

# The SMEFT at NLO: Two-Loop Effects Matter for New Physics

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Anders Eller Thomsen

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Based on work w/ *L. Born, J. Fuentes-Martín, and A. Greljo*

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<sup>b</sup>  
UNIVERSITÄT  
BERN

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ALBERT EINSTEIN CENTER  
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*Charting EFT Structures to Characterize NP*  
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**Swiss National  
Science Foundation**

# SM Effective Field Theory

High-energy BSM physics manifests as contact interactions in the SMEFT

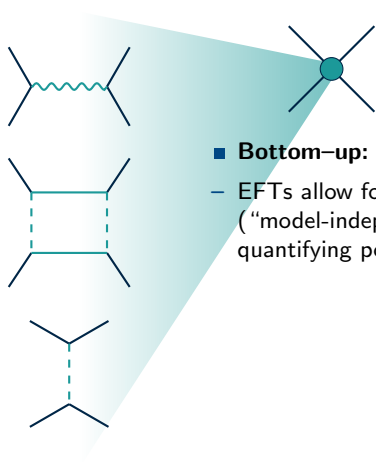
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d=5} \sum_k \frac{C_{d,k}}{\Lambda^{d-4}} \mathcal{O}_{d,k}$$

UV Physics

NP scale

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UV Physics  
NP scale

- **Bottom-up:**
- EFTs allow for a **model-comprehensive** (“model-independent”) analysis of deviations from the SM, quantifying possible deviations as an expansion in  $E/\Lambda$

# SM Effective Field Theory

High-energy BSM physics manifests as contact interactions in the SMEFT


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UV Physics  
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## ■ Bottom-up:

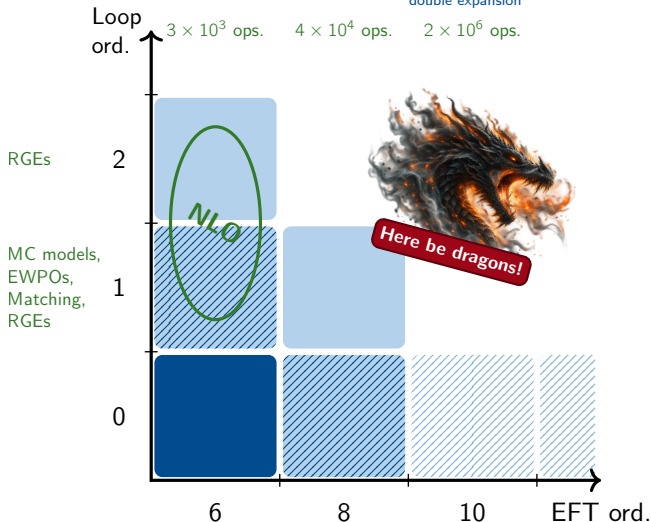
- EFTs allow for a **model-comprehensive** (“model-independent”) analysis of deviations from the SM, quantifying possible deviations as an expansion in  $E/\Lambda$

## ■ Top-down:

- **Precision calculations** necessitates the use of EFTs to separate the large BSM energy scales
- Many BSM models results in the same EFT, ensuring that computation are **reusable**: you only need to compute once in the EFT

# SMEFT as a double expansion

$$\mathcal{L}_{\text{SMEFT}}(\phi) = \mathcal{L}_{\text{SM}}(\phi) + \sum_{d=5}^{\infty} \sum_{\ell=0}^{\infty} \sum_k \underbrace{\frac{C_{d,k}^{(\ell)}}{(16\pi^2)^\ell \Lambda^{d-4}}}_{\text{double expansion}} \mathcal{O}_{d,k}^{(\ell)}(\phi)$$



# Why higher-order effects matter

The dimension-six operators change with the renormalization scale:

$$\frac{dC_i}{d \ln \mu} = \overset{\text{loop level}}{\gamma_{ij}} C_j$$

$\gamma_{ij}$  is non-diagonal (operator mixing): **leading effects, not only precision**

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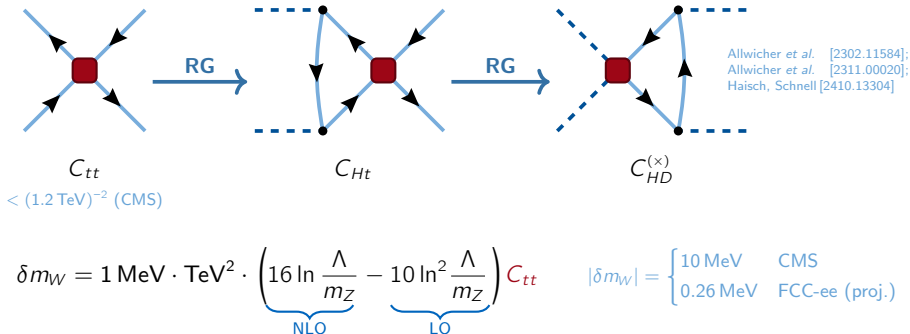
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Hard to access at LHC

Constrained by EWPOs



Biekötter, Pecjak [2503.07724]; Born, Fuentes-Martín, AET [2601.19974]

# 2-Loop SMEFT RGEs

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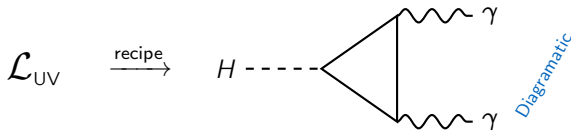
/w Lukas Born and Javier Fuentes-Martín [arXiv:2601.19974]

(~~B~~ part by Banik *et al.* [arXiv:2510.08682])

# Why functional methods? The master formula

One-loop matching is the classical use-case of functional methods

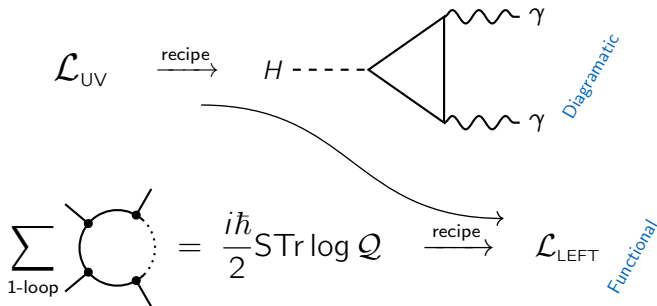
*Henning et al.* [1412.1837]; *Drozdz et al.* [1512.03003]; *Fuentes-Martín et al.* [1607.02142];...



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*Henning et al. [1412.1837]; Drozd et al. [1512.03003]; Fuentes-Martín et al. [1607.02142];...*



- **Bookkeeping:** organization by topology/internal propagators
- **Manifest covariance:** There is no need to separate out

$$D_\mu = \partial_\mu - iA_\mu$$

- **Recycling:** the methods recycle known loop-integral techniques

## ■ EFT matching—extraction of EFT couplings

$$\text{Loop Diagram} \sim \int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2} \sim \frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2} + \text{local EFT vertex}$$

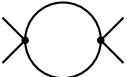
## ■ EFT matching—extraction of EFT couplings



$$\sim \int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2} \xrightarrow{\text{Changed high-}k \text{ int}} \sim \frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2} + \text{local EFT vertex}$$

The diagram shows a bubble loop with two external lines. A blue arrow points to the top arc of the loop, labeled "UV field". The integral is shown as  $\int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2}$ . A blue arrow points from the second denominator to the next term, labeled "Changed high- $k$  int". The next term is  $\sim \frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2}$ . A blue arrow points from the text "local EFT vertex" to a blue circle with four external lines.

## ■ Renormalization/RG functions—extraction of counterterms



$$\sim \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2} \longrightarrow \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)^2} + \delta\lambda$$

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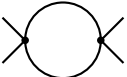
# Matching and RG

- EFT matching—extraction of EFT couplings



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- Renormalization/RG functions—extraction of counterterms



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local UV counterterm

- Amplitudes? Why not?!

But there might be little point... and some *trouble*  $\leftrightarrow$  non-local

# Semi-classical approximation

The loop-expansion is a polynomial of tensor contractions

Fuentes-Martín, Palavrić, **AET** [2311.13630]; Fuentes-Martín, Moreno-Sánchez, Palavrić, **AET** [2412.12270]

$$\Gamma[\hat{\eta}] = \hat{S} + \frac{i\hbar}{2} \overbrace{\zeta_{II} (\log \hat{Q})_{II}}^{\text{STr log } Q}$$

or as supergraphs

$$\Gamma = S + \frac{i}{2} \log \bigcirc$$

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$$\begin{aligned}\Gamma[\hat{\eta}] = & \hat{S} + \frac{i\hbar}{2} \overbrace{\zeta_{II} (\log \hat{Q})_{II}}^{\text{STr log } Q} + \frac{i\hbar^2}{2} \zeta_{II} \hat{Q}_{IJ}^{-1} \hat{V}_{JI}^{(1)} - \frac{\hbar^2}{8} \hat{Q}_{IJ}^{-1} \hat{Q}_{KL}^{-1} \hat{V}_{IJKL}^{(0)} \\ & + \frac{\hbar^2}{12} \zeta_{IM} \zeta_{JN} \zeta_{IN} \hat{Q}_{LI}^{-1} \hat{Q}_{MJ}^{-1} \hat{Q}_{NK}^{-1} \hat{V}_{IJK}^{(0)} \hat{V}_{LMN}^{(0)} + \mathcal{O}(\hbar^3)\end{aligned}$$

Spin statistics:  $\pm 1$

or as supergraphs

$$\Gamma = S + \frac{i}{2} \log \bigcirc + \frac{i}{2} \overset{(1)}{\bigcirc} - \frac{1}{8} \bigcirc \bigcirc + \frac{1}{12} \bigcirc \text{---} \bigcirc + \mathcal{O}(\hbar^3)$$

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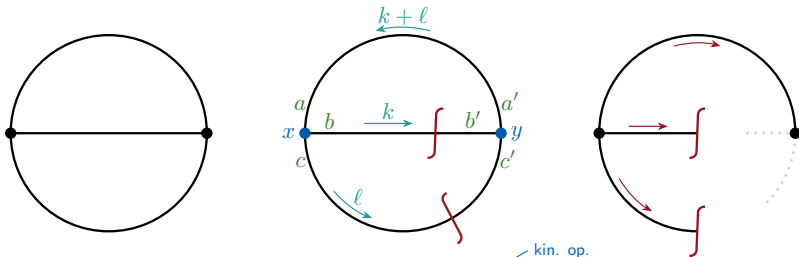
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All **vertices and propagators are fully dressed** with background fields

$$Q_{IJ}[\hat{\eta}] = \frac{\delta^2 S^{(0)}}{\delta \eta^I \delta \eta^J}[\hat{\eta}] = \left( \text{---} \text{---} \right)^{-1}$$

$$\mathcal{V}_{IJK}^{(0)}[\hat{\eta}] = \text{---} \text{---} \text{---}$$

# Sunset formula



$$E_{a_i b_j}^{(\underline{m}_i, \underline{n}_j)}(x, P_x + k) \equiv (-1)^{m_i} (P_x + k)^{\underline{m}_i} Q_{a_i b_j}^{-1}(x, P_x + k) (P_x + k)^{\underline{n}_j}$$

Sunset master formula:

$$\frac{\hbar^2}{12} \sum_{m_i, n_i} \int_x \int_{kl} E_{aa'}^{(0,0)}(y, P_y - k - \ell) V_{abc}^{(\underline{m}_1, \underline{n}_1)}(x) V_{a'b'c'}^{(\underline{m}_2, \underline{n}_2)}(y) \times [E^{(\underline{m}_1, \underline{m}_2)}(x, P_x + k) U(x, y)]_{bb'} [E^{(\underline{n}_1, \underline{n}_2)}(x, P_x + \ell) U(x, y)]_{cc'} \Big|_{y=x}$$

Grassmanian factor:  $\zeta_{aa'} \zeta_{bb'} \zeta_{cc'} \zeta_{ab} \zeta_{ac} \zeta_{a'b} \zeta_{a'c} \zeta_{bc'} \zeta_{a'b'} \zeta_{a'c'} = \pm 1$

# Two-loop counterterms

An application of functional methods at multi-loop order

Born, Fuentes-Martín, Kvedaraitė, **AET** [2410.07320]

$$\delta\hat{S}^{(2)} = \frac{1}{8}\mathbf{K}\bar{\mathbf{R}}^*\mathbf{T}\left[\hat{\mathcal{V}}_{IJKL}^{(0)}\hat{\mathcal{Q}}_{IJ}^{-1}\hat{\mathcal{Q}}_{KL}^{-1}\right] - \frac{1}{12}\mathbf{K}\bar{\mathbf{R}}^*\mathbf{T}\left[\hat{\mathcal{V}}_{IJK}^{(0)}\hat{\mathcal{Q}}_{IL}^{-1}\hat{\mathcal{Q}}_{JM}^{-1}\hat{\mathcal{Q}}_{KN}^{-1}\hat{\mathcal{V}}_{LMN}^{(0)}\right]$$

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Evaluation recipe:

- i) Construct gauge-invariant graphs w/ full background field dependence
- ii) Expand in the hard loop-momentum region
- iii) Subtract subdivergences w/ a local version of  $\mathbf{R}^*$ -operation
- iv) Evaluate single-scale tadpole loop diagrams

Herzog, Ruijl [1703.03776]

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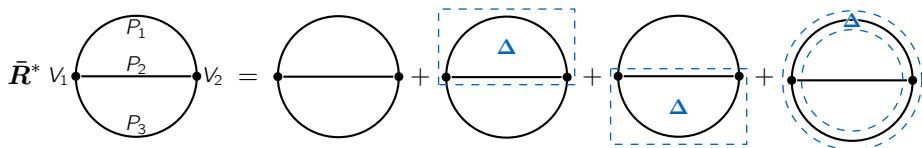
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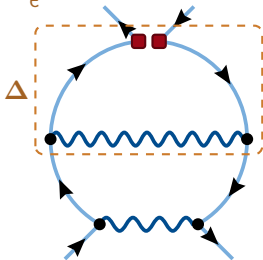
# Keeping track of the operator basis

|           |                                                                                   |
|-----------|-----------------------------------------------------------------------------------|
| Scheme    | $d$ -dimensional $\overline{\text{MS}}$<br>$\overline{d}$                         |
| Op. basis | $\{O_i\}$                                                                         |
| Examples  | $(\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma^\mu e)$<br>$(\bar{\ell}e)(\bar{e}\ell)$ |
| Cts.      | $\{\delta g_i\}$                                                                  |

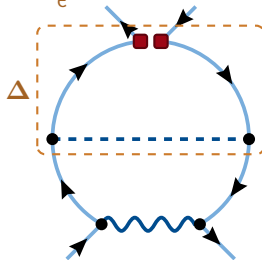
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$$\sim \frac{1}{\epsilon} (\bar{\ell}\gamma_\mu\gamma_\nu\gamma_\rho\ell)(\bar{e}\gamma_\rho\gamma_\nu\gamma^\mu e)$$



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$\bar{R}^*$  counterterm insertions not in the “correct” operator basis!

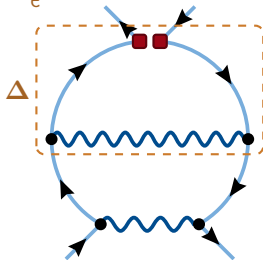
Naterop, Stoffer [2412.13251]; Born, Fuentes-Martín, AET [2601.19974]

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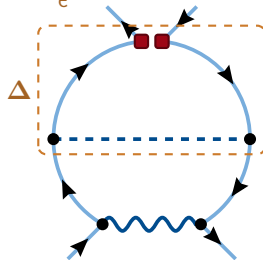
| Scehme    | $d$ -dimensional $\overline{\text{MS}}$<br>$\overline{d}$                         | phys./ev. $\overline{\text{MS}}$<br>$\overline{p}$                                                                                             |
|-----------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| Op. basis | $\{O_i\}$                                                                         | $\{Q_a, E_\alpha\}$                                                                                                                            |
| Examples  | $(\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma^\mu e)$<br>$(\bar{\ell}e)(\bar{e}\ell)$ | $(\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma^\mu e)$<br>$(\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma^\mu e) + \frac{1}{2}(\bar{\ell}e)(\bar{e}\ell)$ |
| Cts.      | $\{\delta g_i\}$                                                                  | $\{\delta\lambda_a, \delta\eta_\alpha\}$                                                                                                       |

$(1\text{-loop})^2$

$$\sim \frac{1}{\epsilon} (\bar{\ell}\gamma_\mu\gamma_\nu\gamma_\rho\ell)(\bar{e}\gamma_\rho\gamma_\nu\gamma^\mu e)$$



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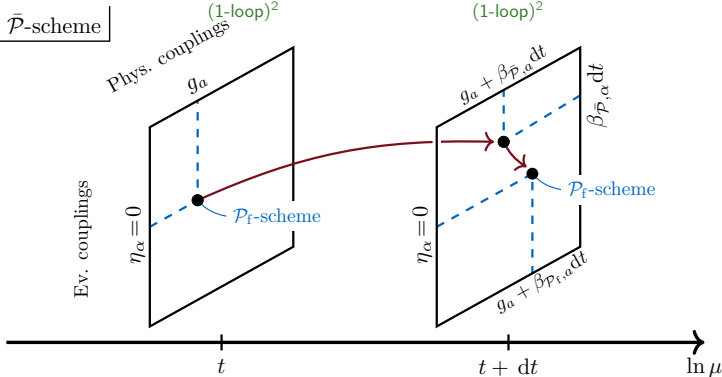
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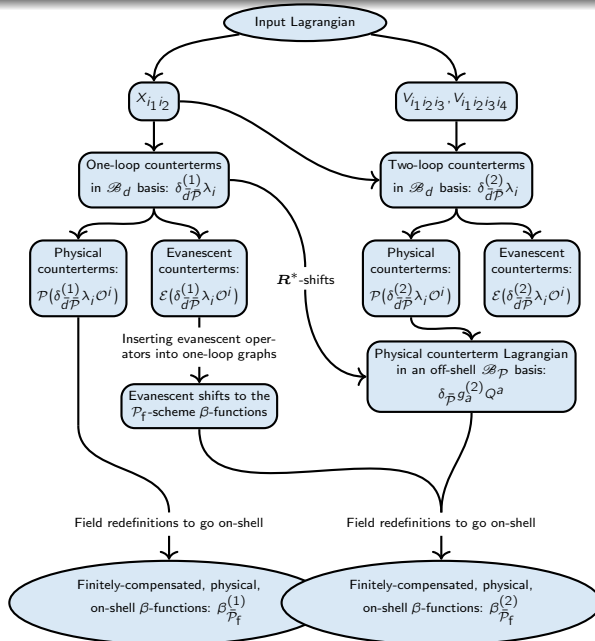
# Keeping track of the operator basis

| Scehme    | $d$ -dimensional $\overline{\text{MS}}$<br>$\overline{d}$                         | phys./ev. $\overline{\text{MS}}$<br>$\overline{p}$                                                                                             | finitely compensated ( $\overline{\text{MS}}$ )<br>$\overline{p}_f$                                                                            |
|-----------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| Op. basis | $\{O_i\}$                                                                         | $\{Q_a, E_\alpha\}$                                                                                                                            | $\{Q_a, E_\alpha\}$                                                                                                                            |
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| Cts.      | $\{\delta g_i\}$                                                                  | $\{\delta\lambda_a, \delta\eta_\alpha\}$                                                                                                       | $\{\delta\lambda_a + \Delta\lambda_a, \delta\eta_\alpha\}$                                                                                     |

$\overline{p}$ -scheme



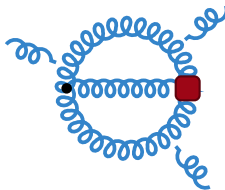
# Computation



~ 60 hours @ 1 core (dim-6 SMEFT)

# Fully automated counterterm calculation

Bespoke version  
of **MATCHETE**



```
L = FreeLag[] + cGt[] Gtilde;  
L // NiceForm
```

*iceForm=*

$$-\frac{1}{4} G^{\mu\nu X^2} + (D_\mu \bar{c}_G^X \cdot D_\mu c_G^X) + g G^{\mu\nu} (\bar{c}_G^X \cdot D_\mu c_G^Z) f^{XYZ} + \frac{1}{6} cGt G^{\mu\rho Y} G^{\nu\rho Z} G^{\sigma\kappa X} f^{XYZ} \epsilon^{\mu\nu\sigma\kappa}$$

```
ctlag = CountertermLagrangian[L, EFTOrder → 6] // EchoTiming;  
ctlag // NiceForm
```

» Added new CG cg1 with indices {SU3c[adj], SU3c[adj], SU3c[adj], SU3c[adj]}

» Added new CG cg2 with indices {SU3c[adj], SU3c[adj], SU3c[adj], SU3c[adj], SU3c[adj]}

13 695.1 — [Mathematica, single core](#)

*/NiceForm=*

$$-\frac{11}{4} \frac{\hbar}{\epsilon} g^2 G^{\mu\nu X^2} - \frac{51}{4} \frac{\hbar^2}{\epsilon} g^4 G^{\mu\nu X^2} - 3 \frac{\hbar}{\epsilon} cGt g^2 G^{\mu\nu X} G^{\rho\sigma Y} G^{\sigma\kappa Z} f^{XYZ} \epsilon^{\mu\nu\rho\kappa} -$$

$$\frac{21}{2} \frac{\hbar^2}{\epsilon^2} cGt g^4 G^{\mu\nu X} G^{\rho\sigma Y} G^{\sigma\kappa Z} f^{XYZ} \epsilon^{\mu\nu\rho\kappa} - \frac{425}{24} \frac{\hbar^2}{\epsilon} cGt g^4 G^{\mu\nu X} G^{\rho\sigma Y} G^{\sigma\kappa Z} f^{XYZ} \epsilon^{\mu\nu\rho\kappa}$$

Born, Fuentes-Martin, Kvedaraitė, **AET** [2410.07320]

# $\gamma_5$ -ambiguities and the limits of NDR

NDR assumptions:

$$(d = 4 - 2\epsilon)$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \quad \{\gamma_\mu, \gamma_5\} = 0, \quad \gamma_5^2 = \mathbb{1},$$
$$\text{Tr}[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5] \xrightarrow{\epsilon \rightarrow 0} -4i\epsilon_{\mu\nu\rho\sigma}.$$

**Reading point ambiguity** / loss of cyclicity (for  $\gamma_5$ -odd traces):

$$\text{Tr}[\gamma_\alpha \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma^\alpha \gamma_5] = 4(4 + 2\epsilon)i\epsilon_{\mu\nu\rho\sigma}$$
$$- \text{Tr}[\gamma^\alpha \gamma_\alpha \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5] = 4(4 - 2\epsilon)i\epsilon_{\mu\nu\rho\sigma}$$



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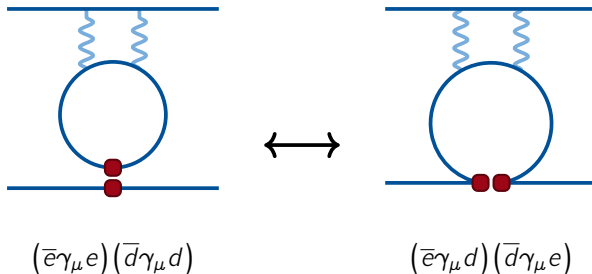


| Operator class                            | Problematic diagrams | Affected $\beta$ -functions                       |
|-------------------------------------------|----------------------|---------------------------------------------------|
| Reading point at the dimension-six vertex |                      |                                                   |
| Scalar $\psi^4$                           | —                    | —                                                 |
| Vector $\psi^4$                           | —                    | —                                                 |
| $\psi^2 H^2 D$                            | —                    | —                                                 |
| $\psi^2 H^3$                              | 1                    | CP-odd $X^2 H^2$                                  |
| $\psi^2 XH$                               | 11                   | $\psi^2 XH$ , CP-odd $X^2 H^2$ , and CP-odd $X^3$ |
| Averaging over vertex reading points      |                      |                                                   |
| Scalar $\psi^4$                           | 1                    | CP-odd $X^2 H^2$                                  |
| $\psi^2 XH$                               | 1                    | $\psi^2 XH$                                       |
| $H^2 X^2$                                 | 1                    | $\psi^2 H^3$                                      |
| $X^3$ and $H^4 D^2$                       | —                    | —                                                 |
| Weinberg operator                         | —                    | —                                                 |

# Example: how some ambiguities can be resolved

The reading-point for almost all four-fermion operators can be **verified through Fierzing** to another operator basis

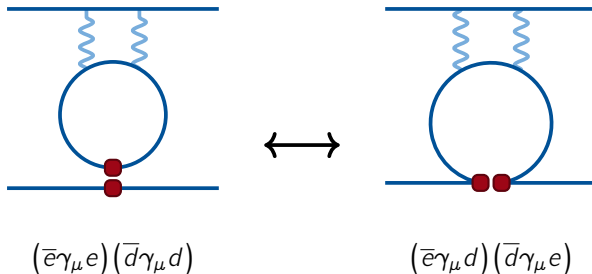
Strategy devised for the LEFT: Buras *et al.* [hep-ph/9211304]; Aebischer *et al.* [2501.08384]



# Example: how some ambiguities can be resolved

The reading-point for almost all four-fermion operators can be **verified through Fierzing** to another operator basis

Strategy devised for the LEFT: Buras *et al.* [hep-ph/9211304]; Aebischer *et al.* [2501.08384]



Fierzing tensor-current operators involves an ambiguous evanescent contribution causing the trick to fail: **we abandon the Warsaw basis**

# The “Mainz” basis

| $(\bar{L}L)(\bar{R}R)$      |                                                                   |
|-----------------------------|-------------------------------------------------------------------|
| $O_{\ell e}^{prst}$         | $(\bar{\ell}^p \gamma_\mu \ell^r) (\bar{e}^s \gamma^\mu e^t)$     |
| $O_{\ell u}^{prst}$         | $(\bar{\ell}^p \gamma_\mu \ell^r) (\bar{u}^s \gamma^\mu u^t)$     |
| $O_{\ell d}^{prst}$         | $(\bar{\ell}^p \gamma_\mu \ell^r) (\bar{d}^s \gamma^\mu d^t)$     |
| $O_{qe}^{prst}$             | $(\bar{q}^p \gamma_\mu q^r) (\bar{e}^s \gamma^\mu e^t)$           |
| $O_{qu}^{(\parallel) prst}$ | $(\bar{q}^p \gamma_\mu q^r) (\bar{u}^s \gamma^\mu u^t)$           |
| $O_{qu}^{(\times) prst}$    | $(\bar{q}_a^p \gamma_\mu q^{br}) (\bar{u}_b^s \gamma^\mu u^{at})$ |
| $O_{qd}^{(\parallel) prst}$ | $(\bar{q}^p \gamma_\mu q^r) (\bar{d}^s \gamma^\mu d^t)$           |
| $O_{qd}^{(\times) prst}$    | $(\bar{q}_a^p \gamma_\mu q^{br}) (\bar{d}_b^s \gamma^\mu d^{at})$ |

| $(\bar{R}R)(\bar{R}R)$      |                                                                   |
|-----------------------------|-------------------------------------------------------------------|
| $O_{ee}^{prst}$             | $(\bar{e}^p \gamma_\mu e^r) (\bar{e}^s \gamma^\mu e^t)$           |
| $O_{uu}^{prst}$             | $(\bar{u}^p \gamma_\mu u^r) (\bar{u}^s \gamma^\mu u^t)$           |
| $O_{dd}^{prst}$             | $(\bar{d}^p \gamma_\mu d^r) (\bar{d}^s \gamma^\mu d^t)$           |
| $O_{eu}^{prst}$             | $(\bar{e}^p \gamma_\mu e^r) (\bar{u}^s \gamma^\mu u^t)$           |
| $O_{ed}^{prst}$             | $(\bar{e}^p \gamma_\mu e^r) (\bar{d}^s \gamma^\mu d^t)$           |
| $O_{ud}^{(\parallel) prst}$ | $(\bar{u}^p \gamma_\mu u^r) (\bar{d}^s \gamma^\mu d^t)$           |
| $O_{ud}^{(\times) prst}$    | $(\bar{u}_a^p \gamma_\mu u^{br}) (\bar{d}_b^s \gamma^\mu d^{at})$ |

| $(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$ |                                                                    |
|---------------------------------------------------|--------------------------------------------------------------------|
| $O_{\ell edq}^{prst}$                             | $(\bar{\ell}_j^p e^r) (\bar{d}^s q_j^t)$                           |
| $O_{quqd}^{(\parallel) prst}$                     | $(\bar{q}_i^p u^r) \varepsilon^{ij} (\bar{q}_j^s d^t)$             |
| $O_{quqd}^{(\times) prst}$                        | $(\bar{q}_{ai}^p u^{br}) \varepsilon^{ij} (\bar{q}_{bj}^s d^{at})$ |
| $O_{\ell equ}^{prst}$                             | $(\bar{\ell}_i^p e^r) \varepsilon^{ij} (\bar{q}_j^s u^t)$          |
| $O_{\ell uqe}^{prst}$                             | $(\bar{\ell}_i^p u^r) \varepsilon^{ij} (\bar{q}_j^s e^t)$          |

| $(\bar{L}L)(\bar{L}L)$          |                                                                         |
|---------------------------------|-------------------------------------------------------------------------|
| $O_{\ell\ell}^{prst}$           | $(\bar{\ell}^p \gamma_\mu \ell^r) (\bar{\ell}^s \gamma^\mu \ell^t)$     |
| $O_{qq}^{(\parallel) prst}$     | $(\bar{q}^p \gamma_\mu q^r) (\bar{q}^s \gamma^\mu q^t)$                 |
| $O_{qq}^{(\times) prst}$        | $(\bar{q}_i^p \gamma_\mu q^{jr}) (\bar{q}_j^s \gamma^\mu q^{it})$       |
| $O_{\ell q}^{(\parallel) prst}$ | $(\bar{\ell}^p \gamma_\mu \ell^r) (\bar{q}^s \gamma^\mu q^t)$           |
| $O_{\ell q}^{(\times) prst}$    | $(\bar{\ell}_i^p \gamma_\mu \ell^{jr}) (\bar{q}_j^s \gamma^\mu q^{it})$ |

See also <https://github.com/NewsMEFTBasis/basis-proposal> and discussions in the LHC EFT Working Group

# The “Mainz” basis

| $(\bar{L}L)(\bar{R}R)$     |                                                                  | $(\bar{R}R)(\bar{R}R)$     |                                                        |
|----------------------------|------------------------------------------------------------------|----------------------------|--------------------------------------------------------|
| $O_{\ell e}^{prst}$        | $(\bar{\ell}^p \gamma_\mu \ell^r)(\bar{e}^s \gamma^\mu e^t)$     | $O_{ee}^{prst}$            | $(\bar{e}^p \gamma_\mu e^r)(\bar{e}^s \gamma^\mu e^t)$ |
| $O_{\ell u}^{prst}$        | $(\bar{\ell}^p \gamma_\mu \ell^r)(\bar{u}^s \gamma^\mu u^t)$     | $O_{uu}^{prst}$            | $(\bar{u}^p \gamma_\mu u^r)(\bar{u}^s \gamma^\mu u^t)$ |
| $O_{\ell d}^{prst}$        | $(\bar{\ell}^p \gamma_\mu \ell^r)(\bar{d}^s \gamma^\mu d^t)$     | $O_{dd}^{prst}$            | $(\bar{d}^p \gamma_\mu d^r)(\bar{d}^s \gamma^\mu d^t)$ |
| $O_{qe}^{prst}$            | $(\bar{q}^p \gamma_\mu q^r)(\bar{e}^s \gamma^\mu e^t)$           | $O_{eu}^{prst}$            | $(\bar{e}^p \gamma_\mu e^r)(\bar{u}^s \gamma^\mu u^t)$ |
| $O_{qu}^{(\parallel)prst}$ | $(\bar{q}^p \gamma_\mu q^r)(\bar{u}^s \gamma^\mu u^t)$           | $O_{ed}^{prst}$            | $(\bar{e}^p \gamma_\mu e^r)(\bar{d}^s \gamma^\mu d^t)$ |
| $O_{qu}^{(\times)prst}$    | $(\bar{q}_a^p \gamma_\mu q^{br})(\bar{u}_i^s \gamma^\mu u^{at})$ | $O_{ud}^{(\parallel)prst}$ | $(\bar{u}^p \gamma_\mu u^r)(\bar{d}^s \gamma^\mu d^t)$ |

Avoiding uncontrolled  $\gamma_5$  traces from tensor currents:

$$Q_{\ell equ}^{(3)} = (\bar{\ell}_i \sigma_{\mu\nu} e) \varepsilon^{ij} (\bar{q}_j \sigma^{\mu\nu} u) \longrightarrow O_{\ell uqe} = (\bar{\ell}_i u) \varepsilon^{ij} (\bar{q}_j e)$$

| $(LR)(RL)$ and $(LR)(LR)$    |                                                                    | $(LL)(LL)$                     |                                                                        |
|------------------------------|--------------------------------------------------------------------|--------------------------------|------------------------------------------------------------------------|
| $O_{\ell edq}^{prst}$        | $(\bar{\ell}_j^p e^r)(\bar{d}^s q_j^t)$                            | $O_{\ell\ell}^{prst}$          | $(\bar{\ell}^p \gamma_\mu \ell^r)(\bar{\ell}^s \gamma^\mu \ell^t)$     |
| $O_{quqd}^{(\parallel)prst}$ | $(\bar{q}_i^p u^r) \varepsilon^{ij} (\bar{q}_j^s d^t)$             | $O_{qq}^{(\parallel)prst}$     | $(\bar{q}^p \gamma_\mu q^r)(\bar{q}^s \gamma^\mu q^t)$                 |
| $O_{quqd}^{(\times)prst}$    | $(\bar{q}_{ai}^p u^{br}) \varepsilon^{ij} (\bar{q}_{bj}^s d^{at})$ | $O_{qq}^{(\times)prst}$        | $(\bar{q}_i^p \gamma_\mu q^{jr})(\bar{q}_j^s \gamma^\mu q^{it})$       |
| $O_{\ell equ}^{prst}$        | $(\bar{\ell}_i^p e^r) \varepsilon^{ij} (\bar{q}_j^s u^t)$          | $O_{\ell q}^{(\parallel)prst}$ | $(\bar{\ell}^p \gamma_\mu \ell^r)(\bar{q}^s \gamma^\mu q^t)$           |
| $O_{\ell uqe}^{prst}$        | $(\bar{\ell}_i^p u^r) \varepsilon^{ij} (\bar{q}_j^s e^t)$          | $O_{\ell q}^{(\times)prst}$    | $(\bar{\ell}_i^p \gamma_\mu \ell^{jr})(\bar{q}_j^s \gamma^\mu q^{it})$ |

See also <https://github.com/NewSMEFTBasis/basis-proposal> and discussions in the LHC EFT Working Group

# The “Mainz” basis

| $X^3$               |                                                                      |
|---------------------|----------------------------------------------------------------------|
| $O_W$               | $g_L^3 f^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$             |
| $O_G$               | $g_s^3 f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$             |
| $O_{\widetilde{W}}$ | $g_L^3 f^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |
| $O_{\widetilde{G}}$ | $g_s^3 f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$ |

| $H^4 D^2$ and $H^6$    |                                          |
|------------------------|------------------------------------------|
| $O_H$                  | $(H^\dagger H)^3$                        |
| $O_{HD}^{(\parallel)}$ | $(H^\dagger H)(D_\mu H^\dagger D^\mu H)$ |
| $O_{HD}^{(\times)}$    | $H_i^* H^j D_\mu H_j^* D^\mu H^i$        |

| $\psi^2 H^3$  |                                              |
|---------------|----------------------------------------------|
| $O_{eH}^{pr}$ | $(H^\dagger H)(\bar{\ell}^p e^r H)$          |
| $O_{uH}^{pr}$ | $(H^\dagger H)(\bar{q}^p u^r \widetilde{H})$ |
| $O_{dH}^{pr}$ | $(H^\dagger H)(\bar{q}^p d^r H)$             |

| $X^2 H^2$             |                                                                 |
|-----------------------|-----------------------------------------------------------------|
| $O_{HG}$              | $g_s^2 (H^\dagger H) G^{A\mu\nu} G_\mu^A G_\nu^A$               |
| $O_{H\widetilde{G}}$  | $g_s^2 (H^\dagger H) G^{A\mu\nu} \widetilde{G}_\mu^A G_\nu^A$   |
| $O_{HW}$              | $g_L^2 (H^\dagger H) W^{I\mu\nu} W_\mu^I W_\nu^I$               |
| $O_{H\widetilde{W}}$  | $g_L^2 (H^\dagger H) W^{I\mu\nu} \widetilde{W}_\mu^I W_\nu^I$   |
| $O_{HB}$              | $g_Y^2 (H^\dagger H) B^{\mu\nu} B_{\mu\nu}$                     |
| $O_{H\widetilde{B}}$  | $g_Y^2 (H^\dagger H) B^{\mu\nu} \widetilde{B}_{\mu\nu}$         |
| $O_{HWB}$             | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} W_{\mu\nu}^I$             |
| $O_{H\widetilde{W}B}$ | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} \widetilde{W}_{\mu\nu}^I$ |

| $\psi^2 X H$  |                                                                      |
|---------------|----------------------------------------------------------------------|
| $O_{eW}^{pr}$ | $g_L (\bar{\ell}^p \sigma^{\mu\nu} e^r t^I H) W_{\mu\nu}^I$          |
| $O_{eB}^{pr}$ | $g_Y (\bar{\ell}^p \sigma^{\mu\nu} e^r H) B_{\mu\nu}$                |
| $O_{uG}^{pr}$ | $g_s (\bar{q}^p \sigma^{\mu\nu} T^A u^r \widetilde{H}) G_{\mu\nu}^A$ |
| $O_{uW}^{pr}$ | $g_L (\bar{q}^p \sigma^{\mu\nu} u^r t^I \widetilde{H}) W_{\mu\nu}^I$ |
| $O_{uB}^{pr}$ | $g_Y (\bar{q}^p \sigma^{\mu\nu} u^r \widetilde{H}) B_{\mu\nu}$       |
| $O_{dG}^{pr}$ | $g_s (\bar{q}^p \sigma^{\mu\nu} T^A d^r H) G_{\mu\nu}^A$             |
| $O_{dW}^{pr}$ | $g_L (\bar{q}^p \sigma^{\mu\nu} d^r t^I H) W_{\mu\nu}^I$             |
| $O_{dB}^{pr}$ | $g_Y (\bar{q}^p \sigma^{\mu\nu} d^r H) B_{\mu\nu}$                   |

| $\psi^2 H^2 D$              |                                                                                |
|-----------------------------|--------------------------------------------------------------------------------|
| $O_{H\ell}^{(\parallel)pr}$ | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{\ell}^p \gamma^\mu \ell^r)$    |
| $O_{H\ell}^{(\times)pr}$    | $i(H_i^* \overleftrightarrow{D}_\mu H^j)(\bar{\ell}_j^p \gamma^\mu \ell^{ir})$ |
| $O_{He}^{pr}$               | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}^p \gamma^\mu e^r)$          |
| $O_{Hq}^{(\parallel)pr}$    | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{q}^p \gamma^\mu q^r)$          |
| $O_{Hq}^{(\times)pr}$       | $i(H_i^* \overleftrightarrow{D}_\mu H^j)(\bar{q}_j^p \gamma^\mu q^{ir})$       |
| $O_{Hu}^{pr}$               | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}^p \gamma^\mu u^r)$          |
| $O_{Hd}^{pr}$               | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}^p \gamma^\mu d^r)$          |
| $O_{Hud}^{pr}$              | $i(\widetilde{H}^\dagger D_\mu H)(\bar{u}^p \gamma^\mu d^r)$                   |

See also <https://github.com/NewSMEFTBasis/basis-proposal> and discussions in the LHC EFT Working Group

# The “Mainz” basis

| $X^3$               |                                                                      |
|---------------------|----------------------------------------------------------------------|
| $O_W$               | $g_L^3 f^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$             |
| $O_G$               | $g_s^3 f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$             |
| $O_{\widetilde{W}}$ | $g_L^3 f^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |
| $O_{\widetilde{G}}$ | $g_s^3 f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$ |

| $H^4 D^2$ and $H^6$    |                                           |
|------------------------|-------------------------------------------|
| $O_H$                  | $(H^\dagger H)^3$                         |
| $O_{HD}^{(\parallel)}$ | $(H^\dagger H) (D_\mu H^\dagger D^\mu H)$ |
| $O_{HD}^{(\times)}$    | $H_i^* H^j D_\mu H_j^* D^\mu H^i$         |

| $\psi^2 H^3$  |                                               |
|---------------|-----------------------------------------------|
| $O_{eH}^{pr}$ | $(H^\dagger H) (\bar{\ell}^p e^r H)$          |
| $O_{uH}^{pr}$ | $(H^\dagger H) (\bar{q}^p u^r \widetilde{H})$ |
| $O_{dH}^{pr}$ | $(H^\dagger H) (\bar{q}^p d^r H)$             |

| Remove derivatives acting on multiple fields |                                                                                                                                  |                                                                                                |
|----------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| $O_{HG}$                                     | $g_s^2 (H^\dagger H) G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$                                                                   | $O_{eW}^{pr} (\bar{\ell}^p \sigma^{\mu\nu} e^r t^I \widetilde{H}) W_{\mu\nu}^I$                |
| $O_{H\widetilde{G}}$                         | $Q_{\square H} = (H^\dagger H) \square (H^\dagger H) \rightarrow O_{HD}^{(\parallel)} = (H^\dagger H) (D_\mu H^\dagger D^\mu H)$ | $O_{uH}^{pr} (\bar{q}^p \sigma^{\mu\nu} u^r t^I \widetilde{H}) W_{\mu\nu}^I$                   |
| $O_{HW}$                                     | $g_L^2 (H^\dagger H) W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$                                                                   | $O_{dH}^{pr} (\bar{q}^p \sigma^{\mu\nu} d^r t^I H) W_{\mu\nu}^I$                               |
| $O_{H\widetilde{W}}$                         | $g_L^2 (H^\dagger H) W_\mu^{I\nu} \widetilde{W}_\nu^{J\rho} W_\rho^{K\mu}$                                                       | $O_{uq}^{(\parallel)pr} i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{q}^p \gamma^\mu q^r)$ |
| $O_{HB}$                                     | $g_Y^2 (H^\dagger H) B^{\mu\nu} B_{\mu\nu}$                                                                                      | $O_{uB}^{pr} g_Y (\bar{q}^p \sigma^{\mu\nu} u^r \widetilde{H}) B_{\mu\nu}$                     |
| $O_{H\widetilde{B}}$                         | $g_Y^2 (H^\dagger H) B^{\mu\nu} \widetilde{B}_{\mu\nu}$                                                                          | $O_{dq}^{(\times)pr} i (H_i^* \overleftrightarrow{D}_\mu H^j) (\bar{q}_j^p \gamma^\mu q^{ir})$ |
| $O_{HWB}$                                    | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} W_{\mu\nu}^I$                                                                              | $O_{Hu}^{pr} i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{u}^p \gamma^\mu u^r)$            |
| $O_{H\widetilde{WB}}$                        | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} \widetilde{W}_\nu^{J\rho} W_\rho^{K\mu}$                                                   | $O_{Hd}^{pr} i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{d}^p \gamma^\mu d^r)$            |
|                                              | $O_{dW}^{pr} g_L (\bar{q}^p \sigma^{\mu\nu} d^r t^I H) W_{\mu\nu}^I$                                                             | $O_{Hud}^{pr} i (\widetilde{H}^\dagger D_\mu H) (\bar{u}^p \gamma^\mu d^r)$                    |
|                                              | $O_{dB}^{pr} g_Y (\bar{q}^p \sigma^{\mu\nu} d^r H) B_{\mu\nu}$                                                                   |                                                                                                |

See also <https://github.com/NewSMEFTBasis/basis-proposal> and discussions in the LHC EFT Working Group

# The “Mainz” basis

| $X^3$               |                                                                      |
|---------------------|----------------------------------------------------------------------|
| $O_W$               | $g_L^3 f^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$             |
| $O_G$               | $g_s^3 f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$             |
| $O_{\widetilde{W}}$ | $g_L^3 f^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |
| $O_{\widetilde{G}}$ | $g_s^3 f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$ |

| $H^4 D^2$ and $H^6$    |                                          |
|------------------------|------------------------------------------|
| $O_H$                  | $(H^\dagger H)^3$                        |
| $O_{HD}^{(\parallel)}$ | $(H^\dagger H)(D_\mu H^\dagger D^\mu H)$ |
| $O_{HD}^{(\times)}$    | $H_i^* H^j D_\mu H_j^* D^\mu H^i$        |

| $\psi^2 H^3$  |                                              |
|---------------|----------------------------------------------|
| $O_{eH}^{pr}$ | $(H^\dagger H)(\bar{\ell}^p e^r H)$          |
| $O_{uH}^{pr}$ | $(H^\dagger H)(\bar{q}^p u^r \widetilde{H})$ |
| $O_{dH}^{pr}$ | $(H^\dagger H)(\bar{q}^p d^r H)$             |

| $X^2 H^2$             |                                                                 |
|-----------------------|-----------------------------------------------------------------|
| $O_{HG}$              | $g_s^2 (H^\dagger H) G^{A\mu\nu} G_{\mu\nu}^A$                  |
| $O_{H\widetilde{G}}$  | $g_s^2 (H^\dagger H) G^{A\mu\nu} \widetilde{G}_{\mu\nu}^A$      |
| $O_{HW}$              | $g_L^2 (H^\dagger H) W^{I\mu\nu} W_{\mu\nu}^I$                  |
| $O_{H\widetilde{W}}$  | $g_L^2 (H^\dagger H) W^{I\mu\nu} \widetilde{W}_{\mu\nu}^I$      |
| $O_{HB}$              | $g_Y^2 (H^\dagger H) B^{\mu\nu} B_{\mu\nu}$                     |
| $O_{H\widetilde{B}}$  | $g_Y^2 (H^\dagger H) B^{\mu\nu} \widetilde{B}_{\mu\nu}$         |
| $O_{HWB}$             | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} W_{\mu\nu}^I$             |
| $O_{H\widetilde{W}B}$ | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} \widetilde{W}_{\mu\nu}^I$ |

| $\psi^2 X H$  |                                                                      |
|---------------|----------------------------------------------------------------------|
| $O_{eW}^{pr}$ | $g_L (\bar{\ell}^p \sigma^{\mu\nu} e^r t^I H) W_{\mu\nu}^I$          |
| $O_{eB}^{pr}$ | $g_Y (\bar{\ell}^p \sigma^{\mu\nu} e^r H) B_{\mu\nu}$                |
| $O_{uG}^{pr}$ | $g_s (\bar{q}^p \sigma^{\mu\nu} T^A u^r \widetilde{H}) G_{\mu\nu}^A$ |
| $O_{uW}^{pr}$ | $g_L (\bar{q}^p \sigma^{\mu\nu} u^r t^I \widetilde{H}) W_{\mu\nu}^I$ |
| $O_{uB}^{pr}$ | $g_Y (\bar{q}^p \sigma^{\mu\nu} u^r \widetilde{H}) B_{\mu\nu}$       |
| $O_{dG}^{pr}$ | $g_s (\bar{q}^p \sigma^{\mu\nu} T^A d^r H) G_{\mu\nu}^A$             |
| $O_{dW}^{pr}$ | $g_L (\bar{q}^p \sigma^{\mu\nu} d^r t^I H) W_{\mu\nu}^I$             |
| $O_{dB}^{pr}$ | $g_Y (\bar{q}^p \sigma^{\mu\nu} d^r H) B_{\mu\nu}$                   |

| $\psi^2 H^2 D$              |                                                                                |
|-----------------------------|--------------------------------------------------------------------------------|
| $O_{H\ell}^{(\parallel)pr}$ | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{\ell}^p \gamma^\mu \ell^r)$    |
| $O_{H\ell}^{(\times)pr}$    | $i(H_i^* \overleftrightarrow{D}_\mu H^j)(\bar{\ell}_j^p \gamma^\mu \ell^{ir})$ |
| $O_{He}^{pr}$               | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}^p \gamma^\mu e^r)$          |
| $O_{Hq}^{(\parallel)pr}$    | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{q}^p \gamma^\mu q^r)$          |
| $O_{Hq}^{(\times)pr}$       | $i(H_i^* \overleftrightarrow{D}_\mu H^j)(\bar{q}_j^p \gamma^\mu q^{ir})$       |
| $O_{Hu}^{pr}$               | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}^p \gamma^\mu u^r)$          |
| $O_{Hd}^{pr}$               | $i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}^p \gamma^\mu d^r)$          |
| $O_{Hud}^{pr}$              | $i(\widetilde{H}^\dagger D_\mu H)(\bar{u}^p \gamma^\mu d^r)$                   |

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# The “Mainz” basis

| $X^3$               |                                                                      |
|---------------------|----------------------------------------------------------------------|
| $O_W$               | $g_L^3 f^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$             |
| $O_G$               | $g_s^3 f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$             |
| $O_{\widetilde{W}}$ | $g_L^3 f^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |
| $O_{\widetilde{G}}$ | $g_s^3 f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$ |

| $H^4 D^2$ and $H^6$    |                                           |
|------------------------|-------------------------------------------|
| $O_H$                  | $(H^\dagger H)^3$                         |
| $O_{HD}^{(\parallel)}$ | $(H^\dagger H) (D_\mu H^\dagger D^\mu H)$ |
| $O_{HD}^{(\times)}$    | $H_i^* H^j D_\mu H_j^* D^\mu H^i$         |

| $\psi^2 H^3$  |                                               |
|---------------|-----------------------------------------------|
| $O_{eH}^{pr}$ | $(H^\dagger H) (\bar{\ell}^p e^r H)$          |
| $O_{uH}^{pr}$ | $(H^\dagger H) (\bar{q}^p u^r \widetilde{H})$ |
| $O_{dH}^{pr}$ | $(H^\dagger H) (\bar{q}^p d^r H)$             |

| $X^2 H^2$             |                                                                 |
|-----------------------|-----------------------------------------------------------------|
| $O_{HG}$              | $g_s (H^\dagger H) G_{\mu\nu}^A G^{\mu\nu A}$                   |
| $O_{H\widetilde{G}}$  | $g_s^2 (H^\dagger H) G_{\mu\nu}^{AB} \widetilde{G}_{\mu\nu}^A$  |
| $O_{HW}$              | $g_L^2 (H^\dagger H) W_{\mu\nu}^I W^{\mu\nu I}$                 |
| $O_{H\widetilde{W}}$  | $g_L^2 (H^\dagger H) W_{\mu\nu}^I \widetilde{W}_{\mu\nu}^I$     |
| $O_{HB}$              | $g_Y^2 (H^\dagger H) B^{\mu\nu} B_{\mu\nu}$                     |
| $O_{H\widetilde{B}}$  | $g_Y^2 (H^\dagger H) B^{\mu\nu} \widetilde{B}_{\mu\nu}$         |
| $O_{HWB}$             | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} W_{\mu\nu}^I$             |
| $O_{H\widetilde{WB}}$ | $g_Y g_L (H^\dagger t^I H) B^{\mu\nu} \widetilde{W}_{\mu\nu}^I$ |

| $\psi^2 X H$  |                                                                      |
|---------------|----------------------------------------------------------------------|
| $O_{eW}$      | $g_L (\bar{\ell}^p \sigma^{\mu\nu} t^I \ell^r) W_{\mu\nu}^I$         |
| $O_{uG}^{pr}$ | $g_s (\bar{q}^p \sigma^{\mu\nu} T^A u^r H) G_{\mu\nu}^A$             |
| $O_{uW}^{pr}$ | $g_L (\bar{q}^p \sigma^{\mu\nu} u^r t^I \widetilde{H}) W_{\mu\nu}^I$ |
| $O_{uB}^{pr}$ | $g_Y (\bar{q}^p \sigma^{\mu\nu} u^r \widetilde{H}) B_{\mu\nu}$       |
| $O_{dG}^{pr}$ | $g_s (\bar{q}^p \sigma^{\mu\nu} T^A d^r H) G_{\mu\nu}^A$             |
| $O_{dW}^{pr}$ | $g_L (\bar{q}^p \sigma^{\mu\nu} d^r t^I H) W_{\mu\nu}^I$             |
| $O_{dB}^{pr}$ | $g_Y (\bar{q}^p \sigma^{\mu\nu} d^r H) B_{\mu\nu}$                   |

| $\psi^2 H^2 D$           |                                                                                        |
|--------------------------|----------------------------------------------------------------------------------------|
| $O_{He}^{(\parallel)pr}$ | $i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{\ell}^p \gamma^\mu \ell^r)$          |
| $O_{He}^{(\times)pr}$    | $i (H_i^\dagger \overleftrightarrow{D}_\mu H^j) (\bar{\ell}_j^p \gamma^\mu \ell^{ir})$ |
| $O_{He}^{pr}$            | $i (H^\dagger D_\mu H) (\bar{e}^p \gamma^\mu e^r)$                                     |
| $O_{Hq}^{(\parallel)pr}$ | $i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{q}^p \gamma^\mu q^r)$                |
| $O_{Hq}^{(\times)pr}$    | $i (H_i^\dagger \overleftrightarrow{D}_\mu H^j) (\bar{q}_j^p \gamma^\mu q^{ir})$       |
| $O_{Hu}^{pr}$            | $i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{u}^p \gamma^\mu u^r)$                |
| $O_{Hd}^{pr}$            | $i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{d}^p \gamma^\mu d^r)$                |
| $O_{Hud}^{pr}$           | $i (\widetilde{H}^\dagger D_\mu H) (\bar{u}^p \gamma^\mu d^r)$                         |

Canonical gauge coupling normalization:

$$X_{\mu\nu} \longrightarrow g_X X_{\mu\nu}$$

See also <https://github.com/NewSMEFTBasis/basis-proposal> and discussions in the LHC EFT Working Group

$$\begin{aligned}
 \beta_{C_{d\bar{B}}^{pr(2)}}^{(2)} = & \left\{ \frac{1}{72} g_Y^2 Y_d^{pr} + \frac{1}{8} (Y_d Y_u^\dagger Y_d)^{pr} + \frac{1}{4} g_L^2 Y_d^{pr} - \frac{1}{3} (Y_u Y_u^\dagger Y_d)^{pr} \right\} C_{HD}^{(ll)} + \left\{ \frac{1}{3} (Y_u Y_u^\dagger Y_d)^{pr} - \frac{5}{144} g_Y^2 Y_d^{pr} - \frac{1}{8} (Y_d Y_d^\dagger Y_d)^{pr} - \frac{3}{16} g_L^2 Y_d^{pr} \right\} C_{HD}^{(x)} \\
 & + \left\{ \frac{29}{108} g_Y^4 Y_d^{pr} + \frac{1}{2} \lambda g_Y^2 Y_d^{pr} + g_Y^2 \left( \frac{1}{12} (Y_u Y_u^\dagger Y_d)^{pr} + \frac{1}{6} T_2 Y_d^{pr} + \frac{1}{4} (Y_d Y_d^\dagger Y_d)^{pr} \right) + \frac{8}{3} g_Y^2 g_s^2 Y_d^{pr} - \frac{1}{8} g_Y^2 g_L^2 Y_d^{pr} \right\} C_{HB} + \left\{ \frac{i}{8} g_Y^2 g_L^2 Y_d^{pr} \right. \\
 & + g_Y^2 \left( \frac{19i}{12} (Y_u Y_u^\dagger Y_d)^{pr} - \frac{i}{6} T_2 Y_d^{pr} - \frac{i}{4} (Y_d Y_d^\dagger Y_d)^{pr} \right) - \frac{29i}{108} g_Y^4 Y_d^{pr} - \frac{i}{2} \lambda g_Y^2 Y_d^{pr} - \frac{8i}{3} g_Y^2 g_s^2 Y_d^{pr} \left. \right\} C_{H\bar{B}} + \frac{1}{8} g_L^4 C_{HW} Y_d^{pr} - \frac{i}{8} g_L^4 C_{H\bar{W}} Y_d^{pr} \\
 & + \frac{4}{3} g_s^4 C_{HG} Y_d^{pr} - \frac{4i}{3} g_s^4 C_{H\bar{G}} Y_d^{pr} + \left\{ \frac{9}{32} g_Y^2 g_L^2 Y_d^{pr} + \frac{425}{96} g_L^4 Y_d^{pr} - \frac{3}{8} \lambda g_L^2 Y_d^{pr} - g_L^2 \left( \frac{3}{8} T_2 Y_d^{pr} + \frac{9}{16} (Y_d Y_d^\dagger Y_d)^{pr} + \frac{15}{16} (Y_u Y_u^\dagger Y_d)^{pr} \right) \right. \\
 & - 6 g_s^2 g_L^2 Y_d^{pr} \left. \right\} C_{HWB} + \left\{ \frac{3i}{8} \lambda g_L^2 Y_d^{pr} + g_L^2 \left( \frac{3i}{16} (Y_u Y_u^\dagger Y_d)^{pr} + \frac{3i}{8} T_2 Y_d^{pr} + \frac{9i}{16} (Y_d Y_d^\dagger Y_d)^{pr} \right) + 6i g_s^2 g_L^2 Y_d^{pr} - \frac{9i}{32} g_Y^2 g_L^2 Y_d^{pr} \right. \\
 & - \frac{425i}{96} g_L^4 Y_d^{pr} \left. \right\} C_{H\bar{W}\bar{B}} - \frac{21}{4} g_L^6 C_W Y_d^{pr} + \frac{35i}{12} g_L^6 C_{\bar{W}} Y_d^{pr} + 2 g_s^6 C_G Y_d^{pr} - \frac{26i}{9} g_s^6 C_{\bar{G}} Y_d^{pr} + \left\{ \frac{3}{8} \delta^{pa} \delta^{rb} g_Y^2 - \frac{1}{8} \delta^{pa} \delta^{rb} g_Y^2 - \frac{1}{8} \delta^{pa} (Y_d^\dagger Y_d)^{br} \right. \\
 & - \frac{1}{8} \delta^{rb} (Y_d Y_d^\dagger)^{pa} \left. \right\} C_{dH}^{ab} + \left\{ \frac{5}{4} g_Y^2 Y_e^{*ab} Y_d^{pr} + \frac{9}{4} g_L^2 Y_e^{*ab} Y_d^{pr} \right\} C_{eB}^{ab} - \frac{9}{4} g_L^2 Y_e^{*ab} C_{eW}^{ab} Y_d^{pr} + \left\{ \frac{1}{2} T_2 \delta^{pa} (Y_u^\dagger Y_d)^{br} + g_L^2 \left( \frac{9}{4} Y_u^{*ab} Y_d^{pr} - \frac{39}{8} \delta^{pa} (Y_u^\dagger Y_d)^{br} \right) \right. \\
 & + g_Y^2 \left( \frac{5}{12} Y_u^{*ab} Y_d^{pr} - \frac{11}{24} \delta^{pa} (Y_u^\dagger Y_d)^{br} \right) + \frac{5}{2} (Y_u^\dagger Y_d)^{br} (Y_u Y_u^\dagger)^{pa} + \frac{5}{2} \delta^{pa} (Y_u^\dagger Y_u Y_u^\dagger Y_d)^{br} - \frac{1}{2} (Y_u^\dagger Y_d)^{br} (Y_d Y_d^\dagger)^{pa} - \delta^{pa} (Y_u^\dagger Y_d^\dagger Y_d)^{br} \left. \right\} C_{uB}^{ab} \\
 & + C_{uB}^{*ab} \left\{ \frac{1}{2} Y_u^{pb} (Y_u Y_u^\dagger Y_d)^{ar} + T_2 Y_d^{ar} Y_u^{pb} + g_Y^2 \left( \frac{5}{12} Y_d^{pr} Y_u^{ab} - \frac{2}{9} Y_d^{ar} Y_u^{pb} \right) + \frac{5}{2} Y_d^{ar} (Y_u Y_u^\dagger Y_u)^{pb} - Y_u^{pb} (Y_d Y_d^\dagger Y_d)^{ar} - 2 Y_d^{ar} (Y_d Y_d^\dagger Y_u)^{pb} - \frac{9}{4} g_L^2 Y_d^{ar} Y_u^{pb} \right. \\
 & - 8 g_s^2 Y_d^{ar} Y_u^{pb} \left. \right\} - \frac{5}{8} \delta^{pa} g_L^2 C_{uW}^{ab} (Y_u^\dagger Y_d)^{br} + g_L^2 C_{uW}^{*ab} \left\{ \frac{15}{8} Y_d^{pr} Y_u^{ab} - \frac{1}{8} Y_d^{ar} Y_u^{pb} \right\} + \frac{8}{3} \delta^{pa} g_s^2 C_{uG}^{ab} (Y_u^\dagger Y_d)^{br} + \frac{44}{9} g_s^2 C_{uG}^{*ab} Y_d^{ar} Y_u^{pb} \\
 & + \left\{ g_L^2 \left( \frac{9}{16} \delta^{rb} (Y_u Y_u^\dagger)^{pa} + \frac{15}{8} T_2 \delta^{pa} \delta^{rb} + \frac{27}{4} Y_d^{*ab} Y_d^{pr} + 9 \delta^{pa} (Y_d^\dagger Y_d)^{br} + \frac{237}{16} \delta^{rb} (Y_d Y_d^\dagger)^{pa} \right) + g_Y^2 \left( \frac{5}{12} Y_d^{*ab} Y_d^{pr} + \frac{601}{144} \delta^{rb} (Y_d Y_d^\dagger)^{pa} \right) \right. \\
 & + \frac{349}{72} \delta^{pa} (Y_d^\dagger Y_d)^{br} + \frac{39}{8} \delta^{pa} \delta^{rb} \text{Tr} [Y_e^\dagger Y_e] - \frac{3}{8} \delta^{pa} \delta^{rb} \text{Tr} [Y_d^\dagger Y_d] - \frac{101}{144} \delta^{rb} (Y_u Y_u^\dagger)^{pa} - \frac{29}{8} \delta^{pa} \delta^{rb} \text{Tr} [Y_u^\dagger Y_u] \left. \right) + g_s^2 \left( 20 \delta^{pa} \delta^{rb} \text{Tr} [Y_d^\dagger Y_d] \right. \\
 & + 20 \delta^{pa} \delta^{rb} \text{Tr} [Y_u^\dagger Y_u] - \frac{16}{3} \delta^{rb} (Y_d Y_d^\dagger)^{pa} - \frac{16}{3} \delta^{rb} (Y_u Y_u^\dagger)^{pa} - \frac{32}{3} \delta^{pa} (Y_d^\dagger Y_d)^{br} \left. \right) + \frac{5}{4} T_2 \delta^{rb} (Y_u Y_u^\dagger)^{pa} + \frac{5}{4} \delta^{pa} (Y_d^\dagger Y_d Y_d^\dagger Y_d)^{br} + \frac{5}{4} \delta^{rb} (Y_d Y_d^\dagger Y_d Y_d^\dagger)^{pa} \\
 & + \frac{3}{2} \delta^{pa} \delta^{rb} \lambda^2 + \frac{3}{2} \delta^{pa} \delta^{rb} \text{Tr} [Y_d^\dagger Y_d Y_d^\dagger Y_u] + \frac{103}{48} \delta^{pa} \delta^{rb} g_Y^2 g_L^2 + \frac{1}{4} \delta^{rb} (Y_u Y_u^\dagger Y_u Y_u^\dagger)^{pa} + \frac{172}{3} \delta^{pa} \delta^{rb} g_s^4 - \frac{1}{4} \delta^{pa} (Y_d^\dagger Y_u Y_u^\dagger Y_d)^{br} \\
 & - \frac{1}{2} \delta^{rb} (Y_d Y_d^\dagger Y_u Y_u^\dagger)^{pa} - \lambda \left( \frac{3}{2} \delta^{pa} \delta^{rb} g_Y^2 + 6 \delta^{pa} (Y_d^\dagger Y_d)^{br} + 6 \delta^{rb} (Y_d Y_d^\dagger)^{pa} \right) - \delta^{rb} (Y_u Y_u^\dagger Y_d Y_d^\dagger)^{pa} - \frac{31}{27} \delta^{pa} \delta^{rb} g_Y^2 g_s^2 - \frac{1577}{1296} \delta^{pa} \delta^{rb}
 \end{aligned}$$

+ another  
296 pages

## Results: interactive notebook

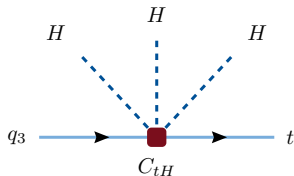
[illegible]

# Modified Top-Yukawa

---

/w Born and Greljo: [arXiv:2604.26817]

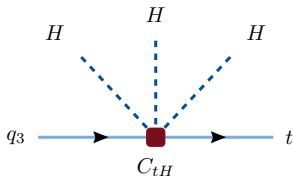
# Top-Yukawa feeding into EWPO @ 2 loops



$$\delta\kappa_t = -\frac{v^2 C_{tH}}{y_t}$$

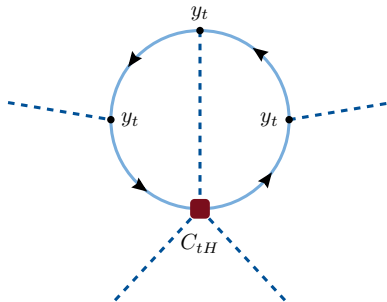
$$|\delta\kappa_t| = \begin{cases} 0.11 & \text{ATLAS, CMS} \\ 0.034 & \text{HL-LHC (proj.)} \end{cases}$$

# Top-Yukawa feeding into EWPO @ 2 loops



$$\delta\kappa_t = -\frac{v^2 C_{tH}}{y_t}$$

$$|\delta\kappa_t| = \begin{cases} 0.11 & \text{ATLAS, CMS} \\ 0.034 & \text{HL-LHC (proj.)} \end{cases}$$



$$\frac{dC_{HD}^{(\times)}}{d\ln\mu} \supset -\frac{18}{(4\pi)^4} |y_t|^2 \text{Re}[y_t^* C_{tH}]$$

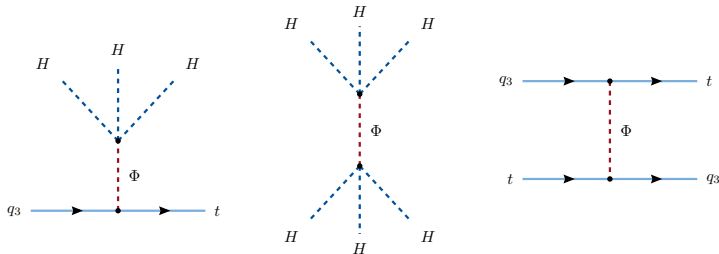
$$\delta m_W = 16.1 \text{ MeV} \cdot \delta\kappa_t \ln \frac{\Lambda}{m_Z}$$

$$|\delta m_W| = \begin{cases} 10 \text{ MeV} & \text{CMS} \\ 0.26 \text{ MeV} & \text{FCC-ee (proj.)} \end{cases}$$

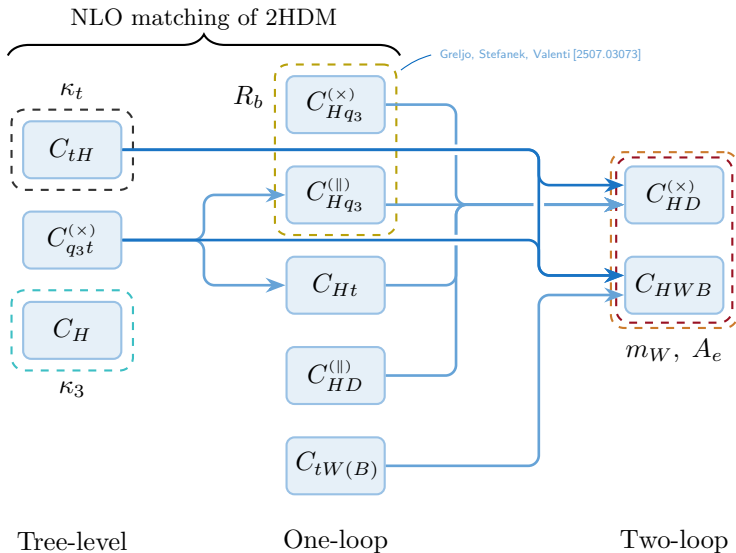
UV realization of the  $C_{tH} \rightarrow C_{HD}^{(\times)}$  effect:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin}}^\Phi - \left[ \lambda_\Phi (\Phi^\dagger H)(H^\dagger H) + y_\Phi \bar{q}_3 \tilde{\Phi} t_R + \text{H.c.} \right] + \dots$$

Integrating the heavy Higgs  $\Phi$  out in the decoupling limit:

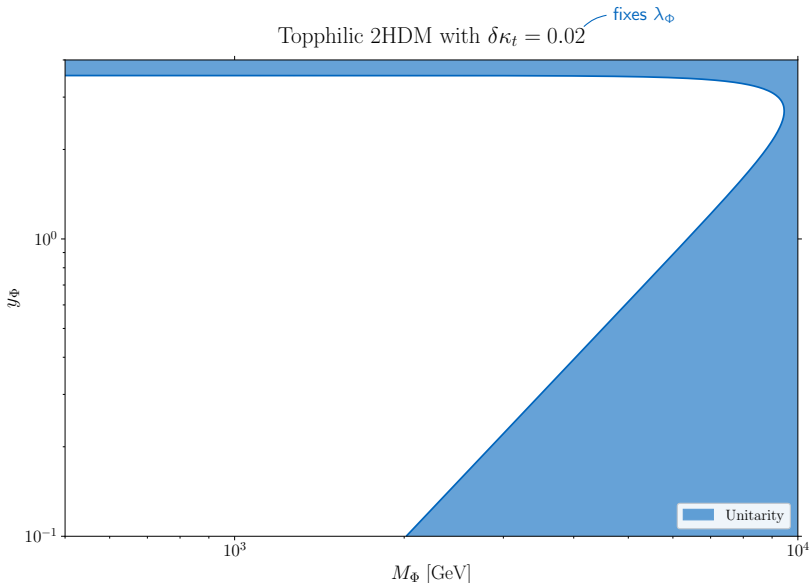


# EWPOs in the top-philic 2HDM

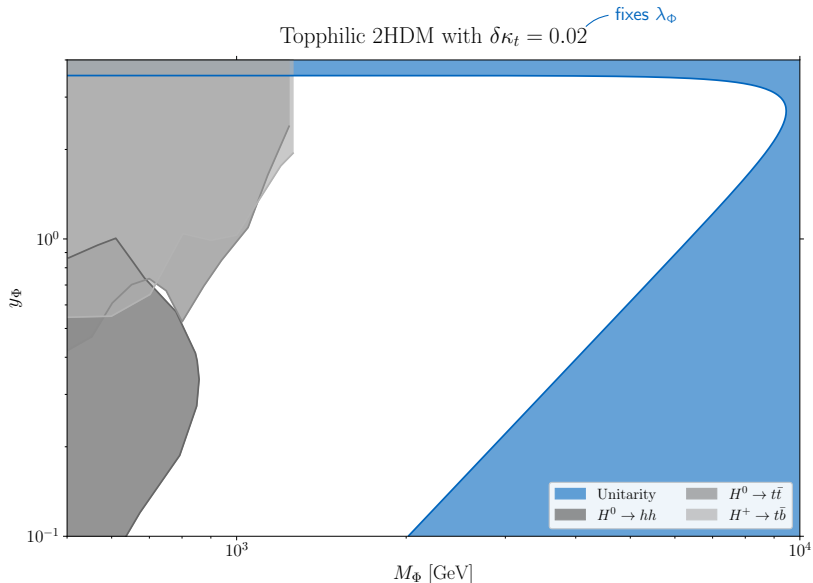


Our analysis uses the 2-loop running (no resummation) of Born, Fuentes-Martín, **AET** [2601.19974],  
1-loop matching with Matchete [2212.04510] and 1-loop determination of EWPOs Biekötter, Pecjak [2503.07724]

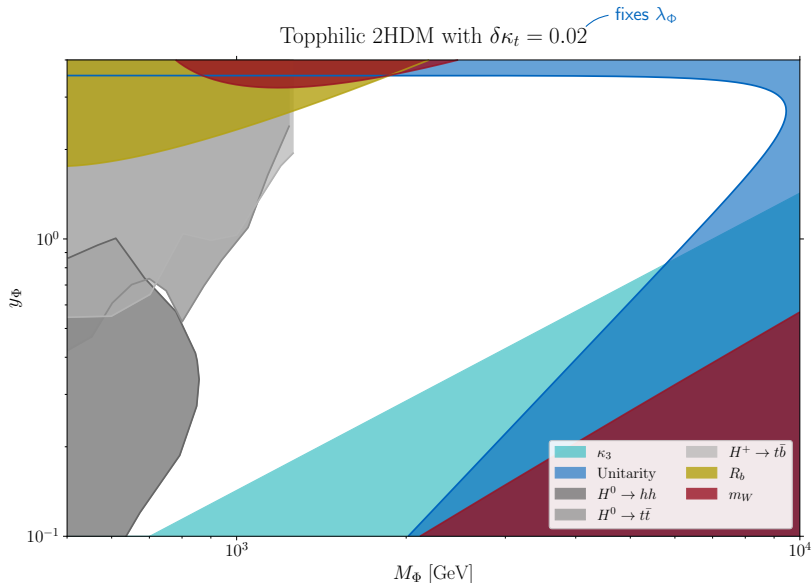
# parameter space for the top-philic 2HDM



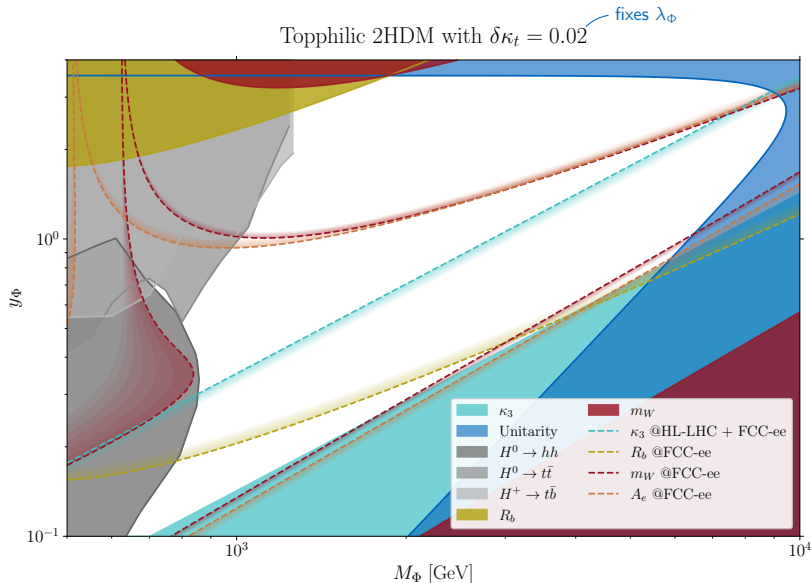
# parameter space for the top-philic 2HDM



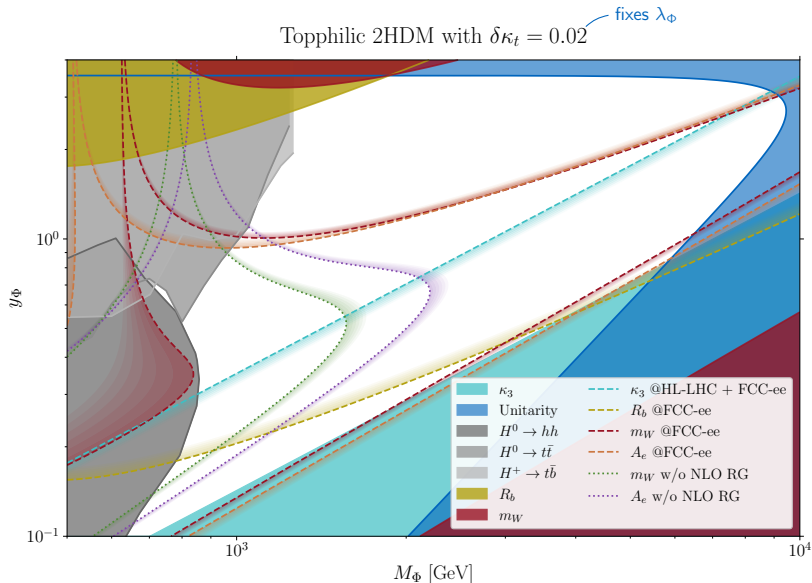
# parameter space for the top-philic 2HDM



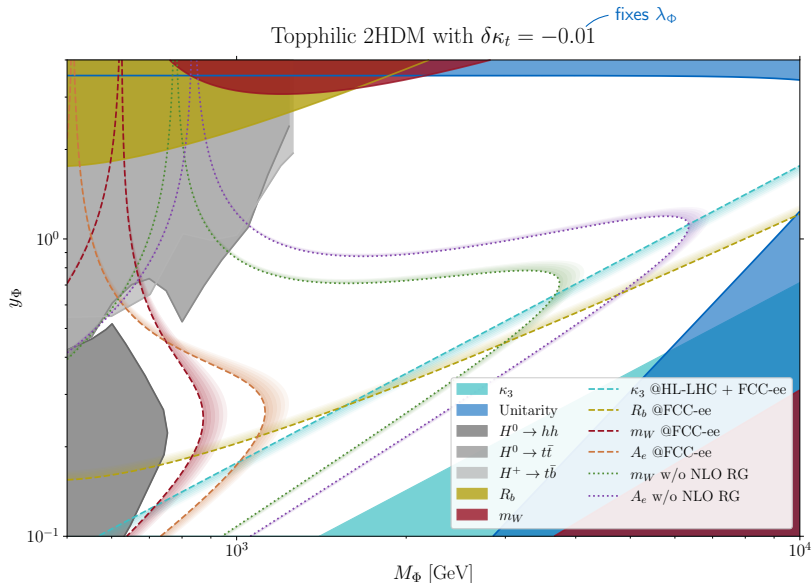
# parameter space for the top-philic 2HDM



# parameter space for the top-philic 2HDM



# parameter space for the top-philic 2HDM



# Summary and outlook

- Functional methods provide a strong basis for automated multi-loop calculations of RGEs and matching quantities.
- Results can be adapted to the Warsaw basis at the price of an extra ambiguity
- Remaining ambiguities in the 2-loop SMEFT (such as they are) would have to be resolved in the BMHV scheme: we hope to integrate the code with a BMHV-scheme implementation [w/ Adrian Moreno-Sánchez](#)
- Full use of the results will require implementation into SMEFT running codes
- NLO contributions to EWPO can be the best probes of some operators, e.g.,  $C_{tt}$  and  $C_{tH}$

# Summary and outlook

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**Thank you!**

# Backup

---

# The R-star operation

Any 'diagram'  $G$  satisfies

Herzog, Ruijl [1703.03776]

$$\text{finite} = \Delta G + \bar{R}^* G = \Delta G + \sum_{\substack{S \in \bar{W}_{UV}(G) \\ S' \in W_R(G) \\ S \cap S' = \emptyset}} \Delta_{IR}(S') * \Delta(S) * G/S \setminus S'$$

counterterm op.

# The R-star operation

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$$\text{finite} = \underbrace{\Delta G}_{\text{counterterm op.}} + \bar{\mathbf{R}}^* G = \Delta G + \sum_{\substack{S \in \bar{\mathcal{W}}_{UV}(G) \\ S' \in \mathcal{W}_R(G) \\ S \cap S' = \emptyset}} \Delta_{\text{IR}}(S') * \Delta(S) * G/S \setminus S'$$

$$\begin{aligned} \text{finite} = & \Delta \left( \text{circle with horizontal line and dots at ends and top} \right) + \Delta \left( \text{upper semi-circle with dots at ends} \right) * \left( \text{circle with dot at top and blue square at bottom} \right) \\ & + \Delta_{\text{IR}} \left( \text{lower semi-circle with open circles at ends} \right) * \left( \text{upper semi-circle with red dots at ends} \right) + \left( \text{circle with horizontal line and dots at ends and top} \right) \end{aligned}$$

It is an extension of the  $\mathbf{R}$  operation, which is used to prove perturbative renormalization

# The R-star operation

Any 'diagram'  $G$  satisfies

Herzog, Ruijl [1703.03776]

$$\text{finite} = \underbrace{\Delta G}_{\text{counterterm op.}} + \bar{\mathbf{R}}^* G = \Delta G + \sum_{\substack{S \in \bar{\mathcal{W}}_{UV}(G) \\ S' \in \bar{\mathcal{W}}_R(G) \\ S \cap S' = \emptyset}} \Delta_{\text{IR}}(S') * \Delta(S) * G / S \setminus S'$$

$\epsilon$ -pole operator

$$\Delta G = -K \bar{\mathbf{R}}^* G$$

The counterterm is polynomial (local)

Taylor expansion

$$\Delta G = -K \bar{\mathbf{R}}^* T_{\{p_i, m_j^2\}}^{(\omega(G))} G$$

IR rearrangement: counterterms are insensitive to momentum routing and masses

$$\Delta G = \Delta G', \quad \text{for } \omega(G) = 0, \quad G' \sim G$$

# The R-star operation

Any 'diagram'  $G$  satisfies

Herzog, Ruijl [1703.03776]

$$\text{finite} = \underbrace{\Delta G}_{\text{counterterm op.}} + \bar{\mathbf{R}}^* G = \Delta G + \sum_{\substack{S \in \bar{W}_{UV}(G) \\ S' \in \bar{W}_R(G) \\ S \cap S' = \emptyset}} \Delta_{IR}(S') * \Delta(S) * G / S \setminus S'$$

The counterterm operator  $\Delta$  applied to  $\Gamma$  yields

## Counterterm master formula

$$\delta S^{(2)} = \frac{1}{8} \mathbf{K} \bar{\mathbf{R}}^* \mathbf{T} \left[ \mathcal{D}_{IJKL}^{(0)} \mathcal{Q}_{IJ}^{-1} \mathcal{Q}_{KL}^{-1} \right] - \frac{1}{12} \mathbf{K} \bar{\mathbf{R}}^* \mathbf{T} \left[ c_{IJK}^{(0)} \mathcal{Q}_{IL}^{-1} \mathcal{Q}_{JM}^{-1} \mathcal{Q}_{KN}^{-1} c_{LMN}^{(0)} \right]$$

Born, Fuentes-Martín, Kvedaraitė, AET: [2410.07320]

- Includes a truncating hard region-like expansion ( $\mathbf{T}$ )
- No need to explicitly calculate counterterm topologies
- *No need to include EOM-vanishing or gauge-violating counterterms!*

# R-star in practice

**Scaleless integrals do not vanish** under the R-star operation

$$\overset{\text{Hard-region expansion}}{-\frac{1}{12}K\bar{R}^*T\left(\bigcirc\right)} \supset -\frac{m^2\lambda^2\phi^2}{4}K\bar{R}^*\int_{k,\ell}\frac{1}{k^4}\frac{1}{\ell^2}\frac{1}{(k+\ell)^2}$$

With **auxiliary mass IR rearrangement**, all IR subdivergences are removed

$$-\frac{m^2\lambda^2\phi^2}{4}K\bar{R}^*\int_{k,\ell}\frac{1}{k^4-a}\frac{1}{\ell^2-a}\frac{1}{(k+\ell)^2-a}$$

$$-\frac{m^2\lambda^2\phi^2}{4}K\left(\int_{k,\ell}\frac{1}{k^4-a}\frac{1}{\ell^2-a}\frac{1}{(k+\ell)^2-a} + \int_k\frac{1}{k^4-a}\overset{K\bar{R}^*T^{(0)}}{\Delta}\overset{i/\epsilon}{\int_\ell\frac{1}{\ell^2-a}\frac{1}{(k+\ell)^2-a}}\right)$$

All dependence on the auxiliary mass vanishes from the counterterm:

$$\delta S^{(2)} \supset \frac{m^2\lambda^2\phi^2}{8}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right)$$

*Each diagram/graph/integral gives a local counterterm!*