Charm quark mass with calibrated uncertainty

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Work in collaboration with
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preliminary results

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter PRISMA

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

MITP, Mainz, 7-12 March, 2016

Outline

- Motivation and Introduction
- Using Sum Rules to extract m_Q
  - overview
  - our proposal
- Conclusions and outlook

Motivation: why precise m_Q?

Techniques

Υ-spectroscopy $\quad m(\Upsilon(1S)) = 2M_b - \mathcal{C}\alpha^2 M_b + \cdots$

lattice: HPQCD '14

$$\overline{m_c}(3\mathrm{GeV}) = 986(6)\mathrm{MeV}$$

$$\overline{m_b}(10\mathrm{GeV}) = 3617(25)\mathrm{MeV}$$

QCD Sum Rules

$$\int \frac{\mathrm{d}s}{s^{n+1}} R_q(s) \sim \left(\frac{1}{m_q}\right)^{2n}$$

Motivation: why precise m$_Q$?

$\overline{m_c}(\overline{m_c})$	method	reference
1275.8 ± 5.8	lattice ($N_f = 4$), PS current	HPQCD, 1408.4169
1348 ± 46	lattice (2+1+1), M_D	ETM, 1403.4504
1274 ± 36	lattice ($N_f = 2$), f_D	ALPHA, 1312.7693
1240 ± 50	$c\bar{c}$ X-section DIS	Alekhin et al, 1310.3059
1260 ± 65	$c\bar{c}$ X-section NLO fit	HI and ZEUS, 1211.1182
1262 ± 17	SR $J/\Psi, \Psi(2S - 6S)$	Narison, 1105.5070
1260 ± 36	lattice (2+1), f_D	PACS-CS, 1104.4600
1278 ± 9	SR $J/\Psi, \Psi, R$	Bodenstain et al, 1102.3835
1282 ± 24	1st moment SR $J/\Psi, \Psi, R$	Dehnadi et al, 1102.2264
1280 ± 70	lattice + pQCD in static potential	Laschka et al, 1102.0945
1279 ± 13	1st moment SR $J/\Psi, \Psi, R$	Chetyrkin et al, 1010.6157
1275 ± 25	PDG average	PDG 2014

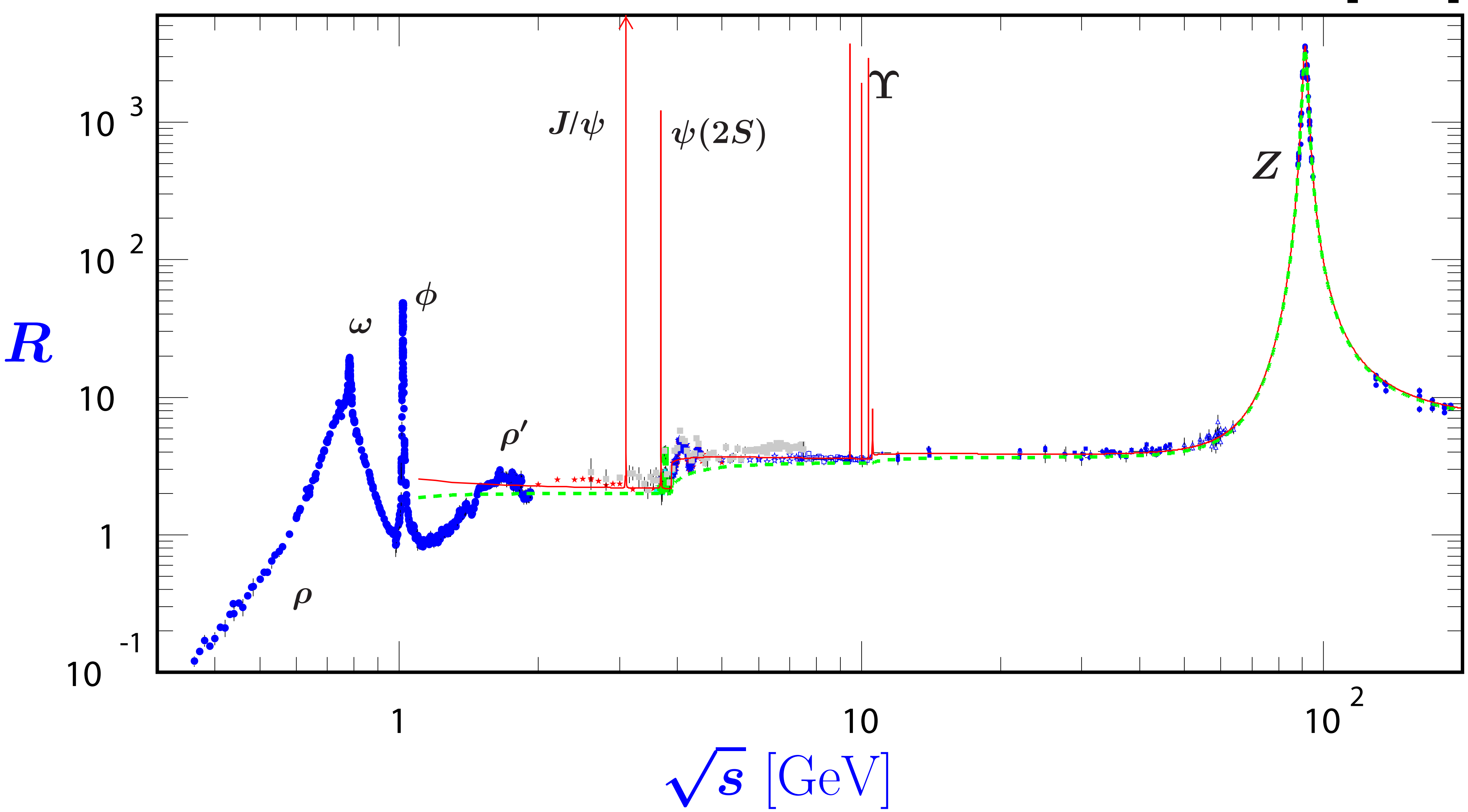
Motivation: why precise m$_Q$?

$\overline{m_b}(\overline{m_b})$	method	reference
4174 ± 24	lattice (N_f = 4), PS current	HPQCD, 1408.4169
4201 ± 43	N^3LO pQCD, M_Υ	Ayala et al, 1407.2128
4169 ± 9	SR $\Upsilon(1S - 6S)$	Penin, Zerf, 1401.7035
4247 ± 34	SR, f_B	Lucha et al, 1305.7099
4166 ± 43	lattice + pQCD, M_Υ, M_{B_s}	HPQCD, 1302.3739
4235 ± 55	SR $\Upsilon(1S - 6S)$, R	Hoang et al, 1209.0450
4171 ± 9	SR $\Upsilon(1S - 6S)$, R	Bodenstain et al, 1111.5742
4177 ± 11	SR $\Upsilon(1S - 6S)$	Narison, 1105.5070
4180 ± 50	lattice + pQCD in static potential	Laschka et al, 1102.0945
4163 ± 16	2nd moment SR $\Upsilon(1S - 6S)$, R	Chetyrkin et al, 1010.6157
4.180 ± 30	PDG average	PDG 2014

QCD Sum Rules

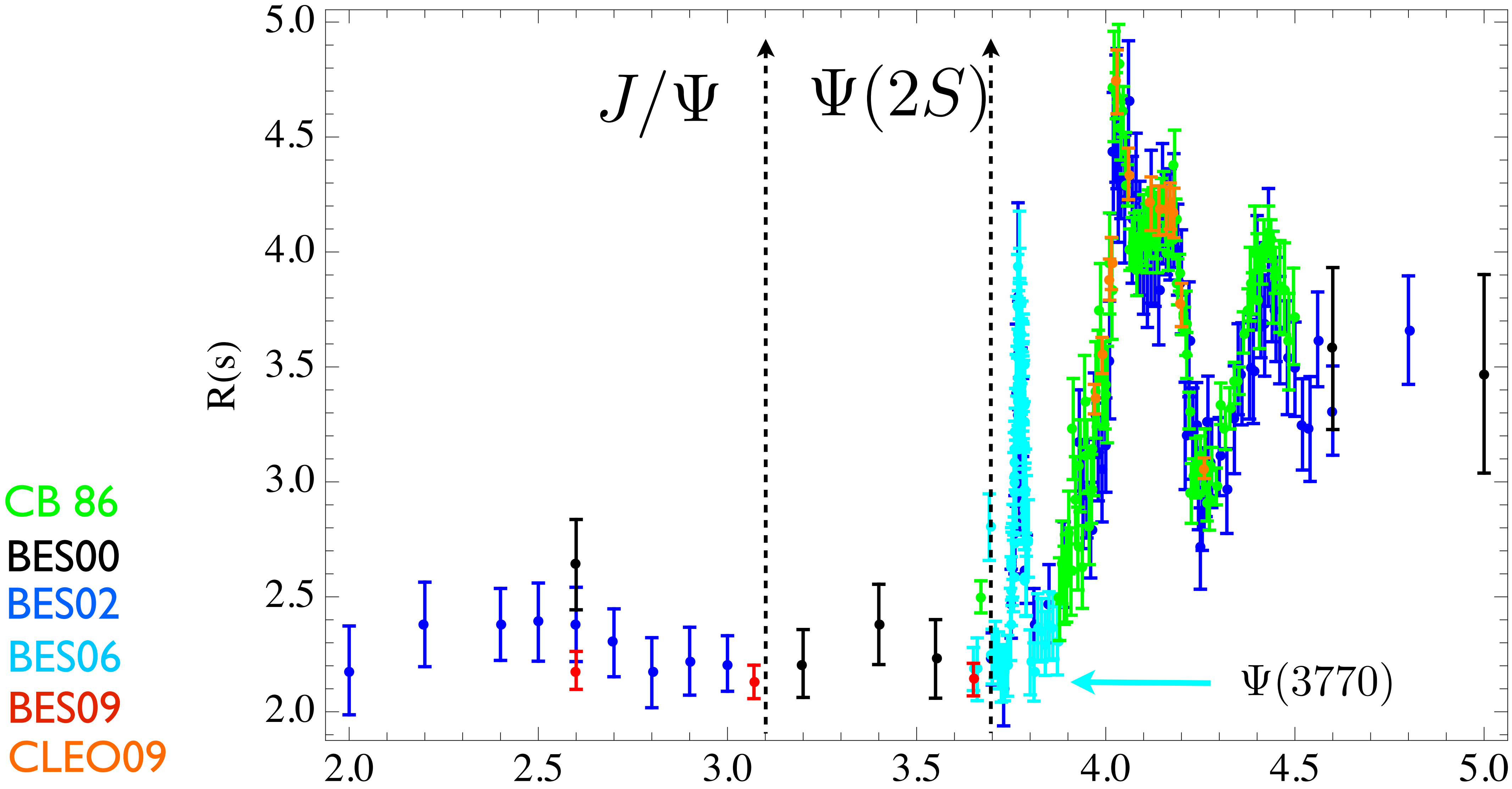
$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \qquad \sigma(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha_{\rm em}(s)^2/3s$$

[PDG]



QCD Sum Rules

$$R(s) = R_{\rm uds}(s) + R_q(s) \qquad R_q(s) = R_q^{\rm Res}(s) + R_q^{\rm th}(s) + R_q^{\rm cont}(s)$$



QCD Sum Rules

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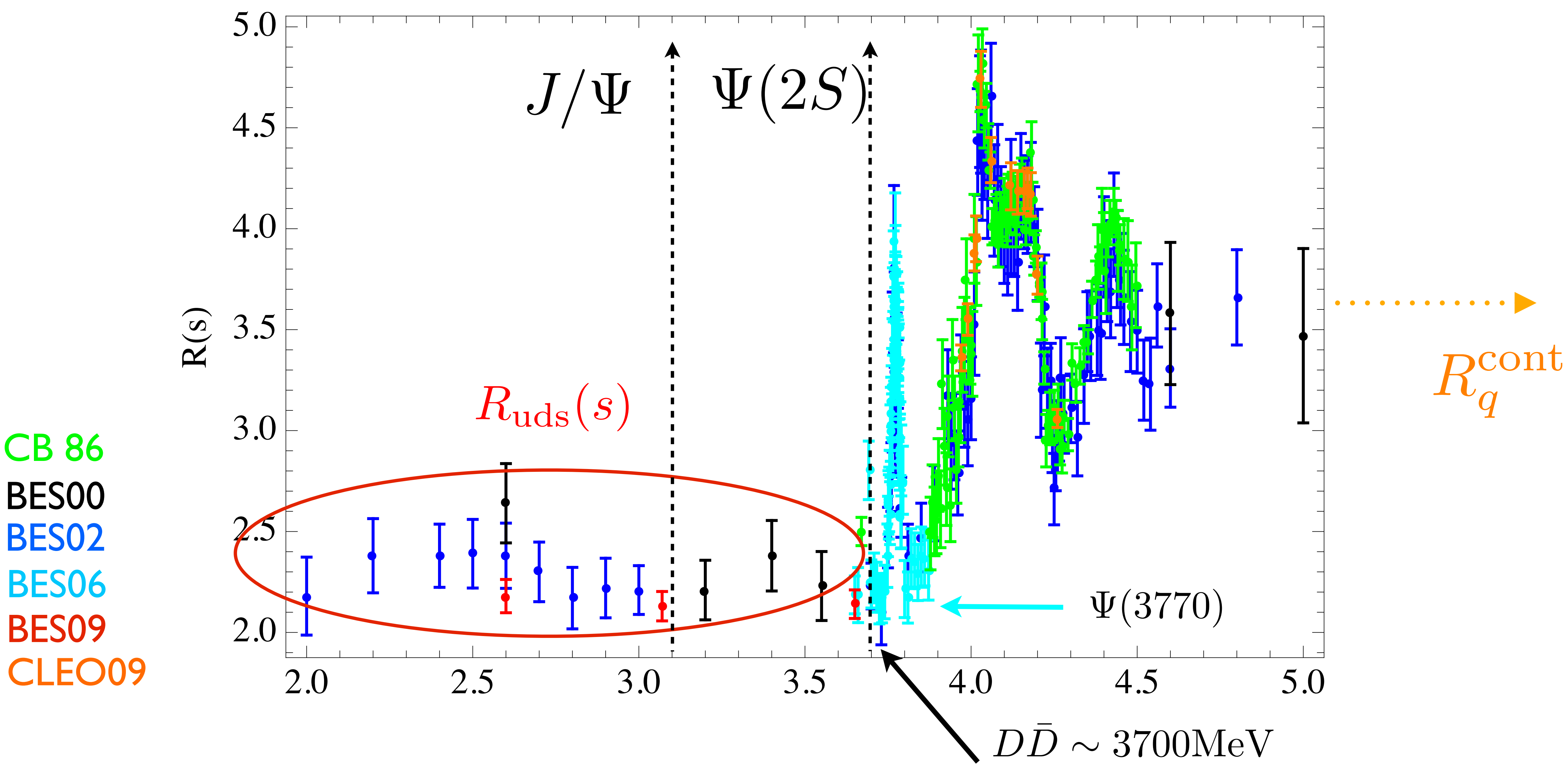
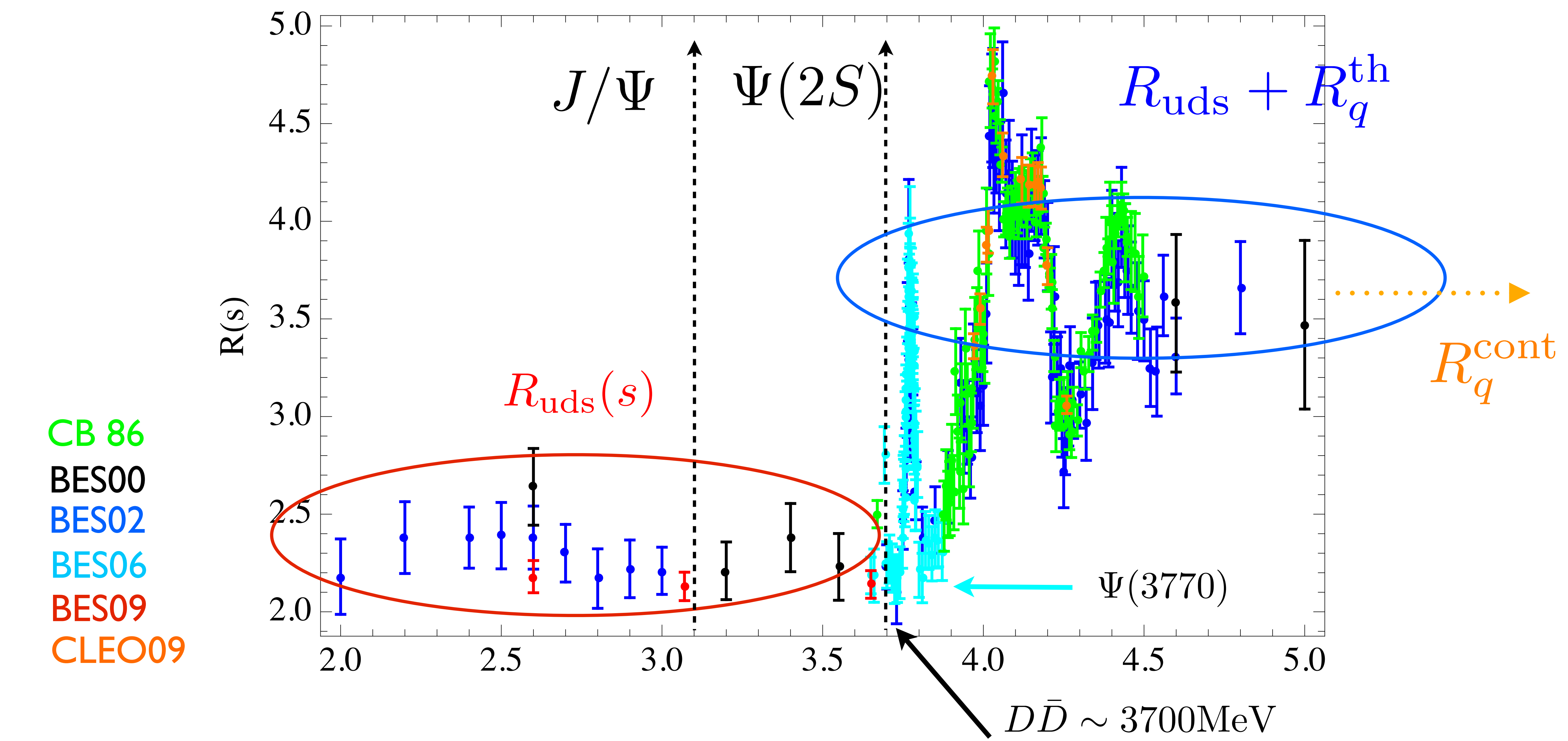


QCD Sum Rules

$R(s) = R_{\mathrm{uds}}(s) + R_q(s)$

$R_q(s) = R_q^{\mathrm{Res}}(s) + R_q^{\mathrm{th}}(s) + R_q^{\mathrm{cont}}(s)$



QCD Sum Rules

Using the optical theorem: [SVZ,'79]

$$R(s) = 12\pi \mathrm{Im}[\Pi(s+i\epsilon)]$$

$\Pi_q(s)$ is the correlator of two heavy-quark vector currents which can be calculated in pQCD order by order and satisfies a Dispersion Relation:

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

$\hat{\Pi}_q(s)$ in $\overline{MS}$

For t→0

$$\boxed{\mathcal{M}_n := \frac{12\pi^2}{n!} \frac{d^n}{dt^n} \hat{\Pi}_q(t)\bigg|_{t=0} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} R_q(s)}$$

QCD Sum Rules

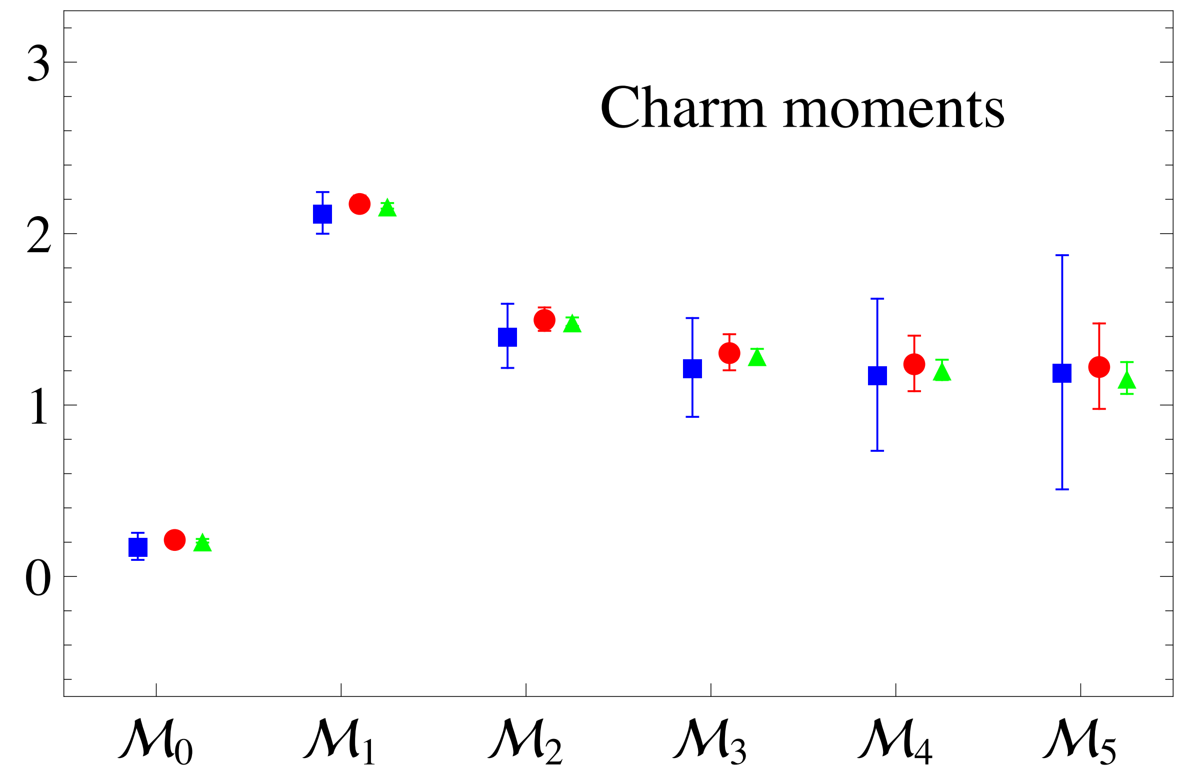
$\hat{\Pi}_q(s)$ can be Taylor expanded:

$$\Pi_q(t) = Q_q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \left(\frac{t}{4\hat{m}_q^2}\right)^n$$

QCD Sum Rules

$\hat{\Pi}_q(s)$ can be Taylor expanded:

$$\Pi_q(t) = Q_q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \left(\frac{t}{4\hat{m}_q^2}\right)^n$$



$$\mathcal{M}_n^{\mathrm{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n} \bar{C}_n$$

$$\hat{\alpha} = \hat{\alpha}(\overline{m_q})$$

$$\bar{C}_n = \bar{C}_n^{(0)} + \left(\frac{\hat{\alpha}}{\pi}\right) \bar{C}_n^{(1)} + \left(\frac{\hat{\alpha}}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\hat{\alpha}}{\pi}\right)^3 \bar{C}_n^{(3)} + \mathcal{O}\left(\frac{\hat{\alpha}}{\pi}\right)^4$$

[Maier et al, '08]
[Chetyrkin, Steinhauser'06]
[Melnikov, Ritberger'03]

[Kiyo et al '09]
[Hoang et al '09]
[Greynat et al '09]

QCD Sum Rules

Sum Rules:

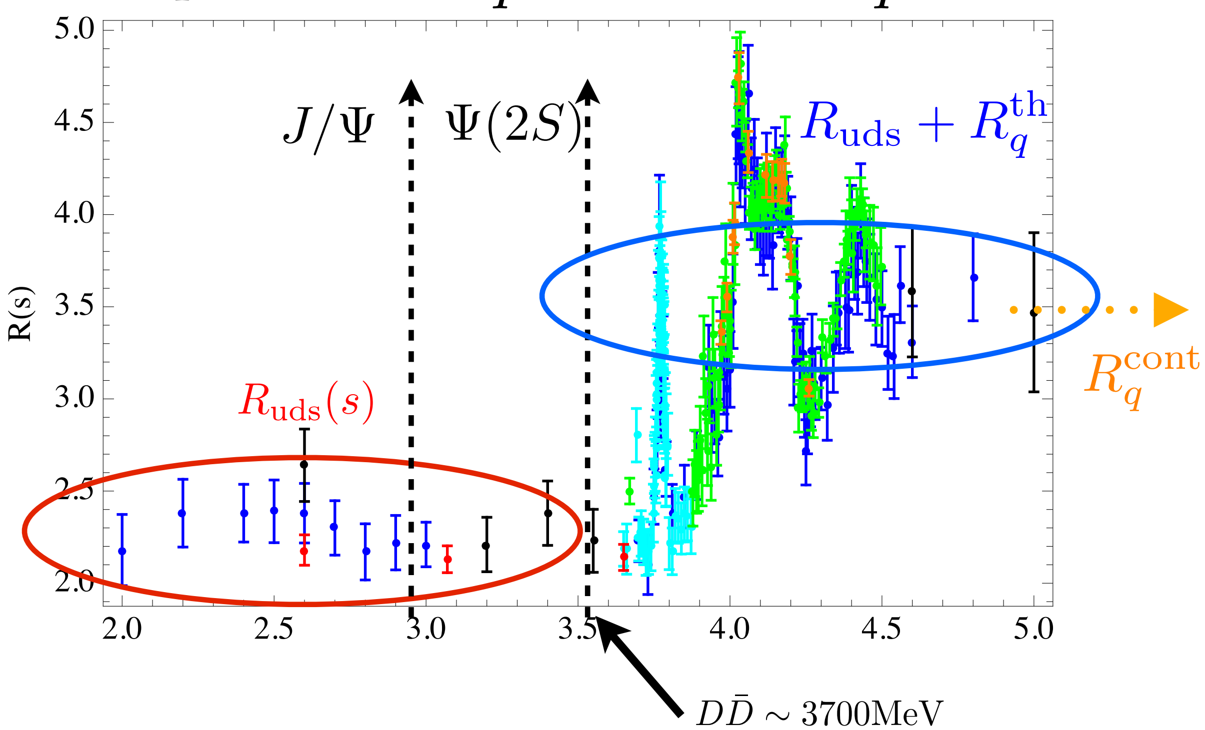
$$\boxed{\mathcal{M}_n = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} R_q(s)}$$

L.h.s. from theory

$$\mathcal{M}_n^{\mathrm{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n$$

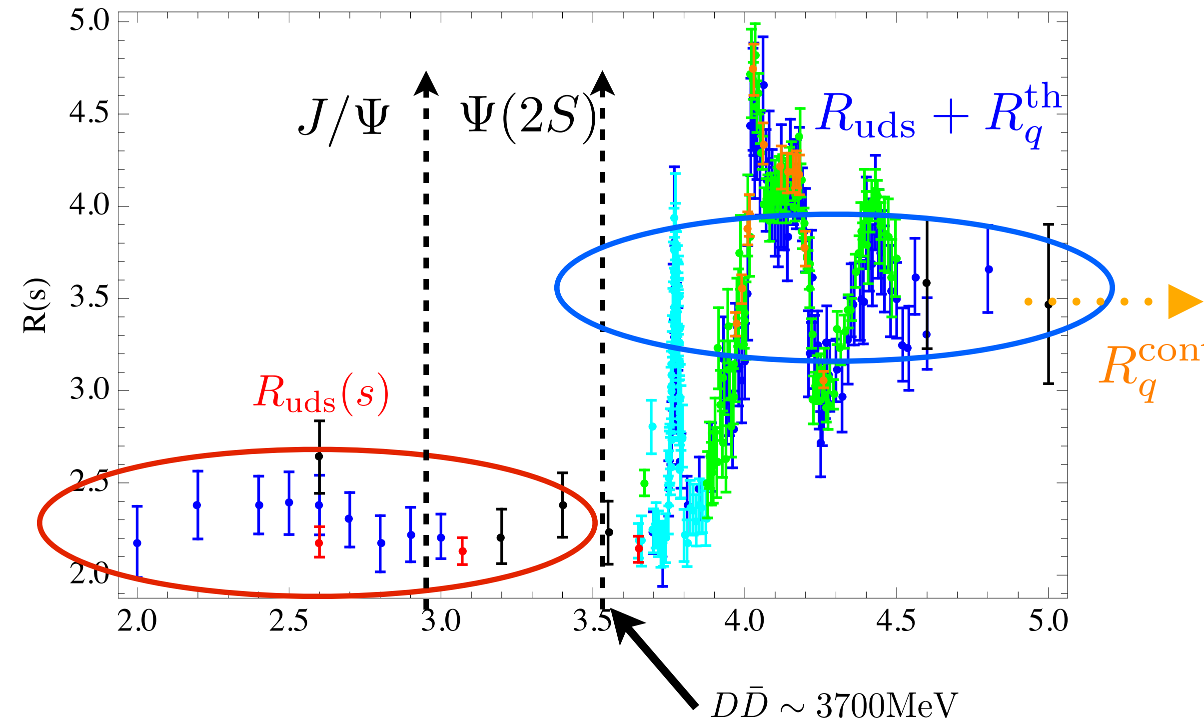
R.h.s. from experiment

$$R_q(s) = R_q^{\mathrm{Res}}(s) + R_q^{\mathrm{th}}(s) + R_q^{\mathrm{cont}}(s)$$



QCD Sum Rules

$$R_q(s) = R_q^{\rm Res}(s) + R_q^{\rm th}(s) + R_q^{\rm cont}(s)$$



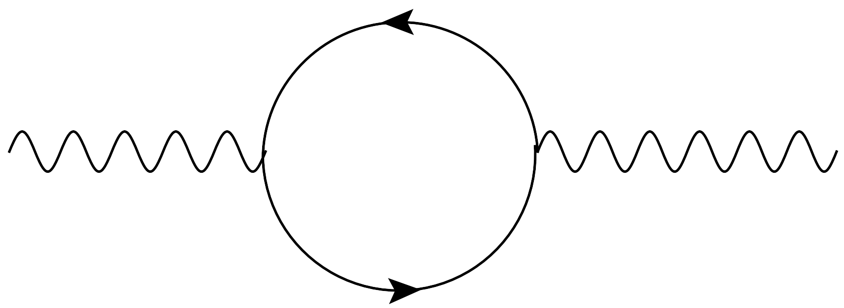
$$R_q^{\rm Res}(s) = \frac{9\pi M_R \Gamma_R^e}{\alpha_{\rm em}^2(M_R)} \delta(s - M_R^2)$$

$$R_q^{\rm th}(s) = R_q(s) - R_{\rm background}$$

$$(2M_D \leq \sqrt{s} \leq 4.8{\rm GeV})$$

$R_q^{\rm cont}(s)$ calculated using pQCD

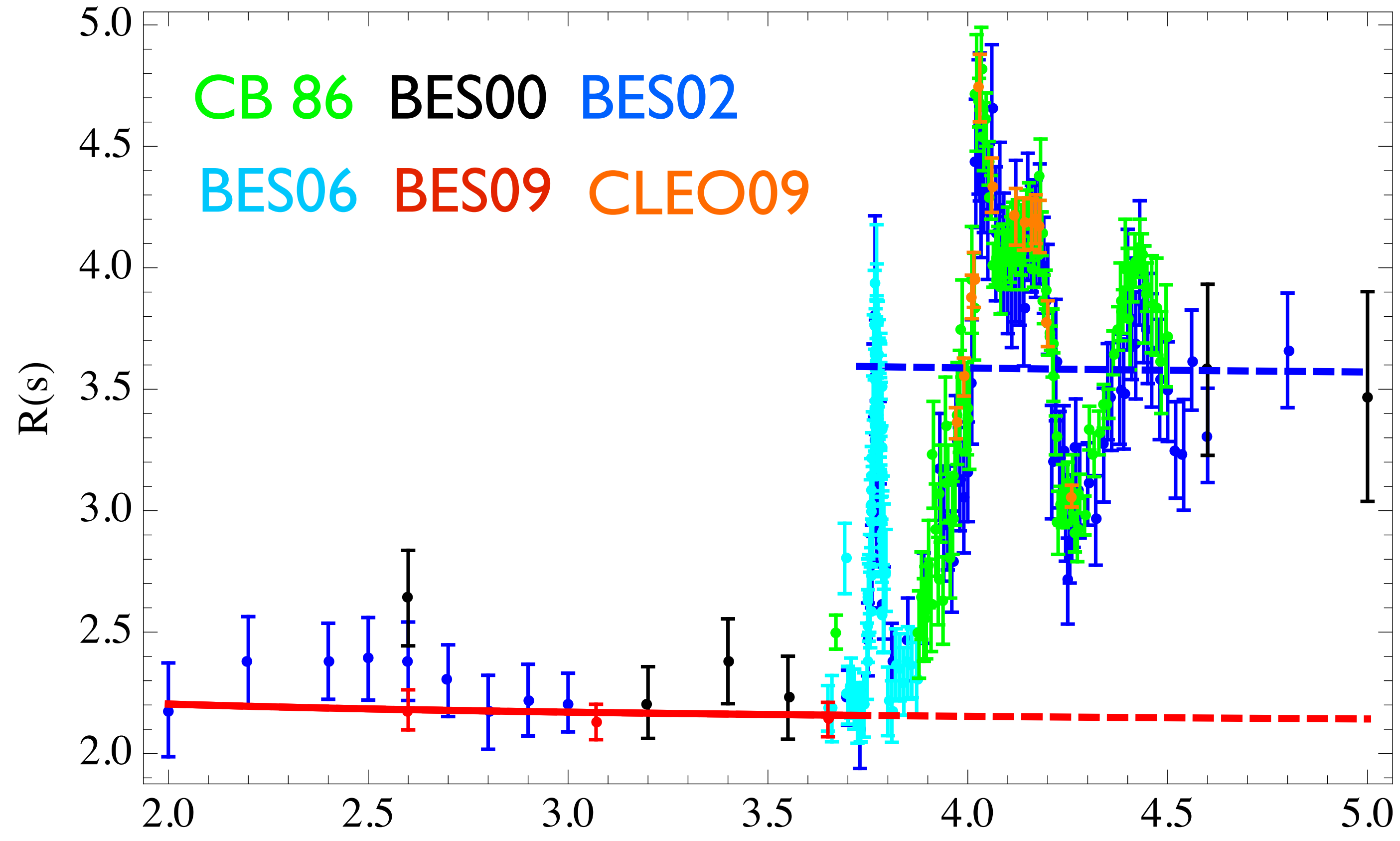
$$(\sqrt{s} \geq 4.8{\rm GeV})$$



Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

Light flavor contribution in charm region

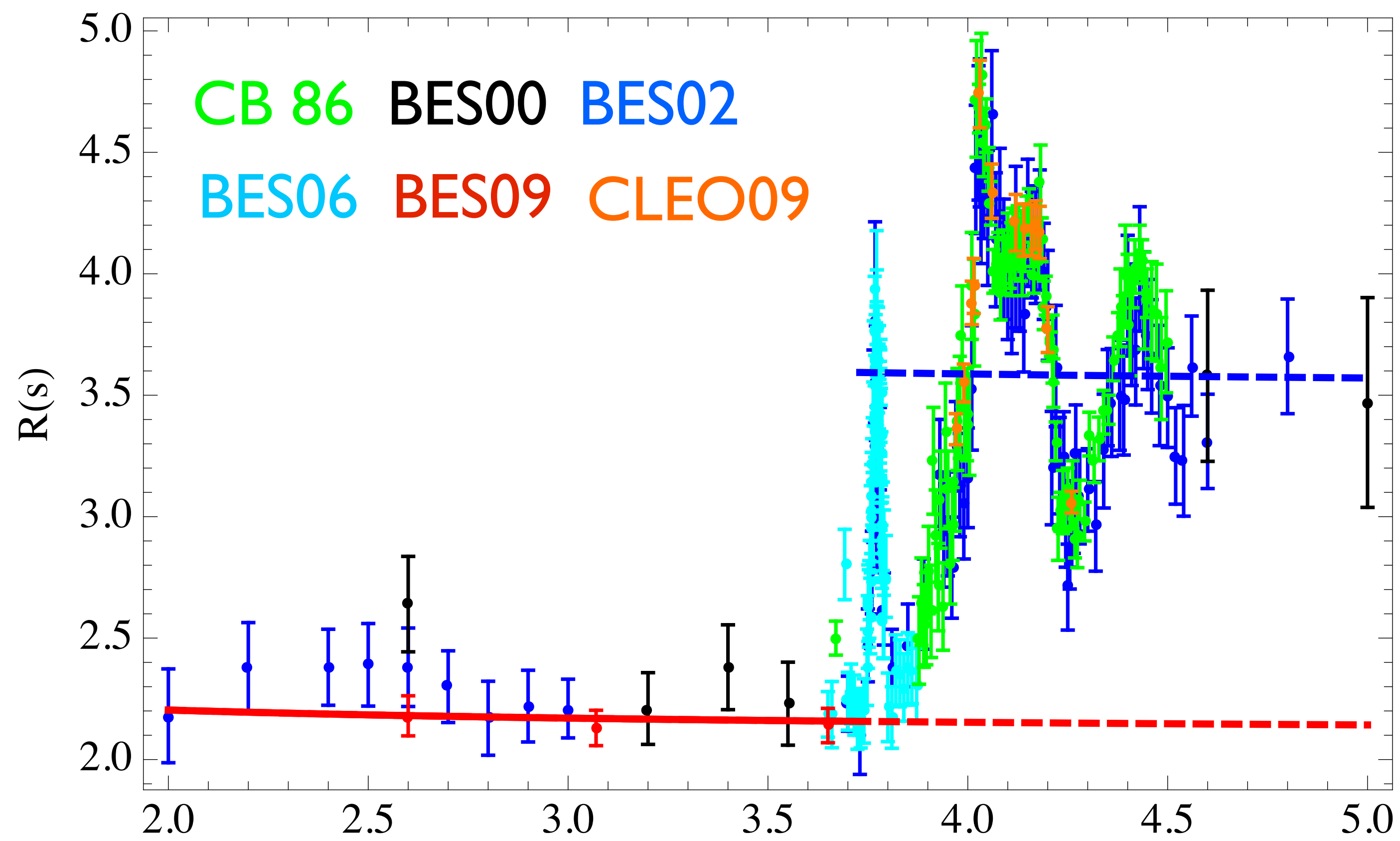


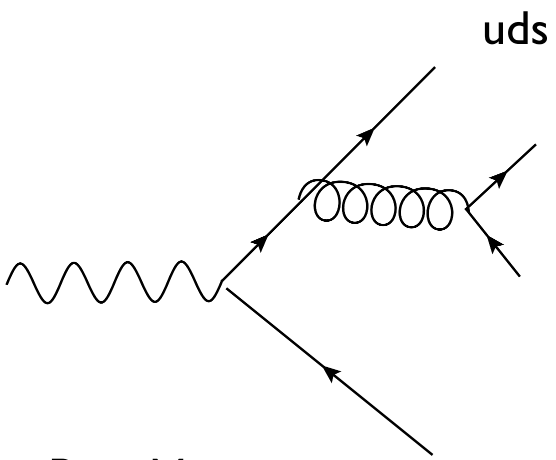
Using pQCD below threshold, calculate R, and extrapolate

Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

Light flavor contribution in charm region
+
secondary production



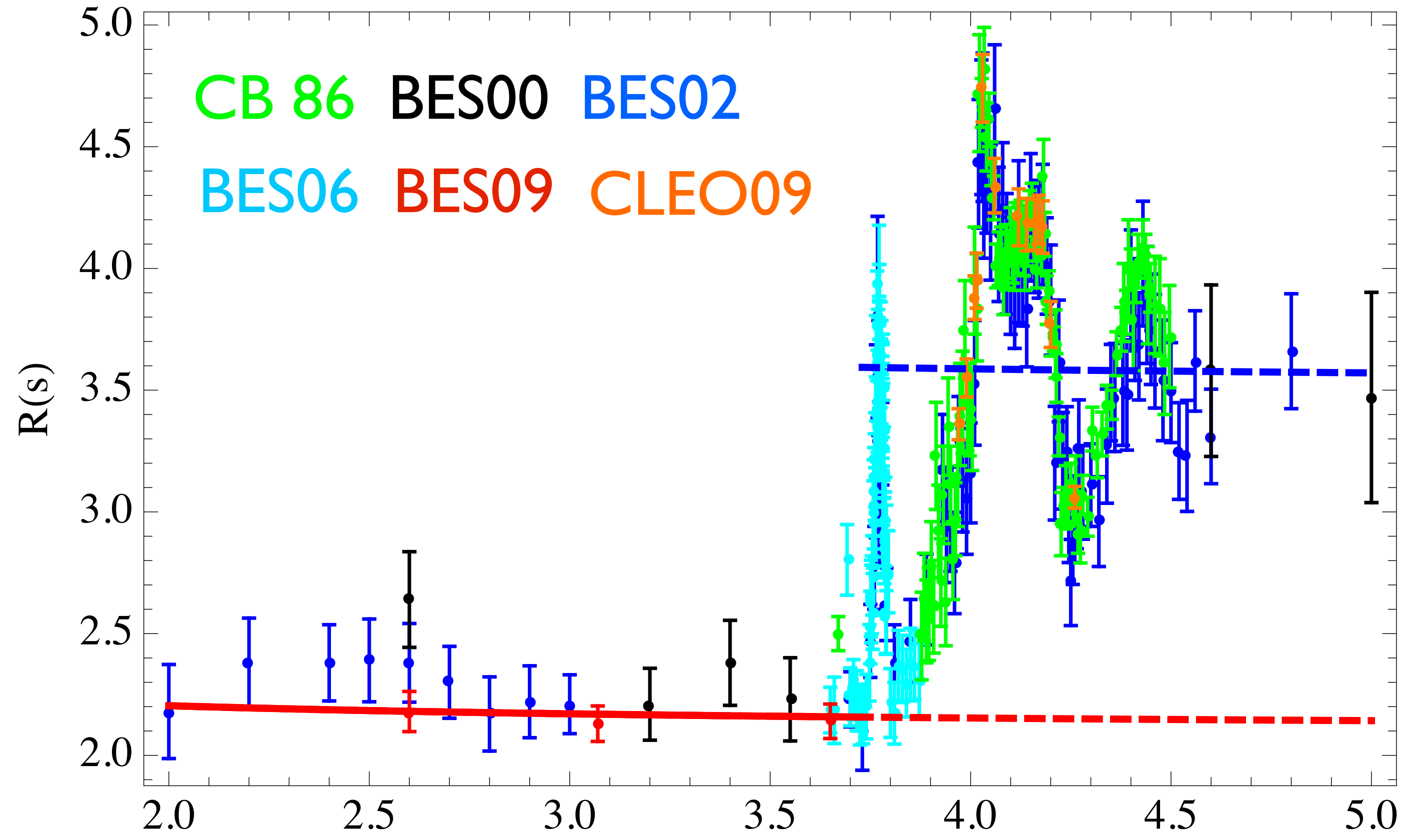


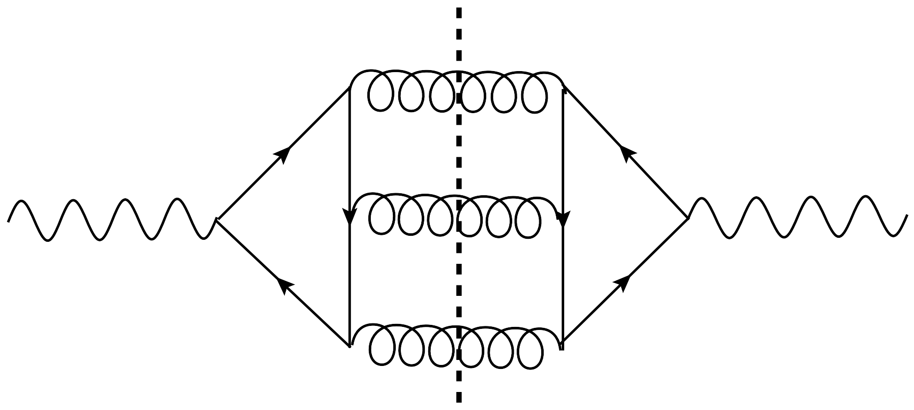
$\sim \alpha^2$

Background

$$R_{\mathrm{background}} = R_{\mathrm{uds}} + R_{\mathrm{uds(cb)}} + R_{\mathrm{sing}} + R_{\mathrm{QED}}$$

Light flavor contribution in charm region
+
secondary production
+
singlet contribution



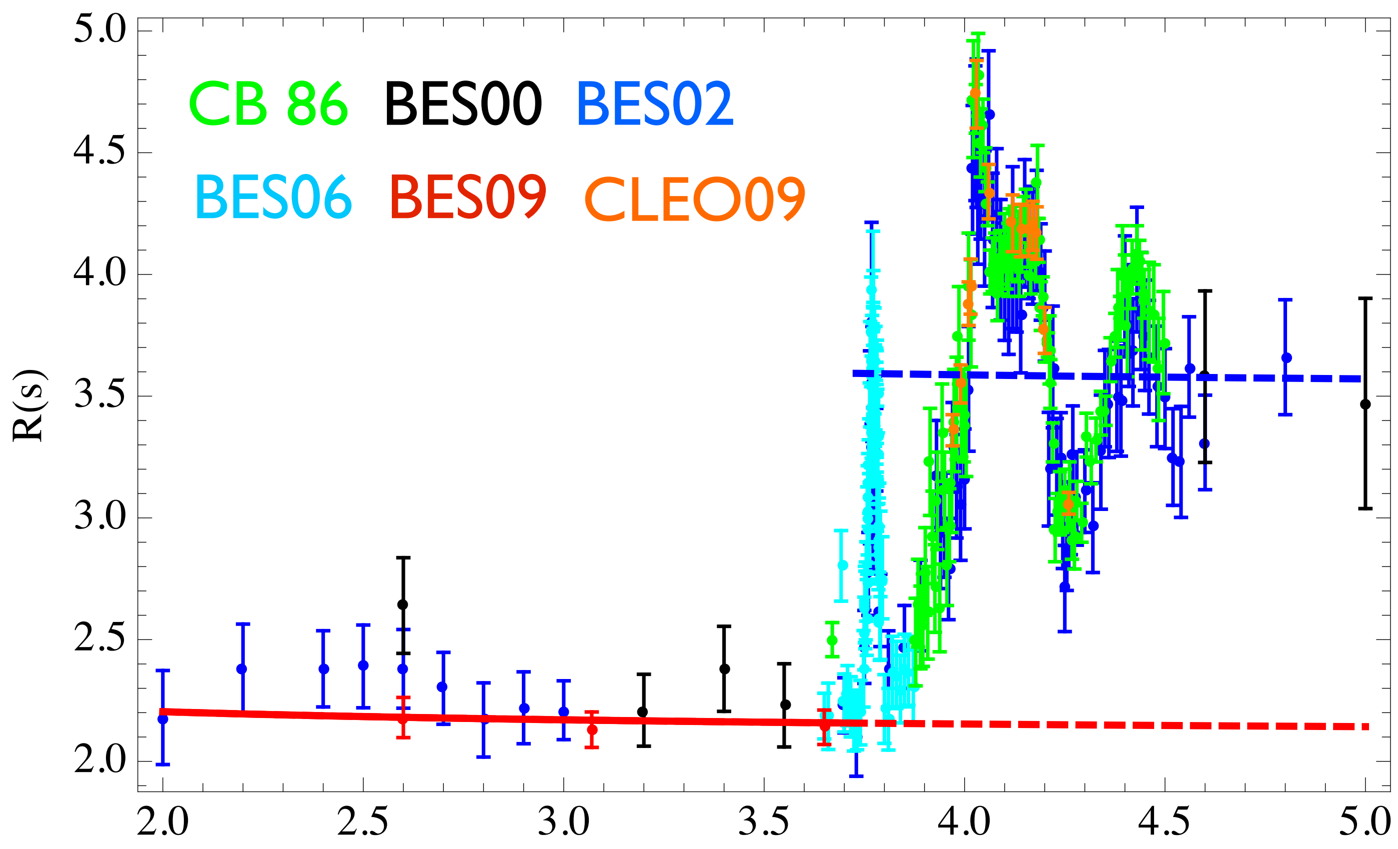


$\sim \alpha^3$

Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

Light flavor contribution in charm region
+
secondary production
+
singlet contribution
+
2loop QED



Non-perturbative effects

Non-perturbative effects due to gluon condensates to the moments are:

[Chetyrkin et al '12]

$$\mathcal{M}_n^{\text{nonp}}(\mu^2) = \frac{12\pi^2 Q_q^2}{(4\hat{m}_q^2)^{n+2}} \text{Cond}\, a_n \left(1 + \frac{\alpha_s(\hat{m}_q^2)}{\pi} b_n\right)$$

$a_n \, , b_n$ are numbers, and $\text{Cond} = \langle \frac{\alpha_s}{\pi} G^2 \rangle = (14 \pm 14) \cdot 10^{-3} \text{GeV}^4$ [Dominguez et al '14]

from fits to tau data

$$\frac{\mathcal{M}_n^{\text{nonp}}(\hat{m}_c)}{\mathcal{M}_n^{\text{th}}} \sim 0.5\% - 2\% \longrightarrow \Delta\hat{m}_c(\hat{m}_c) \sim 2\text{MeV} - 8\text{MeV}$$

QCD Sum Rules

Our approach

- Consider global duality
- Do not use experimental data on threshold region, only resonances
    - Exp data in threshold only for error estimation
- Use two different moments to extract the mass

QCD Sum Rules

Our approach

For a global duality:

$\hat{\Pi}_q(s)$ in $\overline{MS}$

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

$t \to \infty \quad \text{define the } \mathcal{M}_0$

[Erler, Luo '03]

QCD Sum Rules

Our approach

For a global duality:

$\hat{\Pi}_q(s)$ in $\overline{MS}$

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

$t \to \infty \quad \text{define the } \mathcal{M}_0$ (but has a divergent part) [Erler, Luo '03]

$$\lim_{t\to\infty} \hat{\Pi}_q(-t) \sim \log(t) \quad \longleftrightarrow \quad \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} R_q(s) \sim \log(\infty)$$

Fortunately, divergence given by the zero-mass limit of R(s)

QCD Sum Rules

Our approach

[Chetyrkin, Harlander, Kühn, '00]

$$
\begin{aligned}
\lambda_1^q(s) = 1 + \frac{\alpha_s(s)}{\pi} \\
&+ \left[\frac{\alpha_s(s)}{\pi}\right]^2 \left[\frac{365}{24} - 11\zeta(3) + n_q\left(\frac{2}{3}\zeta(3) - \frac{11}{12}\right)\right] \\
&+ \left[\frac{\alpha_s(s)}{\pi}\right]^3 \left[\frac{87029}{288} - \frac{121}{8}\zeta(2) - \frac{1103}{4}\zeta(3) + \frac{275}{6}\zeta(5)\right. \\
&\qquad +n_q\left(-\frac{7847}{216} + \frac{11}{6}\zeta(2) + \frac{262}{9}\zeta(3) - \frac{25}{9}\zeta(5)\right) \\
&\qquad \left. +n_q^2\left(\frac{151}{162} - \frac{1}{18}\zeta(2) - \frac{19}{27}\zeta(3)\right)\right]
\end{aligned}
$$

QCD Sum Rules

Our approach

Zeroth Sum Rule:

$$\sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty} \frac{\mathrm{d}s}{s}\frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s}\lambda_1^q(s)$$

$$= -\frac{5}{3} + \frac{\hat{\alpha}_s}{\pi}\left[4\zeta(3) - \frac{7}{2}\right] \qquad \hat{\alpha}_s = \alpha_s(\hat{m}_q^2)$$

$$+ \left(\frac{\hat{\alpha}_s}{\pi}\right)^2 \left[\frac{2429}{48}\zeta(3) - \frac{25}{3}\zeta(5) - \frac{2543}{48} + n_q\left(\frac{677}{216} - \frac{19}{9}\zeta(3)\right)\right]$$

$$+ \left(\frac{\hat{\alpha}_s}{\pi}\right)^3 \left[-9.86 + 0.40\, n_q - 0.01\, n_q^2\right]$$

n_q active flavors

QCD Sum Rules

Our approach

Zeroth Sum Rule:

$$\boxed{\sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)}} + \int_{4M^2}^{\infty} \frac{\mathrm{d}s}{s}\frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s}\lambda_1^q(s)$$

$$\hat{\alpha}_s = \alpha_s(\hat{m}_q^2)$$

[PDG]

R	M_R [GeV]	Γ_R^e [keV]
J/Ψ	3.096916	$5.55(14)$
$\Psi(2S)$	3.686109	$2.36(4)$

n_q active flavors

QCD Sum Rules

Our approach

Zeroth Sum Rule:

$$\sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s} \lambda_1^q(s)$$

$$\hat{\alpha}_s = \alpha_s(\hat{m}_q^2)$$

$$\hat{\alpha}_{em}(0) \sim 0.98\hat{\alpha}_{em}(M_{J/\Psi})$$

$$\Delta\hat{\alpha}_{em} \to \Delta m_c \sim 12\text{MeV}$$

n_q active flavors

QCD Sum Rules

Our approach

Zeroth Sum Rule: [Erler, Luo '03]

$$R_q^{\rm cont}(s) = 3Q_q^2\lambda_1^q(s)\sqrt{1-\frac{4\,\hat{m}_q^2(2M)}{s'}}\left[1+\lambda_3^q\frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

Simpler version of analytic reconstruction [Greynat, PM, Peris'12] $s' = s + 4(\hat{m}_q^2(2M) - M^2)$

Two parameters to determine: m_q , λ_3^q

QCD Sum Rules

Our approach

Zeroth Sum Rule: [Erler, Luo '03]

$$R_q^{\rm cont}(s) = 3Q_q^2\lambda_1^q(s)\sqrt{1-\frac{4\,\hat{m}_q^2(2M)}{s'}}\left[1+\lambda_3^q\frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

Simpler version of analytic reconstruction [Greynat, PM, Peris'12] $s' = s + 4(\hat{m}_q^2(2M) - M^2)$

Two parameters to determine: $m_q\ ,\ \lambda_3^q$

We need two equations: zeroth moment + nth moment

$$\frac{9}{4}Q_q^2\left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n}\bar{C}_n = \sum_{\rm resonances}\frac{9\pi\Gamma_R^e}{M_R^{2n+1}\hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty}\frac{{\rm d}s}{s^{n+1}}R_q(s)$$

$$n \geq 1$$

QCD Sum Rules

Our approach

Zeroth Sum Rule: [Erler, Luo '03]

$$R_q^{\rm cont}(s) = 3Q_q^2 \lambda_1^q(s) \sqrt{1 - \frac{4\,\hat{m}_q^2(2M)}{s'}} \left[1 + \lambda_3^q \frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

Simpler version of analytic reconstruction [Greynat, PM, Peris'12] $s' = s + 4(\hat{m}_q^2(2M) - M^2)$

Two parameters to determine: $m_q\ ,\ \lambda_3^q$

We use Zeroth + 2nd moments
(no experimental data on R(s) so far)

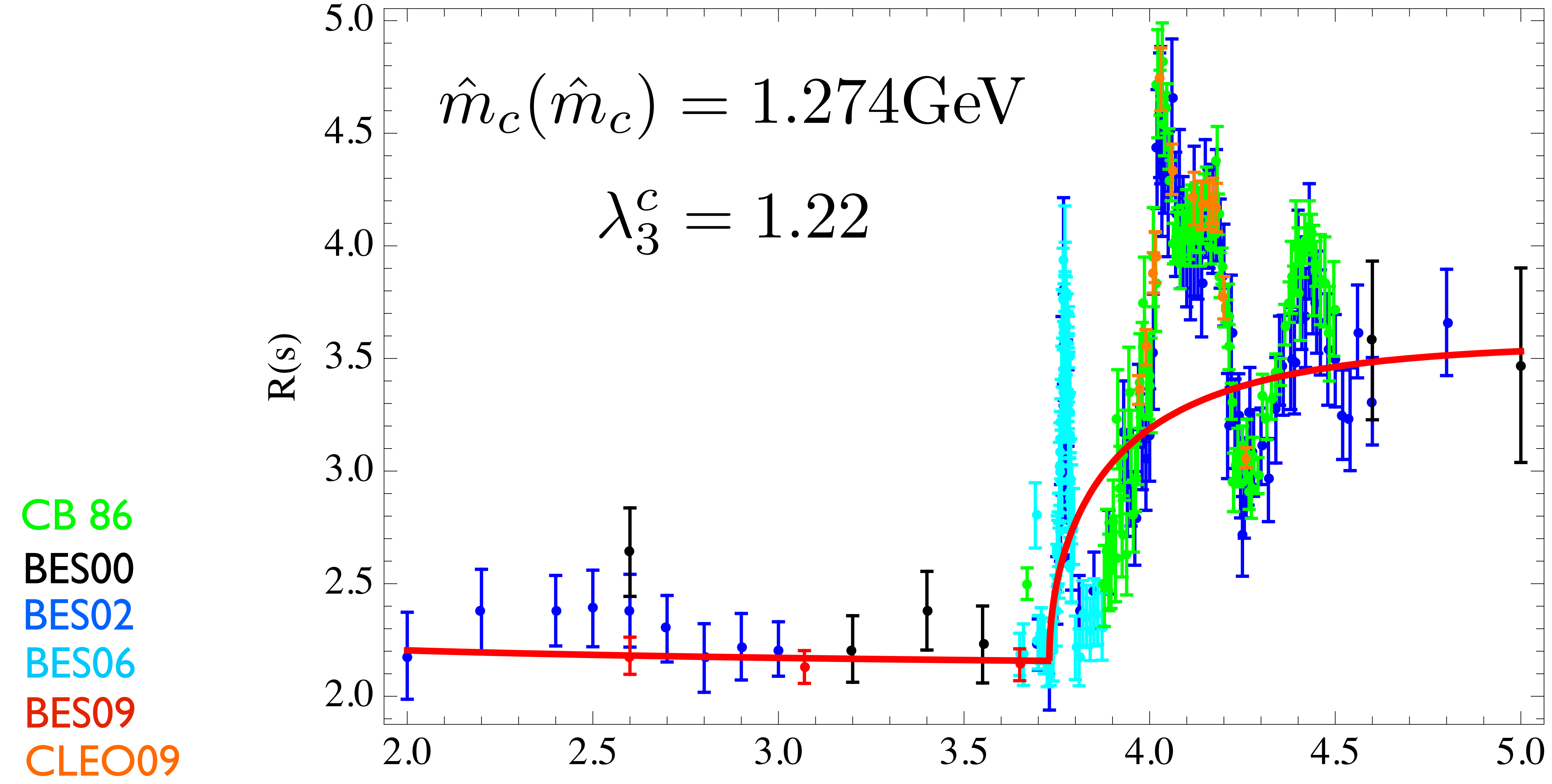
preliminary results

n	Resonances	Continuum	Total	Theory
0	1.231 (24)	−3.230(+31)(43)	−1.999(59)	**Input** (11)
1	1.184 (24)	0.960(+12)(17)	2.144(36)	2.165(16)
2	1.161 (25)	0.327(+6)(8)	1.488(37)	**Input** (25)
3	1.157 (26)	0.149(+3)(4)	1.306(47)	1.291(38)
4	1.167 (27)	0.077(+2)(2)	1.244(65)	1.206(59)
5	1.188 (28)	0.042(+1)(1)	1.230(97)	1.158(93)

QCD Sum Rules

Our approach

preliminary results

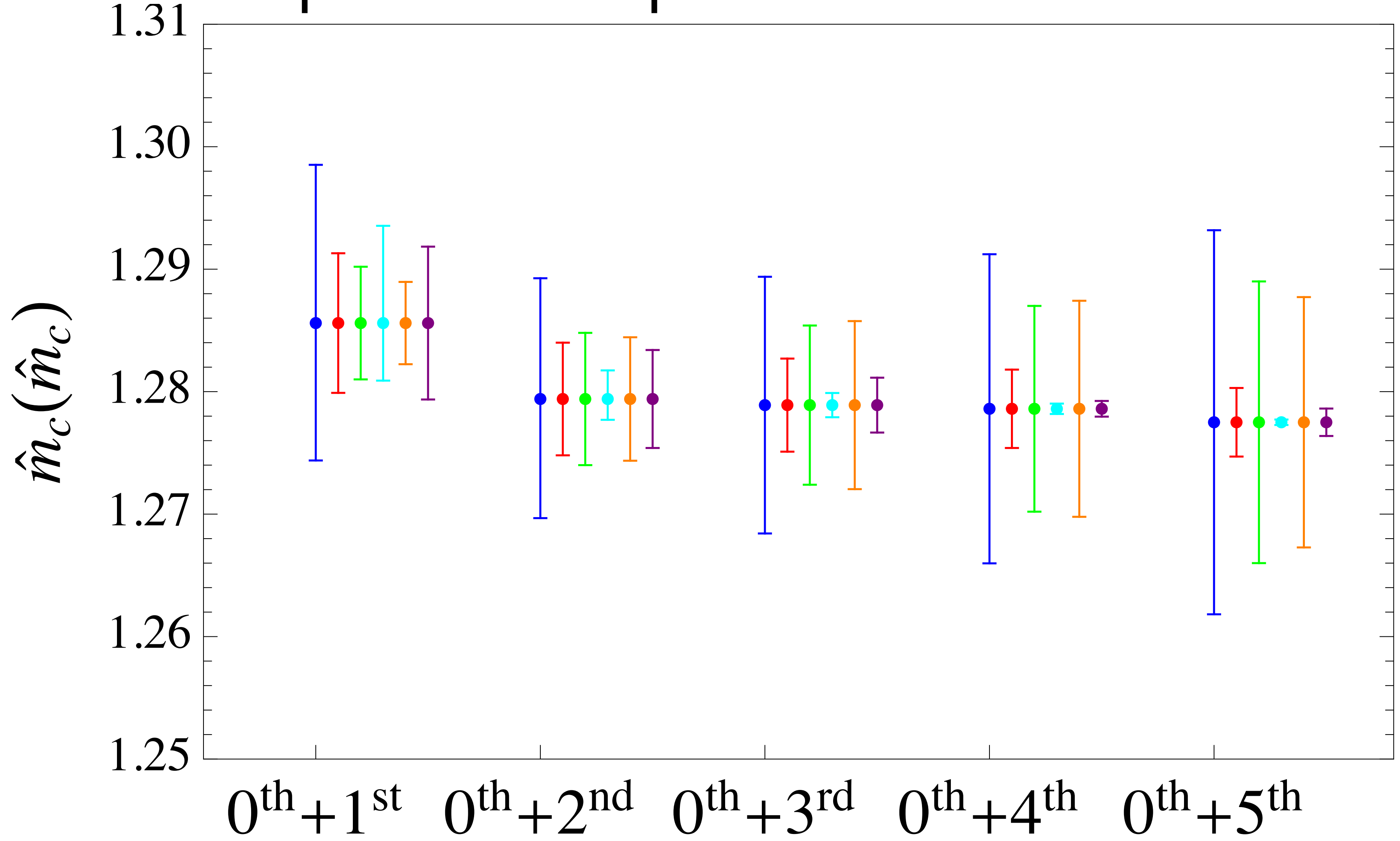


QCD Sum Rules

Our approach

preliminary results

Repeat for each pair Zeroth+nth moment



Resonances

Truncation error

Comparison with R^{Exp} threshold data

Condensates

$\Delta\alpha_s(M_z)$

Good consistency between different sum rules

QCD Sum Rules

Our approach: error budget

Resonances:

$$\frac{9}{4}Q_q^2\left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n}\bar{C}_n = \sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{M_R^{2n+1}\hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty}\frac{\mathrm{d}s}{s^{n+1}}R_q(s)$$

from 6 MeV to 3 MeV

(0th+1st) (0th+5th)

(completely dominated by J/Ψ)

R	M_R [GeV]	Γ_R^e [keV]
J/Ψ	3.096916	5.55(14)
$\Psi(2S)$	3.686109	2.36(4)

QCD Sum Rules

Our approach: error budget

Truncation Error (theory error):

$$\mathcal{M}_n^{\rm pQCD} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n$$

$$\bar{C}_n = \bar{C}_n^{(0)} + \left(\frac{\hat{\alpha}}{\pi}\right) \bar{C}_n^{(1)} + \left(\frac{\hat{\alpha}}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\hat{\alpha}}{\pi}\right)^3 \bar{C}_n^{(3)} + \mathcal{O}\left(\frac{\hat{\alpha}}{\pi}\right)^4$$

[Erler, Luo '03]

$$\boxed{\Delta\mathcal{M}_n^{(4)} = \pm N_C C_F C_A^3\, Q_q^2 \left[\frac{\hat{\alpha}_s(\hat{m}_q)}{\pi}\right]^4 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n}}$$

from 5 MeV to 10 MeV

(0th+1st) (0th+5th)

Example known orders

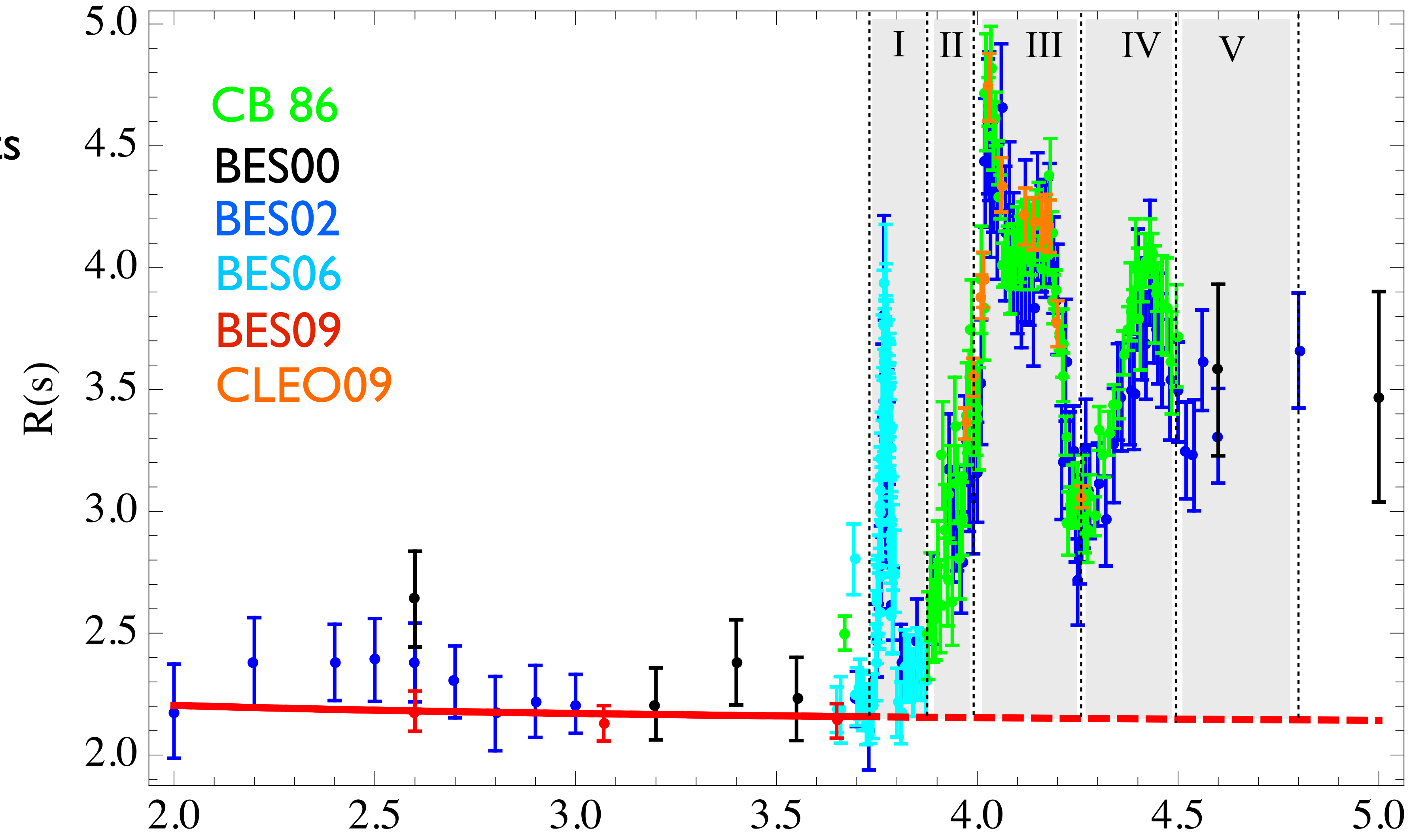
n	$\frac{\Delta\mathcal{M}_n^{(2)}}{\lvert\mathcal{M}_n^{(2)}\rvert}$	$\frac{\Delta\mathcal{M}_n^{(3)}}{\lvert\mathcal{M}_n^{(3)}\rvert}$
0	1.88	3.03
1	2.14	2.84
2	1.92	4.58
3	3.25	5.63
4	6.70	4.30
5	19.18	3.62

QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data: $(2M_D \leq \sqrt{s} \leq 4.8\text{GeV})$

Calculate Exp moments



QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data:

Collab.	n	$2M_{D^0} - 3.872$ GeV	$(3.872 - 3.97)$ GeV	$(3.97 - 4.26)$ GeV	$(4.26 - 4.496)$ GeV	$(4.496 - 4.8)$ GeV
CB86	0	—	0.0339(22)(23)	0.2455(25)(172)	0.1542(27)(108)	—
	1	—	0.0220(14)(15)	0.1459(16)(102)	0.0800(14)(56)	—
	2	—	0.0142(9)(10)	0.0868 (9)(61)	0.0416(7)(29)	—
BES02	0	0.0333(24)(17)	0.0362(29)(18)	0.2361(41)(118)	0.1398(38)(70)	0.1704(63)(85)
	1	0.0232(17)(12)	0.0234(19)(12)	0.1400(24)(70)	0.0726(20)(36)	0.0788(30)(39)
	2	0.0161(12)(8)	0.0152(13)(8)	0.0832(15)(42)	0.0377(10)(19)	0.0365(14)(18)
BES06	0	0.0310(16)(15)	—	—	—	—
	1	0.0216(11)(11)	—	—	—	—
	2	0.0151(8)(7)	—	—	—	—
CLEO09	0	—	—	0.2590(22)(5)	—	—
	1	—	—	0.1538(13)(3)	—	—
	2	—	—	0.0915(8)(2)	—	—
Total	0	0.0319(14)(6)(10)	0.0350(14)(9)(13)	0.2544(18)(40)(14)(18)	0.1447(27)(46)(37)	0.1704(63)(85)
	1	0.0222(9)(4)(7)	0.0226(10)(5)(8)	0.1510(11)(24)(8)(11)	0.0751(14)(24)(12)	0.0788(30)(39)
	2	0.0155(6)(3)(5)	0.0147(7)(3)(5)	0.0898(6)(14)(5)(6)	0.0390(7)(12)(10)	0.0365(14)(18)

QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data:

$$\int_{(2M_{D^0})^2}^{(4.8\mathrm{GeV})^2} \frac{\mathrm{d}s}{s} R_c^{\mathrm{cont}}(s)\Big|_{\hat{m}_c=1.274\mathrm{GeV}} = \mathcal{M}_0^{\mathrm{Data}} = 0.6362(195) \quad \longrightarrow \quad \lambda_3^{\mathrm{c,exp}} = 1.34(17)$$

$$(2M_D \leq \sqrt{s} \leq 4.8\mathrm{GeV})$$

Error induced to Quark mass:

I) $\lambda_3^c = 1.22 \longrightarrow \lambda_3^{\mathrm{c,exp}} = 1.34$

from $+6.4$ MeV to $+0.2$ MeV

II) $\Delta\lambda_3^{\mathrm{c,exp}} = 0.17$

from 4.7 MeV to 0.1 MeV

n	Data	$\lambda_3^c = 1.34(17)$	$\lambda_3^c = 1.22$
0	0.6362(195)	0.6362(195)	0.6222
1	0.3498(101)	0.3506(111)	0.3426
2	0.1955(54)	0.1969(65)	0.1922
3	0.1110(29)	0.1126(38)	0.1099
4	0.0640(16)	0.0656(23)	0.0640
5	0.0375(9)	0.0389(14)	0.0379

QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data:

$$\int_{(2M_{D^0})^2}^{(4.8\mathrm{GeV})^2} \frac{\mathrm{d}s}{s} R_c^{\mathrm{cont}}(s)\Big|_{\hat{m}_c=1.274\mathrm{GeV}} = \mathcal{M}_0^{\mathrm{Data}} = 0.6362(195) \longrightarrow \lambda_3^{c,\mathrm{exp}} = 1.34(17)$$

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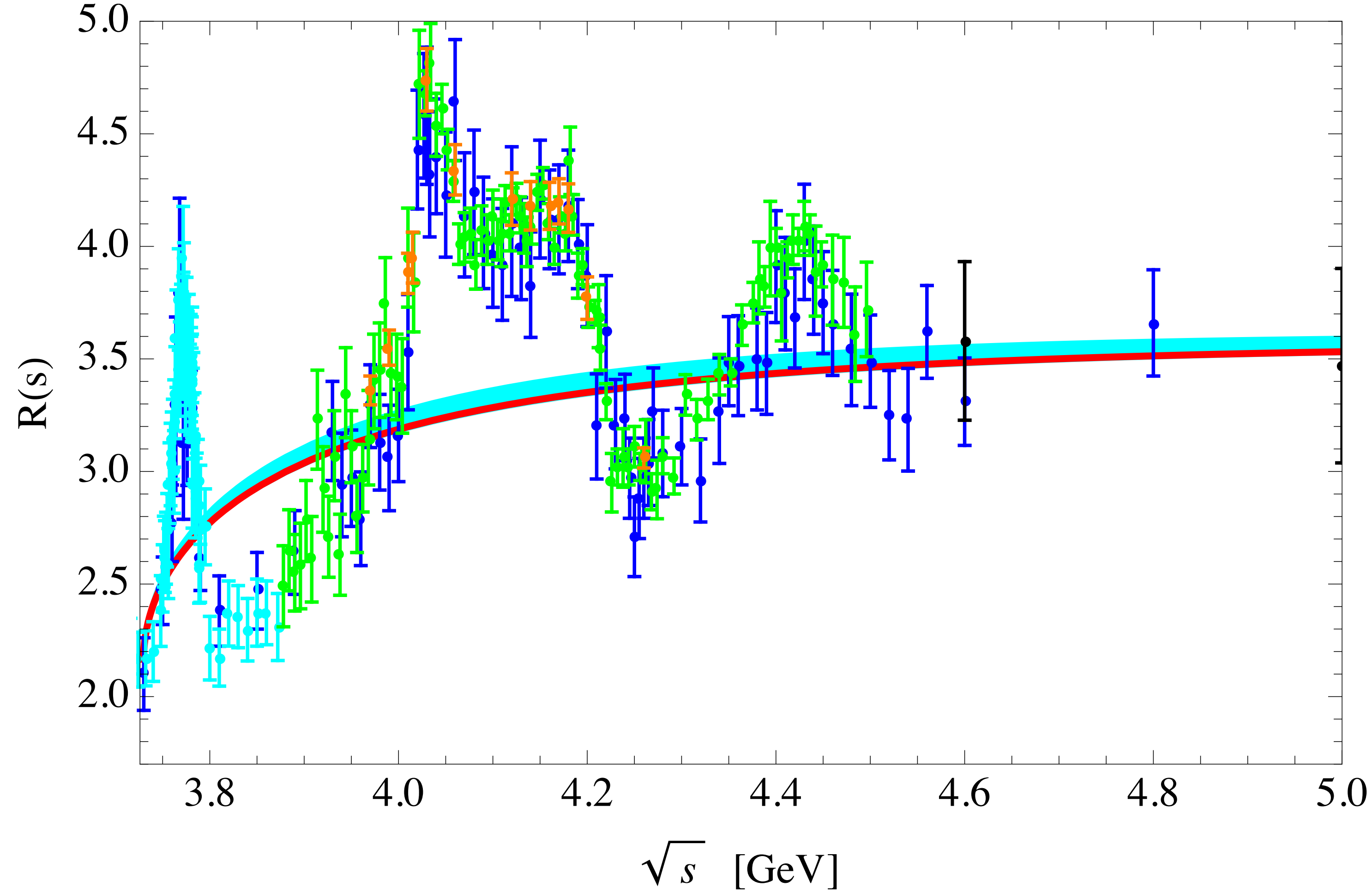


QCD Sum Rules

Our approach: error budget

Condensates:

Non-perturbative effects due to gluon condensates to the moments are: [Chetyrkin et al '12]

$$\mathcal{M}_n^{\mathrm{nonp}}(\mu^2) = \frac{12\pi^2 Q_q^2}{(4\hat{m}_q^2)^{n+2}} \mathrm{Cond}\, a_n \left(1 + \frac{\alpha_s(\hat{m}_q^2)}{\pi} b_n\right)$$

$a_n\,,\, b_n$ are numbers, and $\mathrm{Cond} = \langle \frac{\alpha_s}{\pi} G^2 \rangle = (14 \pm 14) \cdot 10^{-3} \mathrm{GeV}^4$ [Dominguez et al '14]

$$\Delta \langle \frac{\alpha_s}{\pi} G^2 \rangle = 14 \cdot 10^{-3} \mathrm{GeV}^4 \longrightarrow \text{from 3 MeV to 10 MeV}$$

(0th+1st) (0th+5th)

Parametric error:

$$\Delta \hat{m}_c(\hat{m}_c)[\mathrm{MeV}] = +0.24 \cdot 10^3 \frac{\mathrm{MeV}}{\mathrm{GeV}^4} \Delta \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

(but this is only the first condensate)

QCD Sum Rules

Our approach: error budget

$\Delta\alpha_s(M_z)$ $\quad\quad$ $\alpha_s(M_z) = 0.1182(16)$ $\quad\quad$ from PDG

$$\Delta\alpha_s(M_z) = 0.0016 \longrightarrow \text{from 6 MeV to 1 MeV}$$

Parametric error:

(0th+1st) $\quad \Delta\hat{m}_c(\hat{m}_c)[\mathrm{MeV}] = 3.9 \cdot 10^3 \Delta\alpha_s(M_z)$

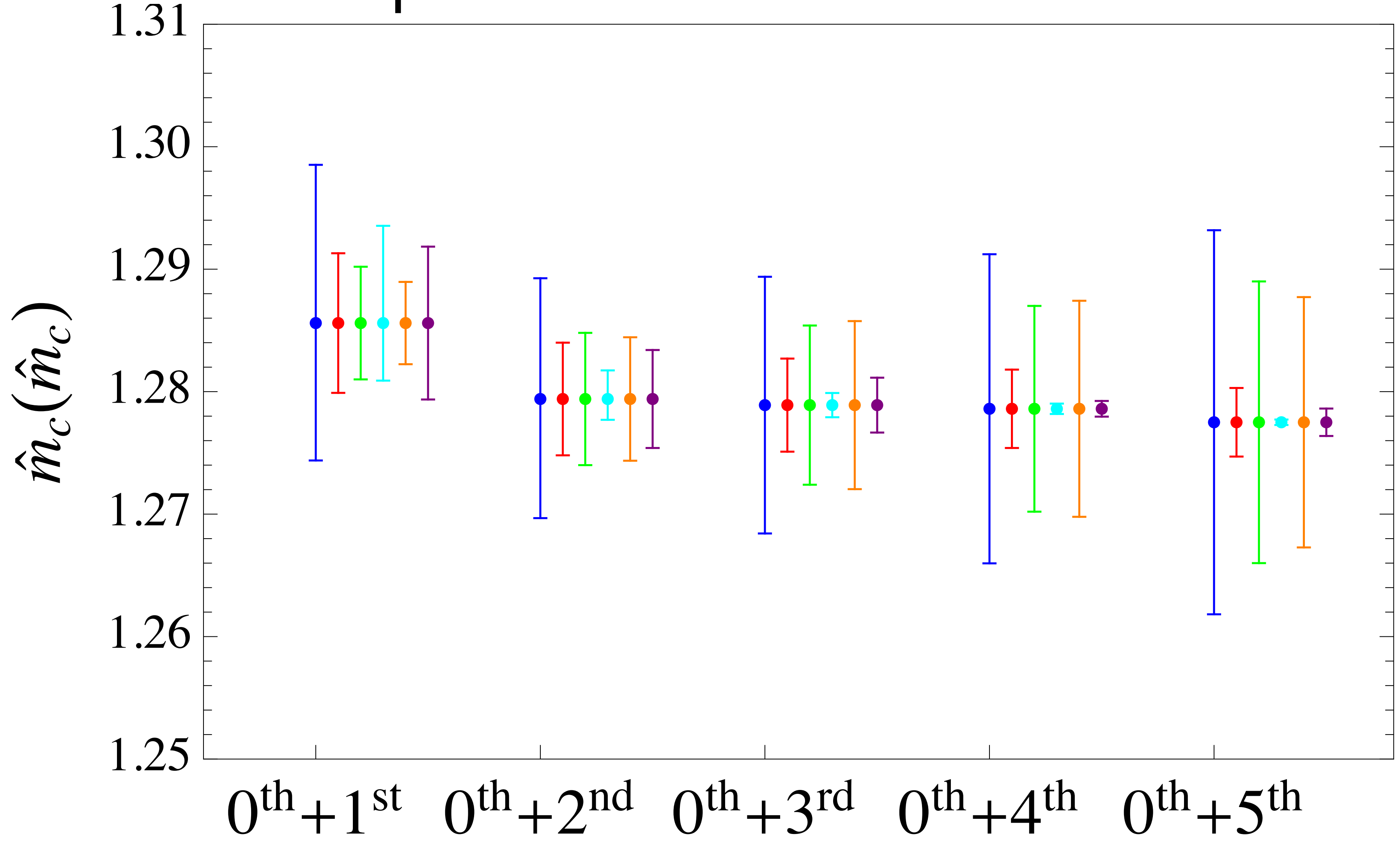
(0th+5th) $\quad \Delta\hat{m}_c(\hat{m}_c)[\mathrm{MeV}] = -0.7 \cdot 10^3 \Delta\alpha_s(M_z)$

QCD Sum Rules

Our approach

preliminary results

What pair/result to choose?



Resonances

Truncation error

Comparison with R^{Exp} threshold data

Condensates

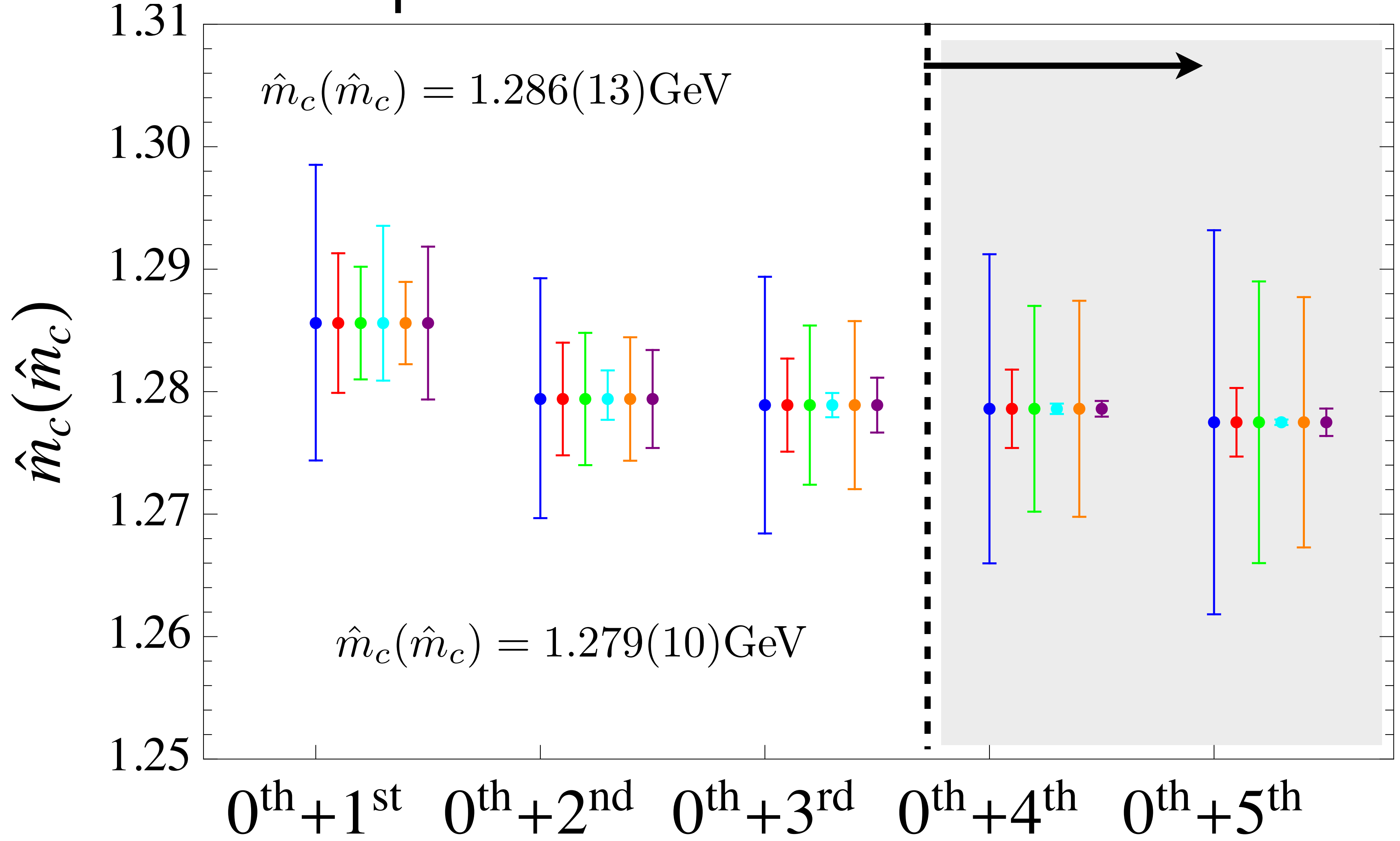
$\Delta\alpha_s(M_z)$

QCD Sum Rules

Our approach

preliminary results

What pair/result to choose?



Resonances

Truncation error

Comparison with R^{Exp} threshold data

Condensates

$\Delta\alpha_s(M_z)$

Large condensate effects
+
new condensates will matter

QCD Sum Rules

Our approach: more than two moments?

Define a χ^2 function:

$$\chi^2 = \frac{1}{2}\sum_{n,m}\left(\mathcal{M}_n - \mathcal{M}_n^{\rm pQCD}\right)\left(\mathcal{C}^{-1}\right)^{nm}\left(\mathcal{M}_m - \mathcal{M}_m^{\rm pQCD}\right) + \chi_c^2$$

$$\mathcal{C} = \frac{1}{2}\sum_{n,m}\rho^{{\rm Abs}(n-m)}\Delta\mathcal{M}_n^{(4)}\Delta\mathcal{M}_m^{(4)}$$

ρ a correlation parameter

$$\chi_c^2 = \left(\frac{\Gamma^e_{J/\Psi(1S)} - \Gamma^{e,\rm exp}_{J/\Psi(1S)}}{\Delta\Gamma^e_{J/\Psi(1S)}}\right)^2 + \left(\frac{\Gamma^e_{\Psi(2S)} - \Gamma^{e,\rm exp}_{\Psi(2S)}}{\Delta\Gamma^e_{\Psi(2S)}}\right)^2 +$$
$$\left(\frac{\hat{\alpha}_s(M_z) - \hat{\alpha}_s(M_z)^{\rm exp}}{\Delta\hat{\alpha}_s(M_z)}\right)^2 + \left(\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle - \langle\frac{\alpha_s}{\pi}G^2\rangle^{\rm exp}}{\Delta\langle\frac{\alpha_s}{\pi}G^2\rangle}\right)^2$$

QCD Sum Rules

Our approach: more than two moments?

Define a χ^2 function:

	Constraints	0th + 1st + 2nd
ρ		0
$m_c(m_c)$		$1.2815^{+0.0092}_{-0.0092}\mathrm{GeV}$
λ_3		$1.151^{+0.081}_{-0.081}$
$\Gamma^e_{J/\Psi}$	$5.55(14)10^{-6}$	$(5.56^{+0.14}_{-0.14})10^{-6}$
$\Gamma^e_{\Psi'}$	$2.36(4)10^{-6}$	$(2.361^{+0.040}_{-0.040})10^{-6}$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$14(14)10^{-3}$	$(16^{+14}_{-14})10^{-3}$
$\alpha_s(M_z)$	$0.1182(16)$	$0.1179^{+0.0015}_{-0.0015}$

Assuming no correlation:

$$\hat{m}_c(\hat{m}_c) = 1.282(9)\mathrm{GeV}$$

QCD Sum Rules

Our approach: more than two moments?

Define a χ^2 function:

Correlation to ensure χ^2 = D.O.F.

	Constraints	0th + 1st + 2nd	0th + (1st + 2nd)$_\rho$
ρ		0.520	0.627
$\hat{m}_c(\hat{m}_c)$ [GeV]		$1.2808^{+0.0088}_{-0.0087}$	$1.2806^{+0.0089}_{-0.0089}$
λ_3^c		$1.137^{+0.071}_{-0.071}$	$1.144^{+0.080}_{-0.080}$
$\Gamma^e_{J/\Psi(1S)}$ [keV]	5.55(14)	$5.57^{+0.12}_{-0.11}$	$5.57^{+0.14}_{-0.14}$
$\Gamma^e_{\Psi(2S)}$ [keV]	2.36(4)	$2.362^{+0.035}_{-0.034}$	$2.362^{+0.040}_{-0.040}$
$\langle\frac{\alpha_s}{\pi}G^2\rangle$ [GeV^4]	0.014(14)	$0.018^{+0.013}_{-0.012}$	$0.018^{+0.013}_{-0.013}$
$\hat{\alpha}_s(M_z)$	0.1182(16)	$0.1176^{+0.0015}_{-0.0015}$	$0.1176^{+0.0015}_{-0.0015}$

$\hat{m}_c(\hat{m}_c) = 1.281(9)\text{GeV}$

QCD Sum Rules

Our approach: more than two moments?

Define a χ^2 function:

Correlation to ensure χ^2 = D.O.F.

	Constraints	0th + (1st + 2nd+3rd)$_\rho$
ρ		0.85
$\hat{m}_c(\hat{m}_c)$ [GeV]		$1.2790^{+0.0083}_{-0.0073}$
λ_3^c		$1.141^{+0.073}_{-0.075}$
$\Gamma^e_{J/\Psi(1S)}$ [keV]	5.55(14)	$5.59^{+0.13}_{-0.14}$
$\Gamma^e_{\Psi(2S)}$ [keV]	2.36(4)	$2.363^{+0.036}_{-0.035}$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$ [GeV^4]	0.014(14)	$0.019^{+0.012}_{-0.012}$
$\hat{\alpha}_s(M_z)$	0.1182(16)	$0.1173^{+0.0013}_{-0.0015}$

$$\hat{m}_c(\hat{m}_c) = 1.279(8)\text{GeV}$$

QCD Sum Rules

Our approach: more than two moments?

Preferred scenario: $0\mathrm{th} + (1\mathrm{st} + 2\mathrm{nd})_\rho$

$$\hat{m}_c(\hat{m}_c) = 1.281(9)\mathrm{GeV}$$

Include Comparison with R^{Exp} threshold data: $\Delta\hat{m}_c(\hat{m}_c) = 8\mathrm{MeV}$

$$\boxed{\hat{m}_c(\hat{m}_c) = 1.281(12)\mathrm{GeV}}$$

preliminary result

Conclusions and Outlook

- Heavy quark masses are interesting: for being fundamental parameters as well as for their implications on many phenomenological scenarios.
- From the different strategies, one of the most precise is the use of SR. Quark mass determinations at the % or sub-% level.

- Using SR + global fit using different moments (χ^2), we extract $\hat{m}_c(\hat{m}_c)$
  - Results still preliminary $\hat{m}_c(\hat{m}_c) = 1.281(12)\mathrm{GeV}$
- Good agreement with other determinations based on SRs and lattice!
- Error sources are understood: seems a clear roadmap for improvements
- Next step: the bottom case

Thanks!