

α_s from Deep-Inelastic Scattering

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in collaboration with S. Alekhin and S.O. Moch, U. Hamburg



- Introduction
- Valence Non-Singlet Analysis
- Combined NS+S Analyses, Inclusion of Collider Data
- Comparison of Results
- Conclusions

$\alpha_s(M_Z^2)$ in 1992

G. Altarelli: QCD - 20 Years Later

NLO World-Average

[23 years ago.]

	$\alpha_s(M_Z^2)$
R_τ	$0.117^{+0.010}_{-0.016}$
DIS	0.112 ± 0.007
Υ Decays	0.110 ± 0.010
$R_{e^+e^-}(s < 62 \text{ GeV})$	0.140 ± 0.020
$p\bar{p} \rightarrow W + jets$	0.121 ± 0.024
$\Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow l\bar{l})$	0.132 ± 0.012
Jets at LEP	0.122 ± 0.009
Average	0.118 ± 0.007

© NLO: still right, but for very different reasons.

NLO error: now down to $\sim 0.0050 - 0.0040$ (TH: scale uncertainty)

Λ_{QCD} and $\alpha_s(M_Z^2)$

older values: 2000 \lesssim date \lesssim 2007

NLO					NNLO				
$\alpha_s(M_Z^2)$	expt	theory	Ref.	$\alpha_s(M_Z^2)$	expt	theory	Ref.		
CTEQ6	0.1165 ± 0.0065		[1]	MRST03	0.1153 ± 0.0020	± 0.0030	[2]		
MRST03	0.1165 ± 0.0020	± 0.0030	[2]	A02	0.1143 ± 0.0014	± 0.0009	[3]		
A02	0.1171 ± 0.0015	± 0.0033	[3]	SY01(ep)	0.1166 ± 0.0013		[8]		
ZEUS	0.1166 ± 0.0049		[4]	SY01(ν N)	0.1153 ± 0.0063		[8]		
H1	0.1150 ± 0.0017	± 0.0050	[5]	GRS	0.111		[10]		
BCDMS	0.110 ± 0.006		[6]	A06	0.1128 ± 0.0015		[11]		
GRS	0.112		[10]	BBG	0.1134	$+0.0019 / - 0.0021$	[9]		
BBG	0.1148 ± 0.0019		[9]	N³LO					
BB (pol)	0.113 ± 0.004	$+0.009 / - 0.006$	[7]	$\alpha_s(M_Z^2)$	expt	theory	Ref.		
				BBG	0.1141	$+0.0020 / - 0.0022$	[9]		

NLO at least: scale errors of ± 0.0050

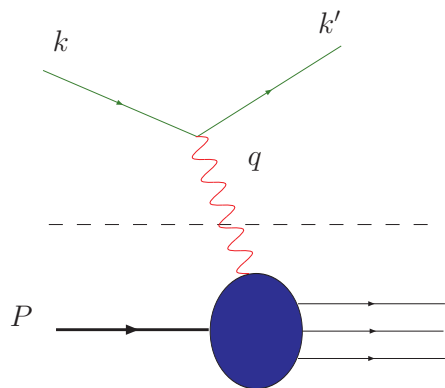
NNLO systematic shifts down

N³LO slight upward shift



BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26 \text{MeV}$

Deep Inelastic Scattering

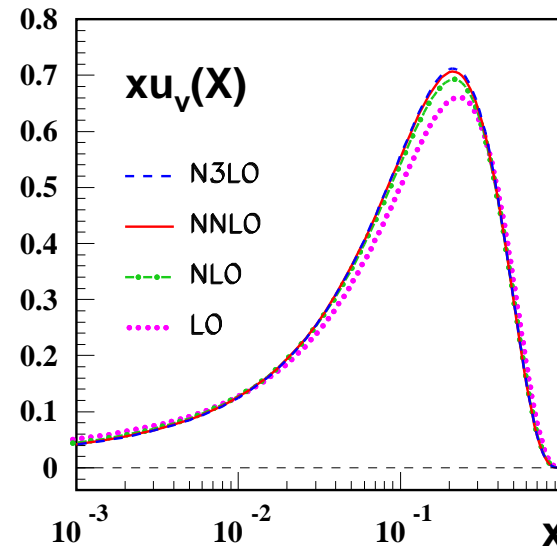
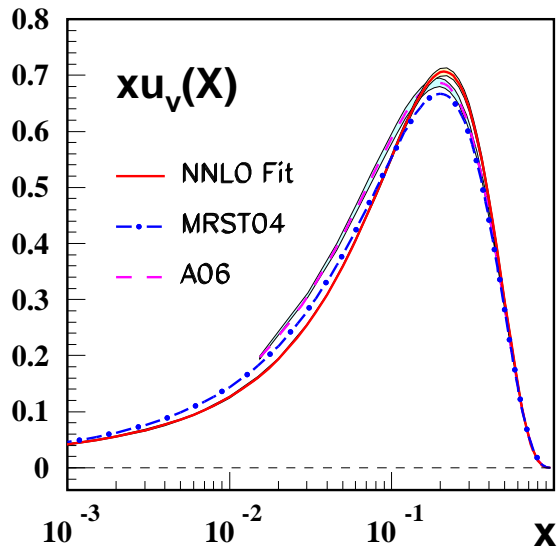


$$\begin{aligned} &\longrightarrow L_{\mu\nu} & Q^2 &:= -q^2, & x &:= \frac{Q^2}{2pq} \\ & & \nu &:= \frac{Pq}{M}, \\ &\longrightarrow W_{\mu\nu} & \frac{d\sigma}{dQ^2 dx} &\sim W_{\mu\nu} L^{\mu\nu} \end{aligned}$$

$$\begin{aligned} W_{\mu\nu}(q, P, s) &= \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, s [J_\mu^{em}(\xi), J_\nu^{em}(0)] P, s \rangle \\ &= \frac{1}{2x} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_L(x, Q^2) \\ &\quad + \frac{2x}{Q^2} \left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu} \right) F_2(x, Q^2) \end{aligned}$$

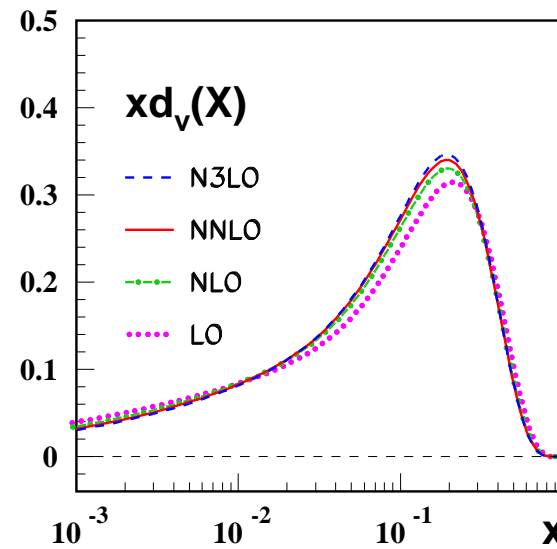
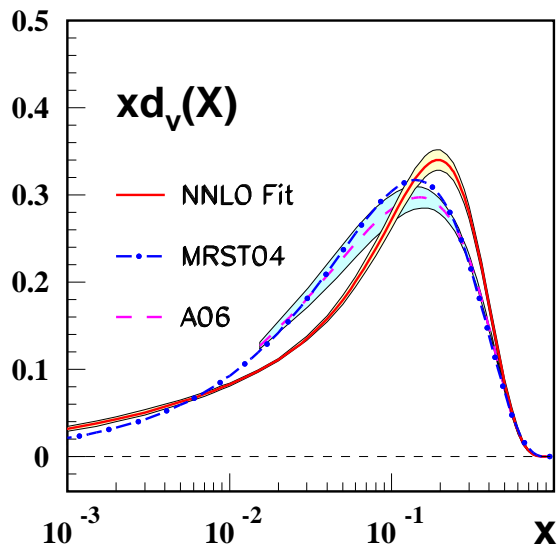
Structure Functions: $F_{2,L}$ contain light and heavy quark contributions
 \implies Further Inclusion of Collider Data: DY (W^\pm, Z), $t\bar{t}$, jets.

World Data Analysis: Valence Distributions (NS)



World data:
NS-analysis

$$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$$



$N^3\text{LO}$:

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher, A. Guffanti
Nucl.Phys. B774 (2007) 182.

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\rightarrow \pm 2$ MeV in Λ_{QCD}

$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)^2}}{\gamma_n^{(1)}}$$

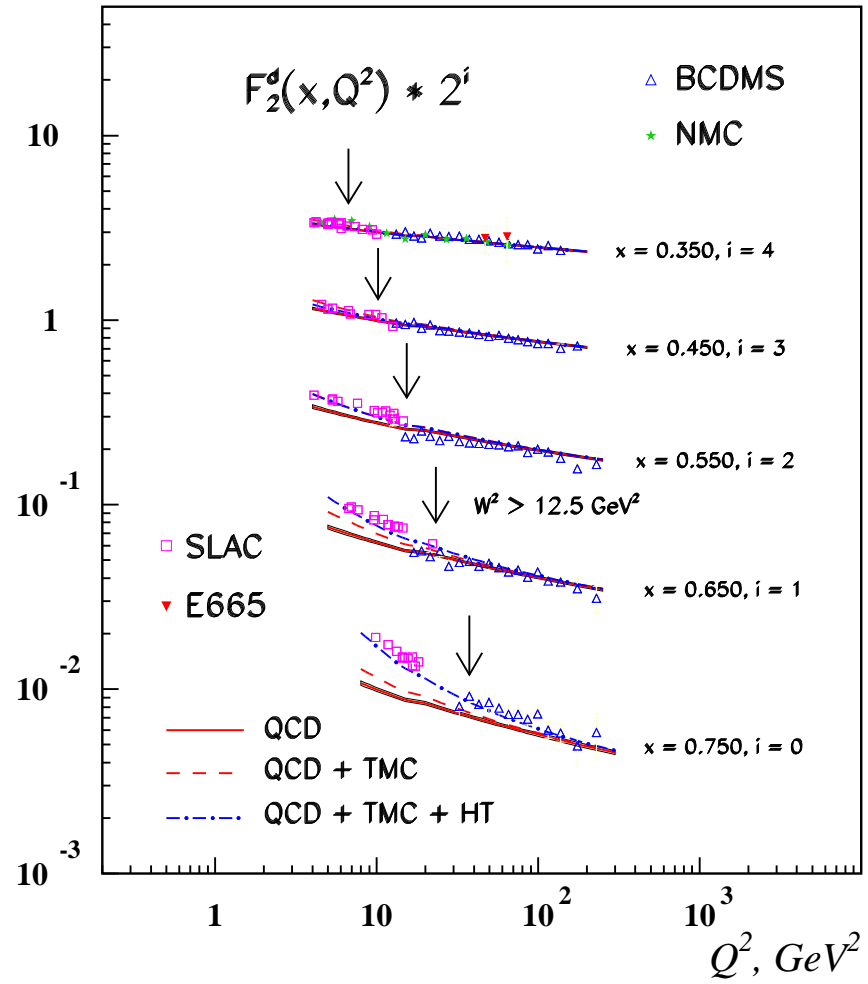
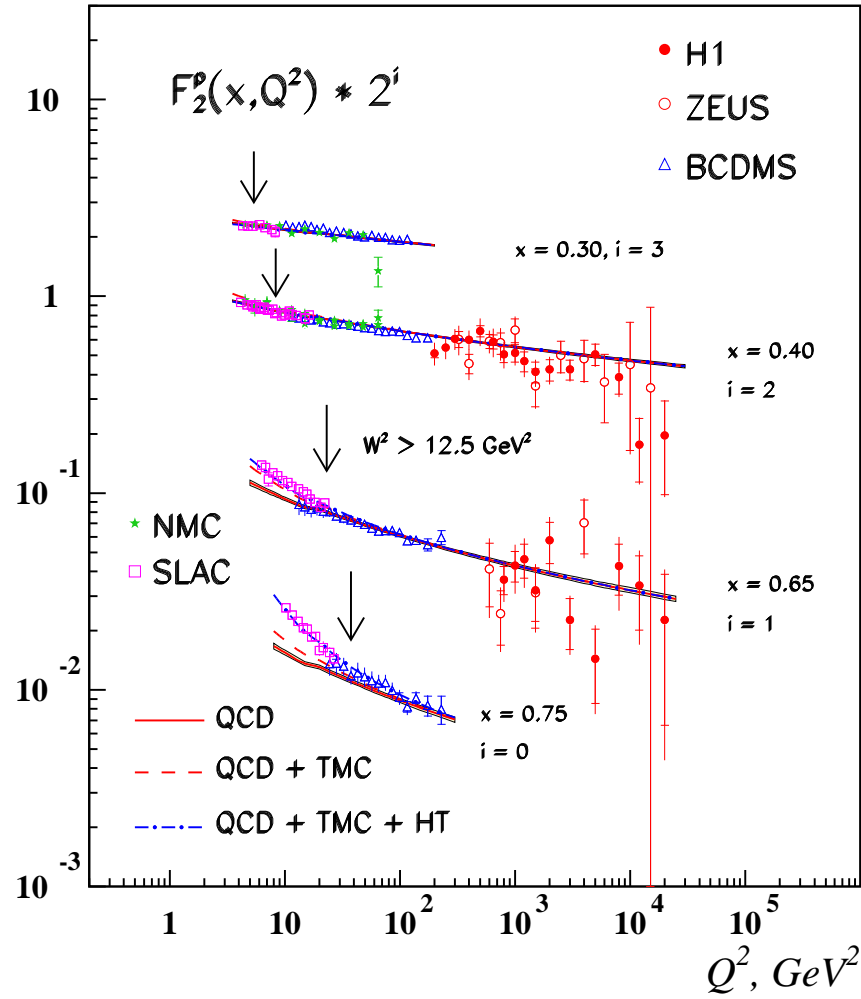
Baikov & Chetyrkin, April 2006:

$$\begin{aligned} \gamma_2^{3;NS} = & \frac{32}{9}a_s + \frac{9440}{243}a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3 \right] a_s^3 \\ & + \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5 \right] a_s^4 \end{aligned}$$

The results agree better than 20%.

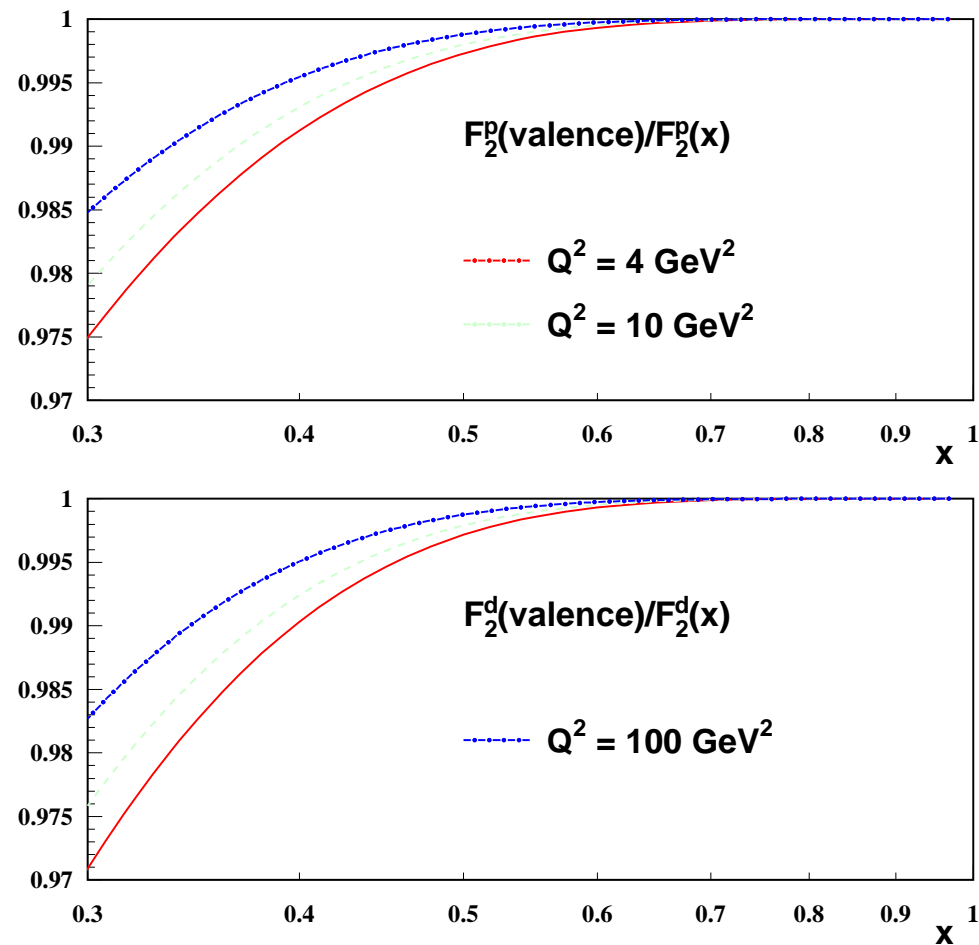
This behaviour is even confirmed for the moments $N = 3, 4$ by Baikov et al. 2013. The moments for $N = 2, 4$ were confirmed by Velizhanin 2012, 2014.

Valence Distributions



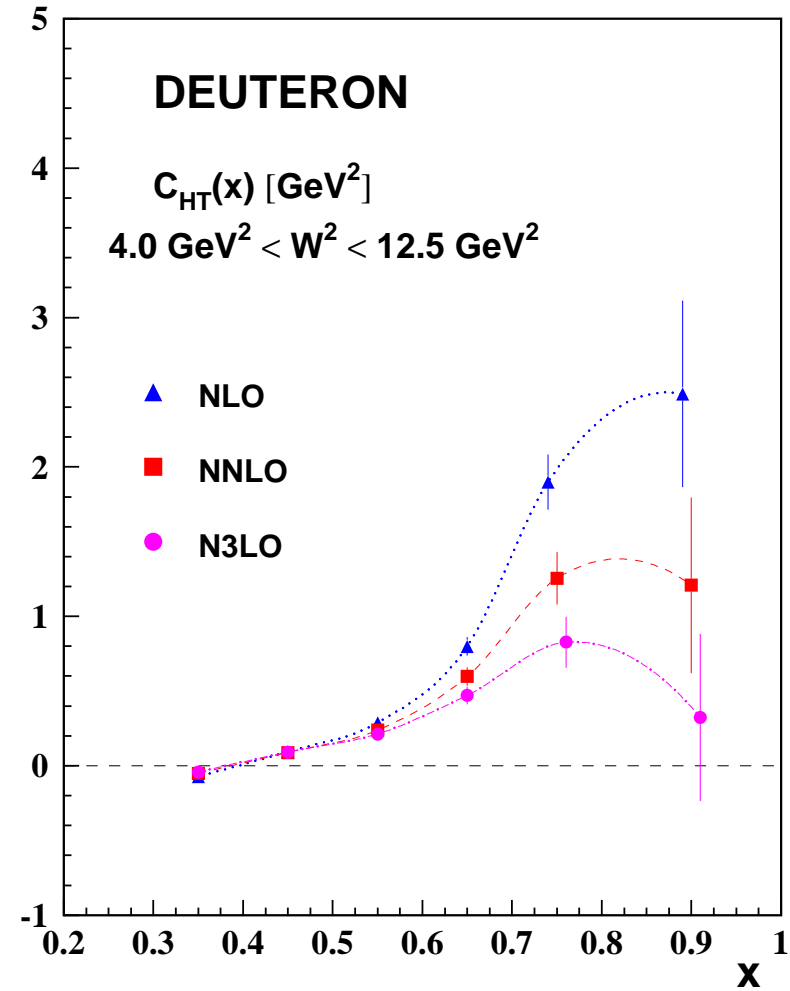
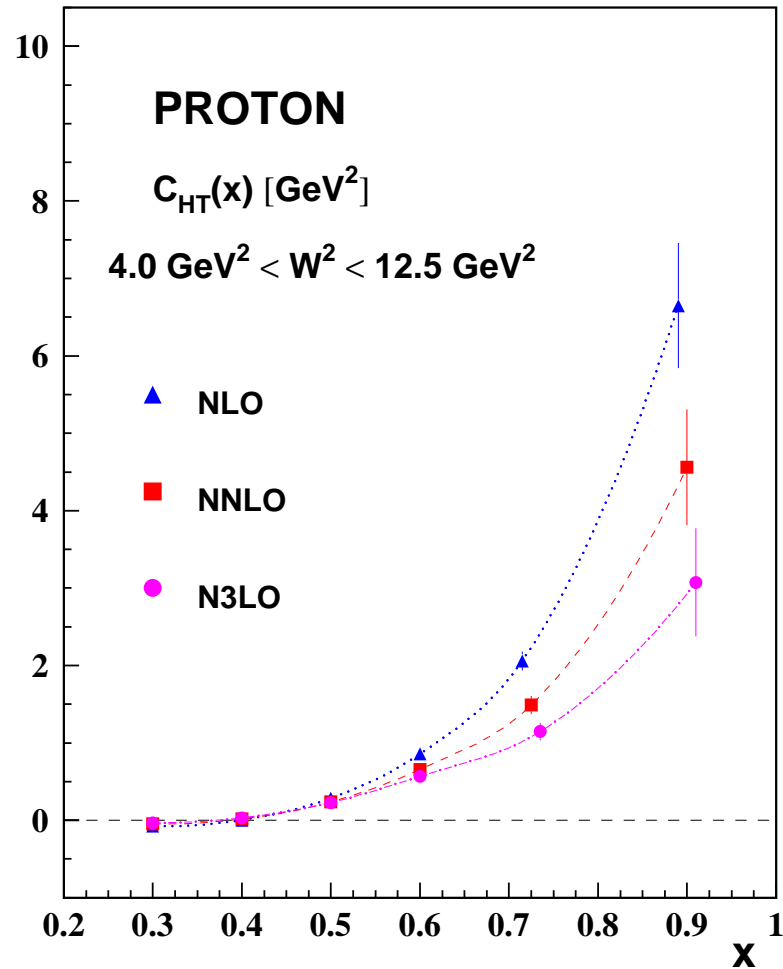
Higher Twist Contributions in the Valence Region

J.B. and H. Böttcher, 2012 (BB) [1207.3170 hep-ph]: NS-tails at NNLO :



Using ABKM09 we corrected for non NS-tails in $F_2(x, Q^2)$.

Valence Distributions: higher twist



- agreement between p and d analysis, J.B., H. Böttcher Phys.Lett. B662 (2008) 336
- LGT determination of interest

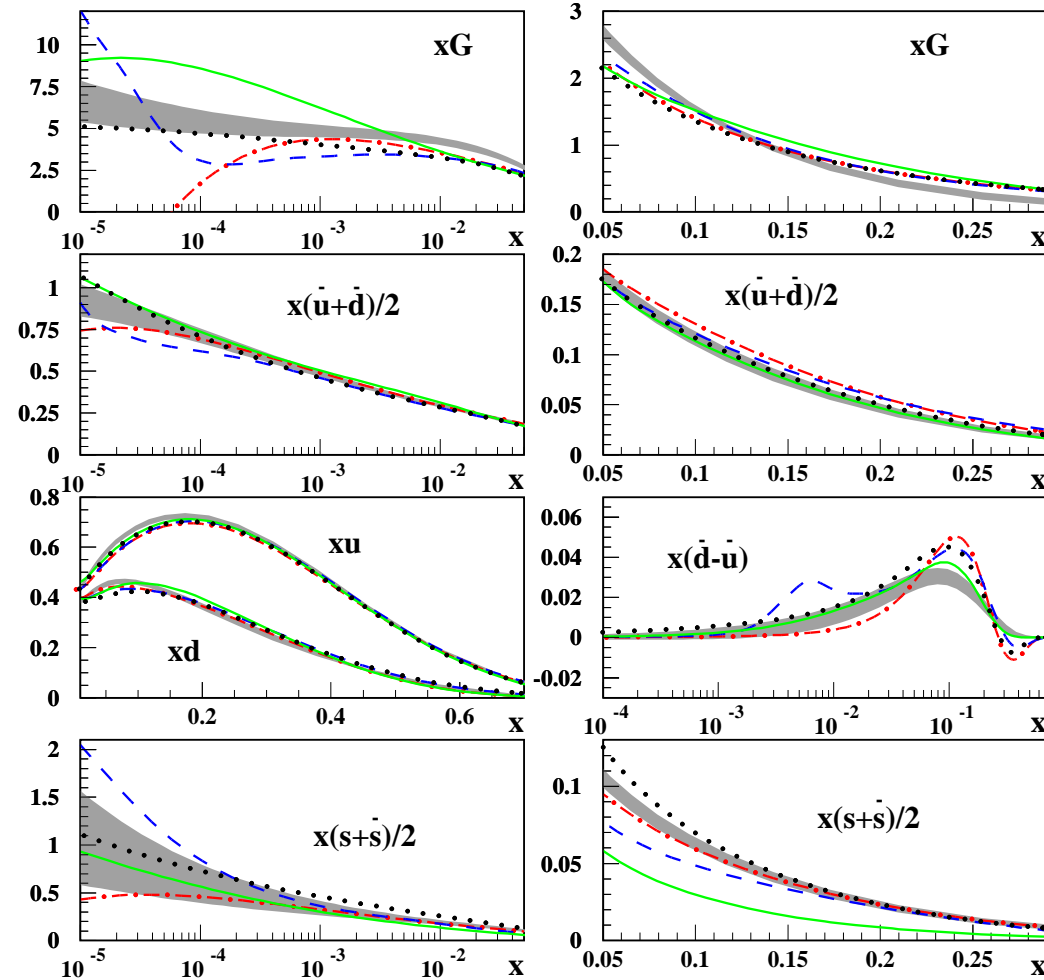
$$\alpha_s(M_Z^2)$$

Experiment	$\alpha_s(M_Z)$			
	NLO_{exp}	NLO	NNLO	N ³ LO*
BCDMS	0.1111 ± 0.0018	0.1138 ± 0.0007	0.1126 ± 0.0007	0.1128 ± 0.0006
NMC	$0.117 \begin{smallmatrix} + 0.011 \\ - 0.016 \end{smallmatrix}$	0.1166 ± 0.0039	0.1153 ± 0.0039	0.1153 ± 0.0035
SLAC		0.1147 ± 0.0029	0.1158 ± 0.0033	0.1152 ± 0.0027
BBG		0.1148 ± 0.0019	0.1134 ± 0.0020	0.1141 ± 0.0021
BB		0.1147 ± 0.0021	0.1132 ± 0.0022	0.1137 ± 0.0022

Table 6: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS and NMC at NLO with the results of the flavor non-singlet fits BBG and BB of the DIS flavor non-singlet world data, at NLO, NNLO, and N³LO* with the response of the individual data sets, combined for the experiments BCDMS NMC SLAC.

S+NS DIS Analysis (NNLO)

$\mu=2 \text{ GeV}, n_f=4$



ABM12: shaded region, JR: full line (green), NN23: dashes (blue), MRST: dash-dotted (red), CT10: dots.

Higher Twist Contributions

ABM11:

$$F_i(x, Q^2) = F_i^{TMC, \tau=2}(x, Q^2) + \frac{H_i^4(x)}{Q^2} + \frac{H_i^6(x)}{Q^4} + \dots$$

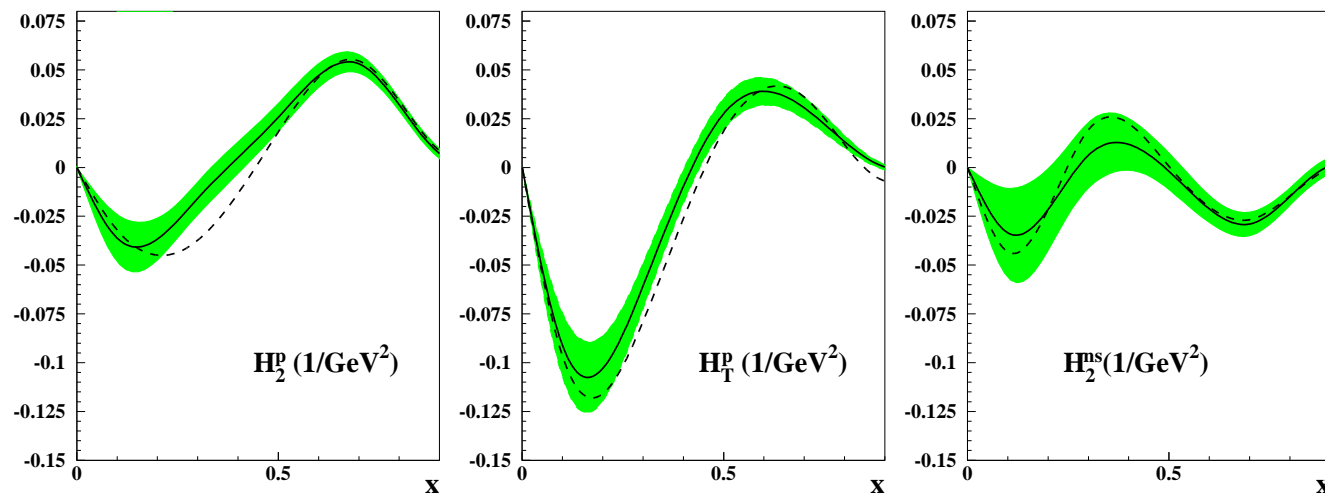


Figure 10: The central values (solid line) and the 1σ bands (shaded area) for the coefficients of the twist-4 terms of the inclusive DIS structure functions obtained from our NNLO fit (left panel: F_2 of the proton, central panel: F_T of the proton, right panel: non-singlet F_2). The central values of the twist-4 coefficients obtained from our NLO fit are shown for comparison (dashes).

$$\alpha_s(M_Z^2)$$

ABM11

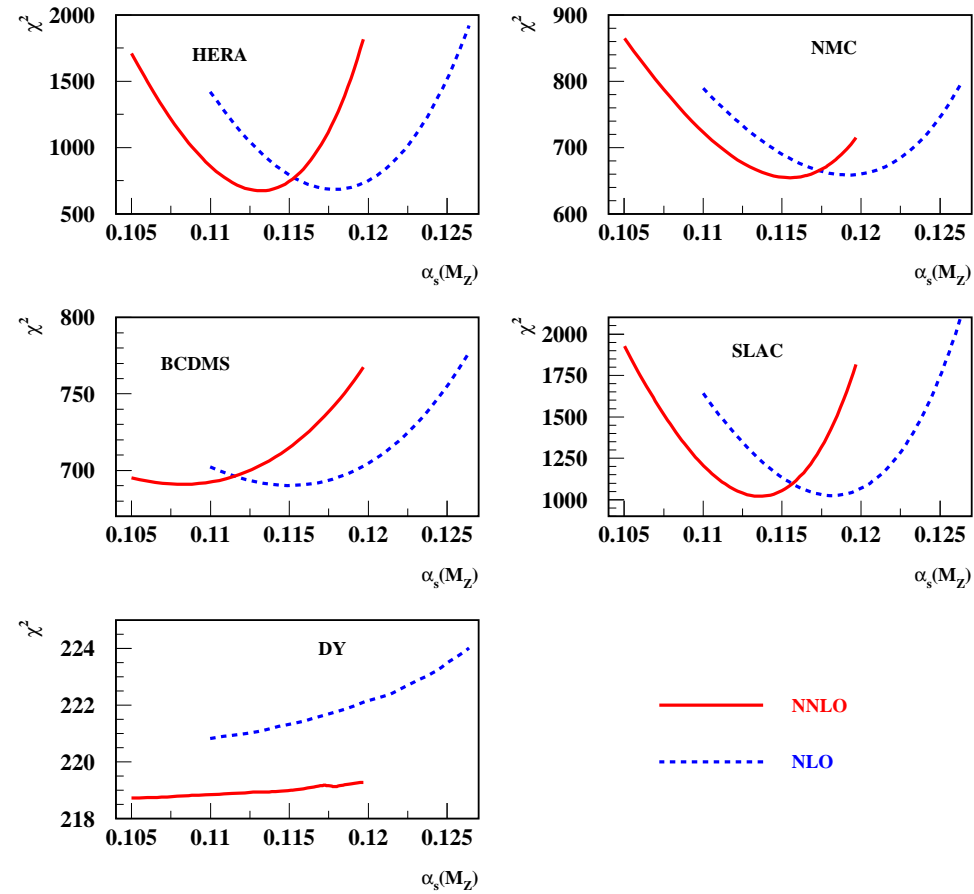


Figure 19: The χ^2 -profile versus the value of $\alpha_s(M_Z)$ for the data sets used, all calculated with the PDF and HT parameters fixed at the values obtained from the fits with $\alpha_s(M_Z)$ released (solid lines: NNLO fit, dashes: NLO one).

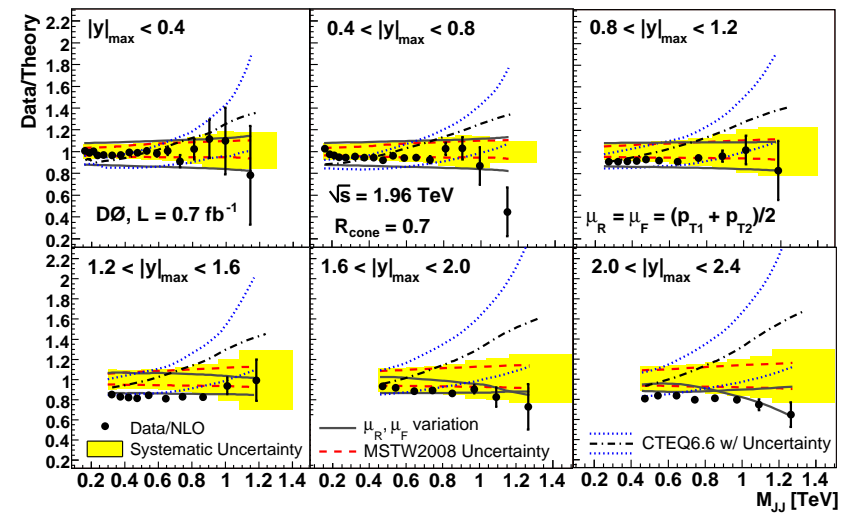
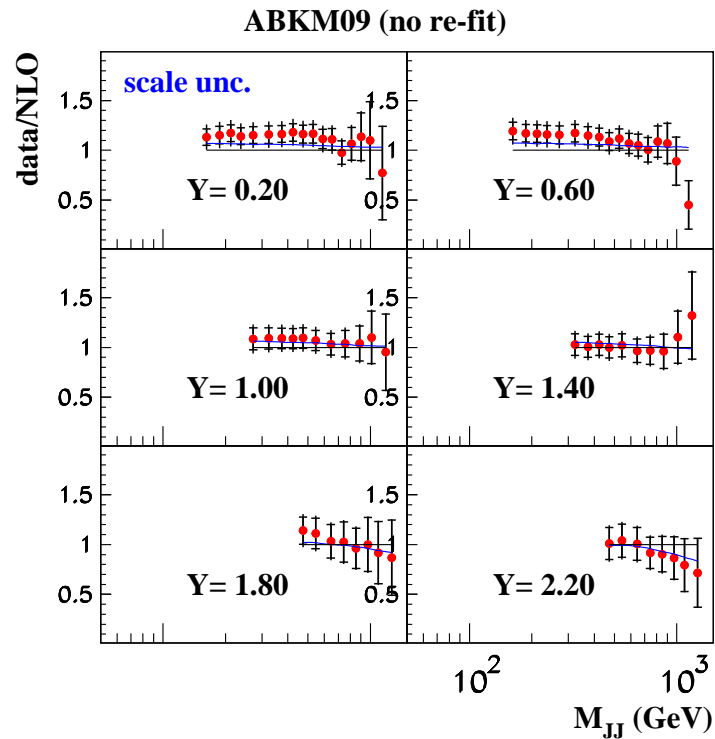
$$\alpha_s(M_Z^2)$$

Experiment	$\alpha_s(M_Z)$		
	NLO_{exp}	NLO	NNLO
BCDMS	0.1111 ± 0.0018	0.1150 ± 0.0012	0.1084 ± 0.0013
NMC	$0.117 \begin{array}{l} + 0.011 \\ - 0.016 \end{array}$	0.1182 ± 0.0007	0.1152 ± 0.0007
SLAC		0.1173 ± 0.0003	0.1128 ± 0.0003
HERA comb.		0.1174 ± 0.0003	0.1126 ± 0.0002
DY		0.108 ± 0.010	0.101 ± 0.025
ABM11		0.1180 ± 0.0012	0.1134 ± 0.0011

Tabelle 4: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS and NMC at NLO with the individual results of the fit in the present analysis at NLO and NNLO for the HERA data the NMC data the BCDMS data the SLAC data and the DY data.

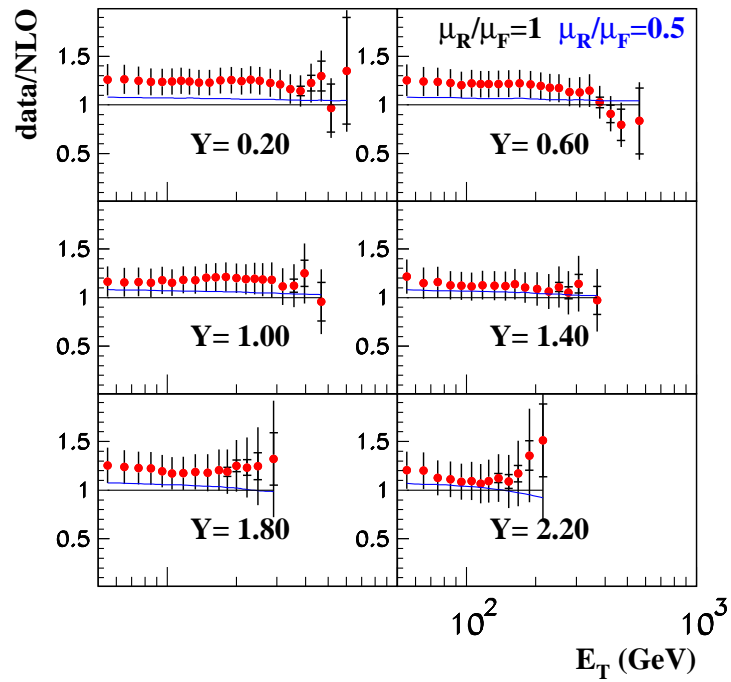
The values of $\alpha_s(M_Z^2)$ in **NLO** and **NNLO** fits are **different**.

ABM fits including Jet Data: D0 run II dijet

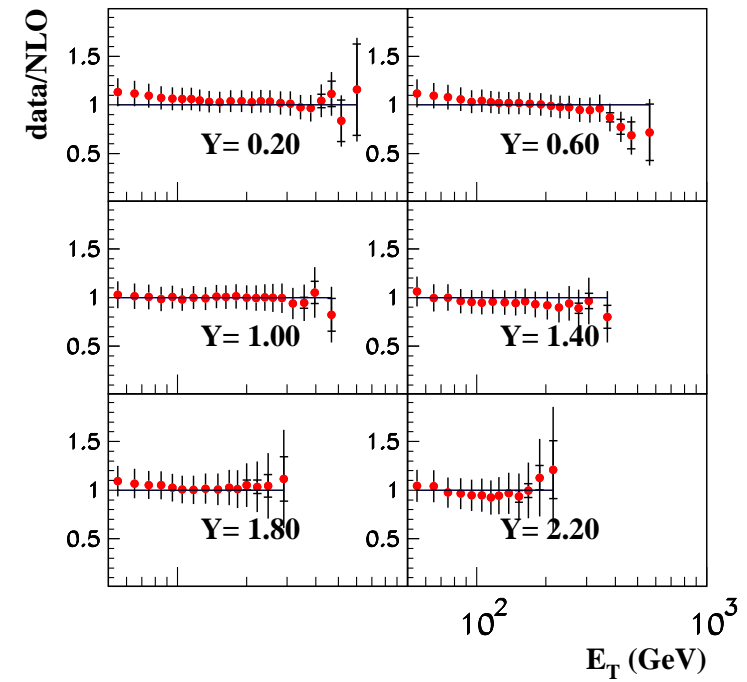


ABM (2011). Note that the cross section is known to NLO only !

D0 run II djet data



before the fit



after the fit

ABM (2011) $\chi^2 = 104/110$

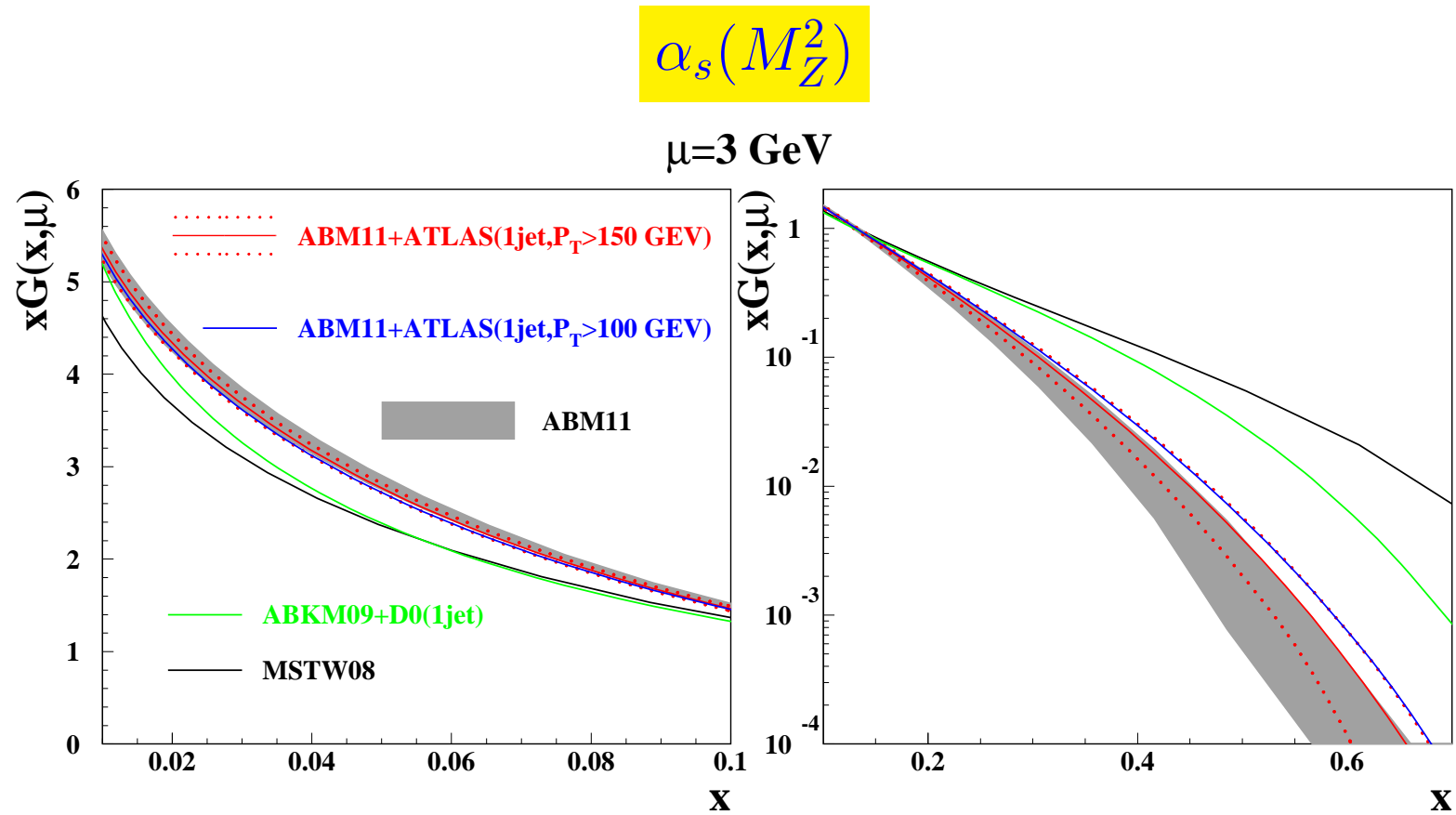


Figure 28: Gluon distribution obtained by including the ATLAS jet data into the ABM11 analysis.

$\alpha_s(M_Z^2)$: Inclusion of Tevatron Jets (NLO)

Experiment	$\alpha_s(M_Z)$		
	NLO_{exp}	NLO	NNLO*
D0 1 jet	$0.1161^{+0.0041}_{-0.0048}$	0.1190 ± 0.0011	0.1149 ± 0.0012
D0 2 jet		0.1174 ± 0.0009	0.1145 ± 0.0009
CDF 1 jet (cone)		0.1181 ± 0.0009	0.1134 ± 0.0009
CDF 1 jet (k_{\perp})		0.1181 ± 0.0010	0.1143 ± 0.0009
ABM11		0.1180 ± 0.0012	0.1134 ± 0.0011

Table 5: Comparison of the values of $\alpha_s(M_Z)$ obtained by D0 with the ones based on including individual data sets of Tevatron jet data into the analysis at NLO. The NNLO* fit refers to the NNLO analysis of the DIS and DY data together with the NLO and soft gluon resummation corrections (next-to-leading logarithmic accuracy) for the 1 jet inclusive data.

S. Alekhin, J.B., S. Moch, Phys.Rev. D86 (2012) 054009

⇒ value depends on data set

⇒ value depends on the jet algorithm

⇒ no large values

$\alpha_s(M_Z^2)$: NNPDF vs Data Sets

Experiment	$\alpha_s(M_Z)$		
	NLO_{exp}	NLO	NNLO
BCDMS	0.1111 ± 0.0018	0.1204 ± 0.0015	0.1158 ± 0.0015
NMC _p	$0.117 \begin{matrix} + 0.011 \\ - 0.016 \end{matrix}$	0.1192 ± 0.0018	0.1150 ± 0.0020
NMC _{pd}			0.1146 ± 0.0107
SLAC		> 0.124	> 0.124
HERA I		0.1223 ± 0.0018	0.1199 ± 0.0019
ZEUS H2		0.1170 ± 0.0027	0.1231 ± 0.0030
ZEUS F2C		0.1144 ± 0.0060	
NuTeV		0.1252 ± 0.0068	0.1177 ± 0.0039
E605		0.1168 ± 0.0100	
E866		0.1135 ± 0.0029	
CDF Wasy		0.1181 ± 0.0060	
CDF Zrap		0.1150 ± 0.0034	0.1205 ± 0.0081
D0 Zrap		0.1227 ± 0.0067	
CDF R2KT		0.1228 ± 0.0021	0.1225 ± 0.0021
D0 R2CON	$0.1161 \begin{matrix} + 0.0041 \\ - 0.0048 \end{matrix}$	0.1141 ± 0.0031	0.1111 ± 0.0029
NN21		0.1191 ± 0.0006	0.1173 ± 0.0007

Table 7: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS, NMC, and D0 at NLO with the results of NN21 for the fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed.

Experiment	$\alpha_s(M_Z)$			
	NLO _{exp}	NLO	NNLO	
BCDMS $\mu p, F_2$	0.1111 \pm 0.0018	–	0.1085 \pm 0.0095	
BCDMS $\mu d, F_2$		0.1135 \pm 0.0155	0.1117 \pm 0.0093	
NMC $\mu p, F_2$		0.117 \pm $^{+0.011}_{-0.016}$	0.1275 \pm 0.0105	0.1217 \pm 0.0077
NMC $\mu d, F_2$			0.1265 \pm 0.0115	0.1215 \pm 0.0070
NMC $\mu n/\mu p$			0.1280	0.1160
E665 $\mu p, F_2$			0.1203	–
E665 $\mu d, F_2$			–	–
SLAC ep, F_2			0.1180 \pm 0.0060	0.1140 \pm 0.0060
SLAC ed, F_2			0.1270 \pm 0.0090	0.1220 \pm 0.0060
NMC,BCDMS,SLAC, F_L			0.1285 \pm 0.0115	0.1200 \pm 0.0060
E886/NuSea pp , DY %citeWebb:2003bj			–	0.1132 \pm 0.0088
E886/NuSea pd/pp , DY			0.1173 \pm 0.107	0.1140 \pm 0.0110
NuTeV $\nu N, F_2$		0.1207 \pm 0.0067	0.1170 \pm 0.0060	
CHORUS $\nu N, F_2$		0.1230 \pm 0.0110	0.1150 \pm 0.0090	
NuTeV $\nu N, xF_3$		0.1270 \pm 0.0090	0.1225 \pm 0.0075	
CHORUS $\nu N, xF_3$		0.1215 \pm 0.0105	0.1185 \pm 0.0075	
CCFR		0.1190	–	
NuTeV $\nu N \rightarrow \mu\mu X$		0.1150 \pm 0.0170	–	
H1 ep 97-00, σ_r^{NC}		0.1250 \pm 0.0070	0.1205 \pm 0.0055	
ZEUS ep 95-00, σ_r^{NC}		0.1235 \pm 0.0065	0.1210 \pm 0.0060	
H1 ep 99-00, σ_r^{CC}		0.1285 \pm 0.0225	0.1270 \pm 0.0200	
ZEUS ep 99-00, σ_r^{CC}		0.1125 \pm 0.0195	0.1165 \pm 0.0095	
H1/ZEUS ep, F_2^{charm}		–	0.1165 \pm 0.0095	
H1 ep 99-00 incl. jets	0.1168 \pm $^{+0.0049}_{-0.0034}$	0.1127 \pm 0.0093		
ZEUS ep 96-00 incl. jets	0.1208 \pm $^{+0.0048}_{-0.0040}$	0.1175 \pm 0.0055		
D0 II $p\bar{p}$ incl. jets	0.1161 \pm $^{+0.0041}_{-0.0048}$	0.1185 \pm 0.0055	0.1133 \pm 0.0063	
CDF II $p\bar{p}$ incl. jets		0.1205 \pm 0.0045	0.1165 \pm 0.0025	
D0 II $W \rightarrow l\nu$ asym.		–	–	
CDF II $W \rightarrow l\nu$ asym.		–	–	
D0 II Z rap.		0.1125 \pm 0.0100	0.1136 \pm 0.0084	
CDF II Z rap.		0.1160 \pm 0.0070	0.1157 \pm 0.0067	
MSTW		0.1202 \pm $^{+0.0012}_{-0.0015}$	0.1171 \pm 0.0014	

Table 8: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS, NMC, HERA-jet and D0 at NLO with the results of the MSTW fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed, cf. Figs. 7a and 7b in MSTW08. Entries not given correspond to $\alpha_s(M_Z)$ central values below 0.110 or above 0.130; in case no errors are assigned these are larger than the bounds provided in form of the plots in MSTW08.

Nuclear Targets

ABM: none

CTEQ: CCFR, CDHSW

JR: none

MMHT: NUTEV, CHORUS

NNPDF: NUTEV, CHORUS

Jet Fits: NLO only!

ABM: singly selected: D0, CDF, ATLAS

CTEQ: CDF, D0, ATLAS, CMS

JR: CDF, D0

MMHT: H1, ZEUS incl., CDF, D0, ATLAS, CMS

NNPDF: CDF, D0, ATLAS, CMS

- Different nuclear targets!

It is likely that the QCD evolution differs off nuclear targets due to the internal composition (π 's, confinement size varies, etc.; various sources leading to the EMC effect.)

Impact on F_L ? Differing EMC effects for different structure functions (NS, S).

- Mixing **NLO** and **NNLO** fits necessarily does not lead to an extraction of α_s^{NNLO} .
- As the NLO values are higher than those at NNLO α_s **increases**.
- In the strict sense MSTW, MMHT, CT14, and NNPDF are not pure NNLO analyses, but contain quite a part of data currently being fitted at NLO only.

$\alpha_s(M_Z^2)$ - Higher Twist Correlation

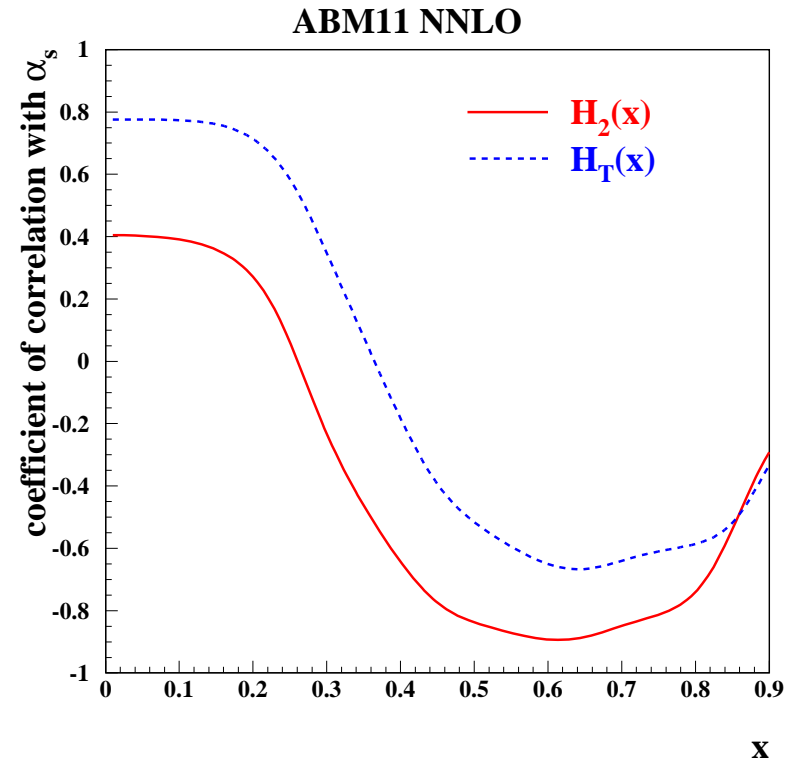


Figure 20: The correlation coefficient of $\alpha_s(M_Z)$ with the nucleon twist-4 coefficients H_2 (solid line) and H_T (dashes) versus x as obtained in our NNLO fit.

\Rightarrow Including scales as $Q^2 < 10 \text{ GeV}^2$ requires fit of higher twist terms (Singlet Analysis).

$\alpha_s(M_Z^2)$: NNLO Comparison ABM, BBG, NNPDF, MSTW

Data Set	ABM11	BBG	NN21	MSTW
BCDMS	0.1048 ± 0.0013	0.1126 ± 0.0007	0.1158 ± 0.0015	0.1101 ± 0.0094
NMC	0.1152 ± 0.0007	0.1153 ± 0.0039	0.1150 ± 0.0020	0.1216 ± 0.0074
SLAC	0.1128 ± 0.0003	0.1158 ± 0.0034	> 0.124	$\left\{ \begin{array}{l} 0.1140 \pm 0.0060 \text{ ep} \\ 0.1220 \pm 0.0060 \text{ ed} \end{array} \right.$
HERA	0.1126 ± 0.0002		$\left\{ \begin{array}{l} 0.1199 \pm 0.0019 \\ 0.1231 \pm 0.0030 \end{array} \right.$	0.1208 ± 0.0058
DY	0.101 ± 0.025	—	—	0.1136 ± 0.0100
	0.1134 ± 0.0011	0.1134 ± 0.0020	0.1173 ± 0.0007	0.1171 ± 0.0014

Table 9: Comparison of the pulls in $\alpha_s(M_Z)$ per data set between the ABM11, BBG, NN21, MSTW analyses at NNLO.

\implies Despite the NNLO values for $\alpha_s(M_Z^2)$ of NNPDF and MSTW are close to each other, the contribution of the different data sets differ considerably.

Heavy Flavor Treatment

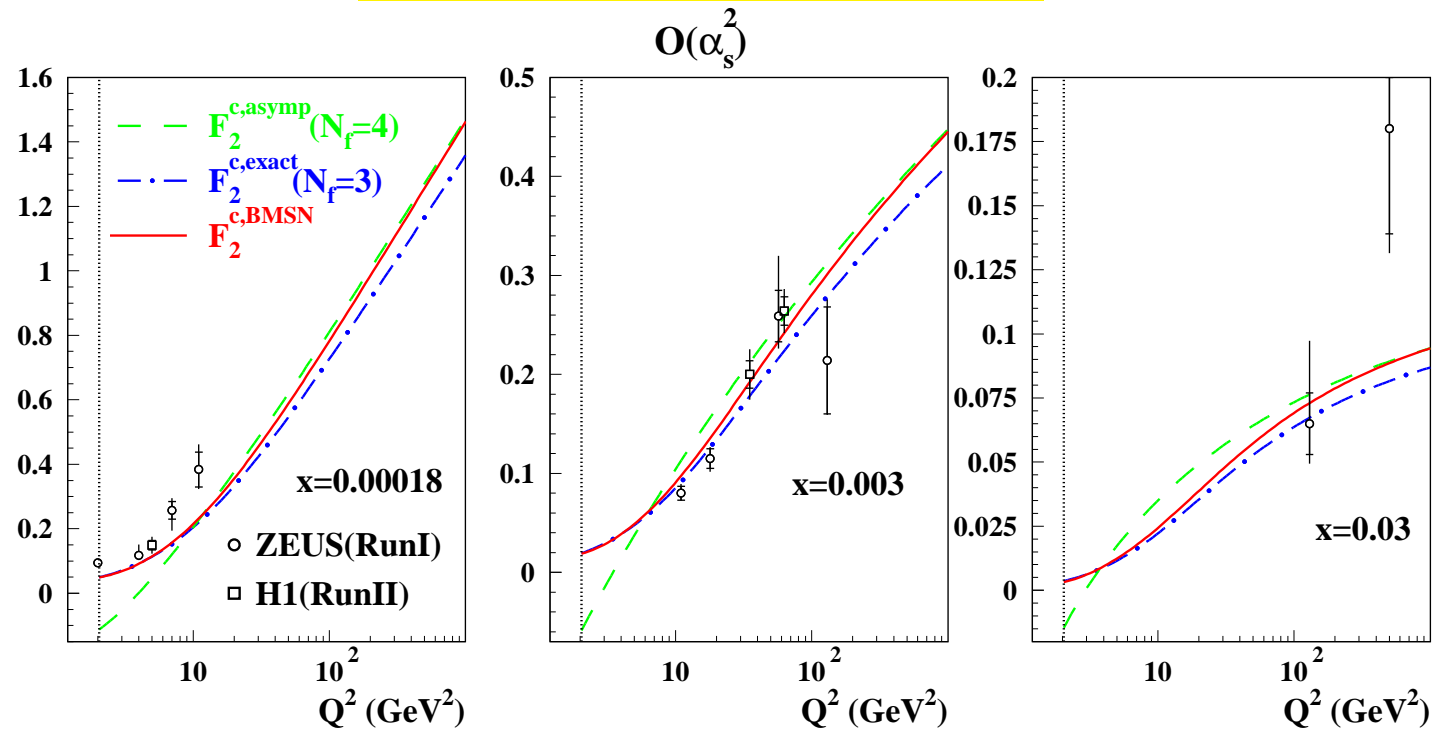


Figure 1: Comparison of F_2^c in different schemes to H1- and ZEUS-data. Solid lines: GMVFN scheme in the BMSN prescription, dash-dotted lines: 3-flavor scheme, dashed lines: 4-flavor scheme. The vertical dotted line denotes the position of the charm-quark mass $m_c = 1.43$ GeV.

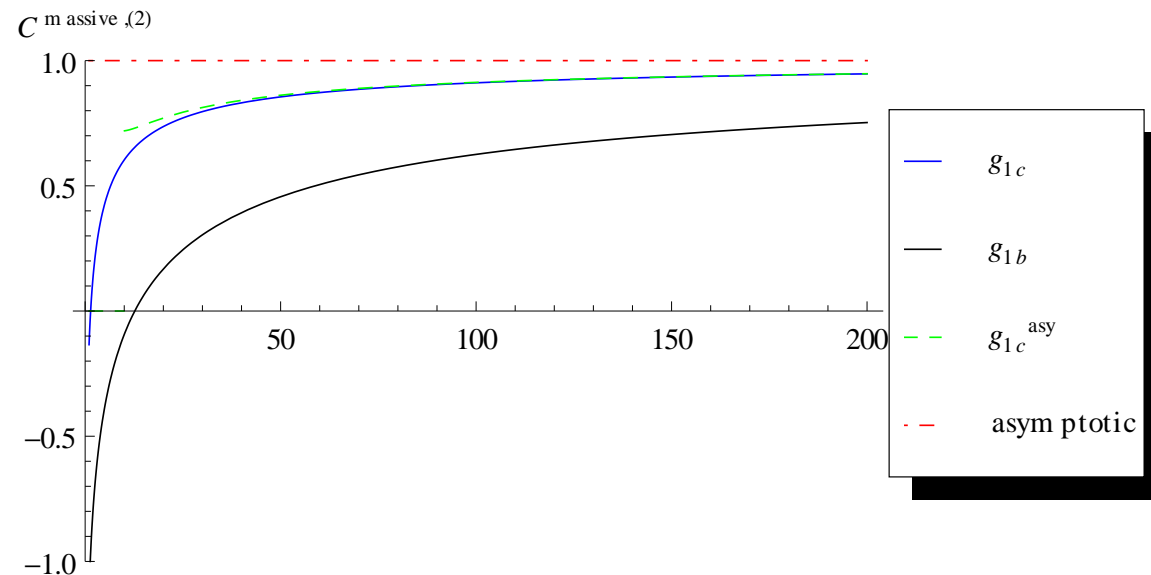
Interpolating scheme: BSMN (1996) to be used; many private models in use (RT, etc.)

Heavy Flavor Treatment: consistent determination of m_c

Alekhin, JB, Daum, Lipka, Moch:Phys.Lett. B720 (2013) 172.

$m_c^{\overline{\text{MS}}}$	$1.24 \pm 0.03 \begin{smallmatrix} +0.03 \\ -0.03 \end{smallmatrix}$	ABDLM, DIS, FFNS, $\chi^2 = 61/52$
$m_c^{\overline{\text{MS}}}$	1.279 ± 0.013	Chetyrkin et al., e^+e^-
$m_c^{\overline{\text{MS}}}$	1.275 ± 0.025	PDG
m_c^{Pole}	1.67 ± 0.07	PDG
m_c^{Pole}	1.25	MMHT [1510.02332] GMVFNS; $\alpha_s = 0.1167$; $\chi^2 = 72/52$

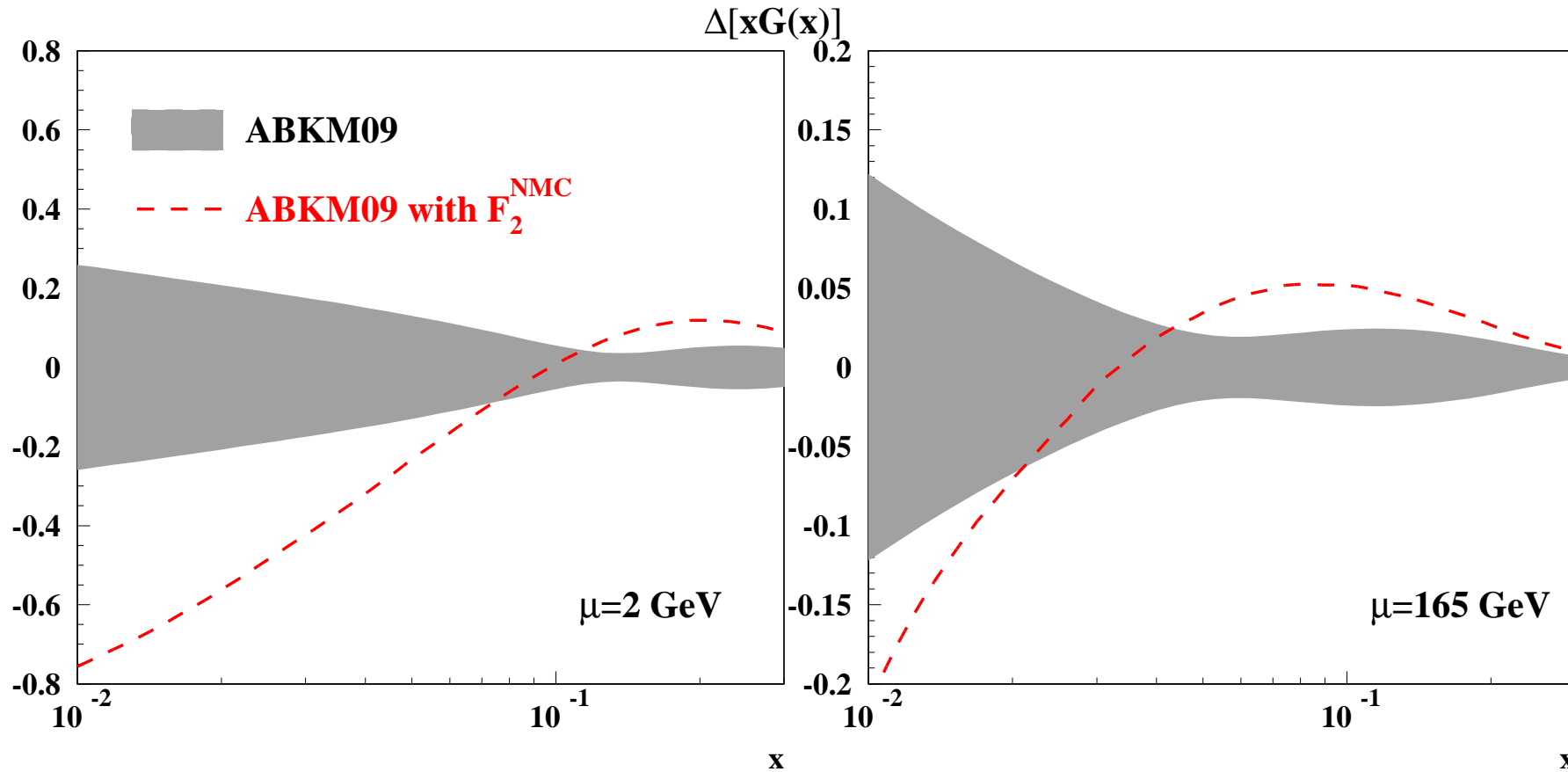
Heavy Flavor Treatment



JB, G. Falcioni, A. De Freitas, DESY 15-171

- Exact $O(\alpha_s^2)$ calculations show the gradual interpolation for charm and bottom effects in DIS as a function of $\xi = Q^2/m_H^2$ bridging from $N_F \rightarrow N_F + 1$. [Note the negative corrections at low scales!]
- The matching at $Q^2 = m_c^2$ for $N_F \rightarrow N_F + 1$ is definitely sub-optimal and may lead to wrong results.
- Adding as complete as possible HQ effects in the fixed flavor scheme in the lower Q^2 range is the better option. Van Neerven 1993; Glück, Reya, Stratmann, 1994; ABM2009–

Effect on the Gluon density



wrong treatment (F_2^{NMC}): larger gluon at $x \simeq 0.1$

⇒ It is important to fit the reduced cross sections, including the correct F_L -behavior to NNLO.

⇒ ABM, JR, NNPDF.

Why is MSTW's $\alpha_s(M_Z^2)$ so high ?

$\alpha_s(M_Z^2)$	with σ_{NMC}	with F_2^{NMC}	difference
NLO	0.1179(16)	0.1195(17)	+0.0026 $\simeq 1\sigma$
NNLO	0.1135(14)	0.1170(15)	+0.0035 $\simeq 2.3\sigma$
NNLO + $F_L O(\alpha_s^3)$	0.1122(14)	0.1171(14)	+0.0050 $\simeq 3.6\sigma$

S. Alekhin, J.B., S. Moch, Eur.Phys.J. C71 (2011) 1723 [arXiv:1101.5261].

\implies also fixed target data shall be analyzed using σ .

\implies This applies to NMC in particular.

- Wrong treatment of $F_L(x, Q^2)$ in NMC F_2 extraction.

\implies also necessary for BCDMS, see BBG (2006).

- MMHT still fits structure functions but includes F_L @ NLO.

- There is still a significant difference from F_L @ NLO and NNLO at low x .

Use: $W^2 > 12.5 \text{ GeV}^2$, $Q^2 > 2.5 \text{ GeV}^2$ and no HT: $\alpha_s(M_Z^2) = 0.1191 \pm 0.0016$

Use: $W^2 > 12.5 \text{ GeV}^2$, $Q^2 > 10 \text{ GeV}^2$ and no HT: $\alpha_s(M_Z^2) = 0.1134 \pm 0.0008$

NNPDF $Q^2 > 5 \text{ GeV}^2$.

NNLO Analyses

		$\alpha_s(M_Z^2)$	
SY	2001	0.1166 ± 0.0013	F_2^{ep}
SY	2001	0.1153 ± 0.0063	$xF_3^{\nu N}$ h. Nucl.
A02	2002	0.1143 ± 0.0020	
MRST03	2003	0.1153 ± 0.0020	
BBG	2004(06,12)	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO
GRS	2006	0.112	valence analysis, NNLO
A06	2006	0.1128 ± 0.0015	
JR	2008	0.1128 ± 0.0010	dynamical approach
JR	2008	0.1162 ± 0.0006	including NLO-jets
ABKM	2009	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$
ABKM	2009	0.1129 ± 0.0014	HQ: BSMN
MSTW	2009	0.1171 ± 0.0014	
Thorne	2013	0.1136	[DIS+DY+HT*]
ABM11 _J	2010	$0.1134 - 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl.
NN21	2011	$0.1174 \pm 0.0006 \pm 0.0001$	+h. Nucl.
ABM12	2013	0.1133 ± 0.0011	
ABM12	2013	0.1132 ± 0.0011	(without jets)
CTEQ	2013	0.1140	(without jets)
CTEQ	2015	$0.1150^{+0.0060}_{-0.0040}$	$\Delta\chi^2 > 1$ +h. Nucl.
MMHT	2015	0.1172 ± 0.0013	+h. Nucl.

Other Lower α_s Values			
NNLO		$\alpha_s(M_Z^2)$	
Gehrmann et al.	2009	$0.1131^{+0.0028}_{-0.0022}$	e^+e^- thrust
Abbate et al.	2010	0.1140 ± 0.0015	e^+e^- thrust
Hoang et al.	2010	0.1123 ± 0.0015	C-param. dist.
Bazavov et al.	2014	$0.1166^{+0.0012}_{-0.0008}$	lattice 2+1 fl.
CMS	2013	$0.1151^{+0.0028}_{-0.0027}$	$t\bar{t}$
NLO		$\alpha_s(M_Z^2)$	
Frederix et al.	2010	$0.1156^{+0.0041}_{-0.0034}$	$e^+e^- \rightarrow 5$ jets
H1	2009	$0.1160^{+0.0095}_{-0.0080}$	ep jets
D0	2010	$0.1156^{+0.0041}_{-0.0034}$	$p\bar{p} \rightarrow$ jets
ATLAS	2012	$0.1151^{+0.0093}_{-0.0087}$	jets
CMS	2013	0.1148 ± 0.0052	3/2 jet ratio

NNLO ep and pp jet analyses are utterly needed.

α_s with the LHeC (FCC-he)

LHeC designed for concurrent operation with LHC in its final phase.

60 GeV * 7 TeV at $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ → Novel Higgs + DIS facility and high precision PDFs.

- High Q^2 to 1 TeV² (CC with high precision) - x to 10⁻⁶ (non-linear i.a.s?)
- high L: direct cross section measurements (NC and CC) up to x=0.8:
- stand-alone determination of complete PDF set for the first time

$$u_v, d_v, \bar{u}, \bar{d}, s, \bar{s}, c, b + top$$

→ This enables high precision determination of α_s in DIS, independent of BCDMS.

case	cut [Q^2 in GeV ²]	relative precision in %
HERA only (14p)	$Q^2 > 3.5$	1.94
HERA+jets (14p)	$Q^2 > 3.5$	0.82
LHeC only (14p)	$Q^2 > 3.5$	0.15
LHeC only (10p)	$Q^2 > 3.5$	0.17
LHeC only (14p)	$Q^2 > 20.$	0.25
LHeC+HERA (10p)	$Q^2 > 3.5$	0.11

Full experimental
uncertainties
(stat+corr+uncorr)

LHeC provides precision info as on charm mass (e.g.) to control systematics at required level.

Jets not studied so far but certainly very precise also – resolve inclusive vs jet question! Wants N³LO

5. Conclusions

- The N³LO DIS analysis yields : $\alpha_s(M_Z^2) = 0.1141 \pm 0.0021$
- Correct NNLO analyses require the fit of $d^2\sigma/dxdQ^2$ and the correct description of $F_L, F_2^{c\bar{c}}$.
- NNLO $\alpha_s(M_Z^2)$ values in the range $0.1122 - 0.1147 \pm 0.0014$ are obtained.
- The various systematic shifts are understood; presently not possible to resolve $\delta\alpha_s < 0.0008$.
- The difference to the MSTW08 value can be explained.
- Consistent α_s and m_c fits are mandatory.
- PDF fits, assuming the value of α_s may lead to biases, as they are not reaching χ_{\min}^2 in general.
- NLO analyses yield systematic higher $\alpha_s(M_Z^2)$ values than NNLO analyses;
averaging of these values is not possible
- Direct relevance for the Higgs search at Tevatron and LHC and likewise for the other standard candle processes ($W/Z, t\bar{t}$).
- Many more $\alpha_s(M_Z^2)$ values at NNLO, and even at NLO, come out lower than the present World average.
- Next important analysis: inclusion of the LHC jet data in complete NNLO fits.