Deconfinement and Polyakov loop in 2+1 flavor QCD

J. H. Weber[1] in collaboration with
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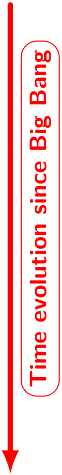
[5]Universität Graz

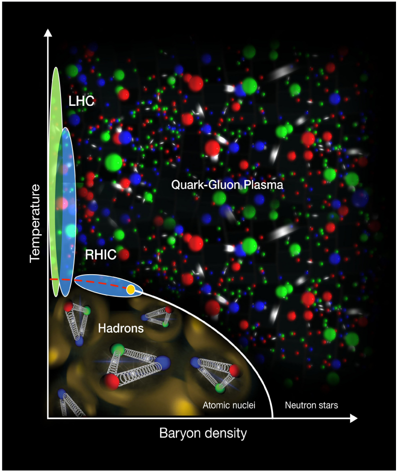
Determination of the Fundamental Parameters in QCD
MITP, 03/11/2016

main results to be published next week

- Overview & introduction
- Polyakov loop in 2+1 flavor QCD
- Static $Q\bar{Q}$ correlators at finite temperature
- Summary

QCD phase diagram





Plasma phase:

quark-gluon-plasma
$T > T_c \approx 160$ **MeV**
deconfinement,
color screening,
iso-vector chiral symmetry,
. . .

Hadronic phase:

dilute hadron gas
$T_c \gg T \approx 0$ **MeV**
confinement,
hidden chiral symmetry,
center symmetry (YM),
. . .

Lattice gauge theory at finite temperature

QCD expectation values

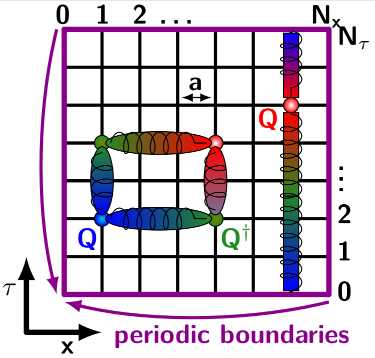
$$\langle\mathcal{O}\rangle_{\rm QCD} = \int \mathcal{D}\phi\ \mathcal{O}[\phi]\ e^{i\int dV_d \mathcal{L}[\phi;\alpha]}$$

- fields $\phi_{\rm QCD} = \{A_\mu, \psi, \bar{\psi}\}$
- parameters $\alpha_{\rm QCD} = \{g, m_q, \ldots\}$
- observable $\mathcal{O}[\phi_{\rm QCD}]$
- Lagrangian $\mathcal{L}_{\rm QCD}[\phi_{\rm QCD}, \alpha_{\rm QCD}]$

Why non-perturbative approaches?

- Non-Abelian group SU(3)
- QCD scale $\Lambda_{QCD} \sim 200$ MeV
- Crossover transition at $T \approx T_c$

LGT on a Euclidean space-time grid



- Interpret finite τ direction as inverse temperature $aN_\tau = 1/T$

Thermal observables in lattice gauge theory

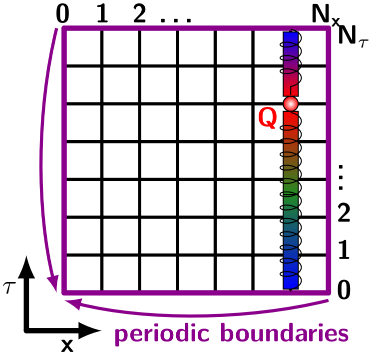
The archetype of thermal observables

- Finite N_τ direction: $aN_\tau = 1/T$
- Loops wrapping around the N_τ direction directly sensitive to T
- Archetype: **Polyakov loop** L

$$W(\beta, N_\tau, \mathbf{x}) = \prod_{x_0=1}^{N_\tau} U_0(x_0, \mathbf{x})$$

$$L(\beta, N_\tau) = \frac{\mathrm{Tr}\, W(\beta, N_\tau, \mathbf{x})}{N_c}$$

The Polyakov loop on the lattice



Interpretation

Free energy of a static quark related to the renormalized Polyakov loop

$$\langle L^{\mathrm{r}} \rangle = e^{-N_\tau a C_Q} \langle L^{\mathrm{b}} \rangle = \exp[-\frac{F_Q}{T}]$$

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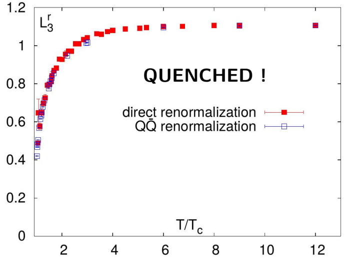
The Polyakov loop in pure YM theory



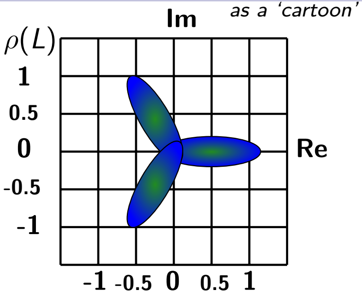
S. Gupta et. al., PRD 77, 034503 (2008)

The renormalized Polyakov loop is an **order parameter** of the transition in pure YM theory.

Due to Z(3) center symmetry

Z(3) center symmetry and Polyakov loop susceptibility

Center symmetry in pure YM theory



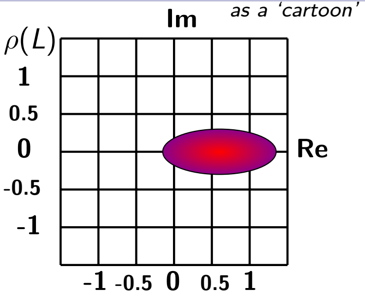
Z(3) center symmetry for $T < T_c$

$$\langle L \rangle = 0 \Leftrightarrow F_Q = \infty$$

Confinement in pure gauge theory

Z(3) center symmetry and Polyakov loop susceptibility

Center symmetry in pure YM theory



No center symmetry for $T > T_c$

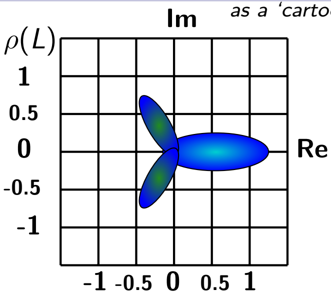
$$\langle L \rangle > 0 \Leftrightarrow F_Q = \text{finite}$$

Deconfinement in pure gauge theory

Z(3) center symmetry and Polyakov loop susceptibility

Center symmetry in pure YM theory

as a 'cartoon'



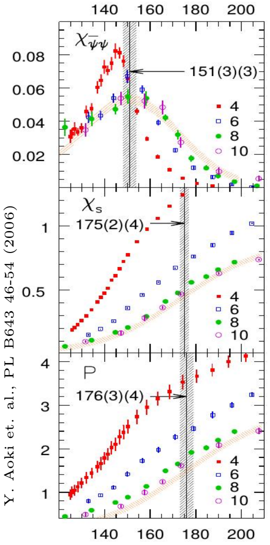
Center symmetry is broken in QCD by **sea quarks** for $T < T_c$

$\langle L \rangle > 0 \Leftrightarrow F_Q = \text{finite}$ **due to string breaking**

$F_Q \simeq \sum_i E_i$ **due to static hadrons with energies** E_i **(cf. HRG models)**

The crossover temperature puzzle in full QCD

The many faces of T_c



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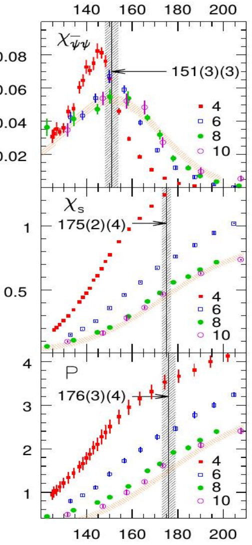
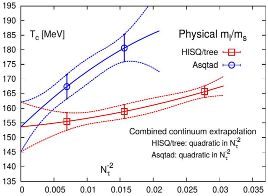


Y. Aoki et. al., PL B643 46-54 (2006)

Newer results from HotQCD collaboration

A. Bazavov et. al., PRD 85 054503 (2012)



N_τ	$\frac{m_l}{m_s} = \frac{1}{27}$	$\frac{m_l}{m_s} = \frac{1}{20}$
8	182(3)	185(3)
12	170(3)	174(3)
∞	161(6)	165(6)
6	168(2)	171(2)
8	161(2)	164(2)
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$\frac{\partial \chi_q}{\partial T}$, $q = l, s$ dominated by regular part of free energy; singular part is not easily accessible.

L has no demonstrated relation to singular part of free energy with massive light quarks.

The crossover temperature puzzle in full QCD

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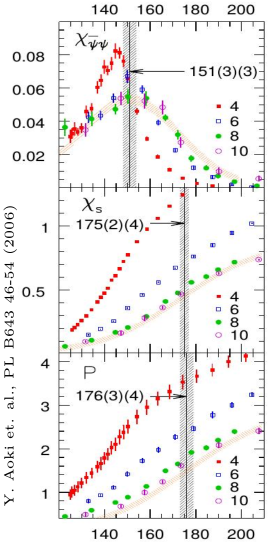

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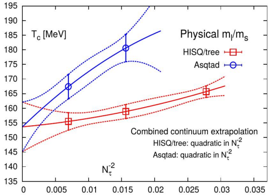

N_τ	$\frac{m_l}{m_s} = \frac{1}{27}$	$\frac{m_l}{m_s} = \frac{1}{20}$
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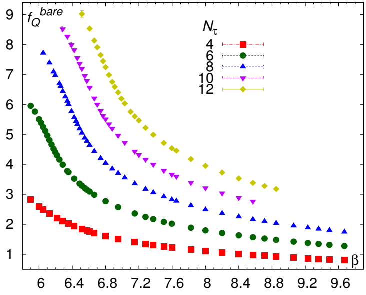
Is the higher value of T_c from L due to physical reasons? Does L provide reliable information about T_c in full QCD?

Bare Polyakov loop and renormalization



Bare free energy:

$$f_Q^{\rm bare} = \frac{F_Q^{\rm bare}}{T} = -\log \langle L^b \rangle$$

31 – 43 lattice spacings for each N_τ

T range from $0.72 T_c$ up to $30 T_c$

Free energy needs renormalization

$$\langle L \rangle = e^{-N_\tau a C_Q} \langle L^{\rm b} \rangle$$

$$\Rightarrow \quad f_Q = f_Q^{\rm bare} + N_\tau a C_Q$$

What is the nature of C_Q?

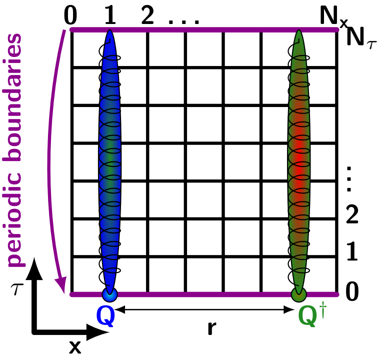
- $C_Q(\beta)$ independent of N_τ
- C_Q diverges as

$$C_Q = 1/a(\beta) c_Q(\beta)$$

- c_Q is related Z_3

$$\exp\left[-c_Q\right] \propto Z_3(g^2)$$

Polyakov loop as asymptotic limit of static meson correlators



Free energy of static $Q\bar{Q}$ pair:
$F_{Q\bar{Q}}(T,r) = Tf_{Q\bar{Q}}(T,r)$

$$f_{Q\bar{Q}}(T,r) = -\log \langle L(T,0)L^\dagger(T,\mathbf{r})\rangle$$

Poylakov loop correlator $C_P(T,r)$

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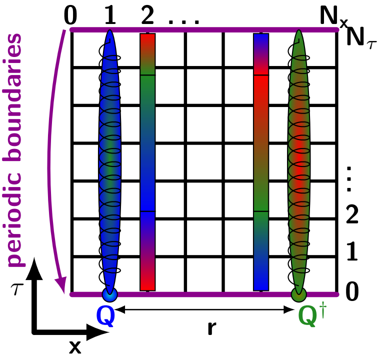

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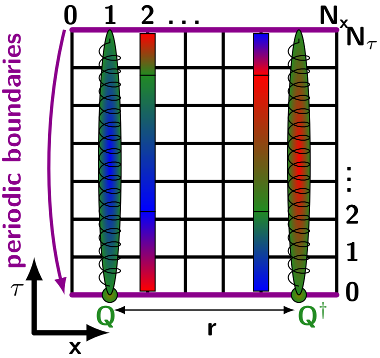
$r \gg 1/T$: static $Q\bar{Q}$ **decorrelate**

$$\lim_{r\to\infty} C_P(T,r) = \langle L(T)\rangle^2$$

Apparent due to **color screening**

Polyakov loop as asymptotic limit of static meson correlators



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Apparent due to **color screening**

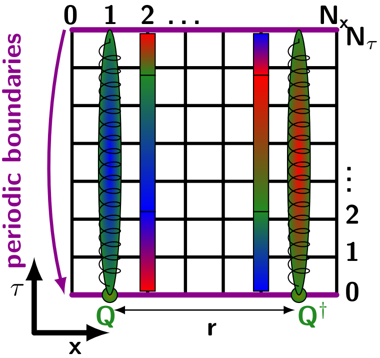
For any color configuration of $Q\bar{Q}$

$$\lim_{r\to\infty} C_S(T,r) = \langle L(T)\rangle^2$$

C_S is defined in Coulomb gauge as

$$C_S(T,r) = \frac{1}{3}\sum_{a=1}^{3} W_a(T,0)W_a^\dagger(T,\mathbf{r})$$

Polyakov loop as asymptotic limit of static meson correlators



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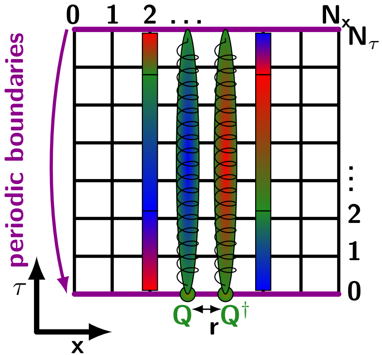
$$\frac{C_S^r}{C_S^b} = \frac{C_P^r}{C_P^b} = \langle\frac{L^r}{L^b}\rangle^2 = \exp\left[-2N_\tau c_Q\right]$$

Static meson correlators at short distances $r \ll 1/T$



$r \ll 1/T$: small thermal effects in

$$F_S(T,r) = -T \log \langle C_S(T,r) \rangle$$

For $r \ll 1/T$: **vacuum-like** due to **asymptotic freedom**

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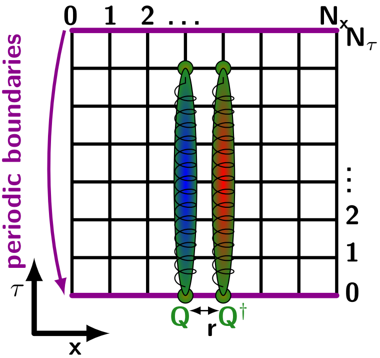

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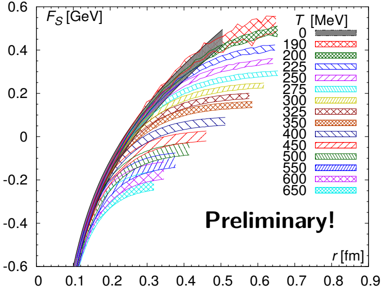
For $r \ll 1/T$: **vacuum-like** due to **asymptotic freedom**

$r \ll 1/T$ is a vacuum-like regime

$$F_S(T,r) = V_{Q\bar{Q}}(r) + \mathcal{O}(rT)$$



$$\frac{F_S^{\mathrm{r}} - F_S^{\mathrm{b}}}{a} = \frac{V_{Q\bar{Q}}^{\mathrm{r}} - V_{Q\bar{Q}}^{\mathrm{b}}}{a} = -2c_Q$$

Renormalization constant c_Q from $Q\bar{Q}$ procedure

$Q\bar{Q}$ procedure:

We fix the static energy ($V_{Q\bar{Q}} \equiv V$)

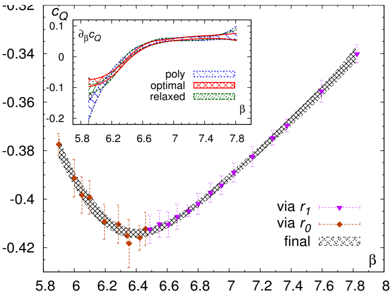
$$V^r(\beta, r) = V^b(\beta, r) + 2c_Q(\beta)$$

for each β (β omitted below) to

$$V^r(r) = \frac{V_i}{r_i}, \quad r^2 \frac{\partial V(r)}{\partial r}\bigg|_{r=r_i} = C_i,$$

with $V_0 = 0.954$, $V_1 = 0.2065$
and $C_0 = 1.65$, $C_1 = 1.0$

- We take $2c_Q$ from HotQCD,
  A. Bazavov et. al., PRD 90 094503 (2014)
- we interpolate in β and
- add $N_\tau c_Q$ to $f_Q^{\text{bare}}(T[\beta, N_\tau])$.



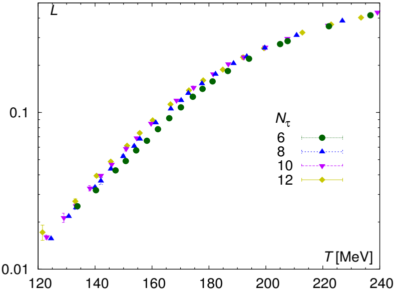
Drawbacks of $T \approx 0$ lattices

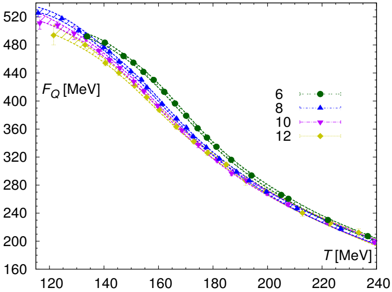
- computationally expensive
- currently limited to $\beta \leq 7.825$

Advantages of $T \approx 0$ lattices

- unambiguous procedure

Renormalized Polyakov loop, free energy

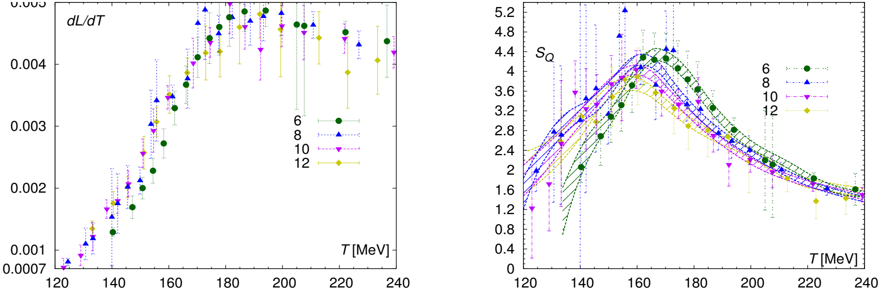




Renormalized Polyakov loop and free energy:

- Cutoff effects are large for $N_\tau = 6$ only in crossover region.
- Cutoff effects on par with errors for $T > 200$ MeV
- Errors due to $N_\tau c_Q$ become dominant for high T

Renormalized Polyakov loop, free energy and temperature derivatives



Expose the 'critical behavior' in the Polyakov loop and in the free energy:

Temperature derivative of L

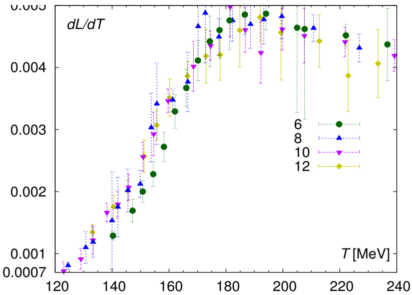
- $\frac{dL}{dT}$ peaks at $T \approx 190$ MeV

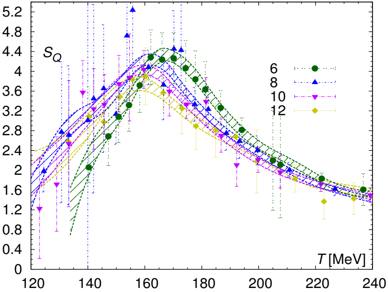
Temperature derivative of F_Q

- $S_Q = -\frac{dF_Q}{dT}$ peaks at $T \approx 160$ MeV

Is there any relation between the maxima and the deconfinement crossover?

Renormalized Polyakov loop, free energy and temperature derivatives





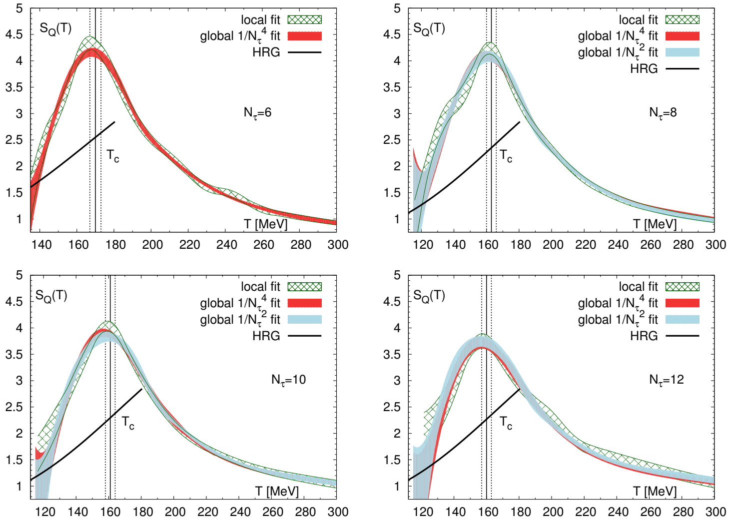
Different inflection points of L and F_Q?

- In principle the entropy $S_Q(T) = -\frac{dF_Q(T)}{dT}$ is a **measurable** quantity.

→ The inflection point of F_Q is **renormalization scheme independent**.

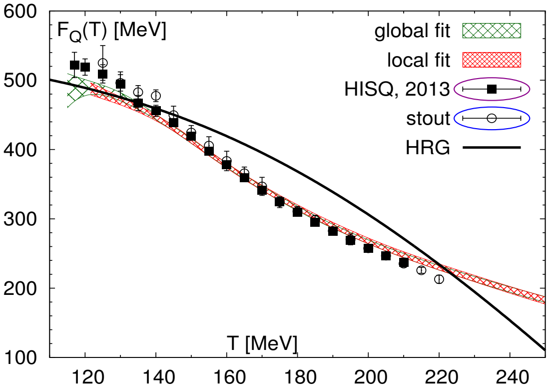
- The inflection point of L is scheme dependent with no physics implied.

Relation between entropy and pseudocritical temperature



T_c is from $O(2)$ scaling fits to chiral susceptibilities
A. Bazavov et. al., PRD 85 054503 (2012)

Continuum limit of the free energy

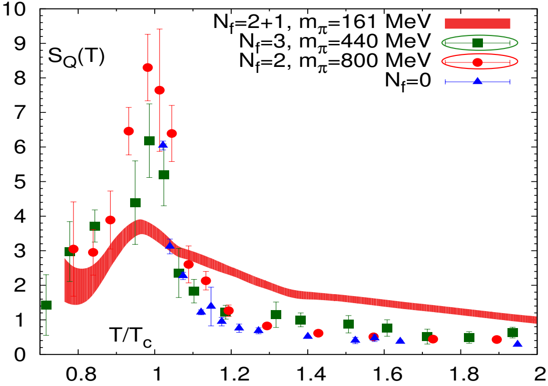


A. Bazavov,P. Petreczky, Phys.Rev. D87, 094505

S. Borsanyi et al., JHEP 09, 073 (2010)

- F_Q lies for low T below and for high T above the older HISQ result.
- Hadron resonance gas agrees with our data up to $T \lesssim 135$ MeV.

The entropy at the (almost) physical point



O. Kaczmarek, F. Zantow, hep-lat/0506019 (2005)
P. Petreczky, K. Petrov Phys. Rev. D70, 054503 (20

- The peak decreases for lower quark masses and for finer lattices.
- The entropy peaks at $T_S = 153^{+6.5}_{-5}$ MeV in the continuum limit.

Alternative renormalization scheme with gradient flow

Gradient flow approach

M. Lüscher, JHEP 08, 071(2010), ...

- Artificial fifth dimension t
- Diffusion-type field evolution

$$\dot{V}_\mu = -g_0^2 \{\partial_\mu S[V]\} V_\mu$$

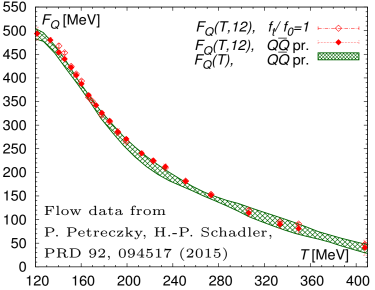
- Fields at finite flow time

$$V_\mu \equiv V_\mu(x,t), \;\; V_\mu(x,0) = U_\mu(x)$$

- Fields are smeared out over length scale $f_t = \sqrt{8t}$, no short distance singularities
- flow time t defines a specific renormalization scheme if

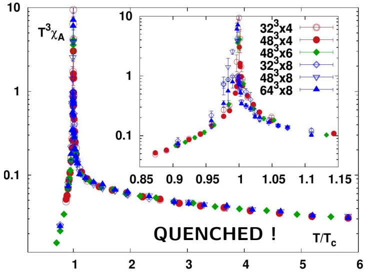
$$a \ll f_t = \sqrt{8t} \ll 1/T = aN_\tau$$

- adapt flow time for higher T



- f_t dependent cutoff effects are quite mild for $T \lesssim 400$ MeV.
- Hence, results at flow time f_t differ only by a constant and cross-check $Q\bar{Q}$ procedure.

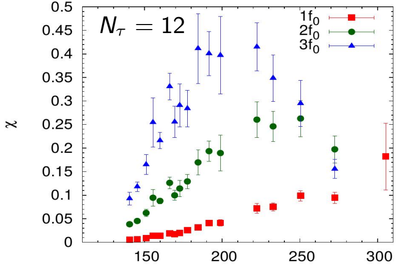
Polyakov loop susceptibility



P.M. Lo et. al., PRD 88, 074502 (2013)

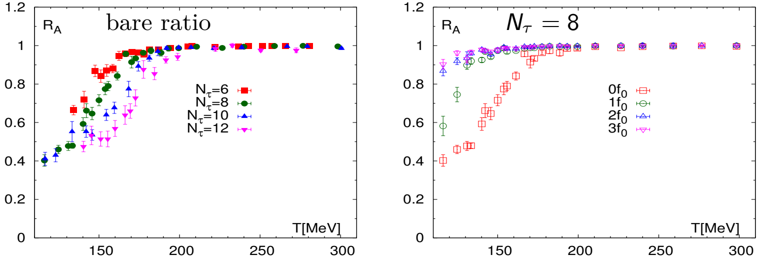
- Polyakov loop susceptibility: $\chi_A = (VT)^3 \left(\langle|L|^2\rangle - \langle|L|\rangle^2\right)$
- Mixes representations: $\langle|L_3|^2\rangle = \langle|L_6|\rangle - \langle|L_3|\rangle$
- Casimir scaling violations prohibit application of $Q\bar{Q}$ procedure

Polyakov loop susceptibility



- Polyakov loop susceptibility: $\chi_A = (VT)^3 \left(\langle |L|^2\rangle - \langle |L|\rangle^2\right)$
- 2+1 flavor HISQ data, renormalized via gradient flow
- χ_A strongly flow time dependent, no indication for critical behavior

Ratios of Polyakov loop susceptibilities

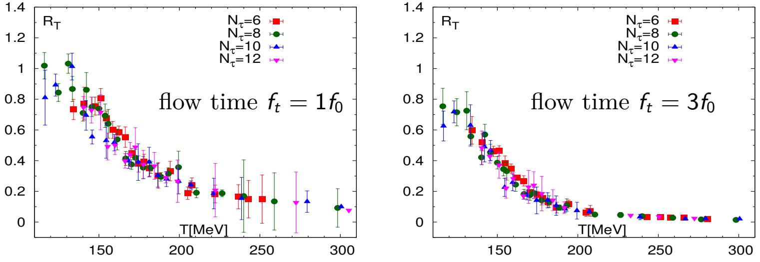


- Longitudinal and transverse Polyakov loop susceptibilities:

$$\chi_L = (VT)^3 \left(\langle \mathrm{Re}\, L^2 \rangle - \langle \mathrm{Re}\, L \rangle^2 \right), \quad \chi_T = (VT)^3 \left(\langle \mathrm{Im}\, L^2 \rangle \right)$$

- $R_A = \chi_A / \chi_L$: step function behavior cannot be related to crossover.

Ratios of Polyakov loop susceptibilities



- Longitudinal and transverse Polyakov loop susceptibilities:

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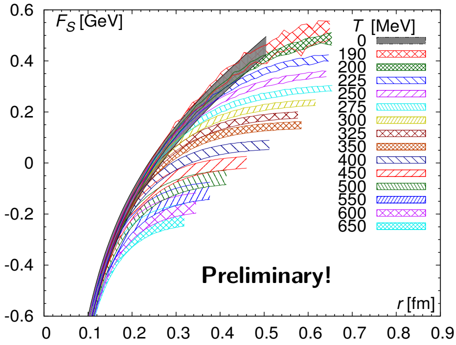
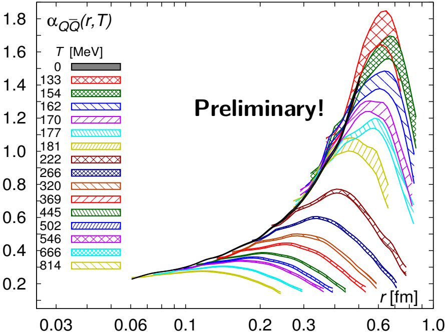
- $R_T = \chi_T / \chi_L$: crossover pattern for $f_t \geq f_0$, exposes critical behavior.

Singlet free energy



- Singlet free energy: $C_S(r,T) = \frac{1}{3}\langle \sum_{a=1}^{3} W_a(T,0) W_a^\dagger(T,\mathbf{r})\rangle = e^{-F_S(r,T)/T}$
- Consistent with $T=0$ static energy $V_{Q\bar{Q}}(r)$ up to $r \sim 0.45/T$
- Deviation from $V_{Q\bar{Q}}(r)$ is driven by the onset of color screening

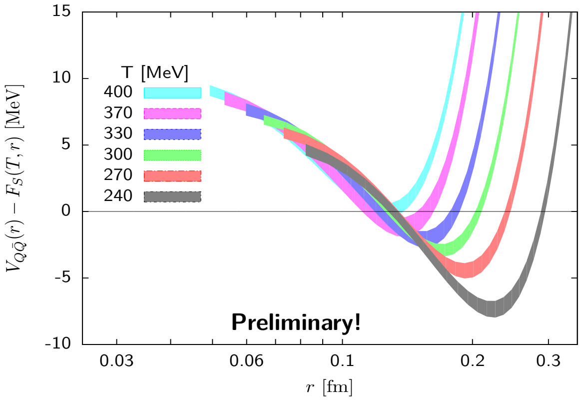
Confining and screening regimes



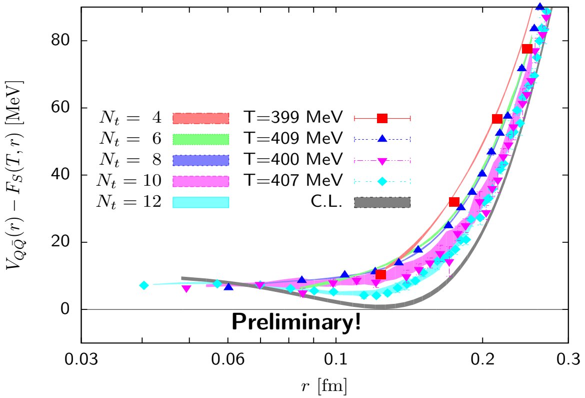
- Effective coupling constant makes different regimes explicit

$$\alpha_{Q\bar{Q}}(r,T) = \frac{r^2}{C_F}\frac{\partial E(r,T)}{\partial r}, \quad E = \{F_S(r,T), V_{Q\bar{Q}}(r)\}$$

- $\alpha_{Q\bar{Q}}(r,T) \gtrsim 0.5$ for $T \lesssim 2T_c$: QGP in HIC is strongly coupled

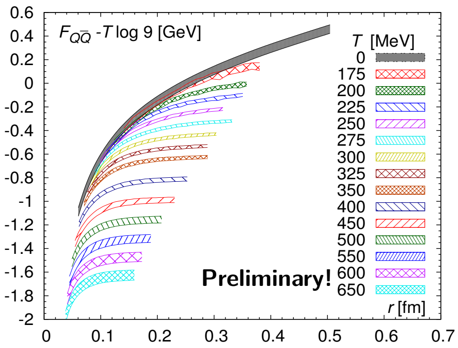


- Thermal modifications small for $r \to 0 \to$ study $V_{Q\bar{Q}}(r) - F_S(r,T)$
- $V_{Q\bar{Q}}(r)$ and $F_S(r,T)$ differ by up to 10 MeV for $r \lesssim 0.27/T$



- Thermal modifications small for $r \to 0 \to$ study $V_{Q\bar{Q}}(r) - F_S(r, T)$
- $V_{Q\bar{Q}}(r)$ and $F_S(r, T)$ differ by up to 10 MeV for $r \lesssim 0.27/T$
- Cutoff effects are rather large, finer lattices (larger N_τ) required

Static $Q\bar{Q}$ free energy



$$C_P(r,T) = \langle L(T,0) L^\dagger(T,\mathbf{r}) \rangle = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}} = \frac{1}{9} e^{-\frac{F_S(r,T)}{T}} + \frac{8}{9} e^{-\frac{F_O(r,T)}{T}}$$

- Consistent with $T=0$ static energy $V_{Q\bar{Q}}(r)$ up to $r \sim 0.15/T$
- This deviation from $V_{Q\bar{Q}}(r)$ is driven by the color-octet contribution

Summary (I)

- We extract different deconfinement observables from the renormalized Polyakov loop. Our analysis is firmly based on the $Q\bar{Q}$ procedure.
- Renormalization scheme dependence leads to an inflection point of the Polyakov loop at higher temperatures $T \approx 200$ MeV.
- We see crossover behavior at $T \approx T_c$ for the entropy $S_Q(T) = -\frac{dF_Q(T)}{d\,T}$ and for the ratio of Polyakov susceptibilities $R_T(T) = \frac{\chi_T(T)}{\chi_L(T)}$.
- We extract $T_c = 153^{+6.5}_{-5}$ MeV from the entropy, in agreement with $T_c = 160(6)$ MeV from chiral susceptibilities (O(2) scaling fits for $m_l/m_s = 1/20$).

N_τ	∞	12	10	8	6
$T_c(S_Q)$	$153^{+6.5}_{-5}$	157.5(6)	159(4.5)	162(4.5)	167.5(4.5)
$T_c(\chi_{m,l})$	160(6)	161(2)	$[162(2)]^*$	164(2)	171(2)

Summary (II)

- Static $Q\bar{Q}$ correlators show remnants of confinement at least up to $4T_c$.
- Onset of thermal effects strongly depends on individual observables, is much faster if color octet states contribute.
- Singlet free energy $(T > 0)$ and static energy $(T = 0)$ differ by $\lesssim 10$ MeV for short distances.
- Precision test of perturbation theory for static correlators at finite T is in preparation.

Thank You for listening!