Strong coupling constant from Adler function in lattice QCD

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1. Introduction

Strong coupling constant, α_s

- High accuracy of α_s is required from the precise SM test and beyond the SM (BSM) physics.

125 GeV Higgs partial widths

LHC HCSWG, 1307.1347

channel	$\Delta\alpha_s$	Δm_b	2loop EW
$\Delta\Gamma(H\to bb)$	±2.3%	±3.2%	±2%
$\Delta\Gamma(H\to cc)$	+7.0% -7.1%	+6.2% -6.0%	±2%
$\Delta\Gamma(H\to gg)$	+4.2% -4.1%	±0.1%	±3%

Higgs production cross-section of gluon fusion at 12TeV

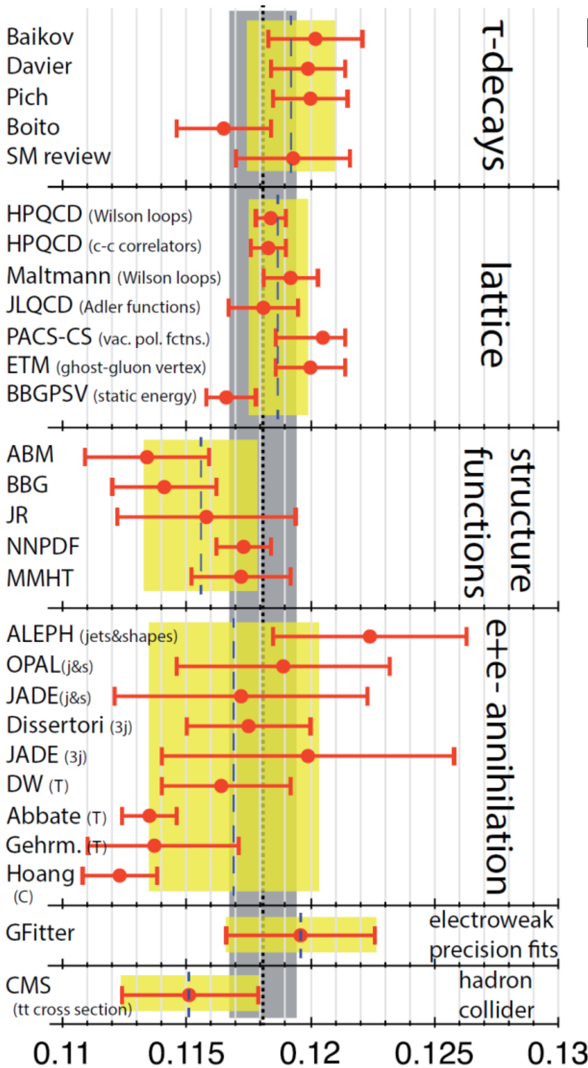
Anastasiou, 1602.00695

σ(theory) = 48.58 pb (+2.22 -3.27 pb)			Δ(PDF+α_s) = ±1.56 pb	
Δ(scale)	Δ(trunc)	Δ(EW)	Δ(PDF)	Δ(α_s)
+0.1 -1.15 pb	±0.18 pb	±0.49 pb	±0.9 pb	+1.27 -1.25 pb

If accuracy of α_s is required to be below EW 2-loop order, it corresponds to 0.5 % accuracy for α_s (and also for m_b).

1. Introduction

Determination of $\alpha_s(M_Z)$



$\alpha_s(M_Z)$

PDG2015

World average (χ^2 average) in 2015

0.1181 ± 0.0013

0.1192 ± 0.0018 (τ decay)

Lattice is still leading the high precision.

0.1184 ± 0.0005 (Lattice, PDG χ^2)

± 0.0012 (Lattice, FLAG13)

0.1156 ± 0.0023 (DIS, unweighted)

0.1169 ± 0.0034 (e^+e^-, unweighted)

0.1196 ± 0.0030 (electroweak, NNLO)

0.1151 ± 0.0028 (tt 7 TeV, CMS, NNLO)

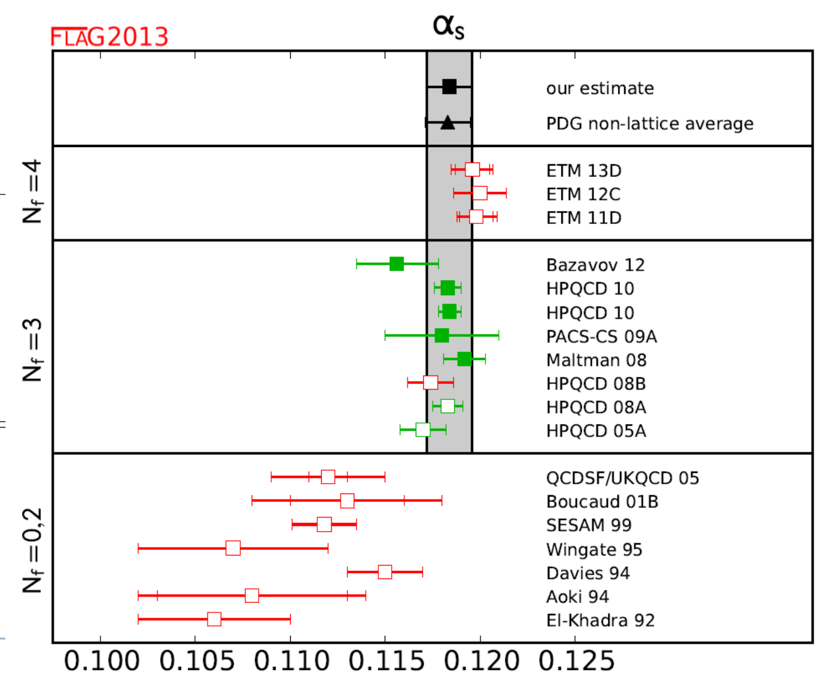
1. Introduction

FLAG report 2013

Collaboration	Ref.	N_f	publication status	renormalisation scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\rm MS}}(M_Z)$	Method	Ta
ETM 13D	[91]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex	
ETM 12C	[92]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex	υ
ETM 11D	[93]	2+1+1	A	○	○	■	0.1198(9)(5)($^{+0}_{-5}$)	gluon-ghost vertex	6
Bazavov 12	[48]	2+1	A	○	○	○	0.1156($^{+21}_{-22}$)	Q-$\bar{Q}$ potential	2
HPQCD 10	[62]	2+1	A	○	○	○	0.1183(7)	current two points	
HPQCD 10	[62]	2+1	A	○	★	★	0.1184(6)	Wilson loops	
PACS-CS 09A	[31]	2+1	A	★	★	○	0.118(3)#	Schrödinger functional	
Maltman 08	[63]	2+1	A	○	○	○	0.1192(11)	Wilson loops	
HPQCD 08B	[75]	2+1	A	■	■	■	0.1174(12)	current two points	
HPQCD 08A	[59]	2+1	A	○	★	★	0.1183(8)	Wilson loops	
HPQCD 05A	[58]	2+1	A	○	○	○	0.1170(12)	Wilson loops	
QCDSF/UKQCD 05	[64]	$0, 2 \to 3$	A	★	■	★	0.112(1)(2)	Wilson loops	
Boucaud 01B	[86]	$2 \to 3$	A	○	○	■	0.113(3)(4)	gluon-ghost vertex	
SESAM 99	[65]	$0, 2 \to 3$	A	★	■	■	0.1118(17)	Wilson loops	
Wingate 95	[66]	$0, 2 \to 3$	A	★	■	■	0.107(5)	Wilson loops	
Davies 94	[67]	$0, 2 \to 3$	A	★	■	■	0.115(2)	Wilson loops	
Aoki 94	[68]	$2 \to 3$	A	★	■	■	0.108(5)(4)	Wilson loops	
El-Khadra 92	[69]	$0 \to 3$	A	★	○	○	0.106(4)	Wilson loops	

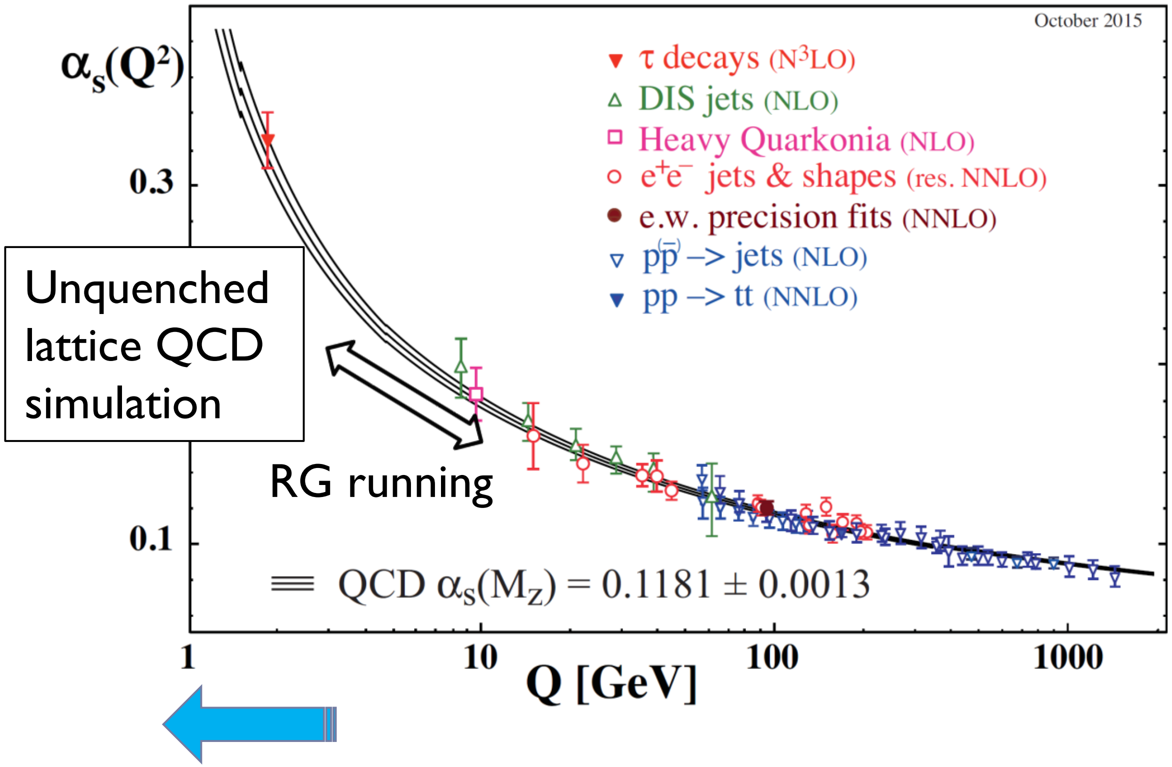
Result with a linear continuum extrapolation in a.

- Flavor Lattice Averaging Group reported such a nice summary of lattice $\alpha_s(M_Z)$ results and combined uncertainty based on their own opinion.



http://itpwiki.unibe.ch/flag

1. Introduction
Lattice study



Reliable region for lattice simulation

1. Lattice calculation of

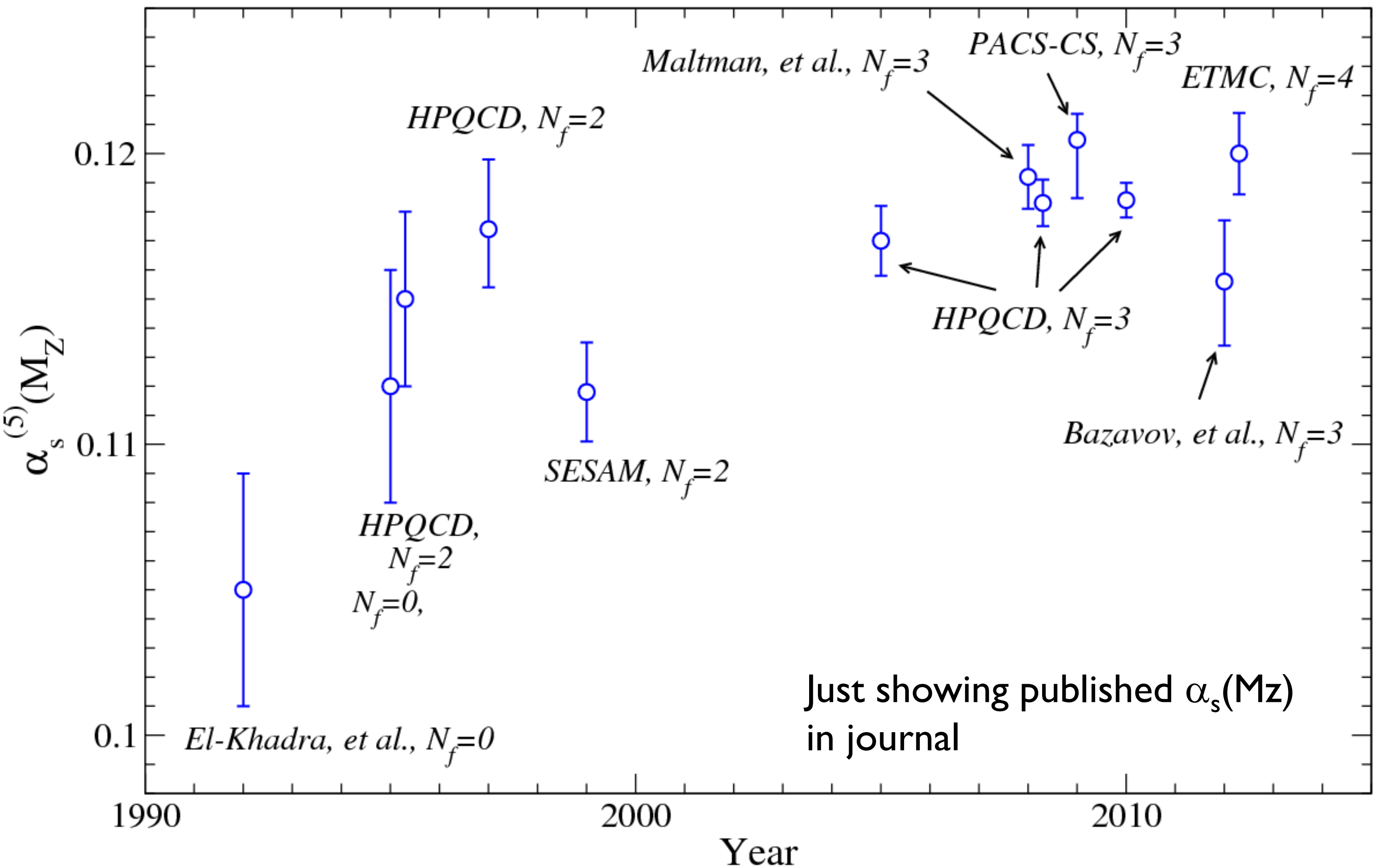
   Wilson loop,
   Heavy current correlator,
   Adler function,
   Schrödinger functional,
   Gluon-gluon (-ghost) vertex,
   Static energy, etc

2. Matching those data with perturbative expansion of α_s(MSbar), α_s(V), α_s(SF), etc, below O(1) GeV.

3. Convert to $\alpha_s(M_Z)$ with renormalization group equation.

1. Introduction

History of $\alpha_s(M_z)$ from lattice QCD

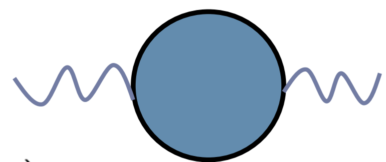


Just showing published α_s(Mz) in journal

2. Lattice calculation of Adler function

Vacuum polarization function (VPF)

- Adler function, given from a derivative of VPF by Q^2
  - N^3LO has been known. Baikov, Chetyrkin, Kuhn, Phys Rev Lett 101, 012002 (2008)
  - OPE describes non-perturbative effect as the expansion of multiple dimension operator condensate.
- Current-current correlator



$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQx} \langle 0 | J^a_\mu(x) J^{b\,\dagger}_\nu(0) | 0 \rangle = \delta^{ab} \Big(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu \Big) \Pi_J$$

Q : Euclidean momentum

Using the analytical expression of Adler function, the perturbative VPF is described as a function of $t = \ln(Q^2/\mu^2)$

$$\Pi_V(Q^2) = \mathrm{const} - \frac{1}{4\pi^2} \left[t + \sum_{k=1}^{5} \left(\frac{\alpha_s(\mu)}{\pi} \right)^k \sum_{m=0}^{k-1} c_{km} \frac{t^{m-1}}{m+1} \right]$$

N.B. c_{50} has not been known from analytical calculation.

2. Lattice calculation of Adler function

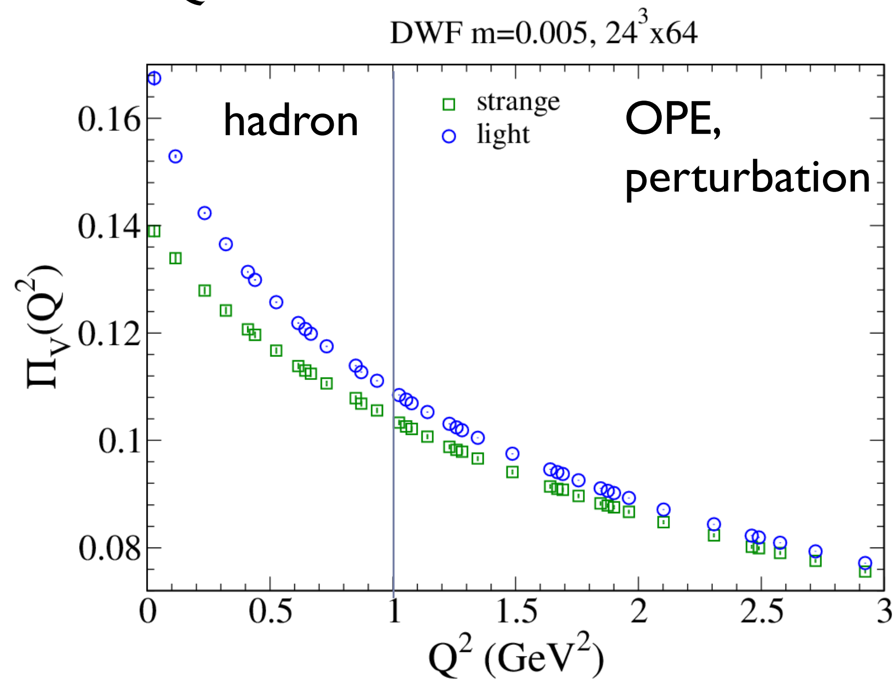
Lattice calculation of VPF

- $\Pi_{\mu\nu}(Q)$ is computed easily, but rich information is contained:
- Long distance (Q < 1 GeV)

  Hadronic contribution to g-2, S-parameter, etc.

  - Taking into account the complicated hadronic state ($\pi\pi$ and ρ).
  - Statistically noisy and sparse Q^2 variation near Q ~ 0 GeV.

  Blum (2003--), JLQCD(2008)
  and see also Wittig's talk.



- Short distance (Q >> 1 GeV)

  OPE, moment, α_s etc.

  - Clear statistical signal, and dense data.
  - Systematic uncertainty due to finite lattice spacing.

  HPQCD (2008), JLQCD(2009,2010)
  and see also Hashimoto's talk.

2. Lattice calculation of Adler function

VPF in short distance

- Q ~ 1 -- 2 GeV : non-perturbative contribution in OPE is important.
  - Quark, gluon condensate in OPE.
  - Relatively small lattice artifact.
- Q > 2 GeV : higher dimensional operator is suppressed as $1/Q^n$.
  - Perturbative expression without quark mass.
  - Large lattice artifact.

According to sum rule analysis, OPE to fit in Q ~ 1 -- 2 GeV is problematic because of comparable size of OPE terms with alternating signs.

Boito, Golterman, Maltman, Osborne, Peris, PRD91,034003(2015)

$$\Pi^{(1+0)}(Q^2) = \sum_k \frac{C_{2k}}{Q^{2k}},$$

$$\begin{aligned} C_{4,V+A} &= +0.00268\,\mathrm{GeV}^4 \\ C_{6,V+A} &= -0.0125\,\mathrm{GeV}^6 \\ C_{8,V+A} &= +0.0349\,\mathrm{GeV}^8 \\ C_{10,V+A} &= -0.0832\,\mathrm{GeV}^{10} \\ C_{12,V+A} &= +0.161\,\mathrm{GeV}^{12} \end{aligned}$$

In this study, we concentrate on Q > 2 GeV.

2. Lattice calculation of Adler function

Managing lattice artifact in Q >> 1 GeV

R. Lewis, lattice 2015

- **Constraint on Q with cylinder cut**

$$(Q_\perp)_\mu \equiv Q_\mu - (\hat{n}\cdot Q)\hat{n}_\mu, \quad n_\mu = (1,1,1,1)/2$$

Here we choose $\Pi_{\mu\nu}(Q)$ in maximum radius $|Q_\perp| < |Q_\perp|_{\max}$

This is to exclude undesirable $\Pi_{\mu\nu}(Q)$ which is, for instance, Q along single axis.

- **Averaging over $\Pi_{\mu\nu}(Q)$ with reflection operator**

$$\Pi_{\text{lat}}(Q) = \frac{1}{12}\sum_{\mu}\sum_{\nu\neq\mu}\frac{\Pi_{\mu\nu}(Q) - \Pi_{\mu\nu}(R_\mu Q)}{2Q_\mu Q_\nu}$$

R_μ : reflection operator in μ direction, O(4) breaking term is possible to remove.

- **Additional term**

$$\Pi_{\text{lat}}(Q) = \Pi(Q) + c_1(aQ)^2 + \cdots$$

$\Pi(Q)$ is comparable with perturbation, and c term is purely lattice artifact

- **Continuum extrapolation**

Adler function

- Differential of $\Pi(Q)$ between Q_1 and Q_2

$$\Delta(Q_1^2, Q_2^2) = -4\pi^2\left(\frac{\Pi(Q_1^2) - \Pi(Q_2^2)}{t_1 - t_2}\right) - 1$$

$$t_1 = \ln(Q_1^2/\mu^2),\ t_2 = \ln(Q_2^2/\mu^2)$$

Need to know α_s^6 term.

Perturbative expression up to α_s^6

$$\begin{aligned}
\Delta(Q_1^2, Q_2^2) &= \left(\frac{\alpha_s(\mu)}{\pi}\right)\Big[1 + 1.639821\left(\frac{\alpha_s(\mu)}{\pi}\right) + 6.371067\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + 49.07688\left(\frac{\alpha_s(\mu)}{\pi}\right)^3 + c_{50}\left(\frac{\alpha_s(\mu)}{\pi}\right)^4 \\
&+ (t_1 + t_2)\left\{-\frac{9}{8}\left(\frac{\alpha_s(\mu)}{\pi}\right) - 5.689597\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 - 33.09141\left(\frac{\alpha_s(\mu)}{\pi}\right)^3 - 346.7778\left(\frac{\alpha_s(\mu)}{\pi}\right)^4\right\} \\
&+ (t_1^2 + t_1 t_2 + t_2^2)\left\{\frac{27}{16}\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + 15.8015\left(\frac{\alpha_s(\mu)}{\pi}\right)^3 + 173.776\left(\frac{\alpha_s(\mu)}{\pi}\right)^4\right\} \\
&+ (t_1^3 + t_1^2 t_2 + t_1 t_2^2 + t_2^3)\left\{-\frac{729}{256}\left(\frac{\alpha_s(\mu)}{\pi}\right)^3 - 42.9221\left(\frac{\alpha_s(\mu)}{\pi}\right)^4\right\} \\
&+ (t_1^4 + t_1^3 t_2 + t_1^2 t_2^2 + t_1 t_2^3 + t_2^4)\frac{6561}{1280}\left(\frac{\alpha_s(\mu)}{\pi}\right)^4\Big]
\end{aligned}$$

- This is a renormalization-independent function, so we use it for fitting function.
- Leading term is α_s and then higher term depends on t.
- No mass dependence.

3. Preliminary result

Lattice parameter

- Domain-wall fermion in RBC/UKQCD collaboration

Lattice size	a^{-1} (GeV)	m_s	m_u	Configs.
$24^3 \times 64$	1.78	0.04	0.005,0.01,0.02	901
$32^3 \times 64$	2.38	0.03	0.004,0.006,0.008	940
$32^3 \times 64$	3.15	0.0186	0.0047	560

RBC/UKQCD, 1411.7017

- DWF has small chiral symmetry violation on the lattice → O(a) suppression
- $\Pi_{\mu\nu}$ is given by the combination of local and conserved current.

$$\sum_\mu \hat{Q}_\mu \Pi_{\mu\nu}(Q) = 0, \ \sum_\nu \hat{Q}_\nu \Pi_{\mu\nu}(Q) \neq 0$$

$$\hat{Q}_\mu = 2e^{iQ_\mu/2} \sin(Q_\mu/2)$$

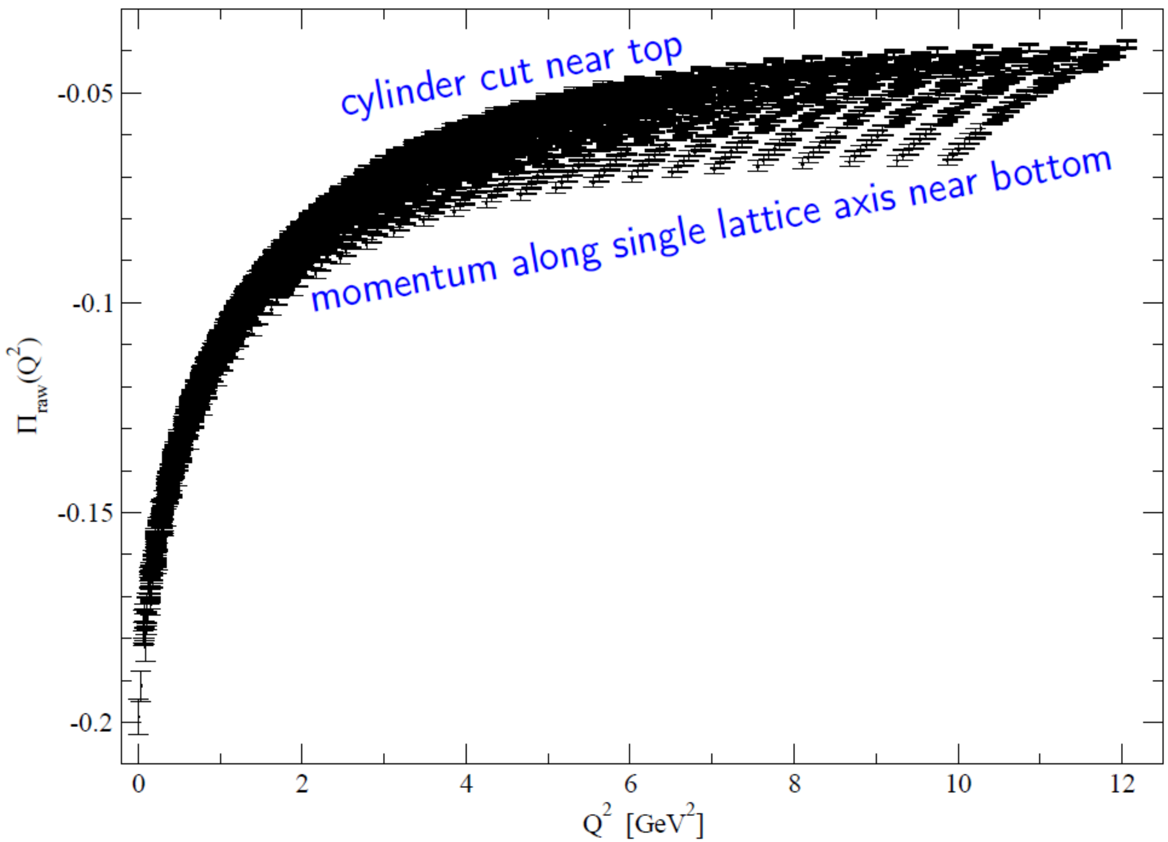
3. Preliminary result

Reduce lattice artifacts

$$\Pi_{\rm raw}(Q^2) = \frac{-1}{3\hat{Q}^2}\left(\delta_{\mu\nu} - \frac{4\hat{Q}_\mu\hat{Q}_\nu}{\hat{Q}^2}\right)\Pi_{\mu\nu}(Q^2)$$

Naïve subtraction shows a “fishbone” pattern of VPF in Q >> 1 GeV².

Near top of this pattern is only relevant.



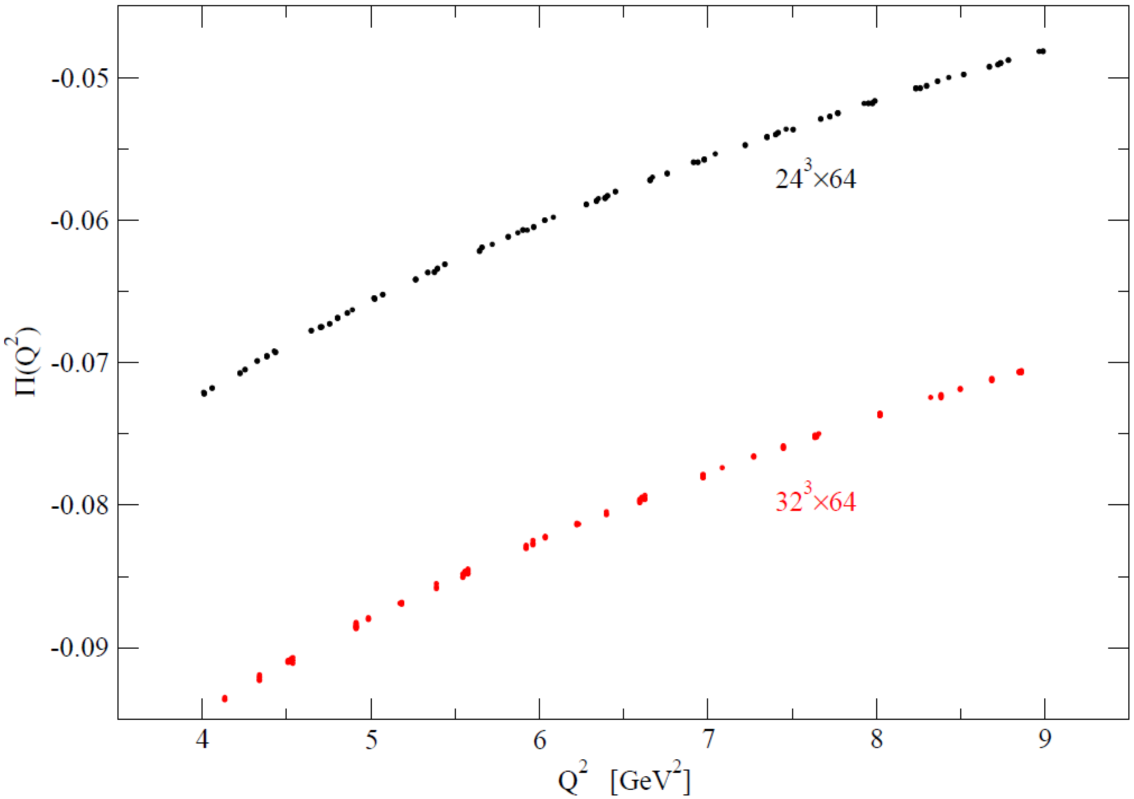
3. Preliminary result

Reduce lattice artifacts

$$\Pi_{\rm lat}(Q) = \frac{1}{12}\sum_{\mu}\sum_{\nu\neq\mu}\frac{\Pi_{\mu\nu}(Q)-\Pi_{\mu\nu}(R_\mu Q)}{2Q_\mu Q_\nu}\Big|_{|Q_\perp|<|Q_\perp^{\rm max}|}$$

Combination of cylinder cut and reflection operator

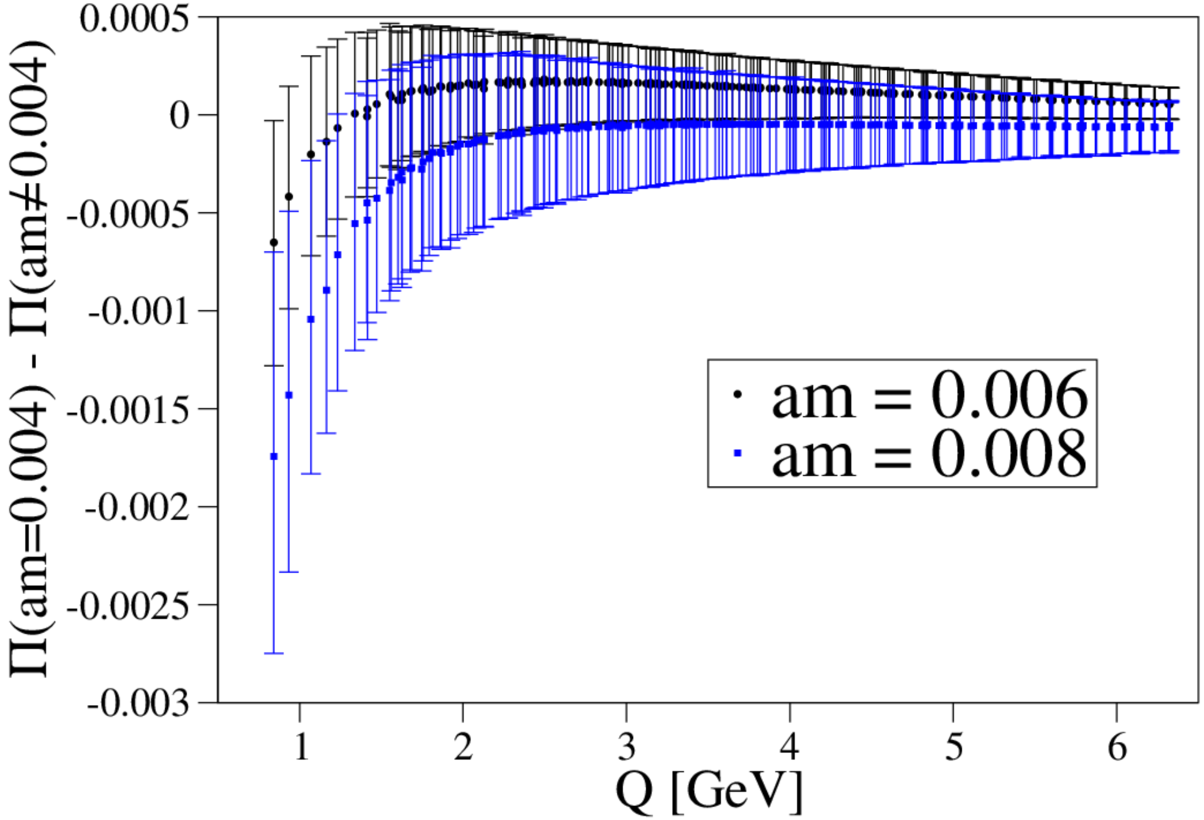
Smooth behavior under restriction on cylindrical region along (1,1,1,1) and subtraction of O(4) breaking term.



3. Preliminary result

Large Q

Q>1.5 GeV, the mass dependence of VPF is negligible.



3. Preliminary result

Fitting with perturbation

- Q_1^2 and Q_2^2 are chosen from region where m dependence of $\Pi(Q)$ is negligible.

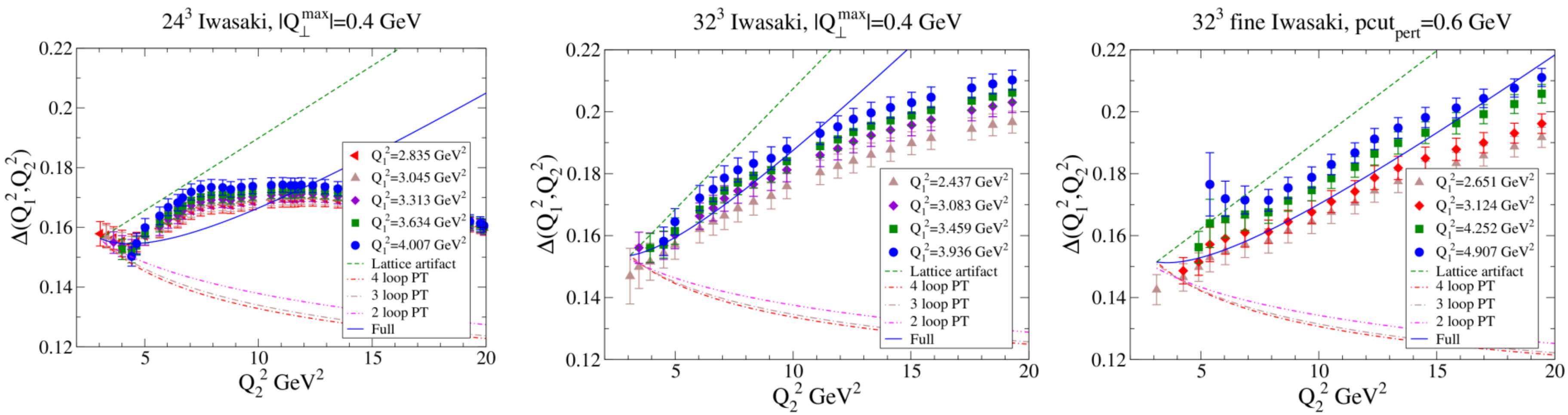
- Fixed Q_1^2 near 4 GeV^2 and fit data in $Q_2^2 > Q_1^2$ with a function combined with $O((aQ)^2)$ term:

$$\Delta(Q_1^2, Q_2^2)|_{Q_1^2<Q_2^2} = \Delta_{\rm pert}(Q_1^2, Q_2^2) + c_1(aQ_2)^2$$

- Determination of fitting window, which is safe from undesirable contribution from higher dimensional operator and lattice artifact, as stable region by changing Q_1^2.

3. Preliminary result

$\Delta(Q_1^2, Q_2^2)$ in $Q_1^2 > 3$ GeV^2



Coarse lattice

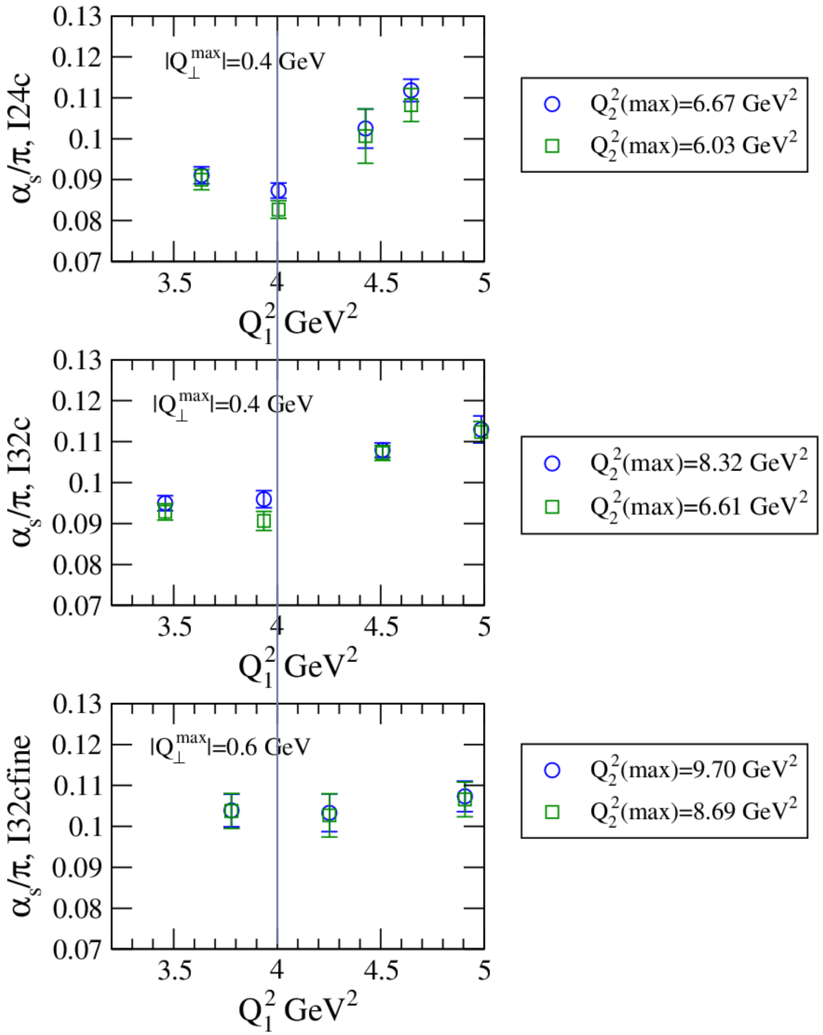
Fine lattice

- In $Q_2^2 < 10$ GeV^2, linear term of $O((aQ)^2)$ is dominant in lattice data rather than perturbation.
- In $Q_2^2 > 10$ GeV^2, the higher order term than $O((aQ)^2)$ is significant in particular for coarse lattice.

3. Preliminary result

Fitting result

Coarse

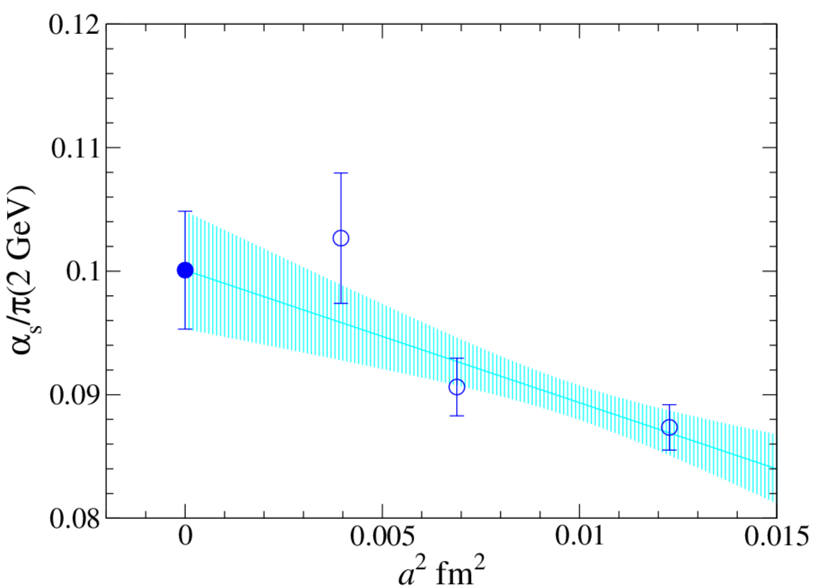


Fine

- In $Q_1^2 > 4$ GeV^2 fitting results is not stable due to non-negligible contribution of $O((aQ)^4)$ lattice artifact.
- Fine lattice is still stable.

Final result

- Continuum extrapolation



Linear ansatz: $a_s + c\, a^2$

α_s(N_f=3, 2 GeV)/π = 0.10008(48)

- Running to $\alpha_s(M_Z)$ in 4-loop

T. van Ritbergen et al., PLB400, 379(1997)

α_s(N_f=3, 2 GeV)

↓

α_s(N_f=4, M_c)

$$\alpha_s^{(4)}(M_c) = \alpha_s^{(3)}(M_c)\Big(1 + \sum_n c_{n0}(\alpha_s^{(3)}(M_c))^n\Big)$$

↓

α_s(N_f=5, M_b)

$$\alpha_s^{(5)}(M_b) = \alpha_s^{(4)}(M_b)\Big(1 + \sum_n c_{n0}(\alpha_s^{(4)}(M_b))^n\Big)$$

↓

α_s(N_f=5, M_Z)

$$\alpha_s(\mu) = \frac{1}{b_0 t}\left(1 - \frac{b_1}{b_0^2}\frac{\ln t}{t} + \cdots\right)$$

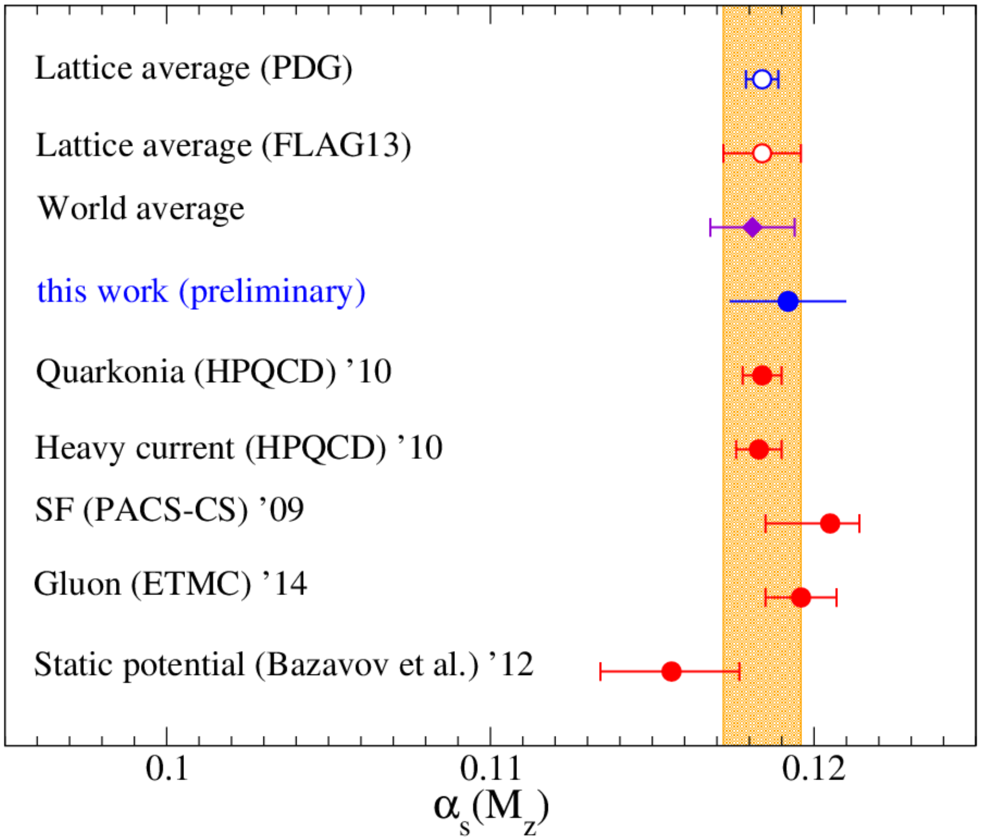
$$t = \ln(\mu^2/\Lambda^2)$$

$$b_0 = \frac{33 - 2n_f}{12\pi}, \quad b_1 = \frac{153 - 19n_f}{24\pi^2}$$

$\alpha_s(M_Z)$ = 0.1192(18)(??)

Statistical error only

Our result



- α_s computation from Adler function with perturbation and lattice.
- It avoids reliance on the OPE and the dangers of an alternating-sign series.
- In high Q^2 lattice artifact is significant.
  $\Rightarrow$ Several techniques to reduce artifact, e.g. cylinder cut.
- Continuum extrapolation
- Uncertainties:
  fitting range, $O(a^2)$ term,…

Thank you for your attention.

Heavy quark correlator

HPQCD (2009, 2010)

- Target : the moments of Heavy-heavy current correlator

$$G(t) = a^6 \sum_{\vec{x}} (am_h)^2 \langle j_5^h(\vec{x},t) j_5^h(0,0)\rangle, \quad G_n = \sum_t (t/a)^n G(t)$$

and ratio to the tree level

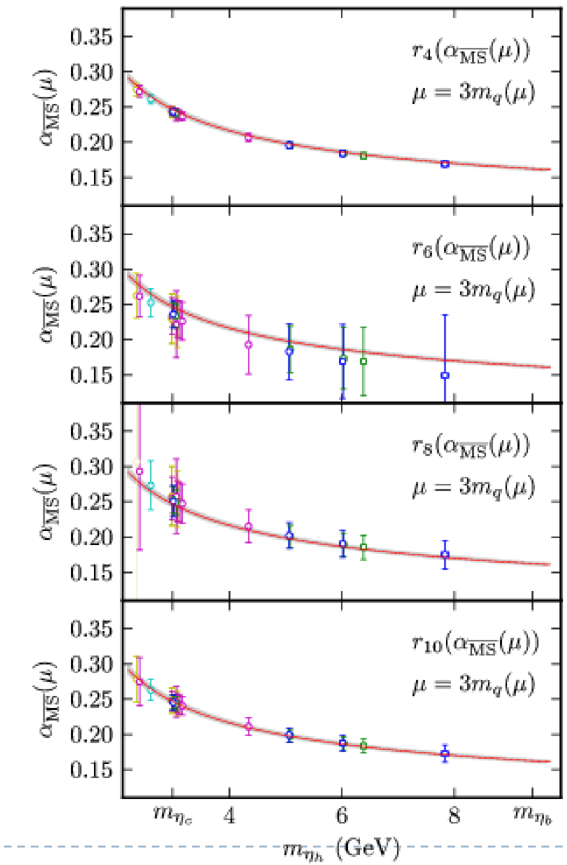
$$R_n = \begin{cases} G_4(t)/G_4(0) & (n=4), \\ \frac{am_{\eta_h}}{2am_h}(G_n/G_n^{(0)})^{1/(n-4)} & (n \geq 6) \end{cases}$$

Fitting $R_{4\text{--}18}$ with PT form:

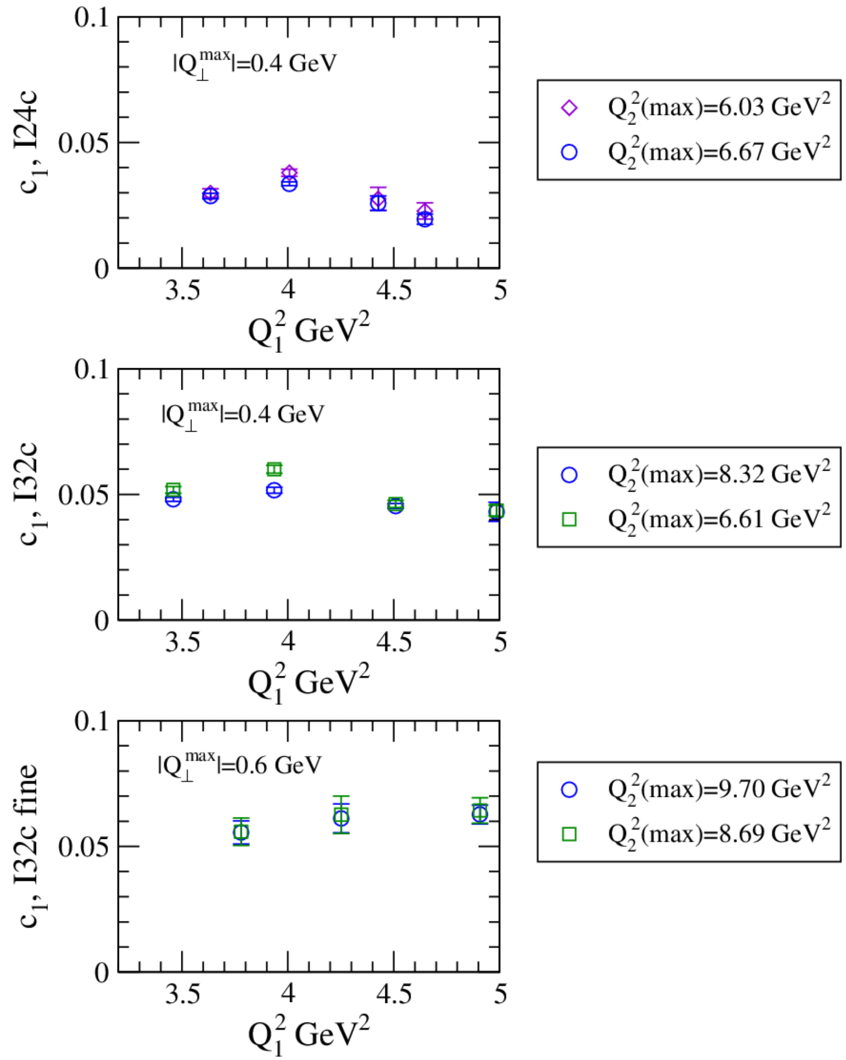
$$r_n(\alpha_s^{\overline{MS}}, \mu) = 1 + \sum_{j=1}^{N_{\rm ph}=6} r_{n\,j}(\mu/m_h)(\alpha_s^{\overline{MS}})^j(\mu)$$

to obtain $\alpha_s^{(5)}(M_Z) = 0.1183(7)$

Also they obtained
m_c(3 GeV, Nf=4) = 0.986(6) GeV,
m_b (10 GeV, Nf=5) = 3.62(3) GeV



c_1 result



0.1
0.05
0
3.5
4
4.5
5
$|Q_\perp^{max}|$=0.4 GeV
c_1, I24c
Q_1^2 GeV^2
Q_2^2(max)=6.03 GeV^2
Q_2^2(max)=6.67 GeV^2
$|Q_\perp^{max}|$=0.4 GeV
c_1, I32c
Q_2^2(max)=8.32 GeV^2
Q_2^2(max)=6.61 GeV^2
$|Q_\perp^{max}|$=0.6 GeV
c_1, I32c fine
Q_2^2(max)=9.70 GeV^2
Q_2^2(max)=8.69 GeV^2

$\alpha_s(M_Z)$ Improvement