-jet rate	$0.1175 \pm 0.0025$	Dissertori et al. 2009	<a href="#">arXiv:0910.4283</a>
$e^+e^-$ thrust	$0.1131^{+0.0028}_{-0.0022}$	Gehrman et al.	<a href="#">arXiv:1210.6945</a>
$e^+e^-$ thrust	$0.1140 \pm 0.0015$	Abbate et al.	<a href="#">arXiv:1204.5746</a>
$C$ -parameter	$0.1123 \pm 0.0013$	Hoang et al.	<a href="#">arXiv:1501.04111</a>
CMS	$0.1151 \pm 0.0033$	$t\bar{t}$	<a href="#">arXiv:1307.1907</a>
NLO Jets ATLAS	$0.111^{+0.0017}_{-0.0007}$		<a href="#">arXiv:1312.5694</a>
NLO Jets CMS	$0.1148 \pm 0.0055$		<a href="#">arXiv:1312.5694</a>

# $\alpha_s(M_Z)$ in Higgs cross sections

PDF sets	$\alpha_s(M_Z)$	method of determination
ABM12 Alekhin, Blümlein, S.M. '13	$0.1132 \pm 0.0011$	fit at NNLO
CJ15 Accardi, Brady, Melnitchouk et al. '16	$0.118 \pm 0.002$	fit at NLO
CT14 Dulat et al. '15	0.118	assumed at NNLO
HERAPDF2.0 H1+Zeus Coll.	$0.1183^{+0.0040}_{-0.0034}$	fit at NLO
JR14 Jimenez-Delgado, Reya '14	$0.1136 \pm 0.0004$ $0.1162 \pm 0.0006$	dynamical fit at NNLO standard fit at NNLO
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	0.118 $0.1172 \pm 0.0013$	assumed at NNLO best fit at NNLO
NNPDF3.0 Ball et al. '14	0.118	assumed at NNLO
PDF4LHC15 Butterworth et al. '15	0.118 0.118	assumed at NLO assumed at NNLO

- Values of  $\alpha_s(M_Z)$  often assumed and not fitted (no correlations)
- Large spread of fitted values at NNLO:  $\alpha_s(M_Z) = 0.1132 \dots 0.1172$
- PDF4LHC: order independent recommendation
  - use  $\alpha_s(M_Z) = 0.118$  at NLO and NNLO

# Differences in $\alpha_s$ determinations

## Why $\alpha_s$ values from MSTW and NNPDF are large

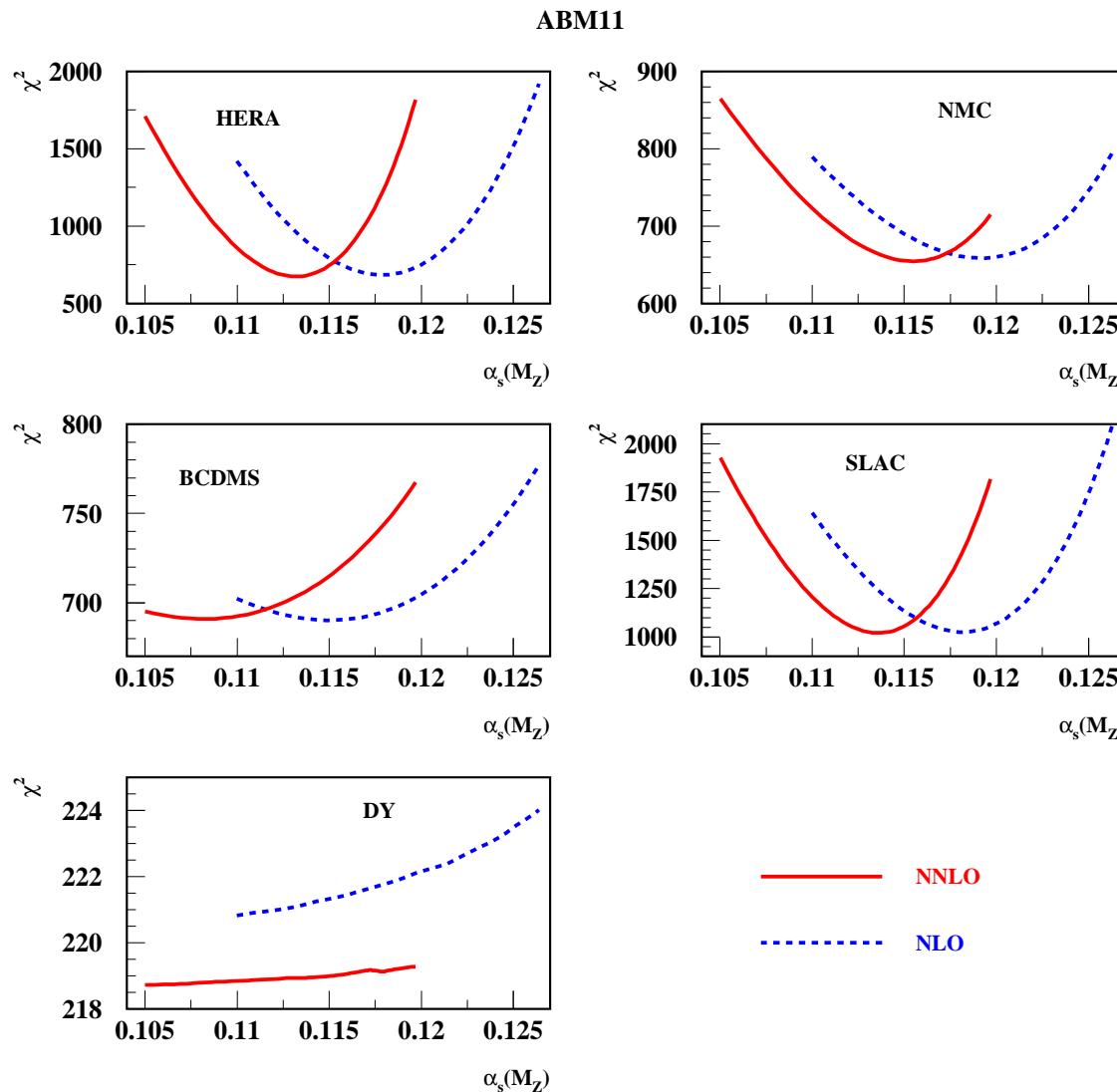
- Differences result from different physics models and analysis procedures
- Fits of DIS data
  - target mass corrections (powers of nucleon mass  $M_N^2/Q^2$ )
  - higher twist  $F_2^{\text{ht}} = F_2 + \text{ht}^{(4)}(x)/Q^2 + \text{ht}^{(6)}(x)/Q^4 + \dots$
  - correlation of errors among different data sets

	$\alpha_s$	NNLO	target mass corr.	higher twist	error correl.
ABM12	$0.1132 \pm 0.0011$	yes	yes	yes	yes
NNPDF21	$0.1173 \pm 0.0007$	(yes)	yes	no	yes
MSTW	$0.1171 \pm 0.0014$	(yes)	no	no	no
MMHT	$0.1172 \pm 0.0013$	(yes)	no	no	–

- Effects for differences are understood
  - variants of ABM with no higher twist etc. reproduce larger  $\alpha_s$  values  
Alekhin, Blümlein, S.M. '11

# Zooming in on ABM

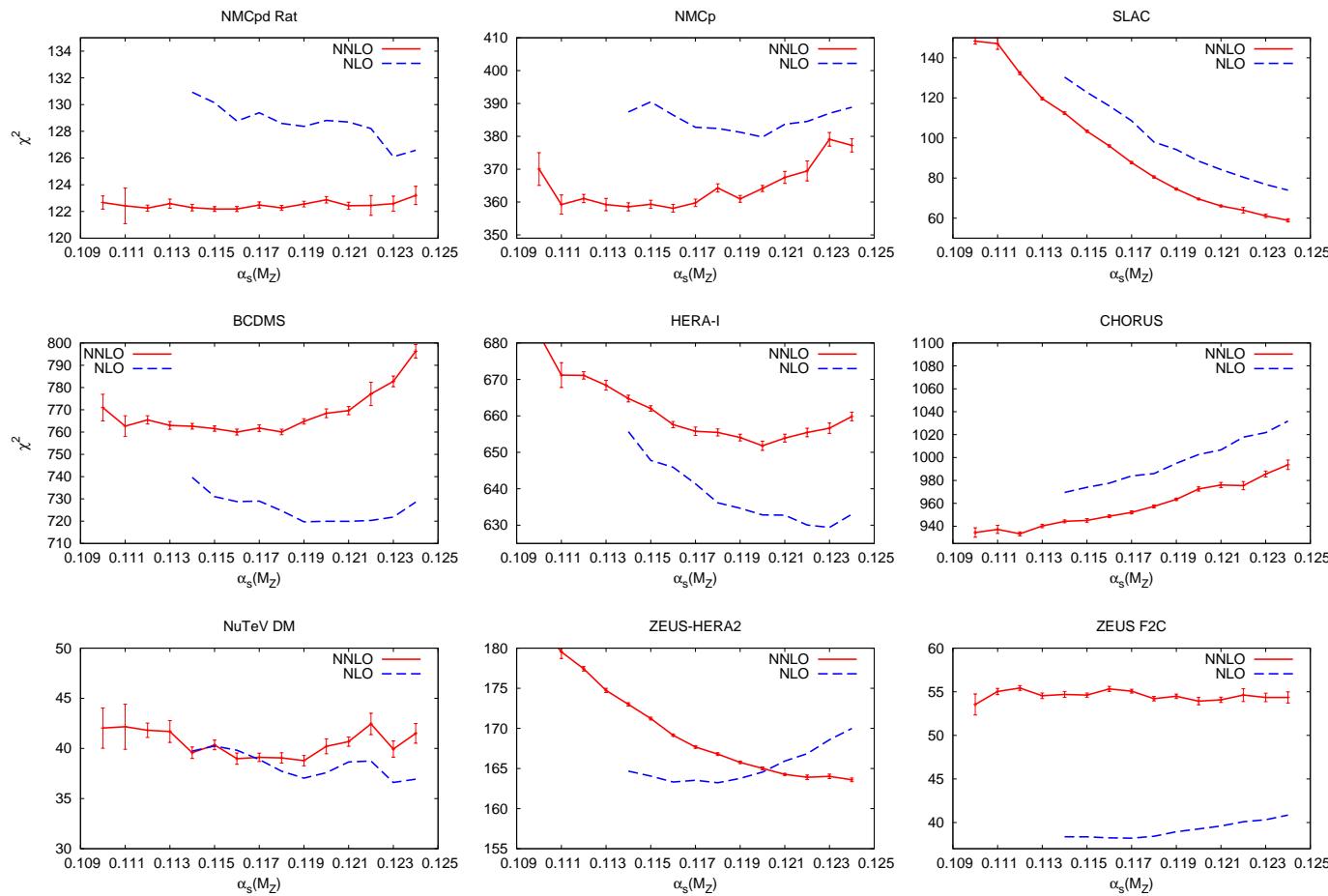
$\alpha_s$  from DIS and PDFs



- Profile of  $\chi^2$  for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. '12

# Zooming in on NNPDF

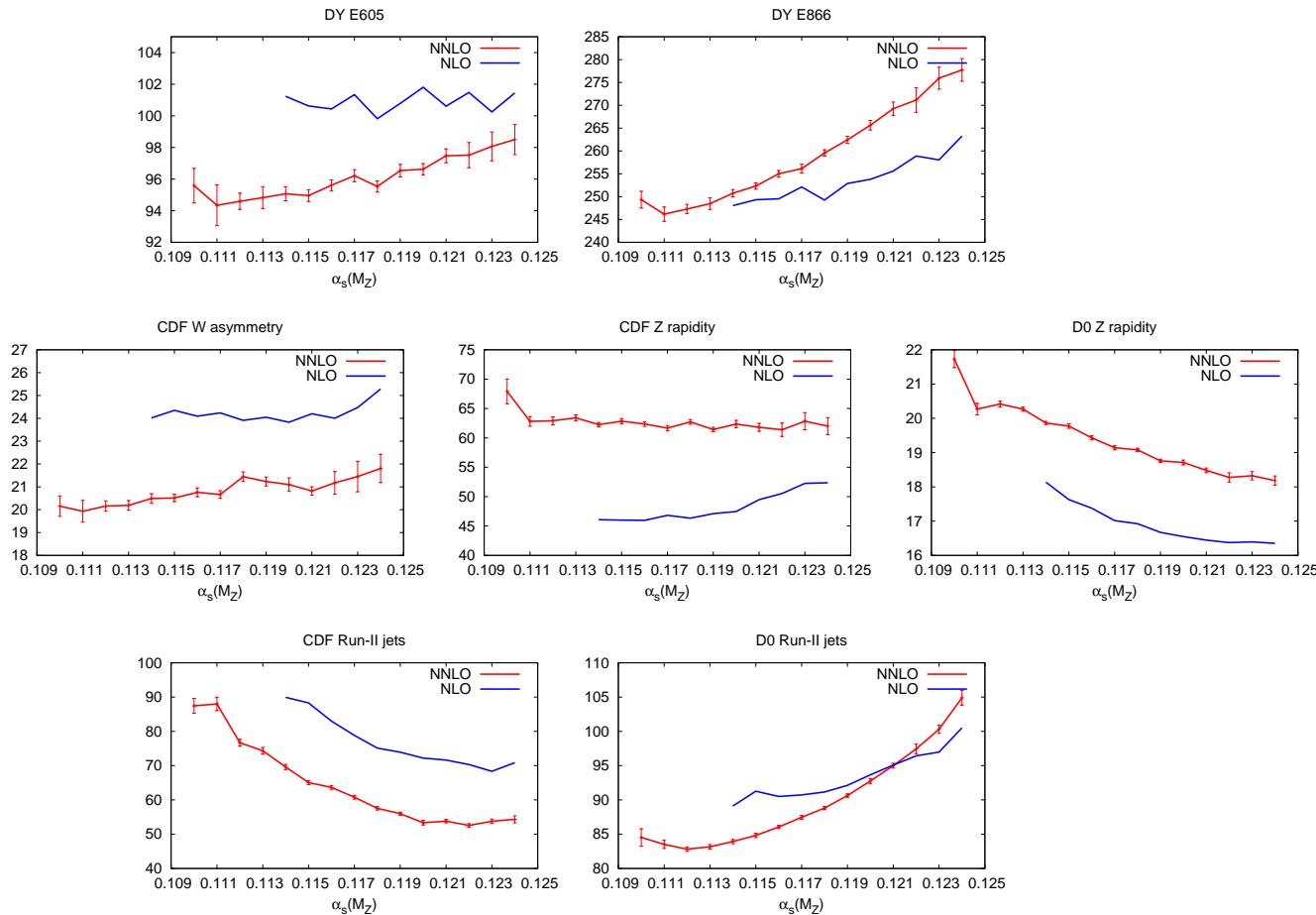
## $\alpha_s$ from DIS and PDFs



- Profile of  $\chi^2$  for different data sets in NNPDF21 fit Ball et al. '11

# Zooming in on NNPDF

$\alpha_s$  from DIS and PDFs



- Profile of  $\chi^2$  for different data sets in NNPDF21 fit Ball et al. '11

## *Parton content of the proton*

# *Data in global PDF fits*

## *Data sets considered in ABM12 analysis*

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC  $(NDP = 2699)$
  - semi-inclusive DIS charm production data HERA  $(NDP = 52)$
  - Drell-Yan data (fixed target) E-605, E-866  $(NDP = 158)$
  - neutrino-nucleon DIS (di-muon data) CCFR/NuTeV  $(NDP = 178)$
  - LHC data for  $W^\pm$ - and  $Z$ -boson production ATLAS, CMS, LHCb  $(NDP = 60)$

## *Iterative cycle of PDF fits*

- i) check of compatibility of new data set with available world data
- ii) study of potential constraints due to addition of new data set to fit
- iii) perform high precision measurement of the non-perturbative parameters
  - parton distributions
  - strong coupling  $\alpha_s(M_Z)$
  - heavy quark masses

## ABM PDF ansatz

- PDFs parameterization at scale  $\mu = 3\text{GeV}$  in scheme with  $n_f = 3$   
Alekhin, Blümlein, S.M. '12
  - ansatz for valence-/sea-quarks, gluon with polynomial  $P(x)$
  - strange quark is taken in charge-symmetric form
  - 24 parameters in polynomials  $P(x)$
  - 4 additional fit parameters:  $\alpha_s^{(n_f=3)}(\mu = 3 \text{ GeV})$ ,  $m_c$ ,  $m_b$  and deuteron correction
  - simultaneous fit of higher twist parameters (twist-4)

$$xq_v(x, Q_0^2) = \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q} (1-x)^{b_q} x^{P_{qv}(x)}$$

$$xu_s(x, Q_0^2) = x\bar{u}_s(x, Q_0^2) = A_{us} x^{a_{us}} (1-x)^{b_{us}} x^{a_{us} P_{us}(x)}$$

$$x\Delta(x, Q_0^2) = xd_s(x, Q_0^2) - xu_s(x, Q_0^2) = A_\Delta x^{a_\Delta} (1-x)^{b_\Delta} x^{P_\Delta(x)}$$

$$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = A_s x^{a_s} (1-x)^{b_s},$$

$$xg(x, Q_0^2) = A_g x^{a_g} (1-x)^{b_g} x^{a_g P_g(x)}$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

# Quality of fit

## Statistical tests

- Goodness-of-fit estimator
  - $\chi^2$  values compared to number of data points (typically a few thousand in global fit)

## Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (I)  
Alekhin, Blümlein, S.M. '12

	$a_u$	$b_u$	$\gamma_{1,u}$	$\gamma_{2,u}$	$a_d$	$b_d$	$A_d$	$b_\Delta$	$A_u$	$a_{us}$	$b_{us}$	$a_G$	$b_G$
$a_u$	1.0000	0.9256	0.9638	-0.2527	0.3382	0.2922	0.1143	-0.4267	0.4706	0.3117	0.1422	0.0982	0.1127
$b_u$		1.0000	0.9574	-0.5608	0.1933	0.1200	0.1058	-0.3666	0.3712	0.2674	0.1537	0.0453	0.1878
$\gamma_{1,u}$			1.0000	-0.4504	0.2328	0.2329	0.0906	-0.3379	0.4106	0.2876	0.0812	0.0491	0.1627
$\gamma_{2,u}$				1.0000	0.3007	0.3119	-0.0242	-0.0118	0.0587	0.0026	-0.0305	0.0949	-0.1876
$a_d$					1.0000	0.8349	-0.2010	-0.3371	0.3786	0.2592	0.1212	-0.0377	0.1305
$b_d$						1.0000	-0.2669	-0.0599	0.2768	0.1941	-0.0698	-0.0926	0.2088
$A_d$							1.0000	-0.2132	0.0549	0.0245	0.2498	-0.0523	0.0614
$b_\Delta$								1.0000	-0.1308	-0.0729	-0.7208	-0.0124	-0.0225
$A_u$									1.0000	0.9240	-0.0723	0.3649	-0.1674
$a_{us}$										1.0000	-0.0144	0.2520	-0.1095
$b_{us}$											1.0000	-0.1274	0.1808
$a_G$												1.0000	-0.6477
$b_G$													1.0000

# Quality of fit

## Statistical tests

- Goodness-of-fit estimator
  - $\chi^2$  values compared to number of data points (typically a few thousand in global fit)

## Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (II)  
**Alekhin, Blümlein, S.M. '12**

	$\gamma_{1,G}$	$\alpha_s(3, 3 \text{ GeV})$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	$A_s$	$b_s$	$a_s$	$a_\Delta$	$m_c$	$m_b$
$a_u$	-0.0727	-0.0611	0.3383	0.6154	0.2320	-0.0724	-0.0681	-0.0763	-0.0935	0.0026	0.0900	-0.0053
$b_u$	-0.1130	-0.1725	0.2992	0.4848	0.0849	0.0720	-0.0723	-0.0618	-0.0926	0.0049	0.0349	-0.0118
$\gamma_{1,u}$	-0.1106	-0.1338	0.2753	0.5638	0.1316	-0.0535	-0.0798	-0.0854	-0.1059	-0.0060	0.0817	0.0003
$\gamma_{2,u}$	0.1174	0.2195	-0.0210	0.0822	0.3712	-0.3310	0.0339	0.0143	0.0381	-0.0098	0.0430	-0.0004
$a_d$	-0.1631	-0.0208	0.0319	0.4974	0.9570	-0.4636	-0.0700	-0.0996	-0.0979	-0.2121	0.1066	-0.0150
$b_d$	-0.2198	-0.0913	-0.1775	0.4092	0.8985	-0.8498	-0.0533	-0.0669	-0.0806	-0.2252	0.0822	-0.0068
$A_d$	-0.0825	0.0188	0.8558	-0.0289	-0.2624	0.2852	-0.0075	-0.0189	-0.0180	0.9602	0.0420	0.0120
$b_\Delta$	0.0530	-0.0801	-0.6666	-0.0904	-0.1981	-0.2532	-0.0022	0.0257	0.0048	-0.0260	-0.0166	-0.0056
$A_u$	0.2502	-0.0157	0.1265	0.7525	0.3047	-0.0668	-0.7064	-0.6670	-0.7267	0.0345	0.2137	0.0358
$a_{us}$	0.1845	-0.0216	0.0683	0.5714	0.2157	-0.0554	-0.8768	-0.8081	-0.8980	0.0145	0.0430	0.0074
$b_{us}$	-0.1619	-0.0715	0.5343	-0.3656	0.0293	0.2430	-0.0345	-0.0132	-0.0356	0.1527	-0.0899	-0.0058
$a_G$	0.8291	0.2306	-0.0260	0.3692	-0.0966	0.1496	0.0087	0.0007	0.0464	-0.0541	-0.0661	0.0417
$b_G$	-0.9184	-0.6145	0.0538	-0.2770	0.1990	-0.2552	0.0381	0.0616	-0.0468	0.0502	0.1847	0.0861

# Quality of fit

## Statistical tests

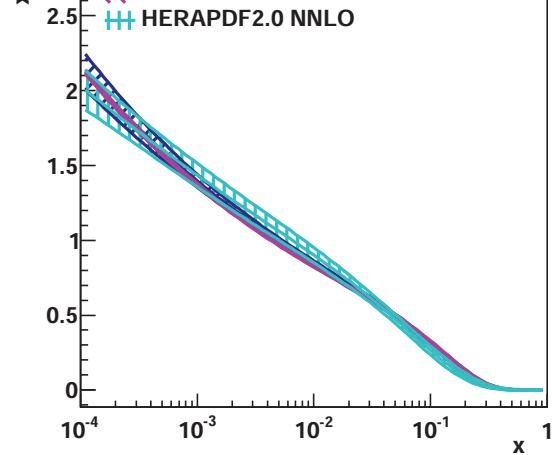
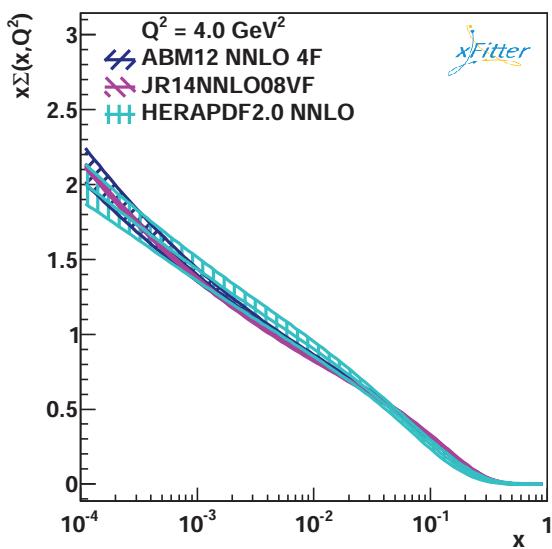
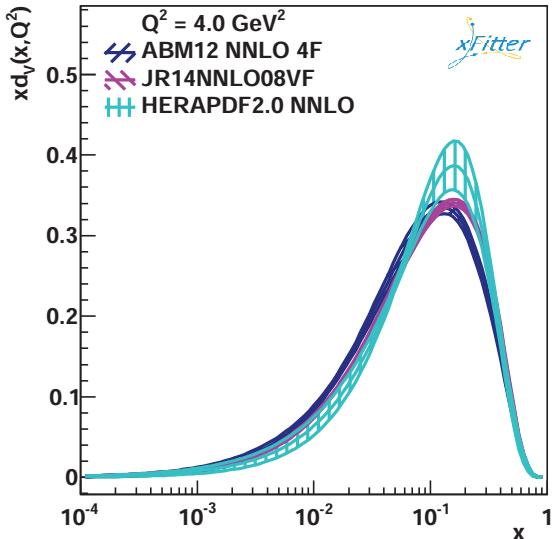
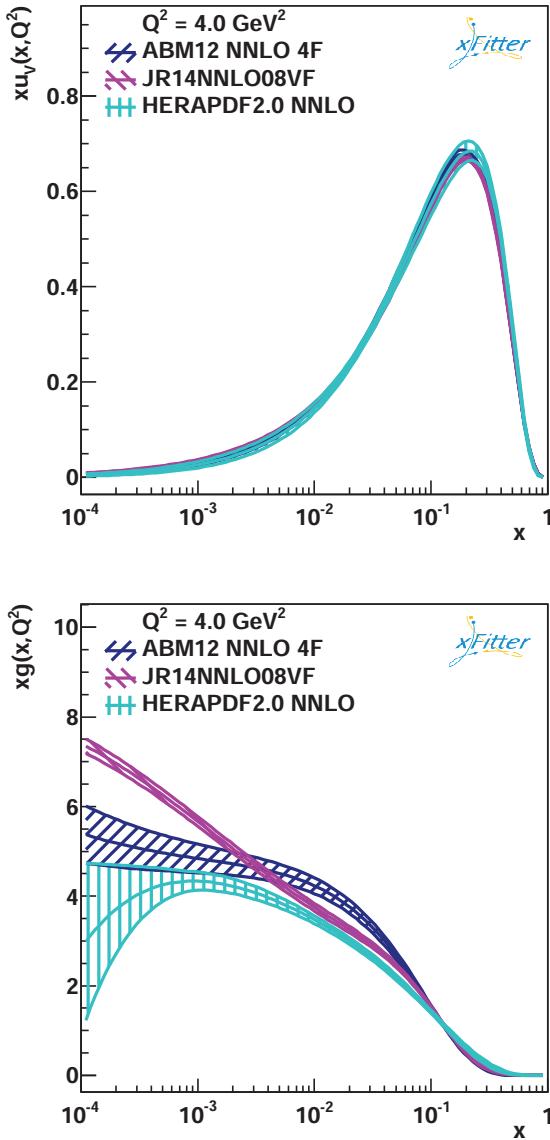
- Goodness-of-fit estimator
  - $\chi^2$  values compared to number of data points (typically a few thousand in global fit)

## Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (III)  
*Alekhin, Blümlein, S.M. '12*

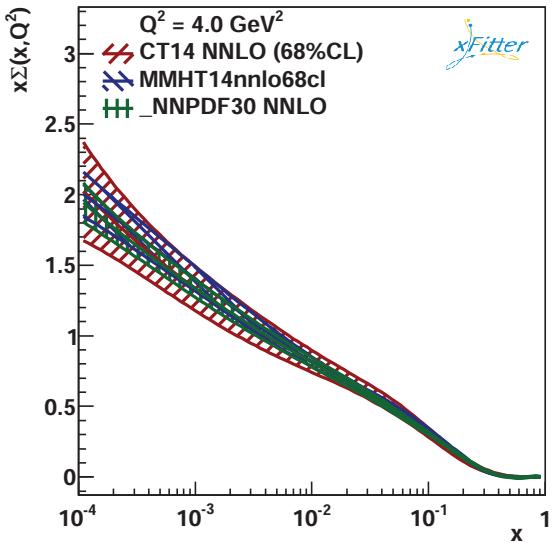
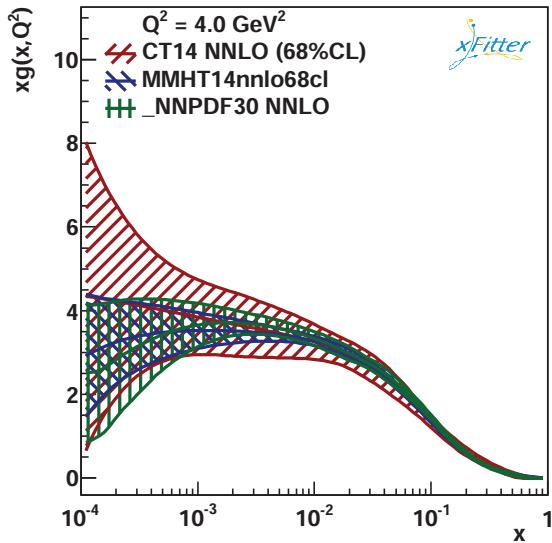
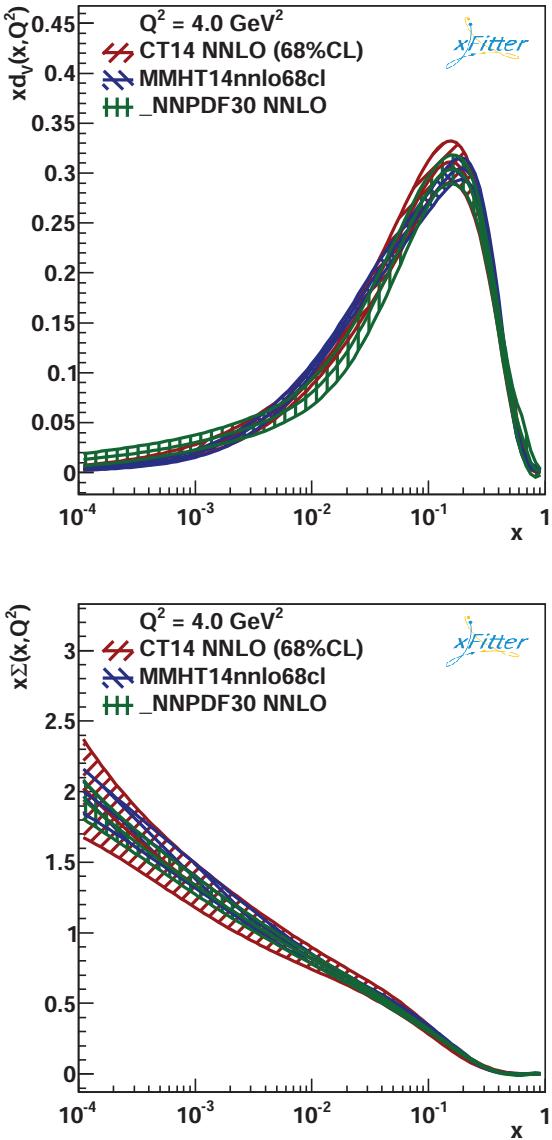
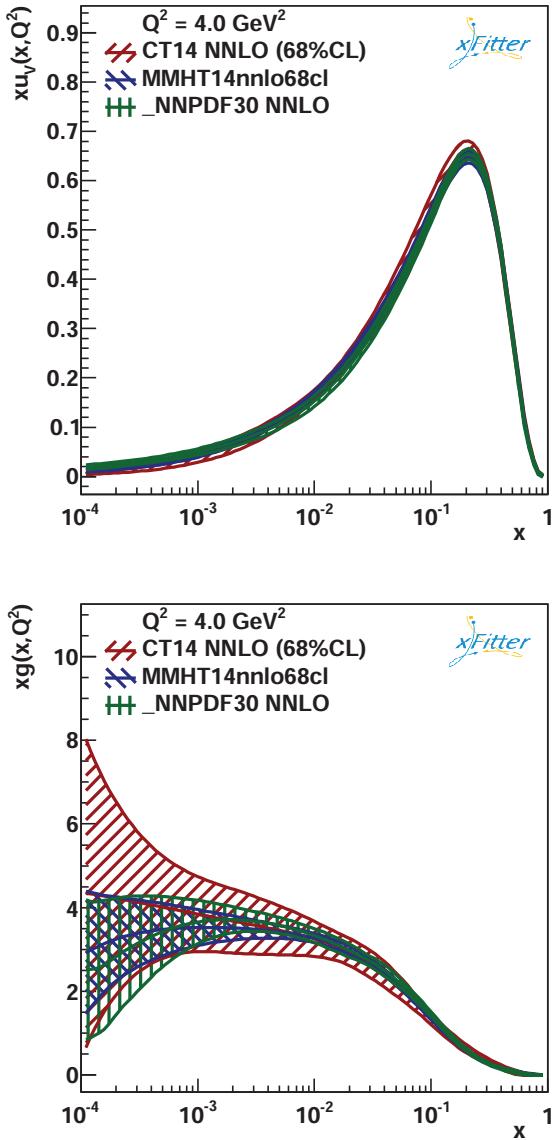
	$\gamma_{1,G}$	$\alpha_s(3, 3 \text{ GeV})$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	$A_s$	$b_s$	$a_s$	$a_\Delta$	$m_c$	$m_b$
$\gamma_{1,G}$	1.0000	0.3546	-0.0876	0.2751	-0.2215	0.2410	-0.0539	-0.0634	0.0122	-0.0658	-0.1149	-0.0474
$\alpha_s(3, 3 \text{ GeV})$		1.0000	0.0601	0.1127	-0.0761	0.1534	-0.0176	-0.0121	0.0883	0.0022	-0.5641	-0.0526
$\gamma_{1,\Delta}$			1.0000	0.0699	-0.1081	0.3796	-0.0050	-0.0329	-0.0175	0.7098	0.0418	0.0113
$\gamma_{1,us}$				1.0000	0.4099	-0.1547	-0.2622	-0.3181	-0.2801	-0.0785	0.1870	0.0103
$\gamma_{1,d}$					1.0000	-0.6540	-0.0688	-0.0892	-0.0974	-0.2332	0.0999	-0.0093
$\gamma_{2,d}$						1.0000	0.0212	0.0128	0.0413	0.1876	-0.0396	-0.0049
$A_s$							1.0000	0.8584	0.9689	-0.0109	0.0596	0.0116
$b_s$								1.0000	0.8826	-0.0173	-0.0777	0.0003
$a_s$									1.0000	-0.0204	-0.0845	-0.0145
$a_\Delta$										1.0000	0.0385	0.0085
$m_c$											1.0000	0.1451
$m_b$												1.0000

# Results for parton distributions



- PDFs with  $1\sigma$  uncertainty bands
  - Comparison of ABM12, HERAPDF2.0, JR14
  - Some interesting observations to be made
- ...

# Results for parton distributions



- PDFs with  $1\sigma$  uncertainty bands
  - Comparison of CT14, MMHT14, NNPDF3.0
  - Some interesting observations to be made
- ...

## *Heavy quarks in deep-inelastic scattering*

# Treatment of heavy-quarks

## Light quarks

- Neglect “light quark” masses  $m_u, m_d \ll \Lambda_{QCD}$  and  $m_s < \Lambda_{QCD}$  in hard scattering process
  - scale-dependent  $u, d, s, g$  PDFs from mass singularities

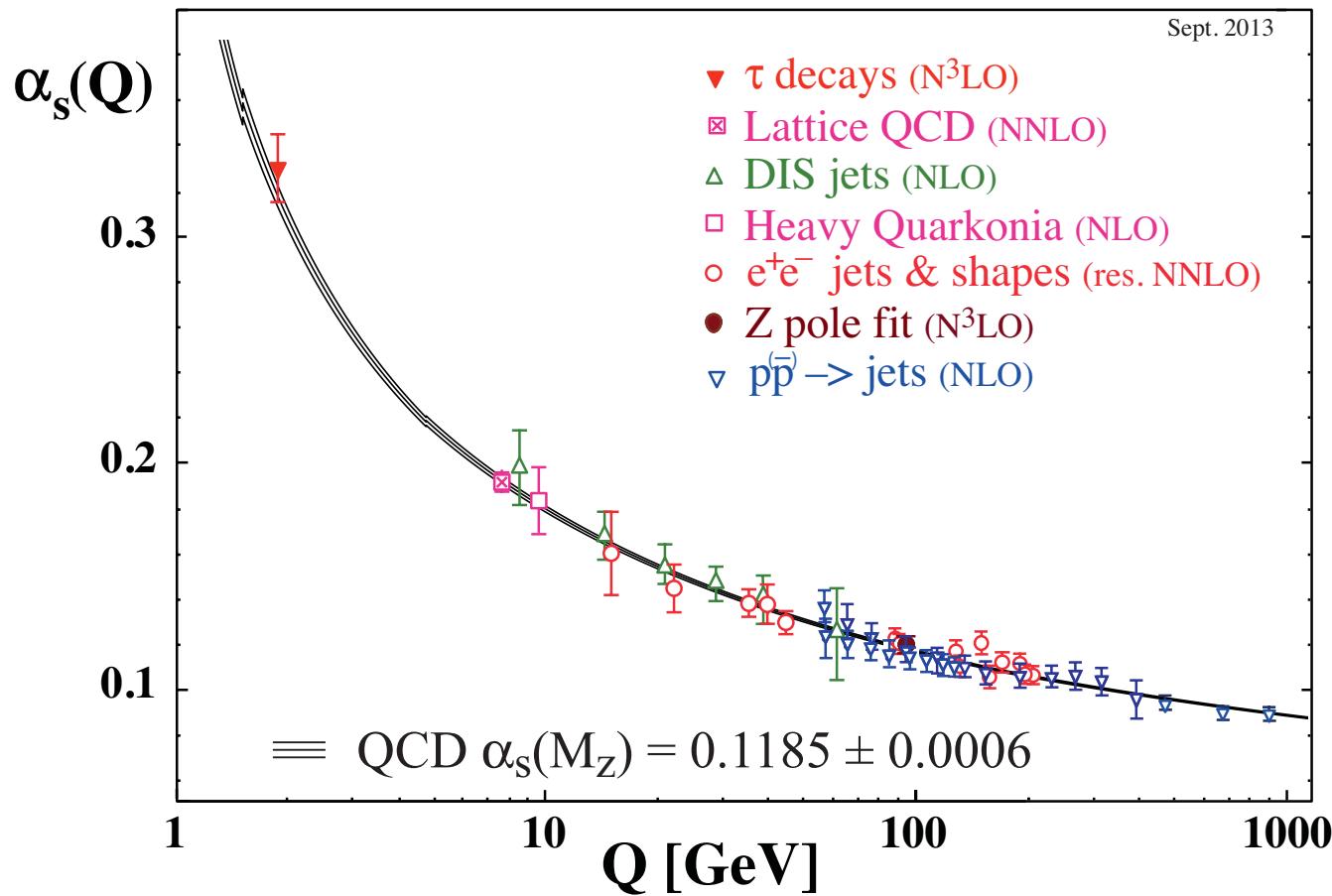
## Heavy quarks

- No mass singularities for  $m_c, m_b, m_t \gg \Lambda_{QCD}$ , no (evolving) PDFs
  - $c$  and  $b$  PDFs for  $Q \ggg m_c, m_b$  generated perturbatively
  - matching of two distinct theories
    - $n_f$  light flavors + heavy quark of mass  $m$  at low scales
    - $n_f + 1$  light flavors at high scales

# *Strong coupling with flavor thresholds*

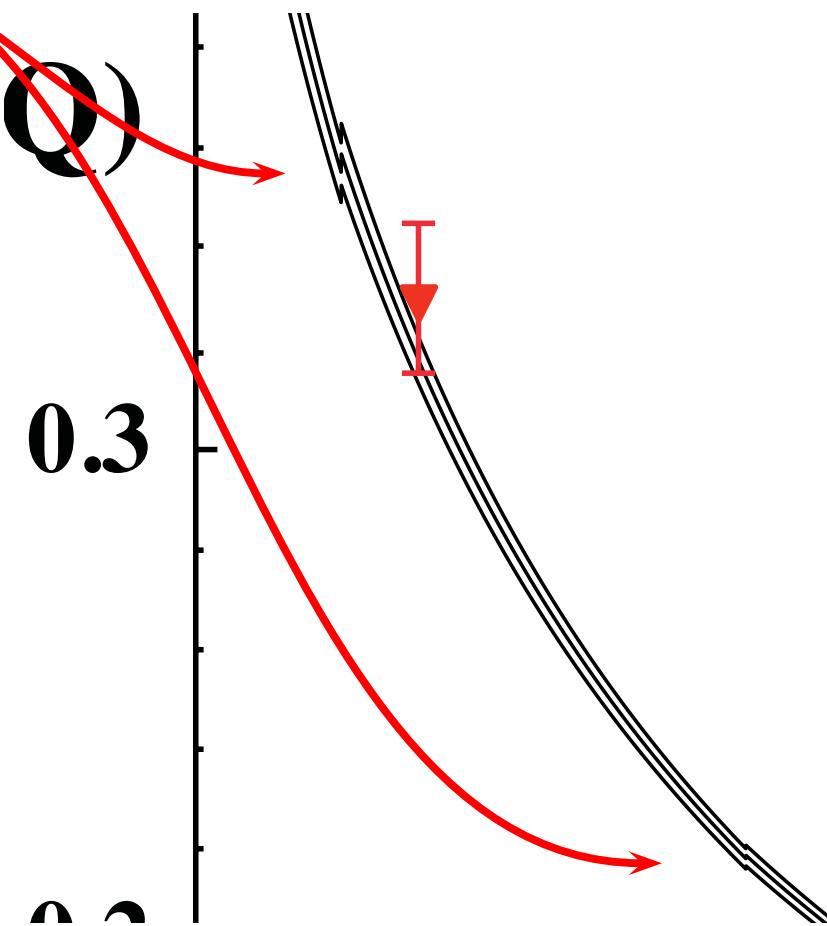
- Solution of QCD  $\beta$ -function for  $\alpha_s^{n_l} \rightarrow \alpha_s^{(n_l+n_h)}$ 
    - discontinuities for  $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
  - Big picture

Bethke for PDG 2014



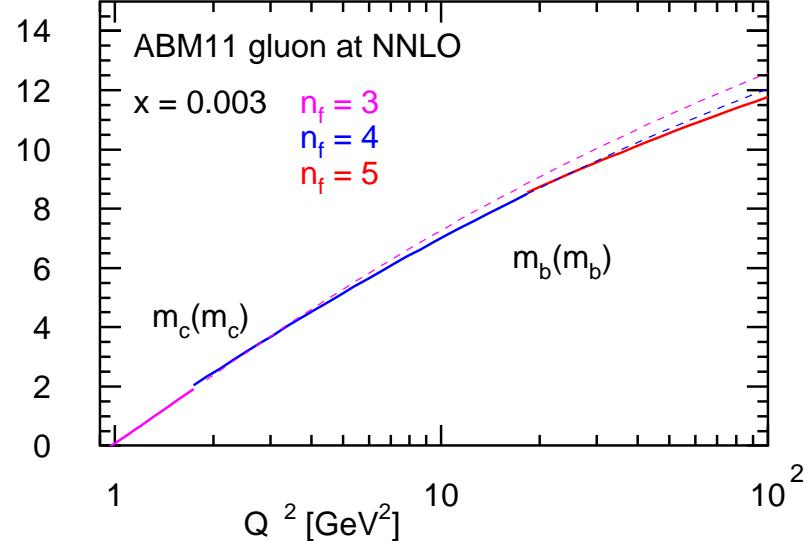
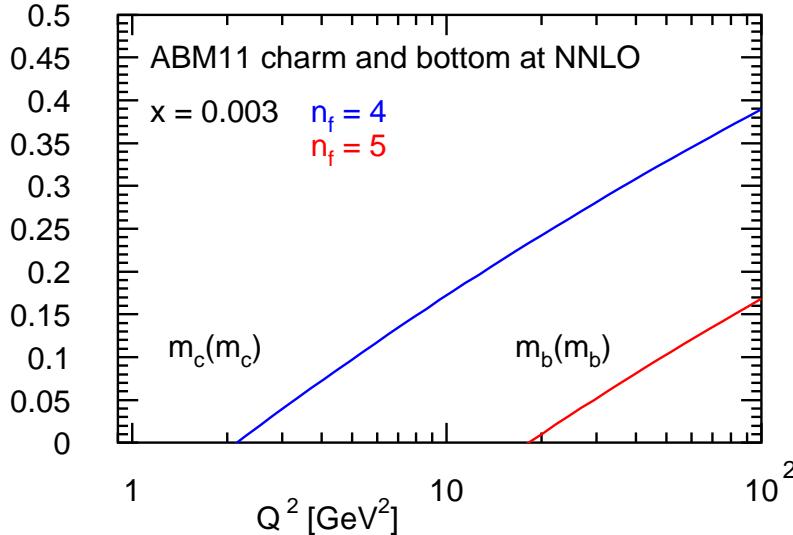
# Strong coupling with flavor thresholds

- Solution of QCD  $\beta$ -function for  $\alpha_s^{n_l} \rightarrow \alpha_s^{(n_l+n_h)}$ 
  - discontinuities for  $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
- Zoom



# PDFs with flavor thresholds

- Generate heavy-quark PDFs  $h^{(n_f+1)}$  from light-flavor PDFs
  - heavy-quark operator matrix elements (OMEs)  $A_{ji}$  at three loops  
Bierenbaum, Blümlein, Klein '09; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14
- $h^{(n_f+1)}(x, \mu) + \bar{h}^{(n_f+1)}(x, \mu) = A_{hq}(x) \otimes \Sigma^{(n_f)}(x, \mu) + A_{hq}(x) \otimes g^{(n_f)}(x, \mu)$
- likewise light-quark PDFs  $l_i^{(n_f)} \rightarrow l_i^{(n_f+1)}$  and gluon and the quark singlet PDFs  $(\Sigma^{(n_f)}, g^{(n_f)}) \rightarrow (\Sigma^{(n_f+1)}, g^{(n_f+1)})$
- Solution of evolution equations between thresholds for  $n_f \rightarrow (n_f + 1)$  with fixed  $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$



# Cross sections with flavor thresholds

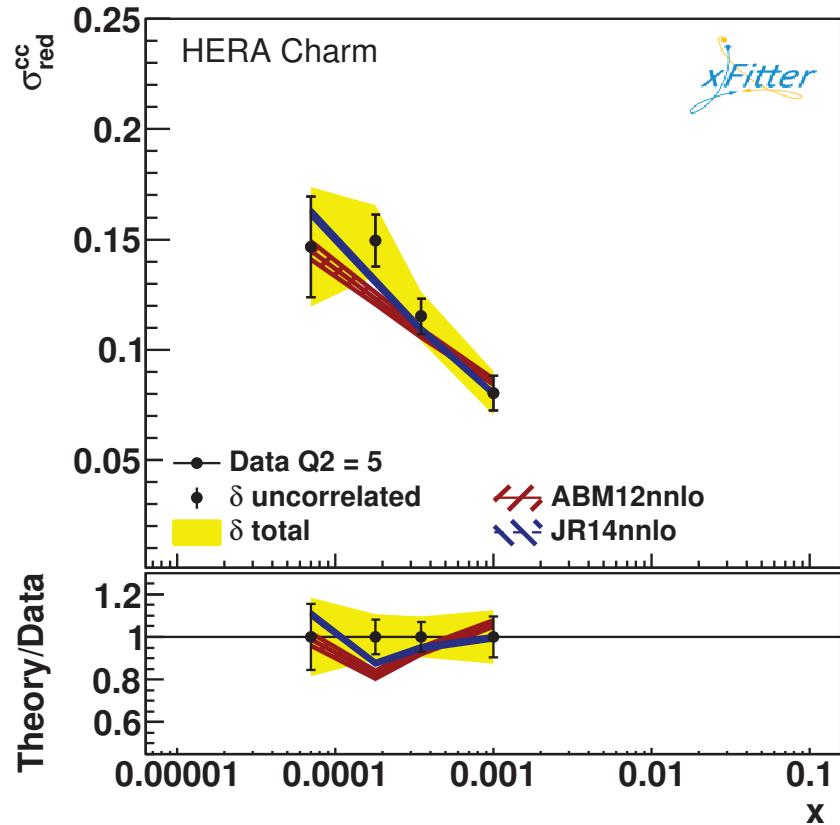
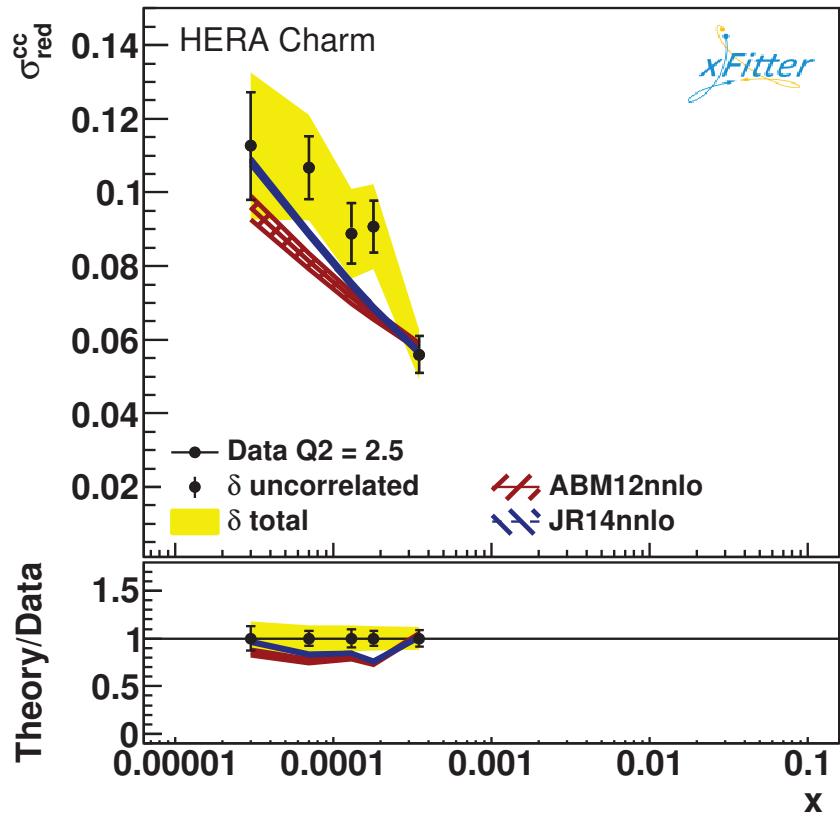
## Fixed flavor number scheme (FFNS) (“do nothing”)

- Cross section with massive quarks at scales  $Q \gg m_c$ 
  - top-quark hadro-production ( $t\bar{t}$  pairs, single top in 4FS or 5FS, ... ]
- $F_2^c$  at HERA with  $u, d, s, g$  partons and massive charm coeff. fcts.
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven ‘92
  - approximations at NNLO Bierenbaum, Blümlein, Klein ‘09; Lo Presti, Kawamura, S.M., Vogt ‘12; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock ‘14

## Variable flavor number scheme (VFNS) (“match something”)

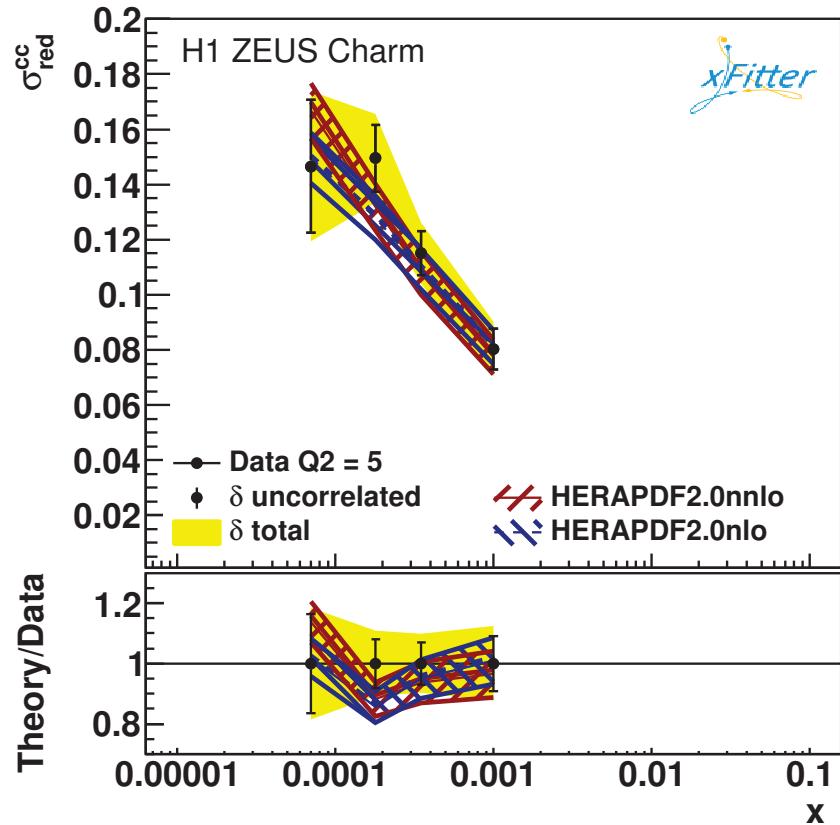
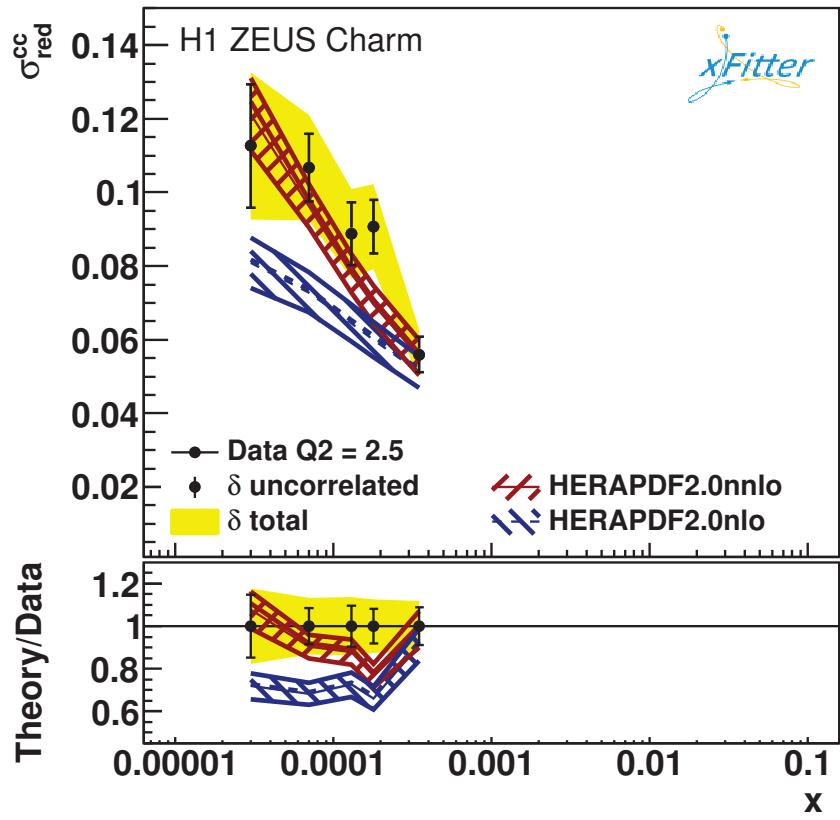
- (Smooth) matching of two distinct theories:  
 $n_f$  light + heavy quark at low scales  $\longrightarrow n_f + 1$  light flavors at high scales
  - Higgs boson production in  $b\bar{b}$ -annihilation (“Santander matching” Harlander, Krämer, Schumacher ‘11)
- $F_2^c$  at HERA with ACOT Aivazis, Collins, Olness, Tung ‘94, BMSN Buza, Matiounine, Smith, van Neerven ‘98, RT Thorne, Roberts ‘98, FONNL Forte, Laenen, Nason, Rojo ‘10
  - model assumptions in matching conditions
  - details of implementation matter in global fits

# Comparision to data



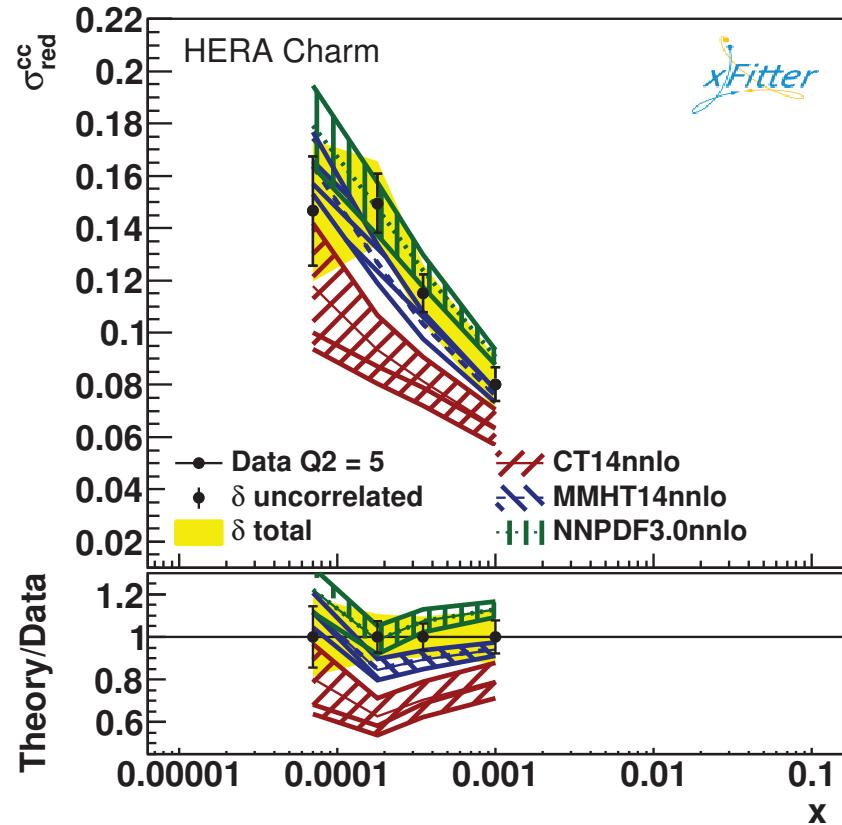
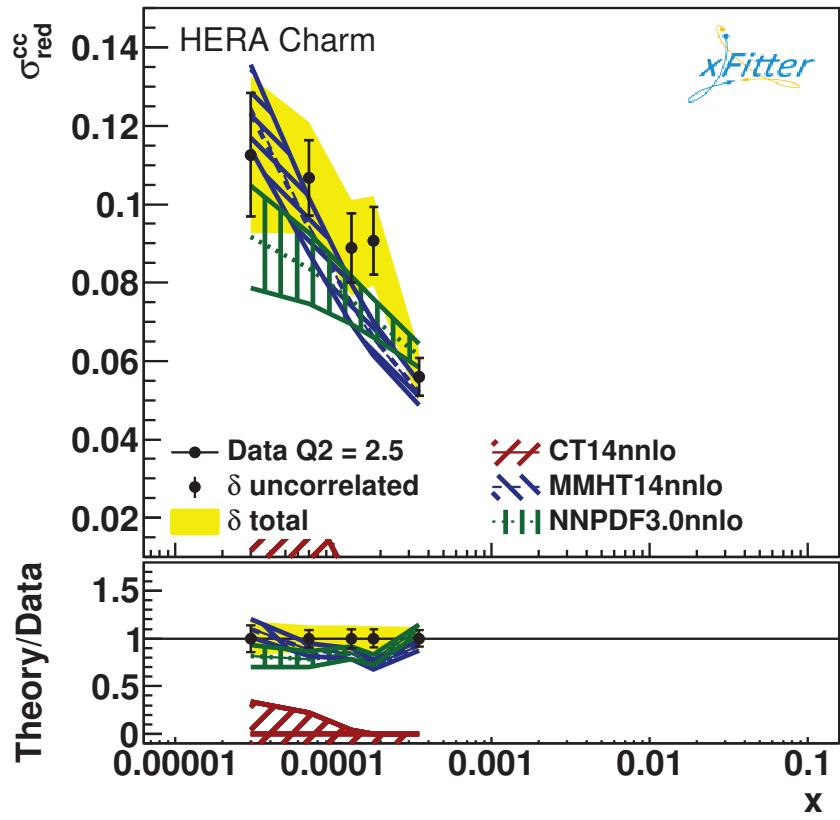
- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - ABM12 and JR14 using FFNS scheme

# Comparision to data



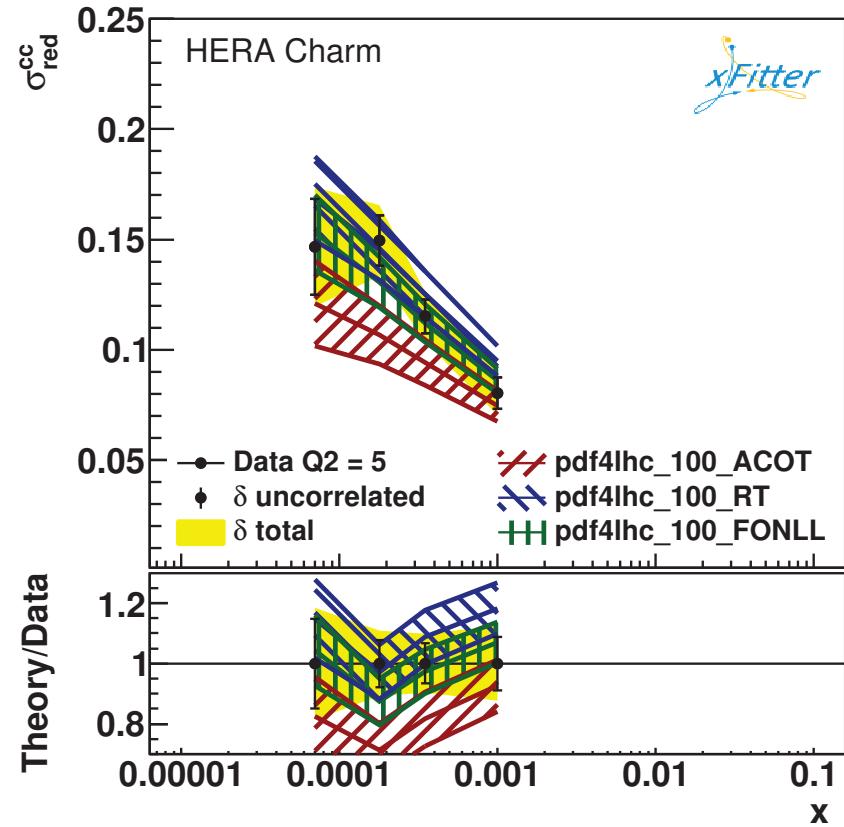
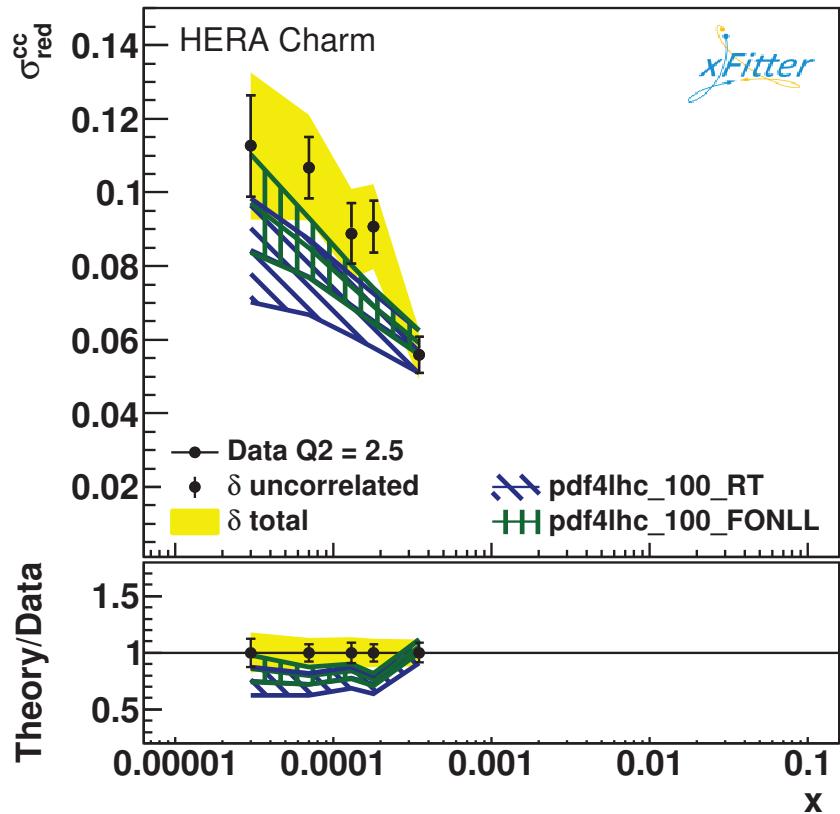
- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - HERAPDF2.0 using the RT optimal GM-VFNS scheme

# Comparision to data



- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - CT14, MMHT14 and NNPDF3.0 using FFNS scheme

# Comparision to data



- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - ABM12 and JR14 using FFNS scheme

# Charm quark mass in PDF fits

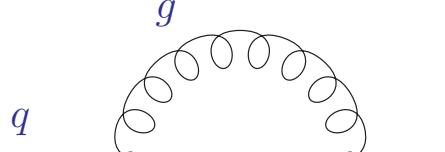
	$m_c$ (GeV)	$m_c$ scheme	$\chi^2/\text{NDP}$ (HERA data)	$F_2^c$ scheme	NNLO Wilson coeff.
ABM12 arXiv:1310.3059	$1.24^{+0.05}_{-0.03}$	$m^{\overline{MS}}$	65/52	FFNS( $n_f = 3$ )	yes
CT14 arXiv:1506.07443	1.3 (assumed)	$m^{\text{pole}}$	582/52 (64/47)	S-ACOT- $\chi$	no
MMHT arXiv:1510.02332	1.25	$m^{\text{pole}}$	75/52	RT optimal	no
NNPDF3.0 arXiv:1410.8849	1.275 (assumed)	$m^{\text{pole}}$	67/52	FONLL-C	no
PDF4LHC15 arXiv:1510.03865	-	-	58/52	FONLL-B	-
	-	-	71/52	RT optimal	-
	-	-	51/47	S-ACOT- $\chi$	-

- PDG quotes running masses:  
charm:  $m_c(m_c) = 1.27^{+0.07}_{-0.11}$  GeV, bottom:  $m_b(m_b) = 4.20^{+0.17}_{-0.07}$  GeV
- Values of charm-quark pole mass for CT14, MMHT14 and NNPDF3.0 not compatible with world average of PDG

# Quark mass renormalization

## Pole mass

- Based on (unphysical) concept of heavy-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2 = m_q^2}$$


- heavy-quark self-energy  $\Sigma(p, m_q)$  receives contributions from regions of all loop momenta – also from momenta of  $\mathcal{O}(\Lambda_{QCD})$
- Renormalon ambiguity in definition of pole mass of  $\mathcal{O}(\Lambda_{QCD})$   
Bigi, Shifman, Uraltsev, Vainshtein '94; Beneke, Braun '94; Smith, Willenbrock '97

## $\overline{MS}$ mass

- Free of infrared renormalon ambiguity
- Conversion between  $m_{\text{pole}}$  and  $\overline{MS}$  mass  $m(\mu_R)$  in perturbation theory known to four loops in QCD Marquard, Smirnov, Smirnov, Steinhauser '15
  - does not converge in case of charm quark

$$m_c(m_c) = 1.27 \text{ GeV} \longrightarrow m_c^{\text{pole}} = 1.47 \text{ GeV} \text{ (one loop)}$$
$$\longrightarrow m_c^{\text{pole}} = 1.67 \text{ GeV} \text{ (two loops)}$$
$$\longrightarrow m_c^{\text{pole}} = 1.93 \text{ GeV} \text{ (three loops)}$$
$$\longrightarrow m_c^{\text{pole}} = 2.39 \text{ GeV} \text{ (four loops)}$$

# *Charm quark mass and the Higgs cross section*

## *MMHT14*

- “Tuning” Charm mass  $m_c$  parameter effects the Higgs cross section
  - linear rise in  $\sigma(H) = 40.5 \dots 42.6 \text{ pb}$  for  $m_c = 1.15 \dots 1.55 \text{ GeV}$  with MMHT14 PDFs Martin, Motylinski, Harland-Lang, Thorne ‘15

$m_c^{\text{pole}}$ [GeV]	$\alpha_s(M_Z)$ (best fit)	$\chi^2/\text{NDP}$ (HERA data on $\sigma^{c\bar{c}}$ )	$\sigma(H)^{\text{NNLO}}$ [pb] best fit $\alpha_s(M_Z)$	$\sigma(H)^{\text{NNLO}}$ [pb] $\alpha_s(M_Z) = 0.118$
1.15	0.1164	78/52	40.48	(42.05)
1.2	0.1166	76/52	40.74	(42.11)
1.25	0.1167	75/52	40.89	(42.17)
1.3	0.1169	76/52	41.16	(42.25)
1.35	0.1171	78/52	41.41	(42.30)
1.4	0.1172	82/52	41.56	(42.36)
1.45	0.1173	88/52	41.75	(42.45)
1.5	0.1173	96/52	41.81	(42.51)
1.55	0.1175	105/52	42.08	(42.58)

# *Charm quark mass and the Higgs cross section*

## NNPDF

- Same trend: lighter charm mass implies smaller Higgs cross section
  - fit range for  $m_c$  too small and no correlation with value of  $\alpha_s(M_Z)$
  - best fits with NNPDF2.1 and NNPDF3.0 give range
$$\sigma(H) = 42.6 \dots 44.2 \text{ pb}$$

PDF sets	$m_c^{\text{pole}}$ [GeV]	$\alpha_s(M_Z)$ (fixed)	$\chi^2/\text{NDP}$ (HERA data on $\sigma^{c\bar{c}}$ )	$\sigma(H)^{\text{NNLO}}$ [pb] fixed $\alpha_s(M_Z)$
NNPDF2.1 [arXiv:1107.2652]	$\sqrt{2}$	0.119	<b>65/52</b>	<b><math>44.18 \pm 0.49</math></b>
	1.5	0.119	78/52	$44.54 \pm 0.51$
	1.6	0.119	92/52	$44.74 \pm 0.50$
	1.7	0.119	110/52	$44.95 \pm 0.51$
NNPDF2.3 [arXiv:1207.1303]	$\sqrt{2}$	0.118	71/52	$43.77 \pm 0.41$
NNPDF3.0 [arXiv:1410.8849]	1.275	0.118	<b>67/52</b>	<b><math>42.59 \pm 0.80</math></b>

# Summary

## Parton distributions at the LHC

- Precision determinations of non-perturbative parameters is essential
  - parton content of proton (PDFs), strong coupling constant  $\alpha_s(M_Z)$ , quark masses  $m_c$ ,  $m_b$ ,  $m_t$
- Experimental precision of  $\lesssim 1\%$  makes theoretical predictions at NNLO in QCD mandatory

## Higgs cross section

- Strong coupling constant  $\alpha_s(M_Z)$ 
  - fixed value of  $\alpha_s(M_Z)$  lacks correlation with parameters of PDF fits
  - $\alpha_s(M_Z) = 0.118$  at NNLO not preferred by data
- Uncertainties due model assumption in PDF fits neglected
  - implementations of variable flavor number schemes use charm-quark mass  $m_c$  to tune cross section
  - low value of pole mass  $m_c^{\text{pole}} \simeq 1.25\text{GeV}$  in contradiction to world average