Heavy quark masses from production near threshold

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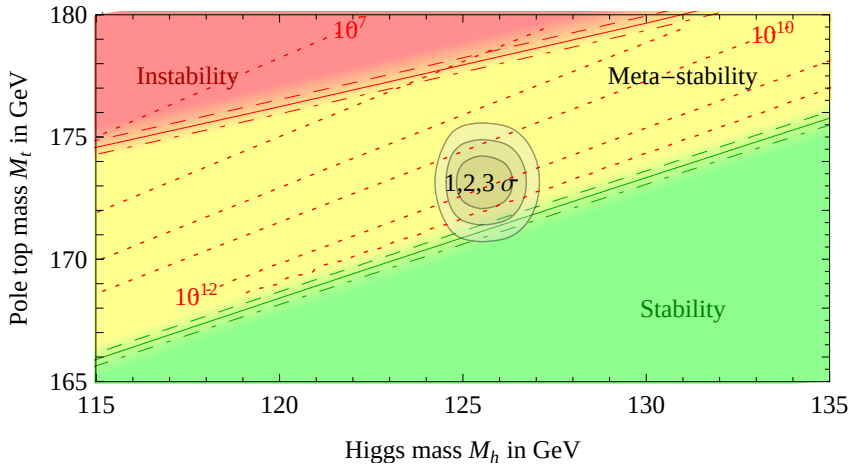
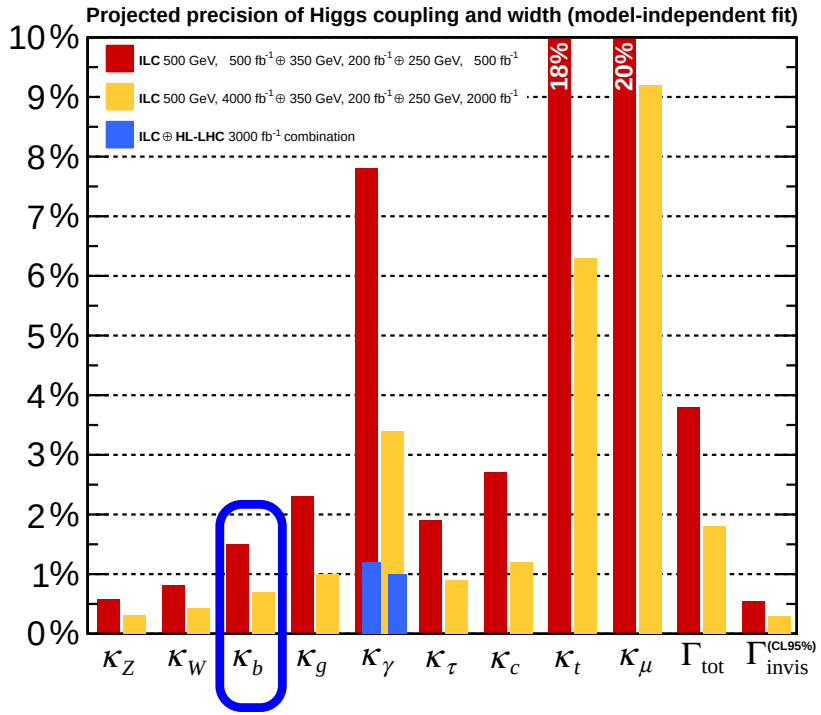
arXiv:1601.02949

Why do we care?

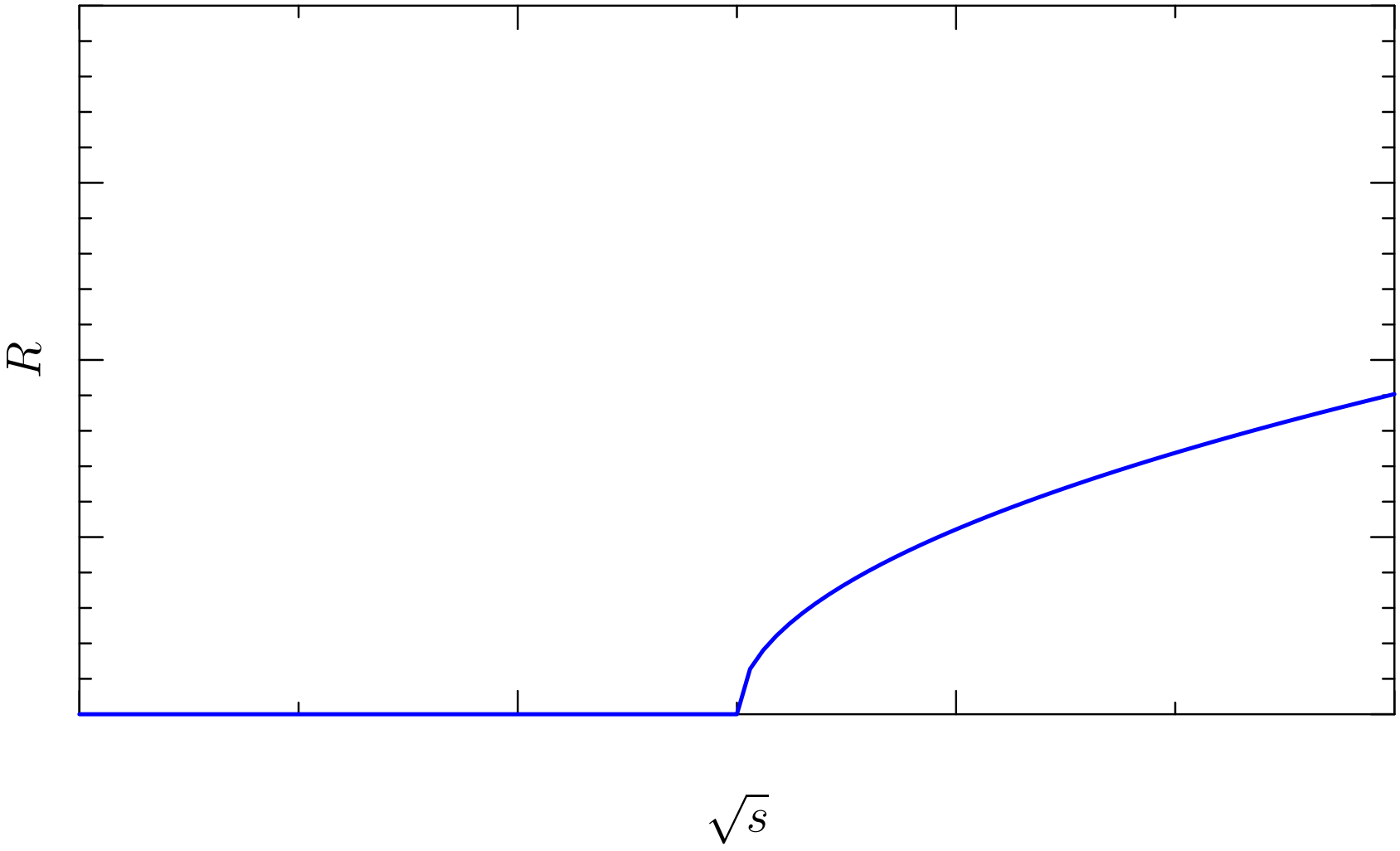
[Degrassi et al. arXiv:1205.6497; Fuji et al. arXiv:1506.05992]





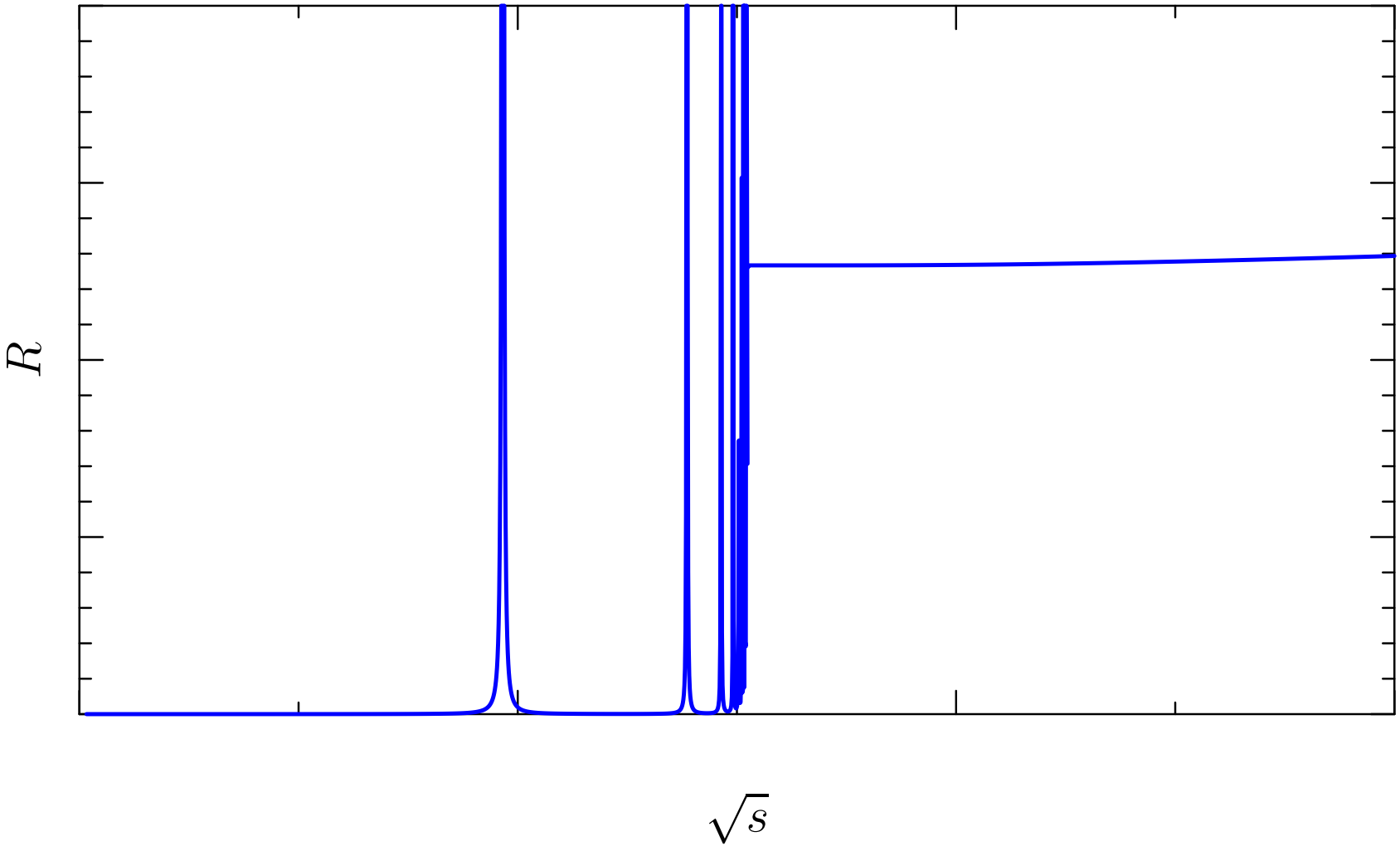
$e^+e^- \to q\bar{q}$ near threshold

Simplistic picture

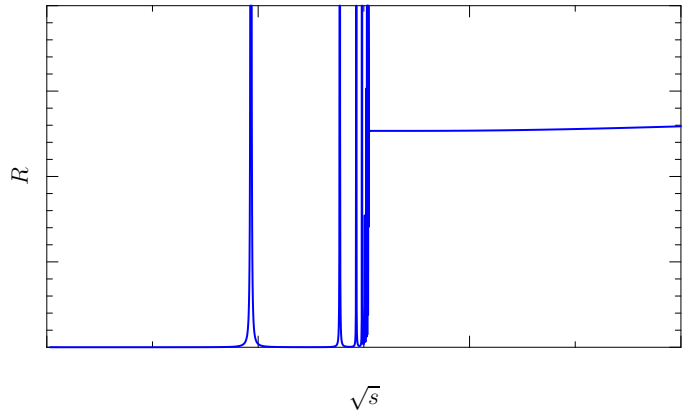


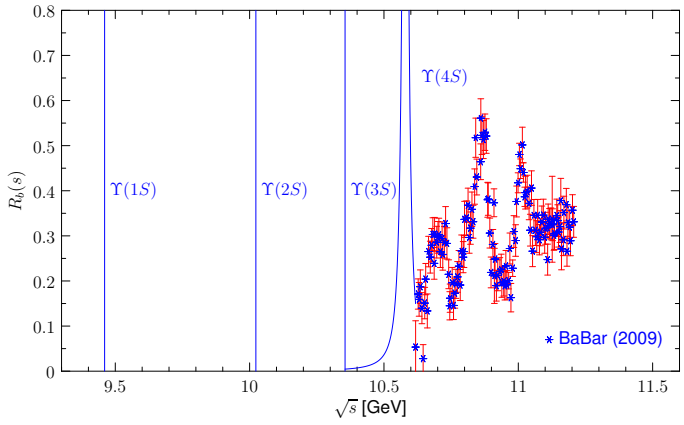
$e^+e^- \to q\bar{q}$ near threshold

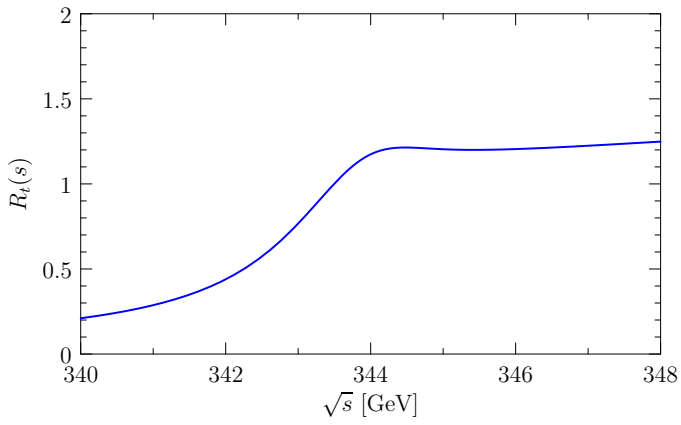
Next-to-simplistic picture



$e^+e^- \to q\bar{q}$ near threshold





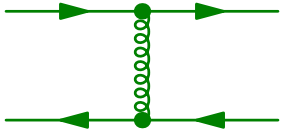


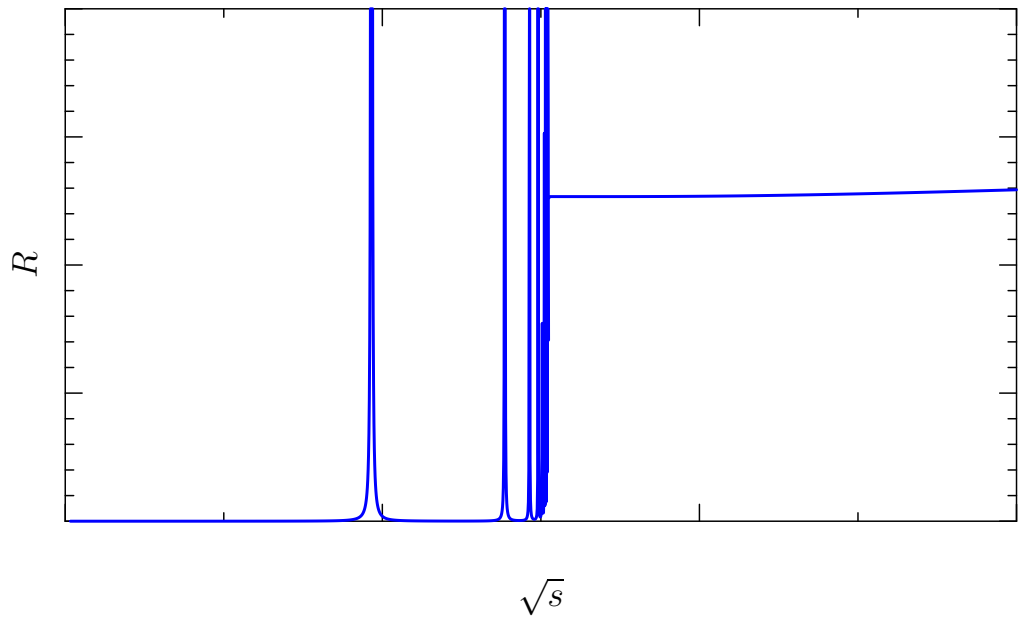
bottom quarks: $\Lambda_{\mathrm{QCD}} \lesssim E_N$

top quarks: $\Gamma \gtrsim E_N$

$e^+e^- \to q\bar{q}$ near threshold

Theory

- Nonrelativistic quarks $v \ll 1 \Rightarrow$ Schrödinger equation
-  $\Rightarrow$ Colour Coulomb potential $-\frac{C_F\alpha_s}{r}$



Higher-order corrections: need effective field theory

Effective theory framework

Scales: $m_q \gg m_q v \gg m_q v^2$ Coulomb potential is LO: $\alpha_s \sim v$

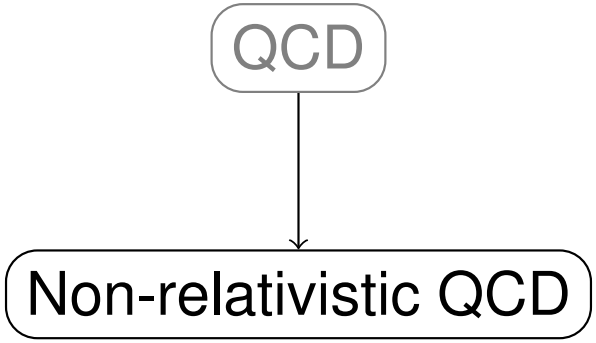
- hard modes: $k \sim m_q$
- soft modes: $k \sim m_q v$
- potential modes: $k_0 \sim m_q v^2, \vec{k} \sim m_q v$
- ultrasoft modes: $k \sim m_q v^2$

QCD

Effective theory framework

Scales: $m_q \gg m_q v \gg m_q v^2$ Coulomb potential is LO: $\alpha_s \sim v$

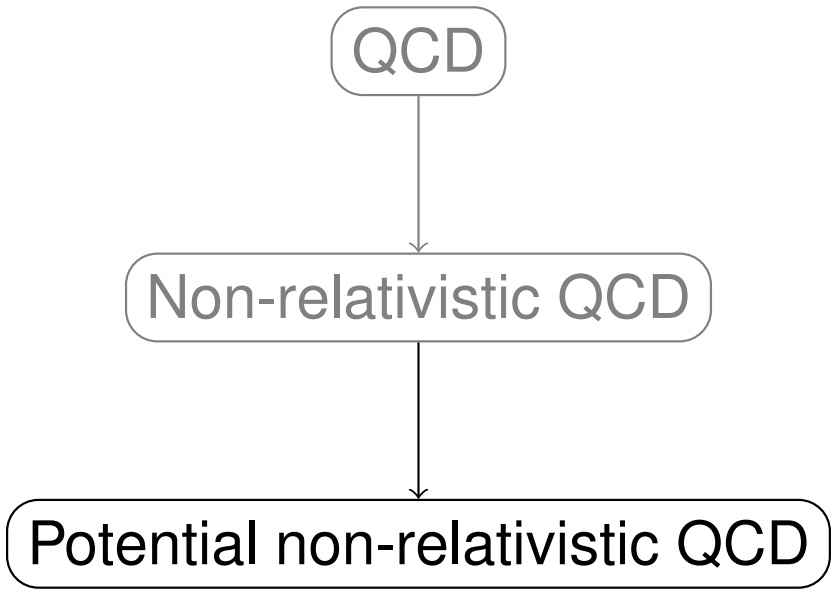
- hard modes: $k \sim m_q \rightarrow$ (local) effective vertices
- soft modes: $k \sim m_q v$
- potential modes: $k_0 \sim m_q v^2, \vec{k} \sim m_q v$
- ultrasoft modes: $k \sim m_q v^2$



Effective theory framework

Scales: $m_q \gg m_q v \gg m_q v^2$ Coulomb potential is LO: $\alpha_s \sim v$

- hard modes: $k \sim m_q \rightarrow$ (local) effective vertices
- soft modes: $k \sim m_q v \rightarrow$ (non-local) potentials
- potential light particle modes $\rightarrow$ (non-local) potentials
- potential heavy quark modes: $k_0 \sim m_q v^2, \vec{k} \sim m_q v$
- ultrasoft modes: $k \sim m_q v^2$



Potential non-relativistic QCD

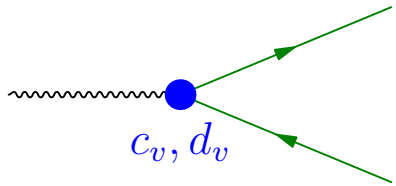
[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = {} & \psi^\dagger \left(i\partial_0 + g_s A_0(t,\mathbf{0}) + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} \right) \psi \\ & + \int d^3\mathbf{r} \, [\psi^\dagger \psi](x+\mathbf{r}) \left(-\frac{C_F \alpha_s}{r} + \delta V(r) \right) [\chi^\dagger \chi](x) \\ & - g_s \psi^\dagger \mathbf{x} \cdot \mathbf{E}(t,\mathbf{0}) \psi \\ & + \mathcal{L}_{\text{anti-quark}} + \{\text{N}^4\text{LO}\}\end{aligned}$$

Beyond LO:

- Corrections to kinetic energy
- Corrections to potential [Anzai, Kiyo, Sumino 09; Smirnov, Smirnov, Steinhauser 09]
- Ultrasoft gluons

Production vertex:

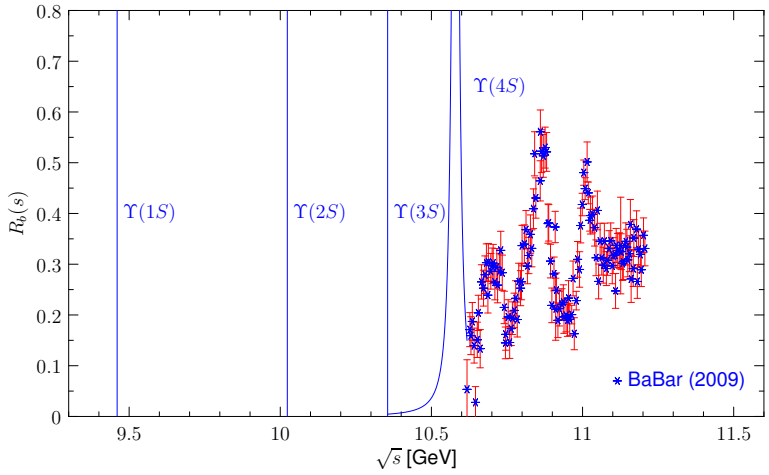


[Marquard, Piclum, Seidel, Steinhauser 14]

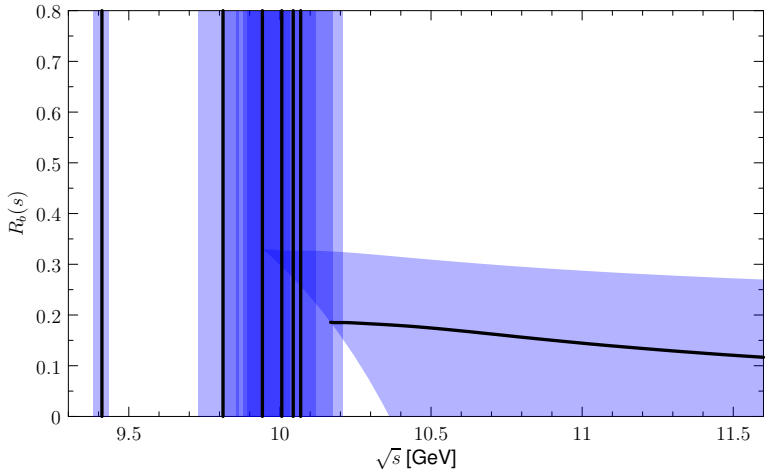
Bottom

Nonrelativistic sum rules

Data



NNNLO PNRQCD



$$\mathcal{M}_n = \int_{s_0}^{\infty} ds\, \frac{R_b(s)}{s^{n+1}}\,, \qquad \mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$$

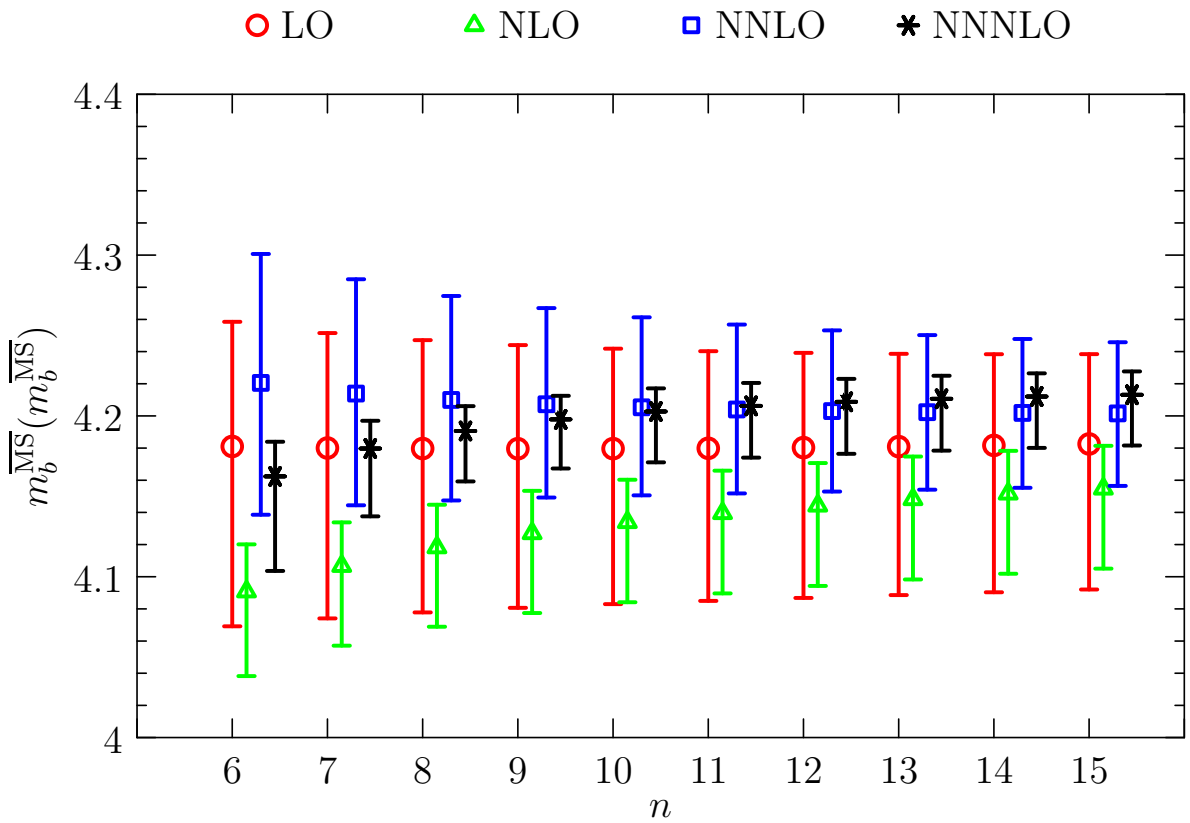
Dominated by $E = mv^2 \sim m_b/n$ for $n \gg 1$

Perturbative: $\Lambda_{\text{QCD}} < m_b/n$

Nonrelativistic: $v \ll 1$

$\Big\}\ n \approx 10$

Bottom quark mass



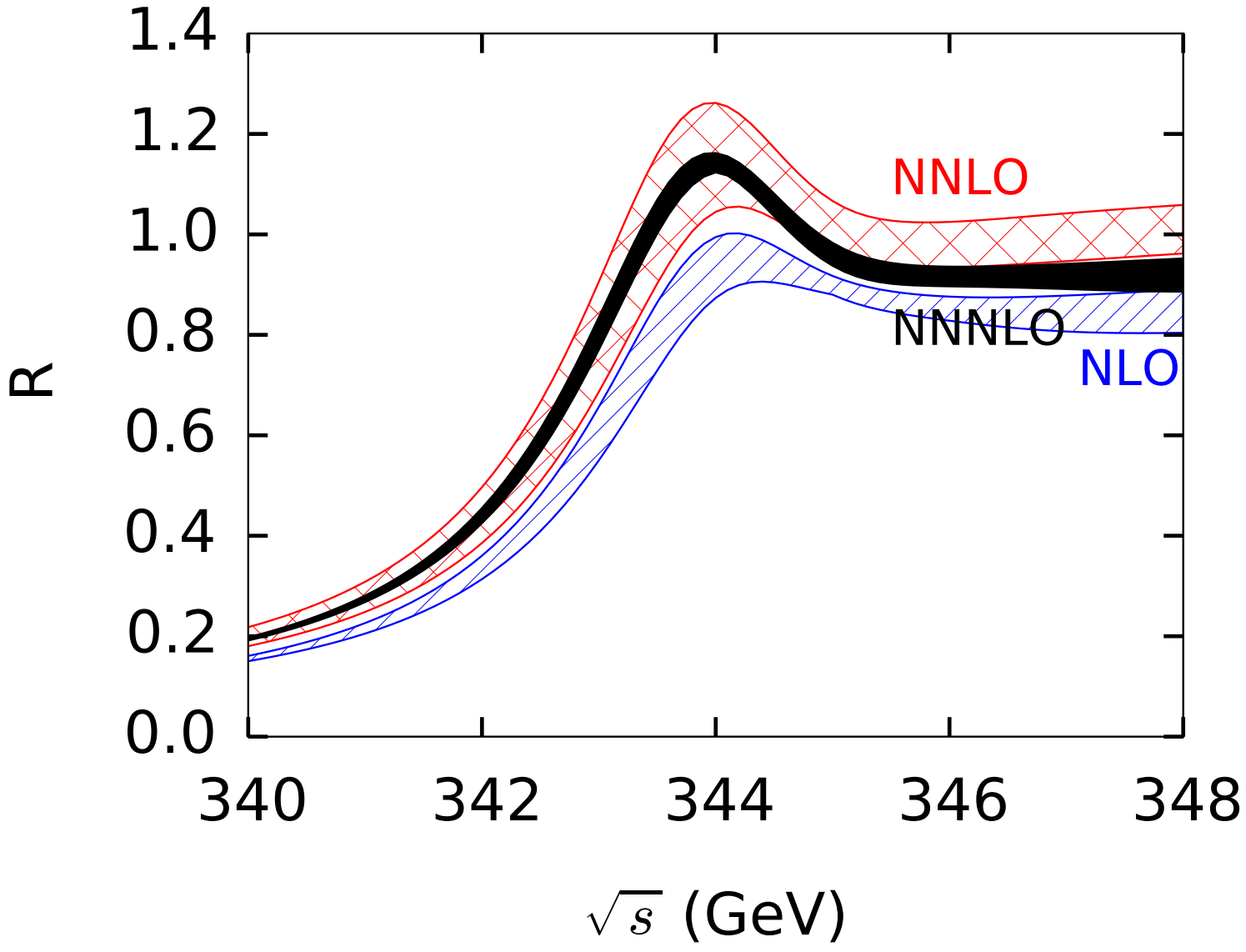
$$m_b^{\rm PS}(2\,{\rm GeV}) = 4.532^{+0.013}_{-0.039}\,{\rm GeV}$$

$$\begin{aligned} m_b^{\overline{\rm MS}}(m_b^{\overline{\rm MS}}) = \big[&4.203^{+0.002}_{-0.031}(\mu) \pm 0.002(\alpha_s)^{+0.003}_{-0}({\rm res})^{+0.013}_{-0.004}({\rm conv}) \\ &\pm 0.002({\rm charm})^{+0.006}_{-0.012}(n) \pm 0.003({\rm exp})\big]\,{\rm GeV} \\ = &\, 4.203^{+0.016}_{-0.034}\,{\rm GeV}\,. \end{aligned}$$

Top

Top production at NNNLO

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]



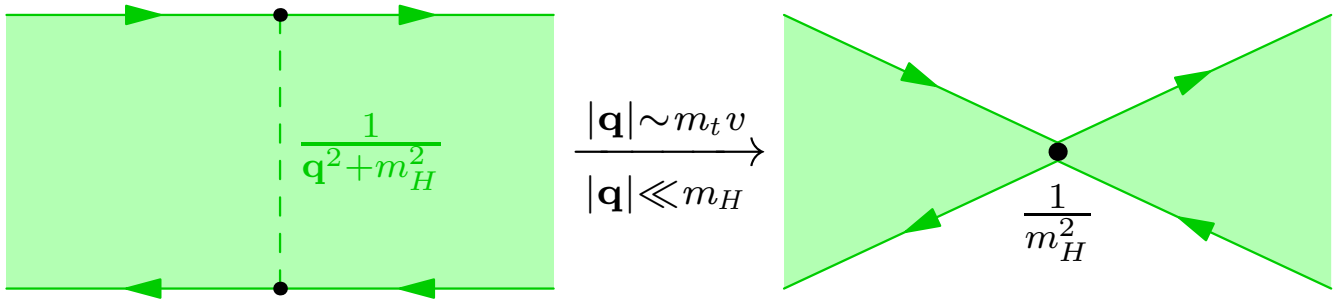
Top production

Going beyond QCD

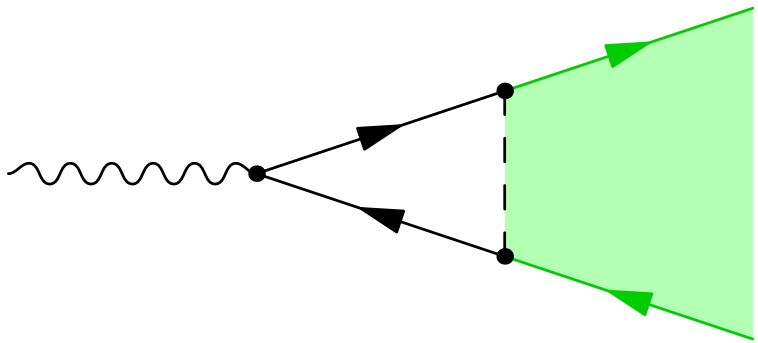
QCD uncertainties under control: $\sim 3\%$

Further corrections $(v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha)$:

- Higgs corrections
  - Potential (NNNLO):



  - Production current (NNLO + NNNLO):



[Grzadkowski, Kühn, Krawczyk, Stuart 1986; Guth, Kühn 1991; Hoang, Reißer 2006; Eiras, Steinhauser 2006]

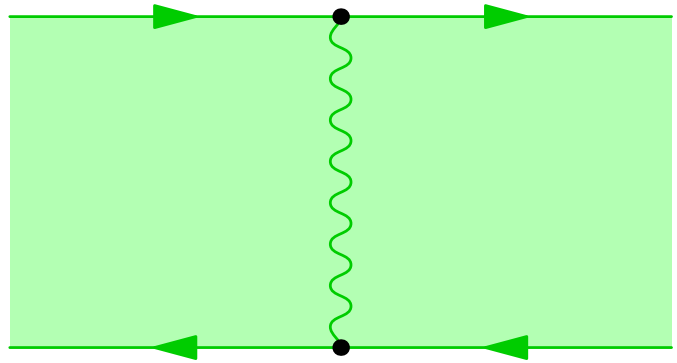
Top production

Going beyond QCD

QCD uncertainties under control: $\sim 3\%$
Further corrections $(v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha)$:

- Higgs corrections
- QED Coulomb potential

Starting at NLO:



Top production

Going beyond QCD

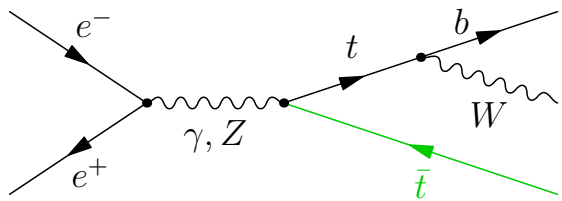
QCD uncertainties under control: $\sim 3\%$

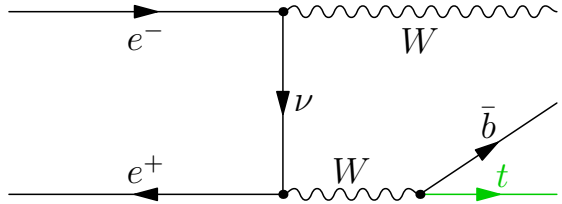
Further corrections $(v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha)$:

- Higgs corrections
- QED Coulomb potential
- Nonresonant production

  Actual final state: $W^+ b W^- \bar{b}$

  - NLO: [Beneke, Jantzen, Ruiz-Femenía 2010]





$\ldots$

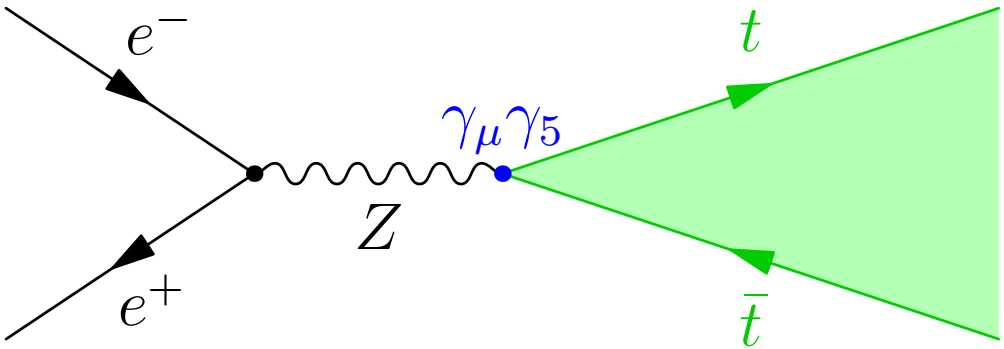
  - NNLO: Partial results [Penin, Piclum 2012; Jantzen, Ruiz-Femenía 2013]
    Work in progress

Top production

Going beyond QCD

QCD uncertainties under control: $\sim 3\%$
Further corrections $(v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha)$:

- Higgs corrections
- QED Coulomb potential
- Nonresonant production
- P-wave production [Penin Pivovarov 1999; Beneke, Piclum, Rauh 2013]



Small contribution at NNLO and NNNLO: $\lesssim 1\%$

Top production

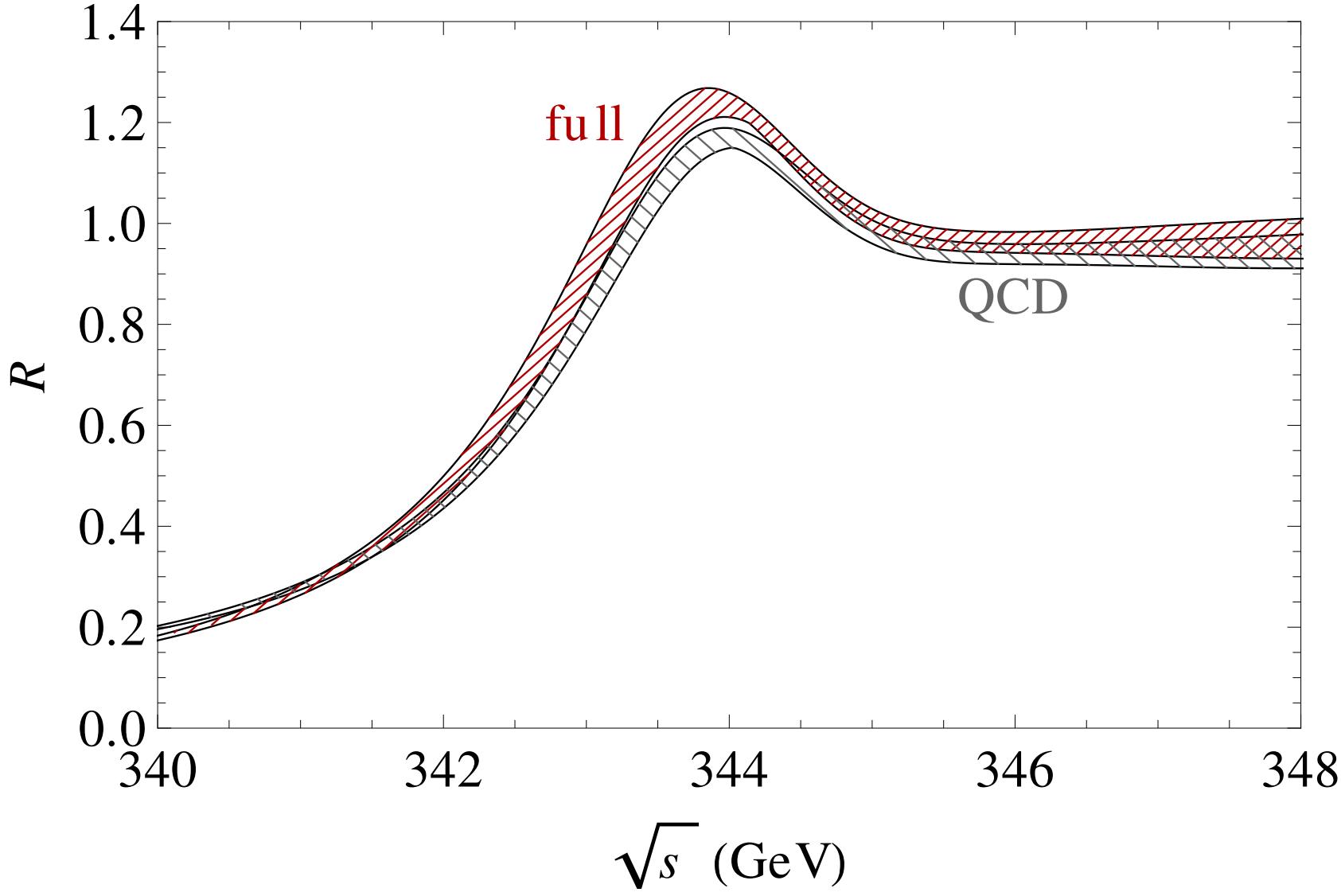
Going beyond QCD

QCD uncertainties under control: $\sim 3\%$
Further corrections $(v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha)$:

- Higgs corrections
- QED Coulomb potential
- Nonresonant production
- P-wave production
- Further NNLO electroweak corrections

[Grzadkowski, Kühn, Krawczyk, Stuart 1986; Guth, Kühn 1991; Hoang, Reißer 2004 & 2006]

Impact of electroweak corrections

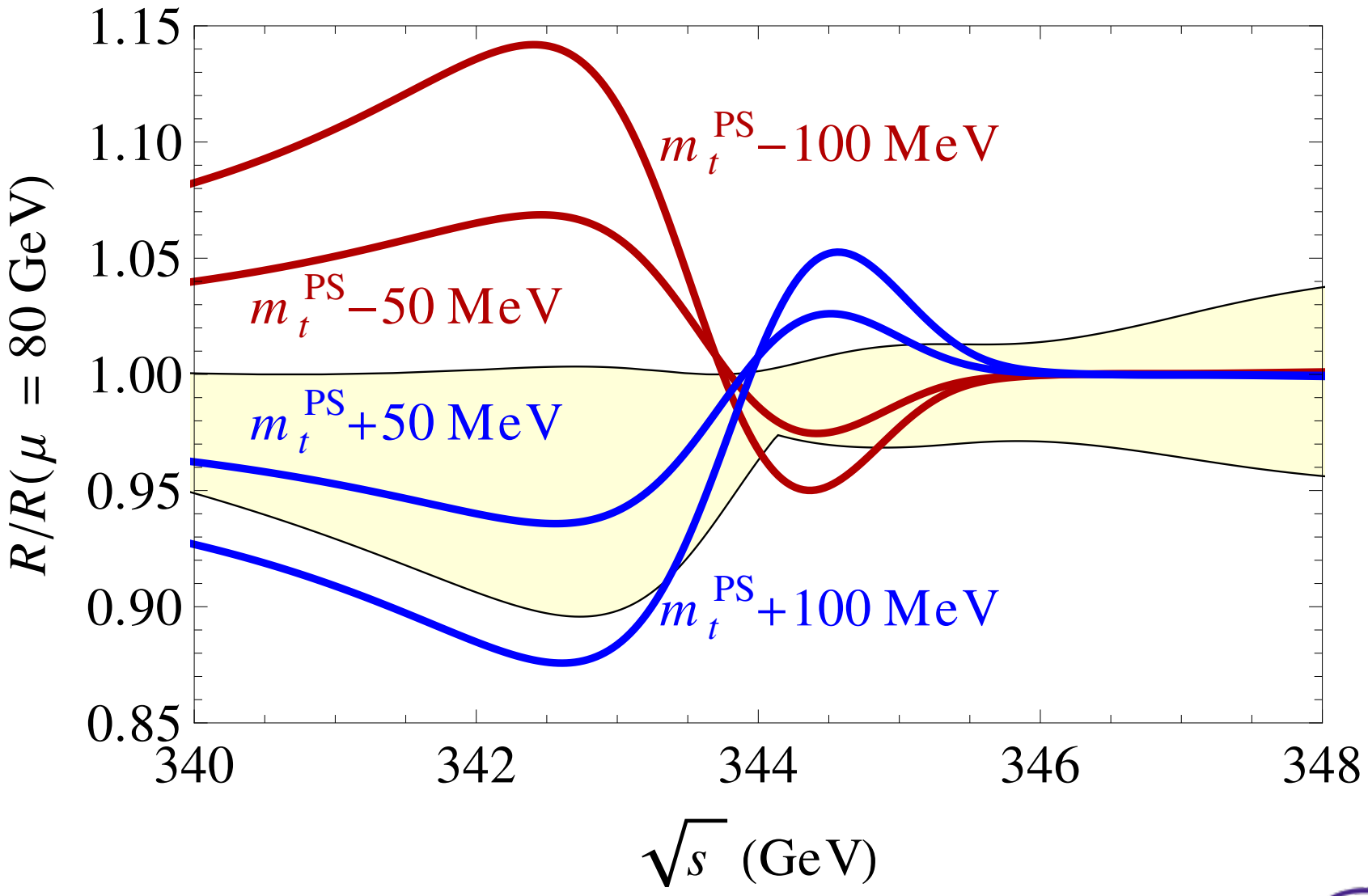


$m_t^{\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}\,,$ $\Gamma_t = 1.33\,\mathrm{GeV}\,,$ $m_H = 171.5\,\mathrm{GeV}\,,$

$\alpha_s(m_Z) = 0.1185\,,$ $\alpha(m_Z) = 1/128.944$ m_W, m_Z

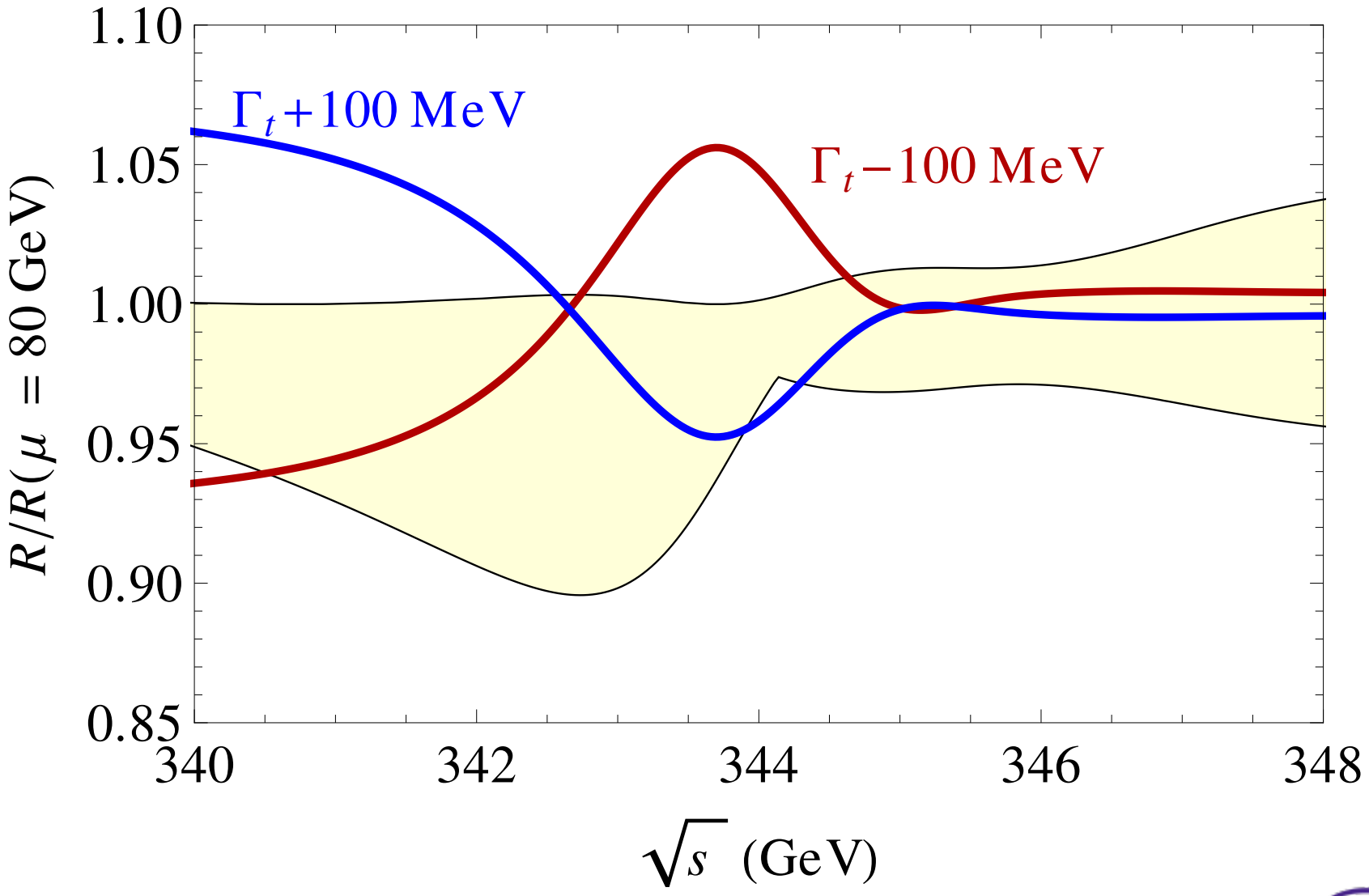
Extracting parameters

Top mass



Extracting parameters

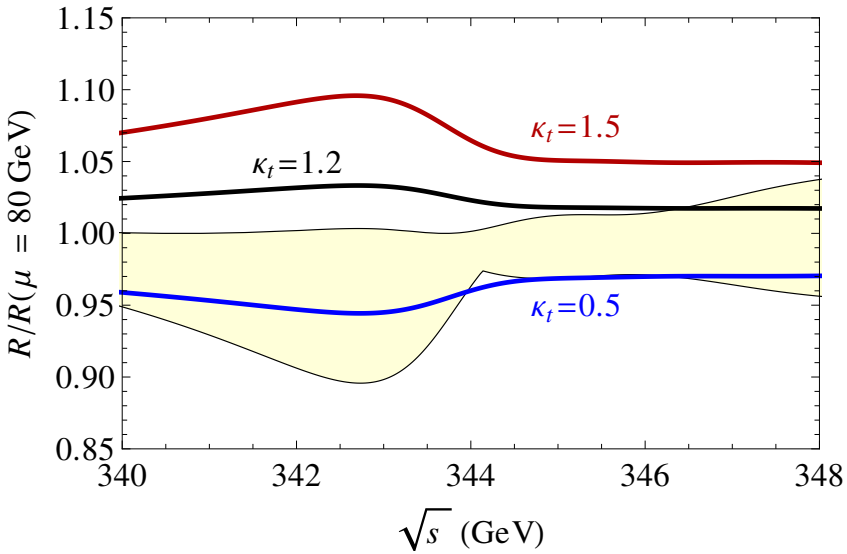
Top width

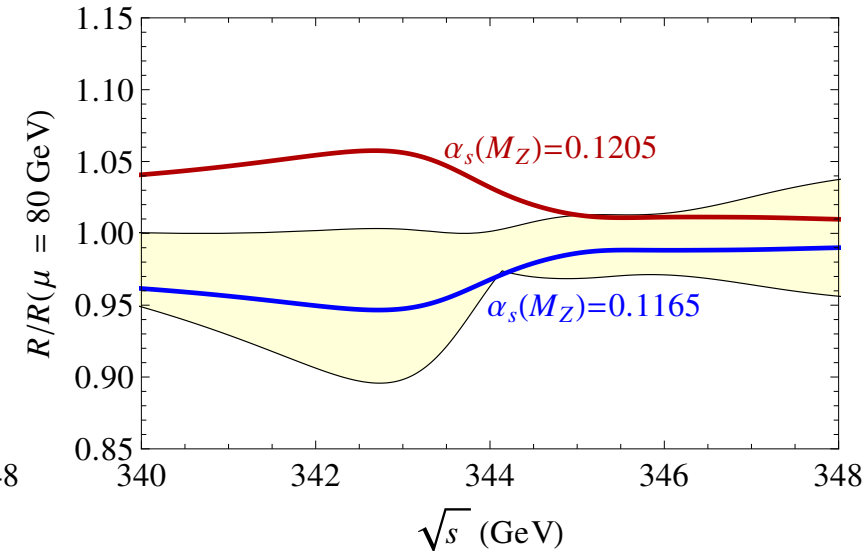


Extracting parameters

Top Yukawa and strong coupling

$$\kappa_t = y_t / y_t^{\mathrm{SM}}, \qquad y_t^{\mathrm{SM}} = \frac{\sqrt{2} m_t}{v}$$





Conclusion

- Precise quark masses from NNNLO production near threshold:

$$m_b^{\mathsf{PS}}(2\,\mathsf{GeV}) = 4.532^{+0.013}_{-0.039}\,\mathsf{GeV}$$
$$m_b^{\overline{\mathsf{MS}}}(m_b^{\overline{\mathsf{MS}}}) = 4.203^{+0.016}_{-0.034}\,\mathsf{GeV}\,,$$
$$\Delta m_t^{\mathsf{PS}}(20\,\mathsf{GeV}) < 100\,\mathsf{MeV}$$

- Error dominated by theory

For top production:

- Up to 10% electroweak effects $>$ QCD uncertainty ($\sim 3\%$)
- Extracting y_t requires precise knowledge of α_s

Outlook

- NNLO electroweak effects
- Renormalisation group improvement
- Coming soon: code

Outlook

- NNLO electroweak effects
- Renormalisation group improvement
- Coming soon: code

```
Needs["QQbarThreshold‘"];
LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
Plot[
   TTbarRRatio[
      sqrts, {80., 350.}, {171.5, 1.33},
      "N3LO"
   ],
   {sqrts, 340, 348}
]
```

```
#include <iostream>
#include "QQbar_threshold/QQbar_threshold.hpp"
using namespace QQbar_threshold;
int main(){
  load_grid(grid_directory() + "ttbar_grid.tsv");
  std::cout << ttbar_R_ratio(
     344., {80., 350.}, {171.5, 1.33}, "N3LO"
  ) << '\n';
}
```