MC top mass calibration with Event Shapes

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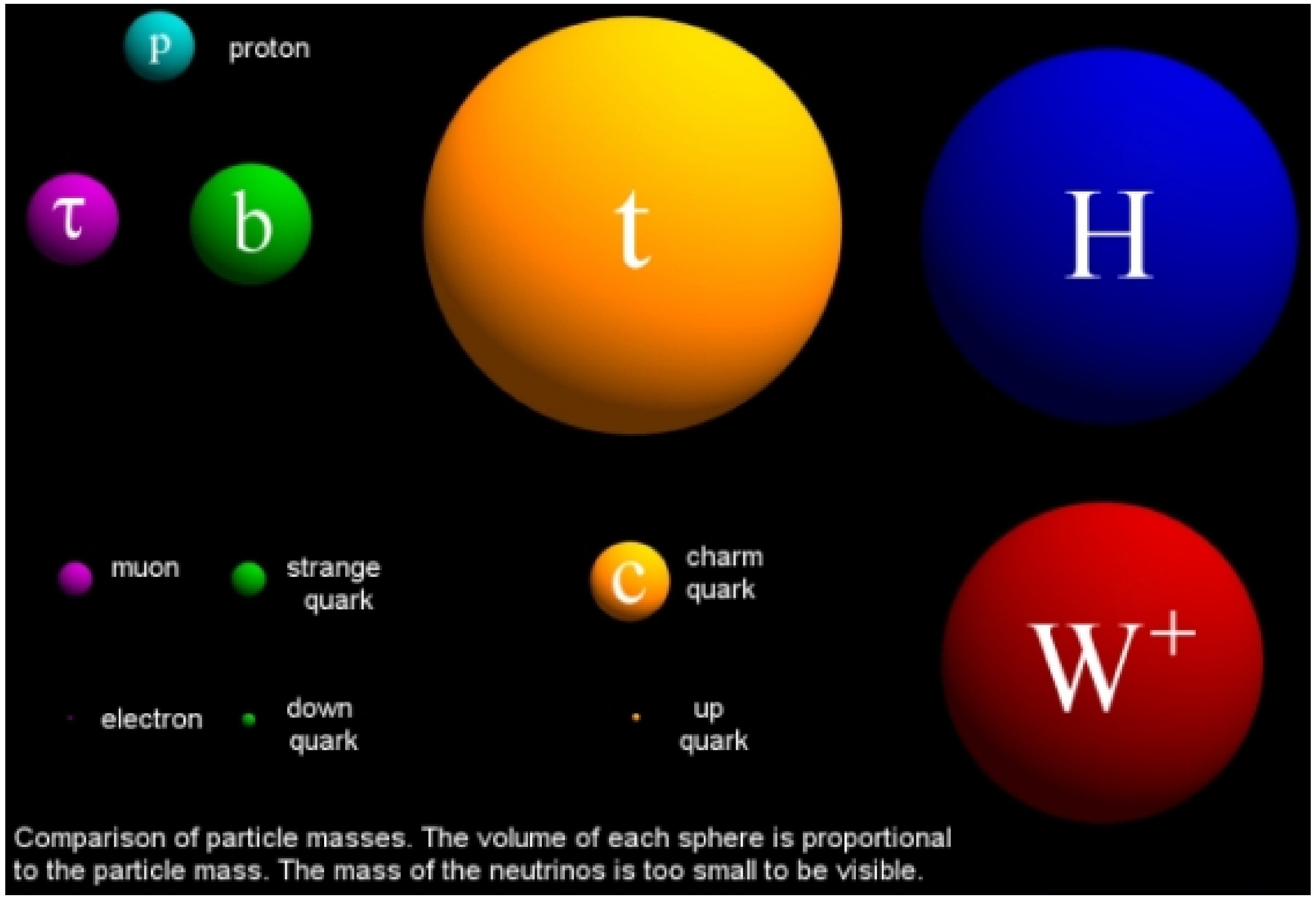
MITP, Mainz – 07-03-2016

Outline

- Motivation: why the top mass is so important.
- The top mass: determination and schemes
- Theoretical setup
- Pythia fits for the top mass
- Conclusions and Outlook

Motivation

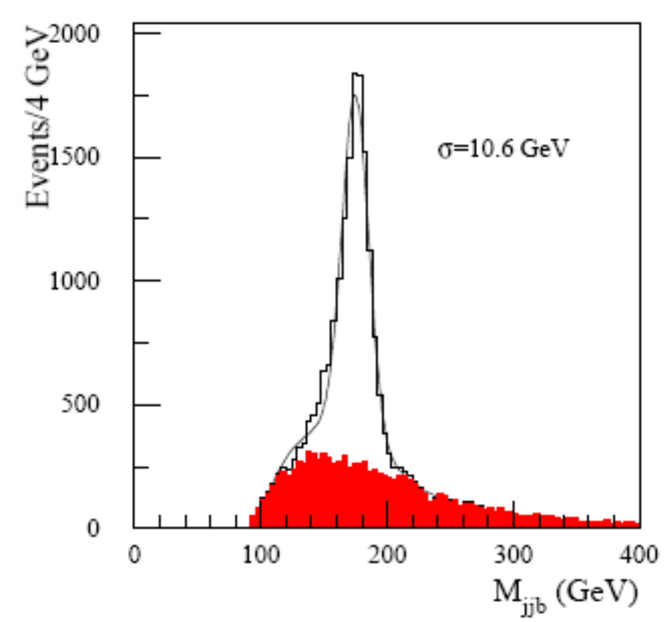
Top quark: more than just "very heavy"



- Heaviest known particle
- Only quark that does not hadronize
- Top mass crucial to EW precision tests
- Top production major background for new-physics searches
- It can form a resonance, almost like "real particles"

Hadronization time $\tau_{\text{had}} \sim 7 \times 10^{-24}\,\text{s}$

Top mean lifetime $\tau_t \sim 5 \times 10^{-25}\,\text{s}$



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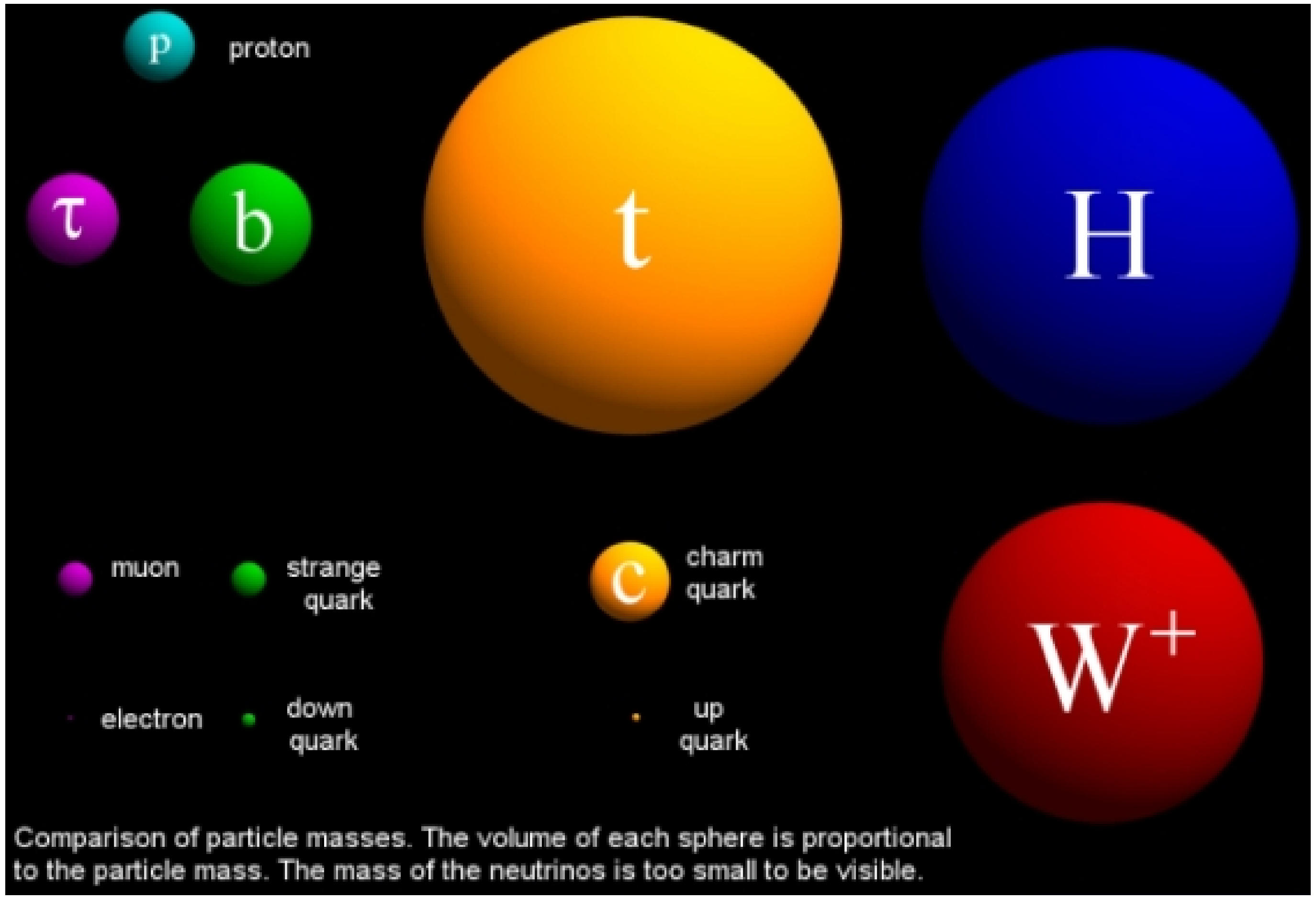

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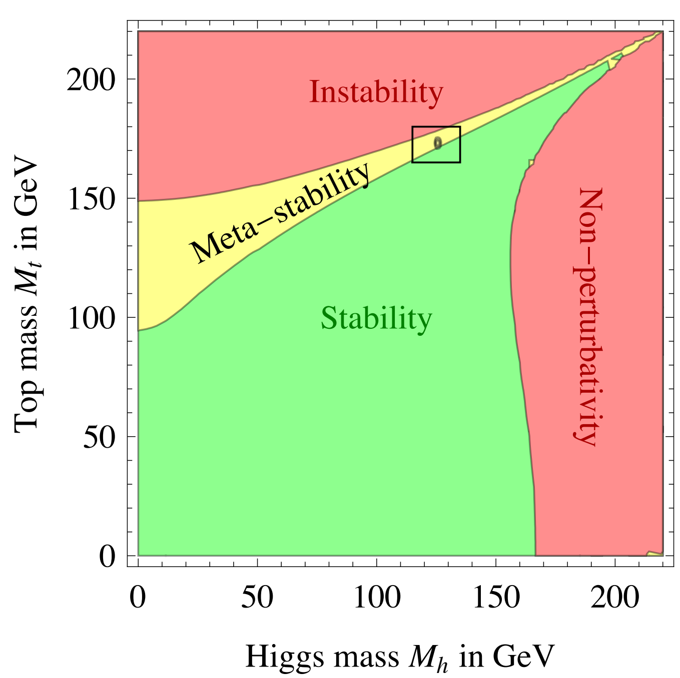
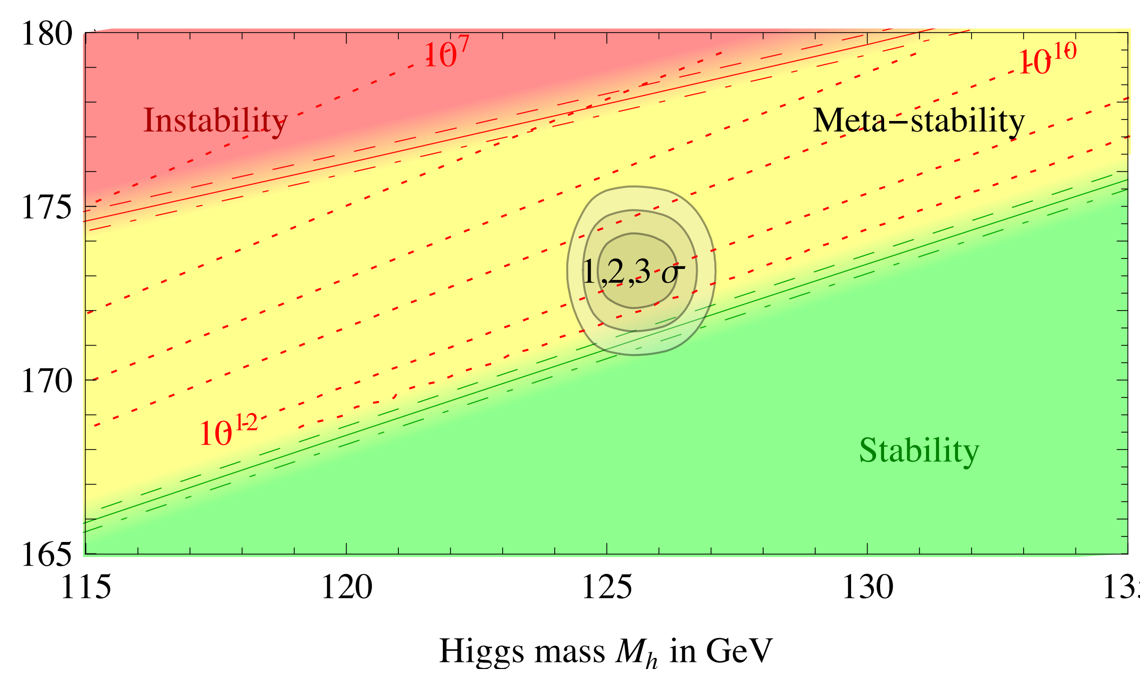
- Top mass crucial to EW precision tests
- Top production major background for new-physics searches
- It can form a resonance, almost like "real particles"
- Precise value of top mass crucial to study the stability of the SM vacuum





Top quark mass: measurements and schemes

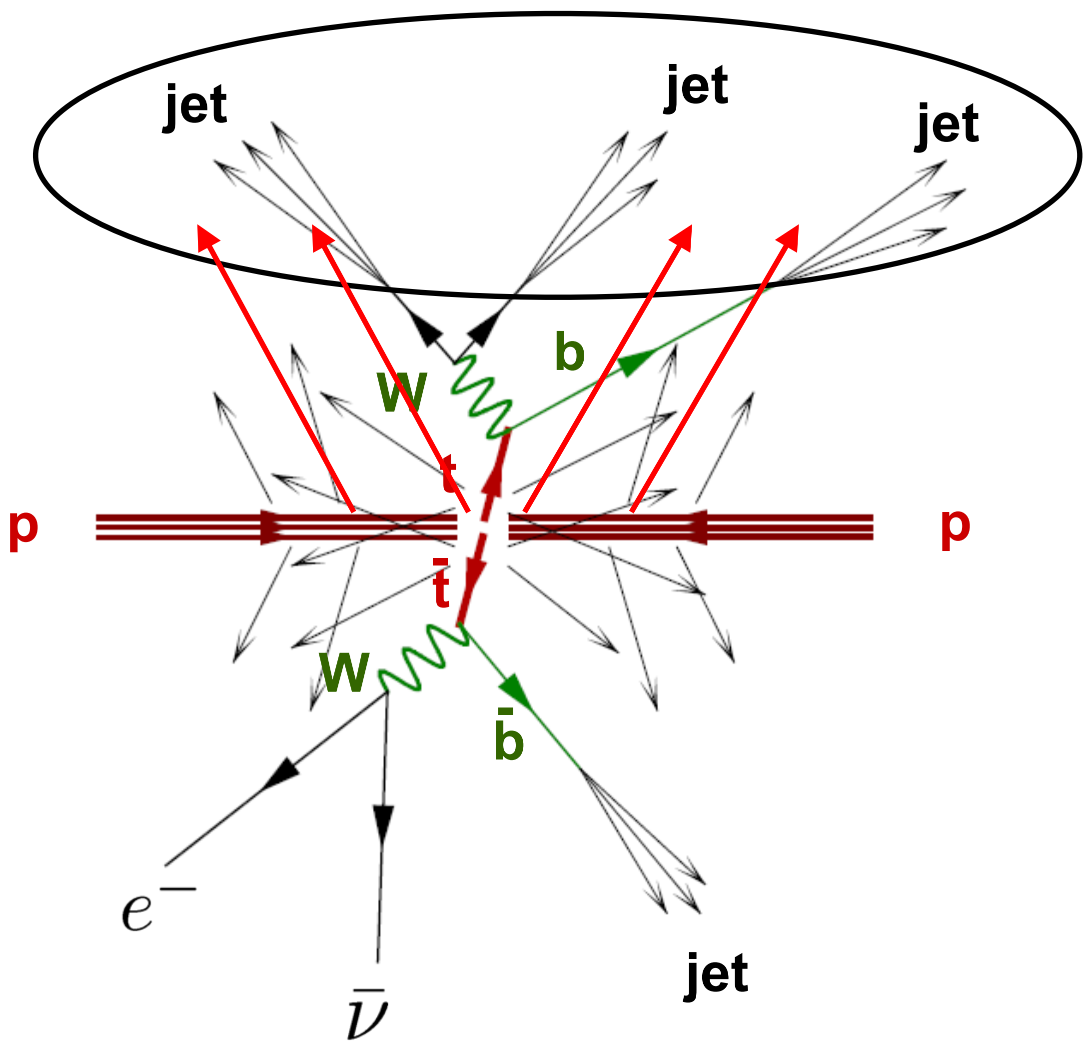
Top quark mass reconstruction

LCH case (Tevatron is similar)

basic idea: identify ALL top decay products

$$\Big(\sum_{i\sim\text{decay}} p_i^\mu\Big)^2 = m_t^2 + \Delta m_t^2$$

pole mass + "contamination"

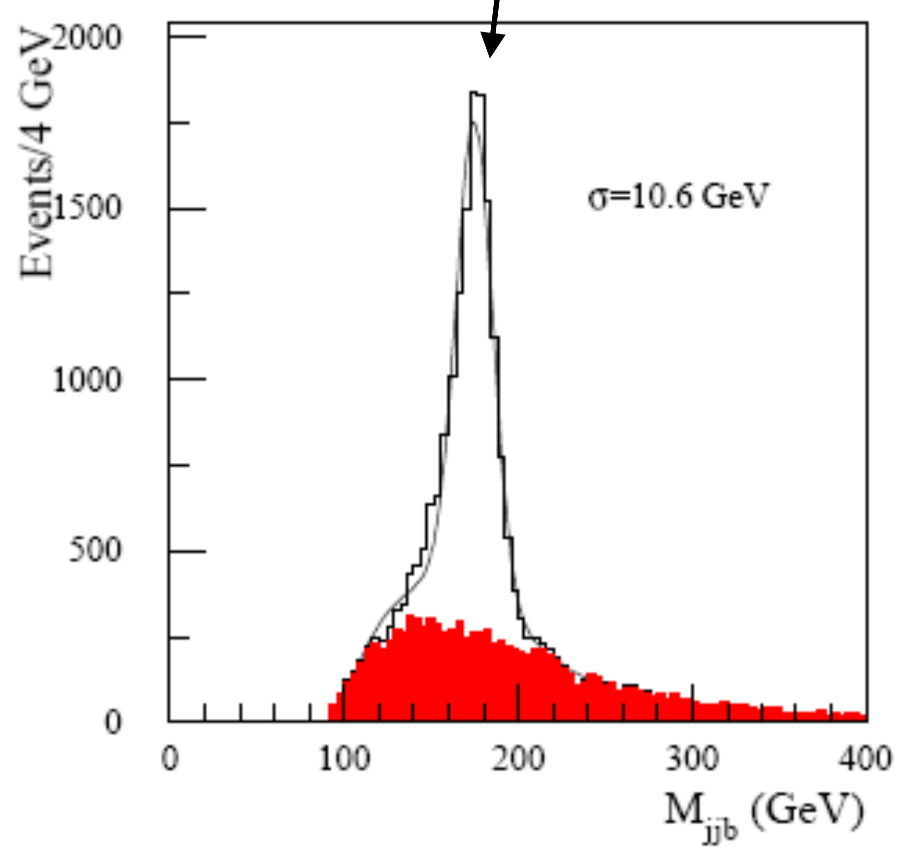


invariant mass distribution

$$m_t + \Delta m_t^{\text{hadronic}}$$

Jet algorithm
hadronization
Soft physics
Underlying event

peak position



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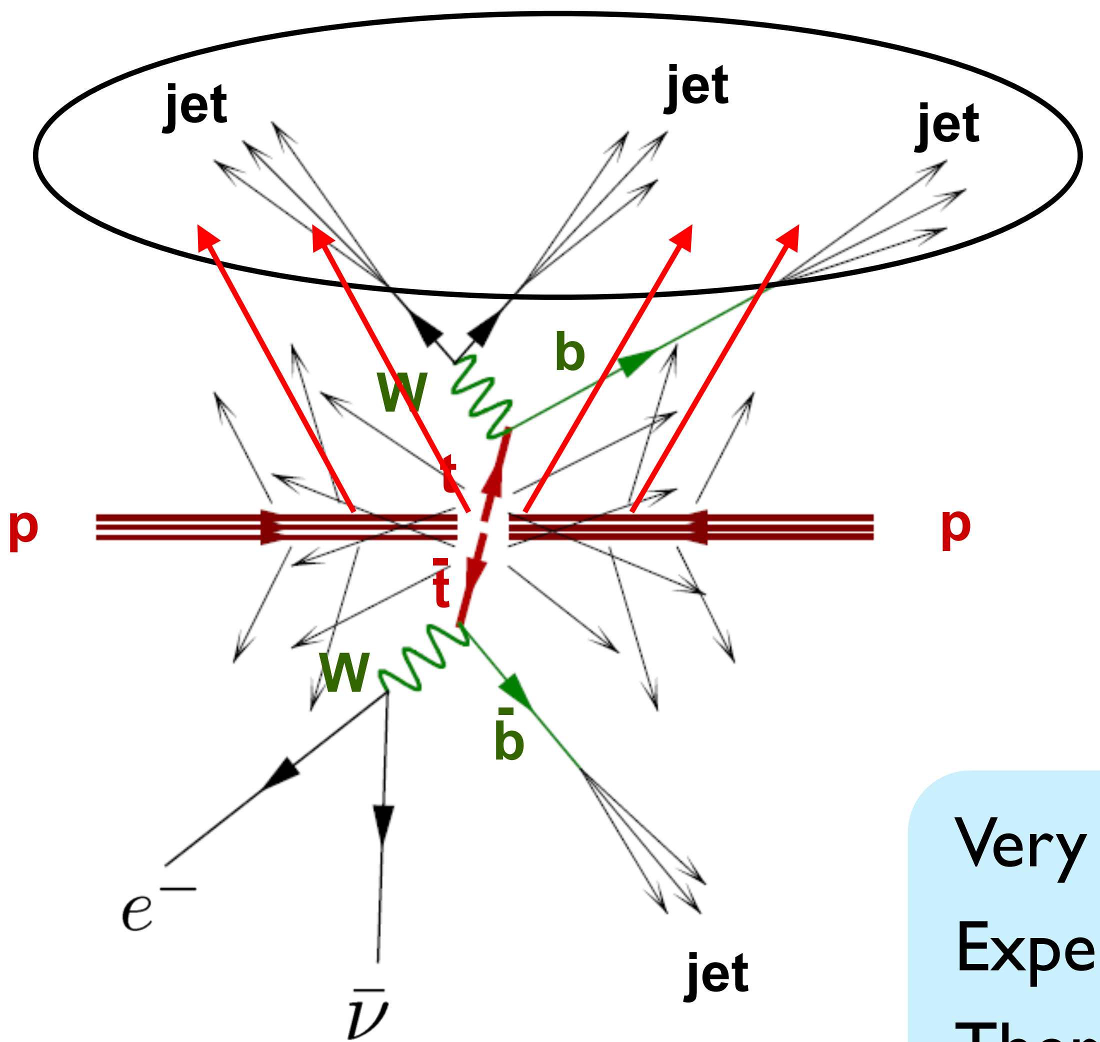

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pole mass + "contamination"

invariant mass distribution

$$m_t + \boxed{\Delta m_t^{\text{hadronic}}}$$

Very hard to compute these effects from first principles

Experimentalist use Parton shower MC to estimate them

Therefore the parameter determined is in fact m_t^{MC}

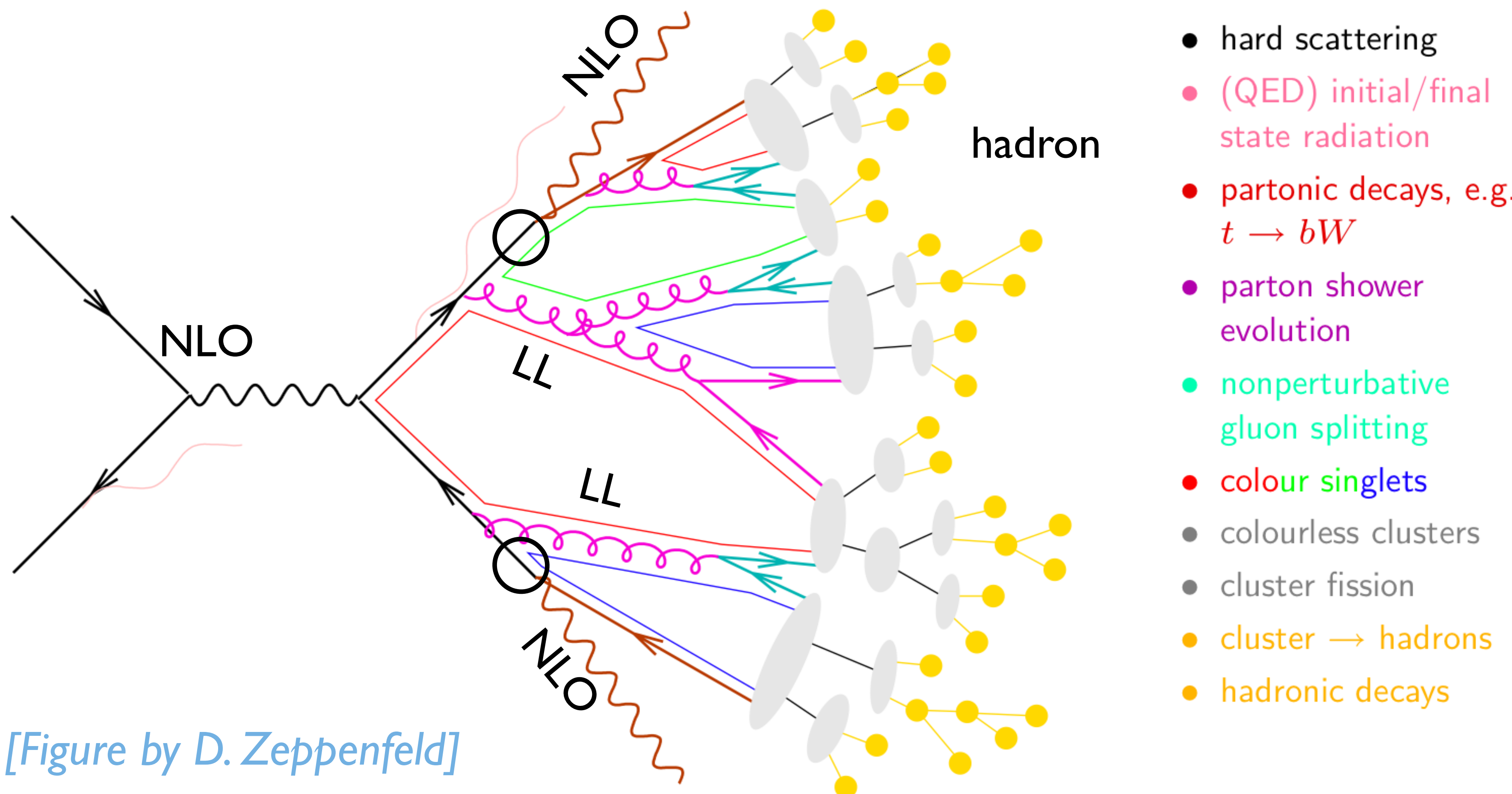
Overview of parton shower MC

Pythia

Sherpa

Herwig

Geneva



[Figure by D. Zeppenfeld]

Hard matrix element: annihilates initial particles into tops + other hard partons

Parton shower: QCD resummation at LL of large Sudakov logs

~ partial NLO matrix elements

Top mass: mass of top propagator prior to top decay… scheme?

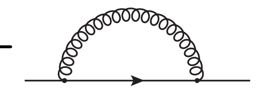
Quark masses and schemes in QFT

$\Sigma(p, m_0)$

independent

has divergences

$+ \quad \cdots \quad = \quad \dfrac{1}{\not{p} - m_0 - \text{(diagram)}}$



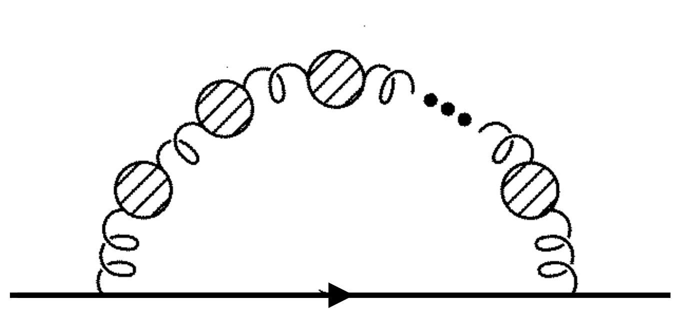
m_0 = bare mass

quark mass defined in context of perturbation theory

Pole scheme: propagator has a pole for $\not{p} \to m_p$

$m_p = m_0 + \Sigma(m_p, m_0)$ pole mass is μ - independent

The whole diagram is absorbed into the mass definition !!!!



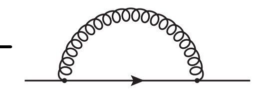
$$\Sigma(m, m) \sim \sum_n \alpha_s^{n+1} (2\beta_0)^n n!$$

Linear sensitivity to infrared momenta leads to factorially growing coefficients in perturbation theory

asymptotic behavior, but impacts lower orders

Similar behavior in other diagrams for a given observable

Quark masses and schemes in QFT

 $+ \quad \cdots \quad = \dfrac{1}{\not p - m_0 - }$ 

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$\overline{\text{MS}}$ scheme: propagator is finite, subtract only $\dfrac{1}{\epsilon}$ in dimensional regularization

$\overline{m}(\mu) = m_0 + \Sigma(m_p, m_0)|_{\frac{1}{\epsilon}}$ MS mass is μ - dependent

no renormalon problem

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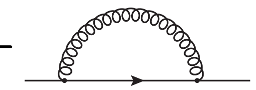
$+ \quad \cdots \quad = \dfrac{1}{\not{p} - m_0 -}$ 

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$m_p - \overline{m}(\mu) = \Sigma(m_p, m_0)|_{\text{finite}} \equiv \delta\overline{m}(\mu)$

μ - dependent

Relation to the pole mass is used to define any other short-distance scheme

Quark masses and schemes in QFT

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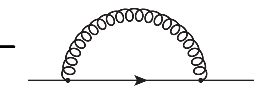
$+ \quad \cdots \quad = \dfrac{1}{\not{p} - m_0 - \text{}}$ 

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$$m_p - m^{\text{MSR}}(R) = \Sigma(R, Z_m^{\overline{\text{MS}}} R)|_{\text{finite}} \equiv \delta m_{\text{MSR}}(R)$$

R - dependent

more on the MSR mass later

no renormalon problem

MSR mass

[Hoang, Jain, Scimemi, Stewart, '08]

$$\delta m_{\overline{\mathrm{MS}}}(\mu) = \overline{m}(\mu) \sum_{n=1} \left[\frac{\alpha_s(\mu)}{4\pi}\right]^n \sum_{k=0}^{n} a_{nk} \log^k\left(\frac{\mu}{\overline{m}(\mu)}\right) \sim \mu \sum_n \alpha_s^{n+1} (2\beta_0)^n\, n!$$

asymptotic behavior
mass-independent
only μ - dependent !!

$$\boxed{\delta m_{\mathrm{MSR}}(R) = R \sum_{n=1} \left[\frac{\alpha_s(R)}{4\pi}\right]^n a_{n0}}$$

Absorbs into mass parameter UV fluctuations from scales > R

MSR mass

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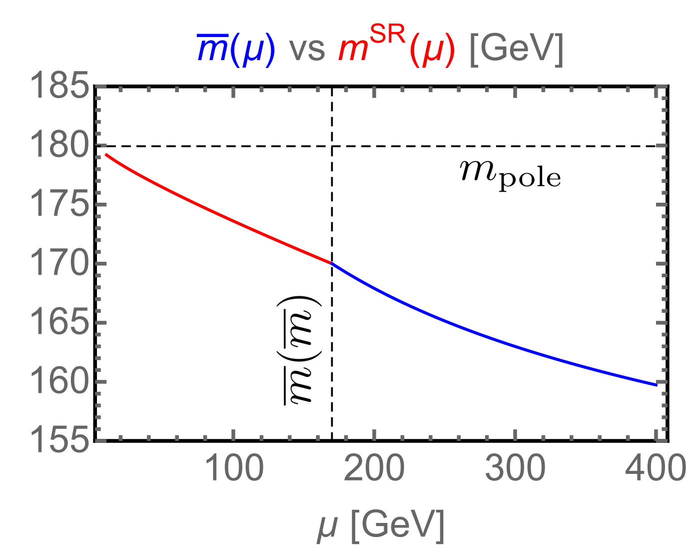
Absorbs into mass parameter UV fluctuations from scales > R

$$m^{\rm MSR}(R=0) = m_{\rm pole}$$

$$m^{\rm MSR}[R=\overline{m}(\overline{m})] = \overline{m}(\overline{m})$$

$m^{\rm MSR}[R \sim \Lambda_{\rm QCD}]$ similar to pole mass or kinetic mass but without renormalon problem!

MSR interpolates between pole and $\overline{\rm MS}$



MC mass

based on [Hoang '14]
[Hoang, Stewart '08]

$m_t^{\rm MC} = (173.34 \pm 0.27_{\rm stat} \pm 0.71_{\rm syst})\,{\rm GeV}$ LHC-Tevatron combination

Let us assume that, to some extent, MC perform ab initio QCD computations

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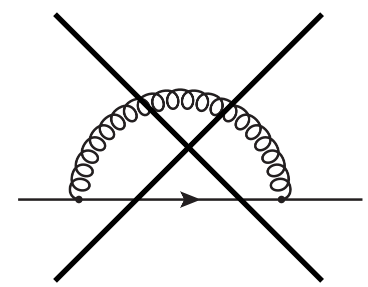
Important fact: MC's do not include quark self-energy corrections

Therefore one can consider these are absorbed into the mass parameter…



… but only for energy scales above the shower cutoff $\Lambda_{\mathrm{shower}} \sim \Lambda_{\mathrm{QCD}}$

$$m_t^{\mathrm{MC}} = m^{\mathrm{MSR}}(R = 1\,\mathrm{GeV})$$

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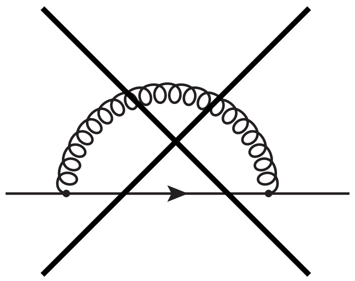

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Parton shower and hadronization model modify the shape of the distribution and further modify the peak location.

$$m_t^{\mathrm{MC}} = m^{\mathrm{MSR}}(R = 1\,\mathrm{GeV}) + \Delta_{t,\mathrm{MC}}(R = 1\,\mathrm{GeV}) \sim 1\,\mathrm{GeV}$$

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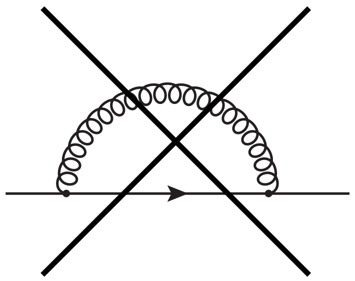

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The MC mass parameter should be process-independent (only MC dependent)

Calibrating the MC mass

[Buttenschön, Dehnadi, Hoang, Preisser, Stewart]

Strategy: "measure" the MC mass using a completely independent hadron level QCD prediction of a strongly mass-dependent observable.

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We will use SCET as our first principle QCD calculator:

Full mass scheme control: $\overline{\text{MS}}$ or MSR
Full hadronization control: shape function
Full resummation perturbation theory + control: N^{2}LL + NLO

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We will use Pythia as our parton-shower MC… to start with

Theoretical setup

Thrust

$$e^+ e^- \to \text{jets}$$

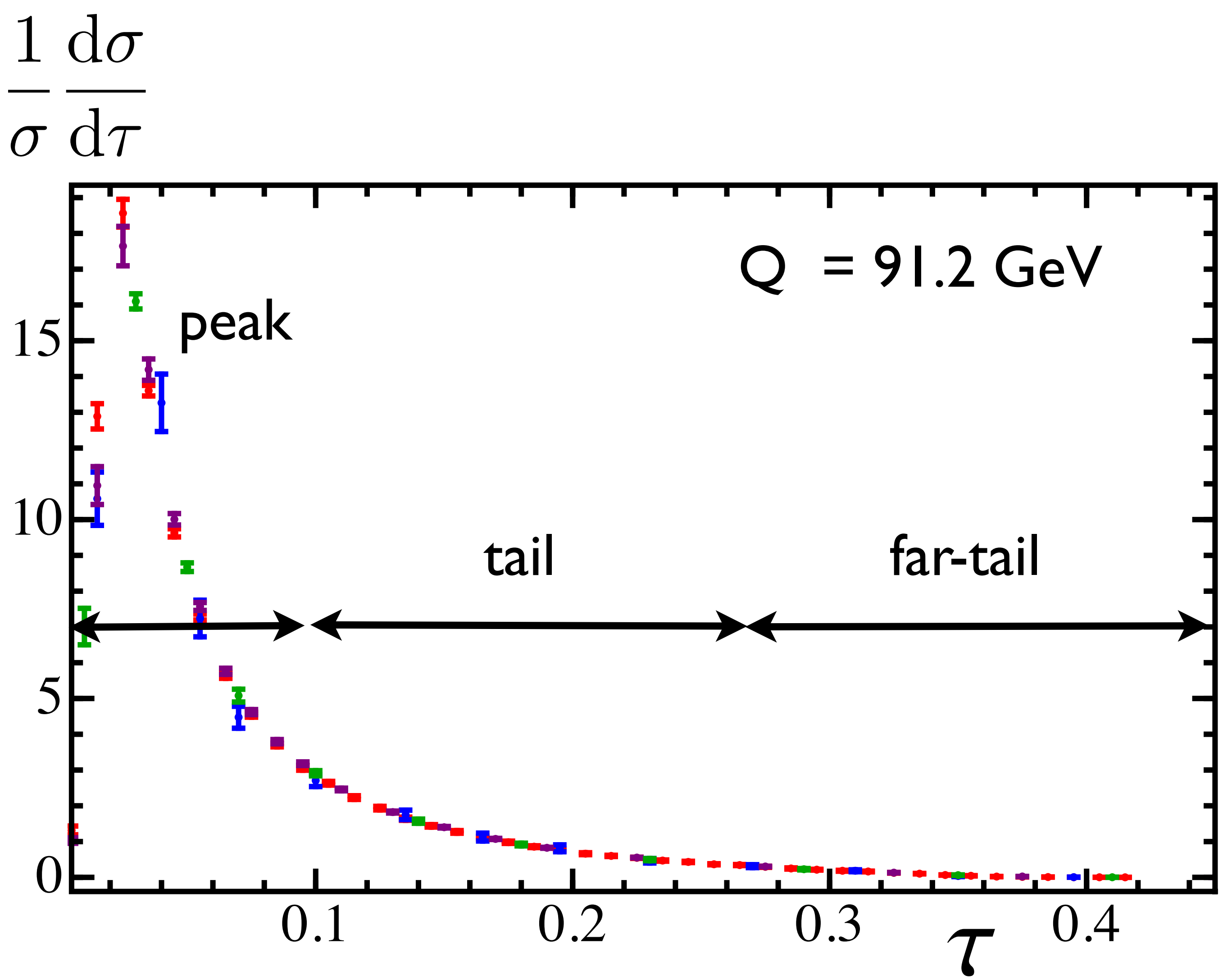
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

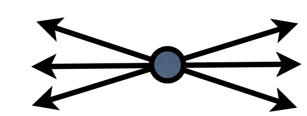
IR and collinear safe

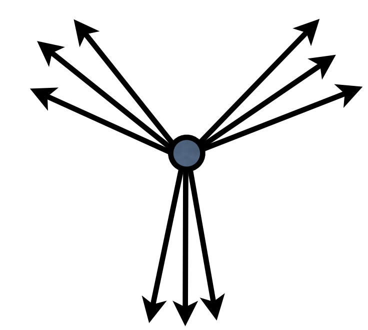
minimizes mass effects, good to fit for α_s

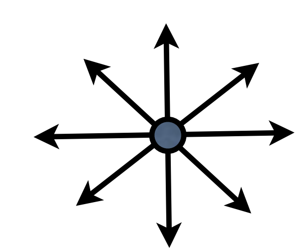
- Single sum
- Requires minimization

Continuous transition from 2-jet to 3-jet, ... multi-jet events



dijet $\tau \sim 0$ 

three jets $\tau \sim 0.3$ 

spherical $\tau \sim 0.5$ 

Continuous transition from 2-jet to 3-jet, ... multi-jet events

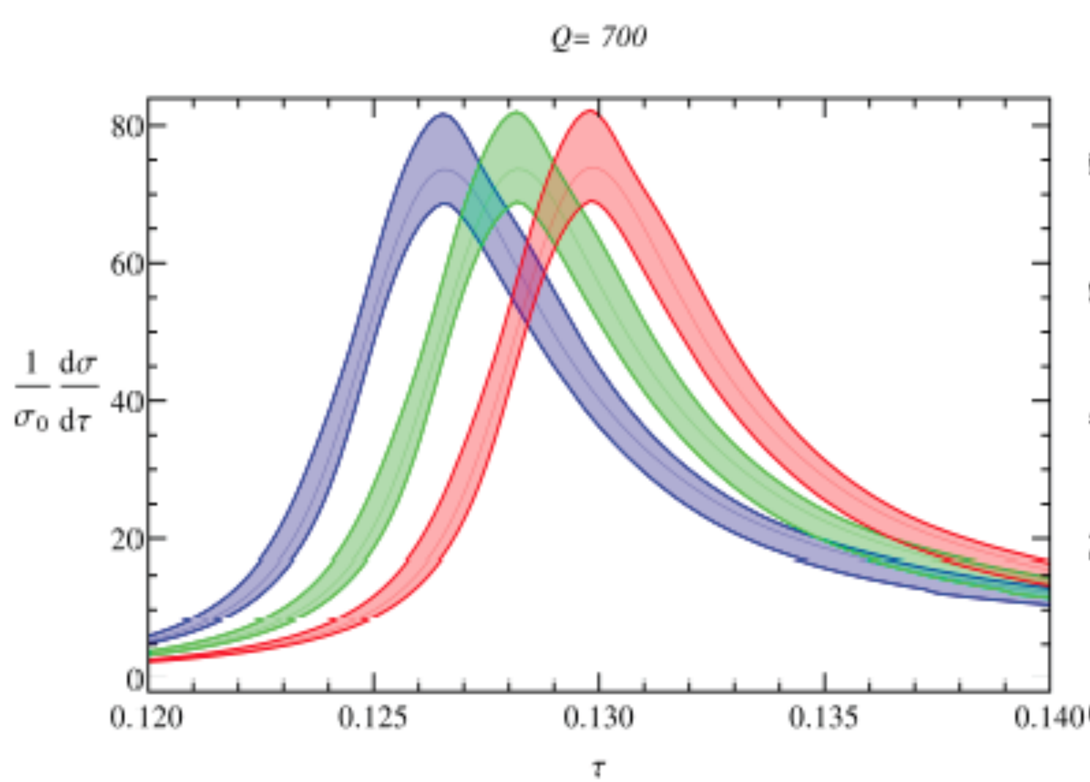
Jettiness $\quad e^+ e^- \to \text{jets}$

$$\tau_J = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q} \underset{Q\,\tau_J \ll Q}{\sim} \frac{M_1^2 + M_2^2}{Q^2}$$

very sensitive to quark mass, good for mass fits !!!

Shifts the whole distribution by ~ $\dfrac{2m^2}{Q^2}$

peak sensitive to mass… good, Pythia can only be trusted in the peak



Additional analyses on the way: C-parameter and HJM, with equally good sensitivity

Jet formation and evolution

hadronization

$\mu_\Lambda \sim \Lambda_{\mathrm{QCD}}$

soft emissions

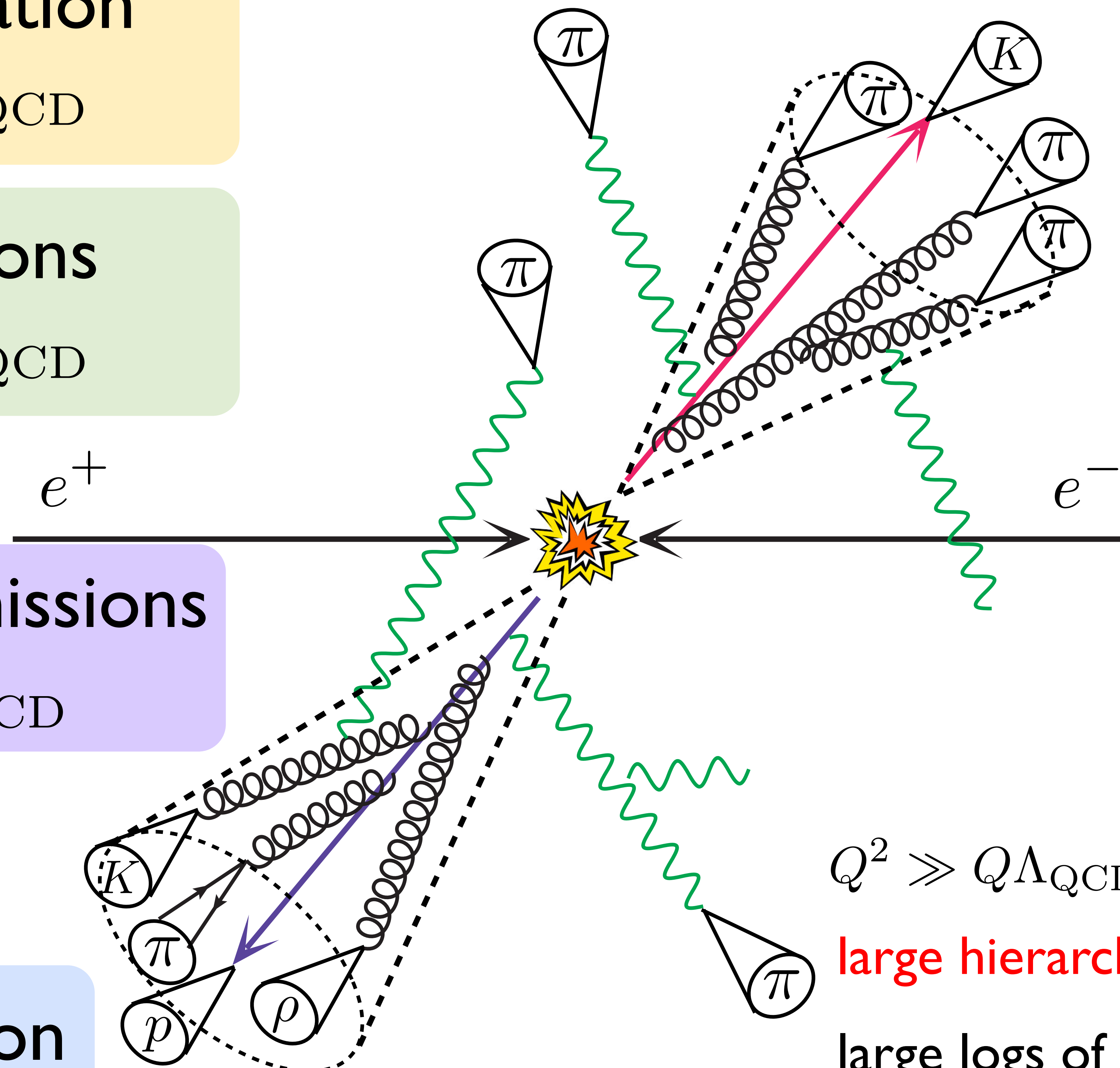
$\mu_S \gtrsim \Lambda_{\mathrm{QCD}}$

collinear emissions

$\mu_J \sim Q\,\Lambda_{\mathrm{QCD}}$

Hard collision

$\mu_h \sim Q$



$Q^2 \gg Q\Lambda_{\mathrm{QCD}} \gg \Lambda^2_{\mathrm{QCD}}$

large hierarchy of scales

large logs of ratios

EFT treatment is called for!

Factorization theorem for massless event shapes

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\rm QCD}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

[Berger, Kuks, Sterman]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

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Leading power correction comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

[Korchemsky, Sterman, Tafat]

perturbative

nonperturbative & [Korchemsky & Sterman]

<u>perturbative</u> [VM, Thaler, Stewart]

$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} \otimes F_e$$

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Perturbative and nonperturbative components

The factorization theorem needs to be modified to include massive particles

- Explicit mass dependence in matrix elements
- Running with different number of flavors according to thresholds
- Matching to a new EFT in the peak region

Renormalization group evolution

large logs

hard scale $\mu_H \sim Q$ — $\log^n\left(\frac{Q}{\mu}\right)$

jet scale $\mu_J \sim Q\sqrt{\tau}$ — $\log^n\left(\frac{Q^2\tau}{\mu^2}\right)$

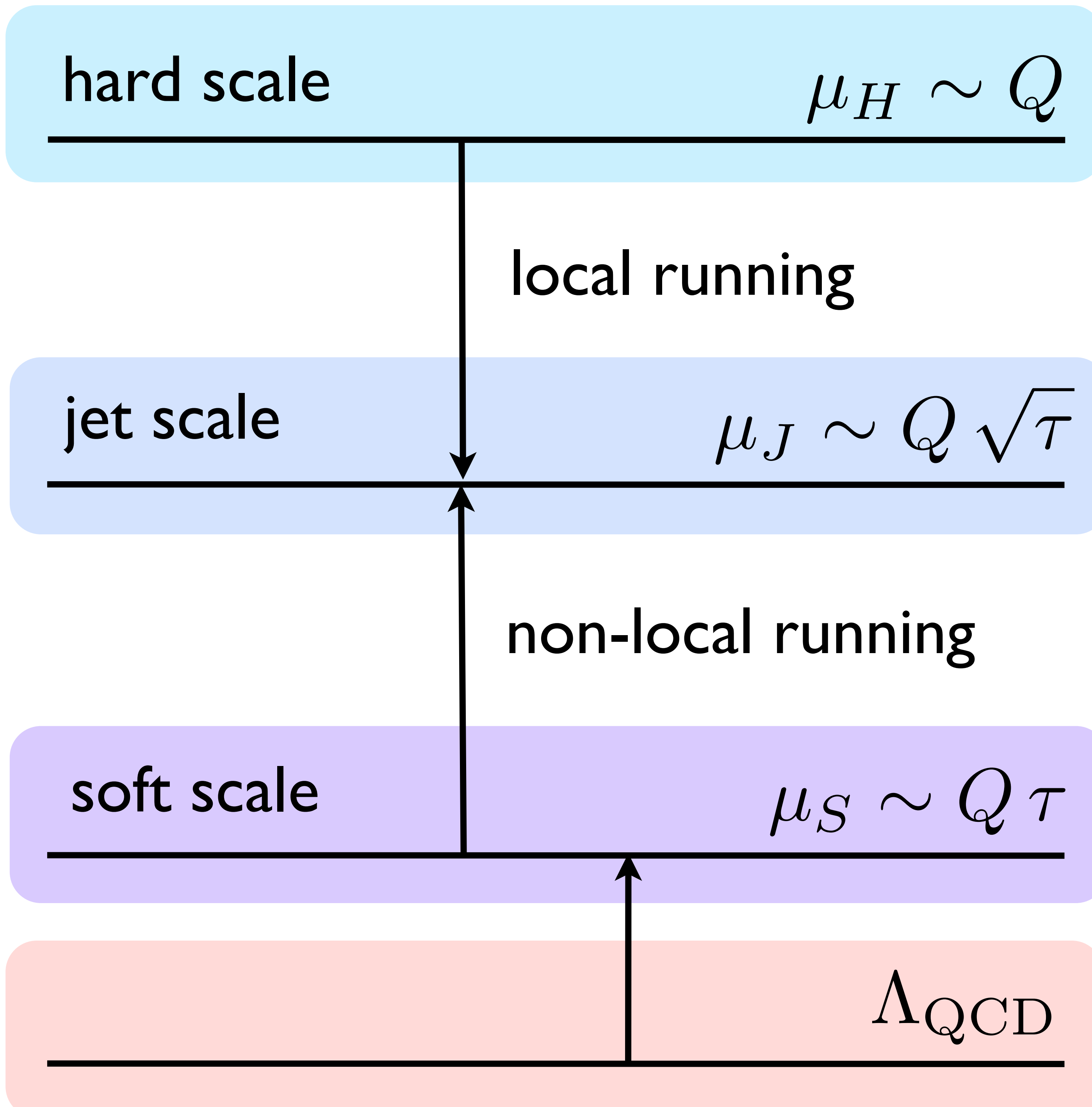
soft scale $\mu_S \sim Q\,\tau$ — $\log^n\left(\frac{Q\tau}{\mu}\right)$

$\Lambda_{\rm QCD}$ — $\log\left(\frac{\Lambda_{\rm QCD}}{Q\tau}\right)$

The hierarchy among the scales depends on the position on the spectrum

Use profile function to describe the whole distribution

Renormalization group evolution



$$\log^n\left(\frac{Q}{\mu_J}\right)$$

$$\log^n\left(\frac{\mu_J}{\mu_S}\right)$$

$$\log^n\left(\frac{\Lambda_{\rm QCD}}{\mu_S}\right)$$

Renormalization group evolution

m

hard scale $\mu_H \sim Q$

jet scale $\mu_J \sim Q\sqrt{\tau}$

soft scale $\mu_S \sim Q\,\tau$

Scenario I

no heavy quark produced

When including massive quarks, a new scale enters the game!

Renormalization group evolution

hard scale $\mu_H \sim Q$

jet scale $\mu_J \sim Q\sqrt{\tau}$

m

soft scale $\mu_S \sim Q\,\tau$

Scenario II

relevant for us,
mas sensitivity

When including massive quarks, a new scale enters the game!

Renormalization group evolution

hard scale $\mu_H \sim Q$

jet scale $\mu_J \sim Q\sqrt{\tau}$

m

soft scale $\mu_S \sim Q\,\tau$

Scenario III

less mass sensitivity

When including massive quarks, a new scale enters the game!

Renormalization group evolution

hard scale $\mu_H \sim Q$

jet scale $\mu_J \sim Q\sqrt{\tau}$

soft scale $\mu_S \sim Q\,\tau$

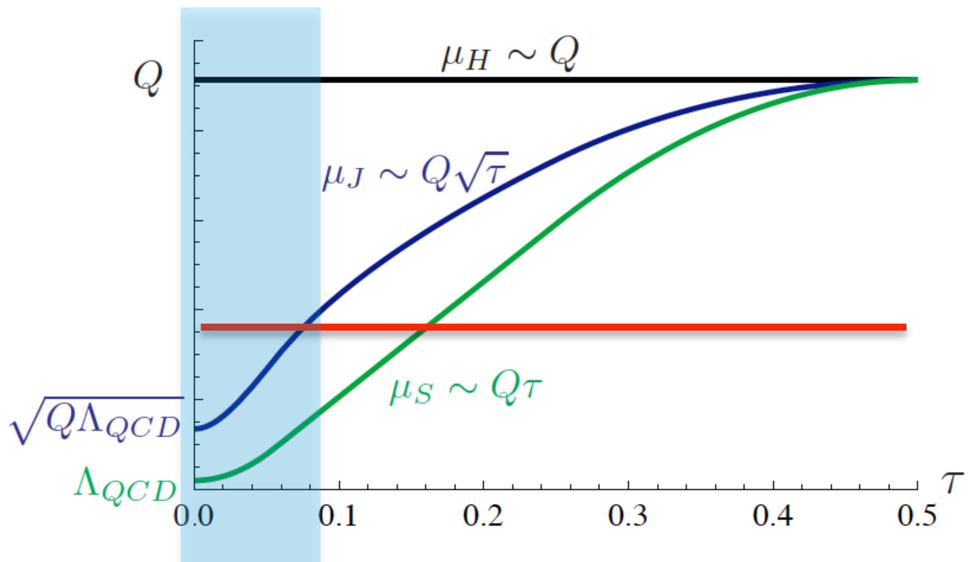
m

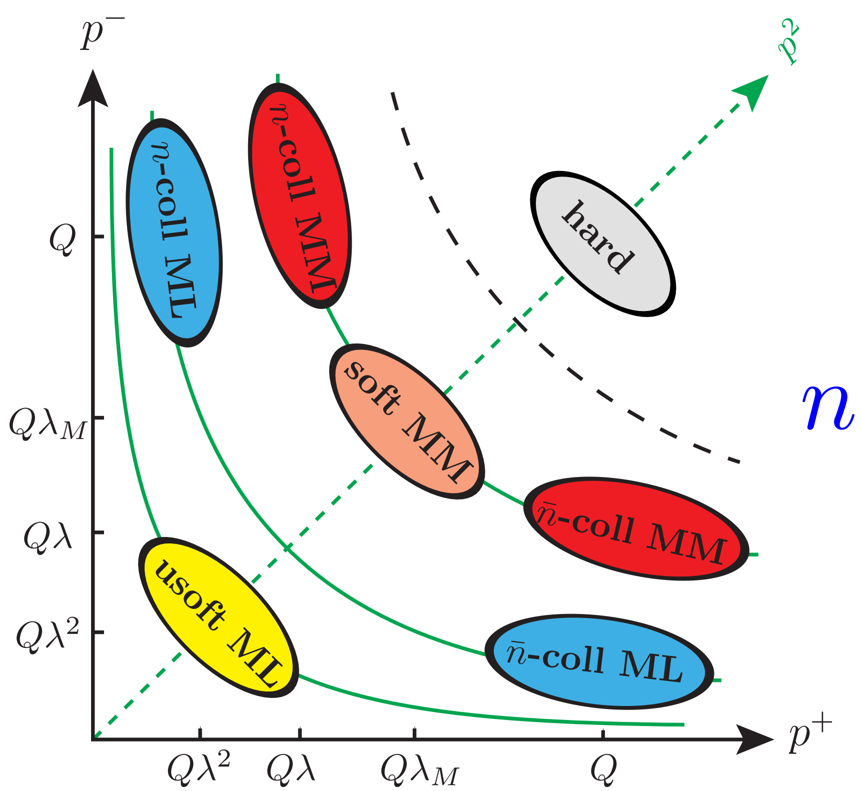
Scenario IV

no mass sensitivity

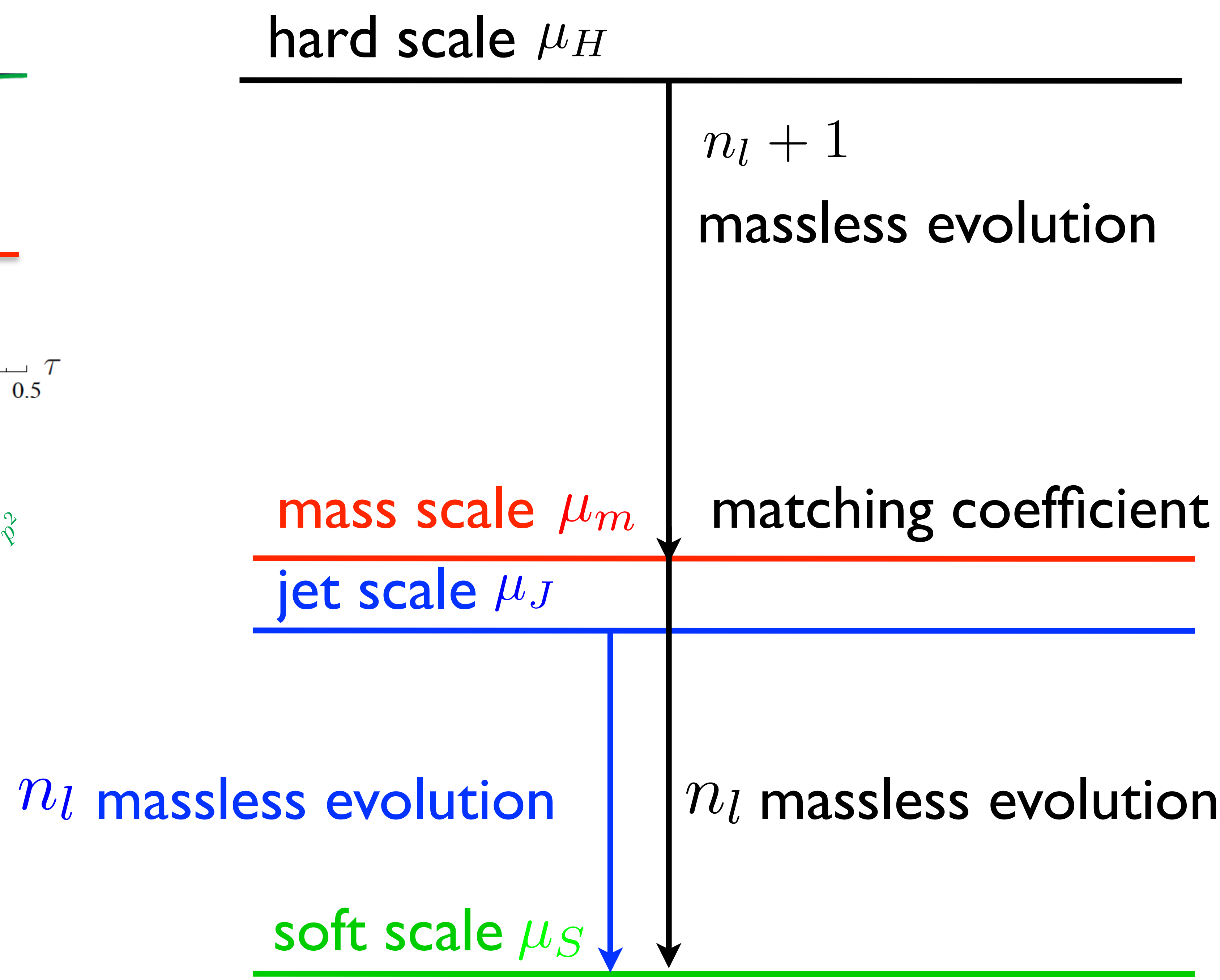
When including massive quarks, a new scale enters the game!

Scenario II





(b)$1 > \lambda_M > \lambda > \lambda^2$



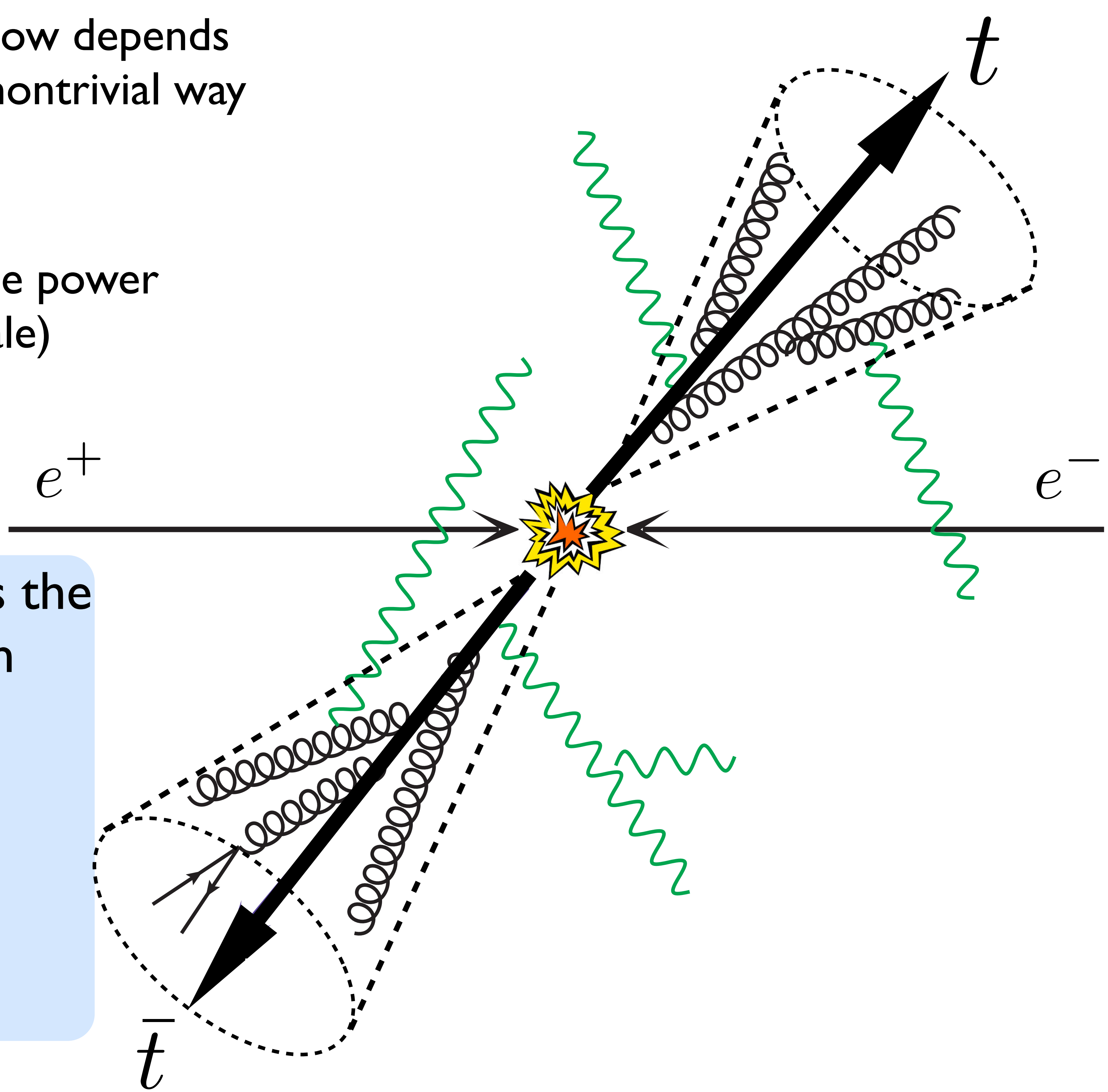
Primary Heavy quark production

[Fleming, Hoang, Mantry, Stewart]

The jet function now depends on the mass in a nontrivial way

$$J_n(s, m, \mu_J)$$

(mass has the same power counting as jet scale)



In particular shifts the whole distribution by $\dfrac{2m^2}{Q^2}$

enhances mass sensitivity !!!

[M. Butenschön, B. Dehnadi, A. Hoang, VM, I. Stewart]

Primary Heavy quark production

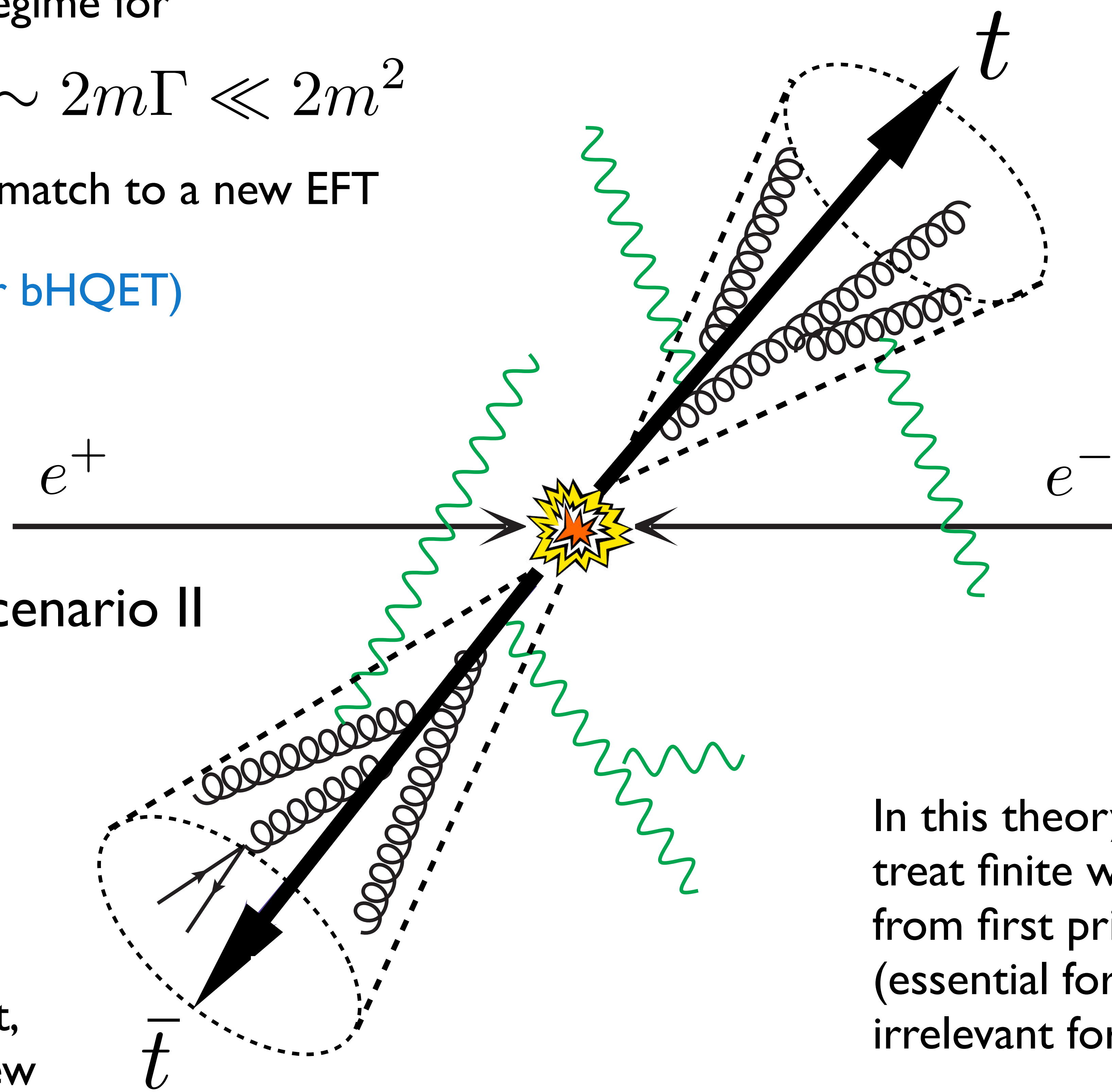
[Fleming, Hoang, Mantry, Stewart]

New kinematical regime for

$$Q^2\tau - 2m^2 \sim 2m\Gamma \ll 2m^2$$

New class of logs, match to a new EFT

boosted HQET (or bHQET)



Takes place in Scenario II

In this theory one can treat finite width effects from first principles (essential for top, irrelevant for bottom)

In practice: new matching coefficient, new jet function, new renormalization scale

[M. Butenschön, B. Dehnadi, A. Hoang, VM, I. Stewart]

boosted HQET Factorization Theorem

[Fleming, Hoang, Mantry, Stewart]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)} \times H_m^{(n_f)} \times J_B^{(n_l)} \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m_t^2}{Q^2}, \frac{\Gamma_t}{m_t}, \tau\right)$$

QCD to SCET matching	SCET to bHQET matching	bHQET jet function	SCET soft function	corrections to fact. theorem (most of them accounted for!)

Hadronization here!

boosted HQET Factorization Theorem

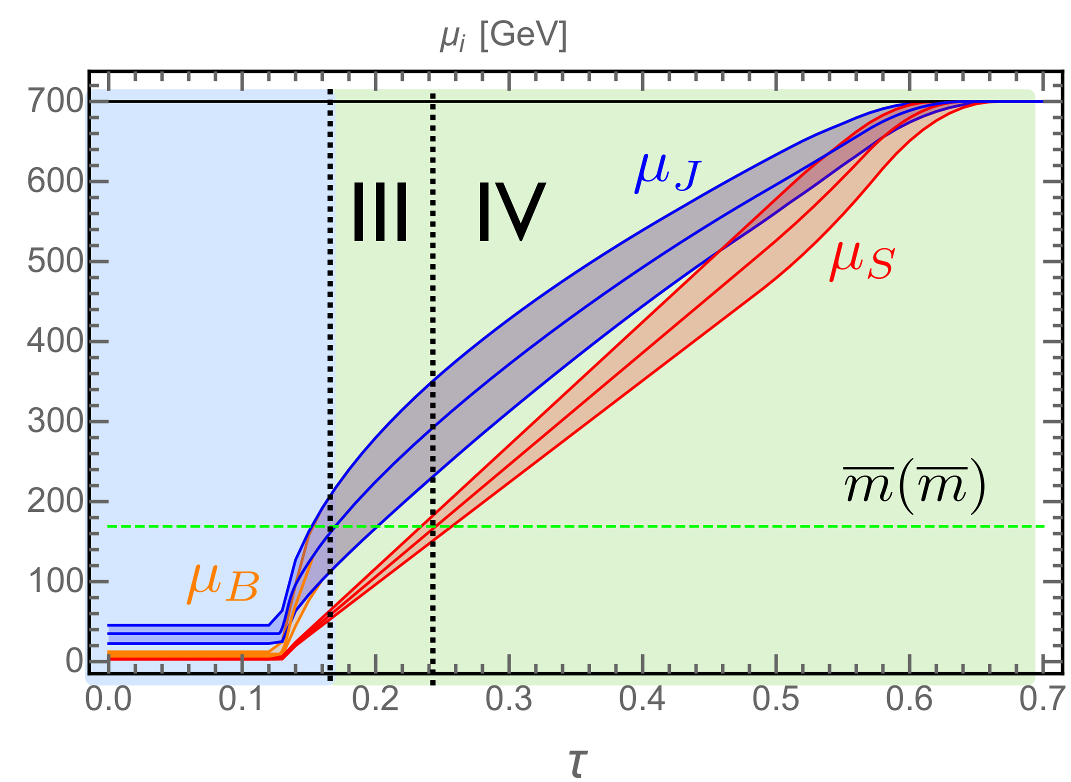
[Fleming, Hoang, Mantry, Stewart]

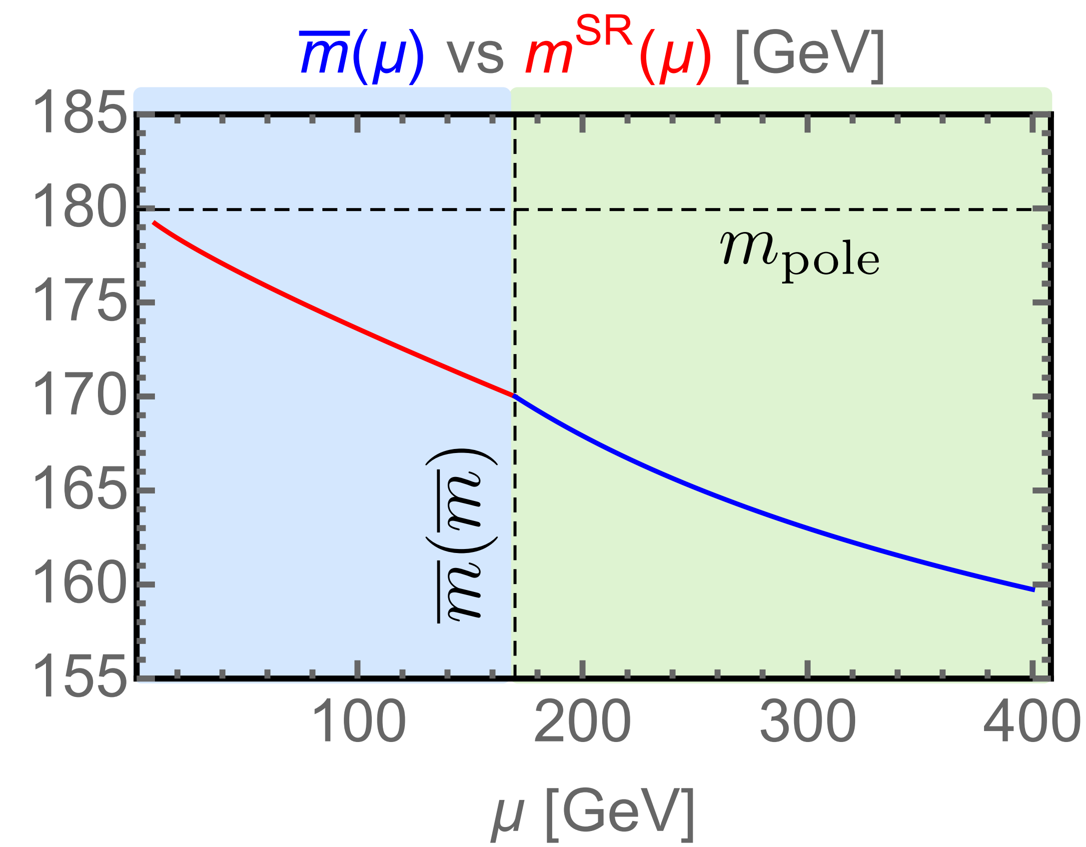
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} \sim H^{(n_l+1)} \times H_m^{(n_f)} \times J_B^{(n_l)} \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m_t^2}{Q^2}, \frac{\Gamma_t}{m_t}, \tau\right)$$

QCD to SCET matching — SCET to bHQET matching — bHQET jet function — SCET soft function — corrections to fact. theorem (most of them accounted for!)

in SCET regime $\overline{\mathrm{MS}}$ mass has correct behavior

in bHQET regime MSR mass has correct behavior

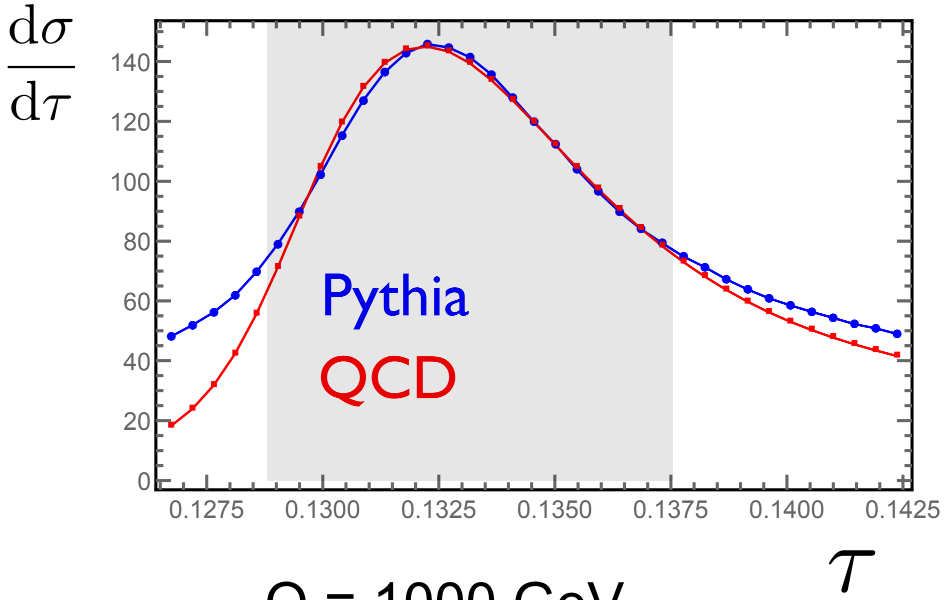




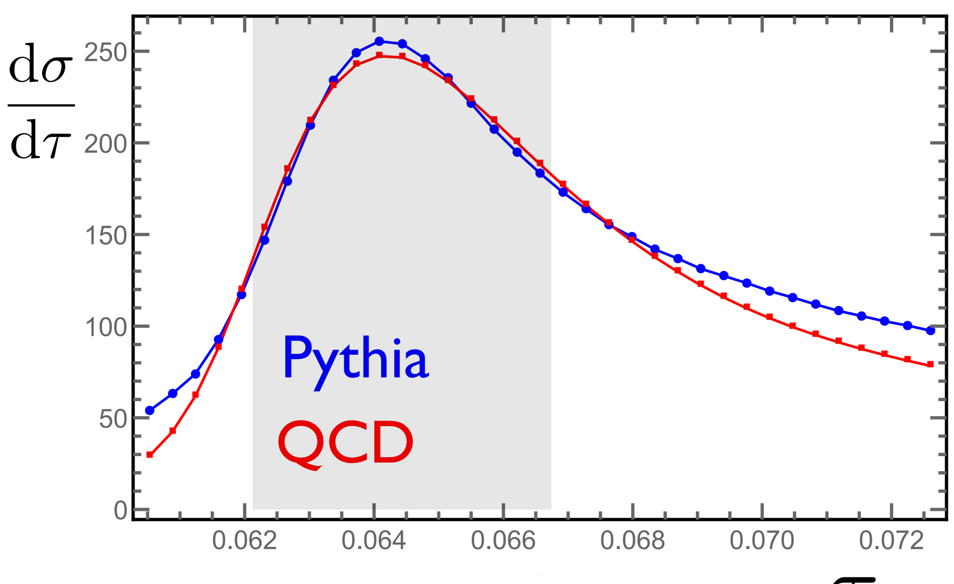
Results

Fits to Pythia data very preliminary

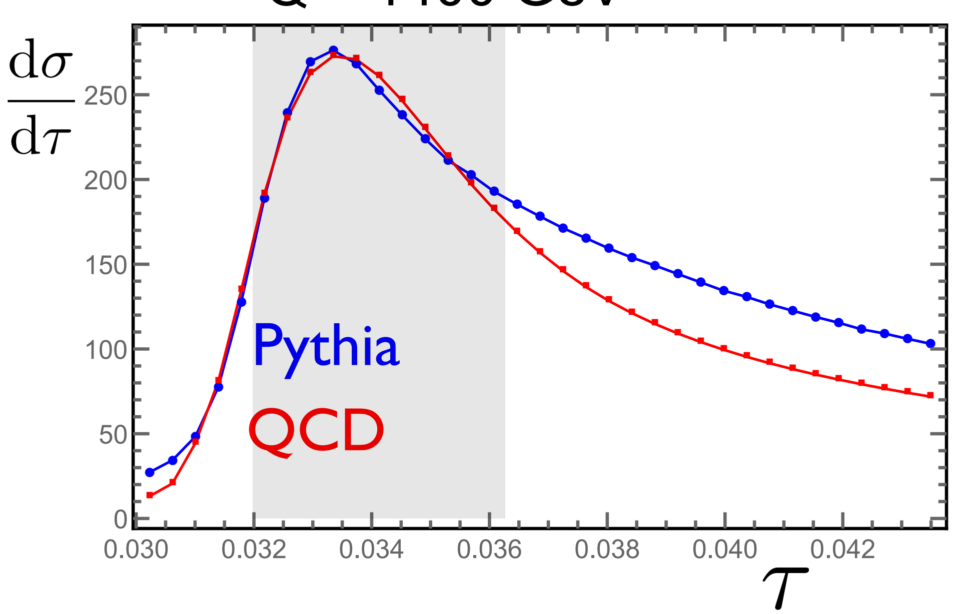
Q = 700 GeV



Q = 1000 GeV



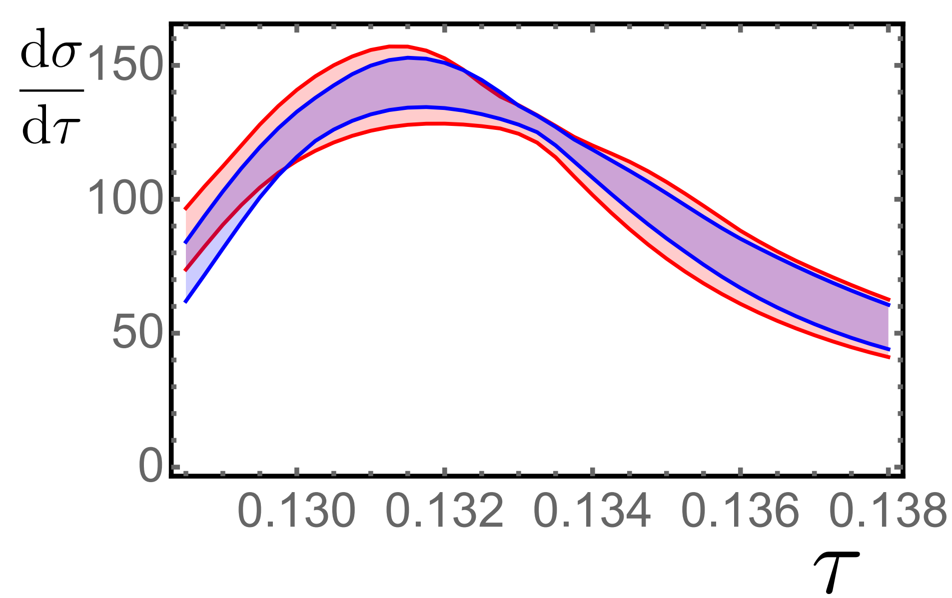
Q = 1400 GeV



We only compare peak data, since otherwise Pythia is not reliable. Also peak gives higher mass sensitivity.

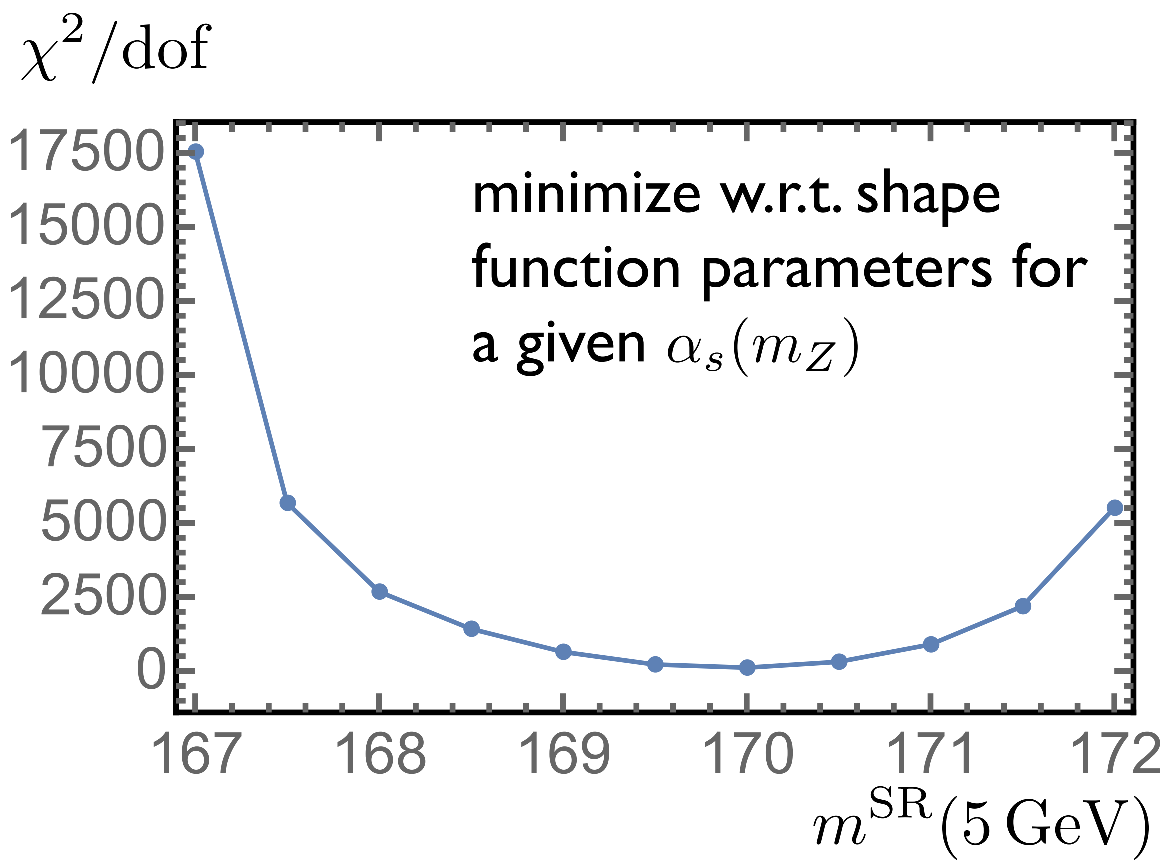
- Good description of Pythia 8.2 default output with default scale setting NNLL + NLO QCD.
- Pythia statistical errors: 10^7 events
- Theory error not included yet.
- Increasing discrepancies in distribution tail and for higher energies due to off shell effects in NS
- Excellent sensitivity to the top quark mass.

Theoretical accuracy at NLL / NNLL order

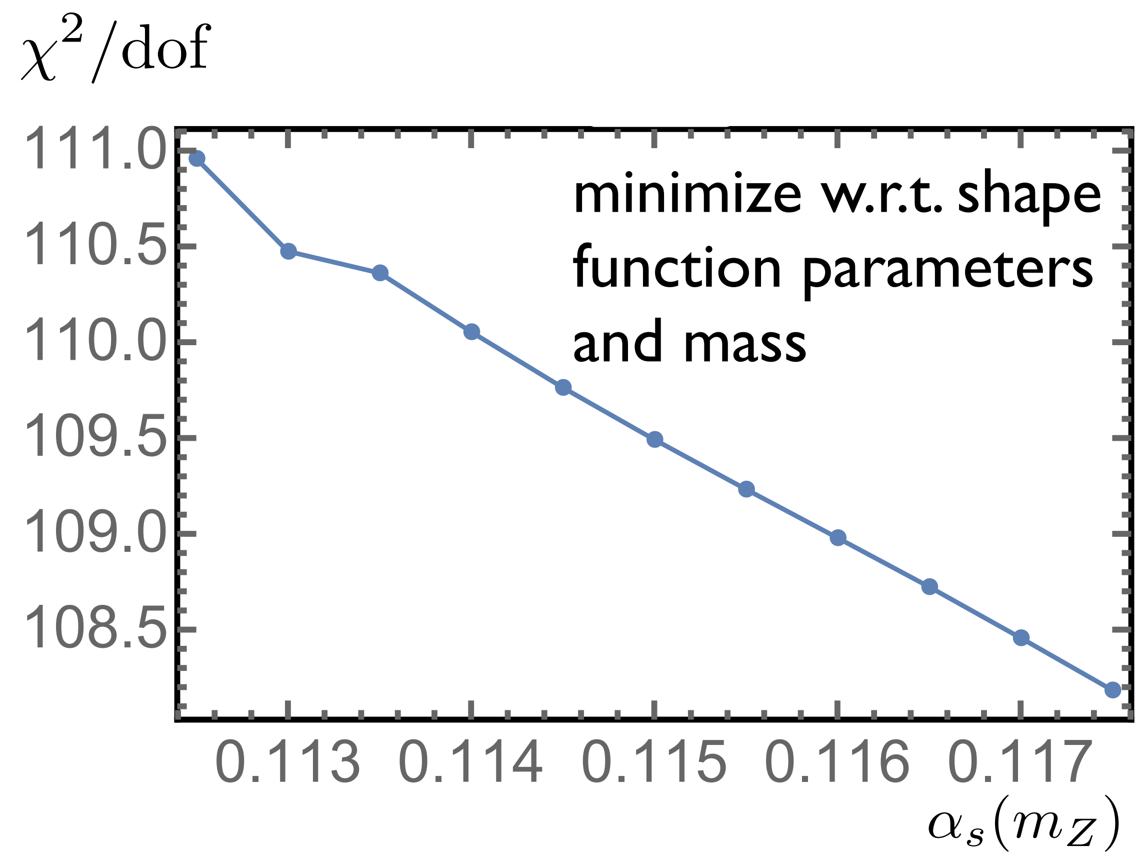


Fits to Pythia data

very preliminary



Excellent sensitivity to the top quark mass



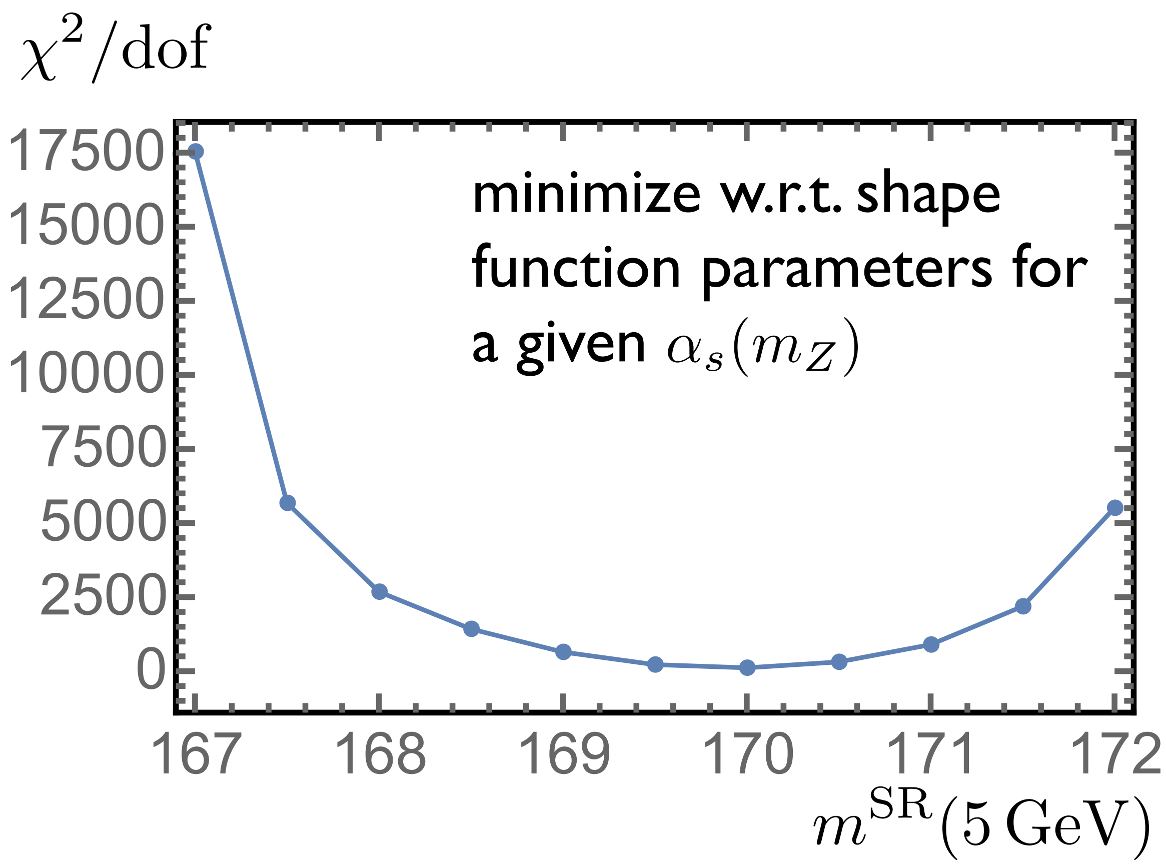
Not very sensitive to strong coupling constant

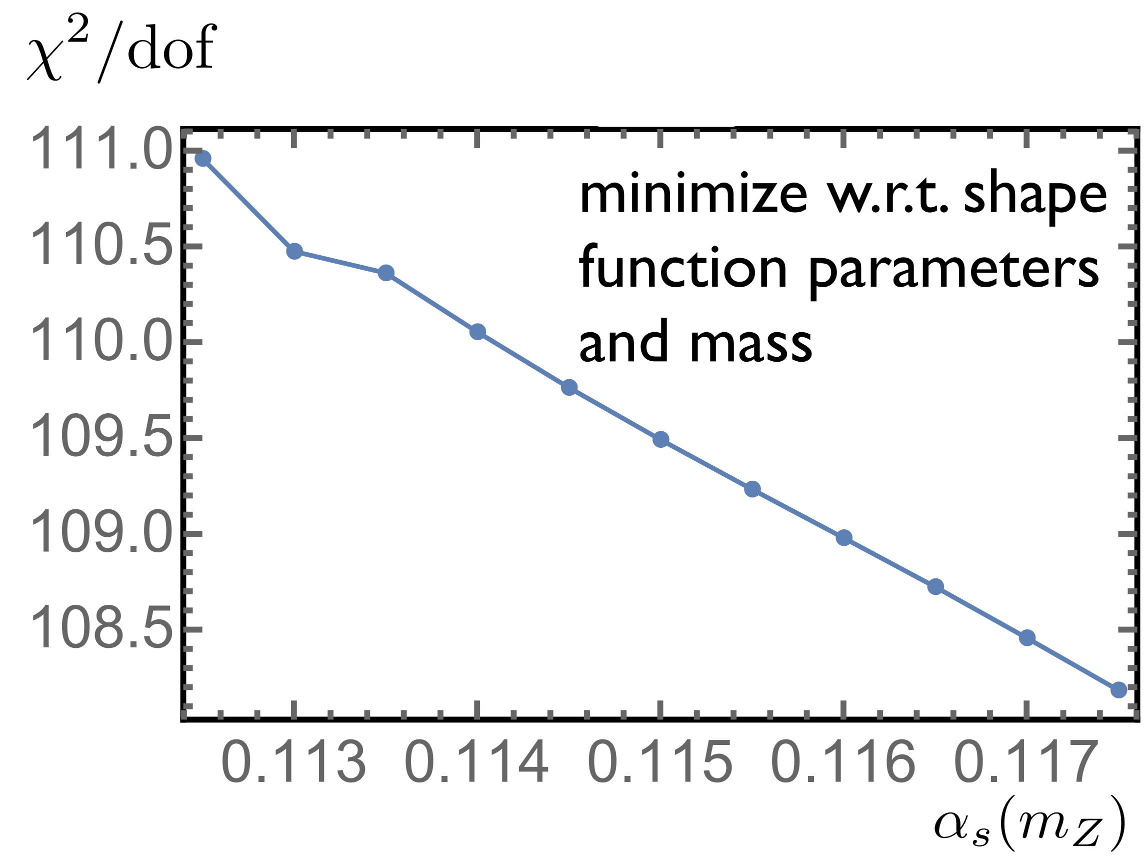
$$m^{\mathrm{Pythia}} = 171\,\mathrm{GeV}$$

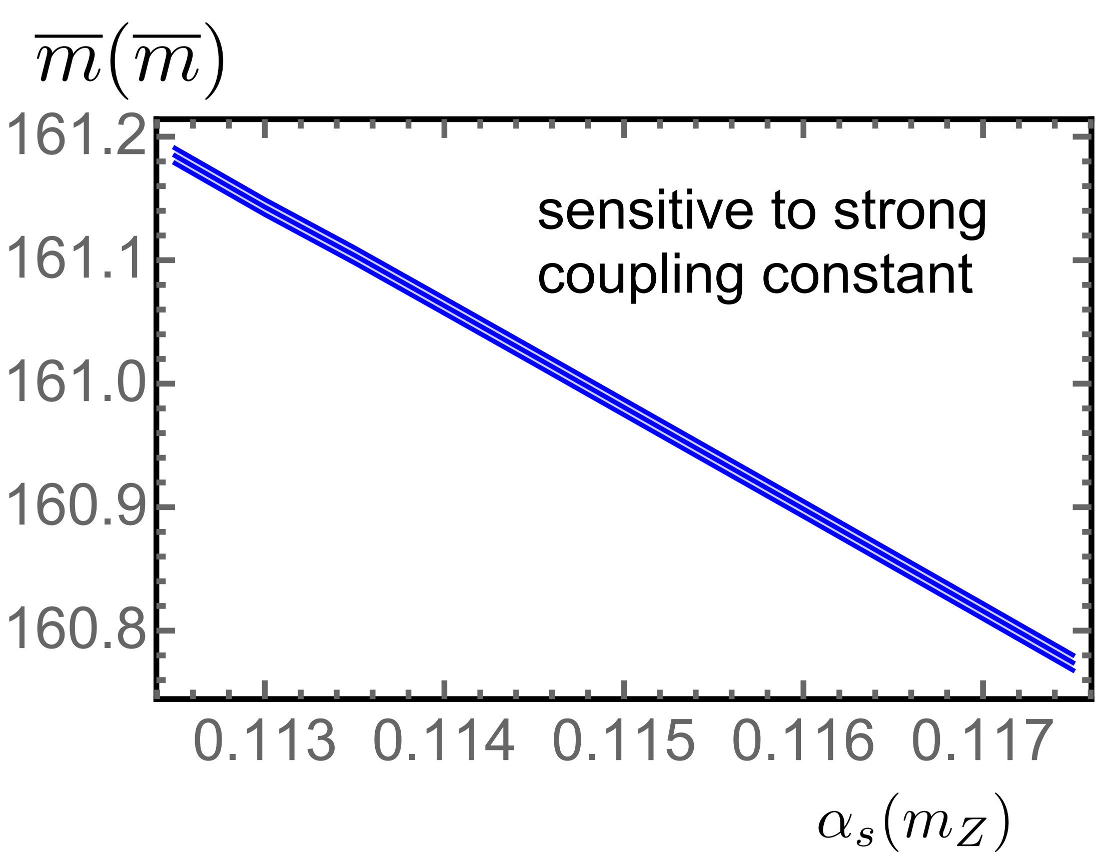
$$m^{\mathrm{SR}}(5\,\mathrm{GeV}) = 169.923 \pm 0.006\,\mathrm{GeV}$$ + theory error

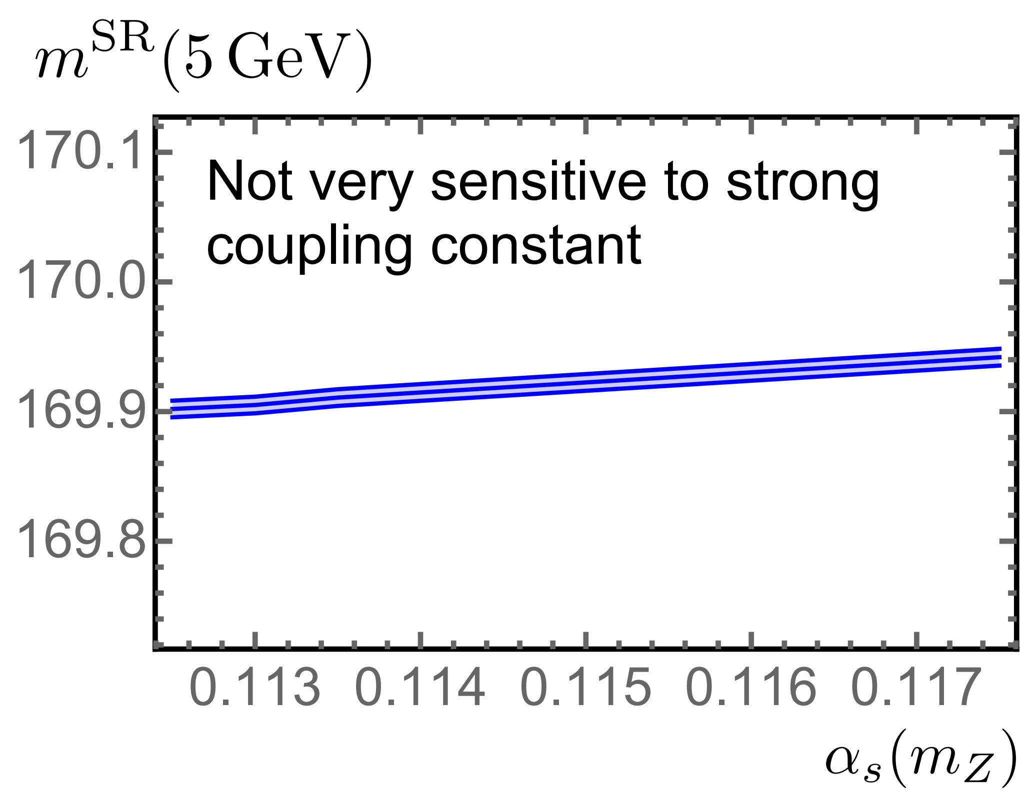
Fits to Pythia data

very preliminary



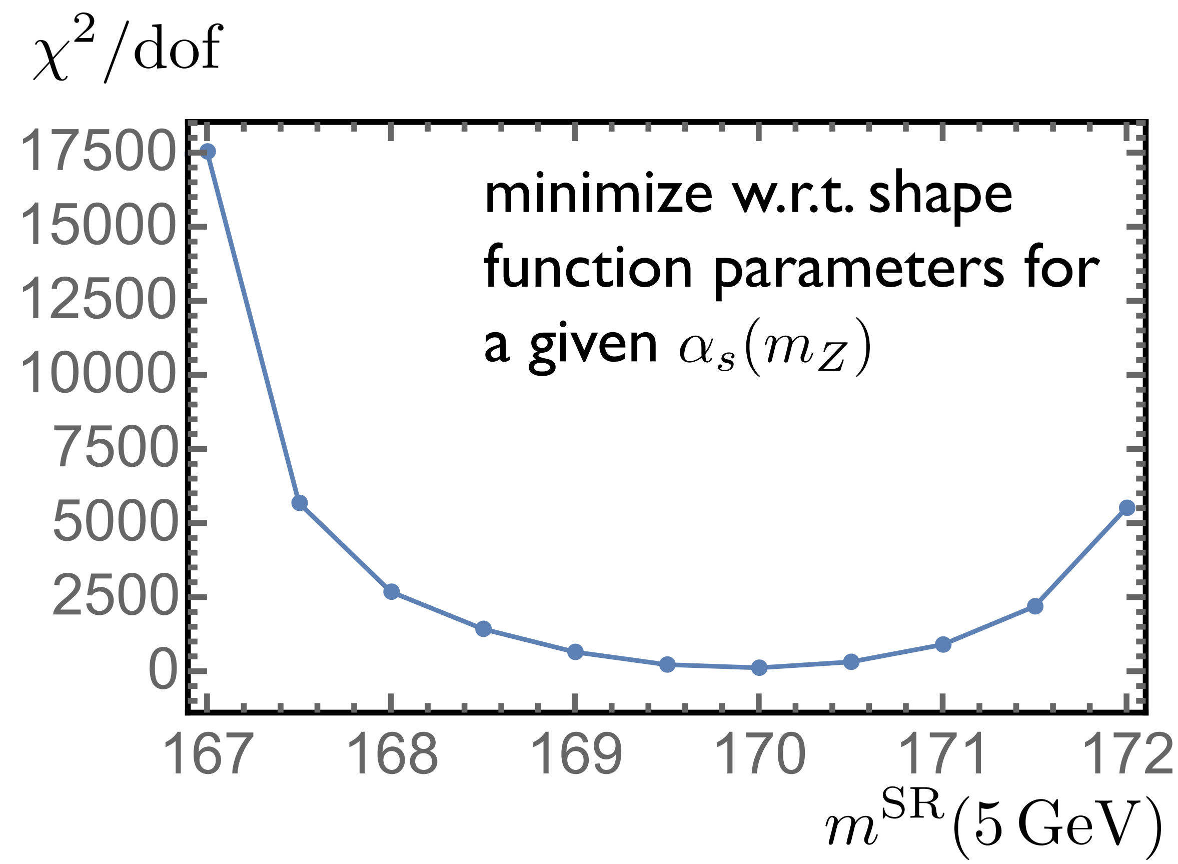


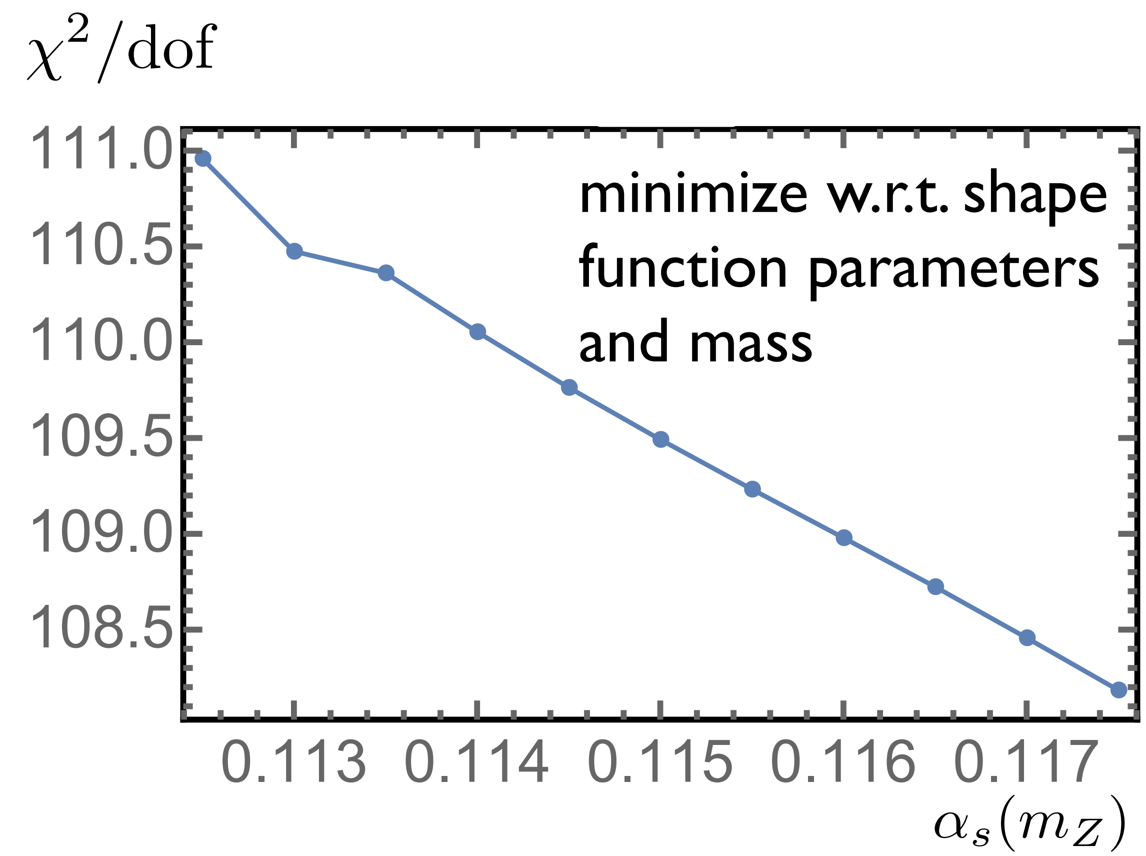


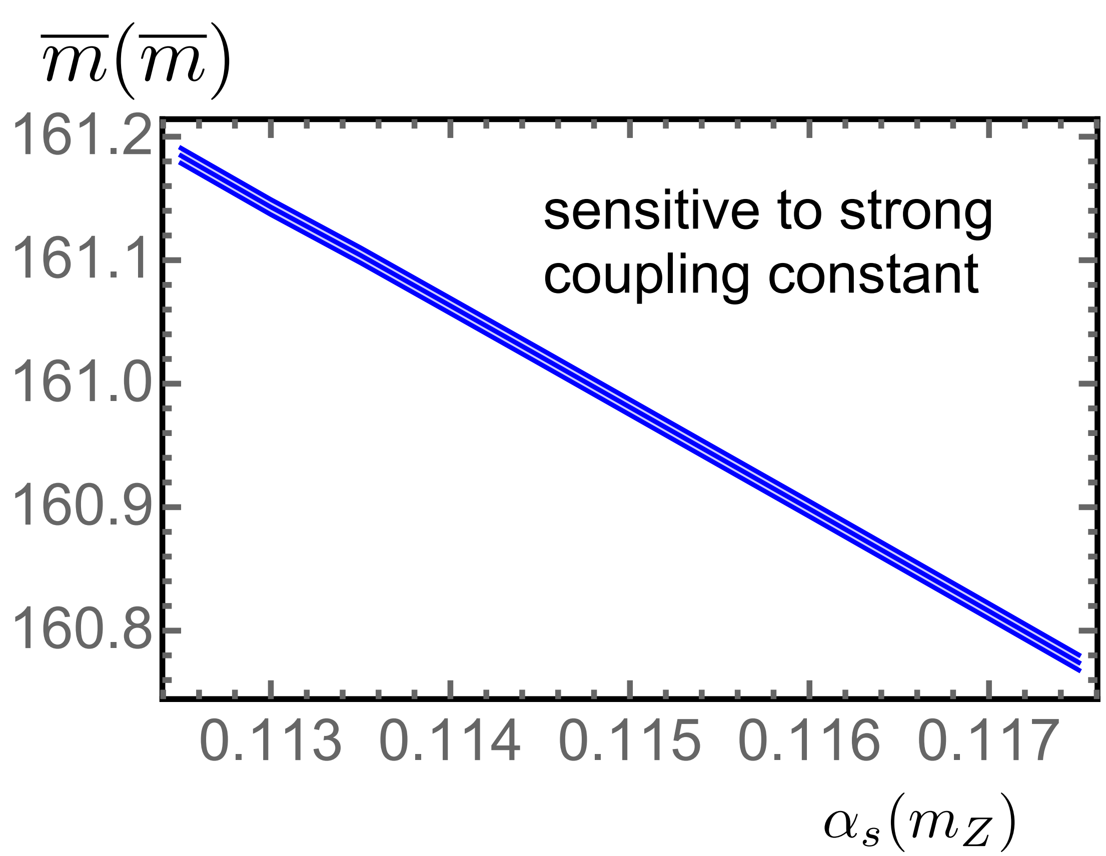


Fits to Pythia data

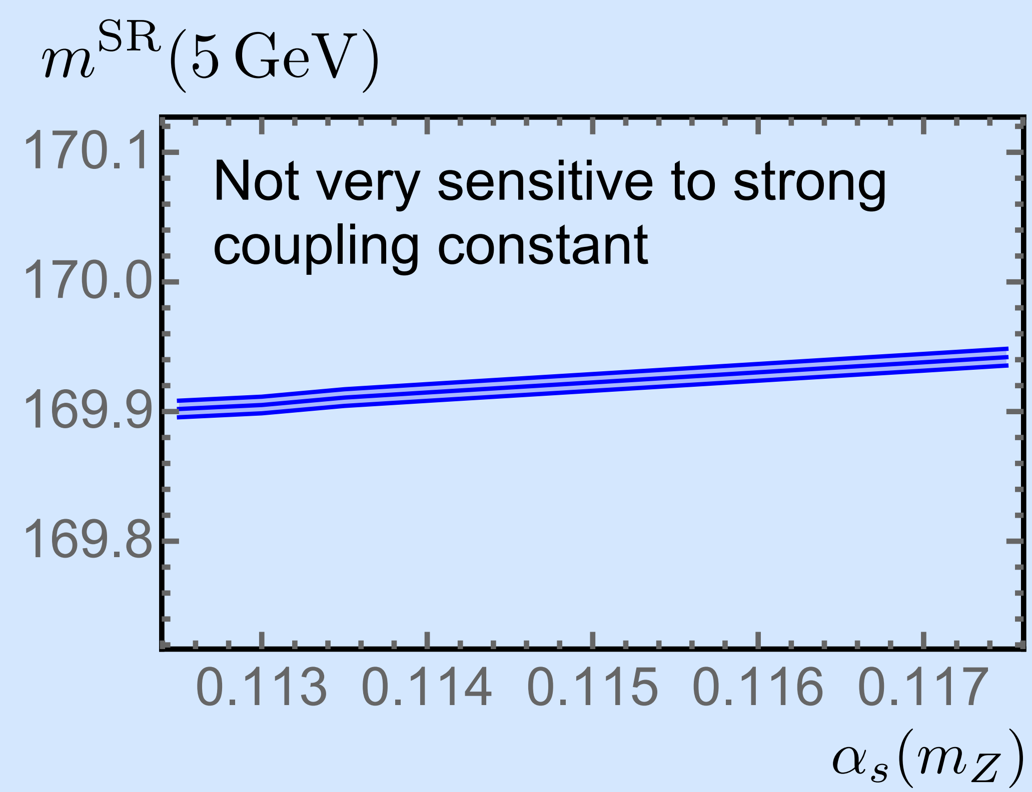
very preliminary







Confirms that MC top mass is closely related to the MSR mass at a scale close to the shower cut !!



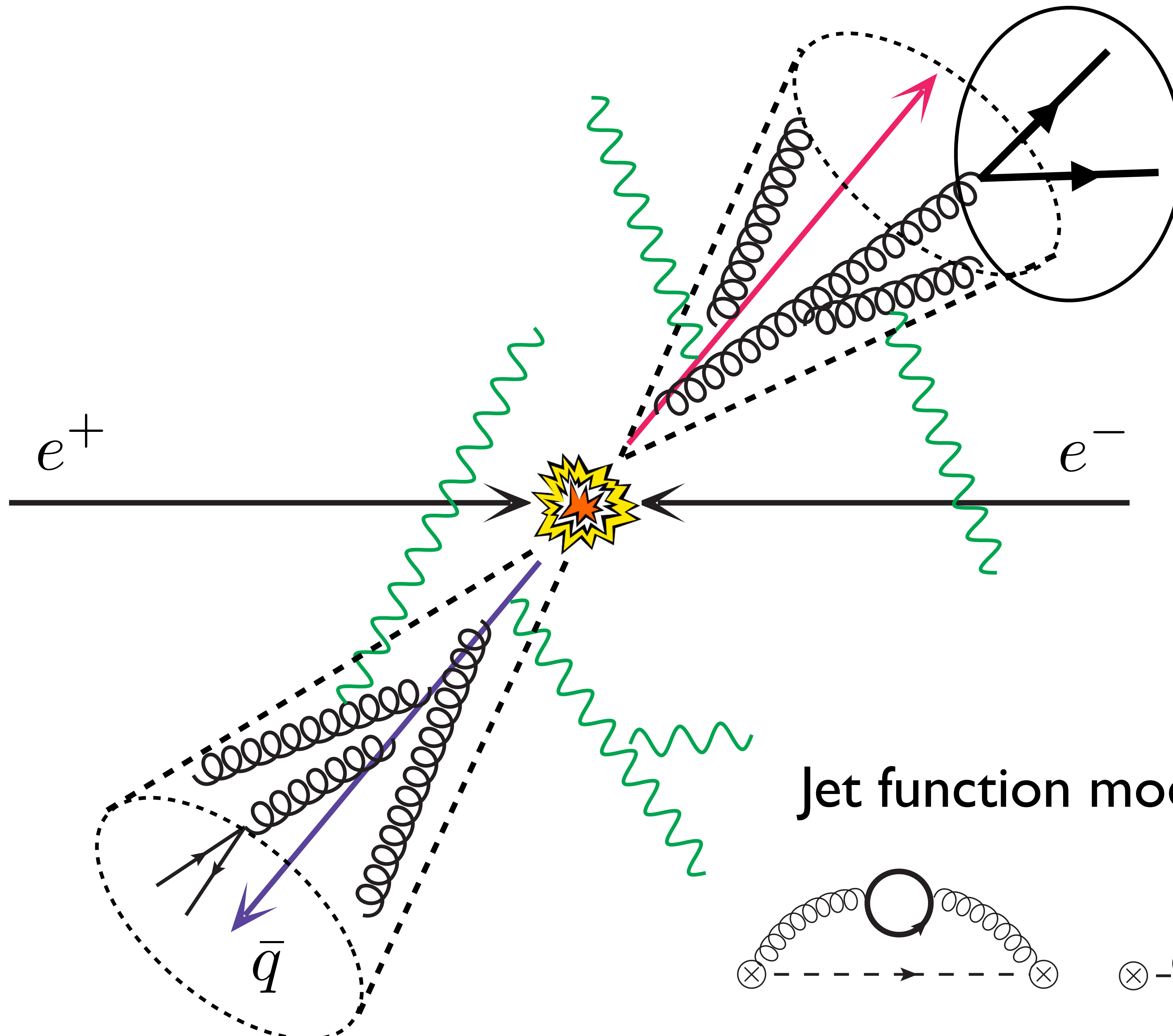
Conclusions

Conclusions

- Top behaves almost as a real particle... but not quite
- Precision physics requires precise top mass scheme knowledge
- MC top mass can be calibrated by comparison to a hadron-level ab initio "QCD calculator" (SCET)
- Electron-positron collision, simplest possible setup.
- Thrust, easy and sensitive observable to start with
- Preliminary fits look promising, full analysis to follow very soon !!!

Backup slides

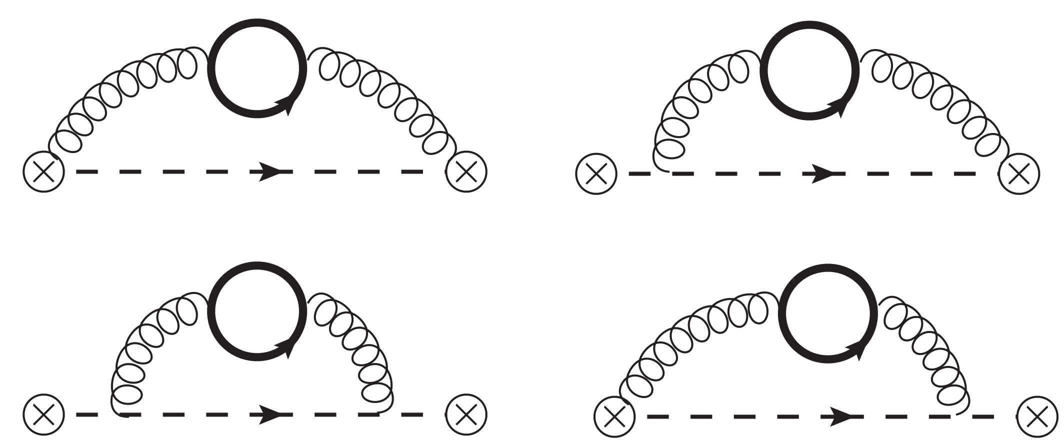
Heavy quark production through gluon splitting



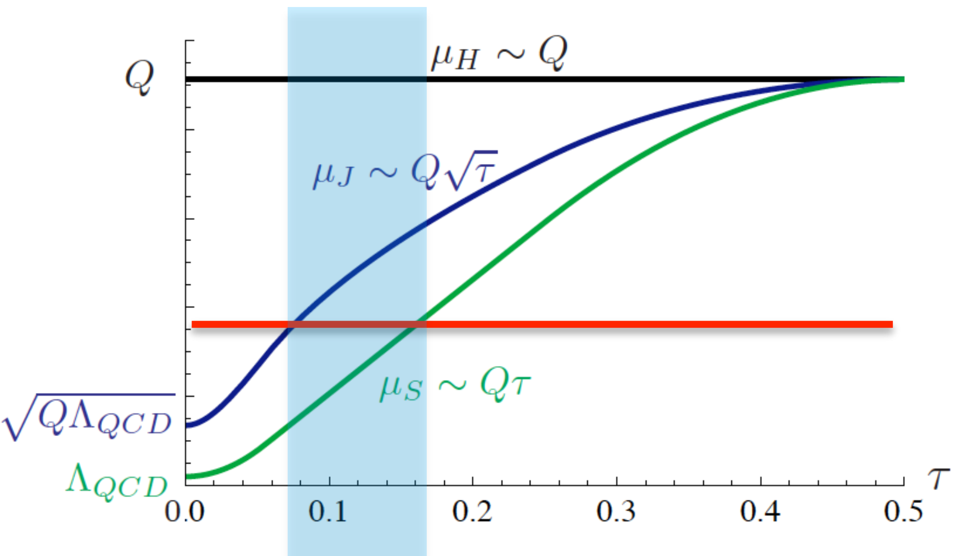
collinear splitting

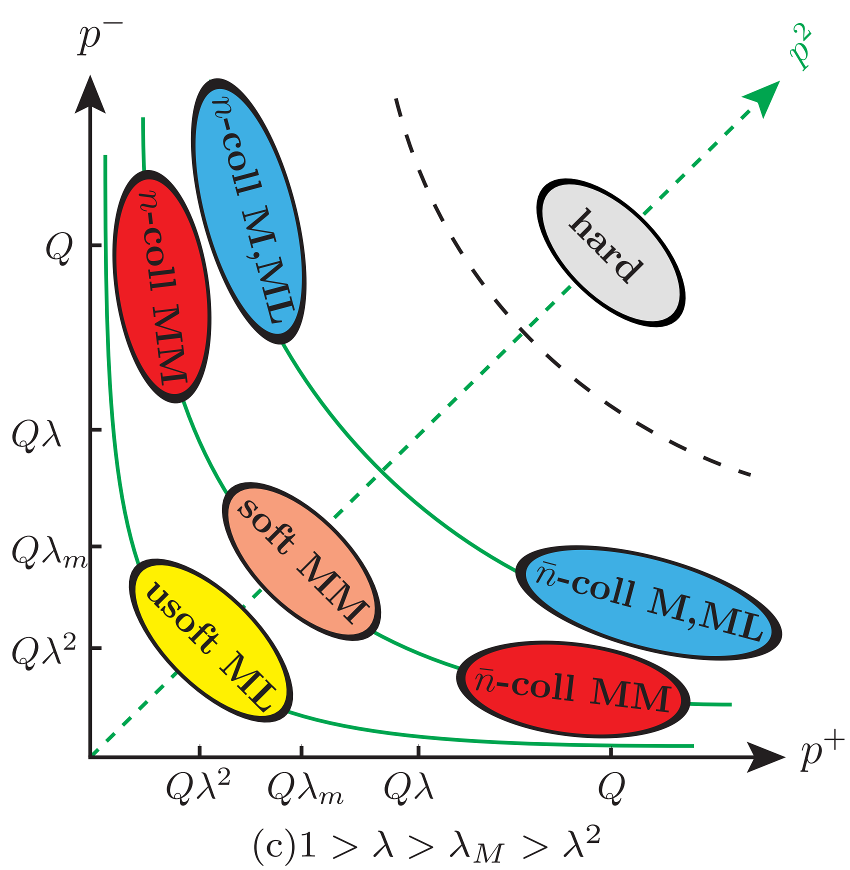
if enough energy to produce pair of heavy quarks

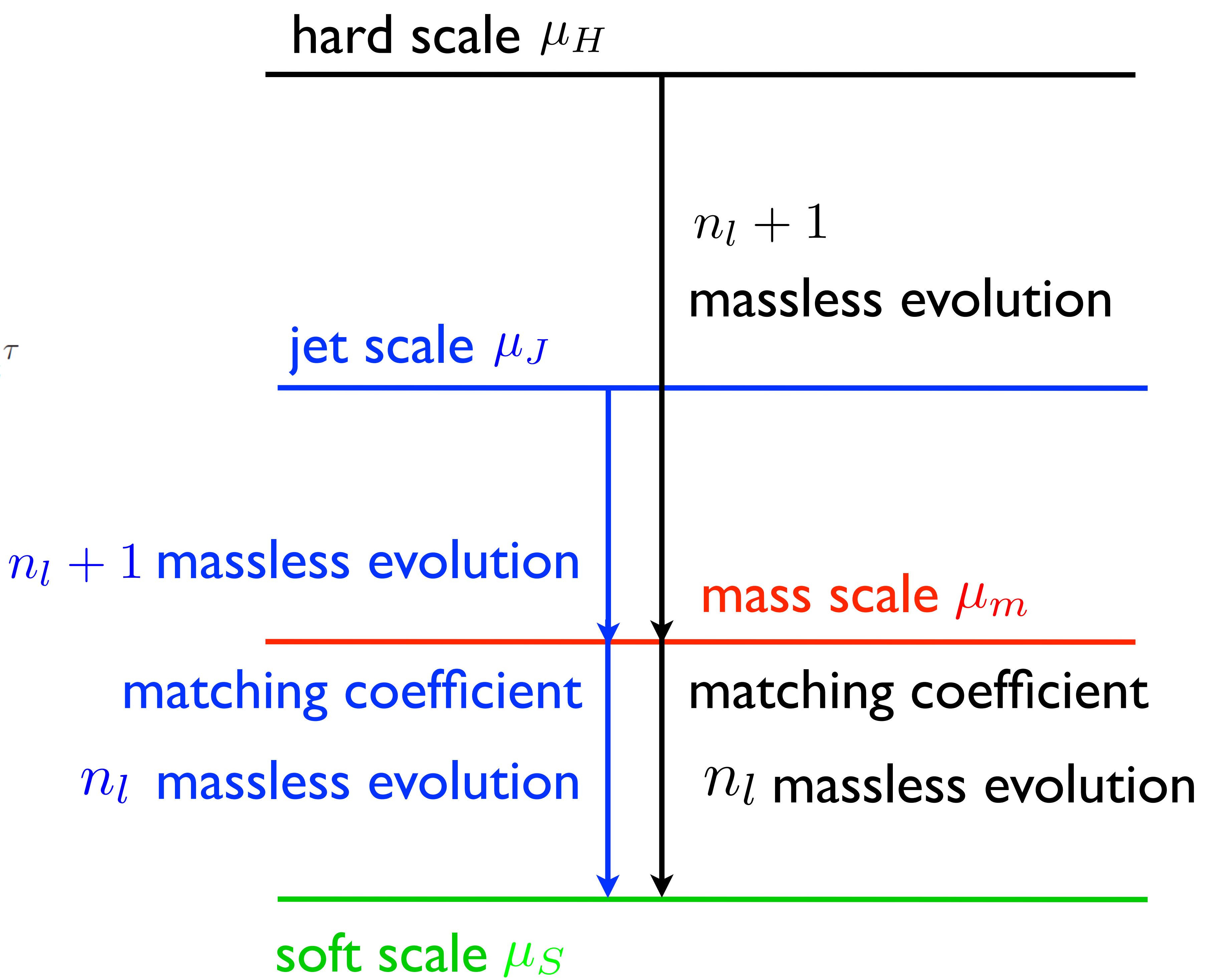
Jet function modification



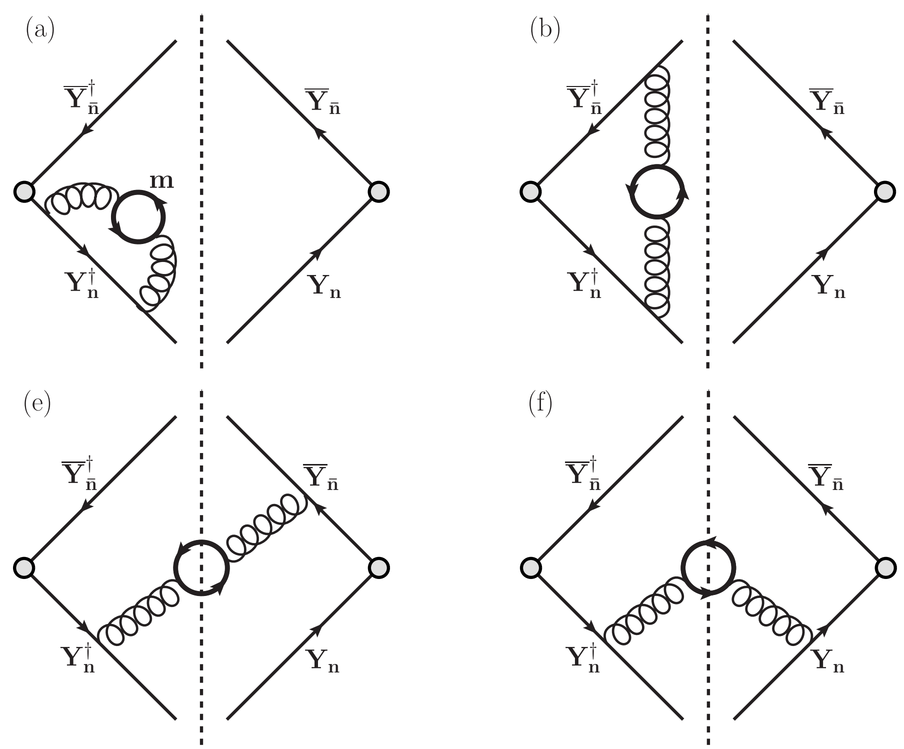
Scenario III



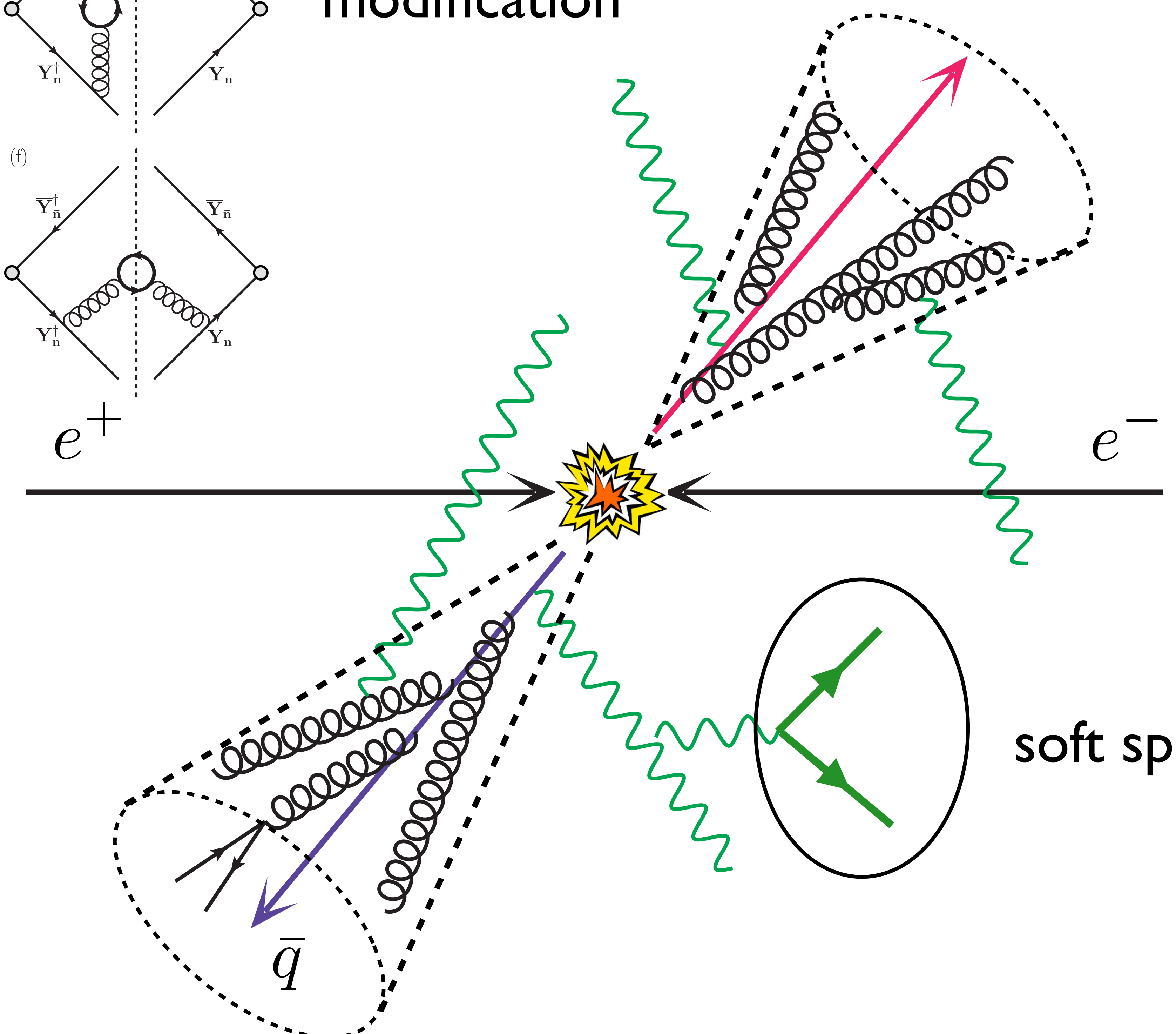




Heavy quark production through gluon splitting

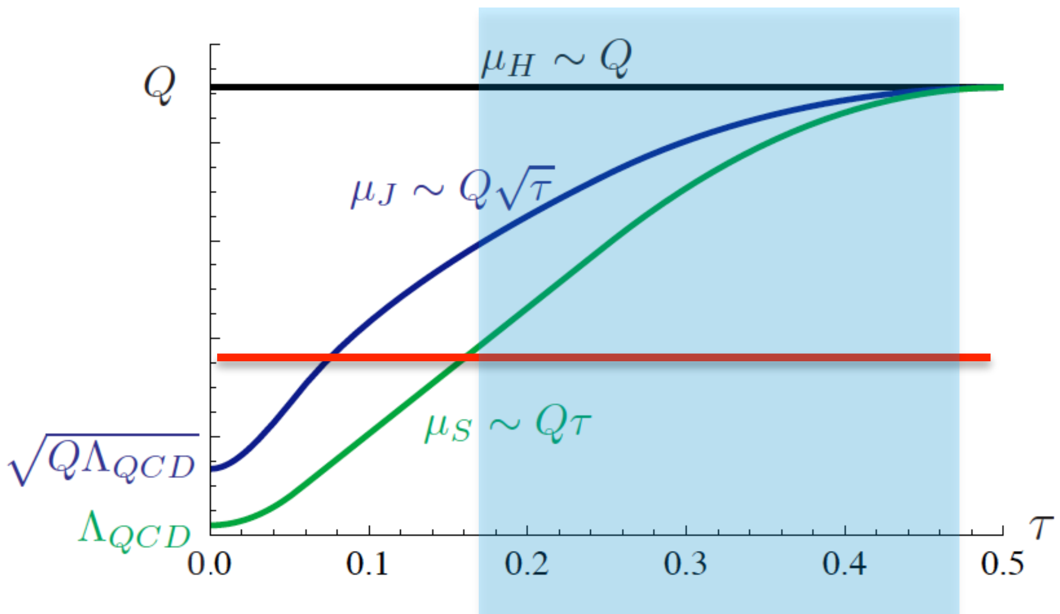


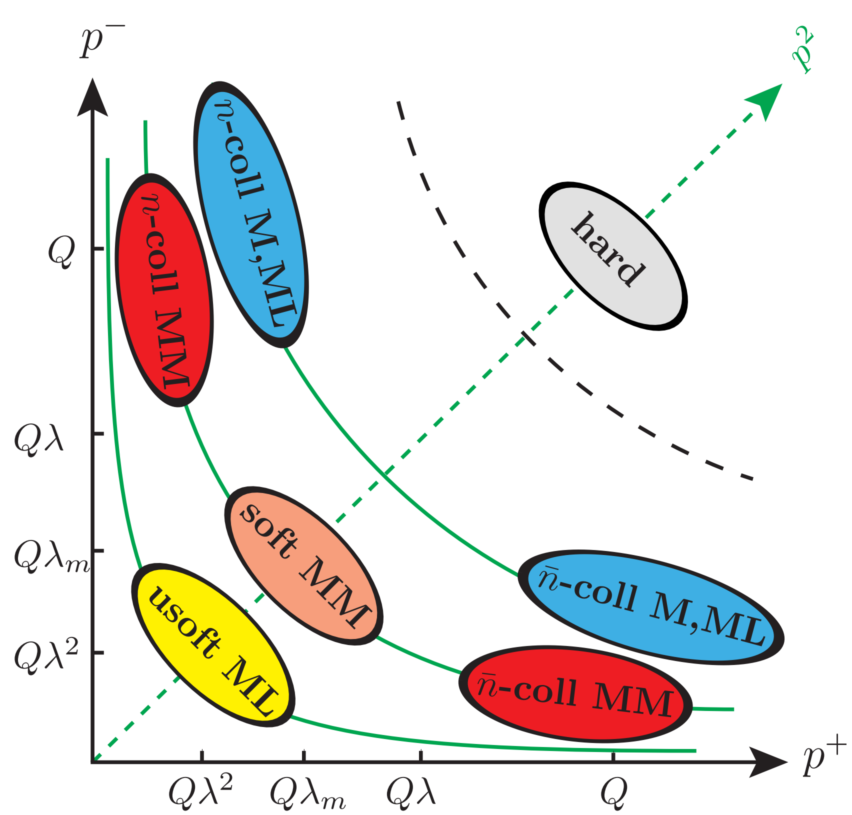
Soft function modification



if enough energy to produce
pair of heavy quarks

Scenario IV





(c)$1 > \lambda > \lambda_M > \lambda^2$

hard scale μ_H

$n_l + 1$

massless evolution

jet scale μ_J

$n_l + 1$

massless evolution

soft scale μ_S

mass scale μ_m

no matching at the mass scale !