V_{us} DETERMINATION(S) USING INCLUSIVE HADRONIC τ DECAY DATA

with

R. J. Hudspith, R. Lewis, J. Zanotti;

R. J. Hudspith, R. Lewis, T. Izubuchi, H. Okhi

MITP Workshop on the Determination of the Fundamental Parameters in QCD

March 7-12, 2016

OUTLINE

- V_{us} *from inclusive FB* τ *FESRs*

  * *The "conventional" implementation and the* $> 3\sigma$ *low* $|V_{us}|$ *puzzle*

  * *OPE issues: continuum and lattice investigations*

  * *A new implementation strategy (and* $|V_{us}|$ *results)*

- *A new approach to* V_{us} *using lattice and* us *V+A* τ *data*

CONTEXT: VARIOUS V_{us} DETERMINATIONS

- From 3-family unitarity: Hardy-Towner 2014 $0^+ \to 0^+$ nuclear β decay analysis $|V_{ud}| = 0.97417(21)(?) \Rightarrow$

$$|V_{us}| = 0.2258(9)(?)$$

- From $K_{\ell 3}$: Current lattice $n_f = 2+1$, $n_f = 2+1+1$ $f_+(0)$ inputs yield

$$|V_{us}| = 0.2235(4)_{exp}(9)_{latt} \quad (RBC/UKQCD \text{ 2015})$$
$$|V_{us}| = 0.2231(4)_{exp}(7)_{latt} \quad (FNAL/MILC \text{ 2014})$$
$$|V_{us}| = 0.2230(4)_{exp}(11)_{latt} \quad (ETM \text{ 2016})$$

- From $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$: Using lattice f_K/f_π, HT2014 $|V_{ud}|$ input

  * FLAG 2015 $n_f = 2+1+1$, $n_f = 2+1$ assessments (both $f_K/f_\pi = 1.194(5)$) $\Rightarrow$

$$|V_{us}| = 0.2250(4)_{exp}(9)_{latt}$$

- C.f. "conventional" FB τ kinematic-weight FESR determination, inclusive ud, us BF input [Gamiz et al.]

  * With HFAG input: $|V_{us}| = 0.2176\,(19)_{exp}(10??)_{th}$ (Passemar, CKM14 τ $|V_{us}|$ summary)

  * 3.4σ low c.f. 3-family unitarity expectations [interesting **if real**, but theory systematics?]

  * In this talk:

    - Continuum, lattice input on OPE input/treatment issues, lattice to quantify OPE errors

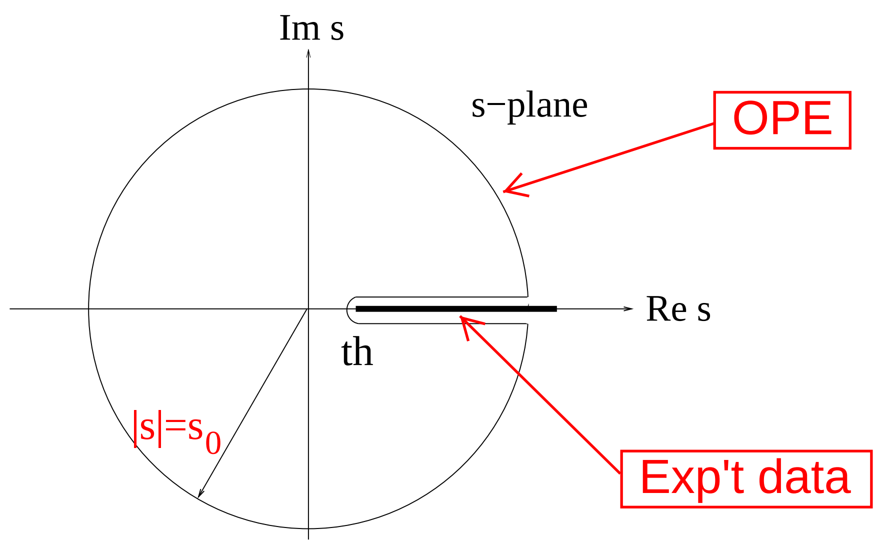
    - Alternate implementation of FB FESR analysis with continuum, lattice lessons in mind

REMINDER RE V_{us} FROM FB FESRs

- Basic theoretical tool: FESRs (*Cauchy's Thm)*

- For any s_0, analytic $w(s)$, kinematic-singularity-free Π:

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$



- Inclusive FB τ $|V_{us}|$ determination: From FESRs for V,A current 2-pt function polarizations $\Pi^{(J)}_{ud-us;V+A}(Q^2)$

  * Experimental spectral input: Scaled spectral functions $|V_{ij}|^2\rho^{(J)}_{ij;V/A}(s)$ from $dR_{ij;V/A}/ds$, with $R_{ij;V/A} \equiv \frac{\Gamma[\tau\rightarrow\nu_\tau \,\text{hadrons}_{ij;V/A}\,(\gamma)]}{\Gamma[\tau^-\rightarrow\nu_\tau e^-\bar{\nu}_e(\gamma)]}$ (+ small us $J=0$ subtraction) [SM "kinematic weight" w_τ]

  * $R^w_{ij;V/A}(s_0)$: Re-weighted $R_{ij;V/A}$ analogue, integrated to variable upper endpoint s_0 in spectrum

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds \, \frac{dR_{ij;V/A}}{ds} \, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

* FESR for $\delta R^w \equiv \frac{R^w_{ud;V+A}}{|V_{ud}|^2} - \frac{R^w_{us;V+A}}{|V_{us}|^2}$ yields

$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

valid for arbitrary analytic $w(y)$, $s_0 \leq m_\tau^2$

* "Self-consistency tests" :

  ◦ $|V_{us}|$ independent of s_0, w (tests control, understanding of OPE, experimental systematics)

  ◦ E.g., integrated $D = 2k+2$ OPE $\sim 1/s_0^k$: errors in higher D treatment $\leftrightarrow$ s_0-instability

- Below: FESRs with variable s_0, w, $\Delta\Pi_\tau \equiv \Pi^{(0+1)}_{ud-us;V+A}$

- The conventional implementation:

  * $w = w_\tau$ only, $s_0 = m_\tau^2$ only

  * w_τ degree 3 $\Rightarrow$ OPE contributions up to $D = 8$; $D = 2$, $D = 4$ known, $D = 6$ OPE estimated with "VSA" (very small), $D = 8$ neglected

  * Resulting $\left[\delta R^{w_\tau}(m_\tau^2)\right]^{OPE}$ $(D \geq 2)$ estimate a few to several % of individual $R_{ud,us;V+A}$

    $\Rightarrow$ modest OPE accuracy enough for precision $|V_{us}|$

  * Advantage of $w = w_\tau$, $s_0 = m_\tau^2$ restriction: $R_{ud,us;V+A}$ from inclusive ud, us BFs only [BUT also precludes $w(s)$-, s_0-independence tests $\Rightarrow$ price to pay in unquantified potential systematic errors]

- Testing the conventional implementation

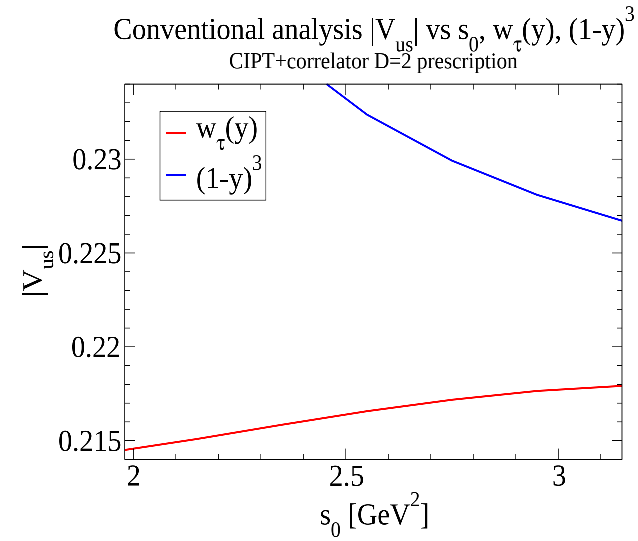
  * Variable $s_0 \leq m_\tau^2$, $|V_{us}|$ s_0-stability check

  * A (targeted) $|V_{us}|$ weight-independence check:

    ◦ $\hat{w}(y)$ c.f. $w_\tau(y)$ $\Delta\Pi_\tau$ $|V_{us}|$ results, $y = (s/s_0)$, $w_\tau(y) = 1 - 3y^2 + 2y^3$, $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$

    ($D = 6$, 8 for $\hat{w}$: -1, $-1/2$ $\times$ those for w_τ)

    ◦ **IF** $D > 4$ assumptions OK for w_τ FESR $\Rightarrow$ also for $\hat{w}$ FESR $\Rightarrow$ $|V_{us}|$ agreement. **If NOT, opposite-sign s_0-instabilities for w_τ, $\hat{w}$ FESRs, decreasing with s_0 for both**



Which is it? You decide ...

- Candidate self-consistency problem sources:

  * Experiment: The less-well-known us distribution

  * Theory: Conventional $D > 4$ assumptions (w_τ, $\hat{w}$ comparison); slow $D = 2$ OPE convergence

- The $D = 2, 4$ series and slow $D = 2$ convergence

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\left[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + \cdots\right]$$

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=4} = \frac{\left[\langle m_\ell\bar{\ell}\ell\rangle - \langle m_s\bar{s}s\rangle\right]}{Q^4}\left(2 - 2\,\bar{a} - \frac{26}{3}\,\bar{a}^2\right)$$

with running $\overline{MS}$ quantities $\bar{a} = \frac{\alpha_s(Q^2)}{\pi}$, $\bar{m}_s = m_s(Q^2)$

  * $\bar{a}(m_\tau^2) > 0.10 \Rightarrow$ slowly converging $D = 2$ series at ALL scales accessible in τ decay

  * Slow $D = 2$ convergence a potential issue for conventional $D = 2$ truncation error estimates

- More on the $D > 4$ assumptions

  * No natural rapid convergence with D for general $w(s)$

  * VSA ($D = 6$) known to be VERY crude, channel-dependent from ud V, A studies

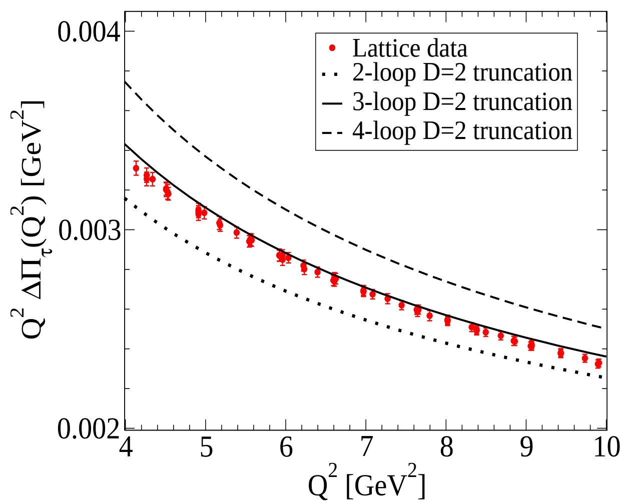
  * Strong cancellation in individual ud, us V+A sums in VSA

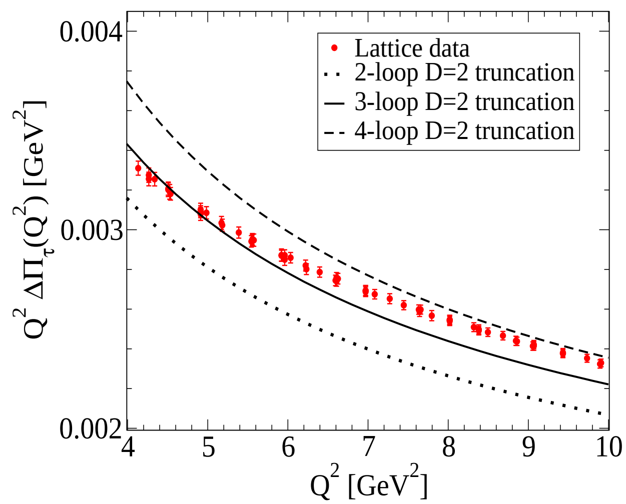
  * Even stronger cancellation in resulting $ud - us$ VSA difference

  * w_τ, $\hat{w}$ comparison STRONGLY suggests problem(s) with $D > 4$ assumptions

LATTICE RE $D=2$ SERIES, $D>4$ OPE ISSUES

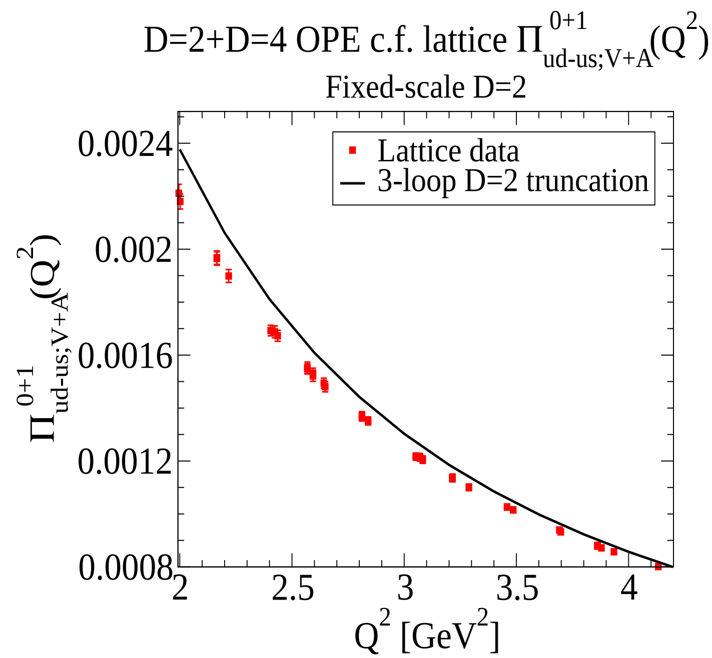
- OPE c.f. RBC/UKQCD lattice $\Delta\Pi_\tau(Q^2)$
  - * Here: lightest m_π, fine ($1/a = 2.38\ GeV$) $32^3\times 64$ $2+1$ ensemble ($m_\pi \sim 300\ MeV$, $m_\pi L \sim 4.1$)
  - * Tight cylinder cut for continuum correlator behavior (width from stabilization in $ud\ V$ channel α_s study)
  - * Large Q^2: $D=2+4$ OPE exploration; lower Q^2: re possible non-negligible $D>4$ at τ decay scales
  - * Fixed- or local-scale $D=2$? [c.f. "FOPT" vs "CIPT" issue for FESRs]



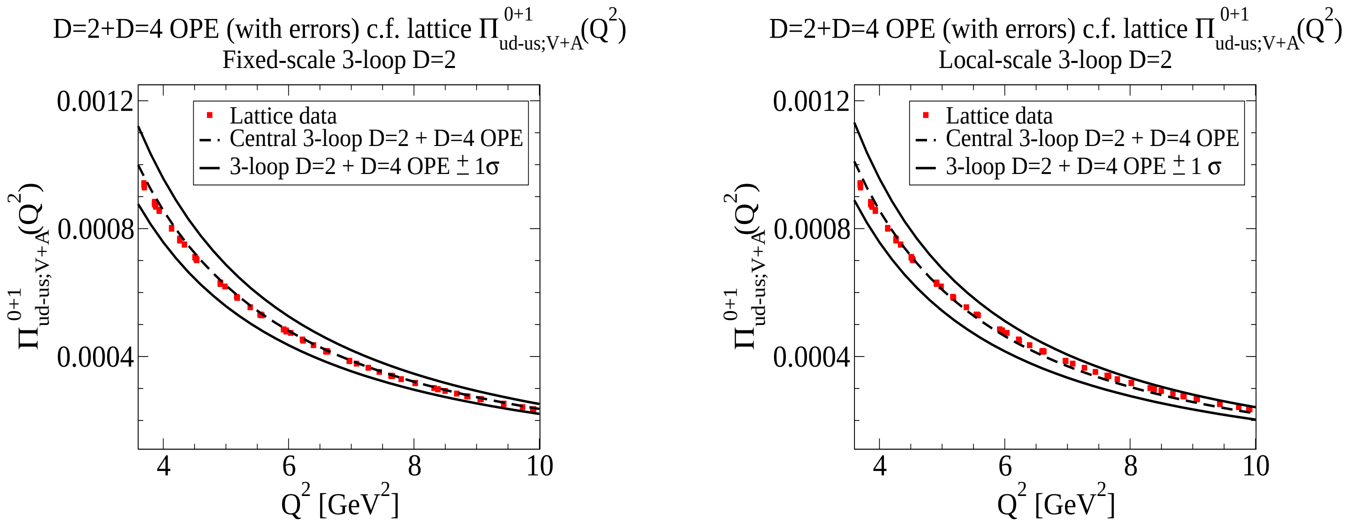


- Higher Q^2: best (excellent) lattice vs $D = 2 + 4$ OPE match for 3-loop-truncated, fixed-scale $D = 2$

- Fixed scale suggests FOPT for FESR $D = 2$



- Onset of $D > 4$ contributions below $\sim 4\ GeV^2$



- Standard $D = 2$, $D = 4$ error estimates conservative, despite very slow convergence of the $D = 2$ series

A NEW IMPLEMENTATION STRATEGY BASED ON LATTICE/CONTINUUM LESSONS

- No assumptions re $D > 4$; include contributions and fit effective condensates C_D as part of analysis

- 3-loop-truncated FOPT for $D = 2$ contribution; standard error estimates for $D = 2 + 4$

- Use range of s_0 and fit both $|V_{us}|$, C_D

- Fits simplest for $w(y) = w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ (C_{2N+2}, $|V_{us}|$ as only fit parameters)

- Self-consistency check: $|V_{us}|$ from different w_N

OPE, SPECTRAL INPUT

- PDG, FLAG, HPQCD input for $D = 2, 4$ OPE
- ud V+A spectral data from ALEPH 2013
- us V+A spectral data from sum over exclusive modes [$>$ 90% of B_{us}^{TOT} from $K_{\ell 2}$, Belle, BaBar $K\pi$, $K\pi\pi$, 3K results; residual: 1999 ALEPH]
- Here, for brevity, $K\pi$ normalization including preliminary BaBar $B[\tau \to K^- \pi^0 \nu_\tau]$ update (Adametz thesis)

RESULTS OF THE NEW ANALYSES

- Fitted $|V_{us}|$ (as expected) lies between (s_0-unstable) w_τ and $\hat{w}$ conventional implementation results

- Fitted C_D show FB cancellation in comparison to fitted ud V+A analogues (qualitative self-consistency test)

- Excellent stability of both $|V_{us}|$, C_{2N+2}, wrt variation of fit window size; excellent agreement of central $|V_{us}|$ from different w_N

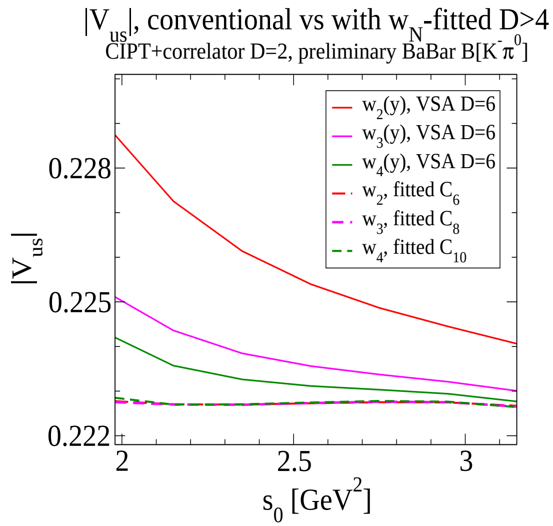
- Results using preliminary $K\pi$ BF update

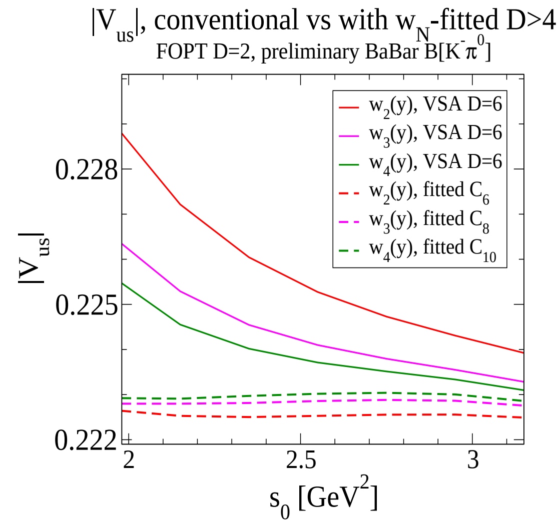
Weight	$\|V_{us}\|$ CIPT+corr $D=2$	$\|V_{us}\|$ FOPT $D=2$
w_2	0.22271(228)	0.22252(228)
w_3	0.22271(228)	0.22282(228)
w_4	0.22271(229)	0.22296(229)

Error budget, 3-loop-truncated FOPT $D=2$ fits

Source	$\delta\|V_{us}\|$ (w_2 FESR)	$\delta\|V_{us}\|$ (w_3 FESR)	$\delta\|V_{us}\|$ (w_4 FESR)	
$\delta\alpha_s$	0.00001	0.00004	0.00004	
$\delta m_s(2\ GeV)$	0.00017	0.00019	0.00019	
$\delta\langle m_s\bar{s}s\rangle$	0.00035	0.00035	0.00035	← **Theory**
$\delta(long\ corr)$	0.00009	0.00009	0.00009	
ud exp	0.00027	0.00028	0.00028	
us exp	0.00226	0.00227	0.00227	← **Exp't**

Fit quality: s_0-stability for conventional implementation, except with fitted C_{2N+2}





SUMMARY, CONVENTIONAL FB FESR ANALYSIS

- Continuum, lattice $\Rightarrow$ conventional $D > 4$ assumptions untenable, must fit $D > 4$ effective condensates

- Fitting $D > 4$ resolves s_0-, $w(y)$-dependence problems

- Fitted $D > 4$, when included, raise $|V_{us}|$ by ~ 0.0020

- $D > 4$ fit needs variable s_0 $\Rightarrow$ **BF-only approach untenable** ($\sim .0020 - 0.0030$ additional systematic)

- Lattice $\Rightarrow$ 3-loop-truncated FOPT for $D = 2$ series

- Lattice $\Rightarrow$ $D = 2 + 4$ OPE error estimate conservative

- $|V_{us}|$ errors then strongly dominated by us experimental uncertainties, subject to experimental improvement

- New preliminary $K^-\pi^0$ BF normalization, 3-weight combined fit result $|V_{us}| = 0.2228(23)_{exp}(5)_{th}$ in agreement with other determinations (especially $K_{\ell 3}$)

  (c.f. $0.2200(23)_{exp}(5)_{th}$ with 2014 HFAG $B[K^-\pi^0]$)

- Theory error (conservative) $\sim$ 0.0005 c.f. 0.0009 for $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ (lattice crucial here)

- **$\Rightarrow$ FB FESR method competitive with sufficient improvement to us experimental errors**

FUTURE PROSPECTS

- Current us spectral integral errors δB normalization uncertainty dominated $\Rightarrow$ improvement through BF error reduction (*B-factory $K^-\pi^0\pi^0$ distribution useful too*)

- An upcoming alternate to the FB FESR approach

  * KM, Hudspith, Izubuchi, Lewis, Ohki + RBC/UKQCD

  * Generalized dispersion rel'ns for products of us V+A polarization, weights with poles at Euclidean Q^2

  * Theory: Lattice us 2-point function data (no OPE)

  * Experiment: $dR_{us;V+A}/ds$ (no $J = 0$ subtraction)

More on the lattice-inclusive us τ approach

- Experimental $dR_{us;V+A}/ds$ distribution yields $|V_{us}|^2\,\tilde{\rho}(s)$,

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1 + 2\frac{s}{m_\tau^2}\right)\rho^{(J=1)}_{us;V+A}(s) + \rho^{(J=0)}_{us;V+A}(s)$$

(no continuum us $J = 0$ subtraction)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1 - 2\frac{Q^2}{m_\tau^2}\right)\Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)$$

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)}$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

$$\int_{th}^{\infty} ds\, w_N(s)\, \tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)}$$

  * Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS

  * LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$

  * $w_N(s)$: rapid fall-off in s if all $Q_k^2 < 1\ GeV^2$ ($\Rightarrow K$, $K\pi$ dominate spectral integral, near-endpoint multi-particle, $s > m_\tau^2$ contributions strongly suppressed)

  * Increasing $\{Q_k^2\}$: decreasing lattice $\tilde{\Pi}_{us;V+A}(Q_k^2)$ error, increasing experimental error

- An illustration of expected precision: $N = 4$, $\{Q_k^2\} = \{0.25, 0.30, 0.35, 0.40\ GeV^2\}$

  * Spectral integral error $\sim 0.5\%$

  * Statistical error, sum of residues for near-physical-point ensembles: $\sim 1/3\,\%$ for $48^3 \times 96$, $1/a = 1.73\,GeV$, $\sim 1/2\,\%$ for $64^3 \times 128$, $1/a = 2.36\ GeV$

  * Quadrature sum $\Rightarrow$ $\boxed{\delta|V_{us}| \sim 0.0007, \sim 0.0008}$ for near-physical-point 48^3, 64^3 ensembles, respectively

- Optimization/systematics under current study

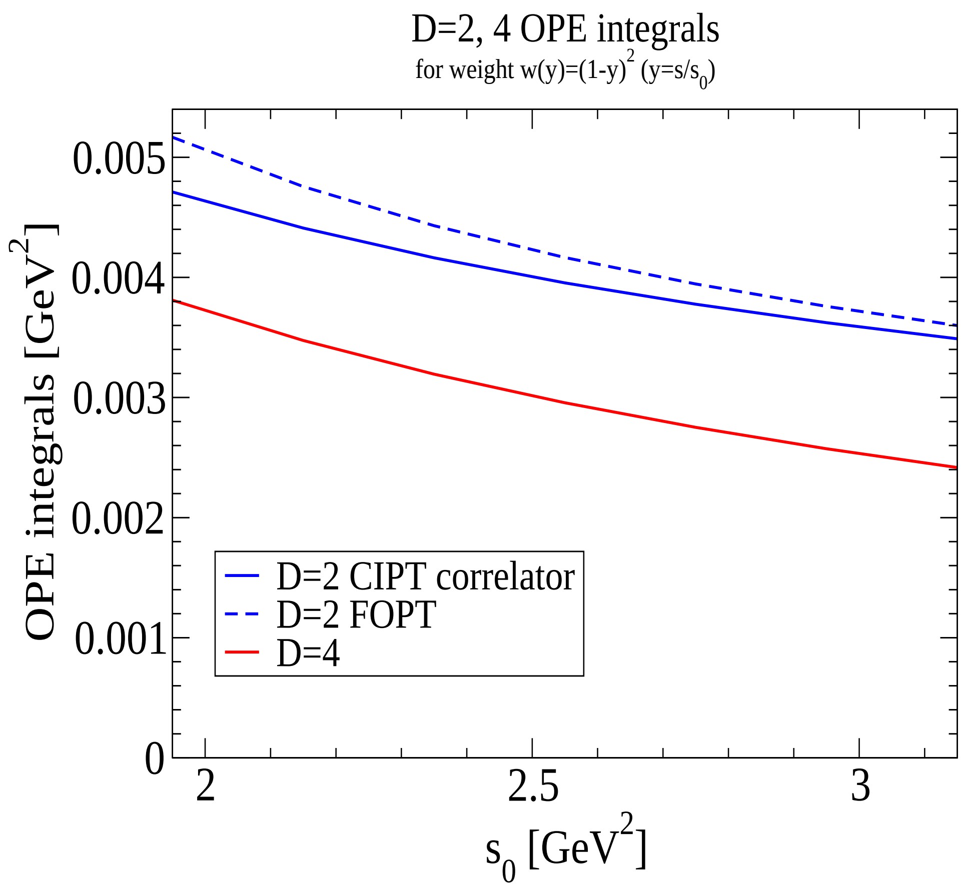
(THE END)

BACKUP SLIDES

- Are $D = 6, 8$ OPE contributions likely to be small?

  * $D = 4 << D = 2$ for $w_\tau(y) = 1 - 3y^2 + 2y^3$, $y = s/s_0$, "accidental" [$O(\alpha_s^2)$ suppression due to absence of term linear in y in $w_\tau(y)$]

  * Comparison of $D = 2, 4$ OPE contributions for $w(y) = (1 - y)^2$ (a case without this suppression) to see natural relative sizes



D=2, 4 OPE integrals
for weight w(y)=(1-y)^2 (y=s/s_0)
OPE integrals [GeV^2]
0.005
0.004
0.003
0.002
0.001
0
2
2.5
3
s_0 [GeV^2]
D=2 CIPT correlator
D=2 FOPT
D=4

- Spectral functions from hadronic τ decay distributions

With $R_{ij;V/A} \equiv \frac{\Gamma[\tau \to \nu_\tau \, \text{hadrons}_{ij;V/A}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2 \, |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[w_T\,(y_\tau)\; \rho^{(0+1)}_{ij;V/A}(s) + w_L\,(y_\tau)\; \rho^{(0)}_{ij;V/A}(s) \right]$$

$$w_T(y) = (1-y)^2(1+2y),\, w_L(y) = -2y(1-y)^2$$

$$y_\tau = s/m_\tau^2$$

More on the $D > 6$ VSA guesstimate

- VSA VERY crude: sizable (e.g. $\sim 4 - 5$), channel-dependent violation in ud V, A channels

- Crudeness $\Rightarrow$ double very close cancellation (in individual ud, us V+A sums, and in FB $ud - us$ difference) dangerous to rely on

- Integrated $D > 4 << $ than integrated $D = 2$ for w_τ (no $O(y)$ term) but comparable for other $w(y)$ $\Rightarrow$ smallness of doubly-cancelled $D = 6$ VSA not "natural"

MORE ON THE us DATA

- K pole via $f_K|V_{us}|$ from $K_{\ell 2}$

- Rather precise unit-normalized $K^-\pi^0$, $\bar{K}^0\pi^-$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$, $3K$ distributions from Belle, BaBar (main uncertainties from BFs)

- K, B-factory modes over 90% of B_{us}^{TOT}

- Residual us exclusive mode contributions from 1999 ALEPH data, covariances

Alternative $K\pi$ BF normalizations

- Existing HFAG $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0126$

- Existing $B[K^-\pi^0\nu_\tau] = 0.00431(15)$ value $\rightarrow$ preliminary BaBar (Adametz thesis) result 0.00500(15) yields $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0133$

- Central $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau]$ from $K_{\ell 3}$, dispersion rel'n expectations [ACLP13] also 0.0133

- 0.07% difference "small" but represents $\sim 2.4\%$ of B_{us}^{TOT}, hence $\sim 1.2\%$ increase in $|V_{us}|$

Results for $|V_{us}|$ for current $K\pi$ BFs:

Weight	$\|V_{us}\|$ CIPT+corr $D=2$	$\|V_{us}\|$ FOPT $D=2$
w_2	0.21985(230)	0.21966(230)
w_3	0.21985(231)	0.21966(231)
w_4	0.21985(231)	0.22009(231)

Error budget, existing $K\pi$ BFs

Source	$\delta\|V_{us}\|$ (w_2 FESR)	$\delta\|V_{us}\|$ (w_3 FESR)	$\delta\|V_{us}\|$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00003	0.00005
$\delta m_s(2\ GeV)$	0.00017	0.00018	0.00020
$\delta\langle m_s\bar{s}s\rangle$	0.00034	0.00034	0.00034
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00027	0.00027
us exp	0.00229	0.00229	0.00230

Stability of $|V_{us}|$ with fitted C_{2N+2} input, existing $K\pi$ BF normalization