Determination of $\alpha_{\rm s}$ from the QCD static energy

Antonio Vairo

Technische Universität München

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Determination of $\alpha_{\rm s}$ from the QCD static energy: an update
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(2) X. Garcia i Tormo
Review on the determination of $\alpha_{\rm s}$ from the QCD static energy
Mod. Phys. Lett. A28 (2013) 1330028 `arXiv:1307.2238`

(3) A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petreczky, J. Soto and A. Vairo
Determination of $\alpha_{\rm s}$ from the QCD static energy
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(4) N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo
Precision determination of $r_0\Lambda_{\overline{\rm MS}}$ from the QCD static energy
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(5) N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo
The QCD static energy at NNNLL
Phys. Rev. D80 (2009) 034016 `arXiv:0906.1390`

(6) N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo
The logarithmic contribution to the QCD static energy at N^4LO
Phys. Lett. B647 (2007) 185 `arXiv:hep-ph/0610143`

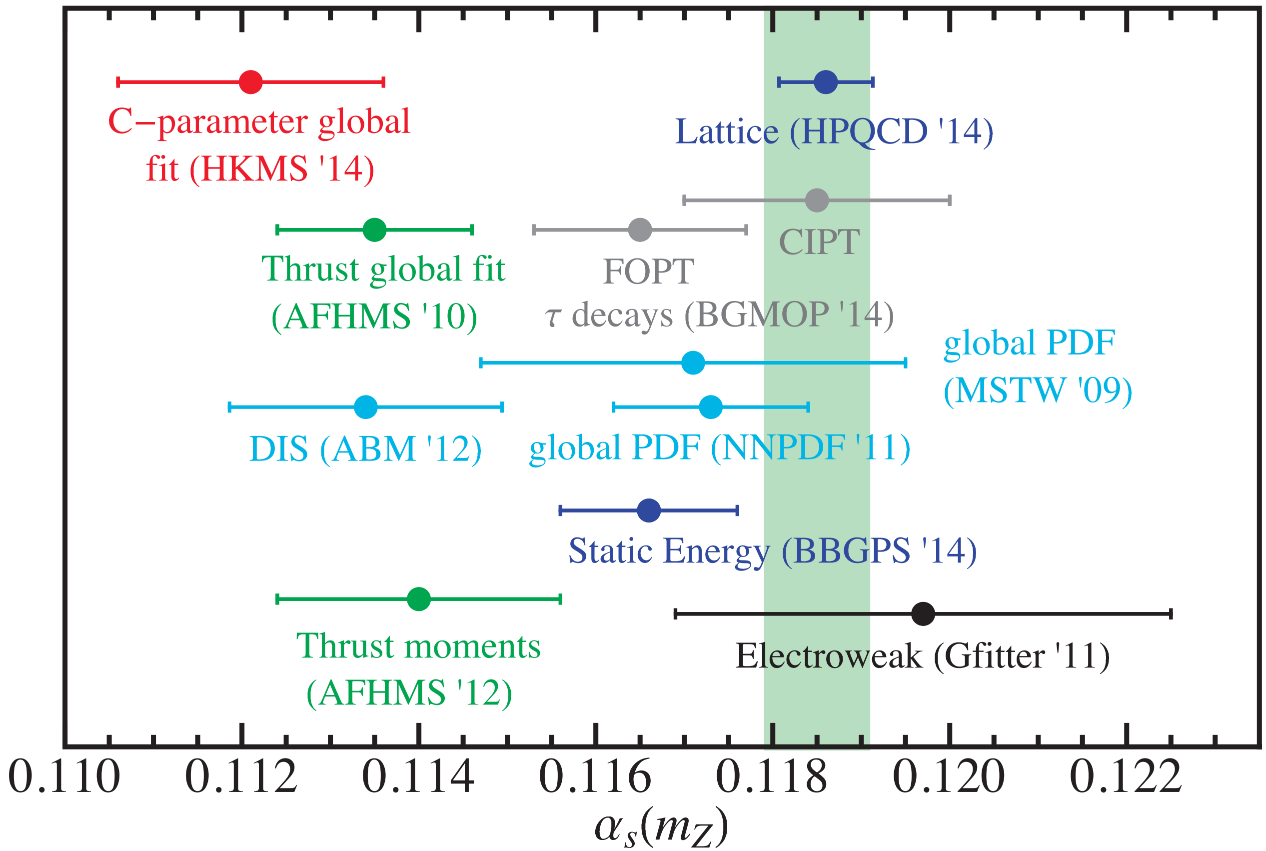
α_s in 2015

Status of α_s

	$\alpha_s(M_Z^2)$	
Alekhin [2001]	0.1143 ± 0.013	DIS [4]
BBG [2004]	$0.1134\ ^{+0.0019}_{-0.0021}$	valence analysis, NNLO [5,6]
GRS	0.112	valence analysis, NNLO [7]
ABKM	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$ [8]
JR14	0.1136 ± 0.0004	dynamical approach [9]
JR14	0.1162 ± 0.0006	including NLO-jets [9]
MSTW	0.1171 ± 0.0014	(2009) [10]
Thorne	0.1136	[DIS+DY, HT*] (2014) [11]
ABM11$_J$	$0.1134 - 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl. [12]
ABM13	0.1133 ± 0.0011	[13]
ABM13	0.1132 ± 0.0011	(without jets) [13]
CTEQ	$0.1159...0.1162$	[14]
CTEQ	0.1140	(without jets) [14]
NN21	$0.1174 \pm 0.0006 \pm 0.0001$	[15]
Gehrmann et al.	$0.1131\ ^{+\ 0.0028}_{-\ 0.0022}$	e^+e^- thrust [16]
Abbate et al.	0.1140 ± 0.0015	e^+e^- thrust [17]
CMS	0.1151 ± 0.0033	$t\bar{t}$ [18]
NLO Jets ATLAS	$0.111\ ^{+0.0017}_{-0.0007}$	[19]
NLO Jets CMS	0.1148 ± 0.0055	[19]
BBG [2004]	$0.1141\ ^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO [5,6]
3-jet rate	0.1175 ± 0.0025	Dissertori et al. 2009 [20]
Z-decay rate	0.1189 ± 0.0026	BCK 2008/12 (N^{3}LO) [21,22]
τ-decay rate	0.1212 ± 0.0019	BCK 2008 (N^{3}LO) [21,22]
τ-decay rate	0.1204 ± 0.0016	Pich 2011 [1]
τ-decay rate	0.325 ± 0.018 (at m_τ)	FOTP: [23]
τ-decay rate	0.374 ± 0.025 (at m_τ)	CIPT: [23]
Lattice	0.1205 ± 0.0010	PACS-CS 2009 (2+1 fl.) [24]
Lattice	0.1184 ± 0.0006	HPQCD 2010 [25]
Lattice	0.1200 ± 0.0014	ETMC 2012 (2+1+1 fl.) [26]
Lattice	0.1156 ± 0.0022	Bazavov et al. (2+1 fl.) [27]
Lattice	$0.1130 \pm 0.0010(stat)$	RBC-UKQCD (preliminary, 2014) [28]

- Moch et al arXiv:1405.4781

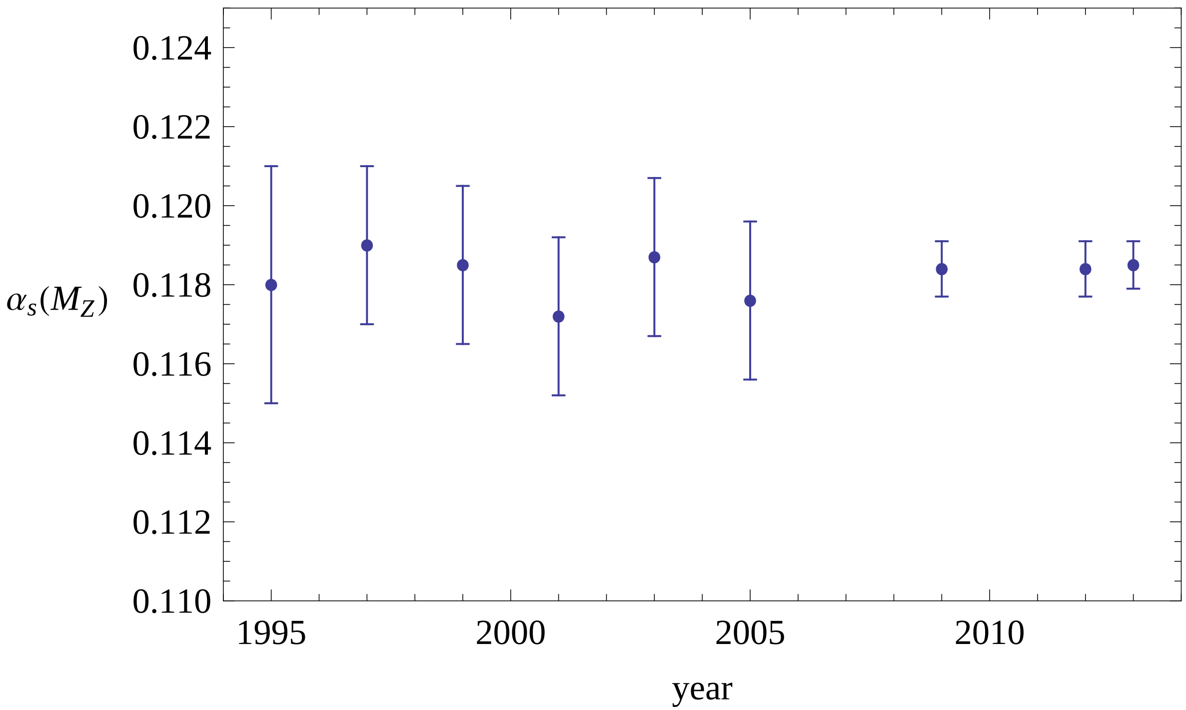
Status of α_s



- Hoang Kolodrubetz Mateu Stewart PRD 91 (2015) 094018

PDG average

Uncertainties of α_s are not fully reflected in the PDG average of 2015...



... but they will in 2016:

for the first time in over 20 years there will be an increase in the error of α_s!

- Bethke Dissertori @ ISMD2015

Theory

Static energy

$$E_0(r) \;=\; \lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle; \qquad \Box = \exp\left\{ ig \oint dz^\mu A_\mu \right\}$$

Perturbation theory describes $E_0(r)$ in the short range ($r\Lambda \ll 1$, $\alpha_s(1/r) < 1$):

$$E_0(r) = \Lambda_s - \frac{C_F \alpha_s}{r}(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln\alpha_s + \#\alpha_s^4 \ln^2\alpha_s + \#\alpha_s^4 \ln\alpha_s + \dots)$$

- $E_0(r)$ is known at three loops.
- $\ln\alpha_s$ signals the cancellation of contributions coming from different energy scales:

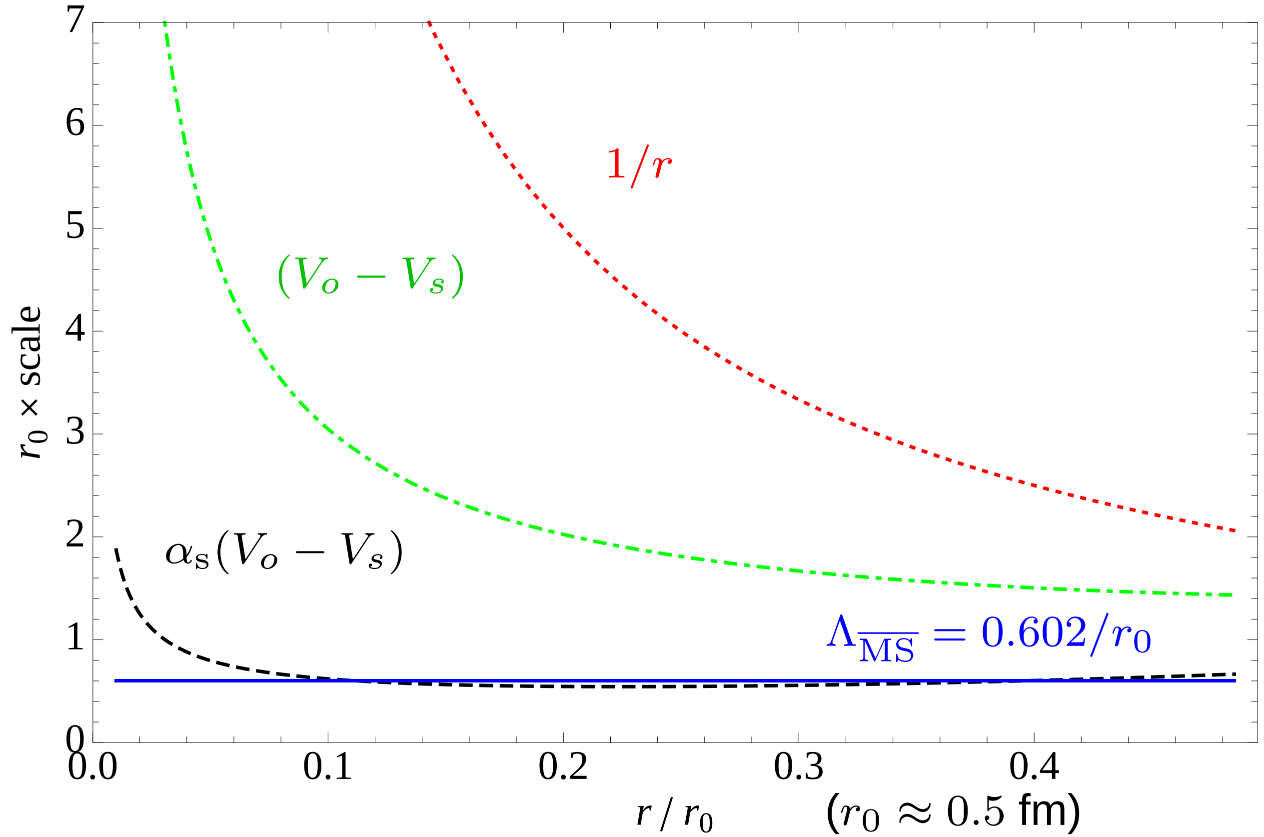
$$\ln\alpha_s = \ln\frac{\mu}{1/r} + \ln\frac{\alpha_s/r}{\mu}$$

◦ Brambilla Pineda Soto Vairo PRD 60 (1999) 091502

Energy scales

In the short range the static Wilson loop is characterized by a hierarchy of energy scales:

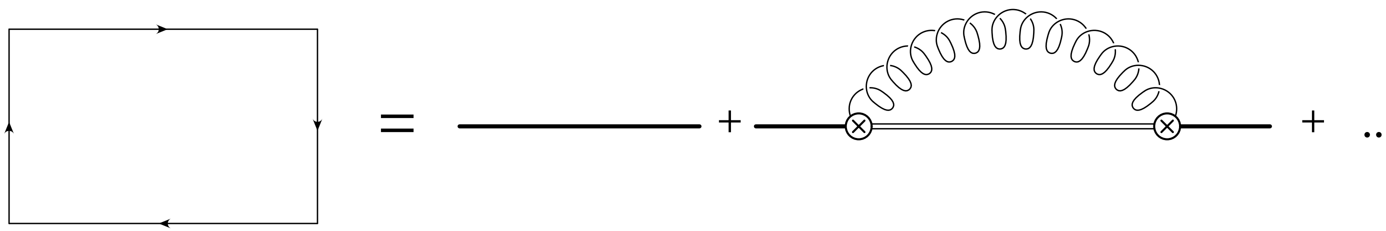
$$1/r \gg V_o - V_s \gg \Lambda; \qquad V_s \approx -C_F \frac{\alpha_{\rm s}}{r}, \quad V_o \approx \frac{1}{2N}\frac{\alpha_{\rm s}}{r}$$



($r_0 \approx 0.5$ fm)

Effective Field Theories

EFTs allow the factorization of contributions from different energy scales.



QCD pNRQCD

$$E_0(r) = \Lambda_s + V_s(r,\mu) - i\frac{g^2}{N}V_A^2\int_0^\infty dt\, e^{-it(V_o-V_s)}\langle \mathrm{Tr}\,\mathbf{r}\cdot\mathbf{E}(t)\,\mathbf{r}\cdot\mathbf{E}(0)\rangle(\mu) + \dots$$

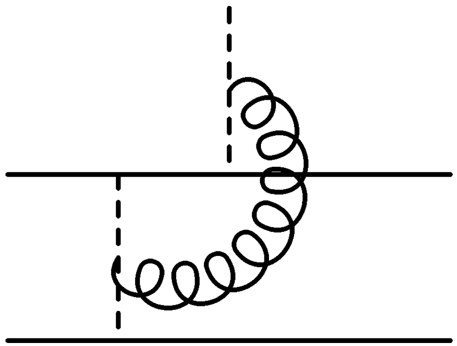
res. mass potential ultrasoft contribution

- Brambilla Pineda Soto Vairo NPB 566 (2000) 275

The μ dependence cancels between
$V_s \sim \ln r\mu, \ln^2 r\mu$, ...
ultrasoft contribution $\sim \ln(V_o - V_s)/\mu, \ln^2(V_o - V_s)/\mu, \dots \ln r\mu, \ln^2 r\mu, \dots$

V_A

The first contributing diagrams are of the type:

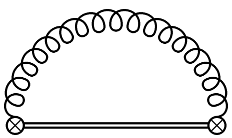


Therefore

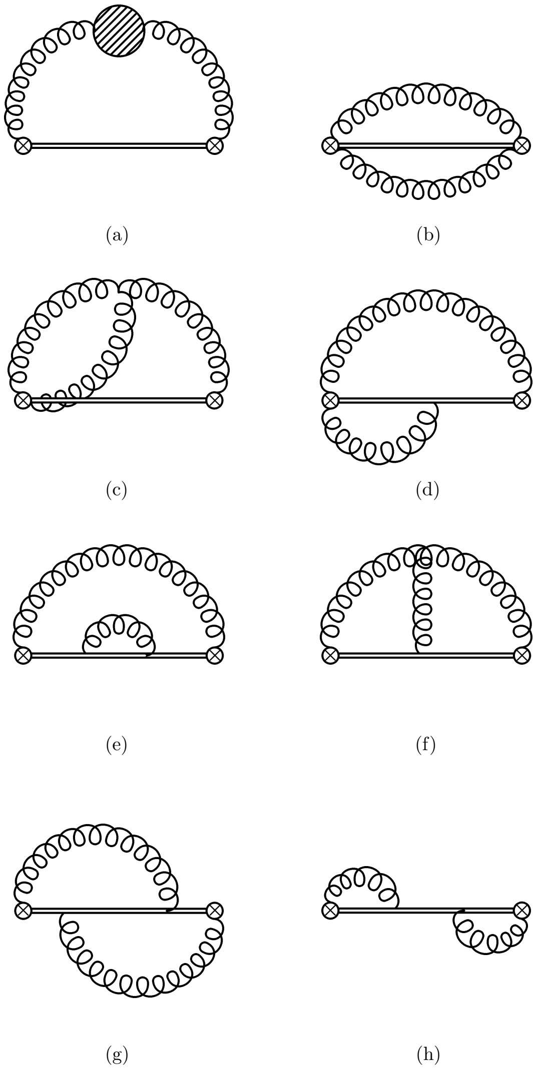
$$V_A(r,\mu) = 1 + \mathcal{O}(\alpha_s^2)$$

Chromoelectric field correlator: $\langle E(t)E(0)\rangle$

Is known at two loops.



LO



NLO

- Eidemüller Jamin PLB 416 (1998) 415

Static octet potential

$$\lim_{T\to\infty} \frac{i}{T} \ln \frac{\langle \square \rangle}{\langle \phi_{ab}^{\mathrm{adj}} \rangle} = \frac{1}{2N} \frac{\alpha_{\mathrm{s}}}{r} \left(1 + \#\alpha_{\mathrm{s}} + \#\alpha_{\mathrm{s}}^2 + \#\alpha_{\mathrm{s}}^3 + \#\alpha_{\mathrm{s}}^3 \ln \mu r + \dots\right)$$

Is known at three loops.

- Anzai Prausa A.Smirnov V.Smirnov Steinhauser PRD 88 (2013) 054030

Static singlet potential at N^4LO

$$
\begin{aligned}
V_s(r,\mu) \;=\; & -C_F \frac{\alpha_s(1/r)}{r} \Bigg\{ 1 + \frac{\alpha_s(1/r)}{4\pi} a_1 + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 a_2 \\
& + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 \left[\frac{16\,\pi^2}{3} C_A^3 \,\ln r\mu + a_3\right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^4 \left[a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\, C_A^3 \beta_0(-5 + 6\ln 2)\right) \ln r\mu + \dots\right] \\
& + \quad \cdots \quad \Bigg\}
\end{aligned}
$$

- Anzai Kiyo Sumino PRL 104 (2010) 112003
  A.Smirnov V.Smirnov Steinhauser PRL 104 (2010) 112002

Static energy at N^{4}LO

$$
\begin{aligned}
E_0(r) \quad = \quad & \Lambda_s - \frac{C_F \alpha_s(1/r)}{r} \Bigg\{ 1 + \frac{\alpha_s(1/r)}{4\pi} \left[a_1 + 2\gamma_E \beta_0 \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 \left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2 \right) \beta_0^2 + \gamma_E \left(4a_1\beta_0 + 2\beta_1 \right) \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \left[\frac{16\pi^2}{3} C_A^3 \ln \frac{C_A \alpha_s(1/r)}{2} + \tilde{a}_3 \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \left[a_4^{L2} \ln^2 \frac{C_A \alpha_s(1/r)}{2} + a_4^{L} \ln \frac{C_A \alpha_s(1/r)}{2} + \ldots \right] \\
& + \qquad \cdots \qquad \Bigg\}
\end{aligned}
$$

Renormalization group equations

$$
\begin{cases}
\mu \dfrac{d}{d\mu} V_s = -\dfrac{2}{3} C_F \dfrac{\alpha_{\rm s}}{\pi} r^2 V_A^2 \left[V_o - V_s\right]^3 \left(1 + \dfrac{\alpha_{\rm s}}{\pi}\, c\right) \\[2ex]
\mu \dfrac{d}{d\mu} V_o = \dfrac{1}{N} \dfrac{\alpha_{\rm s}}{\pi} r^2 V_A^2 \left[V_o - V_s\right]^3 \left(1 + \dfrac{\alpha_{\rm s}}{\pi}\, c\right) \\[2ex]
\mu \dfrac{d}{d\mu} V_A = 0 \\[2ex]
\mu \dfrac{d}{d\mu} \alpha_{\rm s} = \alpha_{\rm s} \beta(\alpha_{\rm s}); \qquad c = \dfrac{-5 n_f + C_A (6\pi^2 + 47)}{108}
\end{cases}
$$

Static singlet potential and energy at N^3LL

$$
\begin{aligned}
V_s(r,\mu) \;=\; & V_s(r,1/r) - \frac{C_F C_A^3}{6\beta_0}\frac{\alpha_s^3(1/r)}{r}\left\{\left(1+\frac{3}{4}\frac{\alpha_s(1/r)}{\pi}a_1\right)\ln\frac{\alpha_s(1/r)}{\alpha_s(\mu)}\right. \\
& \left.\left(\frac{\beta_1}{4\beta_0}-6c\right)\left[\frac{\alpha_s(\mu)}{\pi}-\frac{\alpha_s(1/r)}{\pi}\right]\right\}
\end{aligned}
$$

Summed to the ultrasoft contribution at two loops, it provides the static energy at N^3LL.

Mass renormalon

The perturbative expansion of V_s is affected by a renormalon ambiguity of order Λ.
This ambiguity does not affect the slope of the potential (and the extraction of $\alpha_{\rm s}$).

It may be eliminated from the perturbative series

- either by subtracting a (constant) series in $\alpha_{\rm s}$ to V_s and reabsorb it in a redefinition of the residual mass Λ_s,
- or by considering the force:

$$F(r, \alpha_{\rm s}(\nu)) = \frac{d}{dr} E_0(r, \alpha_{\rm s}(\nu))$$

  - The force $F(r, \alpha_{\rm s}(1/r))$ could be directly compared with lattice,
  - or integrated and compared with the static energy

$$E_0(r) = \int_{r_*}^{r} dr' \, F(r', \alpha_{\rm s}(1/r'))$$

up to an irrelevant constant fixed by the overall normalization of the lattice data.

Note that there are no $\ln \nu r$ ($\nu =$ renormalization scale).

Analysis

Lattice

We use 2+1-flavor lattice QCD obtained from tree-level improved gauge action and Highly-Improved Staggered Quark (HISQ) action by the HotQCD collaboration.
m_s was fixed to its physical value, while $m_l = m_s/20$.
This corresponds to a pion mass of about 160 MeV in the continuum limit.

β	7.373	7.596	7.825
r_1/a	5.172(34)	6.336(56)	7.690(58)
Volume	$48^3 \times 64$	64^4	64^4

The largest gauge coupling, $\beta = 7.825$, corresponds to lattice spacings of $a = 0.041$ fm.
◦ Bazavov et al PRD 90 (2014) 094503

The lattice spacing was fixed using the r_1 scale defined as $r^2 \frac{dE_0(r)}{dr}\Big|_{r=r_1} = 1.0$;
$r_1 = 0.3106 \pm 0.0017$ fm from the pion decay constant f_π.
◦ Bazavov et al PoS LATTICE 2010 (2010) 074

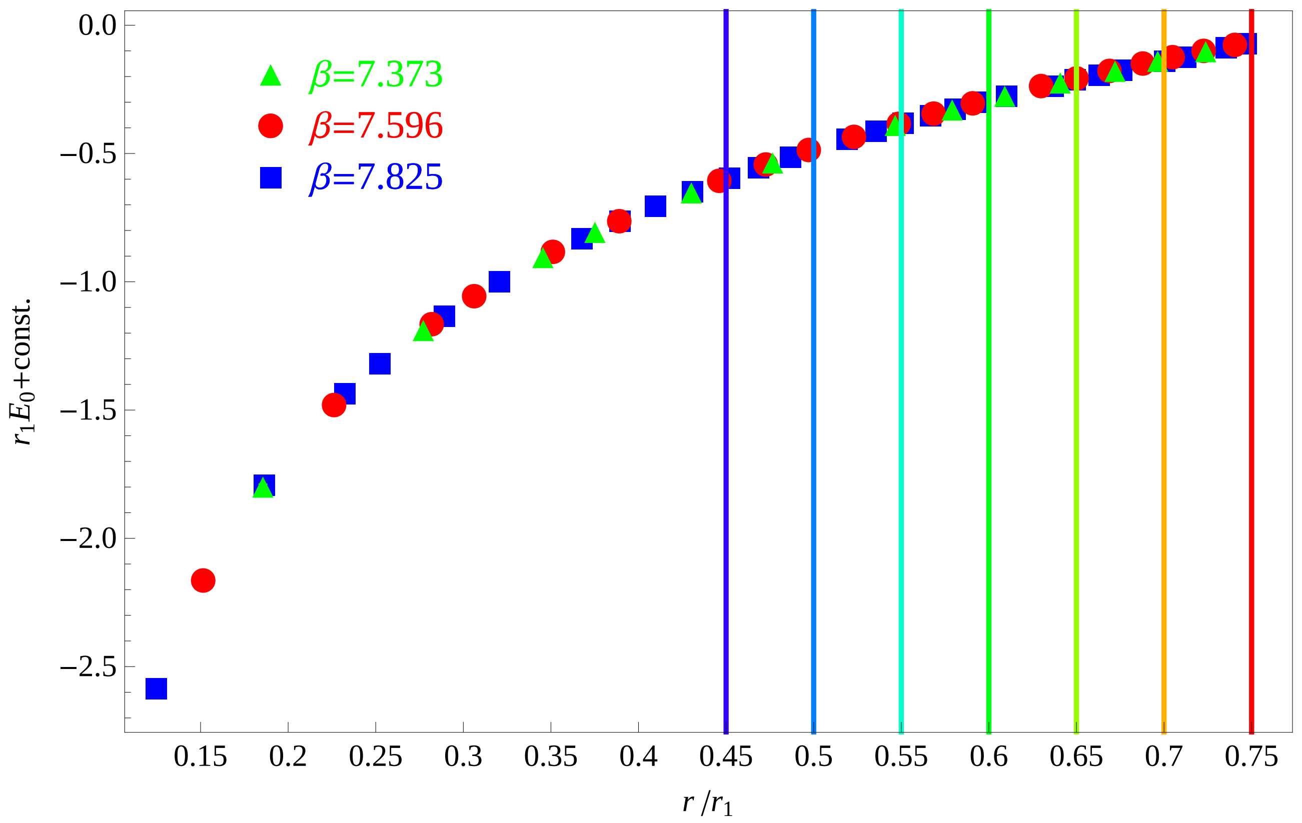
Procedure

We use data for each value of the lattice spacing separately, and at the end perform an average of the different obtained values of $\alpha_{\rm s}$ with the following procedure.

- Perform fits to the lattice data for the static energy $E_0(r)$ at different orders of perturbative accuracy. The parameter of the fits is $\Lambda_{\overline{\rm MS}}$.
- Repeat the above fits for each of the following distance ranges: $r < 0.75r_1$, $r < 0.7r_1$, $r < 0.65r_1$, $r < 0.6r_1$, $r < 0.55r_1$, $r < 0.5r_1$, and $r < 0.45r_1$.
- Use ranges where the reduced χ^2 either decreases or does not increase by more than one unit when increasing the perturbative order, or is smaller than 1.
- To estimate the perturbative uncertainty of the result, repeat the fits
  - by varying the scale in the perturbative expansion, from $\nu = 1/r$ to $\nu = \sqrt{2}/r$ and $\nu = 1/(\sqrt{2}r)$,
  - by adding/subtracting a term $\pm(C_F/r^2)\alpha_{\rm s}^{n+2}$ to the expression at n loops.

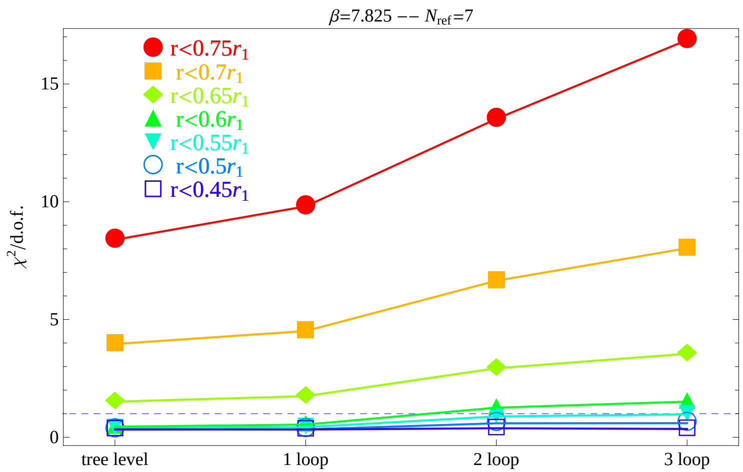
  Take the largest uncertainty.

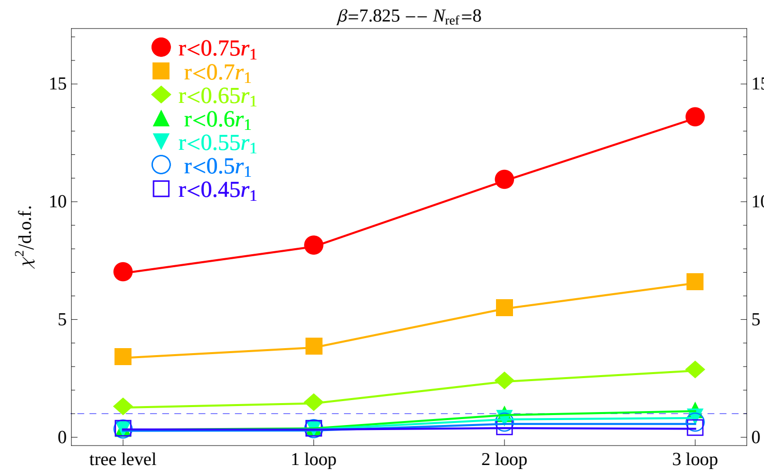
Data ranges

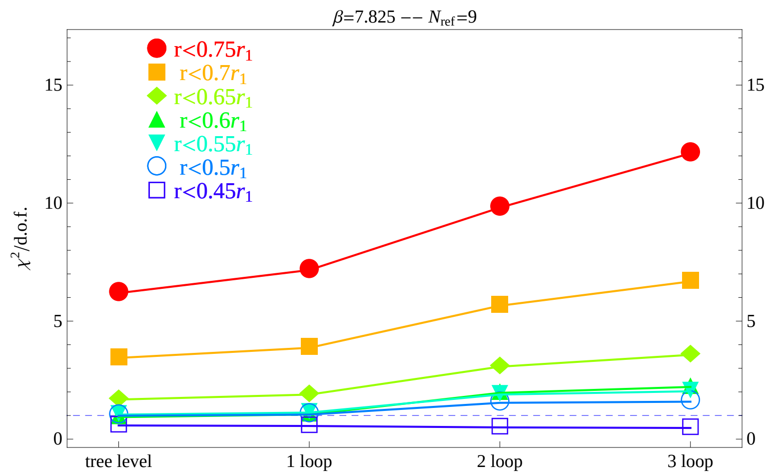


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
$r_1 E_0$+const.
β=7.373
β=7.596
β=7.825
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
r/r_1

$\chi^2/\mathrm{d.o.f.}$ for $\beta = 7.825$

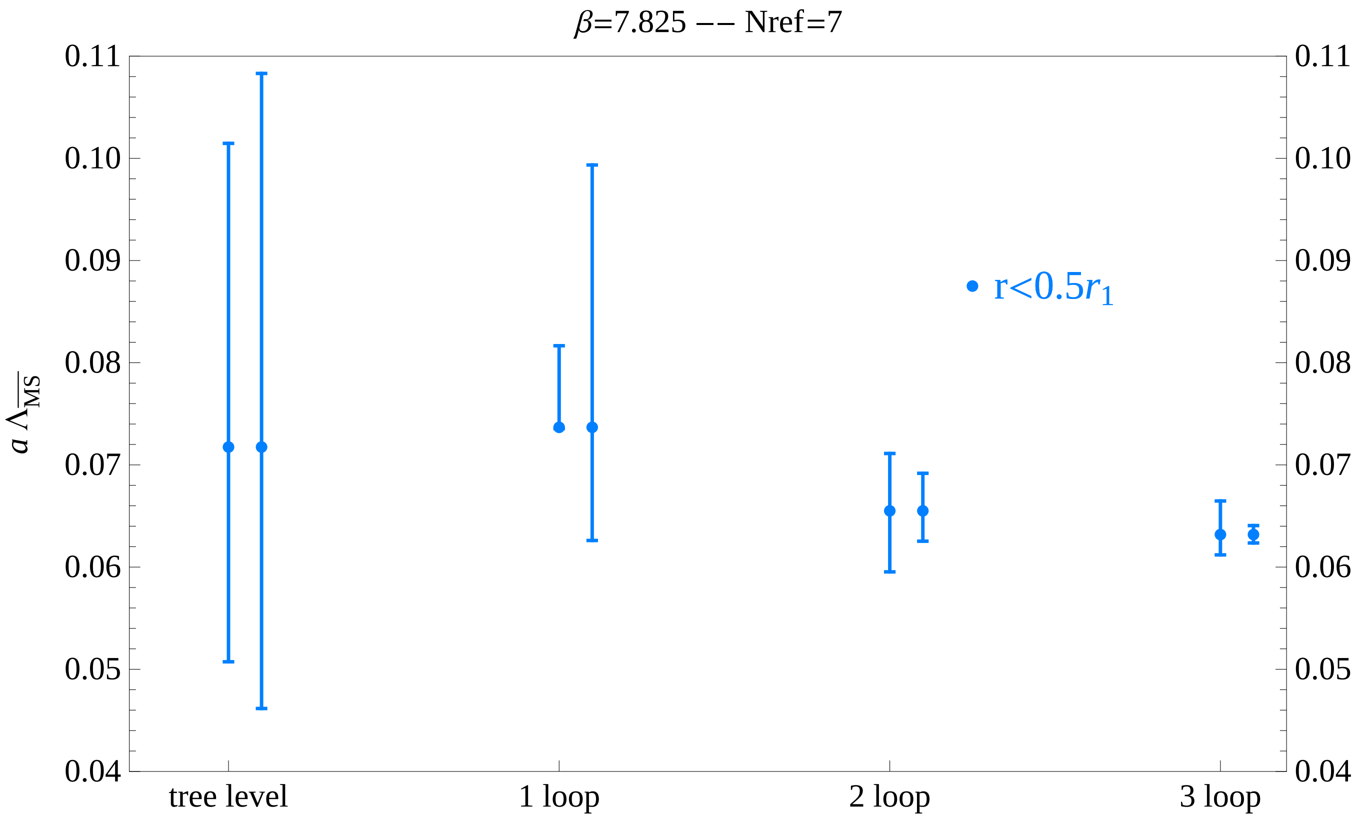




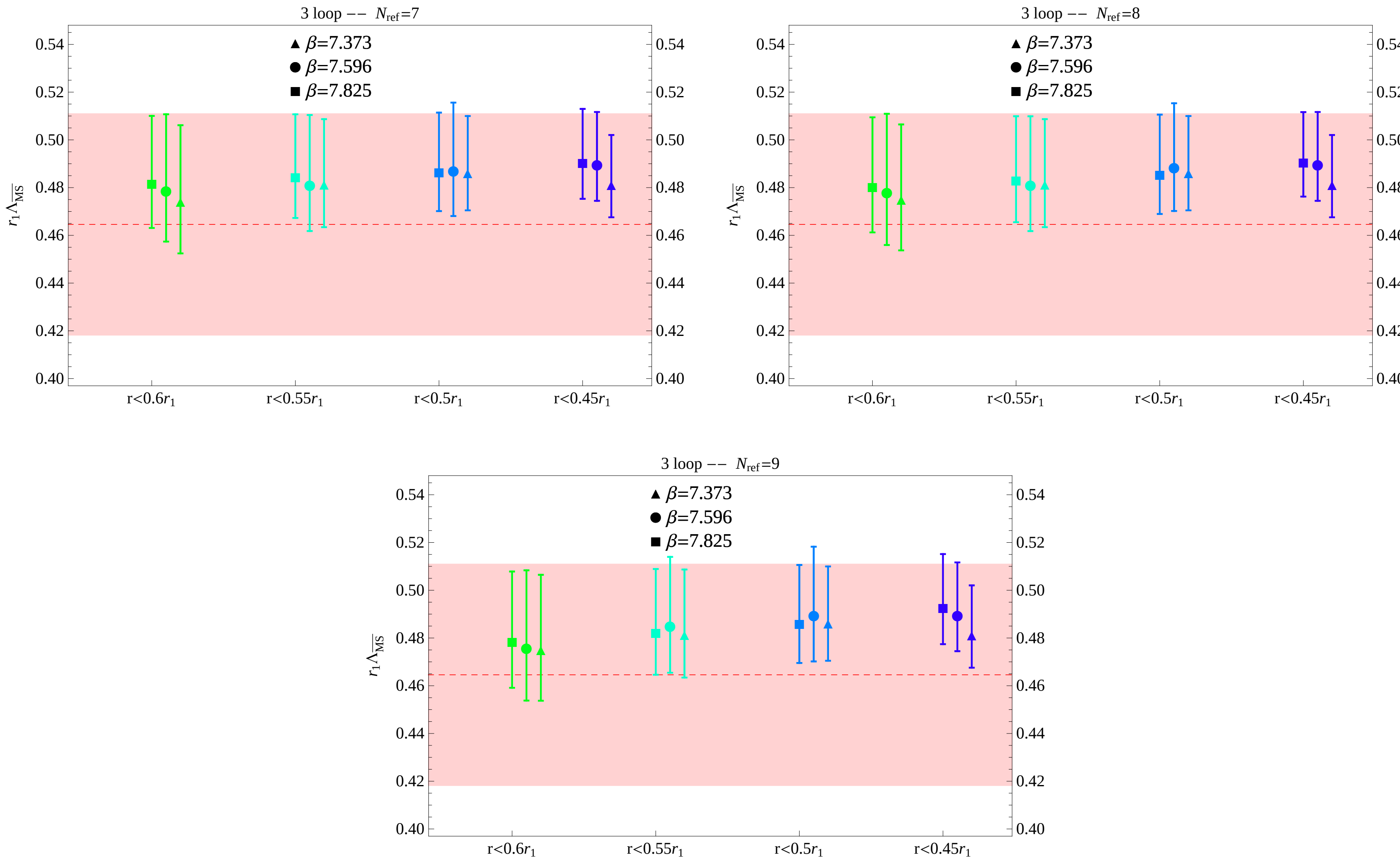


Fits for $r < 0.6r_1$ are acceptable. In the final result we will use only fits for $r < 0.5r_1$.
The fitting curve has been normalized on the 7th, 8th and 9th lattice point respectively.

$a\Lambda_{\overline{\mathrm{MS}}}$ at different orders of perturbative accuracy for $\beta = 7.825$

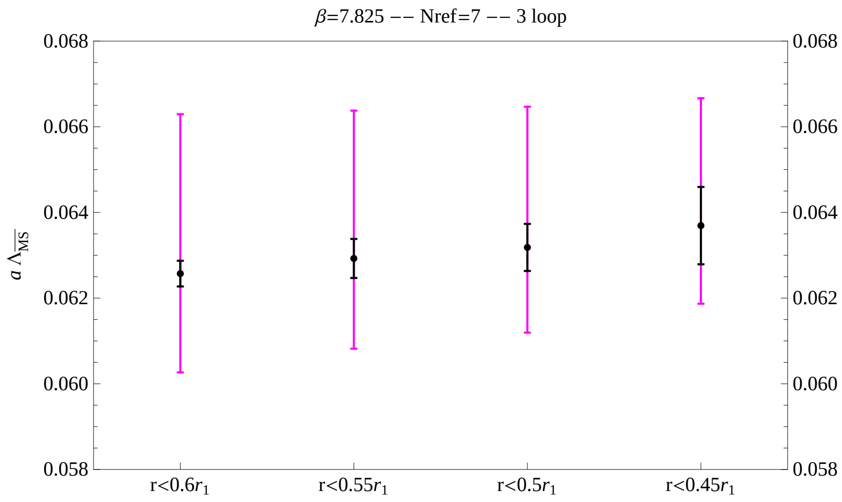


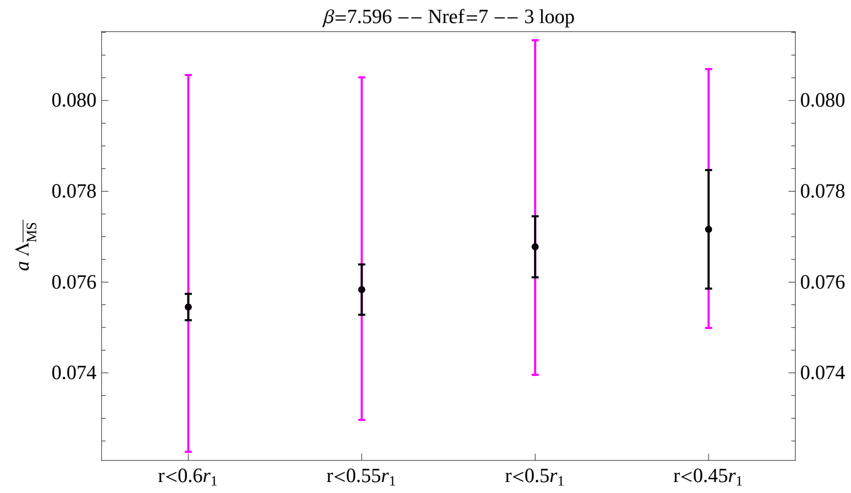
$r_1 \Lambda_{\overline{\mathrm{MS}}}$ at three-loop accuracy

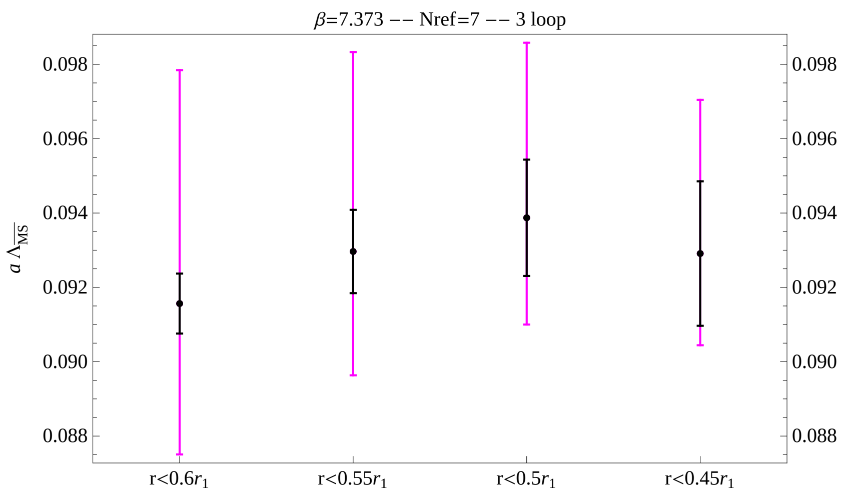


The band shows the determination of 2012.

Statistical error vs perturbative error

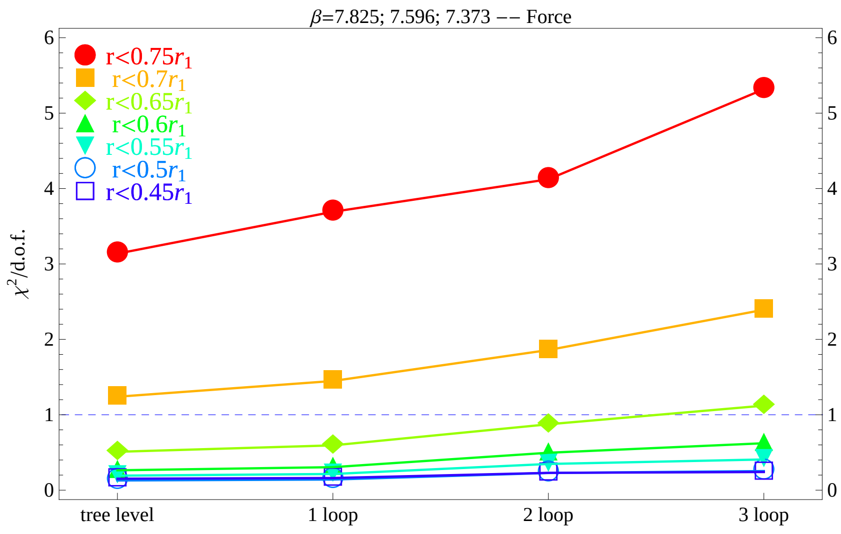


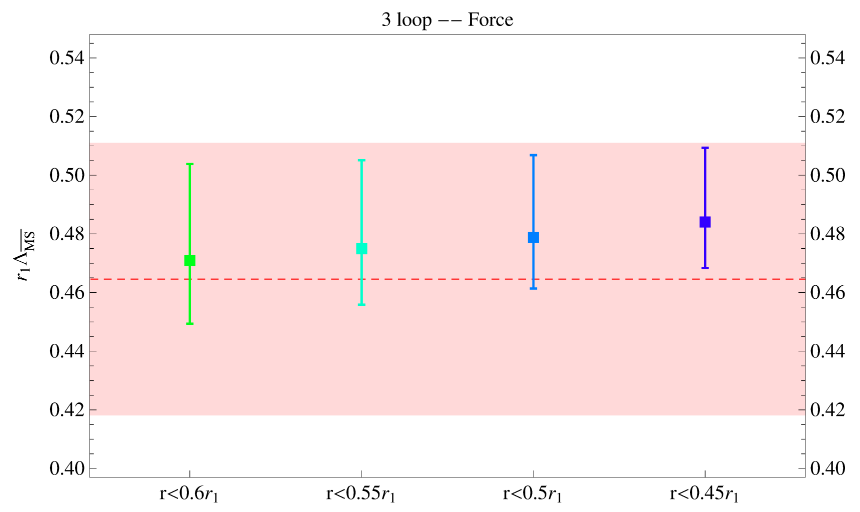




The statistical error is estimated by taking values of $\Lambda_{\overline{MS}}$ at one χ^2 unit above minimum.

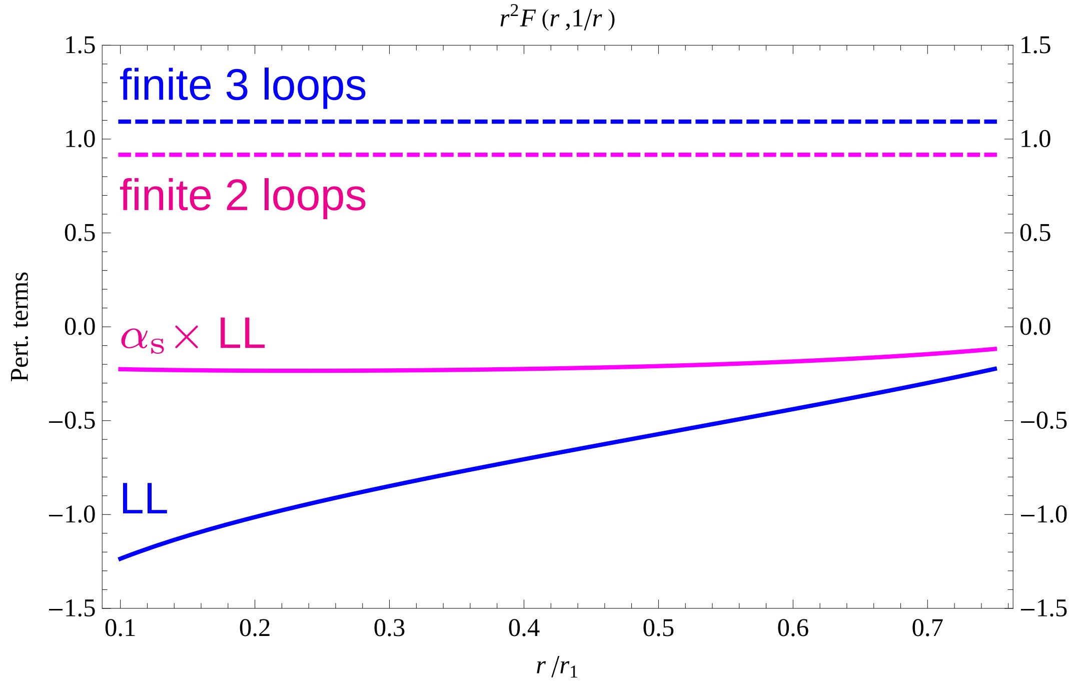
Analysis with the force





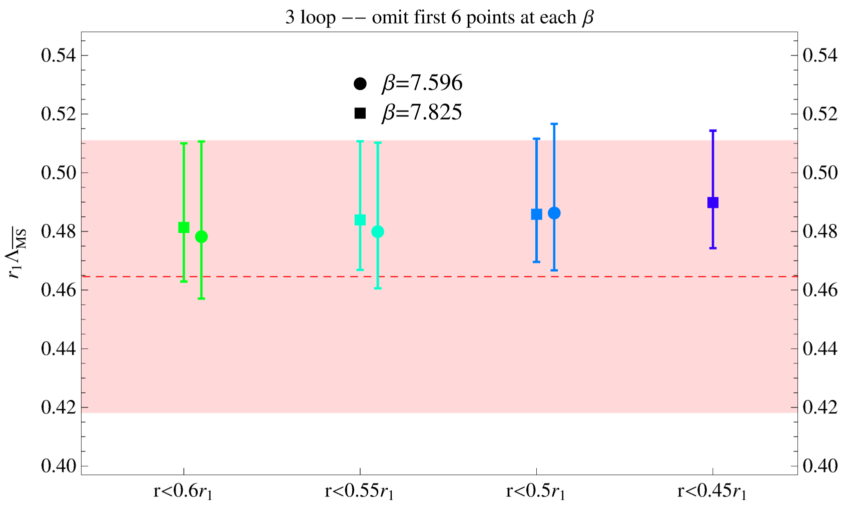
The band shows the determination of 2012.

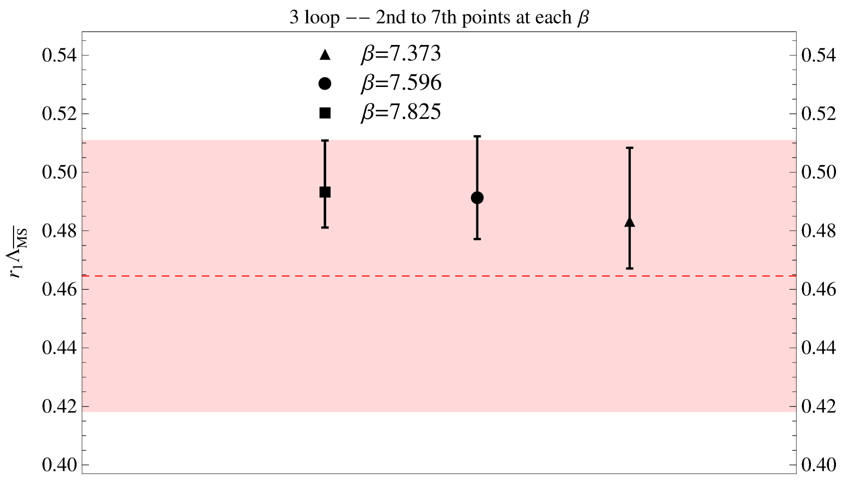
The counting of the ultrasoft contributions

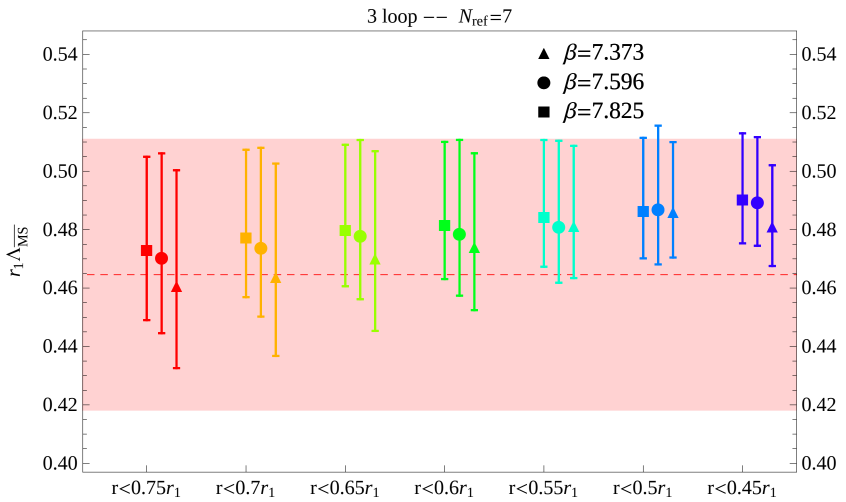


Leading-ultrasoft resummation included along with the three-loop terms is consistent with the observed size of the terms. This goes in our final result.
We chose $\mu = 1.26 r_1^{-1} \sim 0.8$ GeV, for the ultrasoft factorization scale.
Variations of μ only produce small effects on the results.

Short-distance points vs long-distance points

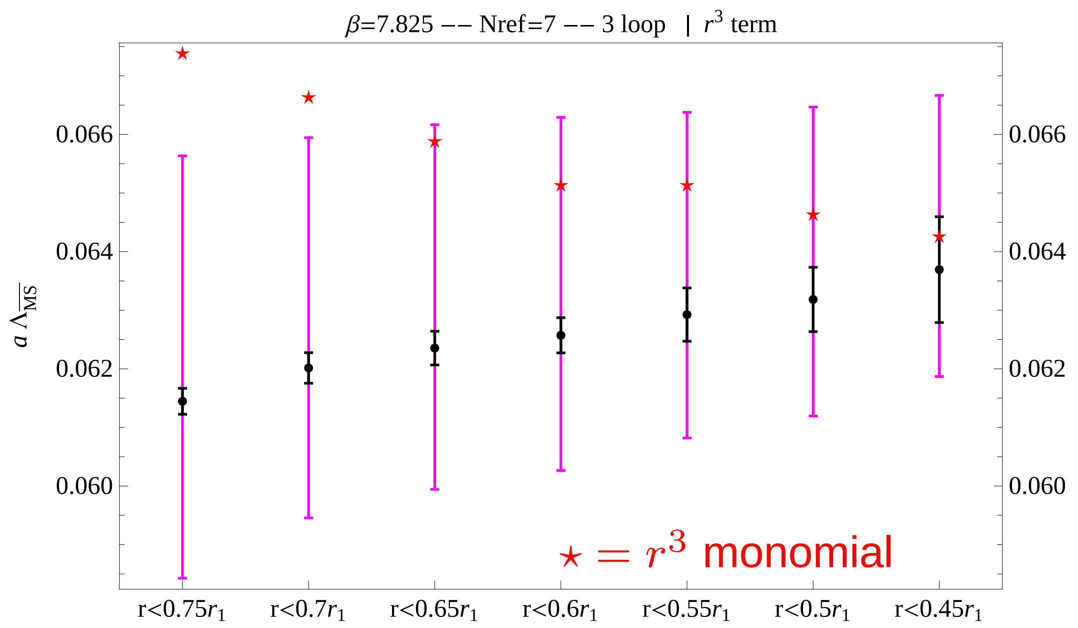






The band shows the determination of 2012.

Looking for condensates



By repeating the fits adding a monomial term proportional to r^3 and r^2, which could be associated with gluon and quark local condensates, and also a term proportional to r, we do not find evidence for a significant non-perturbative term at short distances and the value of $\Lambda_{\overline{\rm MS}}$ remains unchanged.

Results

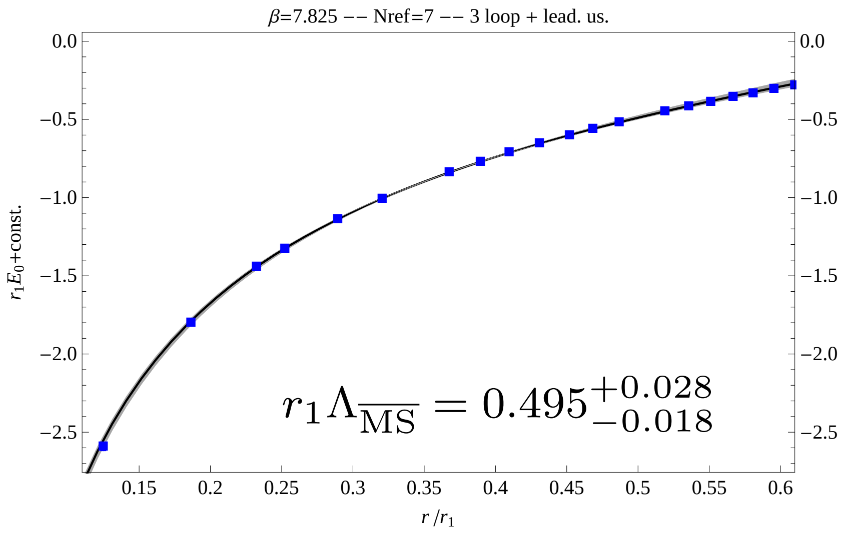
$\Lambda_{\overline{\rm MS}}$

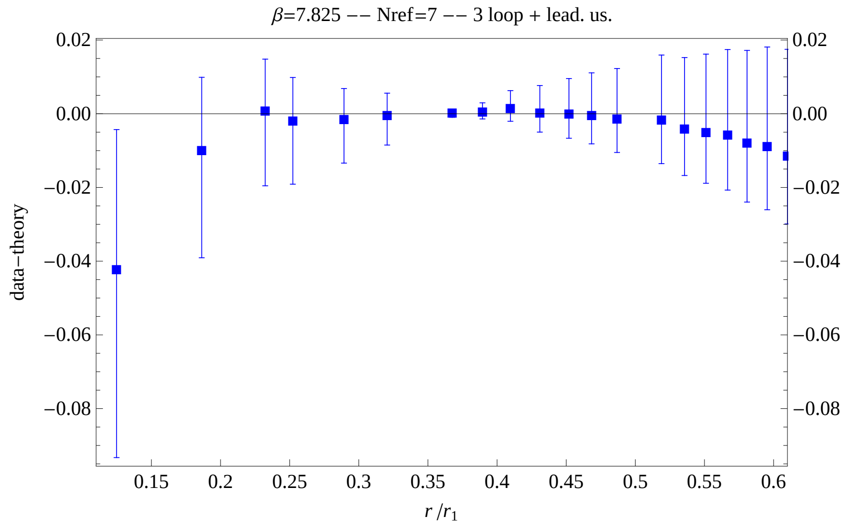
Results at three-loop plus leading-ultrasoft resummation for the $r < 0.5 r_1$ fit range. The final result is the weighted average of different βs with linearly added errors.

	$a\Lambda_{\overline{\rm MS}}$; $N_{\rm ref} = 7$	$a\Lambda_{\overline{\rm MS}}$; $N_{\rm ref} = 8$	$a\Lambda_{\overline{\rm MS}}$; $N_{\rm ref} = 9$	$a\Lambda_{\overline{\rm MS}}$; range spanned	$r_1\Lambda_{\overline{\rm MS}}$; range spanned
$\beta = 7.373$	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.4949^{+0.0240}_{-0.0144} \pm 0.0086 \pm 0.0025$ $= 0.4949^{+0.0256}_{-0.0170}$
$\beta = 7.596$	$0.0781^{+0.0046}_{-0.0029}$ ± 0.0007	$0.0784^{+0.0043}_{-0.0027}$ ± 0.0010	$0.0785^{+0.0046}_{-0.0029}$ ± 0.0007	$0.0783^{+0.0048}_{-0.0031}$ ± 0.0010	$0.4961^{+0.0303}_{-0.0197}{}^{+0.0066}_{-0.0061} \pm 0.0044$ $= 0.4961^{+0.0313}_{-0.0211}$
$\beta = 7.825$	$0.0644^{+0.0032}_{-0.0019}$ ± 0.0006	$0.0642^{+0.0033}_{-0.0020}$ ± 0.0008	$0.0643^{+0.0032}_{-0.0020}$ ± 0.0008	$0.0643^{+0.0033}_{-0.0021}$ ± 0.0008	$0.4944^{+0.0256}_{-0.0159} \pm 0.0065 \pm 0.0037$ $= 0.4944^{+0.0267}_{-0.0175}$
Average					$\mathbf{r_1\Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018}}$

$r_1\Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018}$ which converts to $\Lambda_{\overline{\rm MS}} = 315^{+18}_{-12}$ MeV

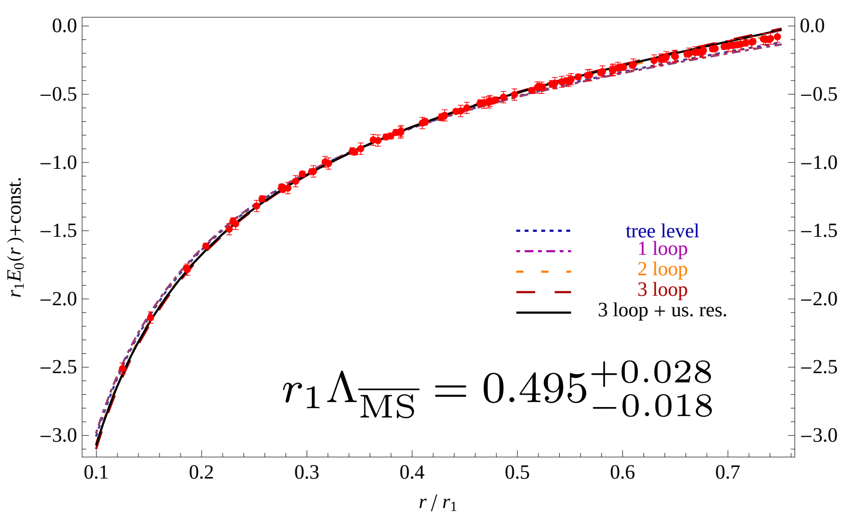
Static energy vs lattice data

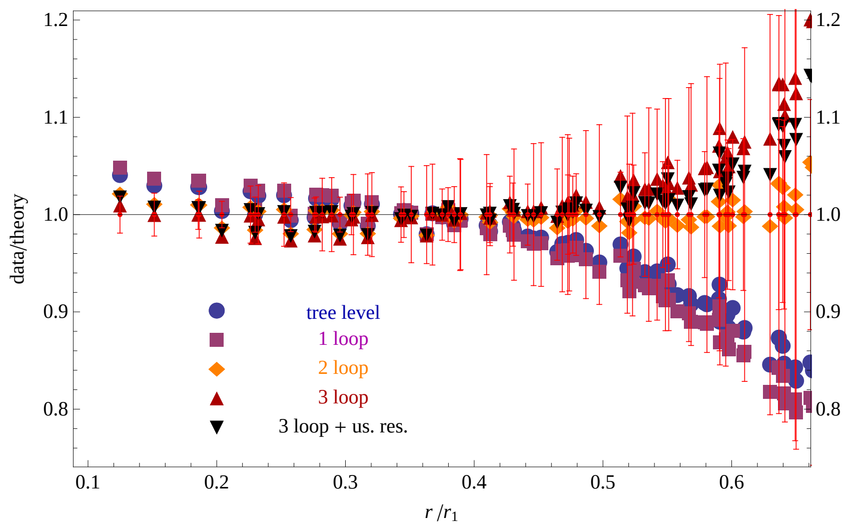




Perturbation theory agrees with lattice data up to about 0.2 fm.

Static energy at different perturbative orders vs lattice data

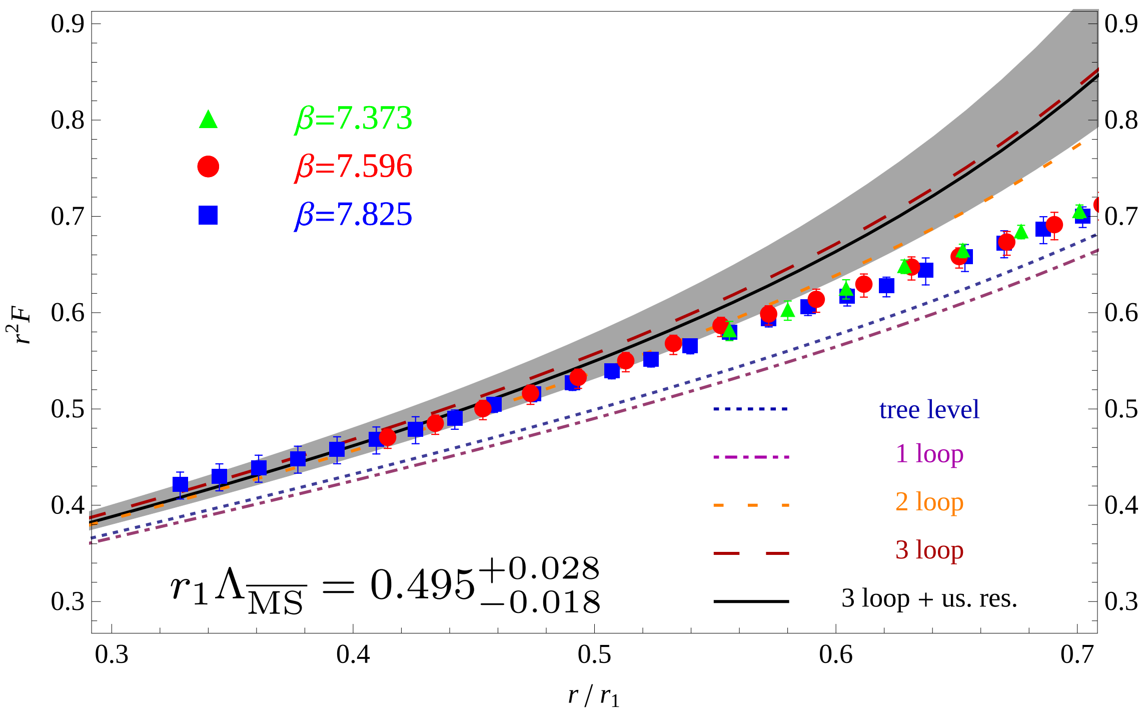




Lattice data with β from 6.664 to 7.825 are displayed.

The red error bars correspond to the errors of the lattice data (include normalization).

Force vs lattice data



α_s

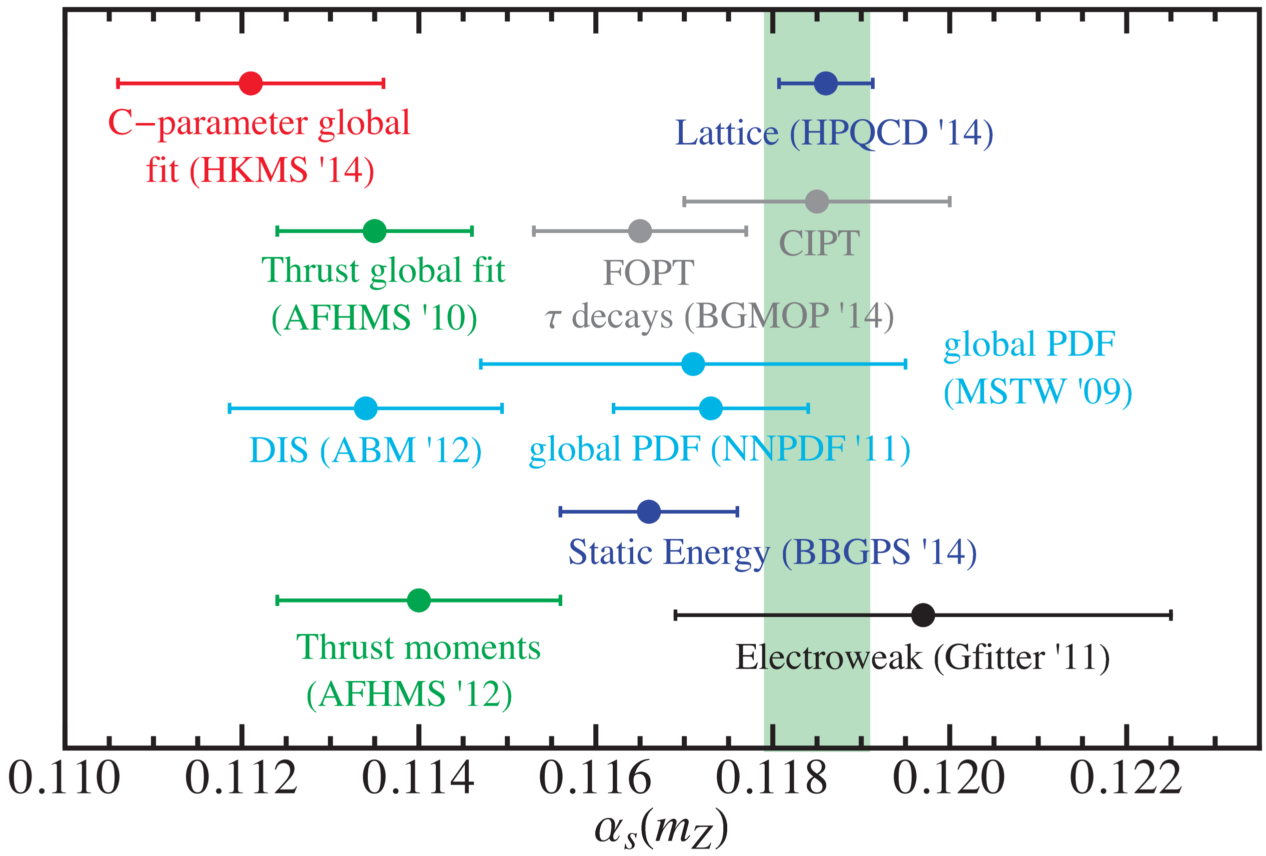
$$\alpha_s(1.5\text{ GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$

which corresponds to

$$\alpha_s(M_Z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$

from four-loop running, $m_c = 1.6$ GeV and $m_b = 4.7$ GeV.

Comparison with other determinations



Outlook

- Not all of the presently available perturbative information has been used. More precise lattice data on finer lattices and with more data points at short distances could take advantage of it.
- It would be important, in order to reduce possible systematic effects, to perform the same study on Wilson loops computed on different lattices with different actions.
- A possible systematic effect is due to the finite lattice spacing. A continuum extrapolation would reduce this effect and allow for a precise determination of the force between static charges along the same lines developed by Necco and Sommer (2001) for the quenched case.
- Compute the force directly from the lattice:

$$F(r) = -\lim_{T\to\infty} \frac{\left\langle \mathrm{Tr}\,\mathrm{P}\,\hat{\mathbf{r}}\cdot g\mathbf{E}(t,\mathbf{r})\exp\left\{ig\oint_{r\times T} dz^\mu\, A_\mu\right\}\right\rangle}{\left\langle \mathrm{Tr}\,\mathrm{P}\exp\left\{ig\oint_{r\times T} dz^\mu\, A_\mu\right\}\right\rangle}$$