Short-distance current correlators on the lattice: light and charm

based on the work by the JLQCD collaboration;
Masaaki Tomii (Sokendai) and Katsumasa Nakayama (Nagoya, KEK), in particular.

Shoji Hashimoto (KEK, Sokendai)

@ "Determination of the Fundamental Parameters in QCD,"
MITP, Mainz, Mar 8, 2016

計算基礎科学連携拠点
Joint Institute for Computational Fundamental Science
JICFuS

国立大学法人
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The Graduate University for Advanced Studies [SOKENDAI]

JLQCD collaboration

- Members
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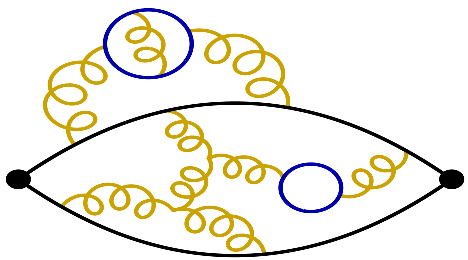
  - Osaka: H. Fukaya, T. Onogi, T. Suzuki
  - Kyoto: S. Aoki
  - Tsukuba: Y. Taniguchi
  - RIKEN: N. Yamanaka
  - Columbia: X. Feng
  - Wuhan: A. Tomiya

- Machines @ KEK
  - Hitachi SR16000 M1
  - IBM Blue Gene /Q

Current correlator

$$\Pi(x) \equiv \langle 0 | \mathrm{T}\, J(x) J(0) | 0 \rangle$$



Hadron spectrum from long distances
- non-perturbative, main use of LQCD

Short distances
- perturbation theory + power corrections (= OPE)
- α_s, quark mass, …

Current correlator

Consider in the coordinate space: no extra divergences

$$\Pi(x) = \frac{c_0(\alpha_s)}{x^6} + \frac{c_2 m_q^2}{x^4} + \frac{c_{4,\bar{q}q} m_q \langle \bar{q}q \rangle + c_{4,G} \langle GG \rangle + \cdots}{x^2} + \cdots$$

Perturbative expansion at m=0; sometimes, known to α_s^4

Finite m correction in PT; typically tiny

OPE: Non-perturbative corrections represented by condensates

Consistent?
LHS: Directly calculable in LQCD.
RHS: Mostly perturbative.

Lattice QCD

- Ab initio calculation of QCD
  → Use as an "experimental" facility of QCD

- Limitation due to finite lattice spacings
  = Window problem

$$a \ll x \ll \Lambda_{\mathrm{QCD}}^{-1}$$

To avoid large discretization effects (→ $a \ll x$)

To avoid too large non-perturbative effects (→ $x \ll \Lambda_{\mathrm{QCD}}^{-1}$)

Need sufficiently small a. How small? See the data.

Lattices with domain-wall

$\beta = 4.17$, $1/a \sim 2.4$ GeV, $32^3 \times 64$ (x12)

m_{ud}	m_π [MeV]	MD time
$m_s = 0.030$		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
$m_s = 0.040$		
0.0035	230	10,000
0.0035 ($48^3 \times 96$)	230	10,000
0.007	320	10,000
0.012	410	10,000
0.019	510	10,000

$\beta = 4.35$, $1/a \sim 3.6$ GeV, $48^3 \times 96$ (x8)

m_{ud}	m_π [MeV]	MD time
$m_s = 0.018$		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
$m_s = 0.025$		
0.0042	300	10,000
0.080	410	10,000
0.0120	510	10,000

$\beta = 4.47$, $1/a \sim 4.6$ GeV, $64^3 \times 128$ (x8)

0.0030	~ 300	10,000

light-light correlator

Definitions

$$\Pi_{\mathbf{S}}(x) = \langle S(x) S(0)^\dagger \rangle, \qquad \Pi_{\mathbf{P}}(x) = \langle P(x) P(0)^\dagger \rangle,$$

$$\Pi_{\mathbf{V},\mu\nu}(x) = \langle V_\mu(x) V_\nu(0)^\dagger \rangle, \qquad \Pi_{\mathbf{A},\mu\nu}(x) = \langle A_\mu(x) A_\nu(0)^\dagger \rangle,$$

$$\Pi_{\mathbf{V/A}}(x) = \sum_\mu \Pi_{\mathbf{V/A},\mu\mu}(x)$$

- Non-singlet local operators are considered

$$S(x) = \bar{\mathrm{u}}\mathrm{d}(x), \qquad P(x) = \bar{\mathrm{u}}\mathrm{i}\gamma_5\mathrm{d}(x),$$

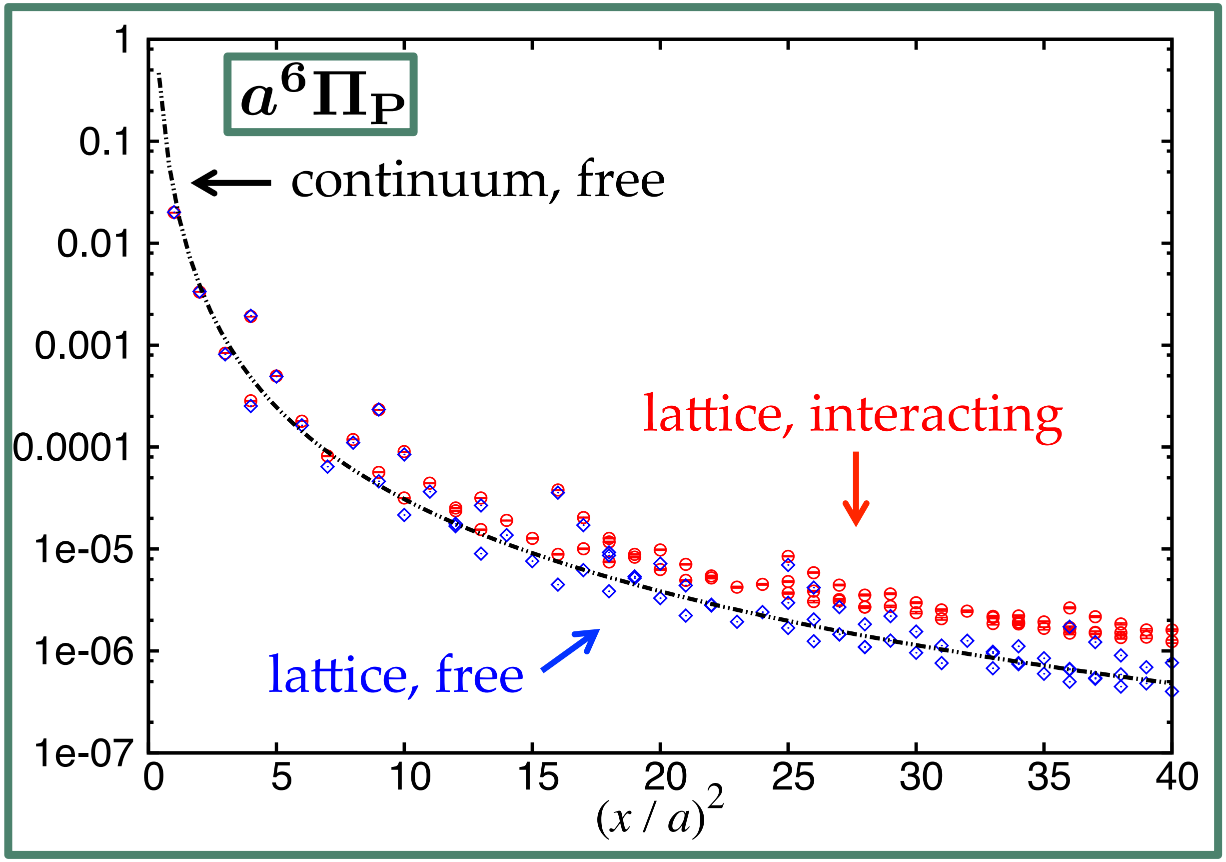
$$V_\mu(x) = \bar{\mathrm{u}}\gamma_\mu\mathrm{d}(x), \qquad A_\mu(x) = \bar{\mathrm{u}}\gamma_\mu\gamma_5\mathrm{d}(x)$$

- Chiral symmetry in perturbation theory (at m=0)

$$\Pi_{\mathbf{S}} = \Pi_{\mathbf{P}}, \quad \Pi_{\mathbf{V},\mu\nu} = \Pi_{\mathbf{A},\mu\nu}$$

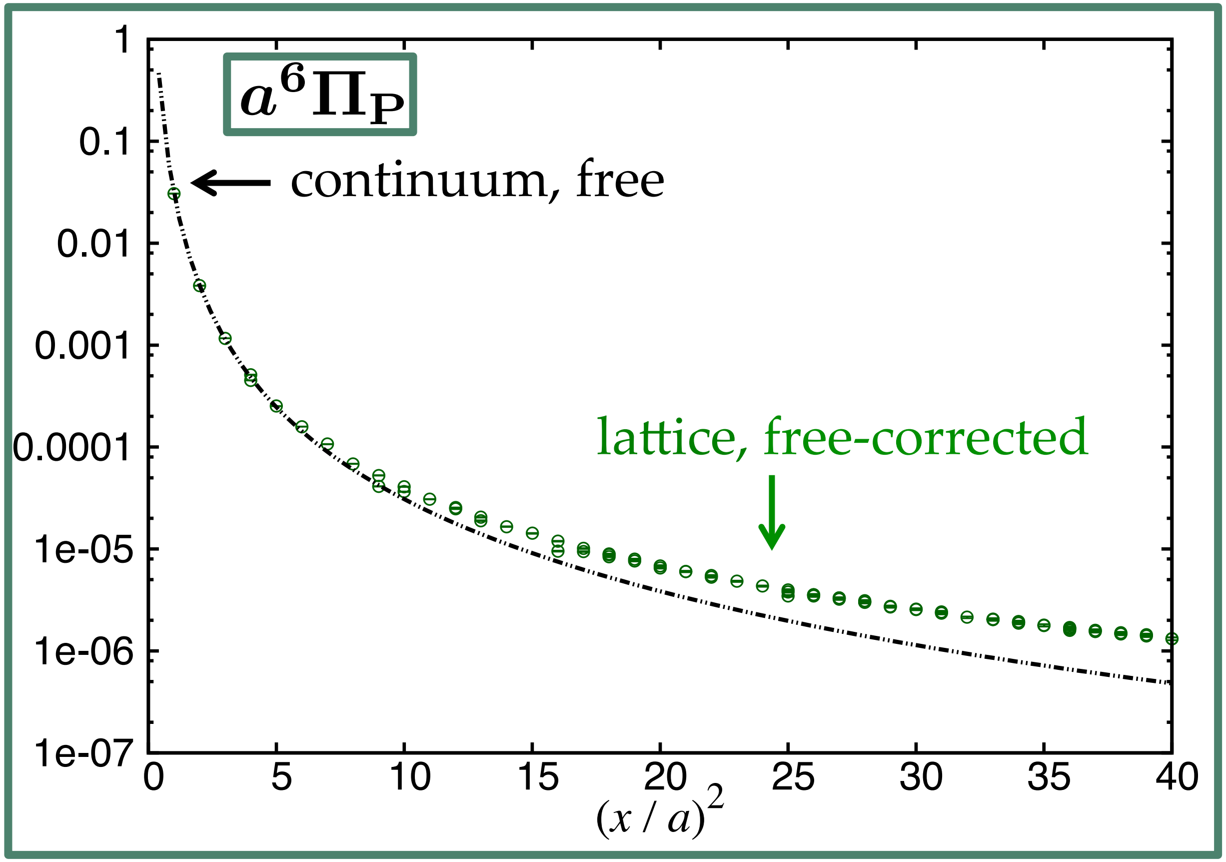
  - Non-perturbative effects breaks these degeneracies

Discretization effect



subtraction:

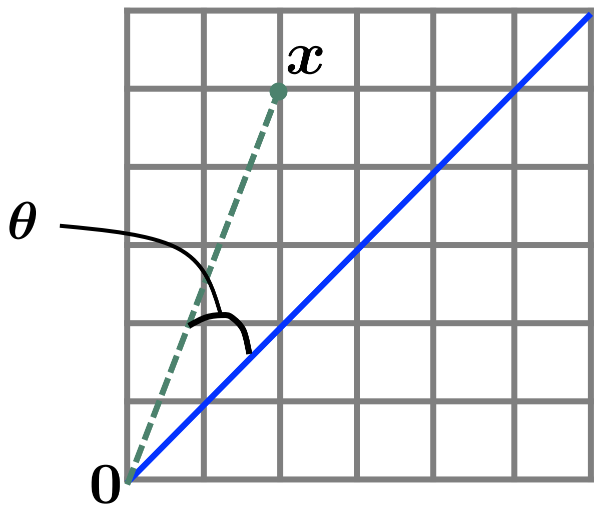
$$\Pi_\Gamma^{lat}(x) \longrightarrow \Pi_\Gamma^{lat}(x) - \left(\Pi_\Gamma^{lat,free}(x) - \Pi_\Gamma^{cont,free}(x)\right)$$



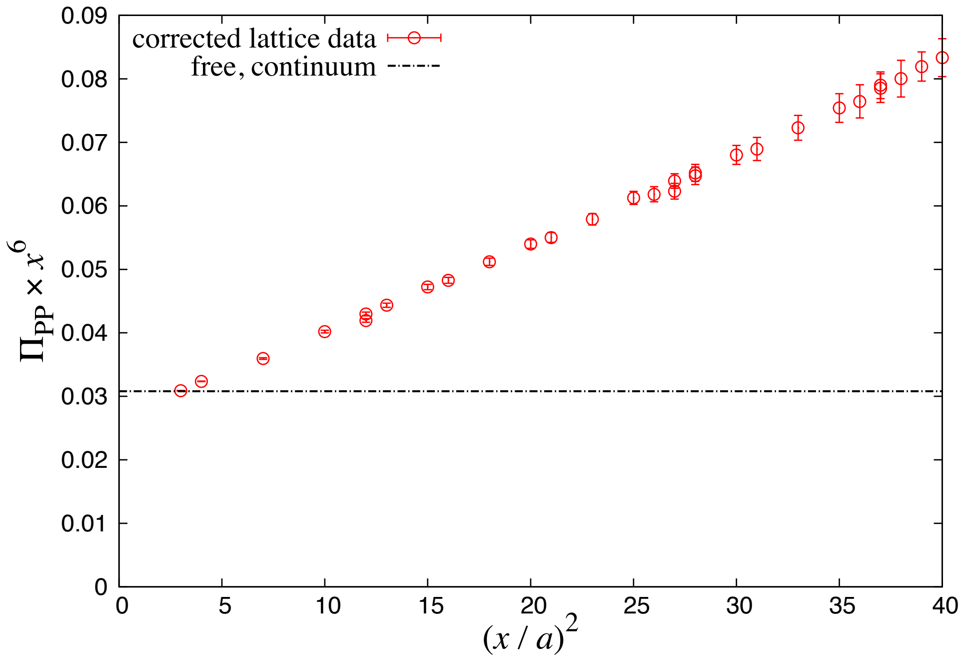
Filtering the points

- Disc error is larger on the direction of lattice coordinates. Filter them out.

Cichy et al., NPB865, 268 (2012).



Cut the points of θ > 30° (angle from (1,1,1,1)).

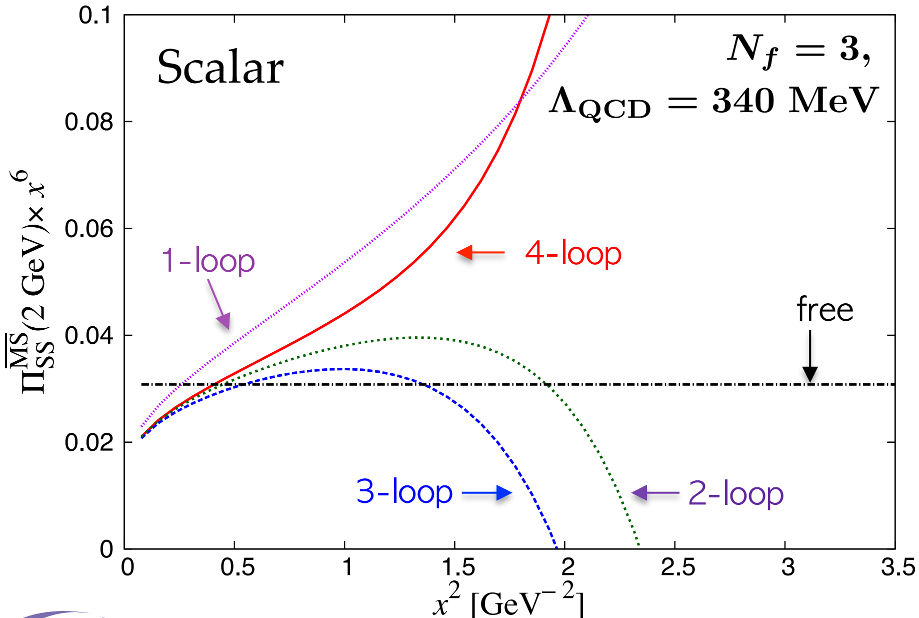


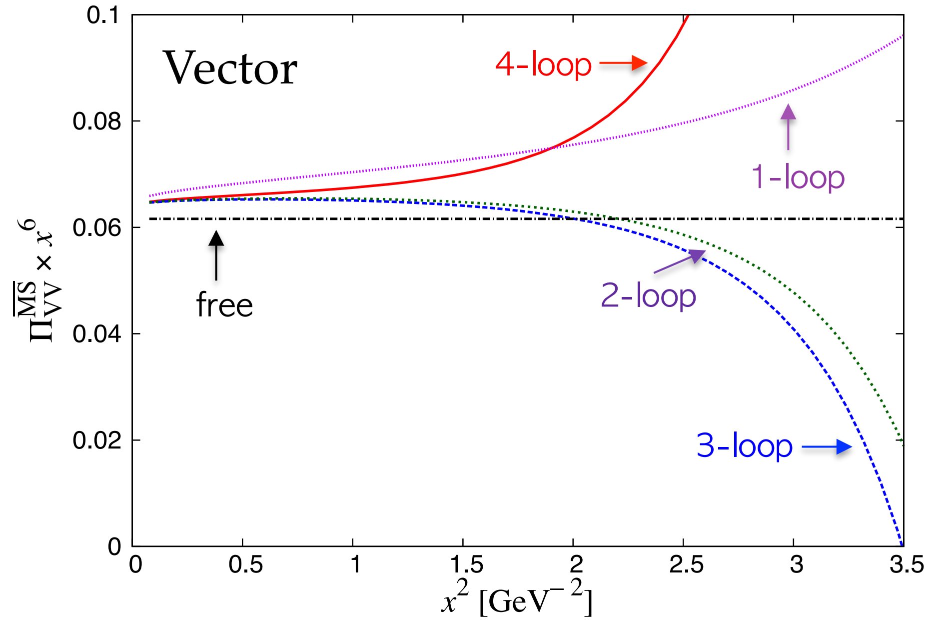
Perturbative expansion

- Available to O($\alpha_s{}^4$)!   Chetyrkin, Maier, NPB844 (2011) 266.
  - Convergence behavior in the region of interest…

$$\Pi_{\mathbf{SS}}^{\widetilde{\overline{\mathrm{MS}}}}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}(1 + 0.67\tilde{a}_s - 16.3\tilde{a}_s^2 - 31\tilde{a}_s^3 + 497\tilde{a}_s^4)$$

$$\Pi_{\mathbf{VV}}^{\widetilde{\overline{\mathrm{MS}}}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s - 4\tilde{a}_s^2 - 1.9\tilde{a}_s^3 + 94\tilde{a}_s^4)$$





Choosing the scale for $\alpha_s(\mu)$

- Well-known example: BLM scale setting

  Brodsky, Lepage, Mackenzie, PRD28, 228 (1983).

  - Improve the perturbative expansion by resuming a class of diagram so that the vacuum polarization diagrams are included to high orders. Amounts to choosing

    $$\tilde{\mu}^* = \tilde{\mu} \exp\big(-11/6 + 2\zeta(3)\big) \simeq 1.8\tilde{\mu}$$

    and rearranging the perturbative expansion.

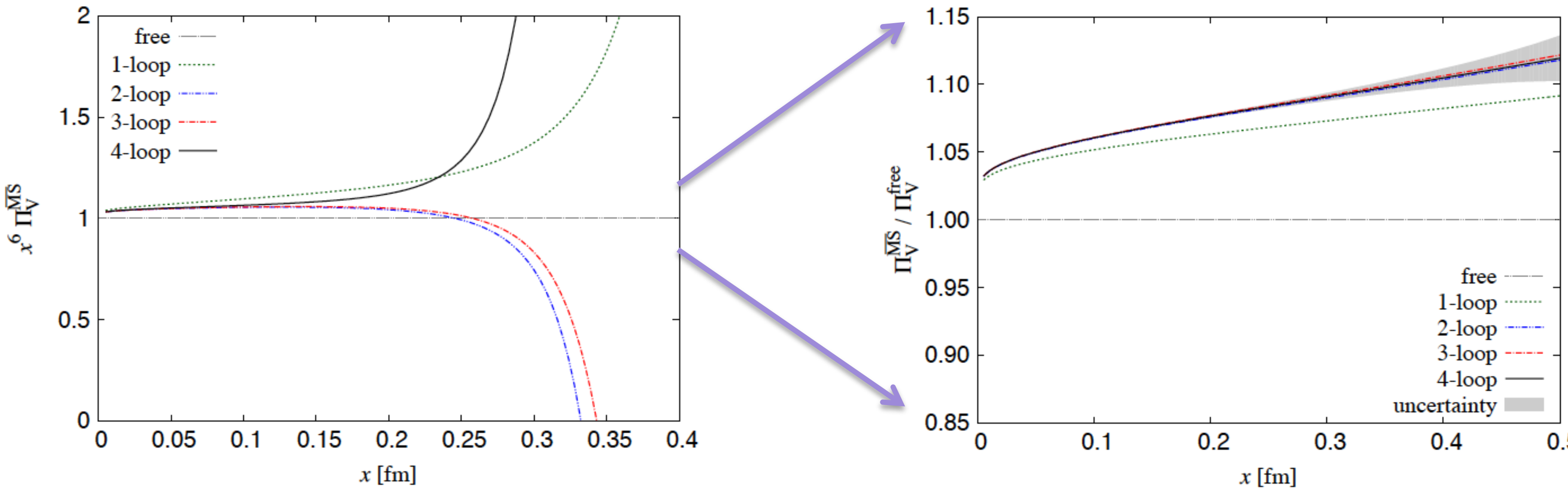
$$\Pi_{VV}^{\widetilde{MS}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s - 4\tilde{a}_s^2 - 1.9\tilde{a}_s^3 + 94\tilde{a}_s^4)$$

$$\Pi_{VV}^{\widetilde{MS}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s^* + 0.083\tilde{a}_s^{*2} - 6\tilde{a}_s^{*3} + 18\tilde{a}_s^{*4}) \qquad \tilde{a}_s^* = a_s(\tilde{\mu}^*)$$

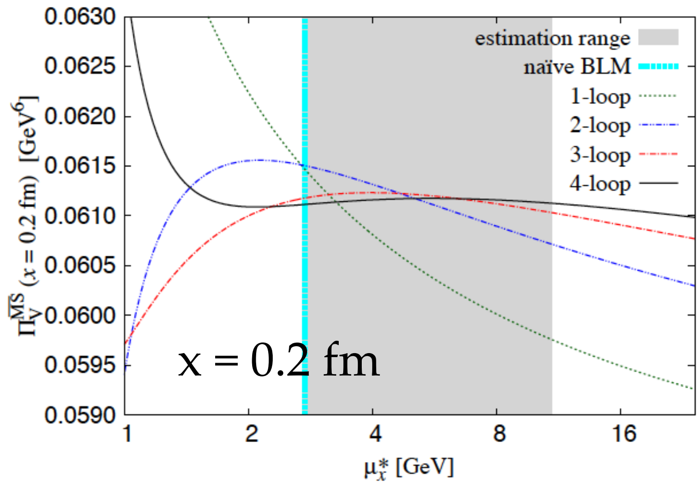
Choosing the scale for $\alpha_s(\mu)$

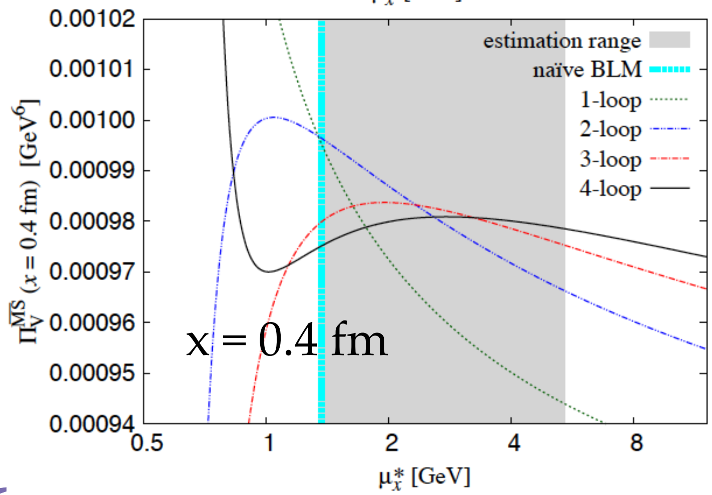
Coupling scale $\mu = x^{-1}$

BLM scale μ^* + variations



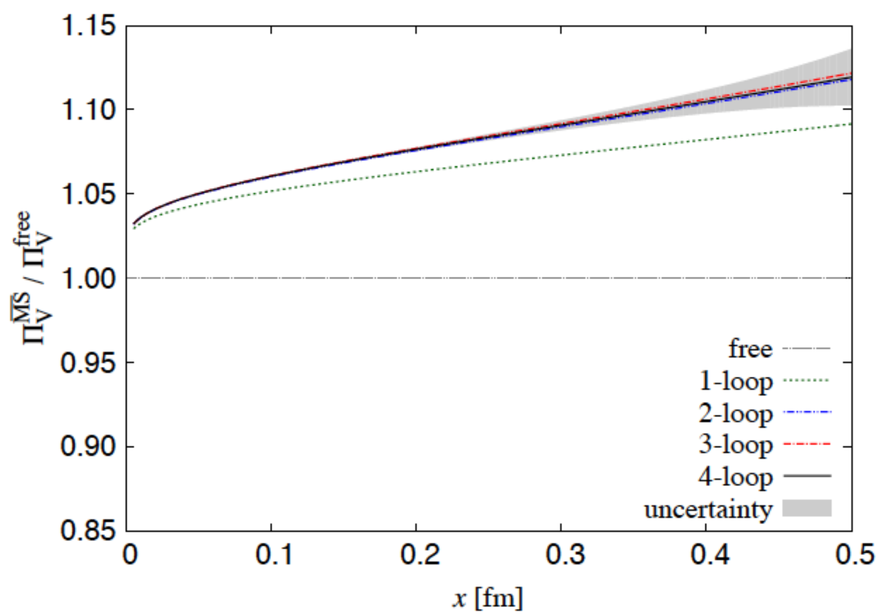
Truncation error





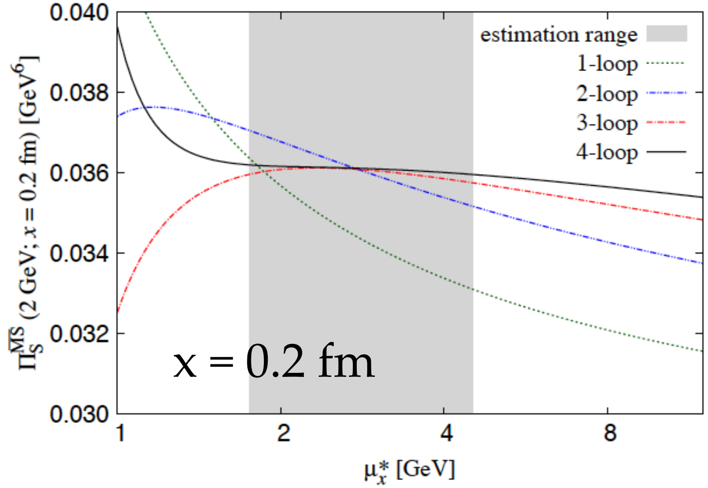
Can be estimated by varying the scale.

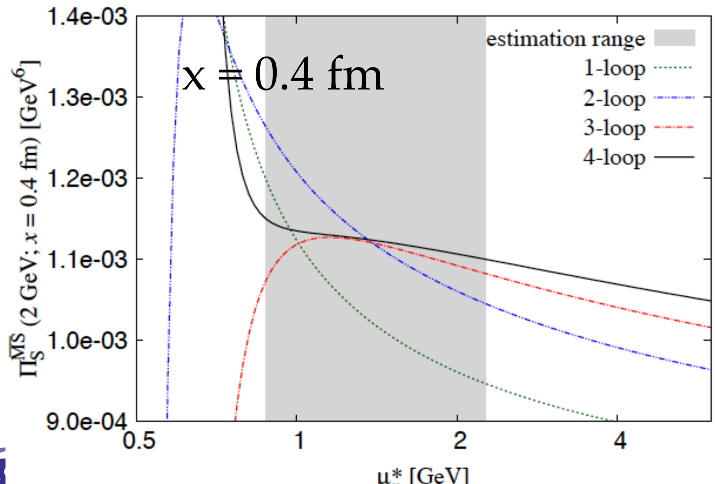
Vector channel



Well under control up to x ~ 0.5 fm.

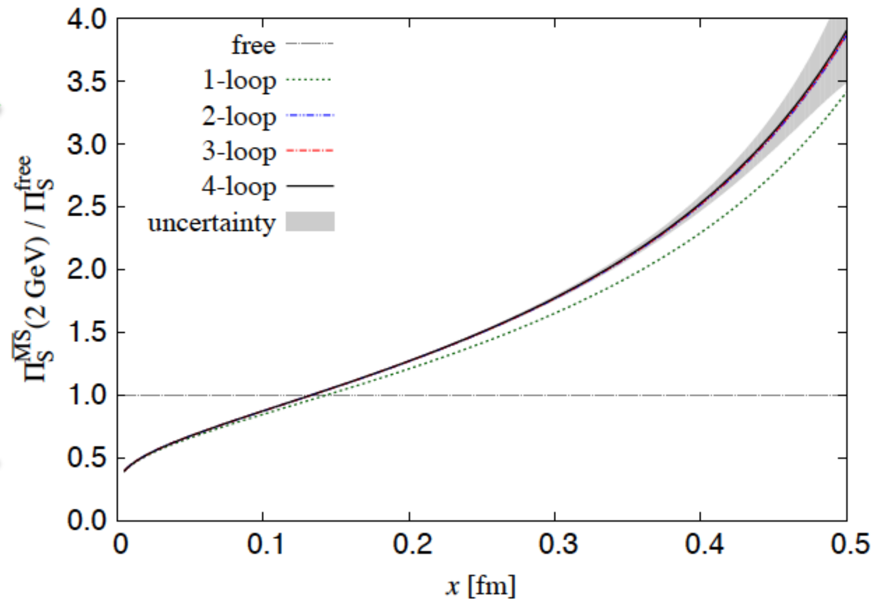
Truncation error





Can be estimated by varying the scale.

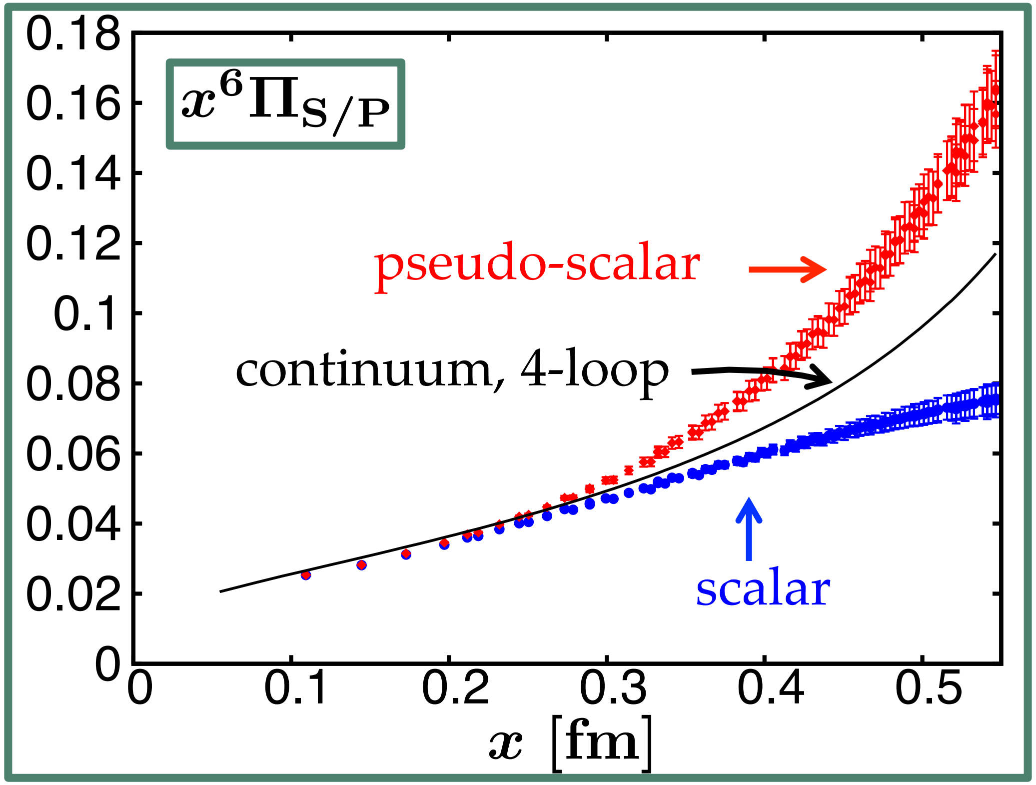
Scalar channel



Well under control up to x ~ 0.4 fm.

Lattice versus PT (S/P)

- Direct comparison at finite *a*.

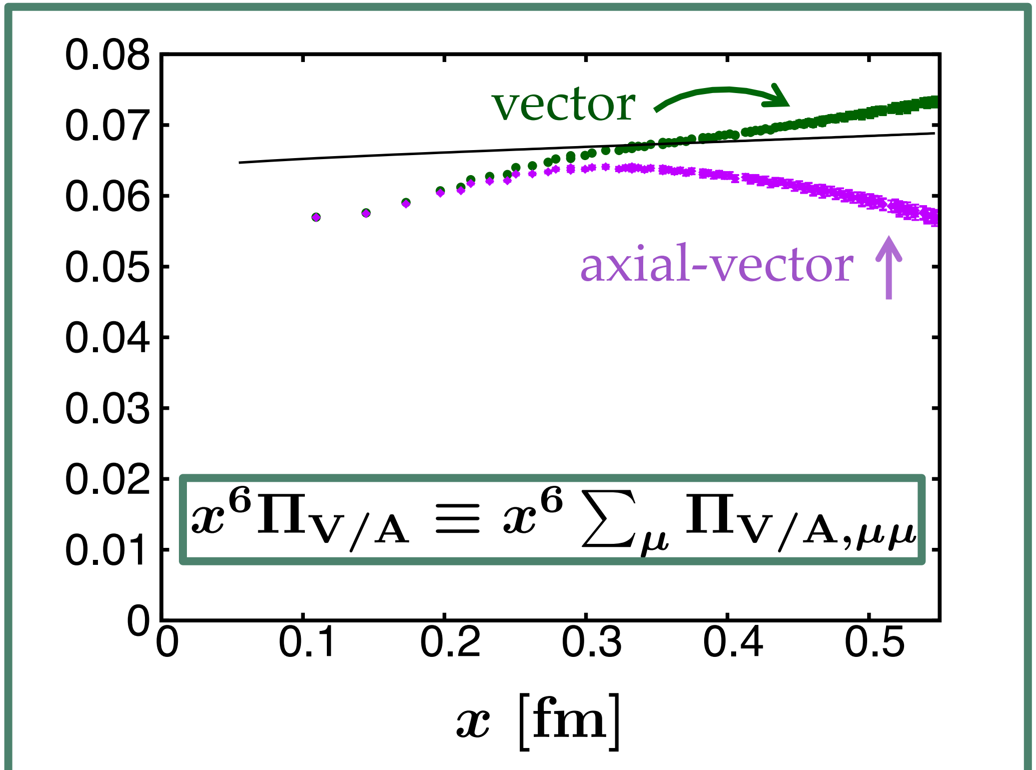


$1/a = 3.6$ GeV,
$M_\pi = 300$ MeV

Non-perturbative effect is significant beyond 0.25 fm.

Lattice versus PT (V/A)

- Direct comparison at finite *a*.



$1/a$ = 3.6 GeV,
M_π = 300 MeV

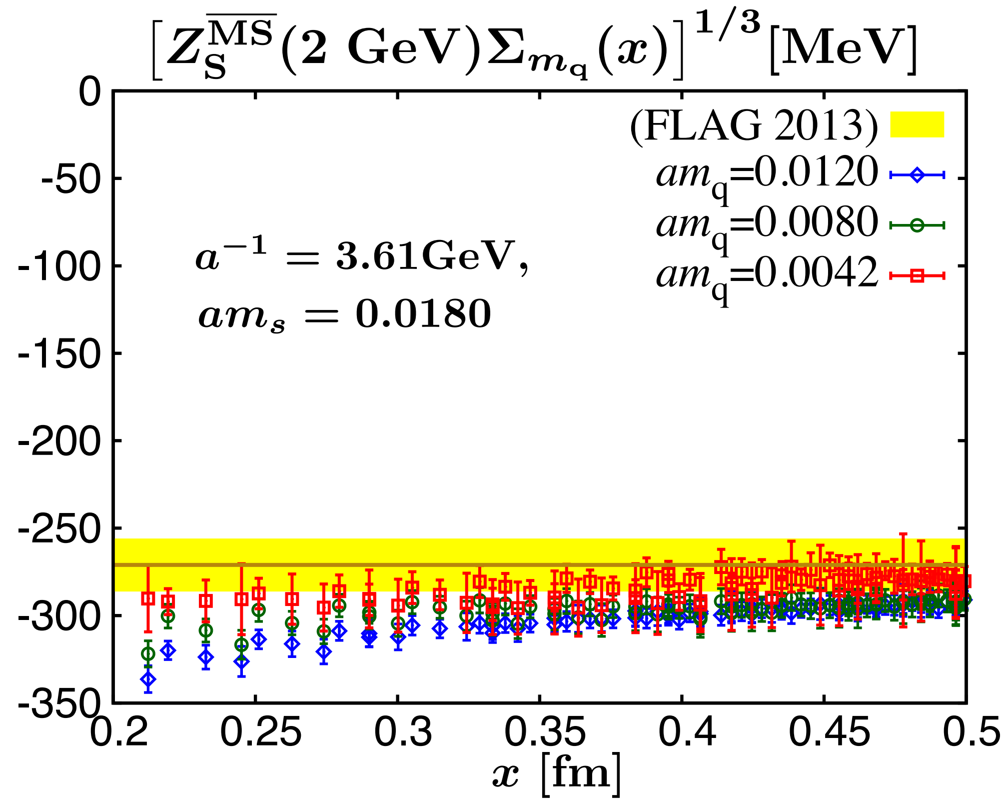
Discretization effect at short distances?

Non-perturbative effect is significant beyond 0.25 fm.

Non-perturbative effect

Looking at the non-conserving part of the axial current

$$\Sigma_{m_q}(x) \equiv -\frac{\pi^2}{2m_q} x^2 x_\nu \partial_\mu \Pi_{A-V,\mu\nu}(x) = \langle \bar{q}q \rangle + O(m_q) \cdot O(x^{-2})$$



Consistent with the FLAG average
<qbar q> = [–271(15) MeV]^3

Operator renormalization

Can be used to renormalize the lattice operators

$$Z_\Gamma^{\overline{MS}/lat}(\mu a)\, O_\Gamma^{lat}(a) = O_\Gamma^{\overline{MS}}(\mu)$$

- Renormalization condition = reproduce the MSbar result at a finite distance x.

Martinelli et al., PLB411, 141 (1997).

$$\Pi_{\mathrm{PP}}(x) = \langle P(x)P(0)\rangle, \quad \Pi_{\mathrm{SS}}(x) = \langle S(x)S(0)\rangle,$$
$$\Pi_{\mathrm{VV}}(x) = \sum_{\mu=1}^{4} \langle V_\mu(x)V_\mu(0)\rangle, \quad \Pi_{\mathrm{AA}}(x) = \sum_{\mu=1}^{4} \langle A_\mu(x)A_\mu(0)\rangle$$

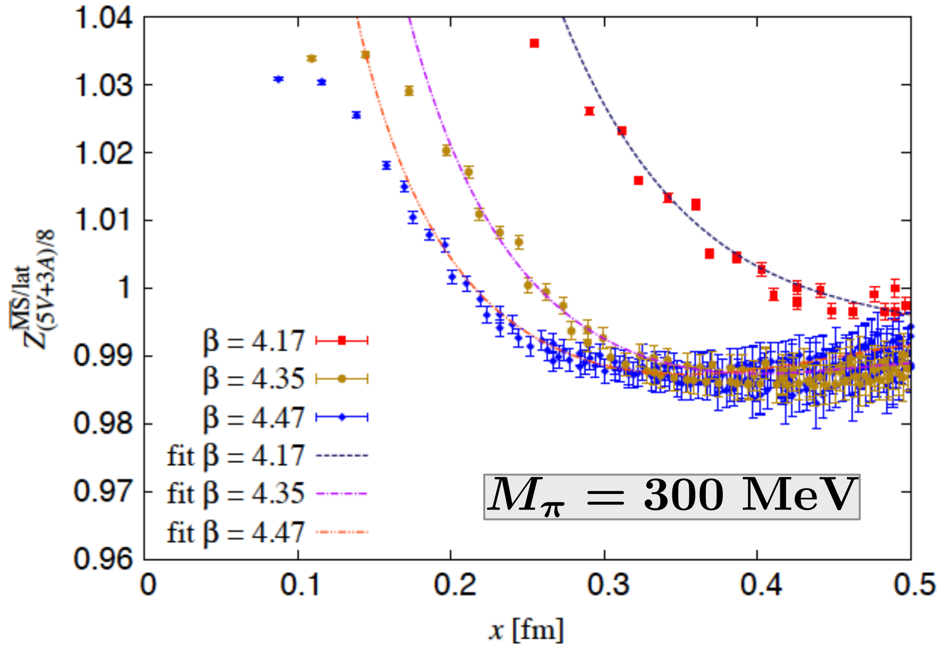
$$\left[Z_\Gamma^{\overline{MS}/lat}(\mu a)\right]^2 \Pi_{\Gamma\Gamma}^{lat}(x) = \Pi_{\Gamma\Gamma}^{\overline{MS}}(x,\mu) \quad \text{or} \quad Z_\Gamma^{\overline{MS}/lat}(\mu a) = \sqrt{\frac{\Pi_{\Gamma\Gamma}^{\overline{MS}}(x,\mu)}{\Pi_{\Gamma\Gamma}^{lat}(x)}}$$

Renormalization of Vector Current

- Eliminate unwanted disc effects and higher order NP effects

$$\widetilde{Z}^{\overline{\rm MS}/lat}_{(5V+3A)/8}(a;x) = Z_V^{\overline{\rm MS}/lat}(a) + c_{-2}(a/x)^2 + c_4 x^4 + (c_6 + c_6' m_q^2)x^6$$

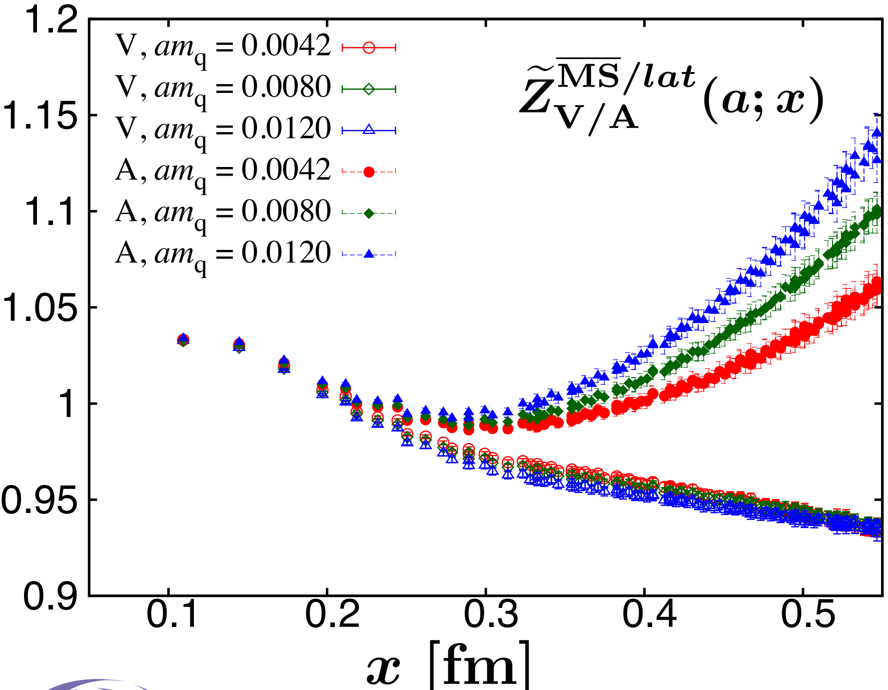
β	$Z_V^{\overline{\rm MS}}$	Errors			
		Stat.	Sys.	μ^*	Λ_{QCD}
4.17	0.9553	(60)	(72)	(9)	(4)
4.35	0.9636	(39)	(45)	(7)	(3)
4.47	0.9699	(29)	(37)	(6)	(4)

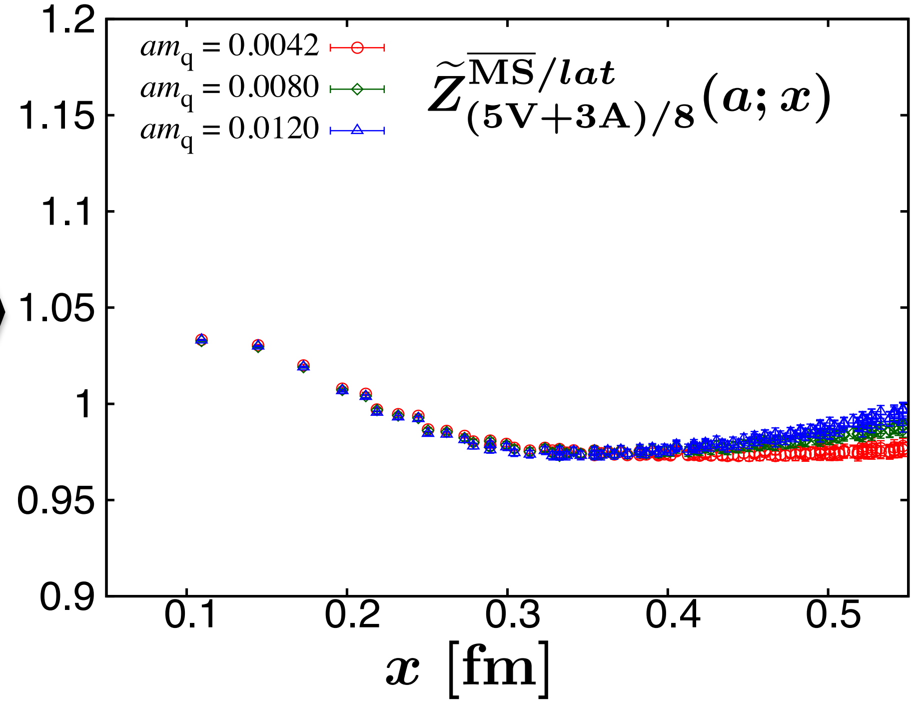


5V+3A ??

- Cancel the term of m<qbar q> in a linear combination

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(5\mathrm{V}+3\mathrm{A})/8}(a;x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(a) + c_{4,\mathrm{G}}\langle \mathrm{GG}\rangle x^4 + \cdots$$

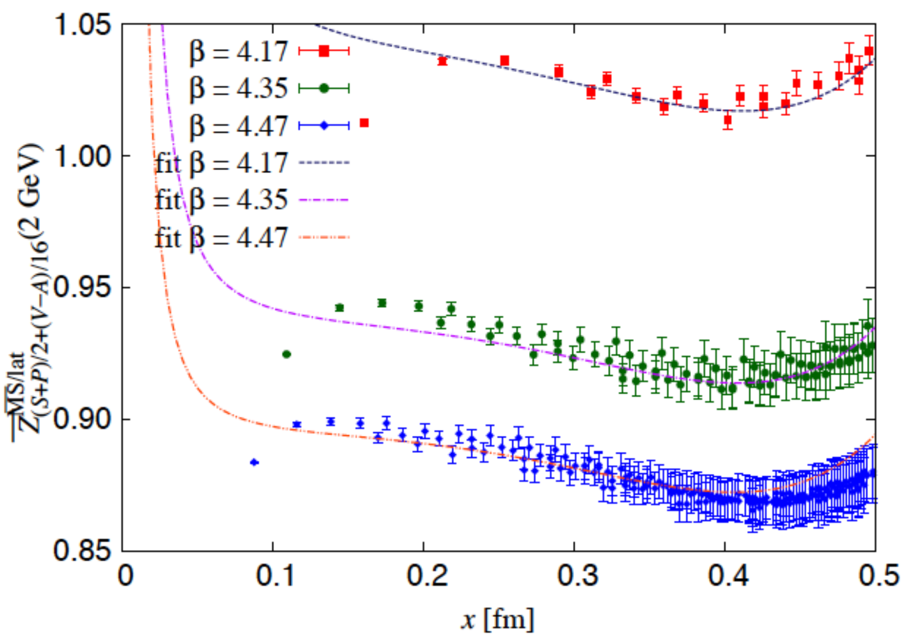




Renormalization of Scalar Density

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(\mathrm{S+P})/2+(\mathrm{V-A})/16}(2\ \mathrm{GeV};a;x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{S}}(2\ \mathrm{GeV};a) + c_{-2}(a/x)^2 + c_4 x^4 + (c_6 + c_6' m_q^2)x^6$$

β	$Z_S^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$	Errors			
		Stat.	Sys.	μ^*	Λ_{QCD}
4.17	1.0347	(92)	(78)	$\binom{+46}{-56}$	(59)
4.35	0.9324	(57)	(55)	$\binom{+30}{-37}$	(37)
4.47	0.8912	(41)	(45)	$\binom{+24}{-31}$	(28)



$M_\pi = 300$ MeV

Summary (light)

- Lattice data at short distances ⇔ pert. theory.
  - "Window" exists around x ~ 1 GeV^{-1} (only after improving both lattice and continuum), but narrow.

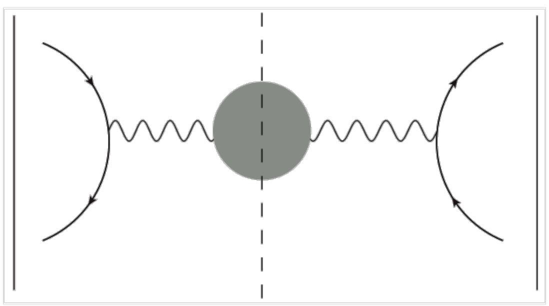
- Applications?
  - Precise calculation of renormalization constants (4-loop available on the continuum side).
  - Extracting α_s from the x-dependence is non-trivial. One has to distinguish the pert x-dep from those of disc effect and non-perturbative contrib.

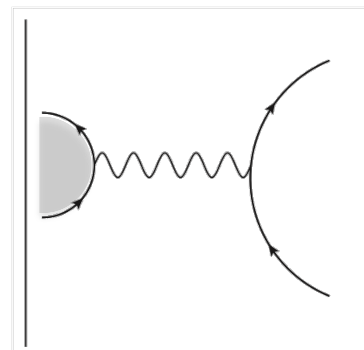
charmonium correlator

Current correlator

- Euclidean: use the optical theorem to relate to "exp"
  - Convenient to use the moments.

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s \frac{1}{s^{n+1}} R(s)_{e^+e^-\to\ \mathrm{hadron}}$$





  - In the coordinate space, it corresponds to the time moments:

$$i\int \mathrm{d}x \frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \mathrm{e}^{iqt} \longrightarrow a^4 \sum_x t^{2n}$$

Time moments on the lattice

- Vector current correlator

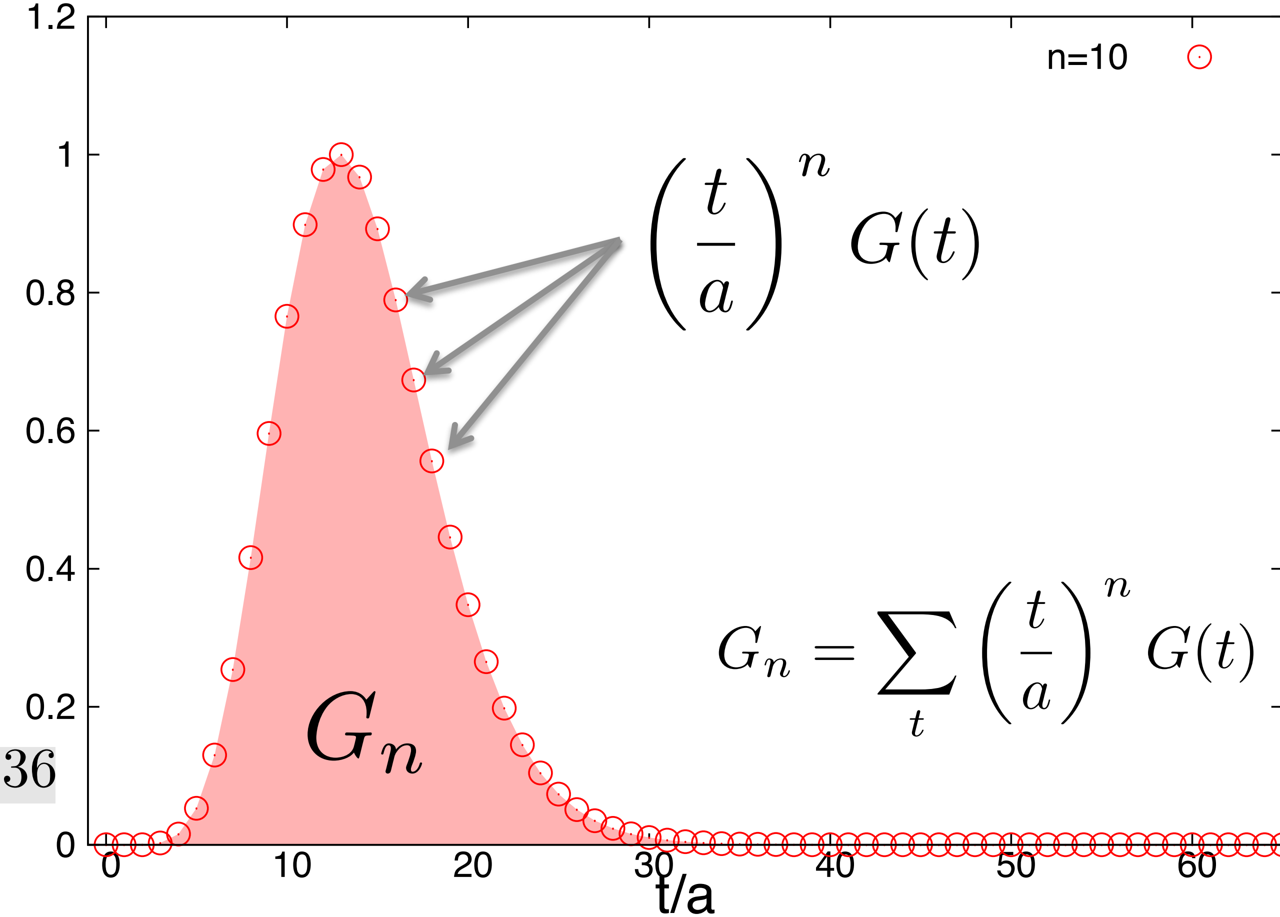
$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V^{(1)}(q^2) = i\int d^4x\, e^{ipx} \langle 0 | T[j_\mu(x) j_\nu(0)] | 0 \rangle$$

- Moments on the lattice

$$G_V(t) = a^6 \sum_x \langle 0 | j_k(x,t) j_k(0,0) | 0 \rangle, \quad G_{V,n} = \sum_t (t/a)^n G_V(t)$$

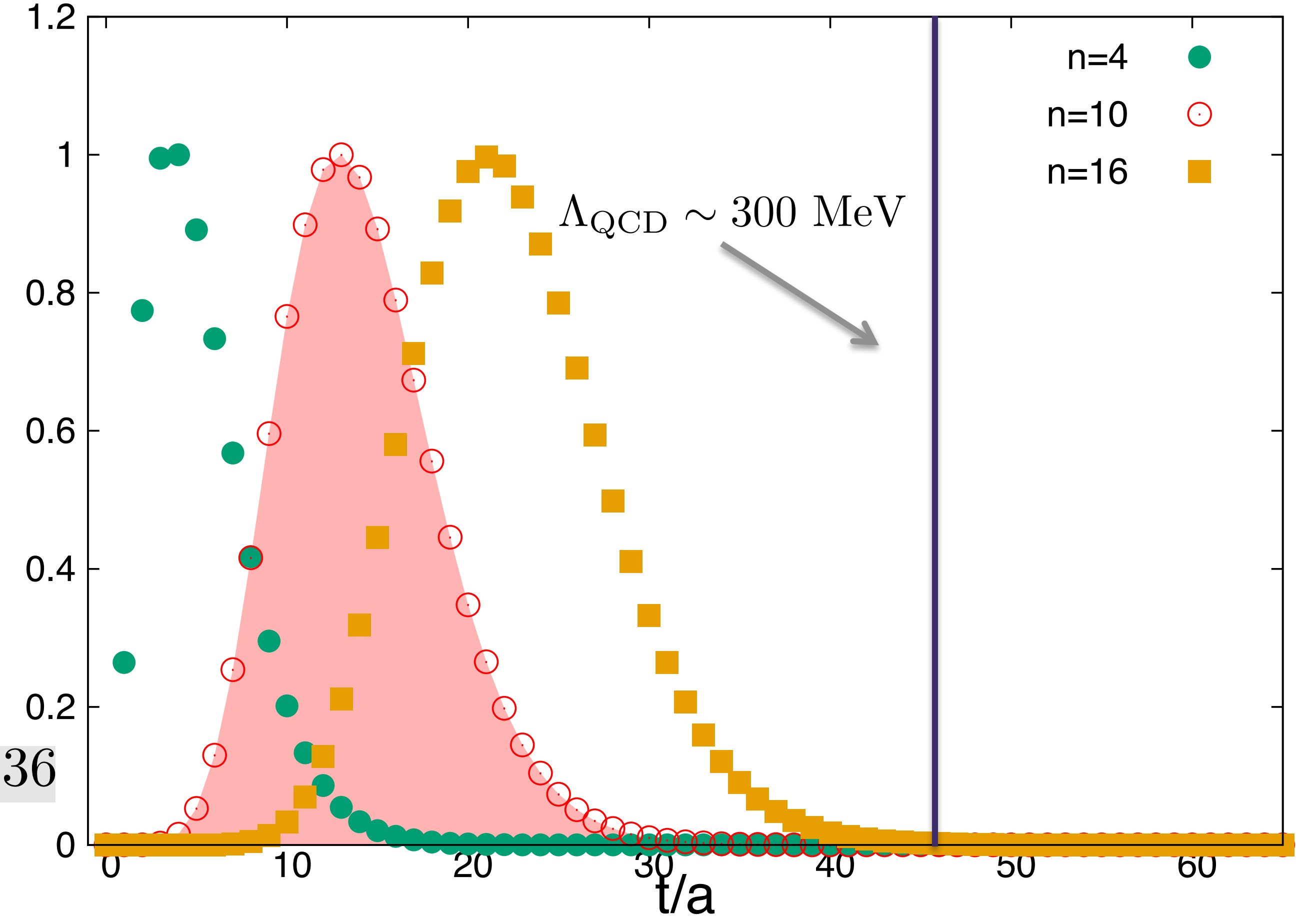
  - $G_V(t)$ represents a J/ψ correlator, ~exp(-$m_{J/\psi}$t), plus its excited states.

Sum to define the moment



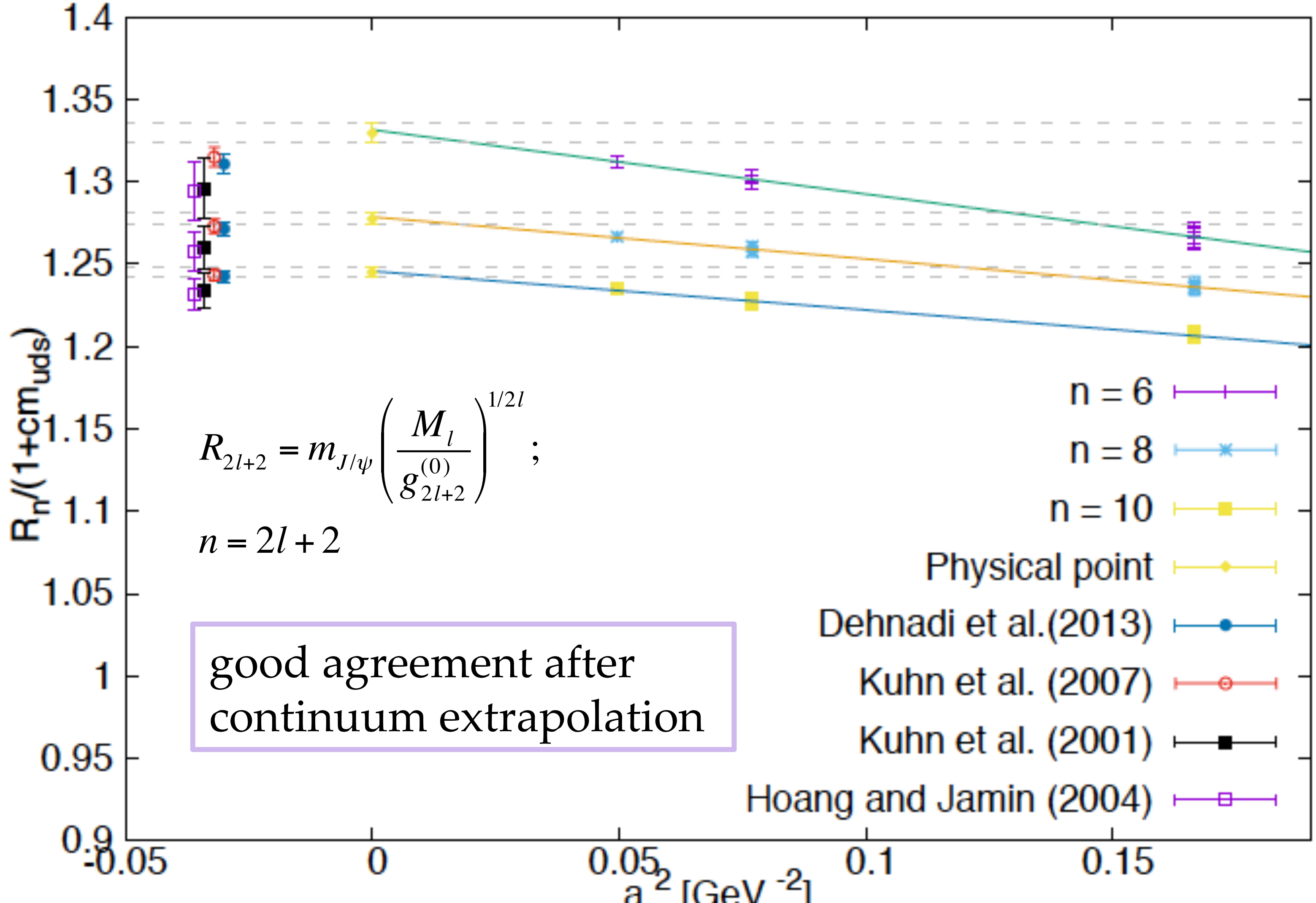
$am_{\eta_c} \sim 0.6636$

Sum to define the moment



$am_{\eta_c} \sim 0.6636$

Lattice versus “exp”



Determination of m_c and α_s

- Following the method of the pioneering work by HPQCD plus the Karlsruhe group (2008~).
  - Allison et al., PRD78, 054513 (2008); McNeile et al., PRD82, 034512 (2010); Chakraborty et al., PRD91, 054508 (2015).

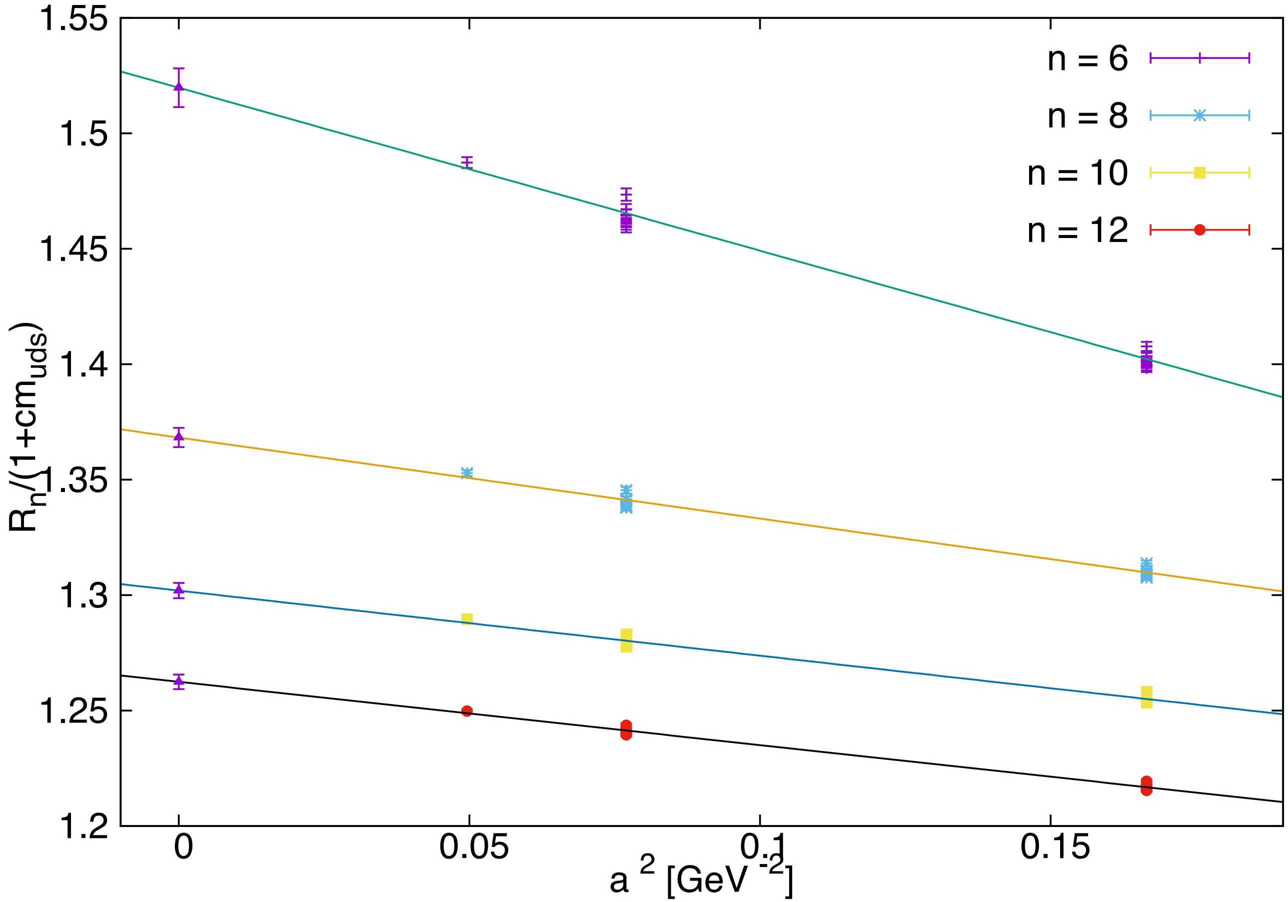
Lattice data after continuum extrap

Continuum perturbation theory known to α_s^3.

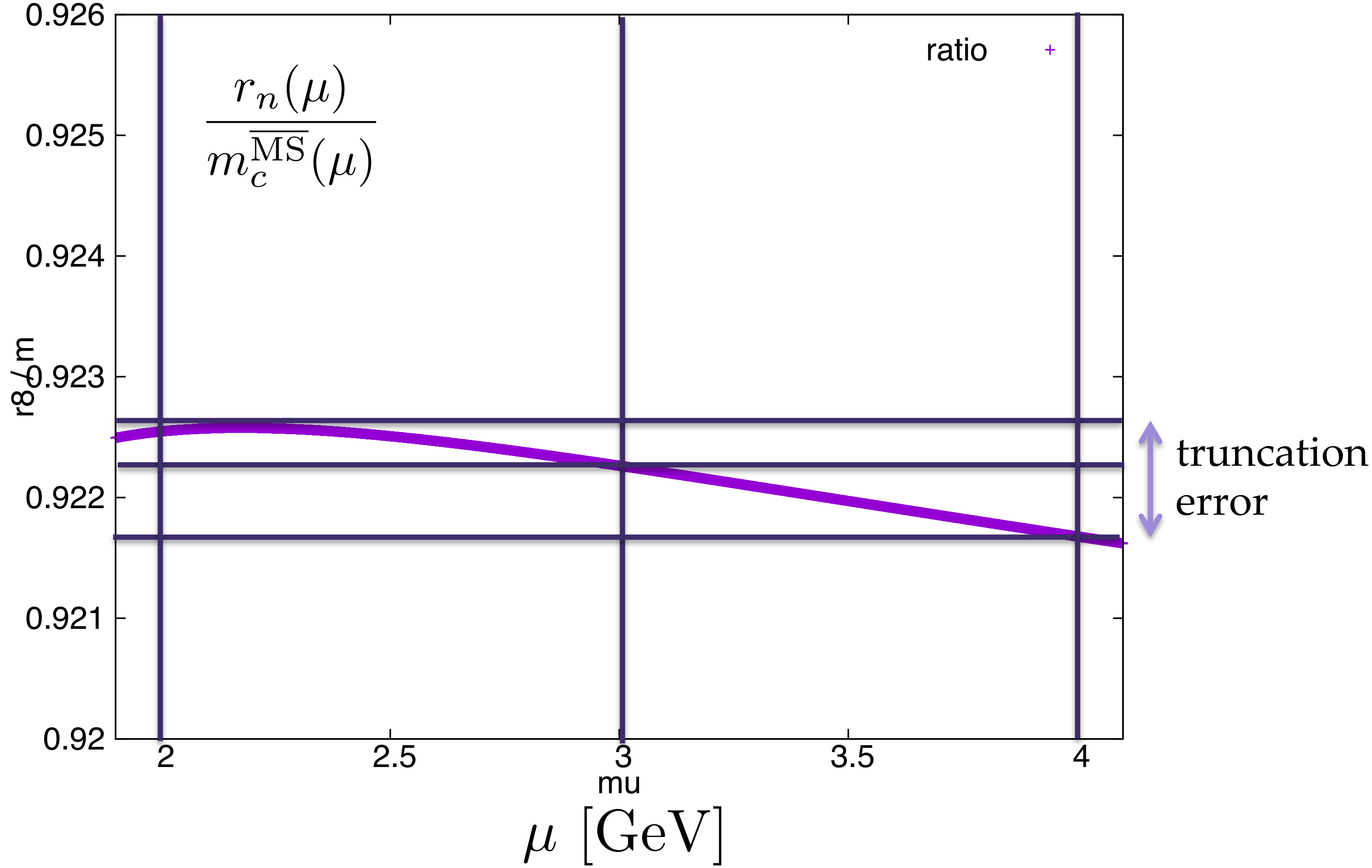
$$R_n = \frac{am_{\eta_c}^{(\mathrm{exp})}}{2a\bar{m}_c(\mu)} r_n(\mu; m_c(\mu), \alpha_s(\mu))$$

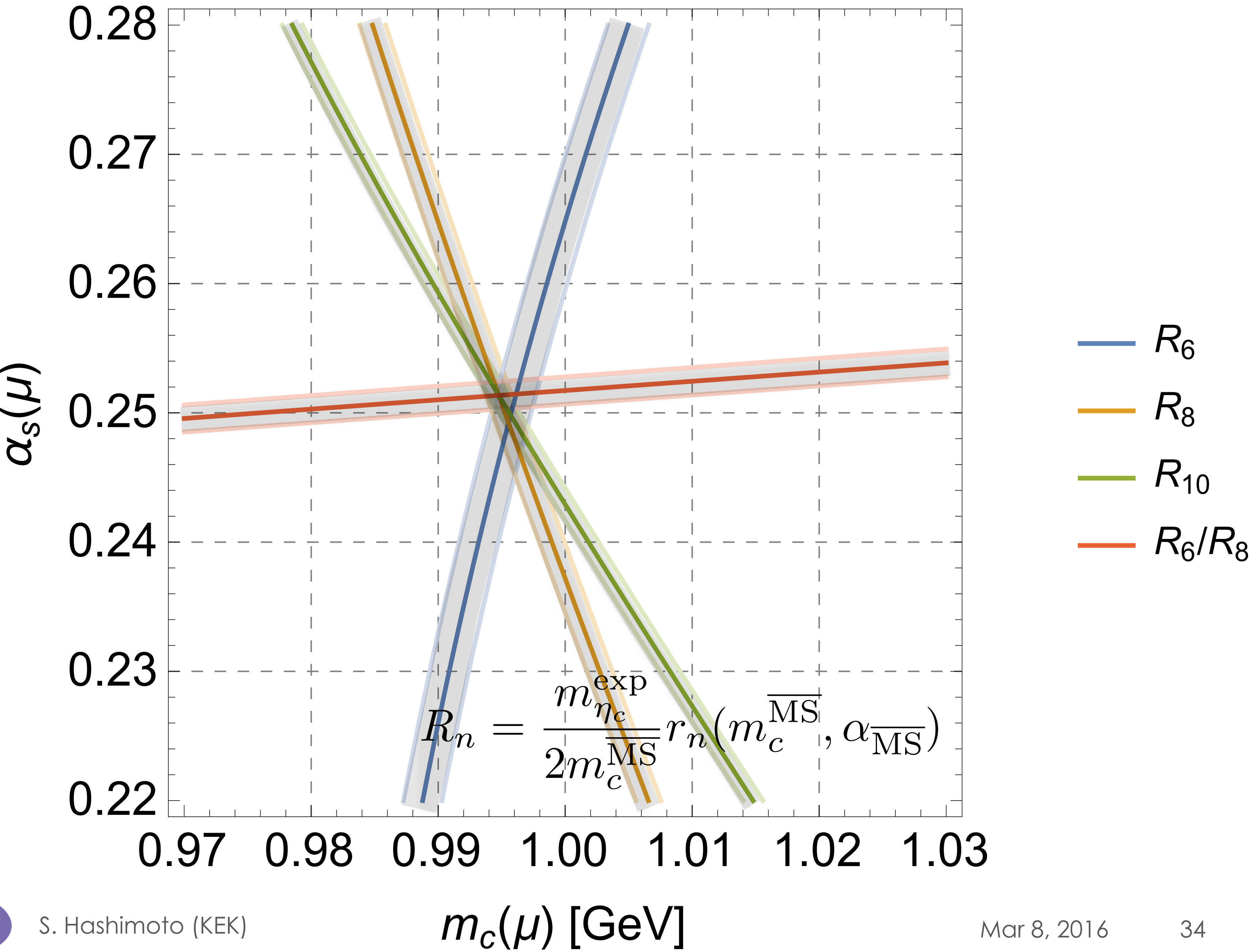
- Solve equations with different n's (n = 6, 8, 10, in this work).
- Use the pseudo-scalar density correlator for both lattice and pert. Perturbative coefficients known to α_s^3.

LHS: Continuum extrapolation of the lattice data



RHS: Estimate of the perterbative truncation error.




0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\alpha_s(\mu)$
0.97
0.98
0.99
1.00
1.01
1.02
1.03
$m_c(\mu)$ [GeV]
R_6
R_8
R_{10}
R_6/R_8
$R_n = \frac{m_{\eta_c}^{\exp}}{2m_c^{\overline{\rm MS}}} r_n(m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}})$

Error budgets

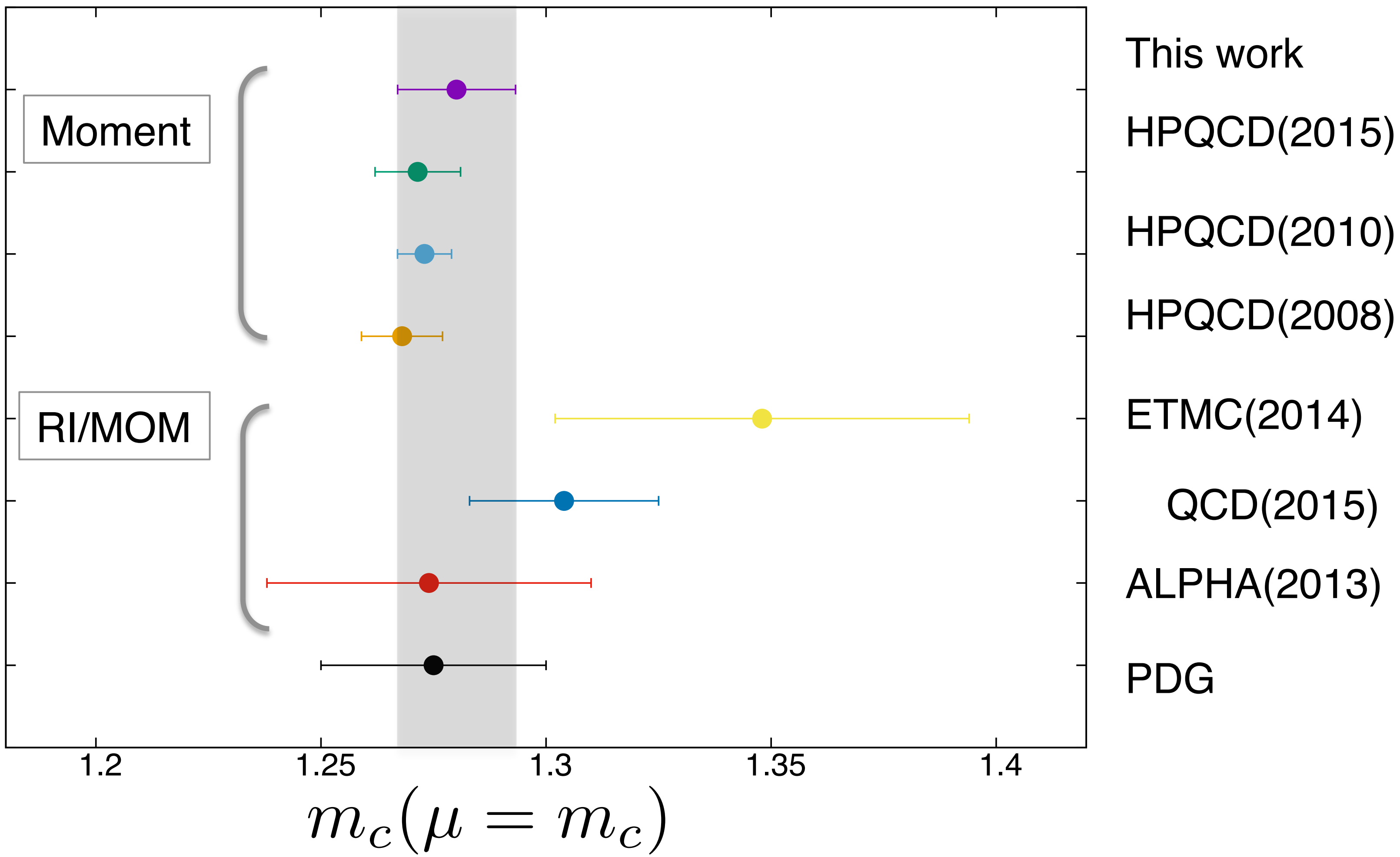
$\mu_m = \mu_\alpha$

		Trunc.	Stat. (Lattice)	a (Lattice)	$O(a^4)$ (Lattice)	FV (Lattice)	$m_{\eta c}^{\rm exp}$ (m_{η_c})	Disc (m_{η_c})	EM (m_{η_c})	$\eta_c J/\psi$ (m_{η_c})	ALL
$m_c(3\text{GeV})$	0.9944 GeV	(26)	(14)	(7)	(53)	(33)	(3)	(3)	(5)	(10)	=(70)
$\alpha_s(3\text{GeV})$	0.2534	(47)	(10)	(4)	(45)	(33)	(0)	(0)	(0)	(1)	=(74)
$\frac{\langle \alpha/\pi G^2 \rangle}{m^4}$	-0.0019 GeV	(38)	(0)	(0)	(3)	(0)	(0)	(0)	(0)	(0)	=(38)

$\mu_m \neq \mu_\alpha$

		Trunc.	Stat. (Lattice)	a (Lattice)	$O(a^4)$ (Lattice)	FV (Lattice)	$m_{\eta c}^{\rm exp}$ (m_{η_c})	Disc (m_{η_c})	EM (m_{η_c})	$\eta_c J/\psi$ (m_{η_c})	ALL
$m_c(3\text{GeV})$	0.9944 GeV	(78)	(14)	(7)	(53)	(33)	(3)	(3)	(5)	(10)	=(102)
$\alpha_s(3\text{GeV})$	0.2534	(119)	(10)	(4)	(45)	(33)	(0)	(0)	(0)	(1)	=(132)
$\frac{\langle \alpha/\pi G^2 \rangle}{m^4}$	-0.0019 GeV	(69)	(0)	(0)	(3)	(0)	(0)	(0)	(0)	(0)	=(69)

Comparison



Moment
RI/MOM
This work
HPQCD(2015)
HPQCD(2010)
HPQCD(2008)
ETMC(2014)
QCD(2015)
ALPHA(2013)
PDG
1.2
1.25
1.3
1.35
1.4
$m_c(\mu = m_c)$

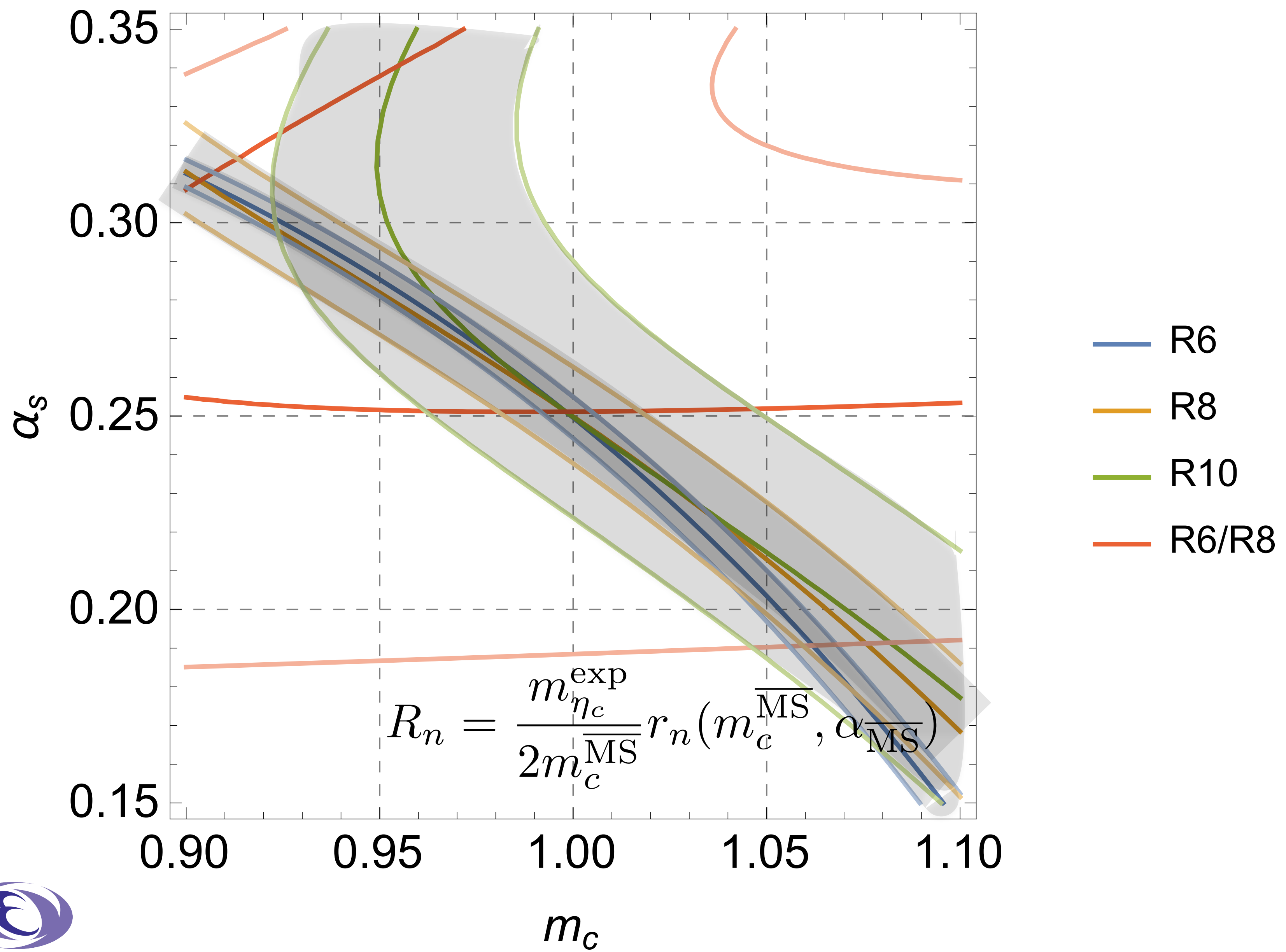
Summary (charm)

- Test of LQCD at short distances
  - Lattice results are consistent with the experimental data, also at short distances.

- Determination of the Fundamental Parameters in QCD
  - LQCD may provide an extra input to feed in the perturbative analysis.
  - The results for m_c and α_s are among the most precise ones.

Backup

From the vector channel:



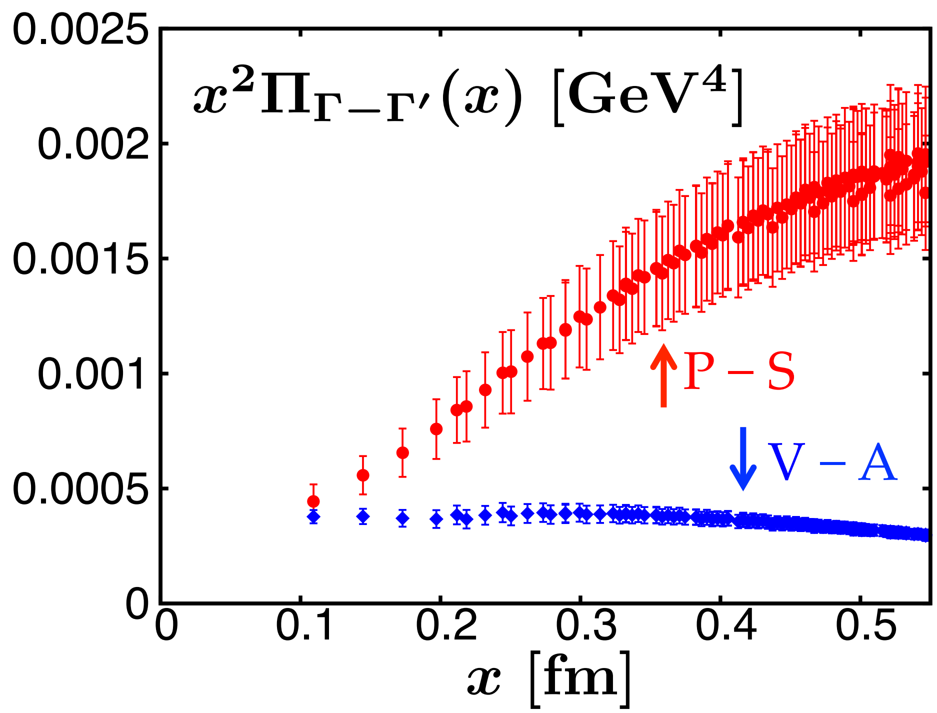
Inconsistency in S/P channel

Non-perturbative effect too large for scalar-pseudoscalar

- OPE predicts the term of m<qbar q>:

  P-S ~ 0.5 x V-A

- Lattice result: P-S >> V-A and different x-dep

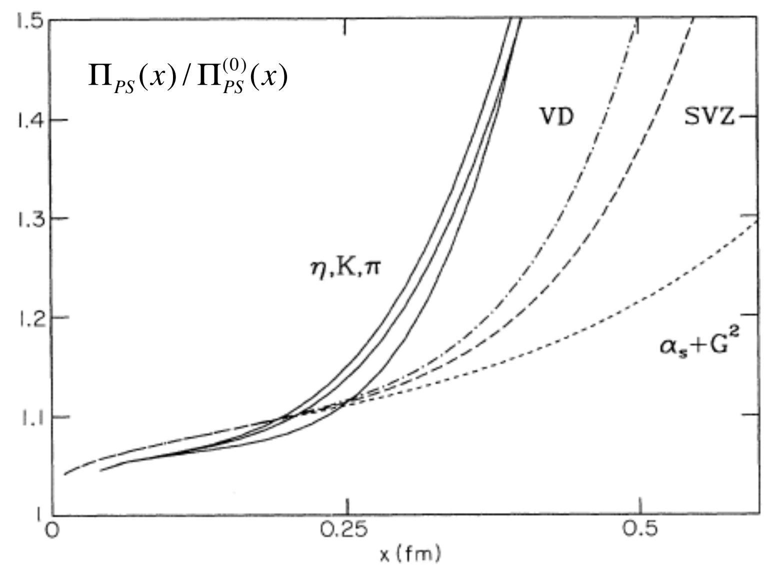


OPE failure?

Known for some time:

- Novikov, Shifman, Vainshtein, Zakharov, *"Are all hadrons alike?"*, NPB191 (1981) 301.
    - Spin-0 correlation functions may deviate from OPE at shorter distances.
- Shuryak, ➔
- Chetyrkin, Narison, Zakharov, *"Short-distance tachyonic gluon mass and 1/Q² corrections,"* NPB550 (1999) 353.
- Narison, Zakharov, *"Hints on the power corrections from current correlators in x-space,"* PLB522 (2001) 266.

Shuryak, RMP65, 1 (1993)



Lattice observations:

- Chu, Grandy Huang, Negele, PRD48 (1993) 3340.
- DeGrand, PRD64 (2001) 094508.