The Mainz (g—2) project: Status and Perspectives

Hartmut Wittig

PRISMA Cluster of Excellence, Institute for Nuclear Physics and Helmholtz Institute Mainz

MITP Workshop
Determination of the Fundamental Parameters in QCD
9 March 2016

HELMHOLTZ ASSOCIATION
Helmholtz Institute Mainz

PRISMA

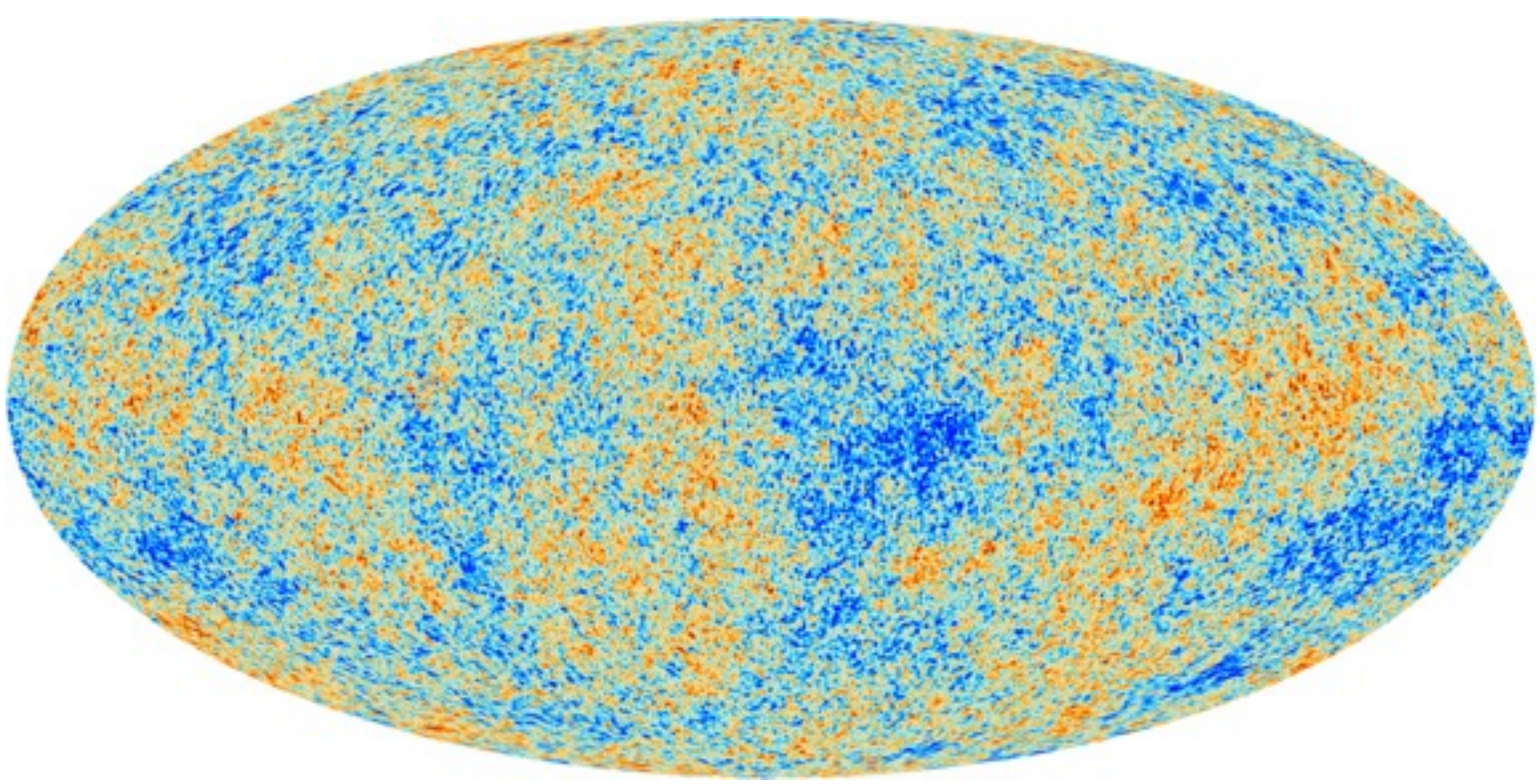
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

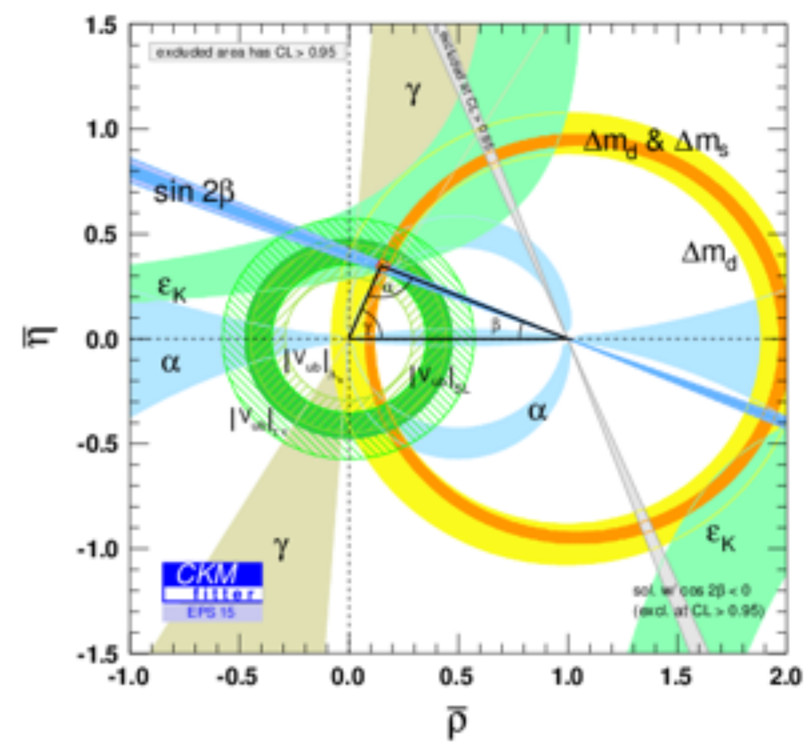
The Standard Model after the Higgs discovery

* Standard Model fully established but cannot account for:
  - Mass and scale hierarchies:

    $$m_{\mathrm{top}}/m_{\nu_e} > 10^{11} \qquad\qquad m_{\mathrm{Higgs}} \ll m_{\mathrm{Planck}}$$

  - Dark matter and dark energy
  - Amount of CP violation to sustain matter/antimatter asymmetry





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  - Search for new particles and phenomena at higher energy
  - Search for enhancement of rare phenomena
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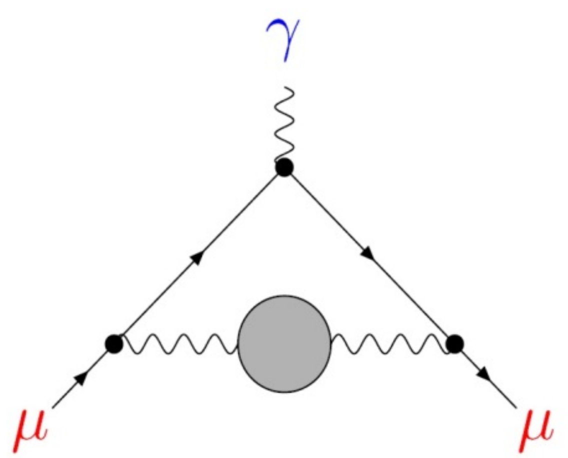
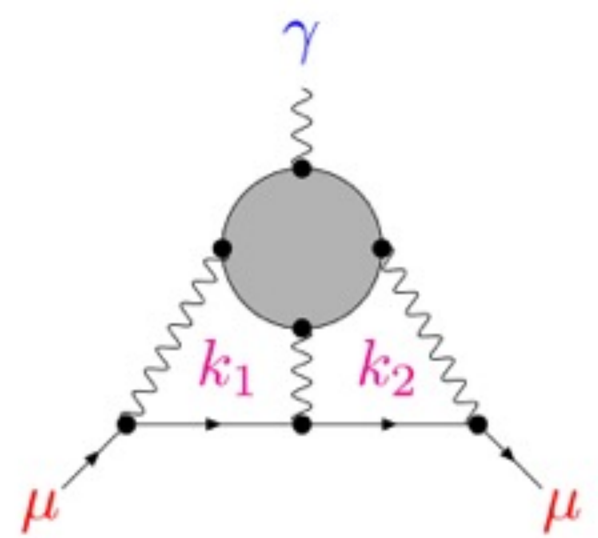
Precision Tests of the Standard Model

Anomalous magnetic moment of the muon: $a_\mu \equiv \frac{1}{2}(g-2)_\mu$

$$a_\mu = \begin{cases} 116\,592\,080(54)(33)\cdot 10^{-11} & \text{Experiment} \\ 116\,591\,802(2)(42)(26)\cdot 10^{-11} & \text{SM prediction} \end{cases}$$





Dispersion theory:

$$a_\mu^{\rm HVP} = (692.3 \pm 4.2 \pm 0.3)\cdot 10^{-10}$$

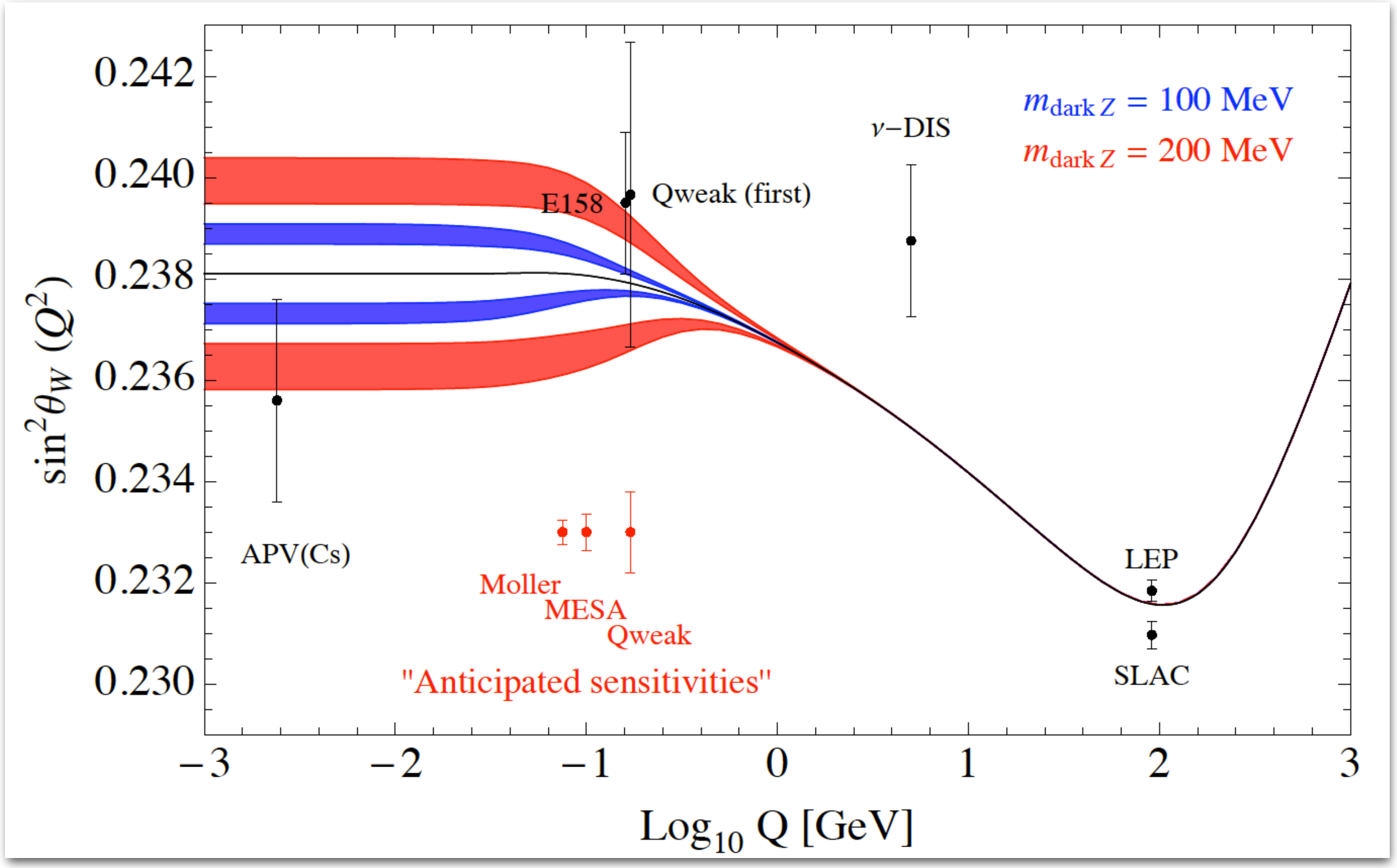
based on $R_{\rm exp}(e^+e^- \longrightarrow \text{hadrons})$

Model estimates:

$$a_\mu^{\rm HLbL} = \begin{cases} (105 \pm 26)\cdot 10^{-11} \\ (116 \pm 39)\cdot 10^{-11} \end{cases}$$

Precision Tests of the Standard Model

Running of electroweak mixing angle



- ✱ Running of $\sin^2\theta_W$ at low energies discriminates between different scenarios for "New Physics"
- ✱ Challenge for theory: hadronic contributions

The Mainz $(g-2)_\mu$ project

Collaborators:

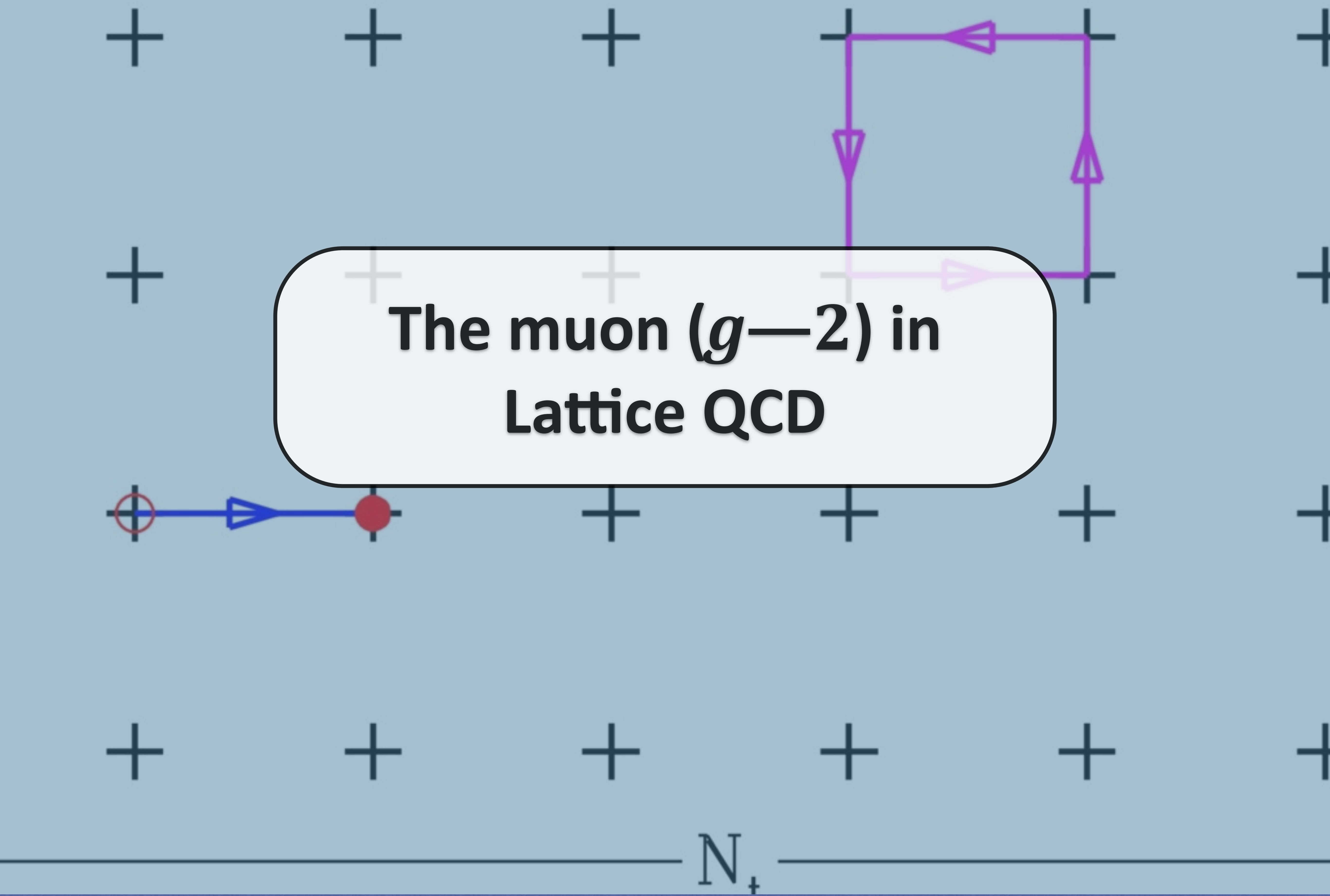
N. Asmussen, A. Gérardin, J. Green, O. Gryniuk, G. von Hippel, H. Horch, H. Meyer, A. Nyffeler, V. Pascalutsa, A. Risch, HW

M. Della Morte, A. Francis, B. Jäger, V. Gülpers, G. Herdoíza

THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Topics:

* Hadronic vacuum polarisation
* Light-by-light scattering
* Running of $\alpha_{\rm em}$ and $\sin^2\theta_{\rm W}$
* Determination of $\alpha_{\rm s}$ from vacuum polarisation function



The muon (g—2) in Lattice QCD

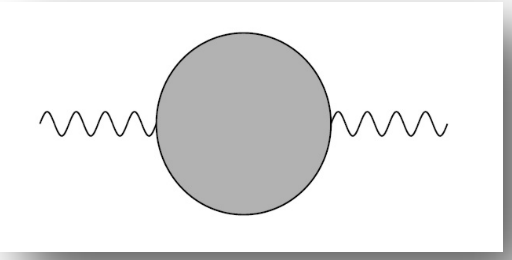
Lattice QCD approach to HVP

* Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\, \{\Pi(Q^2) - \Pi(0)\}$$

$$\Pi_{\mu\nu}(Q) = \int d^4x\, \mathrm{e}^{iQ\cdot(x-y)}\, \langle J_\mu(x) J_\nu(y)\rangle \equiv (Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi(Q^2)$$

$$J_\mu = \tfrac{2}{3}\overline{u}\gamma_\mu u - \tfrac{1}{3}\overline{d}\gamma_\mu d - \tfrac{1}{3}\overline{s}\gamma_\mu s + \ldots$$



* Lattice momenta are quantised: $Q_\mu = \dfrac{2\pi}{L_\mu}$

* Determine VPF $\Pi(Q^2)$ and additive renormalisation $\Pi(0)$

* Statistical accuracy of $\Pi(Q^2)$ deteriorates as $Q \longrightarrow 0$

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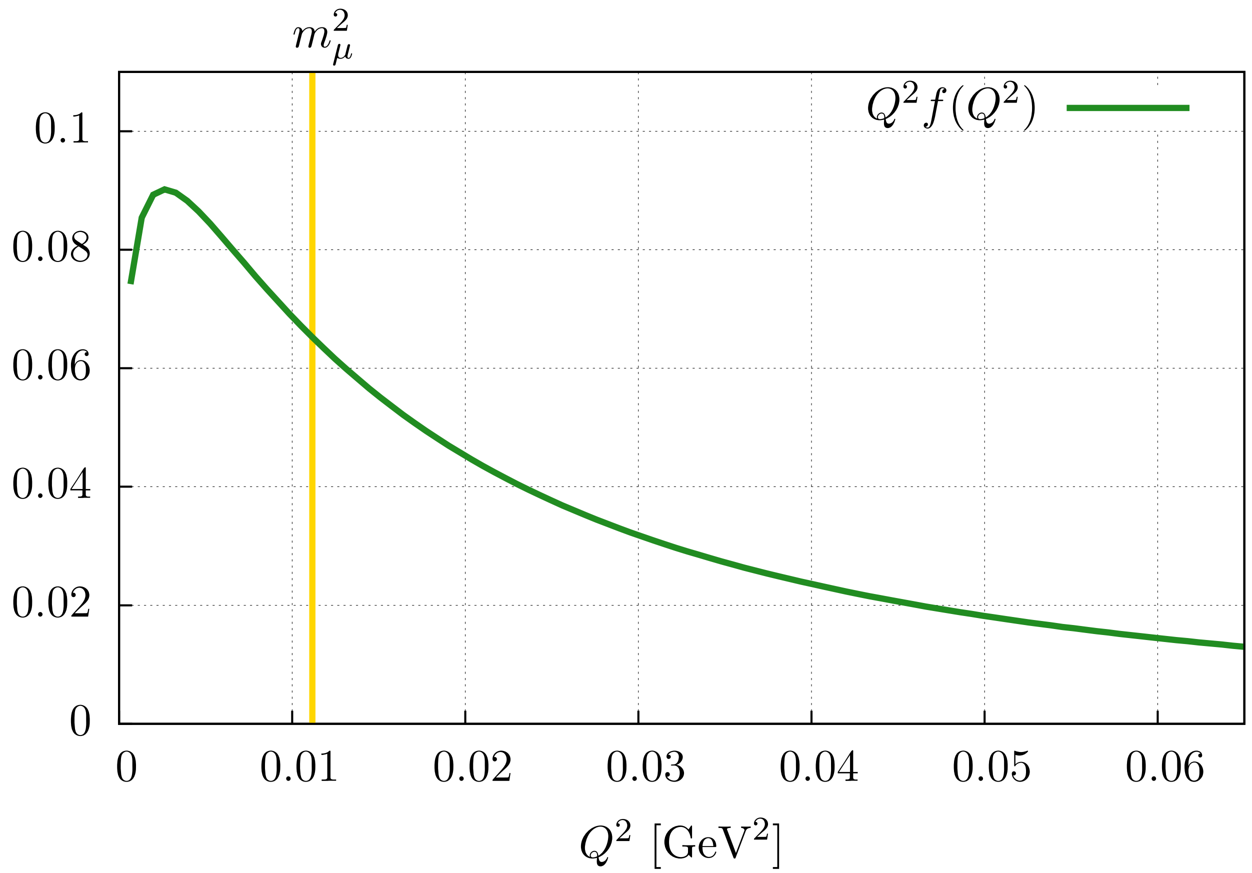
$$a_\mu^{\rm HVP} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\, \{\Pi(Q^2) - \Pi(0)\}$$

- Integrand peaked near $Q^2 \approx (\sqrt{5}-2)m_\mu^2$



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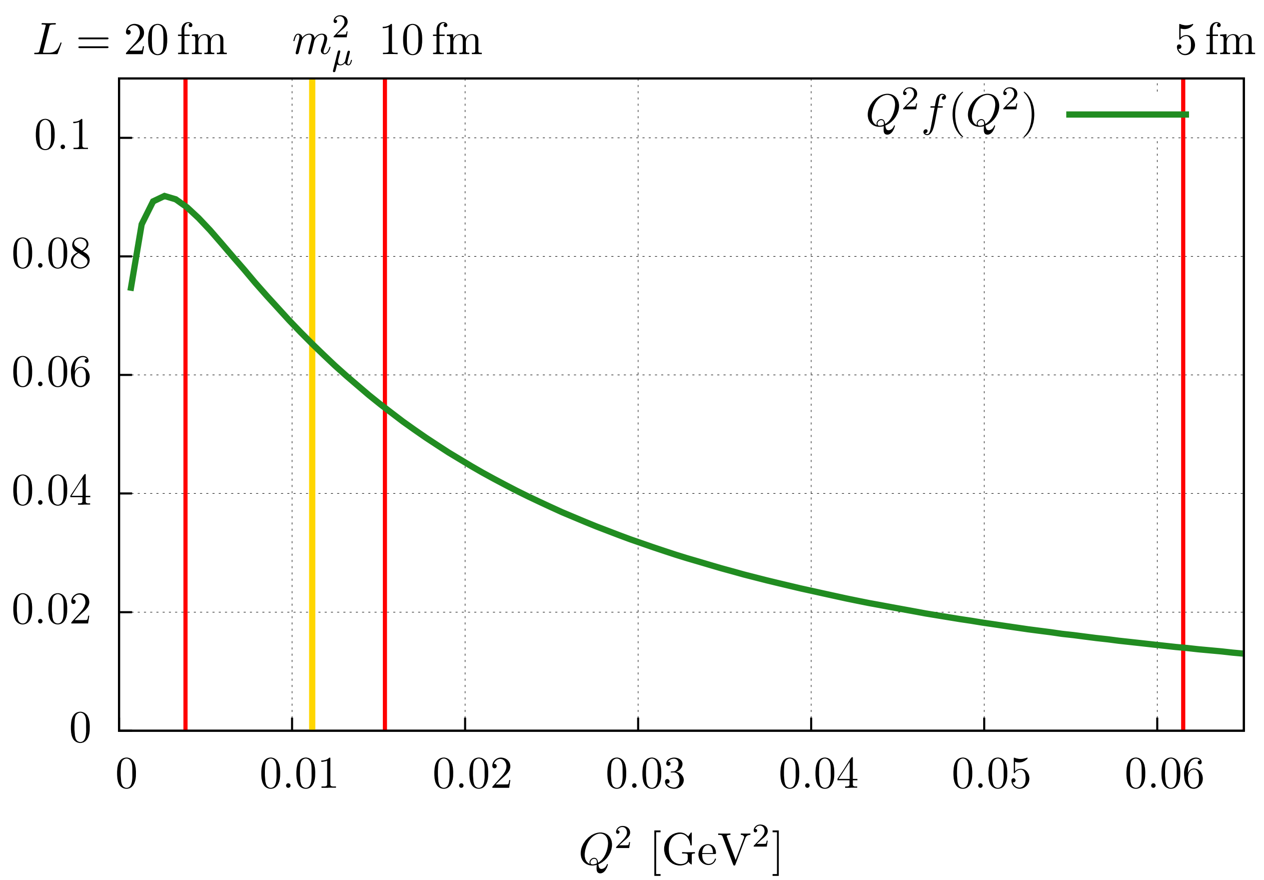
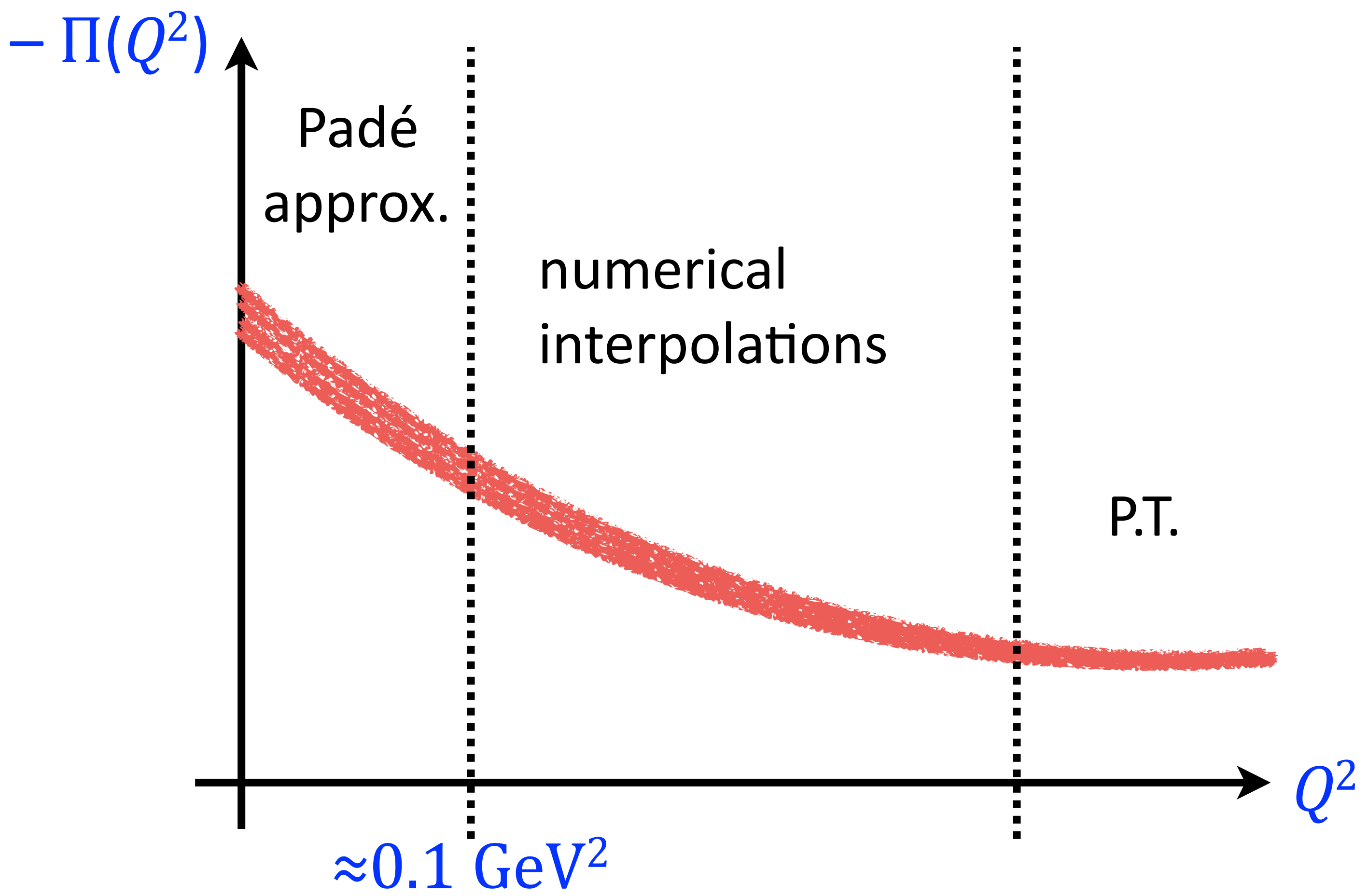


Accurate determination requires large statistics on large volumes!

Lattice QCD approach to HVP

- "Hybrid" method: *[Golterman, Maltman & Peris, Phys Rev D90 (2014) 074508]*



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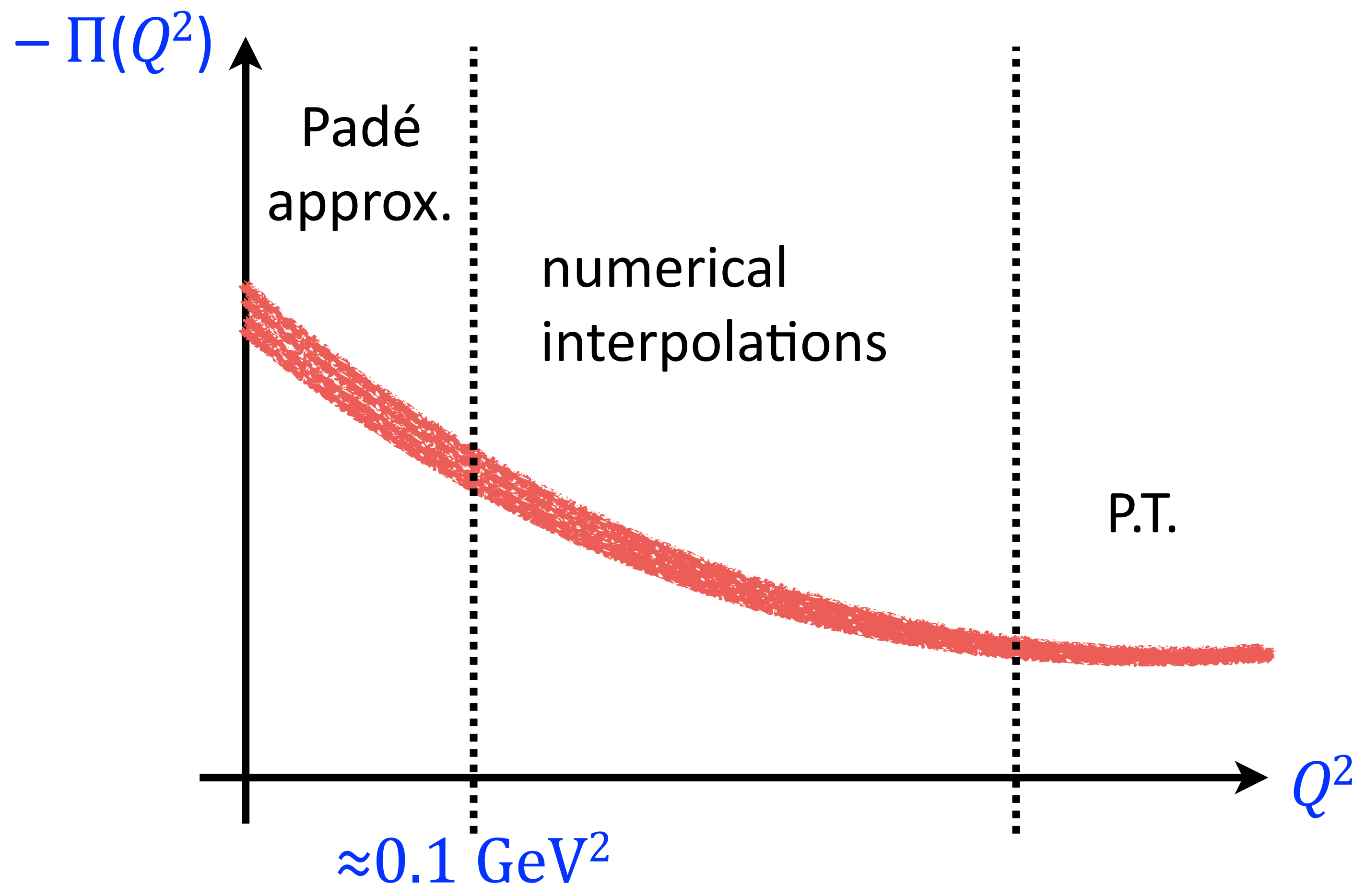

- ✱ Determine $\Pi(0)$ from Padé approximation in small-momentum region

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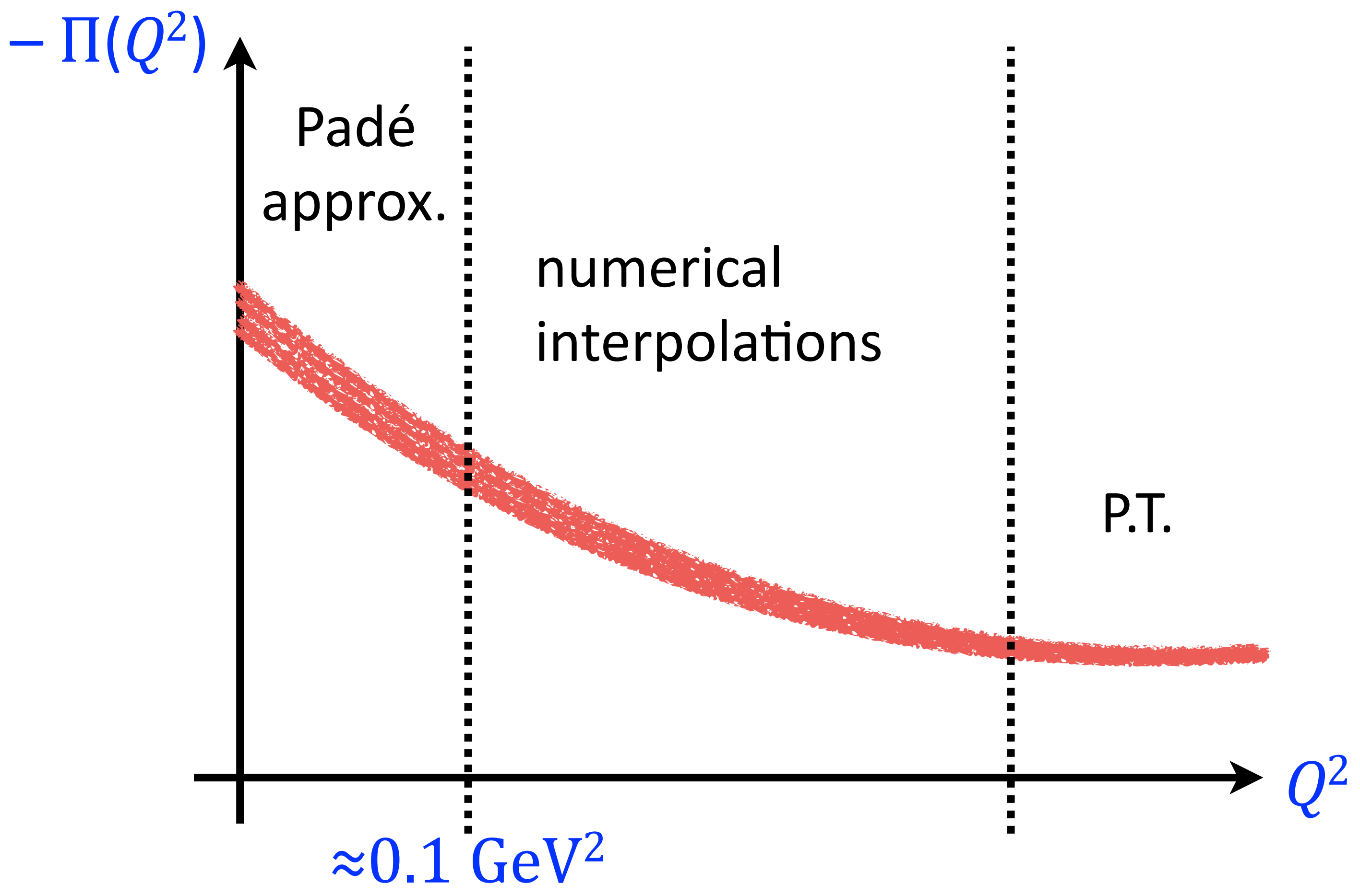


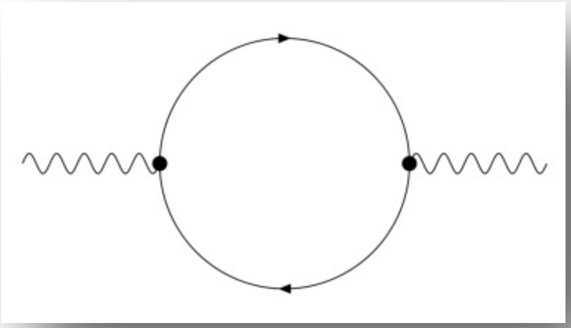
* Determine $\Pi(0)$ from Padé approximation in small-momentum region

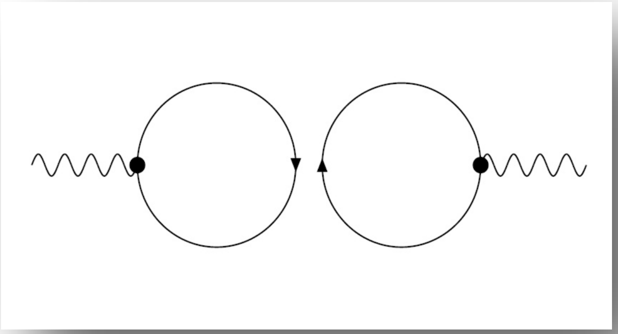
* Requires sub-percent accuracy in u,d-part for $Q^2 = \mathrm{O}(0.1\ \mathrm{GeV}^2)$

Lattice QCD approach to HVP

Main issues:

* Statistical accuracy at the sub-percent level required
* Reduce systematic uncertainty associated with region of small Q^2
  $\Leftrightarrow$ accurate determination of $\Pi(0)$
* Perform comprehensive study of finite-volume effects
* Include quark-disconnected diagrams



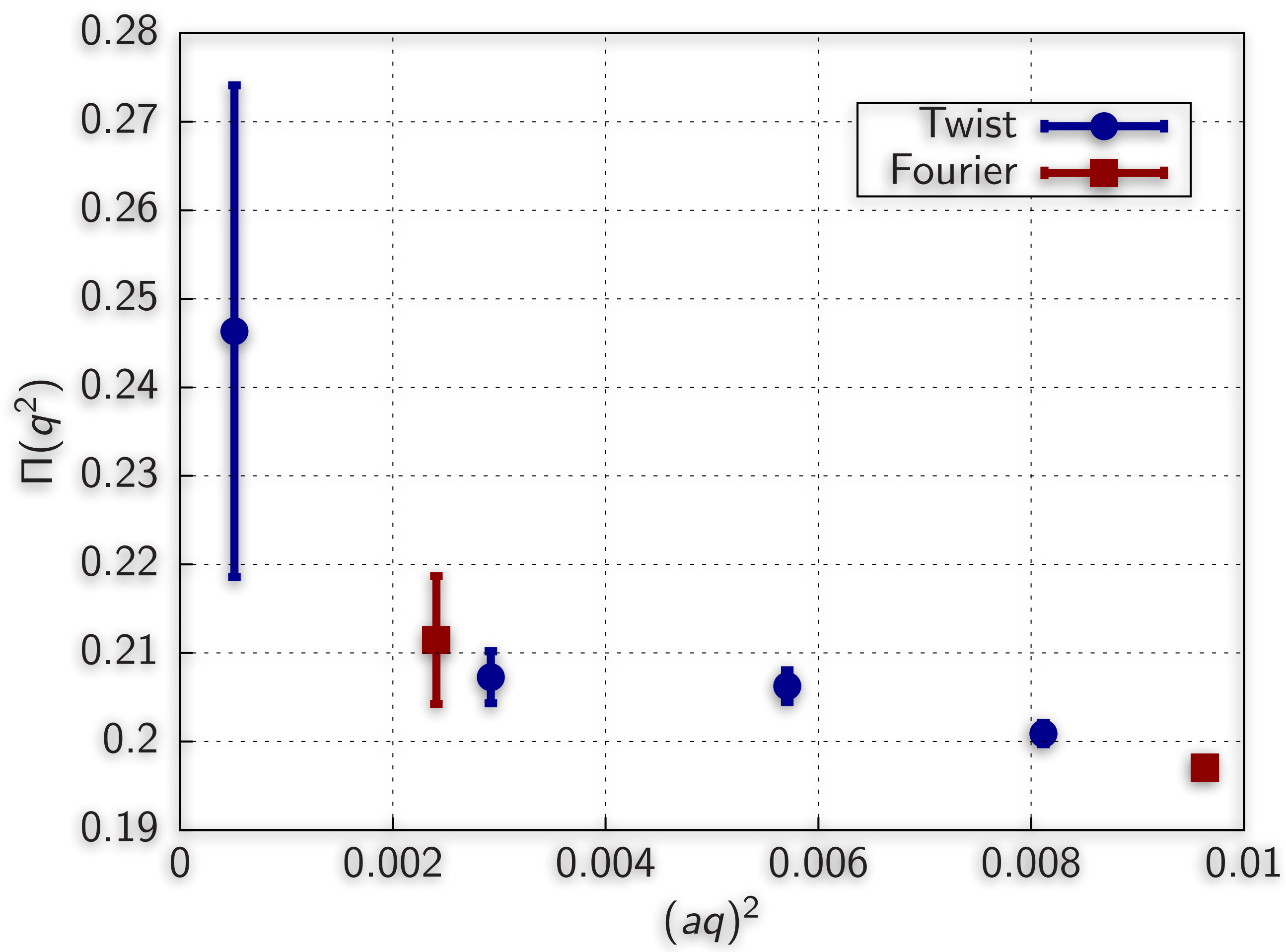


* Include isospin breaking: $m_u \neq m_d$, QED corrections

Low-momentum region of $\Pi(Q^2)$

* Apply twisted boundary conditions to access low-Q^2 regime:

$$\psi(x+Le_\mu) = \mathrm{e}^{i\theta_\mu}\psi(x) \quad \Rightarrow \quad Q_\mu = \frac{2\pi}{L} + \frac{\theta_\mu}{L}$$



[Della Morte, Jäger, Jüttner, H.W., JHEP 1203 (2012) 055; PoS LATTICE2012 (2012) 175]

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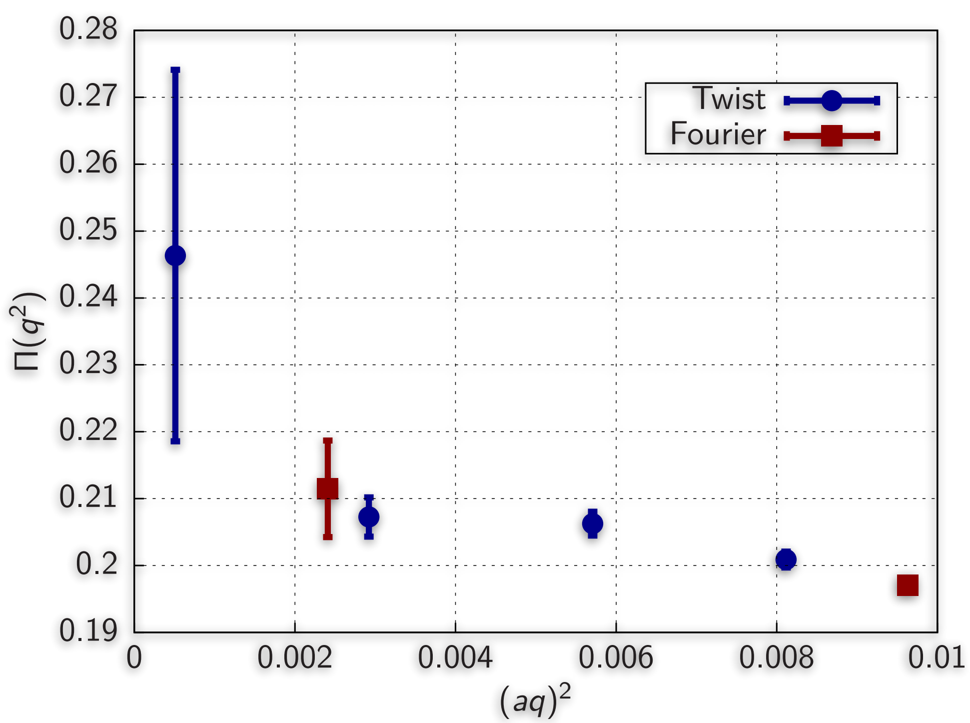

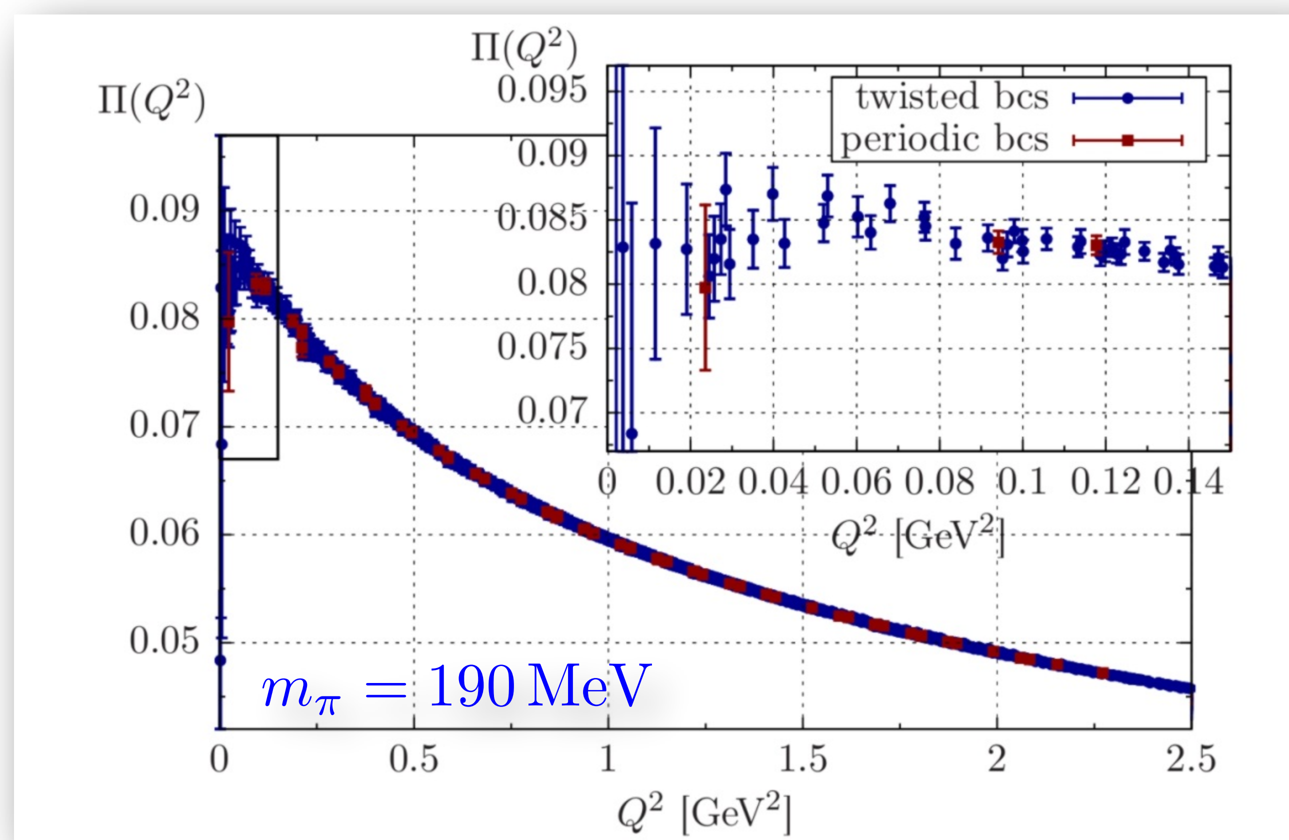

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Low-momentum region: Time moments

- Expansion of VPF at low-Q^2: $$\Pi(Q^2) = \Pi_0 + \sum_{j=1}^{\infty} Q^{2j}\Pi_j$$
- Vacuum polarisation for $Q = (\omega, \vec{0})$:

$$\Pi_{kk}(\omega) = a^4 \sum_{x_0} \mathrm{e}^{i\omega x_0} \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

- Spatially summed vector correlator: $$G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

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* Time moments: *[Chakraborty et al., Phys Rev D89 (2014) 114501]*

$$G_{2n} \equiv a \sum_{x_0} x_0^{2n} G(x_0) = (-1)^n \frac{\partial^{2n}}{\partial \omega^{2n}} \left\{ \omega^2 \hat{\Pi}(\omega^2) \right\}_{\omega^2=0}$$

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* Expansion coefficients: $$\Pi(0) \equiv \Pi_0 = \frac{1}{2} G_2, \quad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$$

Time-Momentum Representation

* Integral representation of subtracted VPF $\hat{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0)$

$$\Pi(Q^2) - \Pi(0) = \frac{1}{Q^2} \int_0^\infty dx_0 \, G(x_0) \left[Q^2 x_0^2 - 4 \sin^2\left(\tfrac{1}{2} Q x_0\right) \right]$$

$$G(x_0) = -\int d^3x \, \langle J_k(x) J_k(0) \rangle$$

[Bernecker & Meyer, Eur Phys J A47 (2011) 148]

[Francis et al. 2013; Feng et al. 2013; Lehner & Izubuchi 2014, Del Debbio & Portelli 2015,…]

* Q^2 is a tuneable parameter

* No extrapolation to $Q^2 = 0$ required; related to time-moments

* Must determine $I = 1$ vector correlator $G(x_0)$ for $x_0 \to \infty$

  $\longrightarrow$ Include two-pion states to capture long-distance behaviour

Time-Momentum Representation

$$\Pi(Q^2) - \Pi(0) = \frac{1}{Q^2}\int_0^\infty dx_0\, G(x_0)\left[Q^2 x_0^2 - 4\sin^2\left(\tfrac{1}{2}Qx_0\right)\right]$$

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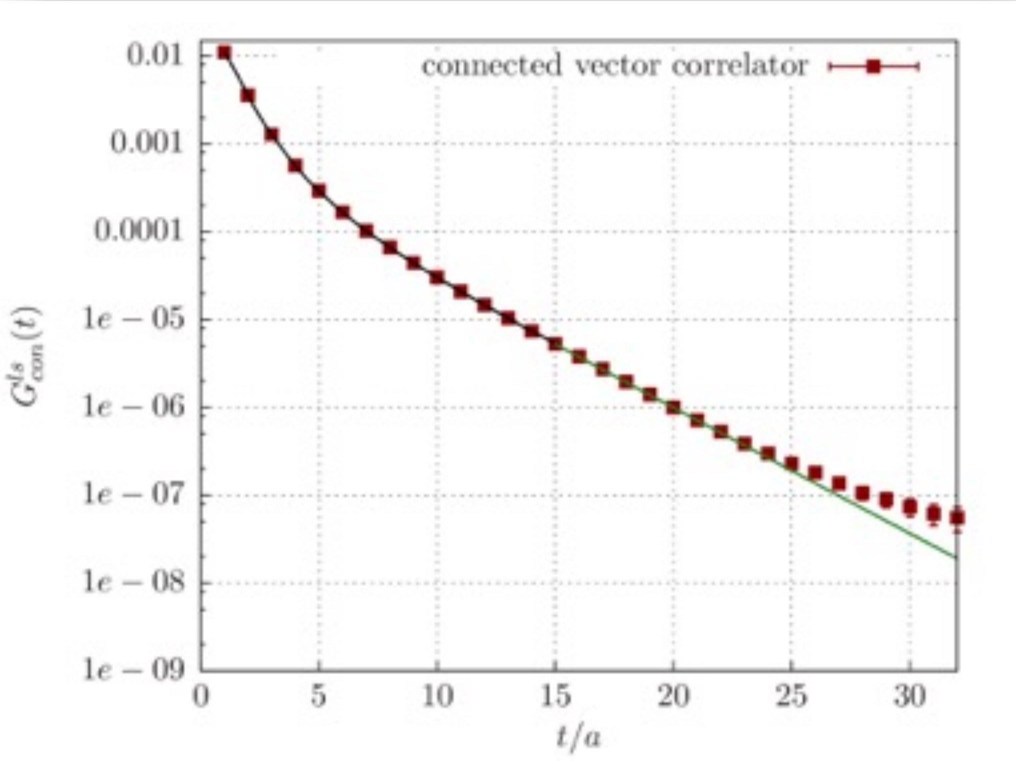
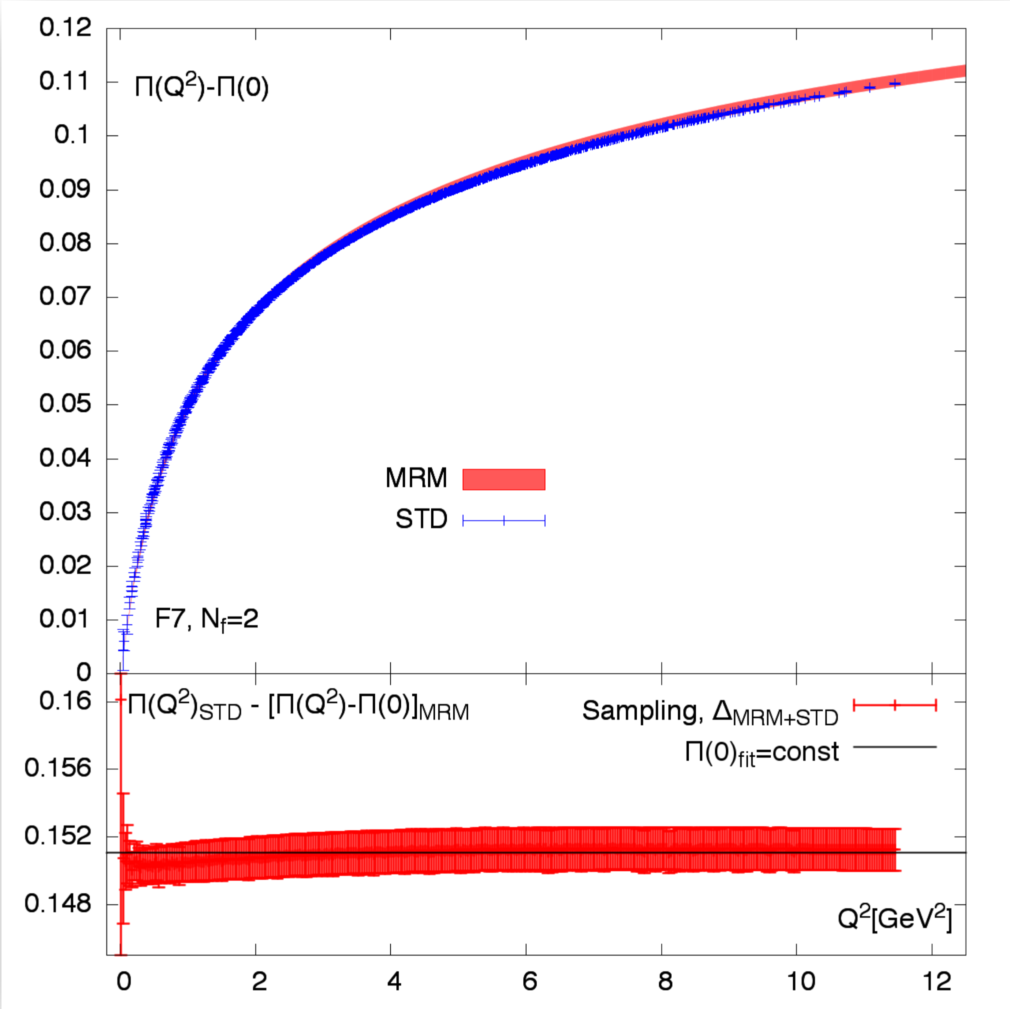




[Gülpers et al., arXiv:1411.7592; Francis et al., arXiv:1410.7491]

Current data sets and statistics

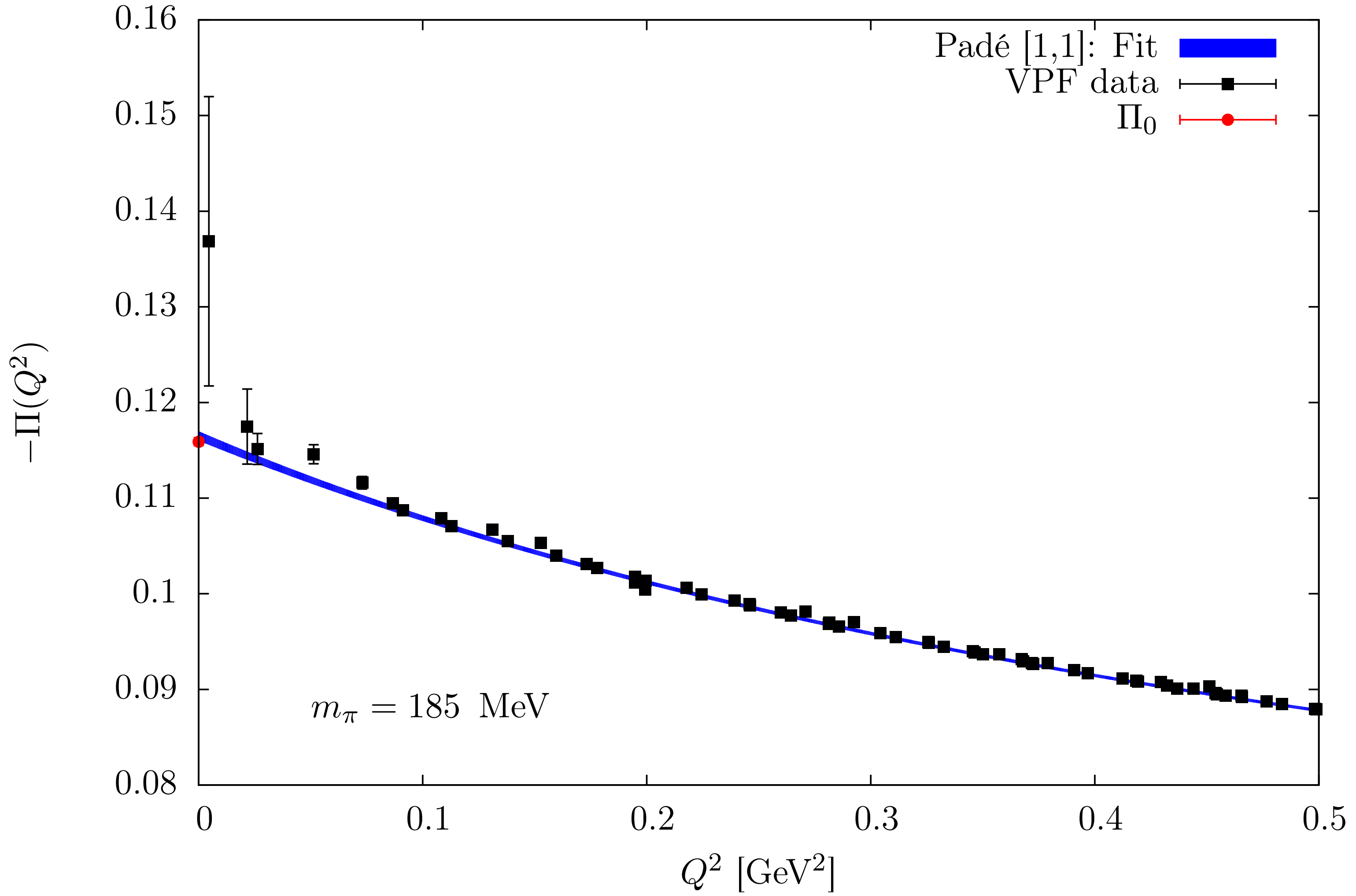
* $N_f = 2$ flavours of $O(a)$ improved Wilson fermions
* Three values of the lattice spacing: $a = 0.076, 0.066, 0.049$ fm
* Pion masses and volumes: $m_\pi^{\min} = 185\,\mathrm{MeV}, \quad m_\pi L > 4$
* 1000–4000 measurements per ensemble

To be processed:

* $N_f = 2+1$ flavours of $O(a)$ improved Wilson fermions; tree-level Symanzik gauge action; open boundary conditions
* Five values of the lattice spacing; physical pion mass

Comparison: Fits versus Time moments

* Construct Padé approximants either from fits or time moments



Fit Padé [1,1] for

$Q^2_{\rm low} \lesssim 0.5\,{\rm GeV}^2$

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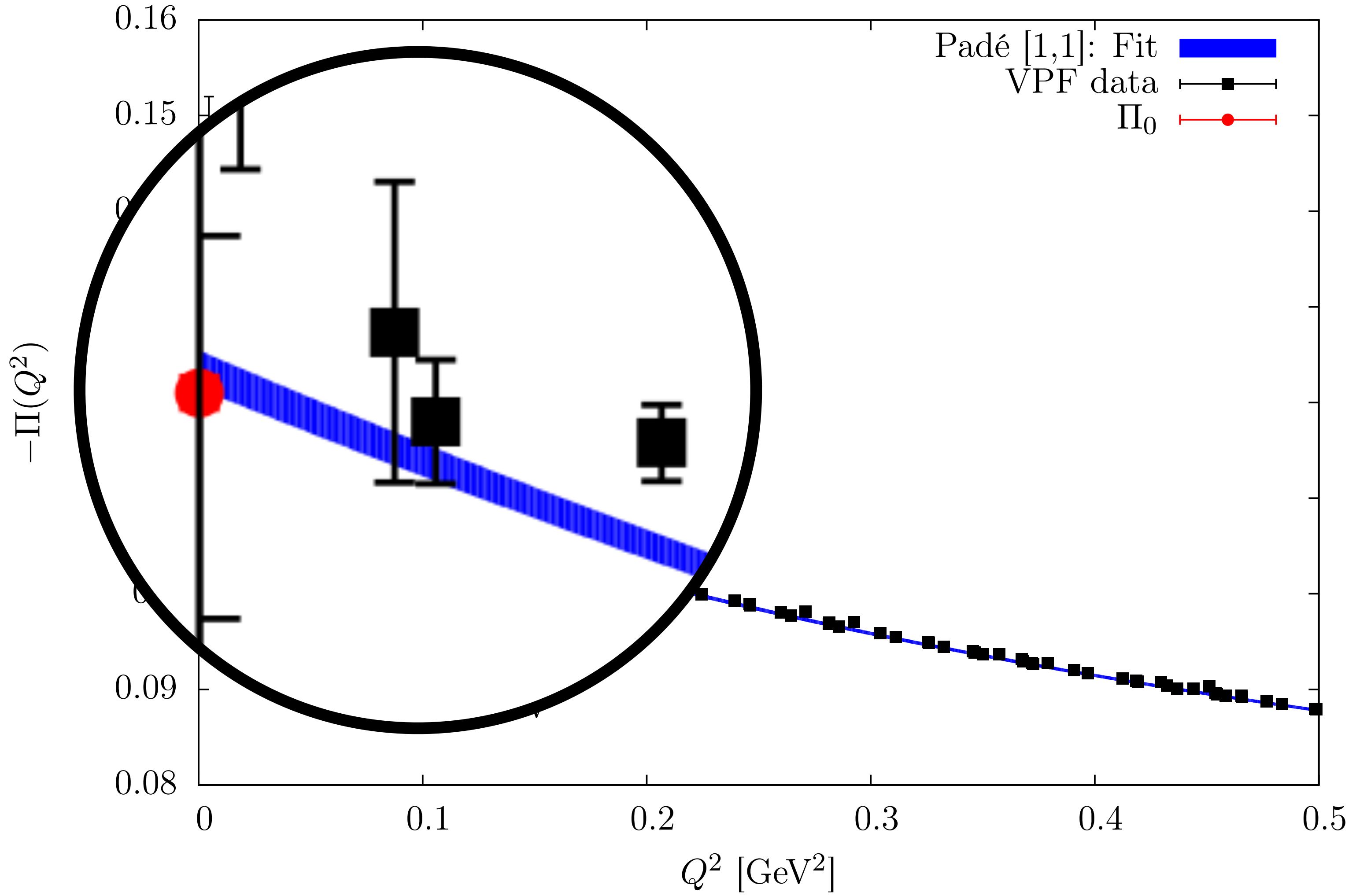

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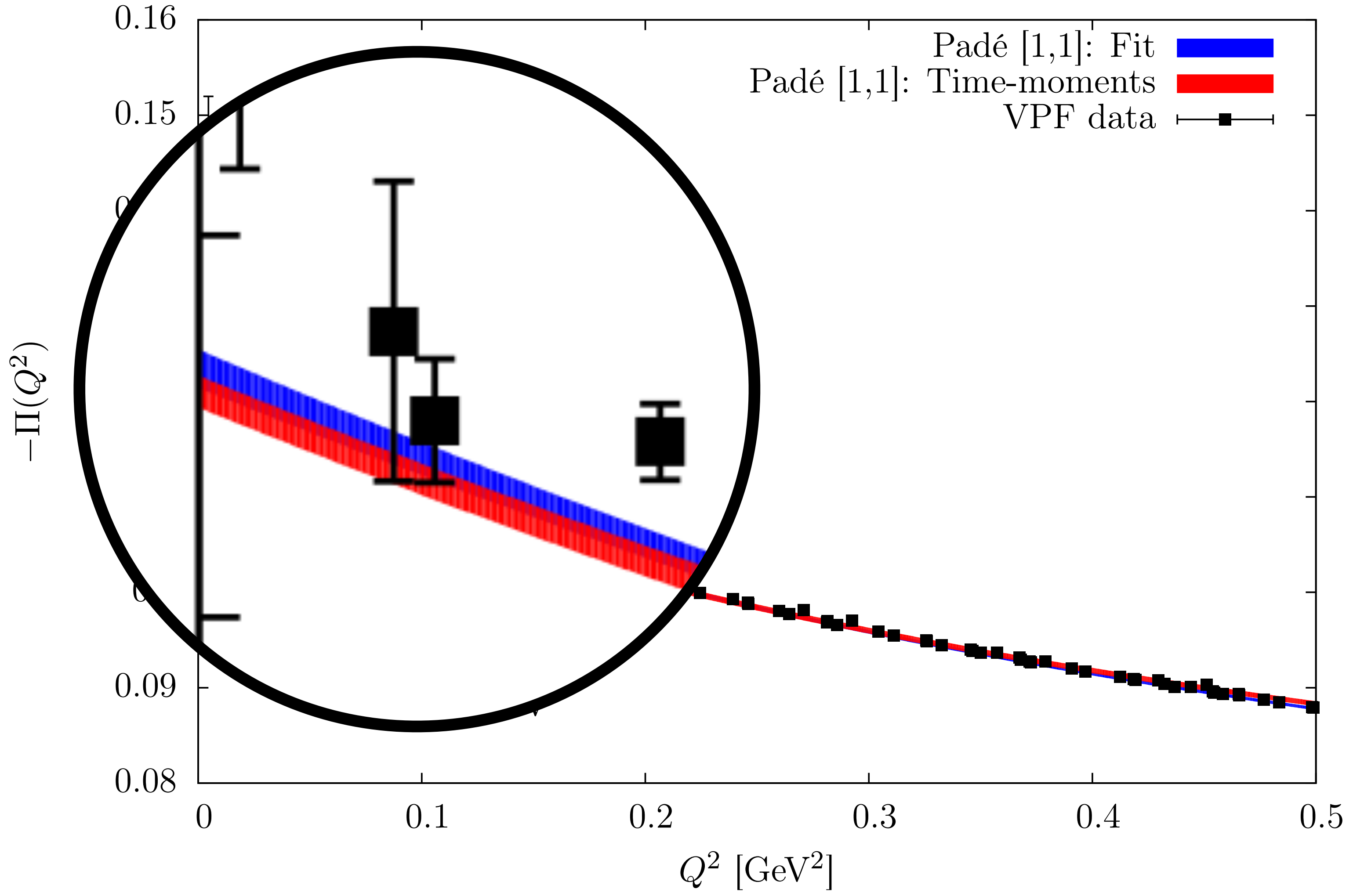

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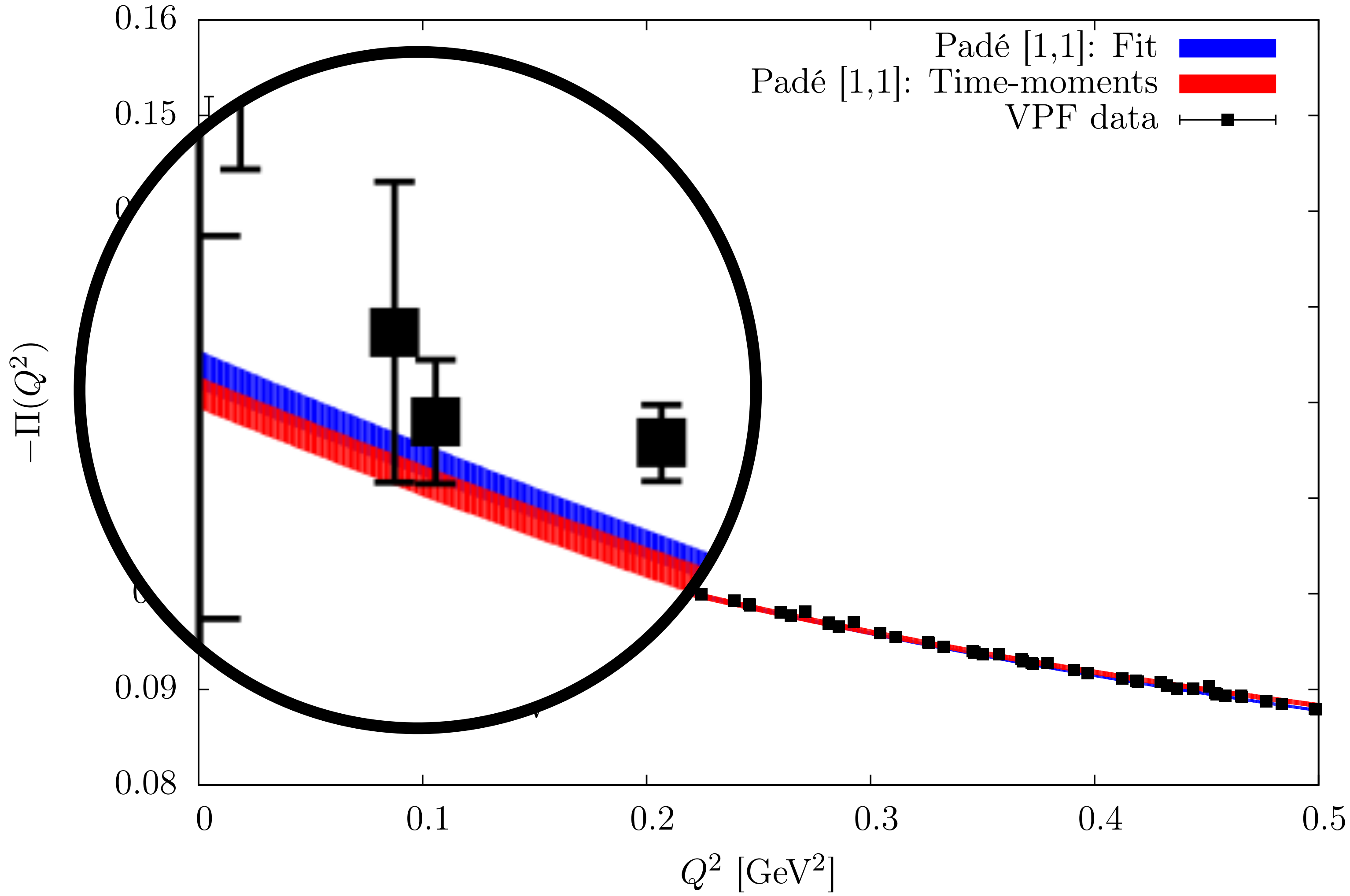

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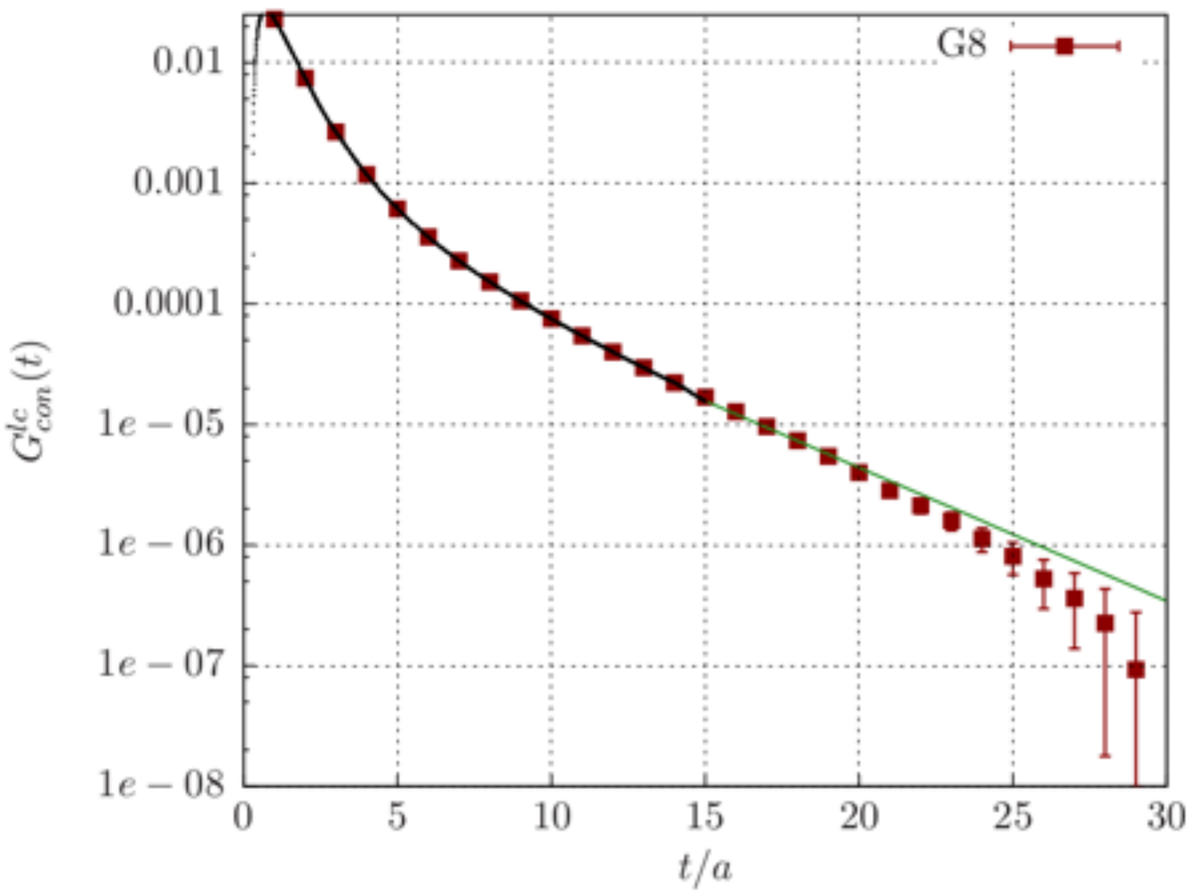
* Low-order Padé approximants consistent for $Q^2 < 0.5$ GeV^2

* Apply trapezoidal rule to evaluate convolution integral for $Q^2 \geq 0.5$ GeV^2

Comparison: TMR versus fits

- Interpolate $G(x_0)$ by cubic spline for $x_0 < x_{0,\mathrm{cut}}$



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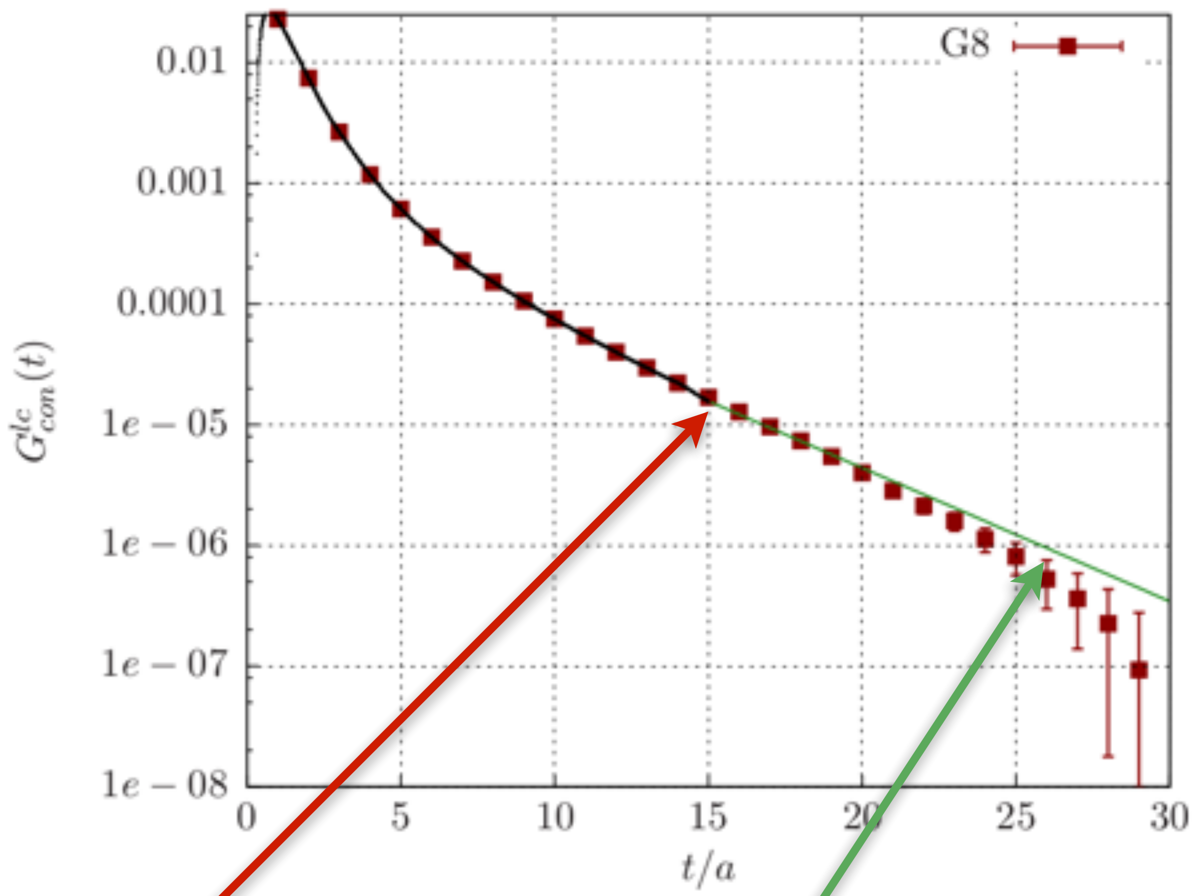

$x_{0,\mathrm{cut}}$

$G(x_0) = A_{\mathrm{eff}}\, \mathrm{e}^{-m_{\mathrm{V}} x_0}$

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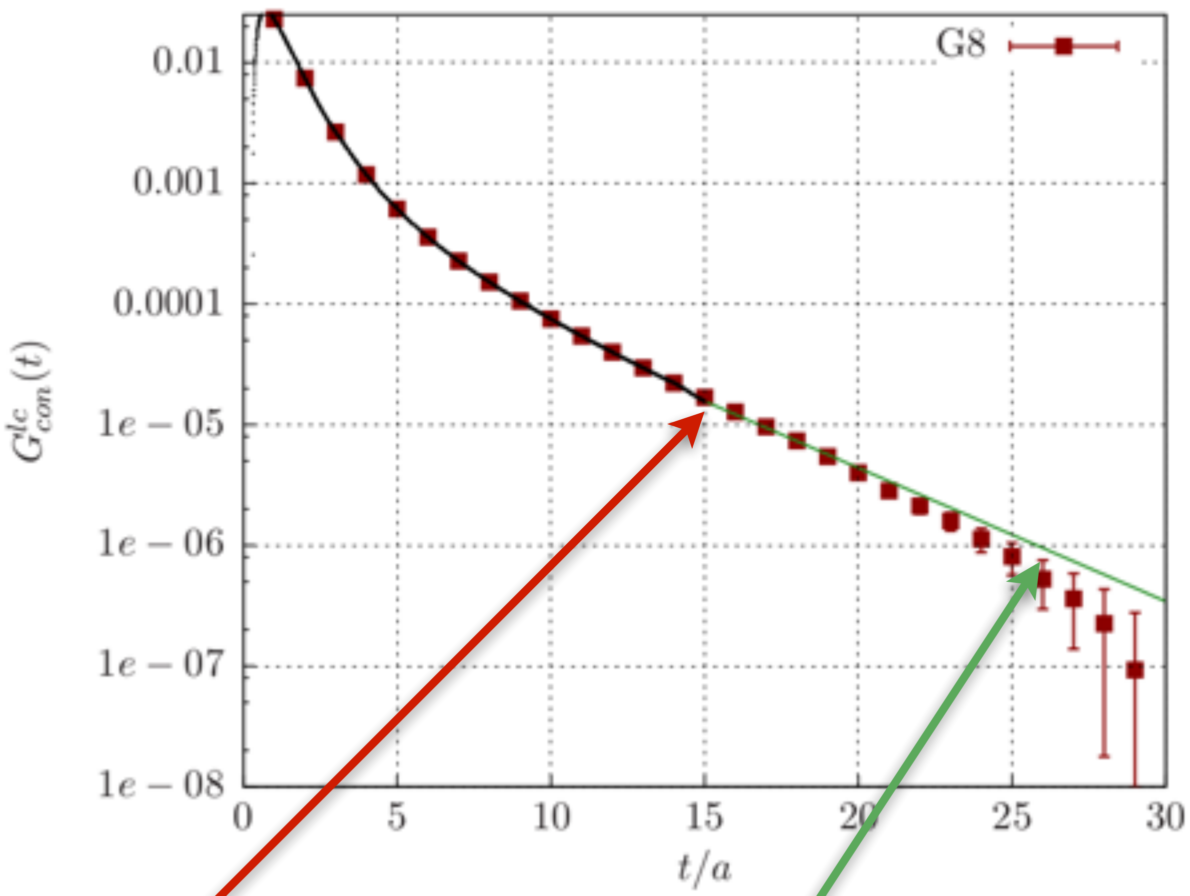

$x_{0,\mathrm{cut}}$ $\qquad$ $G(x_0) = A_{\mathrm{eff}}\, \mathrm{e}^{-m_{\mathrm{V}} x_0}$

* Mild dependence on $x_{0,\mathrm{cut}}$

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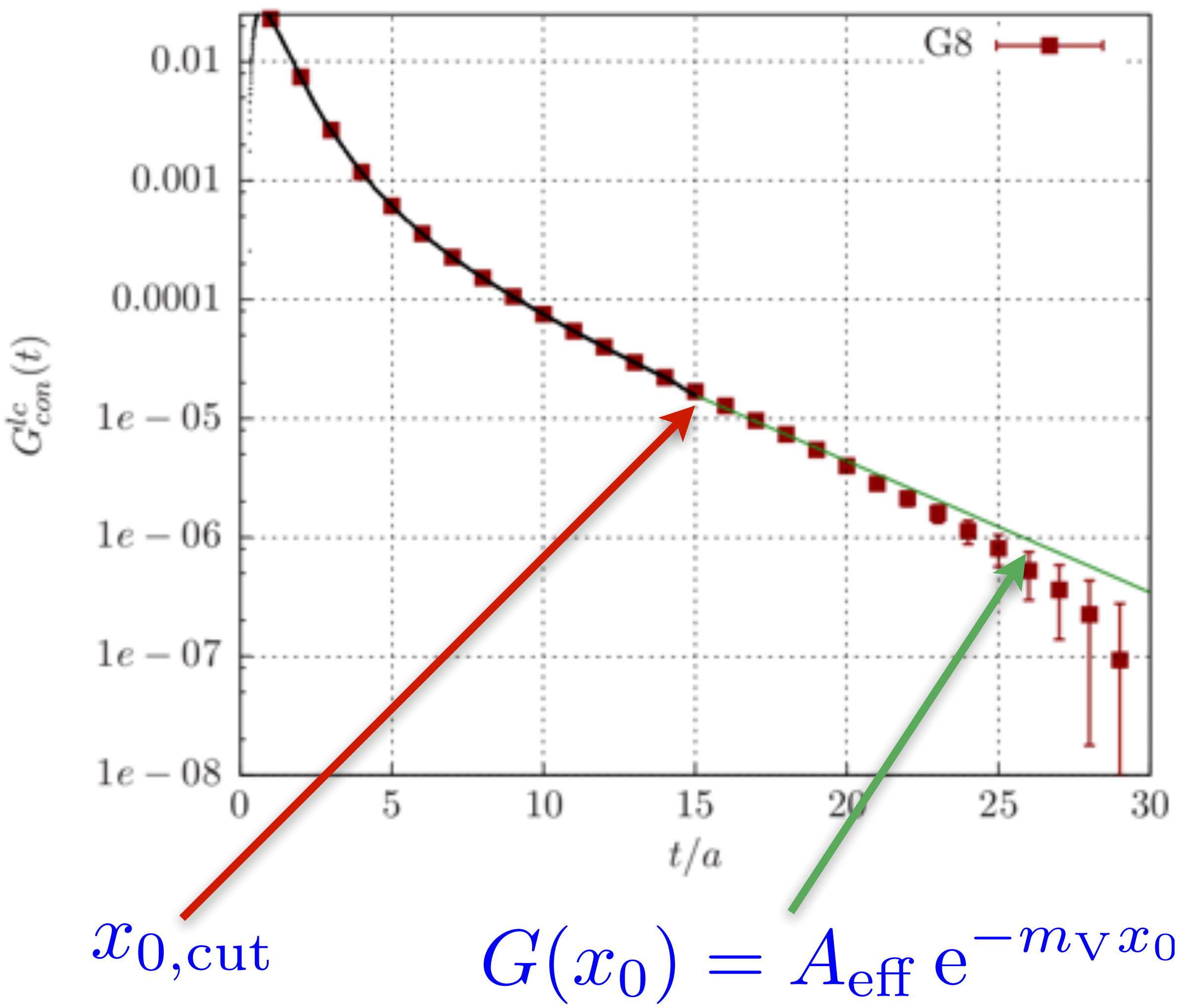

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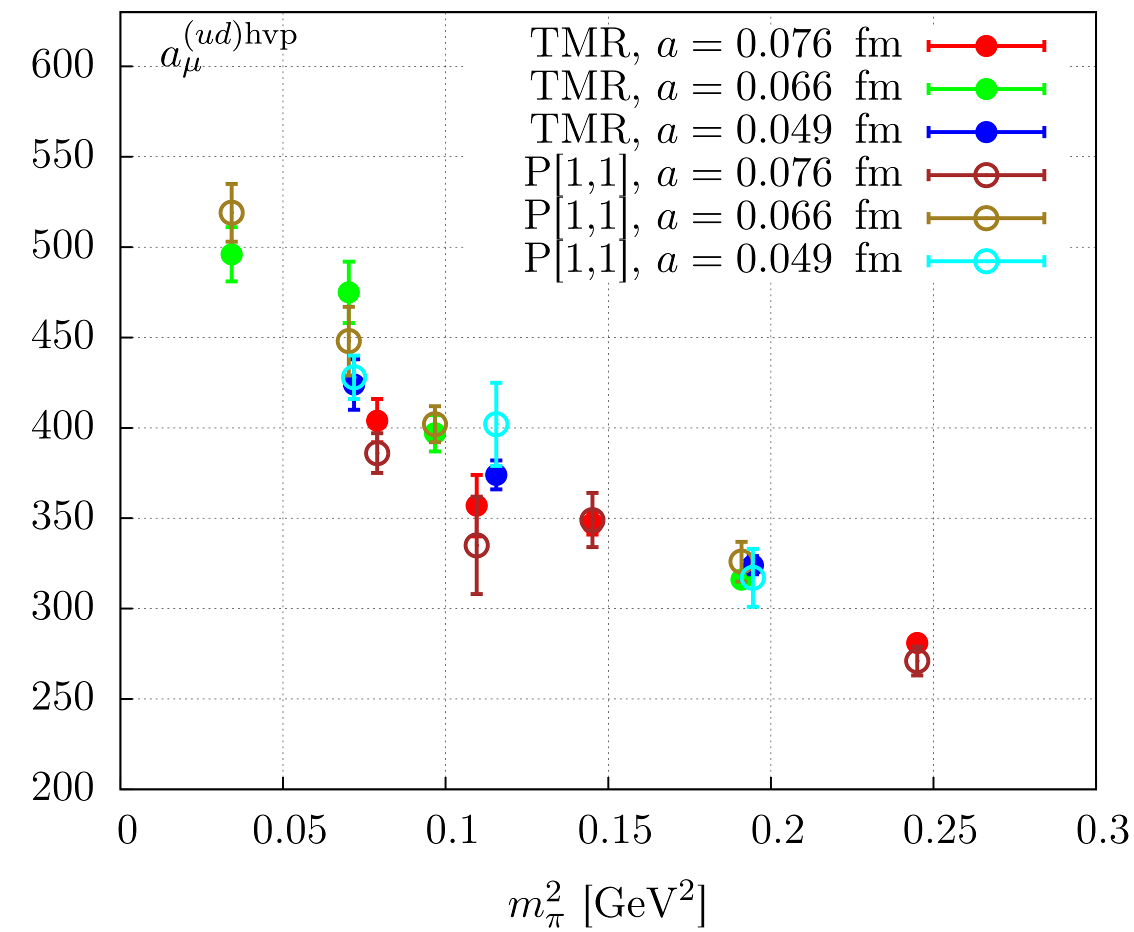

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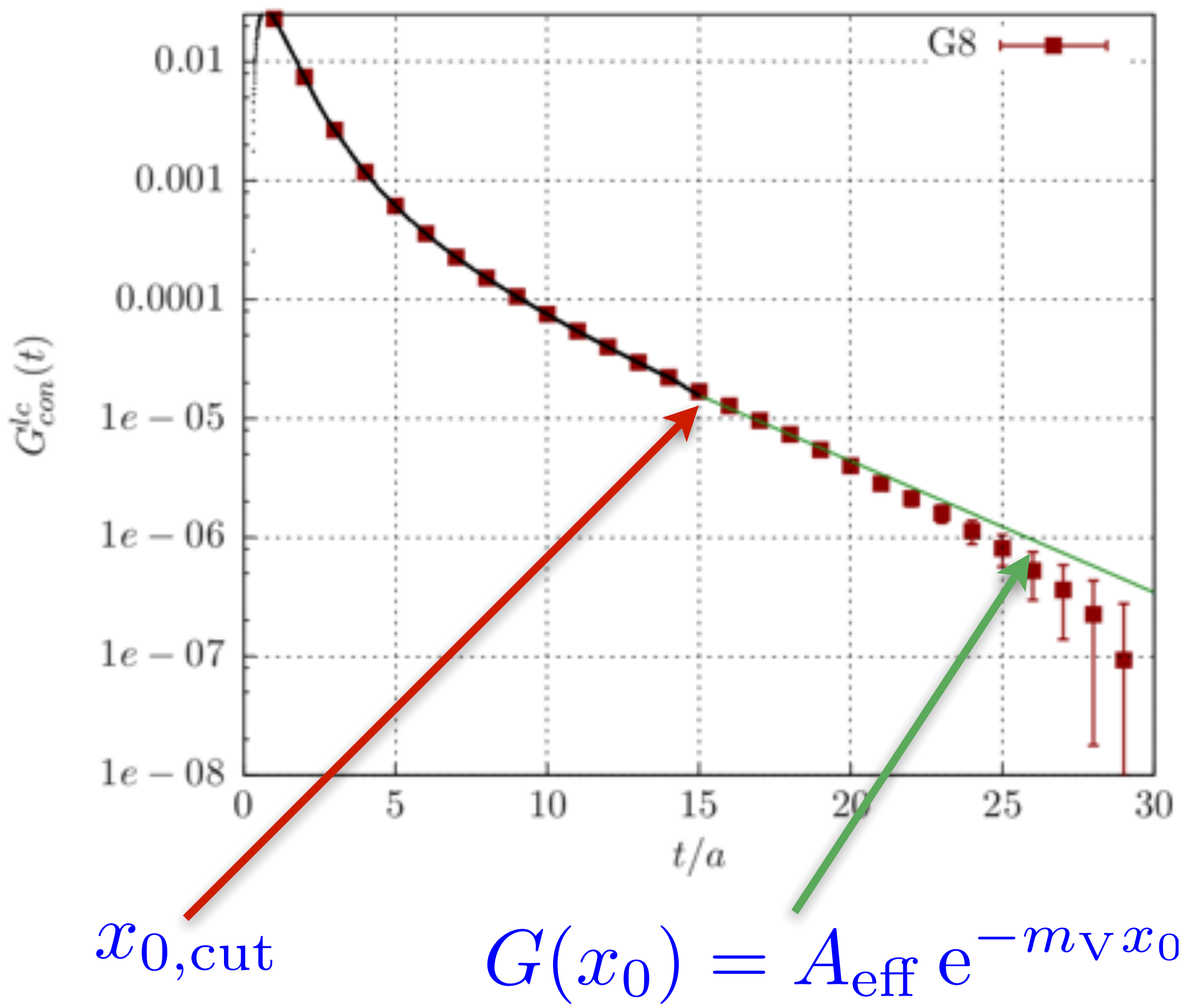

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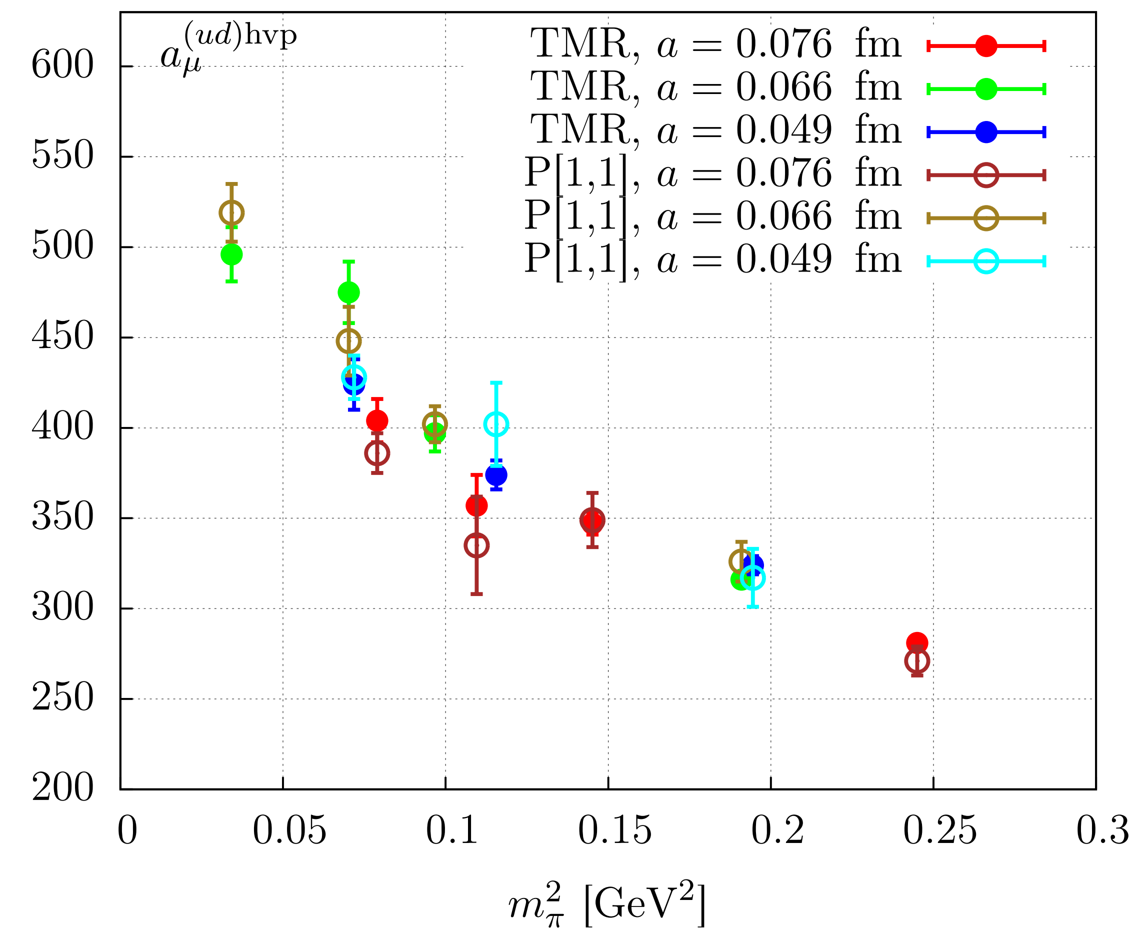
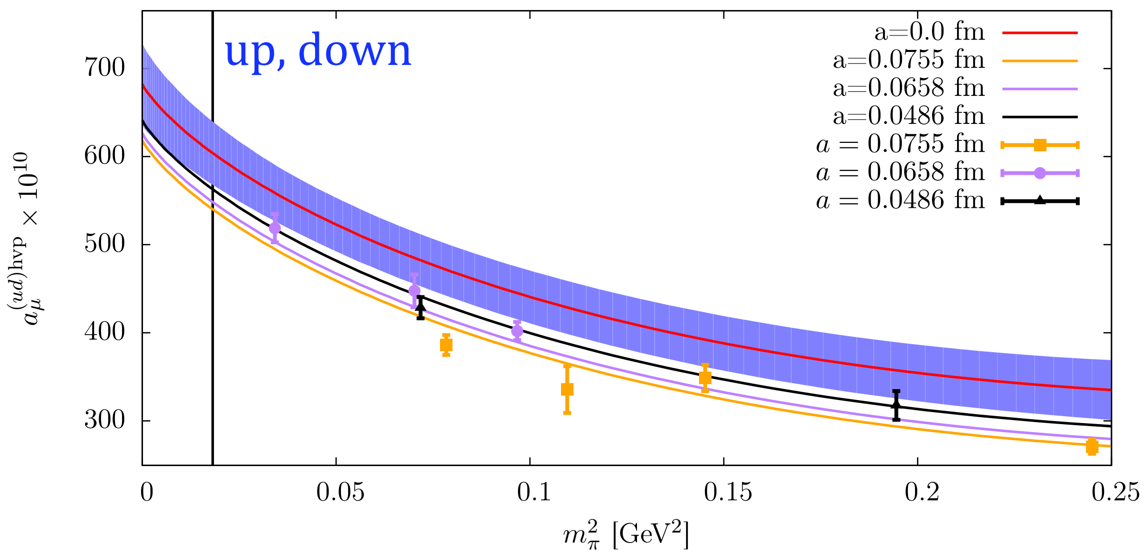


- Consistent results with similar accuracy

Chiral and continuum extrapolations

* Use collection of different functional forms, e.g.



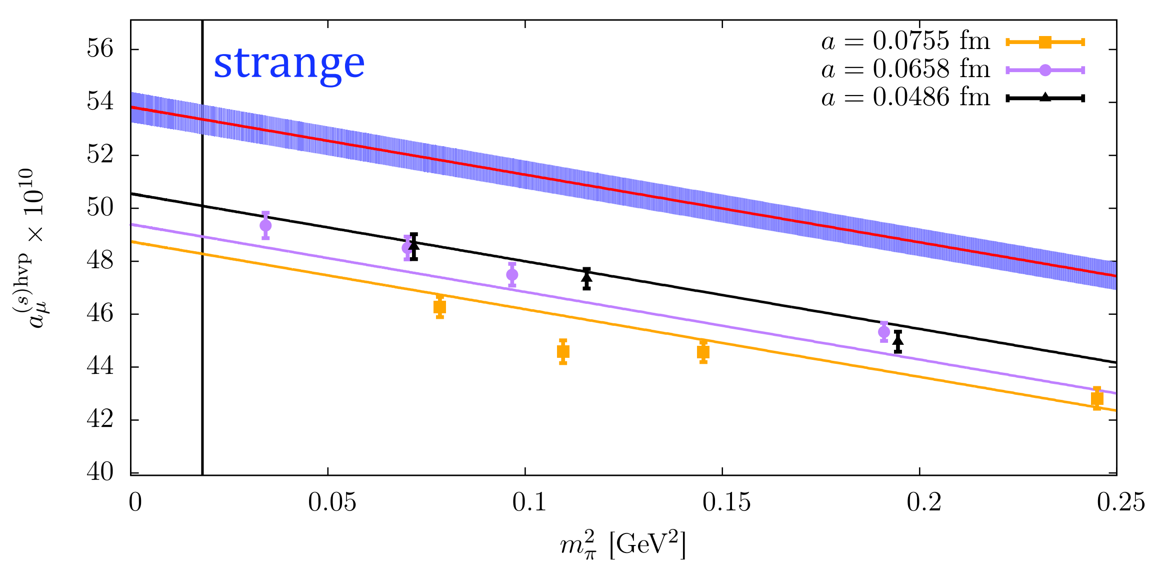
Fit A:

$$b_0 + b_1 m_\pi^2 + b_2 m_\pi^2 \ln(m_\pi^2) + b_3 a$$

Fit B:

$$b_0 + b_1 m_\pi^2 + b_2 m_\pi^4 + b_3 a$$

.....



* Perform cuts in pion mass and lattice spacing

* Lattice spacing effects clearly resolved for larger quark masses

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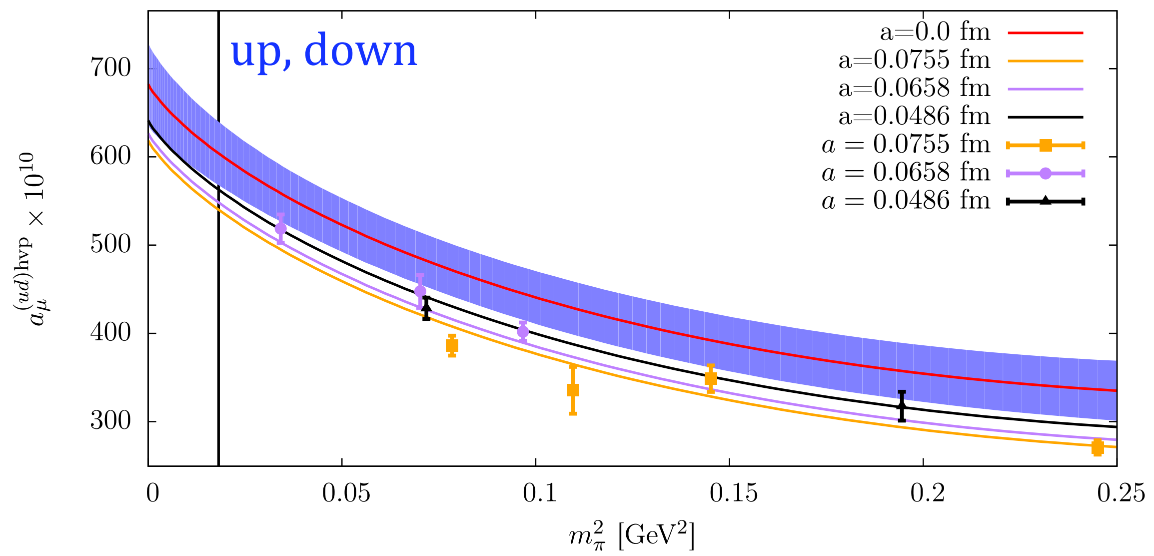

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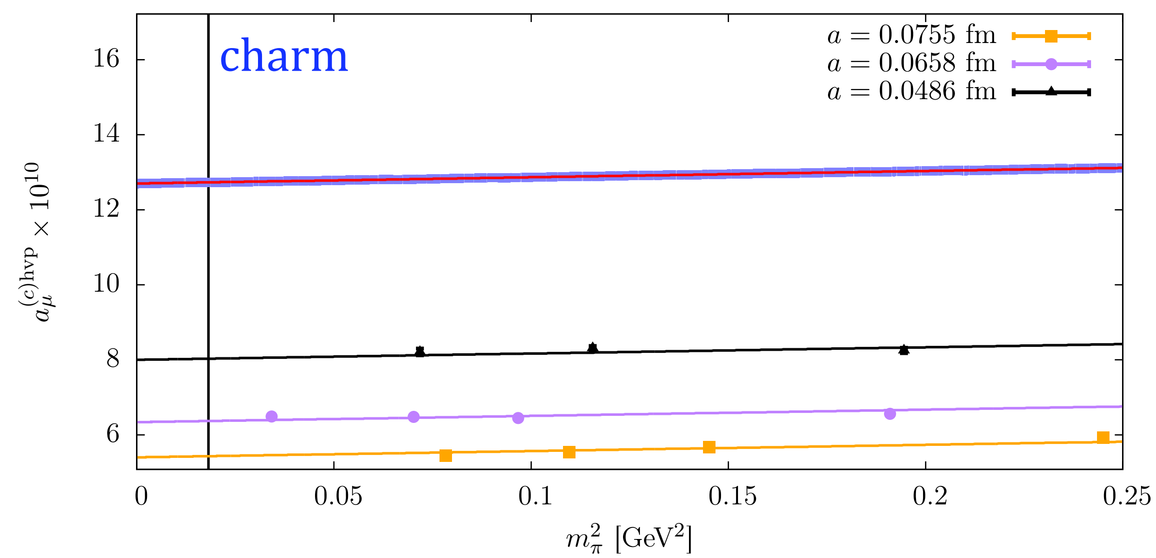

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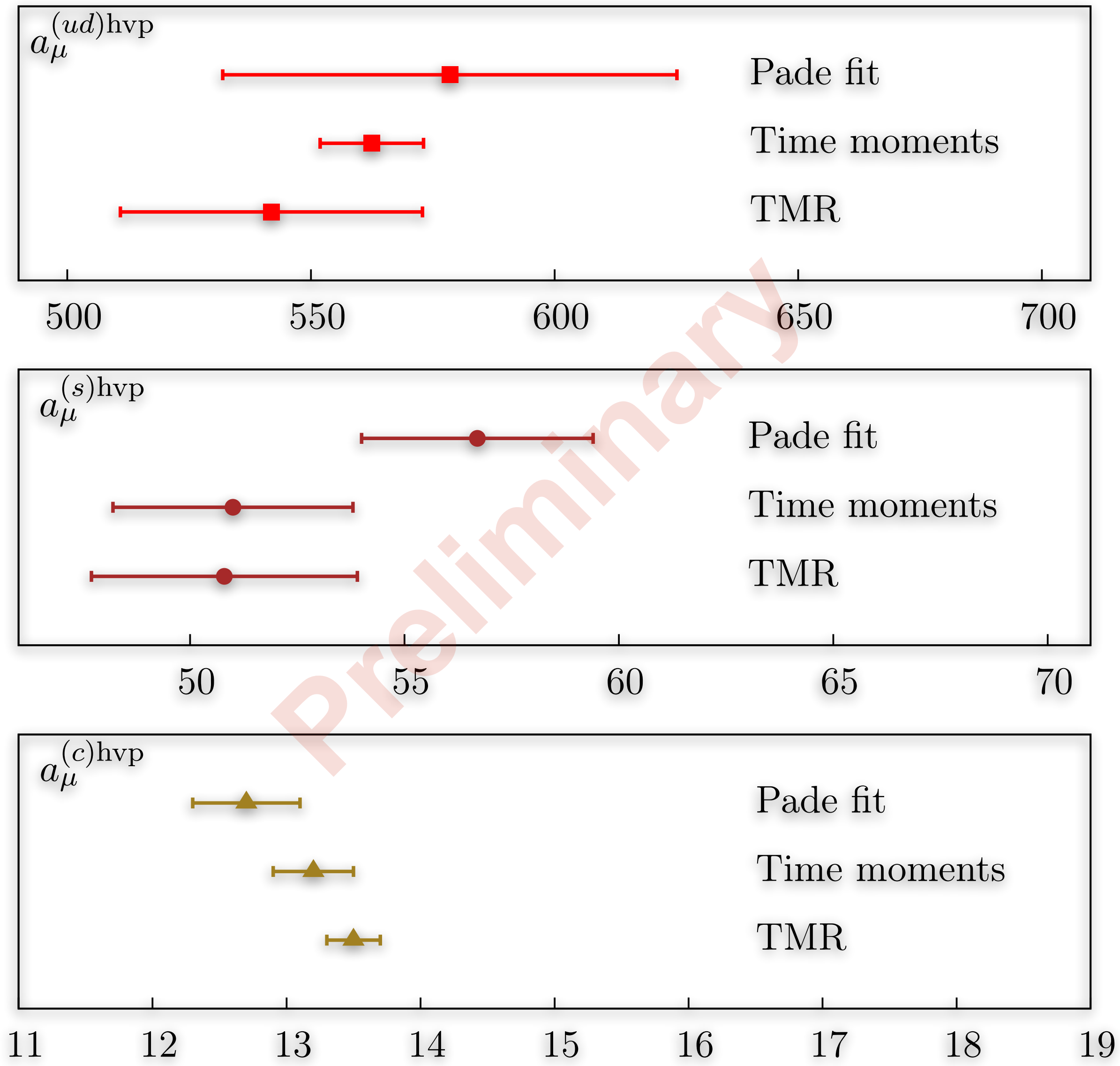
Summary on a_μ^{hvp}

- ✱ Final errors estimated via "extended frequentist method"



- ✱ Different methods consistent at the level of 1σ
- ✱ Time moments statistically most precise
- ✱ Strange quark contribution consistent with other recent results
- ✱ Overall accuracy dominated by u, d contribution

Disconnected Contributions

* Electromagnetic current correlator with *u, d, s* quarks:

$$G^{\ell s}(x_0) := -\int d^3x\,\langle J_k^{\ell s}(x) J_k^{\ell s}(0)\rangle\,, \quad J_\mu^{\ell s} = \tfrac{2}{3}\overline{u}\gamma_k u - \tfrac{1}{3}\overline{d}\gamma_k d - \tfrac{1}{3}\overline{s}\gamma_k s$$

* Identify connected and disconnected contributions:

$$G^{\ell s}(x_0) = \tfrac{5}{9} G^{\ell}_{\rm con}(x_0) + \tfrac{1}{9} G^{s}_{\rm con}(x_0) - \tfrac{1}{9} G^{\ell s}_{\rm disc}(x_0)$$

$$G^{\ell s}_{\rm disc}(x_0) = \int d^3x\,\left\{ {\rm Tr}\left[S^{\ell}(x,x)\gamma_k\right] - {\rm Tr}\left[S^{s}(x,x)\gamma_k\right]\right\} \times \{x \to 0\}$$

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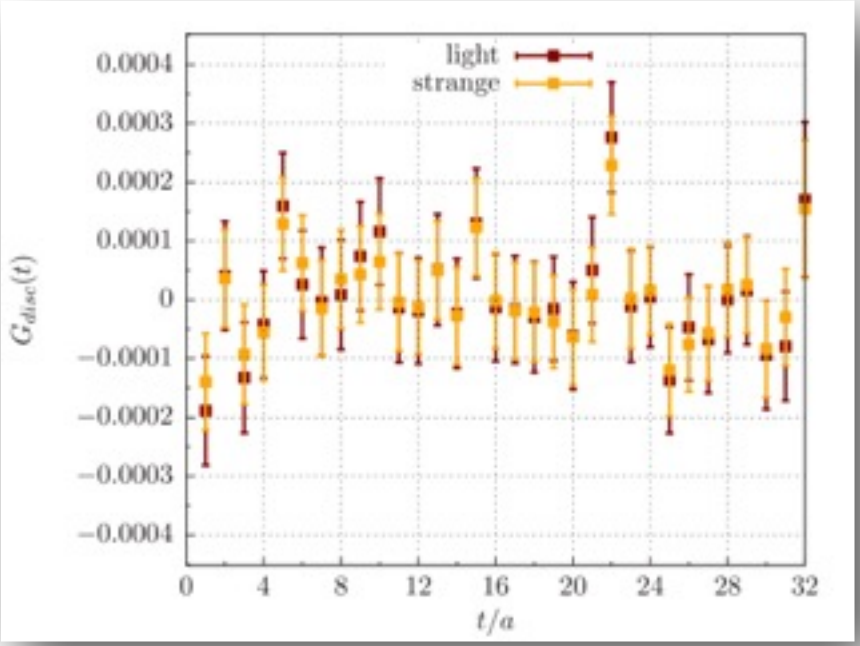

[Gülpers et al., arXiv:1411.7592; V. Gülpers, PhD Thesis 2015]

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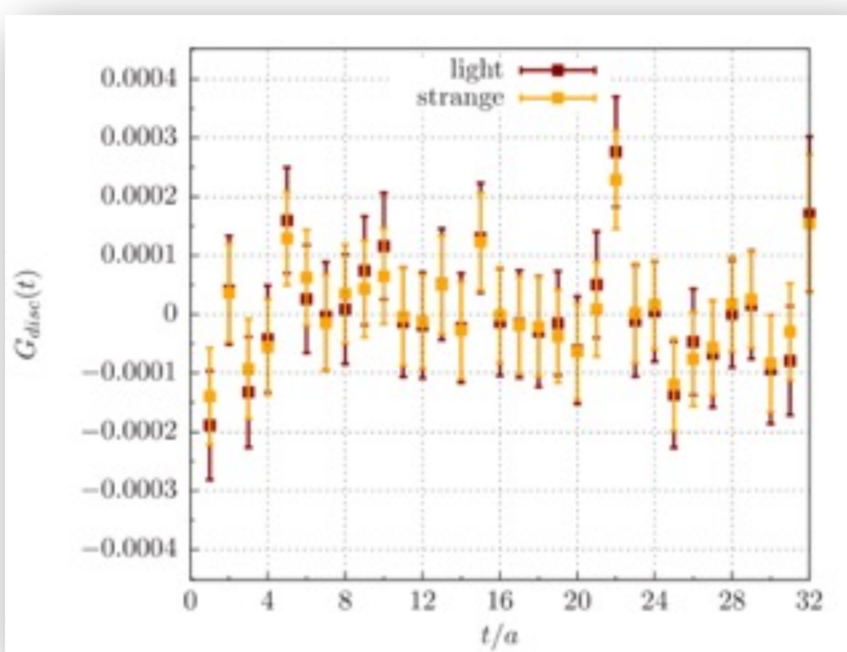
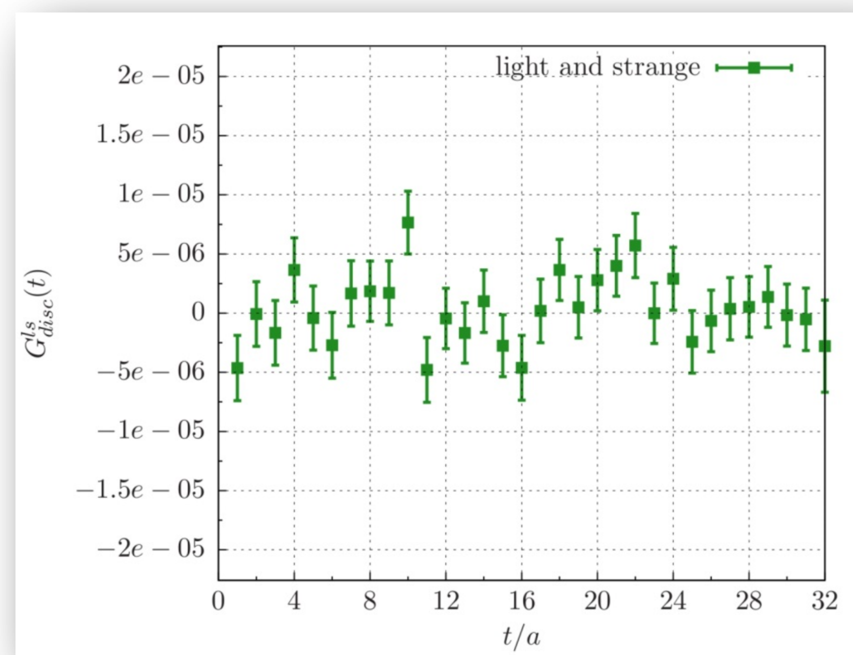
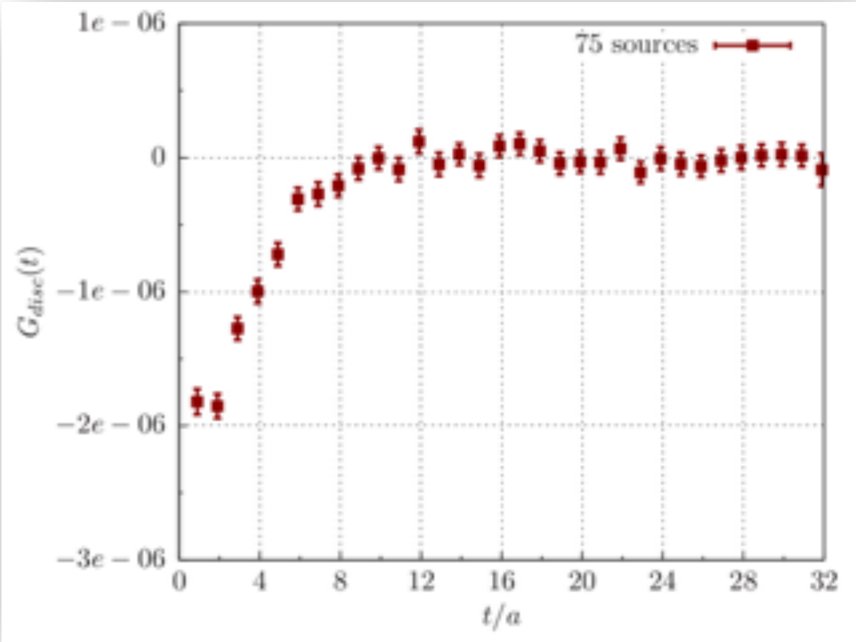


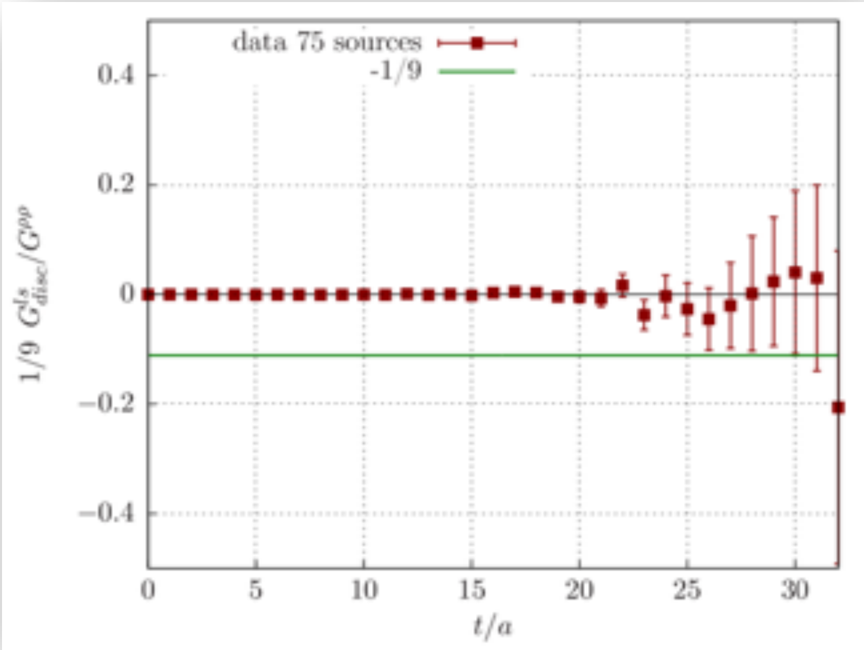


[Gülpers et al., arXiv:1411.7592; V. Gülpers, PhD Thesis 2015]

Disconnected Contributions

* Non-zero disconnected contribution can be resolved:



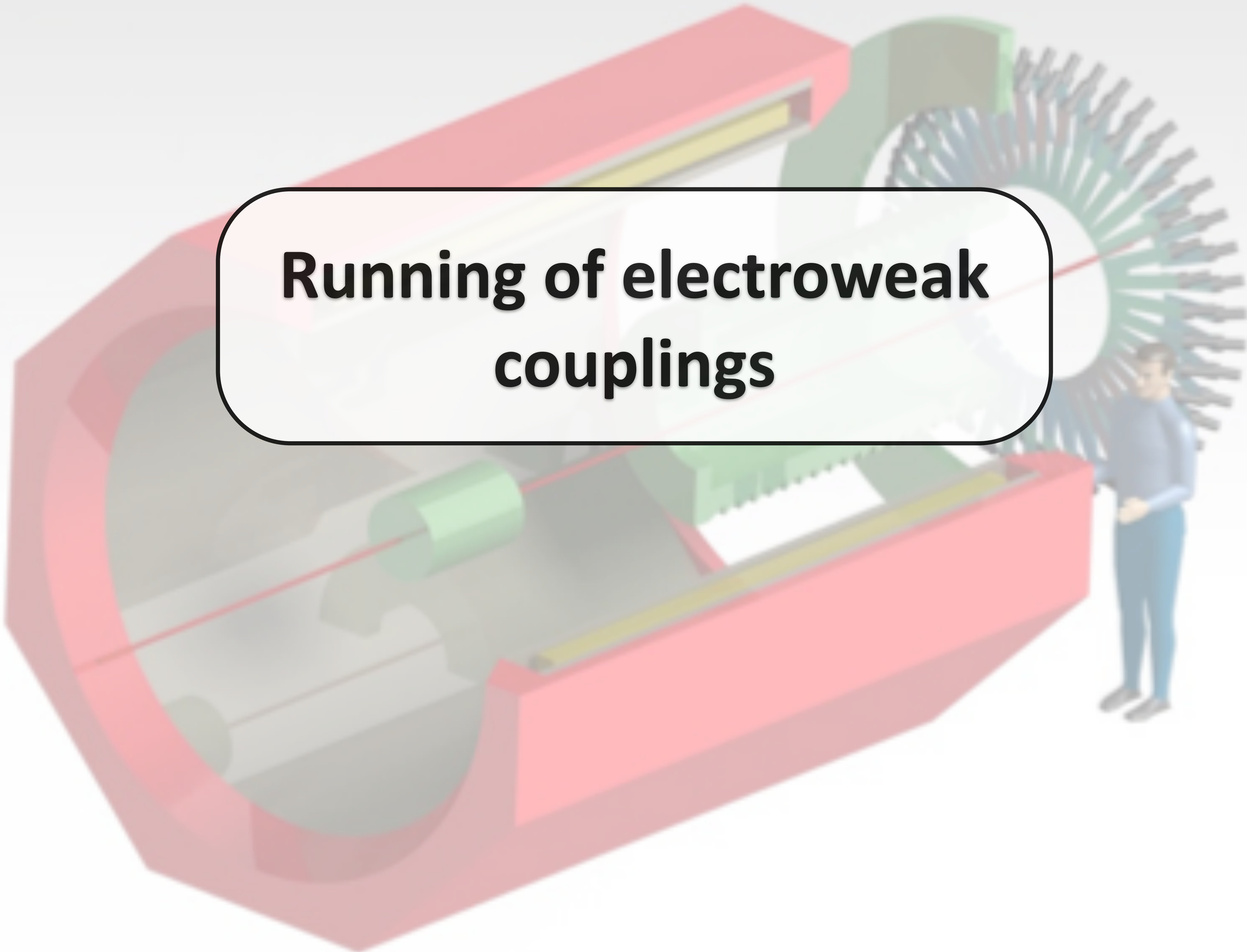


* Disconnected contribution for $x_0 \longrightarrow \infty$:

$$-\frac{1}{9}\frac{G^{\ell s}_{\rm disc}}{G^{\rho\rho}} = \frac{G^{\ell s} - G^{\rho\rho}}{G^{\rho\rho}} - \frac{1}{9}\left(1 - \frac{2G^{s}_{\rm con}}{G^{\ell}_{\rm con}}\right) \xrightarrow{x_0\to\infty} -\frac{1}{9}$$

* Failure to resolve non-zero signal provides systematic error estimate

[Gülpers et al., arXiv:1411.7592; V. Gülpers, PhD Thesis 2015]



Running of electroweak couplings

Running of α — phenomenological approach

* Fine structure constant: $$\alpha(Q^2)=\frac{\alpha}{1-\Delta\alpha(Q^2)}$$

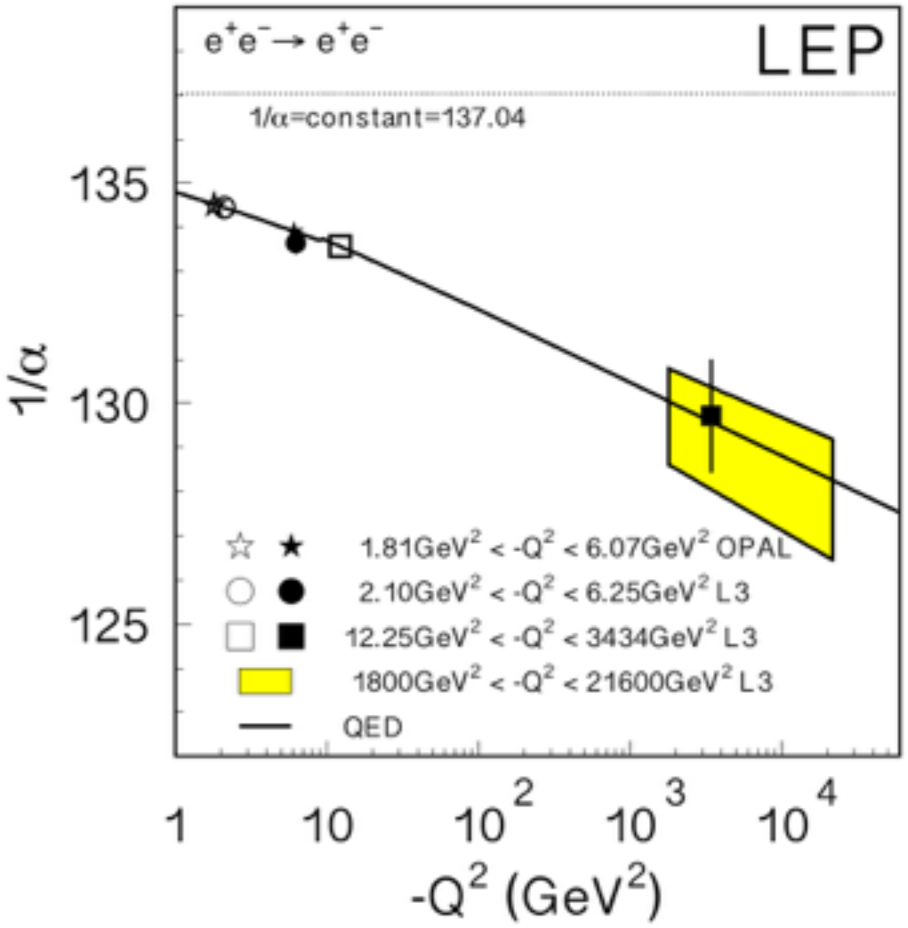
* Hadronic contributions — phenomenological approach:

$$\Delta\alpha_{\rm had}(Q^2)=-\frac{\alpha Q^2}{3\pi}\int_{4m_\pi^2}^{\infty}ds\,\frac{R_{\rm had}(s)}{s(s-Q^2)}$$

c.f.

$$a_\mu^{\rm HVP}=\left(\frac{\alpha m_\mu}{3\pi}\right)^2\int_{4m_\pi^2}^{\infty}ds\,\frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)=(276.26\pm1.38)\cdot10^{-4}$$

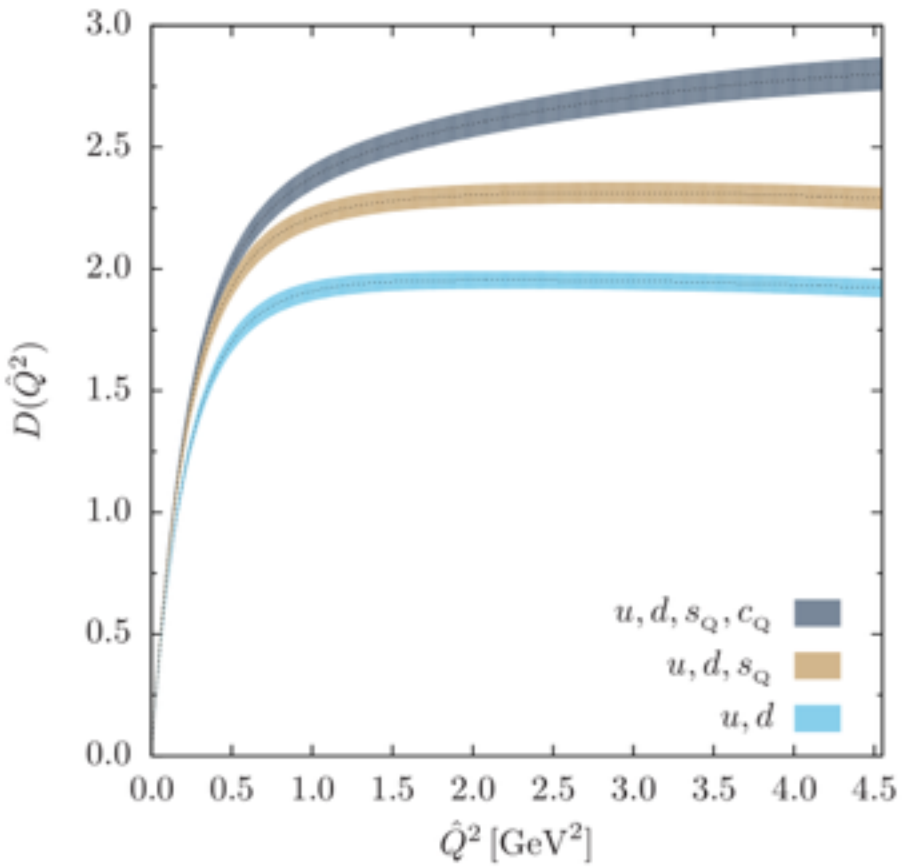


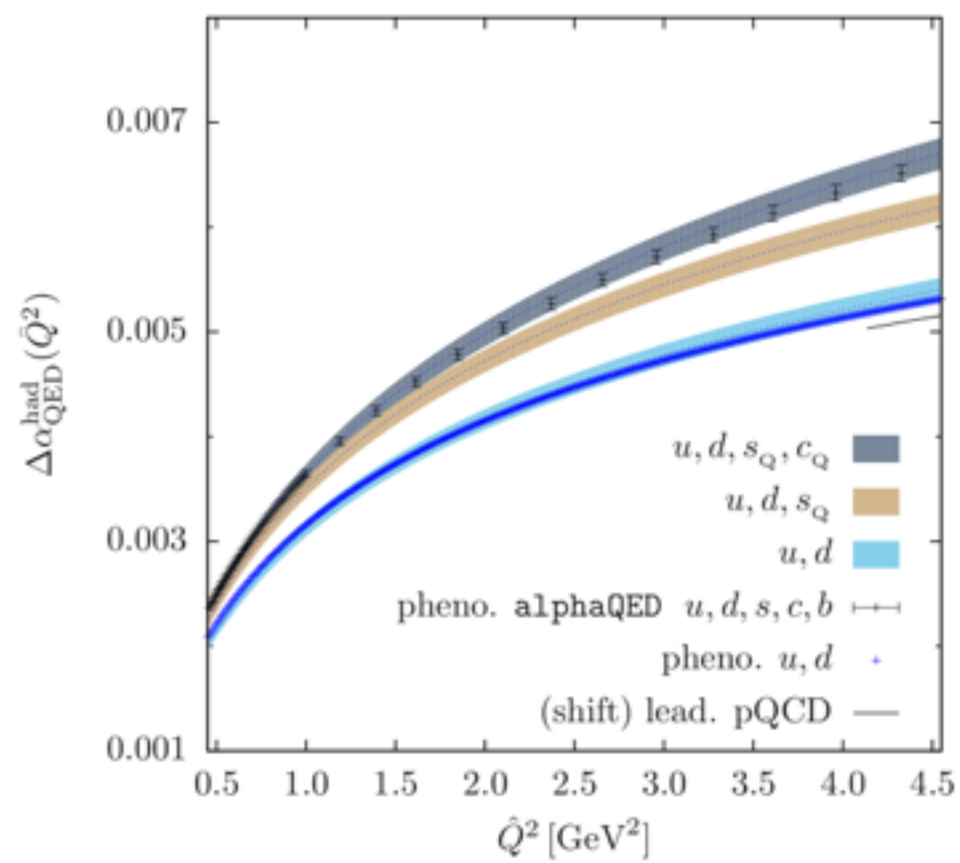
* Error on $\Delta\alpha_{\rm had}$ limits accuracy of Standard Model tests

Running of α — Euclidean approach

- Vacuum polarisation function: $\Delta\alpha_{\rm had}(Q^2) = 4\pi\alpha\Big(\Pi(Q^2) - \Pi(0)\Big)$

- Adler function: $D(Q^2) = -12\pi^2 \dfrac{d\,\Pi(Q^2)}{d\ln Q^2} = \dfrac{3\pi}{\alpha}\dfrac{d}{d\ln Q^2}\Delta\alpha_{\rm had}(Q^2)$



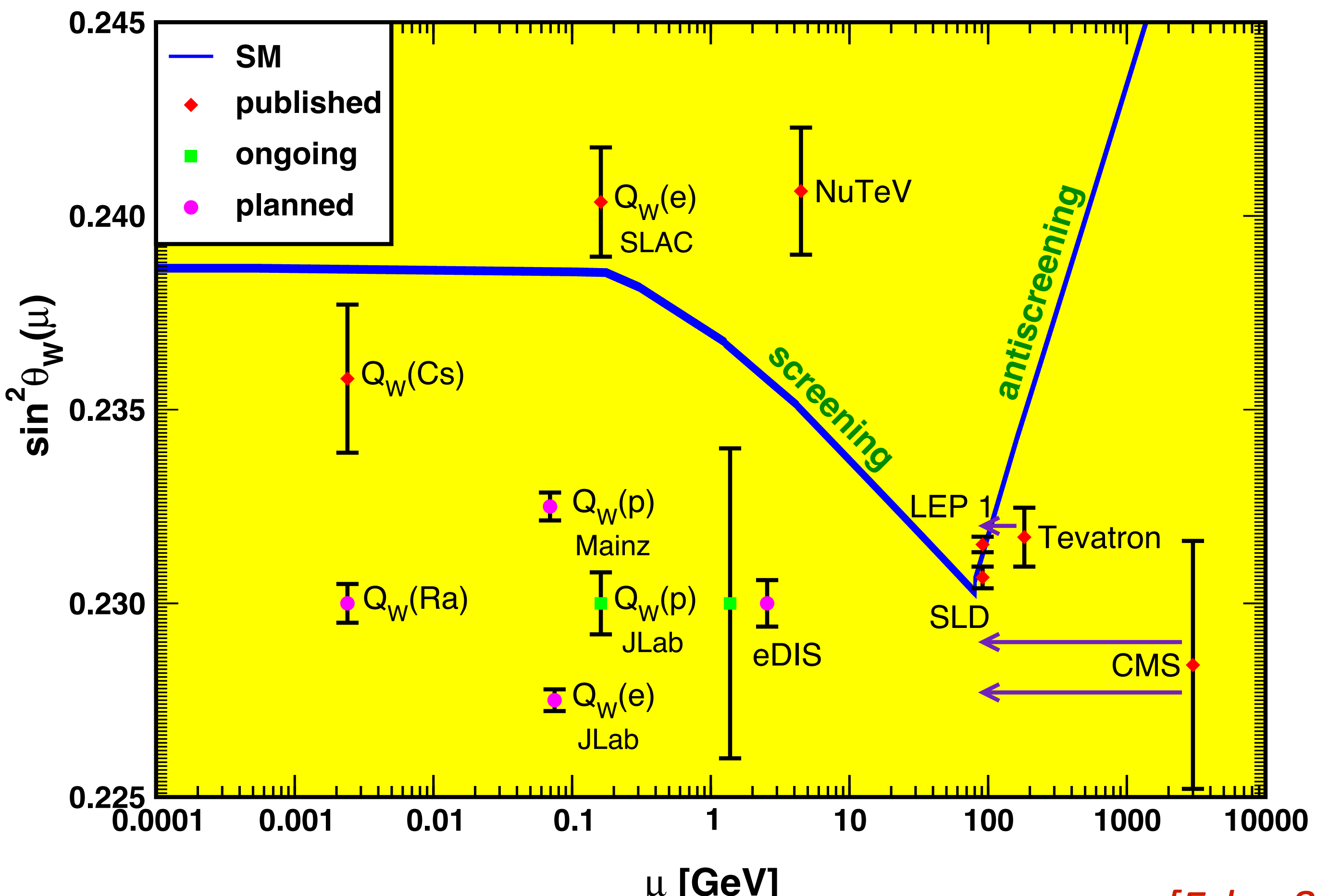


[H. Horch, G. Herdoíza @ Lattice 2015]

- Lattice QCD: similar accuracy as phenomenological approach

Running of $\sin^2\theta_W$

* Definition: $$\sin^2\theta_W(Q^2) = \underbrace{\sin^2\theta(0)}_{0.23864}\left(1 - \Delta\sin^2\theta_W(Q^2)\right)$$

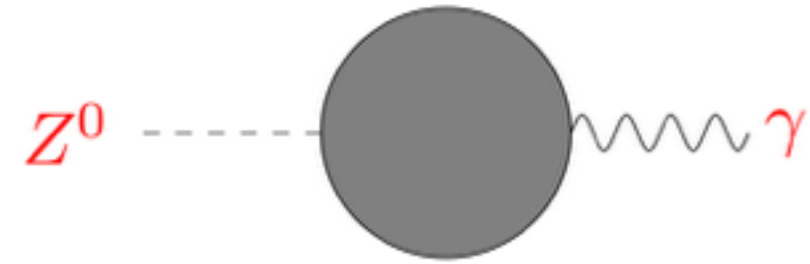


[Erler & Su, PPNP 71 (2013) 119]

* Dispersive approach requires separation of contributions from up/down-type quarks

Running of $\sin^2\theta_W$

- Euclidean approach:



$$\Pi^{\gamma Z}_{\mu\nu}(Q) = \int d^x\, \mathrm{e}^{iQ\cdot x}\, \langle V^Z_\mu(x) J^\gamma_\nu(0)\rangle$$

$$V^Z_\mu = V^3_\mu - \sin^2\theta_\mathrm{W}\, J^\gamma_\mu$$

$$V^3_\mu = \tfrac{1}{4}\left(\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d - \overline{s}\gamma_\mu s + \overline{c}\gamma_\mu c + \ldots\right)$$

$$\Pi^{\gamma Z}(Q^2) = \Pi^{\gamma 3}(Q^2) - \sin^2\theta_\mathrm{W}\Pi^{\gamma\gamma}(Q^2)$$

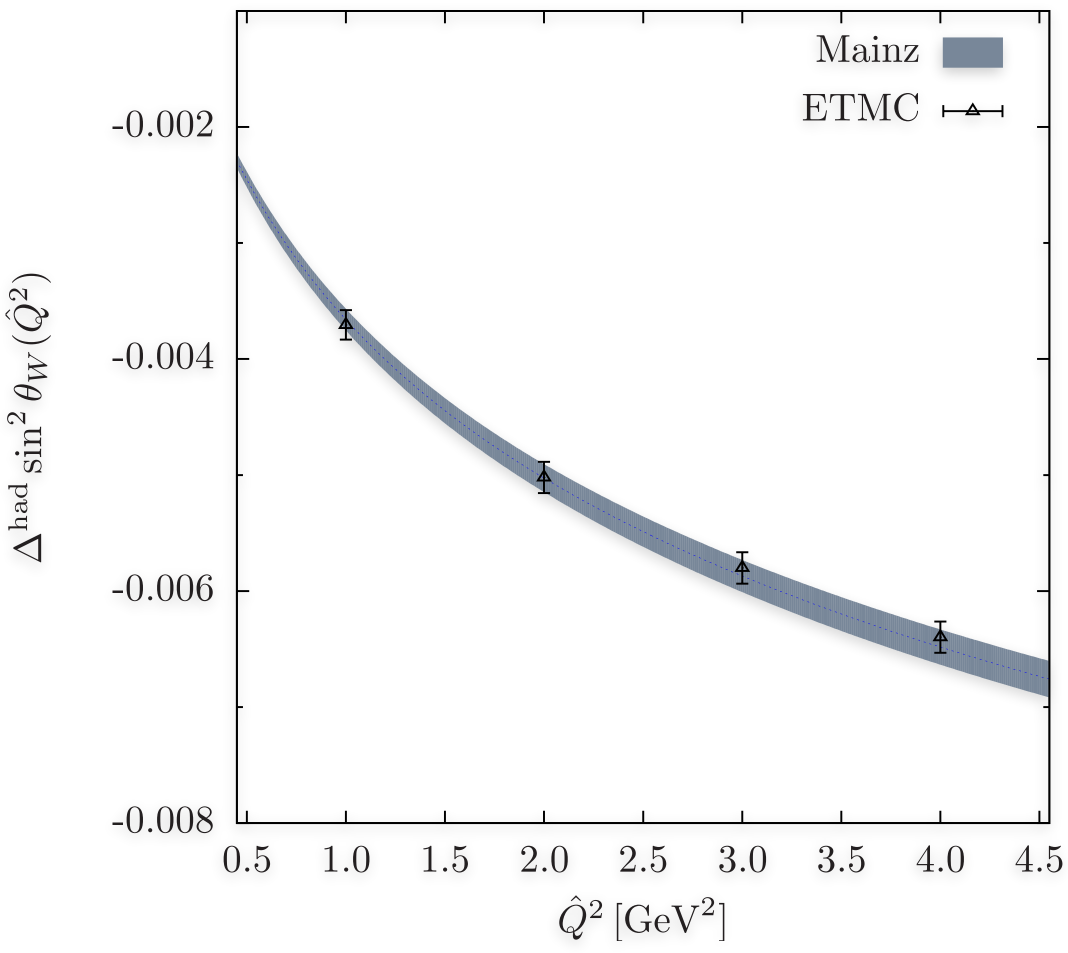
$$\Delta_\mathrm{had}\sin^2\theta_\mathrm{W}(Q^2) = \frac{e^2}{\sin^2\theta_0}\Big(\Pi^{\gamma Z}(Q^2) - \Pi^{\gamma Z}(0)\Big)$$

- Spin-off of calculation of running of $\Delta\alpha_\mathrm{had}$

Running of $\sin^2\theta_W$

* Preliminary results:

Connected contributions:



[H. Horch, G. Herdoíza, V. Gülpers @ Lattice 2015]

Running of $\sin^2\theta_W$

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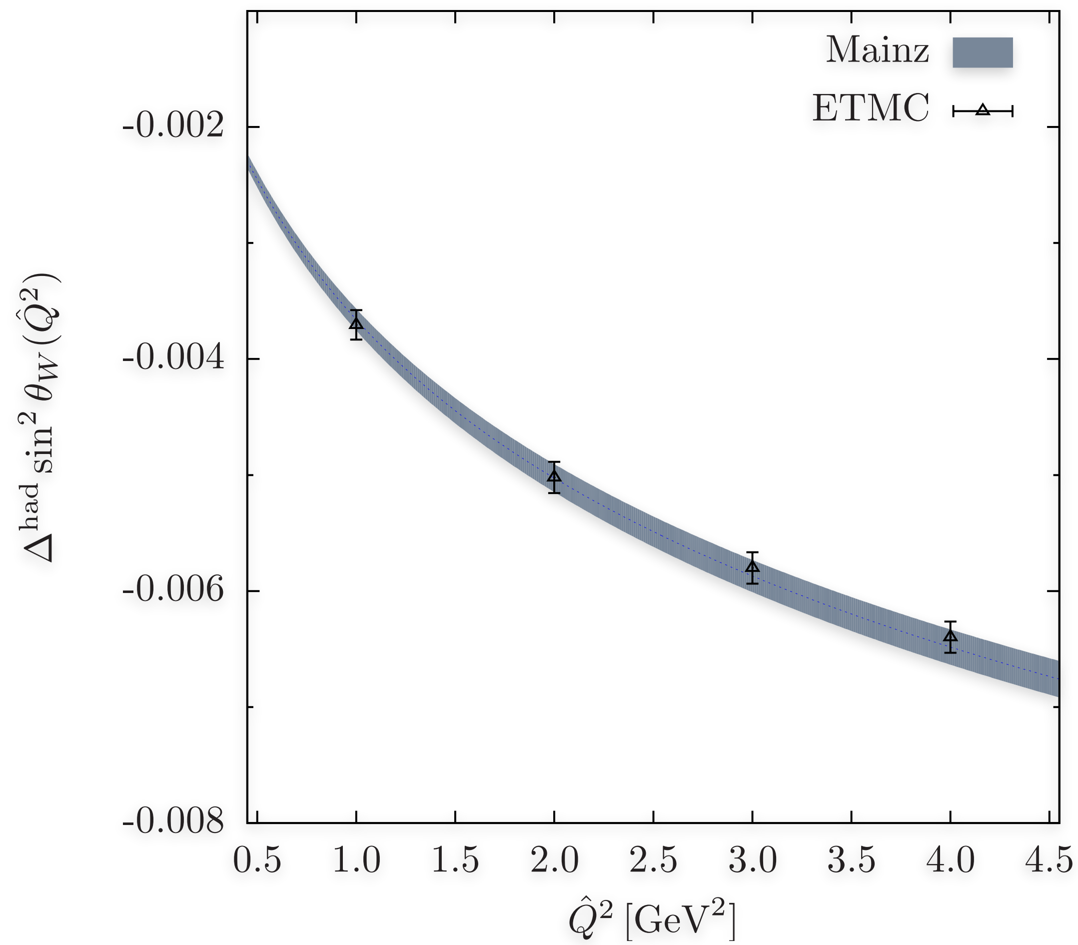

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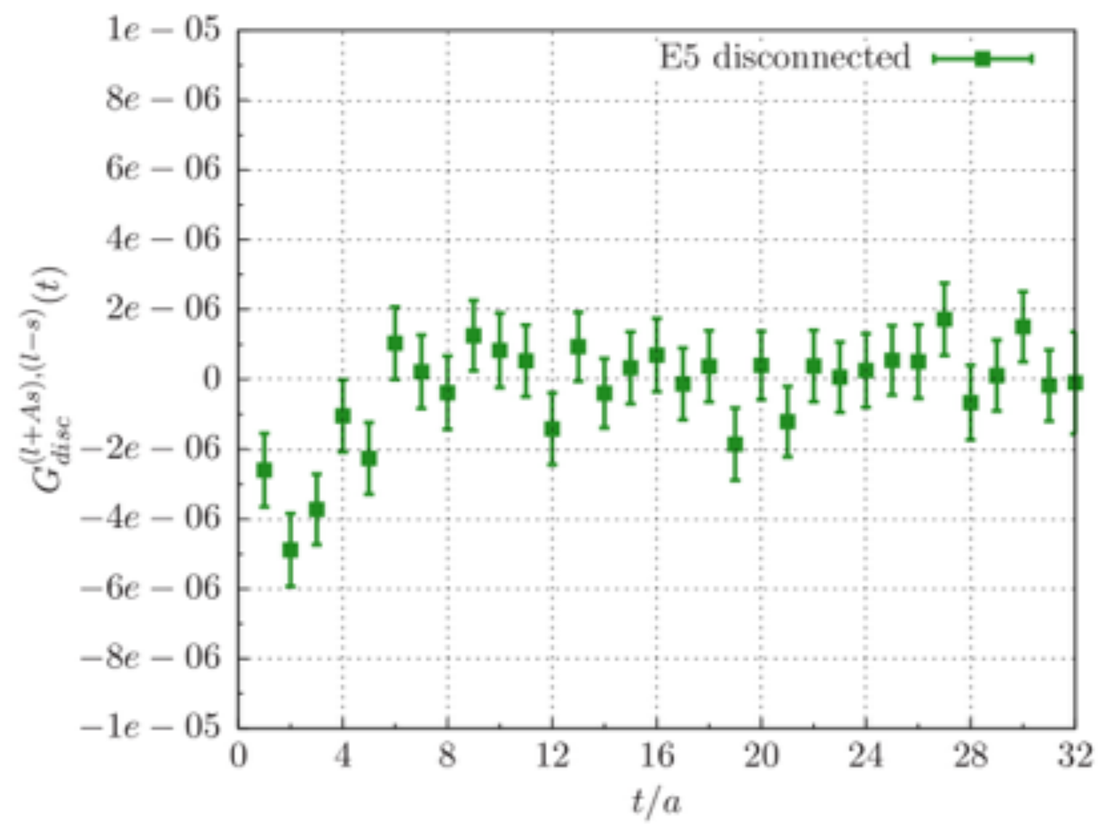

[H. Horch, G. Herdoíza, V. Gülpers @ Lattice 2015]

Running of $\sin^2\theta_W$

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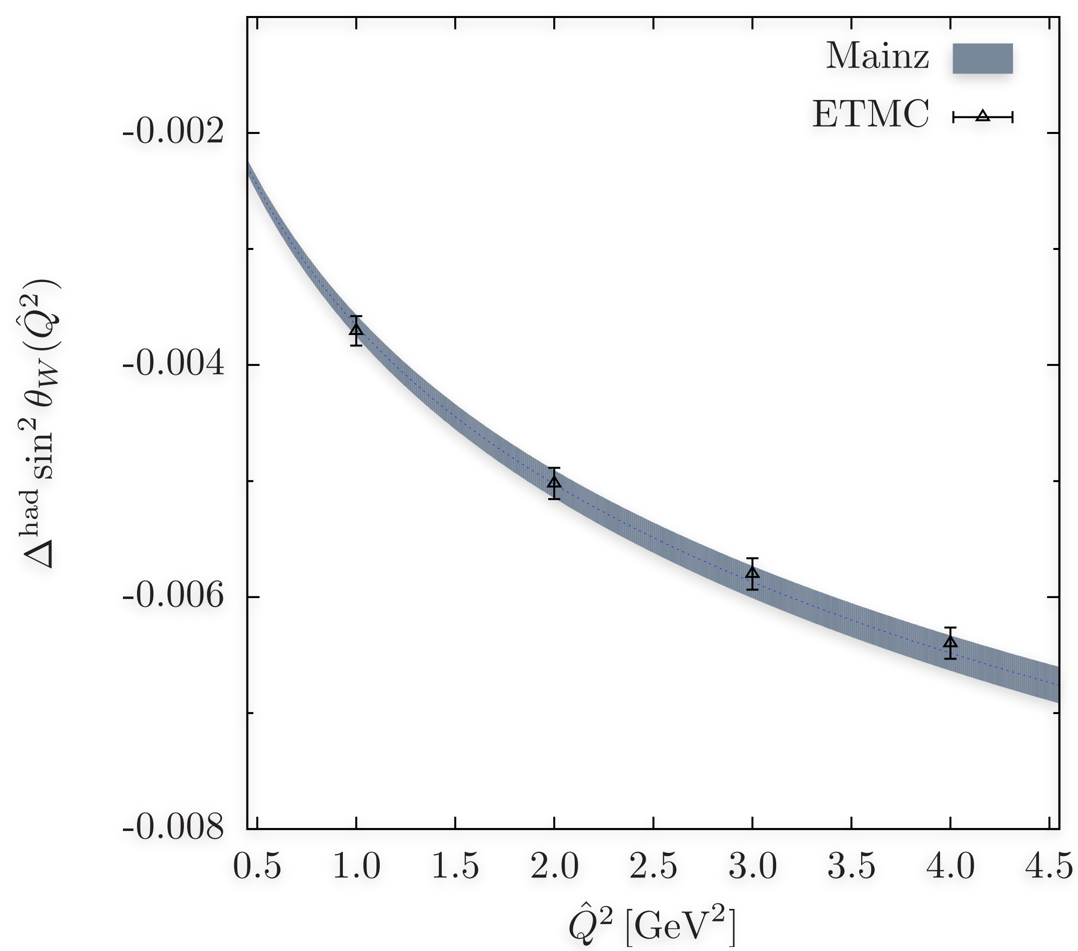

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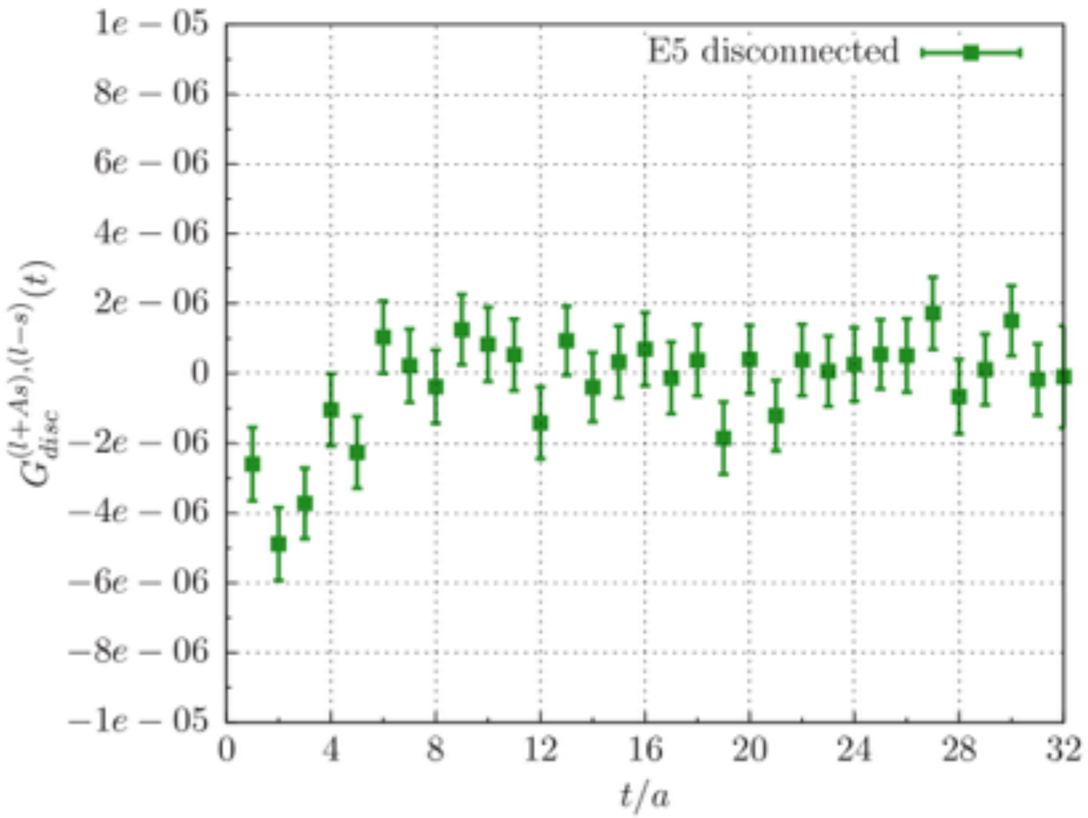
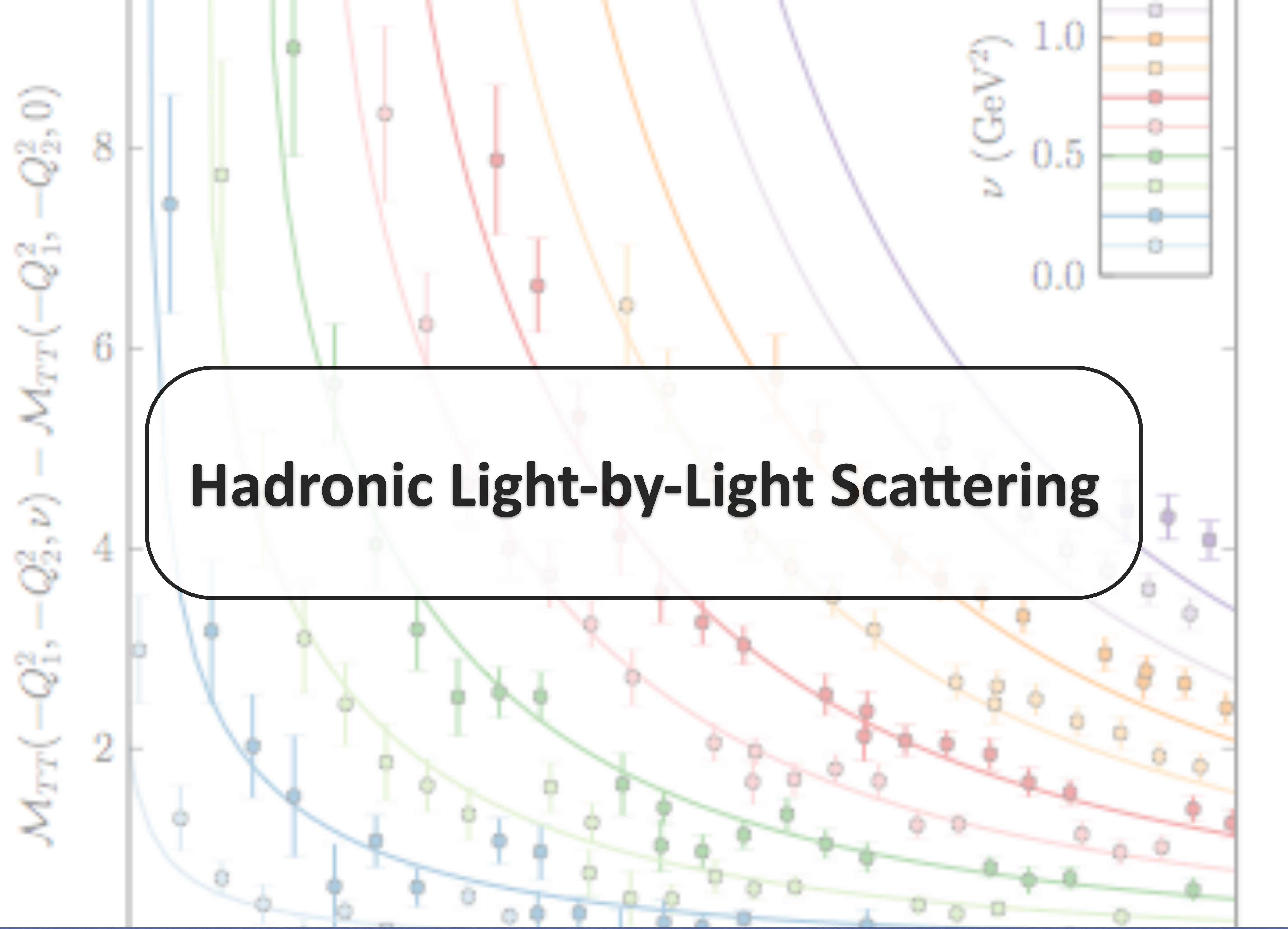


* Long-distance behaviour of total correlator limited by accuracy of disconnected contribution → systematic error estimate

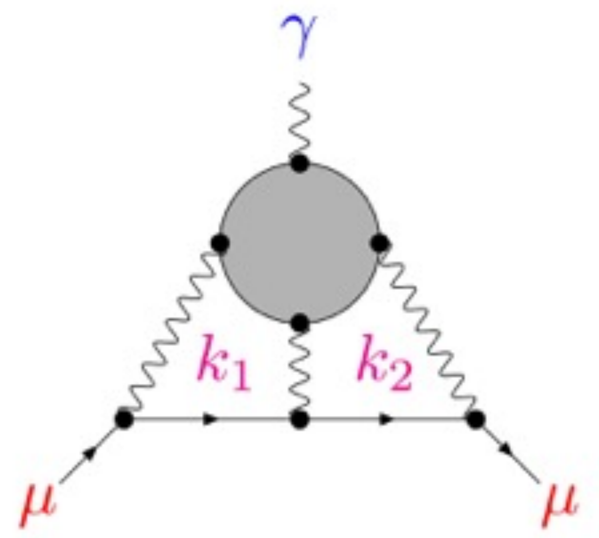
[H. Horch, G. Herdoíza, V. Gülpers @ Lattice 2015]



Hadronic Light-by-Light Scattering

Lattice approaches to HLbL

* Several different methods, e.g. QCD+QED simulations *[T. Blum et al., 2008–...]*



* Here: compute HLbL amplitude from Euclidean four-point function of the vector current

$$\Pi^{\mathrm{E}}_{\mu_1\mu_2\mu_3\mu_4}(P_4; P_1, P_2) =$$

$$\int d^4x_1 d^4x_2 d^4x_4 \mathrm{e}^{-i\sum_a P_a\cdot x_a} \langle J_{\mu_1}(x_1) J_{\mu_2}(x_2) J_{\mu_3}(0) J_{\mu_4}(x_4)\rangle$$

* Pilot study: compute forward scattering amplitude of transversely polarised virtual photons:

$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, \nu) = \frac{e^4}{4} R_{\mu_1\mu_2} R_{\mu_3\mu_4} \Pi^{\mathrm{E}}_{\mu_1\mu_2\mu_3\mu_4}(-Q_2; -Q_1, Q_1)$$

[J. Green, O. Gryniuk, G. von Hippel, H.B. Meyer, V. Pascalutsa, PRL 115 (2015) 222003]

Lattice approaches to HLbL

- Relate forward amplitude to cross section for $\gamma^*\gamma^* \longrightarrow \mathrm{hadrons}$

$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, v) - \mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, 0)$$
$$= \int_{\nu_0}^{\infty} d\nu' \ldots [\sigma_0(\nu') + \sigma_2(\nu')]$$

[J. Green, O. Gryniuk, G. von Hippel, H.B. Meyer, V. Pascalutsa, PRL 115 (2015) 222003]

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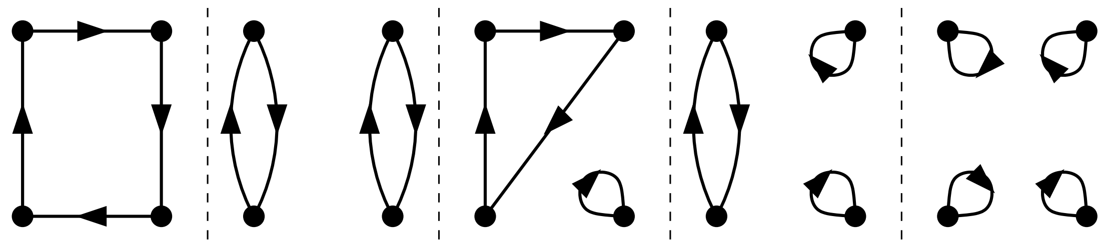

[J. Green, O. Gryniuk, G. von Hippel, H.B. Meyer, V. Pascalutsa, PRL 115 (2015) 222003]

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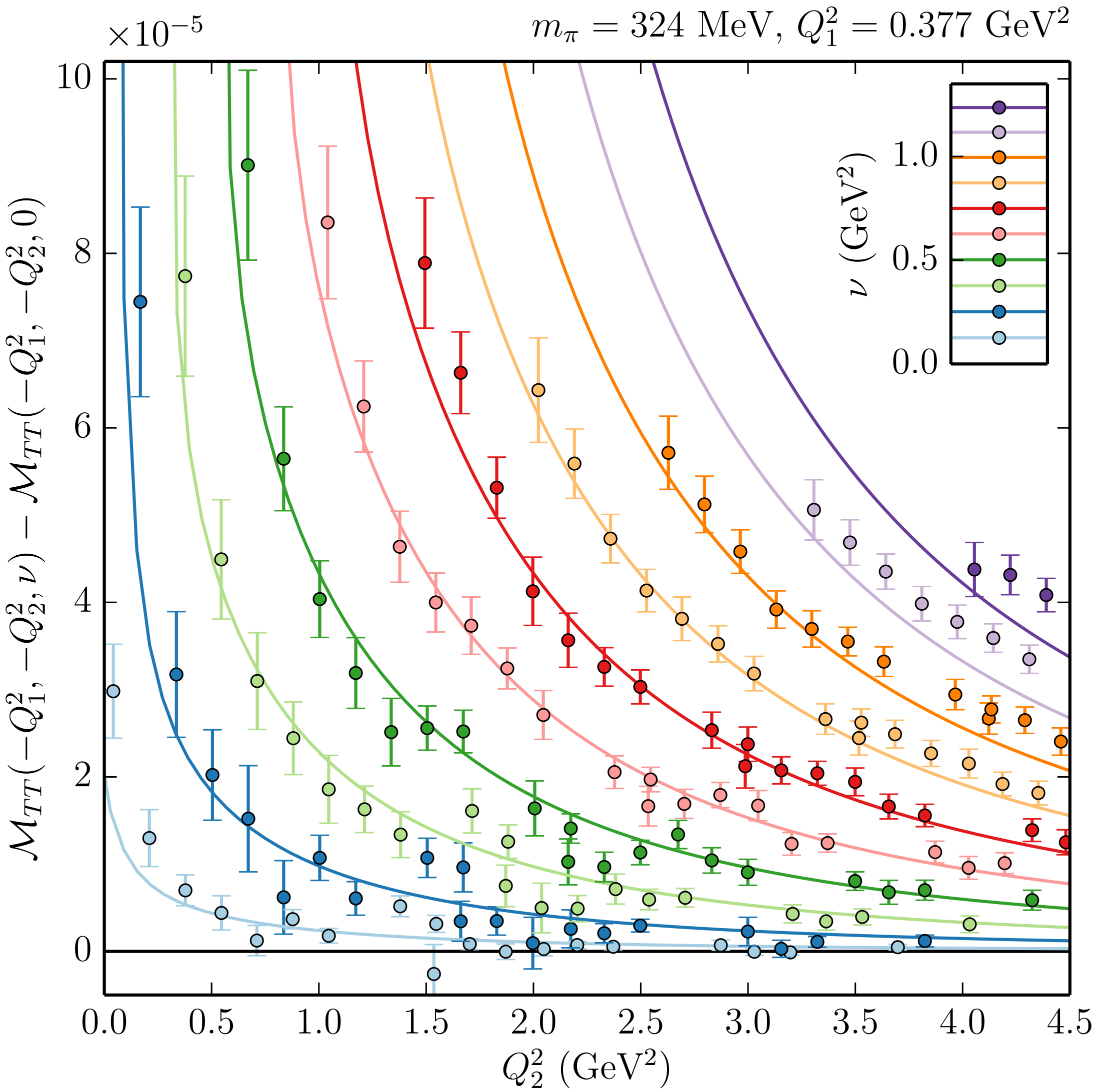

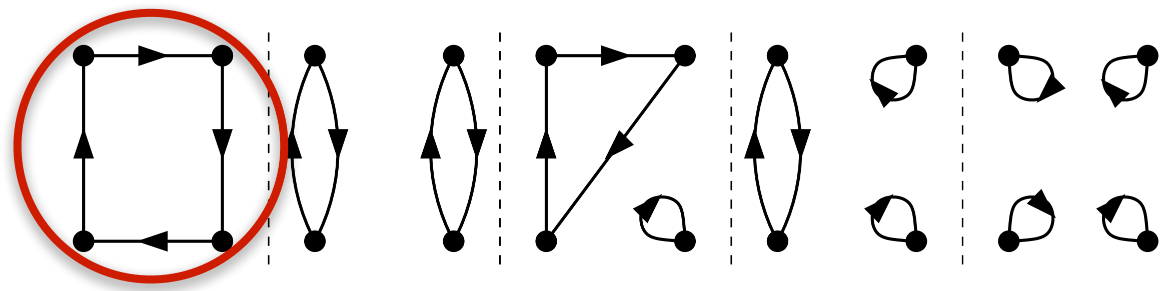

[J. Green, O. Gryniuk, G. von Hippel, H.B. Meyer, V. Pascalutsa, PRL 115 (2015) 222003]

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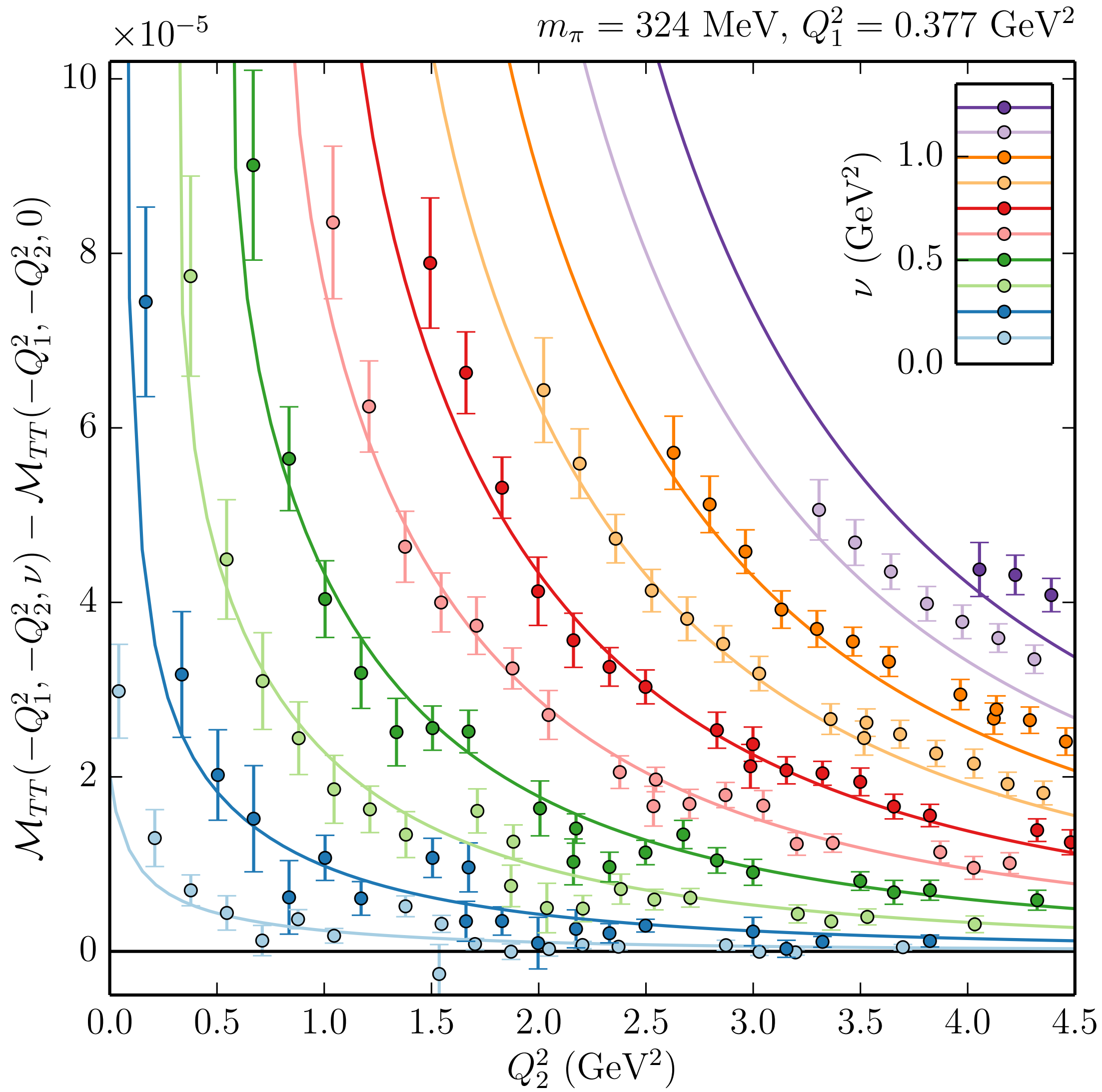

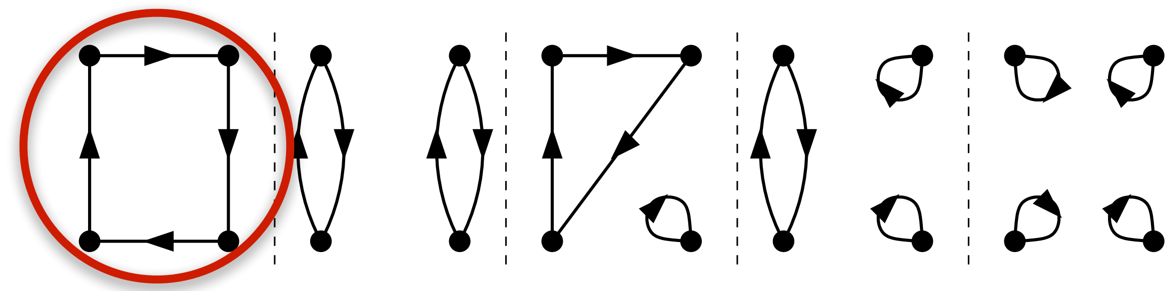

* Comparison with phenomenological model for σ_0 and σ_2

[J. Green, O. Gryniuk, G. von Hippel, H.B. Meyer, V. Pascalutsa, PRL 115 (2015) 222003]


EMMERT

Prospects

Summary

Hadronic vacuum polarisation

* Major effort within Lattice QCD community; focus on reducing overall uncertainty to the sub-percent level
* Several complementary formalisms
* TMR, time moments readily carry over to Lattice QCD with open boundary conditions
* Good progress in computing quark-disconnected contributions

Hadronic light-by-light scattering

* Several concepts are being explored
* Aim for 20–30% overall accuracy