

Threshold Corrections at NLO in SMEFT

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Based on A. Biekötter, LM, and B. Pecjak (2601.15901)

1. Motivation
2. Calculating Threshold Corrections
 - 2.1 NLO SM
 - 2.2 NLO SMEFT
3. Numerical Analysis
 - 3.1 Corrections at the EW scale
 - 3.2 Impact on RG analyses
4. Conclusion

- Program to calculate NLO SMEFT:
 - Process-dependent: cross-sections, decay rates
[Haisch et al. \(2507.21768\)](#), [Bellafronte et al. \(2508.14966\)](#)
 - Universal, Lagrangian level: RG, matching, **threshold corrections**
[Jenkins et al. \(1308.2627/1310.4838/1312.2014\)](#), [Aebischer et al. \(2502.14030\)](#)
- Crucial for consistent RG analyses using NLO SMEFT corrections
[Jenkins et al. \(1308.2627\)](#)
- NNLO SM threshold corrections calculated [Kniehl et al. \(1503.02138\)](#)

Threshold Corrections

What are threshold corrections?

Expressing **symmetric phase parameters** of the Lagrangian as a function of **broken-phase (experimentally measured) parameters**

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Lagrangian Parameters

$$p_i^{\text{sym.}}(\mu) \equiv p_i^{\text{sym.}} \in \{g_1, g_2, \lambda, \mu_H^2, y_t, g_3\}$$

$$p_i^{\text{broken}} \in \{M_W, M_Z, v_\sigma, m_H, m_t, g_s(\mu)\}$$

$$v_\sigma \in \{v_\mu, v_\alpha\} \quad v_\mu = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} \quad v_\alpha = \frac{2M_W s_w}{\sqrt{4\pi\alpha(M_Z)}}$$

Scheme	Inputs
v_μ	G_F, M_W, M_Z
v_α	$\alpha(M_Z), M_W, M_Z$

Threshold Corrections in the SM

What are threshold **corrections**?

At tree level (a few examples):

$$g_2 = \frac{2M_W}{v_\sigma}, \quad \lambda = \frac{m_H^2}{2v_\sigma^2}, \quad y_t = \frac{\sqrt{2}m_t}{v_\sigma}$$

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⇒ Radiative corrections

In the case of λ :

$$\lambda_0 = \frac{m_{H,0}^2}{2v_{\sigma,0}^2} \Rightarrow \lambda^{\overline{\text{MS}}}(\mu) = \frac{m_H^2 \text{ O.S.}}{2v_\sigma^2 \text{ O.S.}} [1 + \delta\lambda(p_i^{\text{broken}}, \mu)]$$

Threshold Corrections in the SM

What are threshold corrections?

At tree level (a few examples):

$\overline{\text{MS}}$ Scheme

- Renormalised parameter depends on scale μ
- Convenient for RG running

On-Shell Scheme

- Renormalised parameter scale-independent
- Physical inputs at the EW scale
- μ -dependence in counterterms

Threshold Corrections in the SM

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Threshold corrections are gauge-independent and finite functions of on-shell renormalisation CT.

A recap of SMEFT

The SMEFT Lagrangian

$$\mathcal{L} = \mathcal{L}^{(4)} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \dots$$

In our work, $d = 6$:

$$\mathcal{L} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}; \quad \mathcal{L}^{(6)} = \sum_i C_i(\mu) Q_i(\mu)$$

Examples of Dimension-6 Operators - Warsaw Basis ([arXiv:1008.4884](https://arxiv.org/abs/1008.4884))

$$\frac{\mathcal{O}_{HD} = (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)}{\mathcal{O}_{HG} = (H^\dagger H) G_{\mu\nu}^A G^{A\mu\nu}} \quad \bigg| \quad \frac{\mathcal{O}_{H\Box} = (H^\dagger H) \Box (H^\dagger H)}{\mathcal{O}_{uH} = (H^\dagger H) (\bar{q}_L u_R \tilde{H}) + \text{h.c.}}$$

Threshold Corrections with SMEFT

We define those expansion coefficients:

$$\lambda = \frac{m_H^2}{2v_\sigma^2} \left[1 + \frac{1}{v_\sigma^2} \delta\lambda^{(4,1,\sigma)} + v_\sigma^2 \delta\lambda^{(\textcolor{red}{6},0,\sigma)} + \delta\lambda^{(\textcolor{red}{6},1,\sigma)} \right]$$

Reminder:

$$p_i^{\text{sym.}}(\mu) \equiv p_i^{\text{sym.}} \in \{g_1, g_2, \lambda, \mu_H^2, y_t, g_3\} \equiv p_i^{\overline{\text{MS}}}$$

$$p_i^{\text{broken}} \in \{M_W, M_Z, v_\sigma, m_H, m_t, g_s(\mu)\} \equiv p_i^{\text{O.S.}}$$

Threshold Corrections in SMEFT

Before EWSB:

$$\mathcal{L}_{\text{sym}} = (D_\mu H)^\dagger (D^\mu H) + C_H (H^\dagger H) \square (H^\dagger H) + C_{HD} (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$
$$V(H) = \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 - C_H (H^\dagger H)^3 \quad \Rightarrow \quad v_T \equiv \left(1 + \frac{3C_H v^2}{8\lambda} \right) v$$

After EWSB:

$$\mathcal{L}_{\text{broken}} \supset \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} m_H^2 h^2 + \dots$$
$$m_H^2 = 2\lambda v_T^2 \left(1 - \frac{3C_H v_T^2}{2\lambda} + 2C_{H\square} v_T^2 - \frac{1}{2} C_{HD} v_T^2 \right)$$

Threshold Corrections in SMEFT

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Leading to:

$$\lambda = \frac{m_H^2}{2v_T^2} \left(1 + v_T^2 \left(\frac{3v_T^2}{m_H^2} C_H + \frac{1}{2} C_{HD} - 2C_{H\square} \right) \right)$$

Threshold Corrections in SMEFT

⇒ Radiative corrections ⇒ Renormalisation procedure to one-loop

$$\lambda_0 = \frac{m_{H,0}^2}{2v_{T,0}^2} \left(1 + v_{T,0}^2 \left(\frac{3v_{T,0}^2}{m_{H,0}^2} C_{H,0} + \frac{1}{2} C_{HD,0} - 2C_{H\Box,0} \right) \right)$$

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For a generic renormalised parameter X : $X_0 = X(1 + \Delta X)$

For Wilson coefficients: $C_{i,0} = C_i + \Delta C_i$

For broken-phase parameters renormalised on-shell, split the counterterms into divergent and finite pieces in the dimensional regulator

$$\Delta X^{\text{O.S.}} = \frac{\Delta X_{\text{div.}}^{\text{O.S.}}}{\epsilon} + \delta X^{\text{O.S.}} \quad X^{\text{O.S.}} \in \{M_W, M_Z, v_\sigma, m_t, m_H\}$$

Threshold Corrections in SMEFT

Reminder:

$$\lambda = \frac{m_H^2}{2v_\sigma^2} \left[1 + \frac{1}{v_\sigma^2} \delta\lambda^{(4,1,\sigma)} + v_\sigma^2 \delta\lambda^{(6,0,\sigma)} + \delta\lambda^{(6,1,\sigma)} \right]$$

For λ :

$$\begin{aligned} \lambda = \frac{m_H^2}{2v_\sigma^2} & \left[1 + v_\sigma^2 \left(\frac{3v_\sigma^2}{m_H^2} C_H - 2C_{H,\text{kin}} - \delta v_\sigma^{(6,0)} \right) + \frac{1}{v} (2\delta m_H^{(4,1)} - \delta v_\sigma^{(4,1)}) \right. \\ & \left. + 2\delta m_H^{(6,1)} - \delta v_\sigma^{(6,1)} - 4\delta v_\sigma^{(6,0)} \delta m_H^{(4,1)} + \frac{3v_\sigma^2}{m_H^2} \delta v_\sigma^{(4,1)} C_H - 4\delta m_H^{(4,1)} C_{H,\text{kin}} \right] \end{aligned}$$

Have calculated all the counterterms $\delta v_\sigma^{(6,0)}$, $\delta m_H^{(6,1)}$...

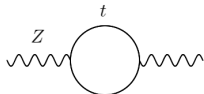
Checks:

- UV divergent parts cancel and answer is gauge invariant
- μ -dependence on LHS and RHS match

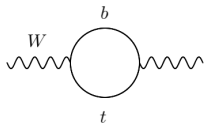
Numerical Analysis at the EW scale

Numerical Analysis at the EW scale

■ Top-quark loop generic size:

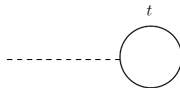
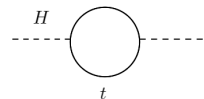


$$\frac{N_c}{16\pi^2} \frac{m_t^2}{v_\sigma^2} = \frac{\Delta\rho_t^{(4,1)}}{v_\sigma^2} \approx 1\%$$



■ Tadpoles scale like:

$$\frac{\Delta\rho_t^{\text{tad}(4,1)}}{v_\sigma^2} \approx \frac{m_t^2}{m_H^2} \Delta\rho_t^{(4,1)} \approx 2\Delta\rho_t^{(4,1)}$$



■ Renormalisation of s_w^2 gives enhanced correction: $\frac{c_w^2}{s_w^2} \frac{\Delta\rho_t^{(4,1)}}{v_\sigma^2} \approx 3.5\%$

At the EW $\mu = M_Z$ scale

Using experimental values (e.g. $M_H = 125.20$ GeV) as input:

$$g_1^{v_\alpha(4,0)} = \frac{\sqrt{4\pi\alpha(M_Z)}}{c_W} = 0.3542, \quad g_1^{v_\mu(4,0)} = \frac{2M_Z s_W}{v_\mu} = 0.3499$$

$$\frac{g_1^{v_\alpha}}{g_1^{v_\alpha(4,0)}} = 1.000 + v_\alpha^2 \left[- (1 \times 1.07) C_{HB} - (0.25 \times 1.08) C_{HD} + \dots \right]$$

$$\frac{g_1^{v_\mu}}{g_1^{v_\mu(4,0)}} = 1.024 + v_\mu^2 \left[- (1.87 \times 1.03) C_{HWB} - (1.12 \times 1.05) C_{HD} + \dots \right]$$

Numerical Analysis at the EW scale

At the EW $\mu = M_Z$ scale

Using experimental values (e.g. $M_H = 125.20$ GeV) as input:

$$\lambda^{\nu_\alpha(4,0)} = \frac{m_H^2}{2v_\alpha^2} = 0.1325, \quad \lambda^{\nu_\mu(4,0)} = \frac{m_H^2}{2v_\mu^2} = 0.1293$$

$$\begin{aligned} \frac{\lambda^{\nu_\alpha}}{\lambda^{\nu_\alpha(4,0)}} &= 1.073 + v_\alpha^2 \left[(11.3 \times 1.07) C_H + (3.73 \times 1.21) C_{H_{WB}} + (2.24 \times 1.17) C_{HD} \right. \\ &\quad \left. - (2 \times 1.00) C_{H\Box} - 0.377 C_{uH}_{33} - 0.159 C_{Hu}_{33} + 0.135 C_{Hq}^{(1)}_{33} - 0.109 C_{Hq}^{(3)}_{33} + \dots \right], \\ \frac{\lambda^{\nu_\mu}}{\lambda^{\nu_\mu(4,0)}} &= 1.119 + v_\mu^2 \left[(11.6 \times 1.02) C_H - (2 \times 1.00) C_{H\Box} - (1 \times 1.27) \left(C_{Hl}^{(3)}_{11} + C_{Hl}^{(3)}_{22} \right) \right. \\ &\quad \left. + (1 \times 1.24) C_{ll}^{1221} + (0.5 \times 1.00) C_{HD} - 0.373 C_{uH}_{33} - 0.029 C_{Hq}^{(3)}_{33} + \dots \right]. \end{aligned}$$

→ **Loop corrections of order $\mathcal{O}(10\%)$**

Threshold Corrections in RGEs

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The bottom-up approach

At fixed order:

$$p_i^{\text{sym.}}(\Lambda) = p_i^{\text{sym.}}(\mu_{EW}) + \dot{p}_i^{\text{sym.}}(\mu_{EW}) \ln \left(\frac{\Lambda}{\mu_{EW}} \right)$$

$$C_i(\Lambda) = c_i(\mu_{EW}) + \dot{C}_i(\mu_{EW}) \ln \left(\frac{\Lambda}{\mu_{EW}} \right)$$

where

$$\dot{p}_i^{\text{sym.}}(\mu_{EW}) \equiv \dot{p}_i^{\text{sym.}}(p_i^{\text{sym.}}(\mu_{EW}), c_i(\mu_{EW}))$$

$$\dot{C}_i^{\text{sym.}}(\mu_{EW}) \equiv \dot{C}_i^{\text{sym.}}(p_i^{\text{sym.}}(\mu_{EW}), c_i(\mu_{EW}))$$

- c_i are fixed initial conditions (at the EW scale)
- $\dot{C}_i$ include the Wilson coefficients that will be generated

Case study 1

RG evolution from $\mu_{EW} = M_Z$ to $\mu_{NP} = 1 \text{ TeV}$

Only 1 non-zero Wilson Coefficient at μ_{EW}

$$c_i(M_Z) = C_{uH}(M_Z) = \frac{1}{1 \text{ TeV}^2}.$$

Reminder: $\mathcal{O}_{uH} = (H^\dagger H) (\bar{q}_L u_R \tilde{H})$

Threshold Corrections in RGEs

$$C_i(\Lambda) = c_i(\mu_{EW}) + \dot{C}_i(\mu_{EW}) \ln \left(\frac{\Lambda}{\mu_{EW}} \right)$$

For our case study $c_i(M_Z) = C_{uH}(M_Z)$, we require:

$$16\pi^2 \dot{C}_H = \left(-\frac{9}{2}g_1^2 - \frac{27}{2}g_2^2 + 108\lambda + 18y_t^2 \right) C_H + (24\lambda y_t - 24y_t^3) C_{uH}_{33}$$

$$16\pi^2 \dot{C}_{uH}_{33} = \left(-\frac{35}{12}g_1^2 - \frac{27}{4}g_2^2 - 8g_3^2 + 24\lambda + \frac{51}{2}y_t^2 \right) C_{uH}_{33}$$

$\Rightarrow g_1, g_2, \lambda, \dots$ are calculated from our threshold corrections

Threshold Corrections in RGEs

$$p_i^{\text{sym.}}(\Lambda) = p_i^{\text{sym.}}(\mu_{EW}) + \dot{p}_i^{\text{sym.}}(\mu_{EW}) \ln \left(\frac{\Lambda}{\mu_{EW}} \right)$$

For the SM parameters:

$$\dot{p}_i = \dot{p}_i^{(4,1)}, \quad p_i \in \{g_1, g_2, g_3, \mu_H^2\},$$

$$\dot{\lambda} = \dot{\lambda}^{(4,1)} + \frac{3\mu_H^2}{4\pi^2} \left(2C_H + y_t C_{uH}_{33} \right),$$

$$\dot{y}_t = \dot{y}_t^{(4,1)} + \frac{3\mu_H^2}{8\pi^2} C_{uH}_{33}.$$

$\Rightarrow \mu_H^2, y_t$ are calculated from our threshold corrections

Threshold Corrections in RGEs

$$\{g_1, g_2, \lambda, \mu_H^2, y_t, g_3\}$$

$$\frac{p_i^{\text{SM}, \nu_\mu}(\Lambda)}{p_i^{\text{SM}, \nu_\mu}(M_Z)} = \{1.01, 0.980, 0.686, 1.07, 0.890, 0.875\}$$

$$\frac{p_i^{\text{SM}+(6,0), \nu_\mu}(\Lambda)}{p_i^{\text{SM}, \nu_\mu}(\Lambda)} = \{1.00, 1.00, 0.925, 1.00, 1.04, 1.00\}$$

$$\frac{p_i^{\text{SM}+(6,0)+(6,1), \nu_\mu}(\Lambda)}{p_i^{\text{SM}+(6,0), \nu_\mu}(\Lambda)} = \{1.00, 1.00, 0.958, 0.981, 1.00, 1.00\}$$

→ Adding NLO SMEFT decrease the values of λ and μ_H^2 at the scale Λ by 4% and 2%

Threshold Corrections in RGEs

Case study

RG evolution from $\mu_{EW} = M_Z$ to $\mu_{NP} = 1 \text{ TeV}$

Only 1 non-zero Wilson Coefficient at μ_{EW}

$$c_i(M_Z) = C_{uH}(M_Z) = \frac{1}{1 \text{ TeV}^2}$$

NLO threshold corrections effect the results for $C_i(\Lambda)$ at below the 1% level

Threshold Corrections in RGEs

Case study

RG evolution from $\mu_{EW} = M_Z$ to $\mu_{NP} = 1 \text{ TeV}$

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NLO threshold corrections effect the results for $C_i(\Lambda)$ at below the 1% level

New scenario

2 non-zero Wilson coefficients

$$c_i(M_Z) = \left\{ C_{uH}(M_Z) = \frac{1}{\Lambda^2}, C_{uG}(M_Z) = \frac{1}{(0.75\Lambda)^2} \right\}$$

The value of C_{uH} (C_H) at that scale is shifted by 3% (-2%) compared to LO SMEFT

- Complete set of NLO SMEFT threshold corrections in analytic form, in two EW input schemes
- Numerical analysis shows changes $\mathcal{O}(5\text{-}10\%)$ in SM symmetric phase parameters values due to NLO threshold corrections
- Would easily be implemented in RG running programs, such as Wilson and RGEsolver [J. Aebischer et al. \(1804.05033\)](#), [S. Di Noi et al. \(2210.06838\)](#)