Scattering-Amplitude Methods in Effective Field Theory

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Selected works with Teng Ma

[1] *SM EFT from On-shell Amplitudes*, Chin. Phys. C 47 (2023) 023105.
[2] *On-shell operator basis for all masses and spins*, Phys. Rev. D 107 (2023) L111901.
[3] *Generic effective field theory for all masses and spins*, Phys. Rev. D 106 (2022) 116010.
[4] *The new formulation of Higgs effective field theory*, JHEP 09 (2023) 101.
[9] *Quantifying EFT Uncertainties in LHC Searches*, JHEP 01 (2026) 094.
[5] *An EFT hunter's guide to two-to-two scattering*, JHEP 05 (2023) 241.
[6] *An Efficient On-shell Framework for EFT Matching*, arXiv:2507.17829.
[7] *Static–Recoil Factorization in Heavy-Baryon Chiral EFTs*, arXiv:2607.12037.
[8] *Dark photons and high spin particles: complete EFT operator basis*, JHEP 07 (2025) 104.

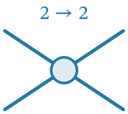
Collaborators Zi-yu Dong, HongKai Liu, Yael Shadmi, Jing Shu, Michael Waterbury, Zi-Zheng Zhou.

Contents

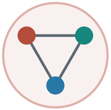
Massless Amplitude Basis

Why effective field theory?

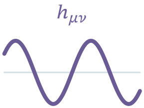
- One framework for physics at a chosen energy scale



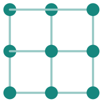
High-energy scattering



Low-energy QCD



Gravity and cosmology



Condensed matter

- Heavy new physics can leave measurable low-energy effects

Electroweak precision tests | **Dark matter searches** | **Flavor physics**

Resolve low-energy effects, constrain short-distance physics.

EFT organizes low-energy effects

- What is EFT?

Wilson coefficients

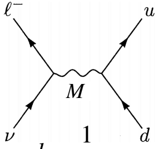
High dimension operator $d > 4$

$$\mathscr{L}_{EFT} = \mathscr{L}_{renormalizable} + \sum \frac{c_i}{\Lambda^{d-4}} \mathscr{O}_i^d$$

Scale

describe IR effects of heavy new physics

- Suppose $E < M$ (New physics scale)



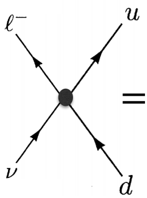
$d \sim \frac{1}{M}$

interaction length

$E < M$

$\sim \frac{1}{E} > d$

probe length

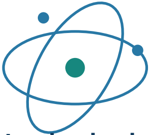


$$= \sum \frac{c_i}{M^{d-4}} \mathscr{O}_i^d$$

S. Weinberg, Physica A 96 (1979) 327; B. Henning, X. Lu and H. Murayama, JHEP 01 (2016) 023.

EFT and scale separation

- The relevant degrees of freedom depend on what we can resolve.

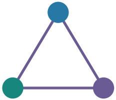


Atomic physics

Electrons and nuclei

Nuclear physics

Nucleons and pions



QCD

Quarks and gluons

Increasing resolution →

- At lower resolution, unresolved structure is encoded in local interactions and their coefficients.

> **Resolve long-distance physics; parametrize short-distance effects.**

What defines an EFT basis?

- At fixed fields, symmetries and operator dimension, list all allowed local interactions.
- Different expressions can describe the same interaction.

Equations of motion	Integration by parts	Index identities and statistics

- IBP example: move a derivative between two factors.

$$\int d^4x\,(\partial_\mu\phi)J^\mu = -\int d^4x\,\phi\,\partial_\mu J^\mu.$$

J^μ: any local vector operator; boundary terms vanish.

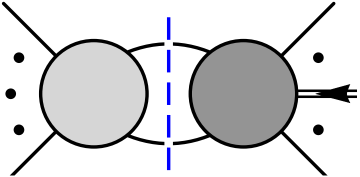
The two operators are related by IBP.

- Hilbert series count independent interactions.
  On-shell amplitudes construct explicit representatives.

B. Henning, X. Lu, T. Melia and H. Murayama, JHEP 10 (2017) 199, arXiv:1706.08520.

What on-shell amplitudes reveal

Unitarity cuts determine RG mixing



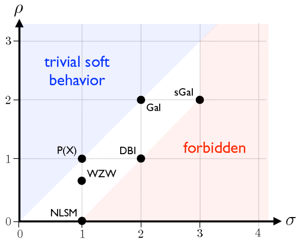
$$\frac{dc_i}{d\log\mu} = \frac{1}{16\pi^2}\sum_j \gamma_{ij} c_j$$

On-shell selection rules expose zeros in γ_{ij}.

Z. Bern, J. Parra-Martinez and E. Sawyer, JHEP 10 (2020) 211, arXiv:2005.12917.

Soft behavior classifies scalar EFTs

$$A_n \sim p^\sigma \qquad (p \to 0)$$



Consistency separates allowed and forbidden EFTs.

C. Cheung, K. Kampf, J. Novotny, C.-H. Shen and J. Trnka, JHEP 02 (2017) 020, arXiv:1611.03137.

Constructing EFTs from on-shell amplitudes

Shadmi–Weiss: scalar/vector couplings to gluons.

Ma–Shu–Xiao: massless amplitude bases ↔ independent SMEFT operators.

Y. Shadmi and Y. Weiss, JHEP 02 (2019) 165, arXiv:1809.09644.
T. Ma, J. Shu and M.-L. Xiao, Chin. Phys. C 47 (2023) 023105, arXiv:1902.06752.

On-shell kinematics and little groups

- Massive spinor and its little group (LG) $SU(2)_i$

$$(p_i)_{\alpha\dot{\alpha}} = (p_i)_\mu(\sigma^\mu)_{\alpha\dot{\alpha}} = |i^I]_{\dot{\alpha}}\langle i_I|_\alpha$$

EOM: $p_i|i^I] = m_i|i^I\rangle$

Lorentz × little group

$SU(2)_l \otimes SU(2)_r \otimes SU(2)_i$

$|i^I] \sim (1,2,2)$
$|i^I\rangle \sim (2,1,2)$

- For massless spinor, its little group is $U(1)_j$

$$|j] \to e^{-i\theta_j}|j], \qquad |j\rangle \to e^{i\theta_j}|j\rangle$$

- Minimal Lorentz scalar: spinor product

$$[ij]^{IJ} = \epsilon^{\dot{\alpha}\dot{\beta}}|i^I]_{\dot{\alpha}}|j^J]_{\dot{\beta}}, \qquad \langle ij\rangle^{IJ} = \epsilon^{\alpha\beta}|i^I\rangle_\alpha|j^J\rangle_\beta$$

- On-shell scattering amplitudes are the functions of spinor products

$$\mathcal{M}_n = \mathcal{M}_n([ij], \langle ij\rangle)$$

N. Arkani-Hamed, T.-C. Huang and Y.-t. Huang, JHEP 11 (2021) 070, arXiv:1709.04891.

Little-group covariance and unitarity

- Little-group covariance fixes spin weights; unitarity constrains poles and cuts.

Tree level **BCFW recursion** **when the large-z boundary vanishes**

Loop level **Unitarity cuts reconstruct loop amplitudes**

- A massless leg of helicity h_j fixes the amplitude's $U(1)_j$ weight:

$$\mathcal{M}(\{e^{-i\theta_j}|j], e^{i\theta_j}|j\rangle, h_j\}) = e^{-i2h_j\theta_j}\mathcal{M}(\{|j], |j\rangle, h_j\})$$

- A massive spin-s_i leg carries $2s_i$ symmetric $SU(2)_i$ indices, giving $2s_i + 1$ spin states:

$$\mathcal{M}(\{w^I{}_J|i]^J, w^I{}_J|i\rangle^J, s_i\}) = w^{I_1}{}_{J_1}\cdots w^{I_{2s_i}}{}_{J_{2s_i}}\mathcal{M}^{\{J_1\cdots J_{2s_i}\}}(\{|i]^J, |i\rangle^J, s_i\}).$$

H. Elvang and Y.-t. Huang, *Scattering Amplitudes*, arXiv:1308.1697.
N. Arkani-Hamed, T.-C. Huang and Y.-t. Huang, JHEP 11 (2021) 070, arXiv:1709.04891.

Spin–kinematic factorization

Polarization tensor

Collect the $2s_i$ symmetric little-group indices of a spin-s_i state.

$$\epsilon_{s_i} = (|i\rangle)^{l_i}(|i])^{2s_i - l_i} \in (2s_i + 1), \qquad l_i = 0, 1, \dots, 2s_i.$$

Angle/square choices are related by the massive EOM: $p|p^I] = m|p^I\rangle$.

Separate spin from kinematics

The little group fixes the spin transformation, allowing the separation into a polarization part and a kinematic part.

$$\mathcal{M}^{\{I\}} = \sum_{\{\alpha\}} \underbrace{\mathcal{A}^{\{I\}}_{\{\alpha\}}(\{\epsilon_{s_i}\})}_{\text{Massive LG charged}} \qquad \underbrace{G^{\{\alpha\}}(|j], |j\rangle, p_i)}_{\text{Massless LG charged}}$$

The spin-dependent factor $\mathcal{A}$ is linear in each ϵ_{s_i}.

From operators to local amplitudes

Local operators modulo redundancies ↔ **Local contact amplitudes** ↔ **Polynomials in spinor products**

- **EOM:** massless external spinors are already on shell.

$$p_i|i] = 0, \qquad p_i|i\rangle = 0.$$

- **IBP:** total derivatives become total momentum.

$$\partial_\mu \mathcal{O} \quad \longleftrightarrow \quad i\Big(\sum_i p_i\Big)_\mu \mathcal{M} = 0.$$

- **Next step:** use $U(N)$ representations to select independent spinor polynomials.

T. Ma, J. Shu and M.-L. Xiao, Chin. Phys. C 47 (2023) 023105; B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015; Y. Shadmi and Y. Weiss, arXiv:1809.09644.

Spinors and Young tableaux

- Treat the particle label $i = 1, \ldots, N$ as an auxiliary $U(N)$ index

Representations under $SU(2)_l \times SU(2)_r \times U(N)$

Square spinor
$\tilde{\lambda}^i_{\dot{\alpha}} \equiv |i]$
$(\mathbf{1}, \mathbf{2}, \mathbf{N})$

Angle spinor
$\lambda_{i\alpha} \equiv |i\rangle$
$(\mathbf{2}, \mathbf{1}, \overline{\mathbf{N}})$

- Antisymmetric spinor products correspond to columns.

$$\begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} = \epsilon^{\dot{\alpha}\dot{\beta}} \tilde{\lambda}^i_{\dot{\alpha}} \tilde{\lambda}^j_{\dot{\beta}} = [ij]$$

$$\begin{array}{|c|}\hline k_1 \\ \hline \cdot \\ \hline \cdot \\ \hline k_{N-2} \\ \hline\end{array} = \frac{\epsilon^{ijk_1\cdots k_{N-2}}}{(N-2)!} \langle ij \rangle$$

Two-box column: square product $[ij]$

Dual column ($N-2$ boxes): $\langle ij \rangle$

B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015, arXiv:1902.06754.

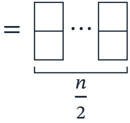
Single-chirality sectors and IBP

- IBP redundancy can be removed by $U(N)$ symmetry

Holomorphic case

Bases with only n
right-handed spinors

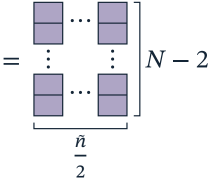
$$f^{(n)}_{\text{hol}}(\{\tilde{\lambda}\}) = g_{SU(2)_r} \otimes g_{U(N)}$$



White column: square spinor product $[ij]$

Bases with only $\tilde{n}$
left-handed spinors

$$f^{(\tilde{n})}_{\text{anti-hol}}(\{\lambda\}) = g_{SU(2)_l} \otimes \tilde{g}_{U(N)}$$



Blue column: angle spinor product $\langle ij\rangle$

- An IBP descendant contains total momentum $P = \sum_i |i]\langle i|$: it needs both chiralities. A single-chirality polynomial cannot contain this factor.

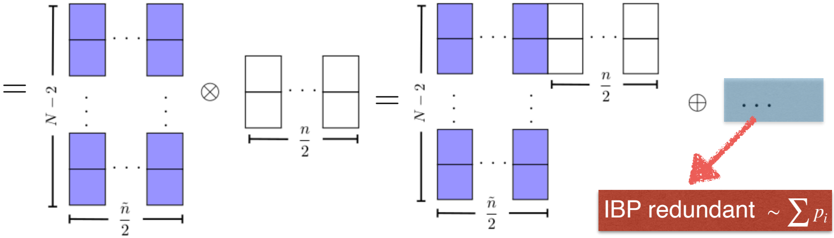
B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015, arXiv:1902.06754.

Mixed chirality: combine two sectors

- IBP redundancy can be removed by $U(N)$ symmetry

Combine n square spinors with $\tilde{n}$ angle spinors

$$f^{(n,\tilde{n})}(\{\lambda,\tilde{\lambda}\}) = f^{(n)}(\{\tilde{\lambda}\})f^{(\tilde{n})}(\{\lambda\}).$$



- Keep the first Young diagram shown above. The other terms contain total momentum and are IBP descendants.

B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015, arXiv:1902.06754.

Four-point examples: both chiralities

- The amplitude bases contain n right-handed spinors and $\tilde{n}$ left-handed spinors. The white and blue box column number is $n/2$ and $\tilde{n}/2$; The length of the blue column is $N-2$ and the white column 2;
- Examples: $N=4$ case:

$\psi_{1R}\psi_{2R}\psi_{3R}\psi_{4R} : (n, \tilde{n}) = (4{,}0)$ White column number is: $n/2 = 2$

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array} = [13][24] + [14][23]$$

$\psi_{1L}\psi_{2L}\psi_{3L}\psi_{4L} : (n, \tilde{n}) = (0{,}4)$ Blue column number is: $\tilde{n}/2 = 2$

Blue column length: $N-2=2$

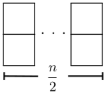
$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array} = \langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle$$

$\psi_{1R}\psi_{2R}\psi_{3L}\psi_{4L} : (\tilde{n}, n) = (2{,}2)$ Blue and white column: $n/2 = 1; \tilde{n}/2 = 1$

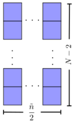
$$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 & 2 \\ \hline\end{array} = \langle 34\rangle[12]$$

Massless basis: takeaways

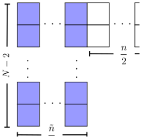
- An independent EFT basis corresponds to an independent local amplitude basis (a polynomial of spinor products)
- Selected $U(N)$ representations remove IBP descendants.
  Their semi-standard Young tableaux (SSYTs) give explicit bases.



Square spinors only
n right-handed



Angle spinors only
$\tilde{n}$ left-handed



Both chiralities
n square, $\tilde{n}$ angle

- Scattering amplitudes provide an efficient route to EFT bases

Massive Amplitude Basis

Why massive EFT bases?

- EFT of massive fields has wide application in particle physics

Higgs EFT $\supset$ SMEFT $\supset$ SM **Dark matter EFT** **Lower energy QCD**

- After electroweak symmetry breaking, measurements are expressed in mass eigenstates

$W^+ \; W^- \; Z$

gauge bosons

Seven massive amplitude bases

SMEFT tower
$W^3(H^\dagger H)^n$

- The challenge is to remove redundancies while keeping physical spin states.

Redundancy: **Equation of motion** **Integration by parts**

Nontrivial EOM $p|p^I] = m|p^I\rangle$

Z.-Y. Dong, **T. Ma**, J. Shu, and Z.-Z. Zhou, Phys. Rev. D 106 (2022) 116010, arXiv:2202.08350.

Massive amplitude factorization

- A massive amplitude has two logically different ingredients: spin/polarization and kinematics

$$\mathcal{M}^I_{m,n} = \sum_{\{\dot\alpha\}} \underbrace{\mathcal{A}^I_{\{\dot\alpha\}}(\{\epsilon_{s_i}\})}_{\text{Massive LG charged}} \underbrace{G^{\{\dot\alpha\}}(|j], |j\rangle, p_i)}_{\text{Massless LG charged}}$$

m massive $\quad$ n massless

- $\mathcal{A}^I(\{\epsilon_{s_i}\})$ carries all massive LG charges and is holomorphic in the massive spinors $|i^I]$

Put massive spin indices in square spinors

$$|i^I\rangle = \frac{p_i|i^I]}{m_i} \qquad \text{(EOM)}$$

Build $\mathcal{A}$ linearly from each polarization tensor

$$\epsilon_{s_i} \equiv |i^{\{I_1}] \cdots |i^{I_{2s_i}\}}] \sim (\mathbf{2s_i+1, 2s_i+1})$$

The spin basis carries no momentum descendants.

Z.-Y. Dong, **T. Ma**, and J. Shu, Phys. Rev. D 107 (2023) L111901, arXiv:2103.15837.

Example: separating spin from kinematics

- Leg 1 is a massive vector; legs 2 and 3 are massless fermions.

$$\mathcal{M}^{IJ} = [1^{\{I}2]\langle 1^{J\}}3\rangle = \frac{[1^{\{I}2][1^{J\}}|p_1|3\rangle}{m_1}.$$

- Use $|1^I\rangle = p_1|1^I]/m_1$ to put the massive spin indices in square spinors.

$$\mathcal{M}^{IJ} = \frac{1}{m_1} \underbrace{[1^{\{I}|_{\dot\alpha}[1^{J\}}|_{\dot\beta}}_{\mathcal{A}^{IJ}_{\dot\alpha\dot\beta}} \underbrace{|2]^{\dot\alpha}(p_1|3\rangle)^{\dot\beta}}_{G^{\dot\alpha\dot\beta}}.$$

- $\mathcal{A}$ carries the vector spin indices I, J; G carries momenta and massless helicity weights.

Contract Lorentz indices; keep the external spin indices free.

Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901, arXiv:2103.15837.

Two ingredients of a massive basis

- Construct spin tensors and kinematic polynomials separately.

Spin tensors $\mathcal{A}^I$

Massive little-group indices
Lorentz $SU(2)_r$ tableaux

Kinematic polynomials G

Momenta and
massless spinors
Auxiliary $U(N)$ tableaux

Lorentz scalar amplitude

$$\mathcal{M}^I = \sum_{\{\dot\alpha\}} \mathcal{A}^I_{\{\dot\alpha\}} G^{\{\dot\alpha\}}$$

- Construct spin and kinematic bases separately, then combine them.

Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901.

Spin basis from massive little groups

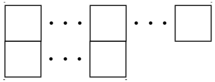
- Massive little-group tensor structures (MLGTS) are linear in the polarization tensors ϵ_{s_i}

$$\epsilon_{s_i} \equiv |i]^{\{I_1}_{\dot{\alpha}_1}, \ldots, |i]^{I_{2s_i}\}}_{\dot{\alpha}_{2s_i}} \sim \underbrace{\boxed{i}\cdots\boxed{i}_{\,r}}_{(2s_i)}$$

- Any MLGTS basis belongs to the tensor product of the $SU(2)_r$ representations carried by all ϵ_{s_i}

$SU(2)_r$ Rep of ϵ_{s_i}

$$\mathscr{A}^I(\{\epsilon_{s_i}\}) \subset \otimes_{i=1}^{m} \underbrace{\boxed{i}\cdots\boxed{i}_{\,r}}_{(2s_i)} =$$



$$\oplus$$

- A complete set of $\{\mathcal{A}^I_{\{\dot{\alpha}\}}\}$ is obtained by decomposing this tensor product into $SU(2)_r$ irreducible representations

Example: $\psi\psi' Zh$ spin structures

- Labels $1, 2, 3$ denote ψ, ψ', Z; the scalar h carries no spin index.

$$\psi \sim \boxed{1} \qquad \psi' \sim \boxed{2} \qquad Z \sim \begin{array}{|c|c|}\hline 3 & 3 \\ \hline \end{array} \qquad h \sim \bullet$$

$$\begin{aligned} &\boxed{1} \times \boxed{2} \times \begin{array}{|c|c|}\hline 3 & 3 \\ \hline \end{array} \times \bullet \\ &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline \multicolumn{1}{|c|}{3} \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 3 & 3 \\ \hline \multicolumn{1}{|c|}{2} \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 3 \\ \hline \end{array} \end{aligned}$$

- The first SSYT gives the following massive little-group tensor structure

$$\mathcal{A}^{I}_{[2,2]} = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} \qquad \longrightarrow \qquad [1^I 3^{\{K_1}][2^J 3^{K_2\}}].$$

- The two columns give $[13]$ and $[23]$. Symmetrize the two Z indices K_1, K_2.

Removing kinematic redundancies

- $G(|j], |j\rangle, p_i)$ bases suffer from EOM and IBP redundancy
- EOM redundancy can be removed by first constructing $G(|j], |j\rangle, p_i)$ massless-limit bases

Construct massless limits

$$p_{i,\dot{\alpha}\alpha} \to |i]_{\dot{\alpha}}\langle i|_{\alpha} \; : \; G(|j], |j\rangle, p_i) \to g \equiv G(|j], |j\rangle, |i]\langle i|)$$

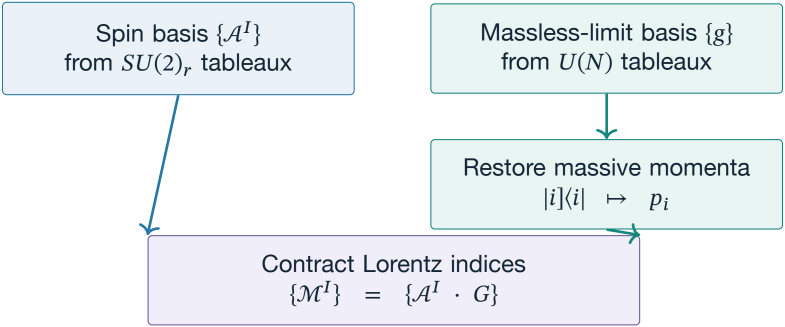
- IBP redundancy can be removed by $U(N)$ symmetry

Select $U(N)$ representations without IBP descendants → Massless-limit polynomials $\{g\}$ → Restore momenta $|i]\langle i| \;\mapsto\; p_i$ to obtain $\{G\}$

Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901.

Assembling independent amplitudes

- The spin and kinematic factors must carry matching Lorentz representations.



- This gives an amplitude basis. Assigning the lowest operator dimension requires one further step.

Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901; Z.-Y. Dong et al., Phys. Rev. D 106 (2022) 116010.

Example: W^+W^-Z amplitude bases

- The Young-tableau construction gives the candidate W^+W^-Z bases

$$
\begin{aligned}
\mathcal{A}^{I}_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset\ & \begin{array}{|c|c|}\hline 1&1\\\hline\end{array} \times \begin{array}{|c|c|}\hline 2&2\\\hline\end{array} \times \begin{array}{|c|c|}\hline 3&3\\\hline\end{array} \\
=\ & \begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2&3&3\\\hline\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1&1&2&2\\\hline 3&3 \\\cline{1-2}\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1&1&2&3\\\hline 2&3 \\\cline{1-2}\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1&1&3&3\\\hline 2&2 \\\cline{1-2}\end{array} \\
\oplus\ & \begin{array}{|c|c|c|c|c|}\hline 1&1&2&2&3\\\hline 3 \\\cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|c|c|}\hline 1&1&2&3&3\\\hline 2 \\\cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|c|c|c|}\hline 1&1&2&2&3&3\\\hline\end{array} \\
\equiv\ & \mathcal{A}^{I}_{[3,3]} \oplus \mathcal{A}^{I}_{[(4,2)^1]} \oplus \mathcal{A}^{I}_{[(4,2)^2]} \oplus \mathcal{A}^{I}_{[(4,2)^3]} \\
& \oplus \mathcal{A}^{I}_{[(5,1)^1]} \oplus \mathcal{A}^{I}_{[(5,1)^2]} \oplus \mathcal{A}^{I}_{[6]}
\end{aligned}
$$

- After kinematic constraints, seven independent bases remain.

$$
\begin{aligned}
\mathcal{A}^{I}_{[3,3]} &: G^{\bullet}_{d=0} = 1 \\
\mathcal{A}^{I}_{[(4,2)^{1,2,3}]} &: G^{[3]}_{d=2} = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array} \\
\mathcal{A}^{I}_{[(5,1)^{1,2}]} &: G^{[6]}_{d=4} = \begin{array}{|c|c|c|c|c|c|}\hline 1&1&2&2&3&3\\\hline\end{array} \\
\mathcal{A}^{I}_{[6]} &: G^{[9]}_{d=6} = \begin{array}{|c|c|c|c|c|c|c|c|c|}\hline 1&1&1&2&2&2&3&3&3\\\hline\end{array}
\end{aligned}
$$

Three-point kinematics

No independent momentum polynomial.

The seven survivors are spin amplitudes.

The seven three-point structures agree with Durieux et al., JHEP 12 (2020) 175 [arXiv:2008.09652].

Massive basis: what is established?

Established: independent amplitude structures

Construct spin tensors $\mathcal{A}$ and kinematic polynomials G,
then contract their Lorentz indices:

$$\mathcal{M}^I = \sum_{\{\dot{\alpha}\}} \mathcal{A}^I_{\{\dot{\alpha}\}} G^{\{\dot{\alpha}\}}.$$

Remaining: lowest operator dimension

The square-spinor representation can hide lower-dimensional interactions behind momenta and inverse masses.

Dimension reduction identifies the lowest-dimensional representatives.

Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901.

Lowest dimensional massive amplitude basis

Why dimension reduction is needed

- A complete $\{\mathcal{A} \cdot G\}$ basis need not display each interaction at its lowest operator dimension.

Trivial mass factors:

$$\mathcal{O}_d = m_i^2 \mathcal{O}_{d-2} + m_i^4 \mathcal{O}_{d-4} + \cdots$$

- For example: 4-pt $\psi_1\psi_2\phi_1\phi_2$ has two dim-5 operator bases

$$\text{dim-5} = \{[1^I 2^J], \langle 1^I 2^J \rangle\}$$

- A square-spinor choice represents angle structures through momenta and inverse masses. Their lowest dimension is then hidden:

Combination of $\{\mathcal{A} \cdot G\}$

$$\langle 1^I 2^J \rangle = \frac{m_2 [1^I 2^J]}{m_1} + \frac{[1^I 3\, 2\, 2^J]}{m_1 m_2} + \frac{[1^I 4\, 2\, 2^J]}{m_1 m_2}$$

Factorized $\{\mathcal{A} \cdot G\}$ route requires dimension reduction

Shadmi et al.: direct massive contact terms no intermediate dimension mismatch

Programmatic realization: *MassiveGraphs*

G. Durieux, T. Kitahara, C. S. Machado, Y. Shadmi and Y. Weiss, arXiv:2008.09652; S. De Angelis, JHEP 08 (2022) 299, arXiv:2202.02681.

Dimension reduction workflow

- The goal is an independent set of lowest-dimensional amplitudes that maps directly to local operators

Complete but redundant $\{\mathcal{C} \cdot F\}$ → **Decompose into the $\{\mathcal{A} \cdot G\}$ basis and order by dimension** → **Remove linear relations independent representatives**

Order candidates by operator dimension.
Keep only linearly independent representatives.

Z.-Y. Dong, **T. Ma**, J. Shu, and Y.-H. Zheng, Phys. Rev. D 106 (2022) 116010, arXiv:2202.08350.

Generate a complete candidate set

- Allow every angle/square choice for each massive polarization tensor.

$$\epsilon_{s_i}^{l_i} = (|i\rangle)^{l_i}(|i])^{2s_i - l_i}, \qquad l_i = 0, 1, \ldots, 2s_i.$$

l_i counts angle spinors, not physical spin projections.

- For fixed $\{l_i\}$, construct the corresponding set $\mathcal{B}_{\{l_i\}}$ using $SU(2)_r \times U(N)$ tableaux.
- Collect all configurations and decompose them to the $\{\mathcal{A}\cdot G\}$ basis; then remove linear relations.

$$\mathcal{B}_{\text{cand}} = \bigcup_{\substack{\{l_i\} \\ 0 \le l_i \le 2s_i}} \mathcal{B}_{\{l_i\}}, \qquad \text{span}\,\mathcal{B}_{\text{cand}} = \text{span}\{\mathcal{C}\cdot F\}.$$

Complete candidate set $\neq$ independent basis.

Z.-Y. Dong et al., Phys. Rev. D 106 (2022) 116010, arXiv:2202.08350.

EFT programs: HEFT + dark photons

- Massive EFT basis construction can be automated: github.com/hamiguazzz/Massive

Applications: HEFT operator bases and dark-photon EFT through $d = 8$.

Z.-Y. Dong, **T. Ma**, C. Yang, and Z. Zhou, JHEP 07 (2025) 104, arXiv:2412.20096; Z.-Y. Dong et al., JHEP 09 (2023) 101; Z. Dong et al., arXiv:2507.17829.

A.91 Type: $ZZZZ$

A.91.1 Dimension = 4, $\mathcal{O}_4^1$

Type: $ZZZZ$ $\quad d=4 \quad \mathcal{O}_4^1$
$Z_\mu Z_\nu Z_\rho Z_\sigma \,\mathrm{Tr}\left(\sigma^\mu\bar\sigma^\rho\right)\mathrm{Tr}\left(\sigma^\nu\bar\sigma^\sigma\right)$

A.91.2 Dimension = 6, $\mathcal{O}_6^{1\sim4}$

Type: $ZZZZ$ $\quad d=6 \quad \mathcal{O}_6^{1\sim4}$
$Z_\mu Z_\nu Z^+_{\rho\sigma} Z^+_{\xi\tau}\,\mathrm{Tr}\left(\sigma^\mu\sigma^\nu\bar\sigma^{\xi\tau}\bar\sigma^{\rho\sigma}\right)$
$Z^-_{\mu\nu} Z_\rho Z_\sigma Z^+_{\xi\tau}\,\mathrm{Tr}\left(\sigma^\rho\sigma^{\mu\nu}\sigma^\sigma\bar\sigma^{\xi\tau}\right)$
$Z^-_{\mu\nu} Z^-_{\rho\sigma} Z_\xi Z_\tau\,\mathrm{Tr}\left(\sigma^\xi\sigma^{\mu\nu}\sigma^{\rho\sigma}\bar\sigma^\tau\right)$
$(D_\mu Z_\rho)\, Z_\sigma\, (D_\nu Z_\xi)\, Z_\tau\,\mathrm{Tr}\left(\sigma^\sigma\bar\sigma^\mu\sigma^\xi\bar\sigma^\tau\right)\mathrm{Tr}\left(\bar\sigma^\nu\sigma^\rho\right)$

A.86 Type: $ZZ\gamma^+\gamma^-$

A.86.1 Dimension = 6, $\mathcal{O}_6^1$

Type: $ZZ\gamma^+\gamma^-$ $\quad d=6 \quad \mathcal{O}_6^1$
$Z_\mu Z_\nu \gamma^+_{\rho\sigma}\gamma^-_{\xi\tau}\,\mathrm{Tr}\left(\sigma^\mu\sigma^{\xi\tau}\sigma^\nu\bar\sigma^{\rho\sigma}\right)$

A.86.2 Dimension = 8, $\mathcal{O}_8^{1\sim4}$

Type: $ZZ\gamma^+\gamma^-$ $\quad d=8 \quad \mathcal{O}_8^{1\sim4}$
$Z^-_{\mu\nu} Z^+_{\rho\sigma}\gamma^+_{\xi\tau}\gamma^-_{\zeta\eta}\,\mathrm{Tr}\left(\bar\sigma^{\rho\sigma}\bar\sigma^{\xi\tau}\right)\mathrm{Tr}\left(\sigma^{\mu\nu}\sigma^{\zeta\eta}\right)$
$Z^-_{\nu\rho} Z_\sigma \gamma^+_{\xi\tau}\left(D_\mu\gamma^-_{\zeta\eta}\right)\mathrm{Tr}\left(\sigma^\sigma\sigma^{\zeta\eta}\sigma^{\nu\rho}\bar\sigma^\mu\bar\sigma^{\xi\tau}\right)$
$Z_\nu Z^+_{\rho\sigma}\left(D_\mu\gamma^+_{\xi\tau}\right)\gamma^-_{\zeta\eta}\,\mathrm{Tr}\left(\bar\sigma^\mu\sigma^{\zeta\eta}\sigma^\nu\right)\mathrm{Tr}\left(\bar\sigma^{\rho\sigma}\bar\sigma^{\xi\tau}\right)$
$Z_\rho\left(D_\nu Z_\sigma\right)\gamma^+_{\xi\tau}\left(D_\mu\gamma^-_{\zeta\eta}\right)\mathrm{Tr}\left(\sigma^\rho\sigma^\mu\bar\sigma^\nu\sigma^{\zeta\eta}\sigma^\sigma\bar\sigma^{\xi\tau}\right)$

Identical-particle symmetry

- For identical bosons (fermions), amplitudes are symmetric (antisymmetric) under exchange
- For example

For three identical bosons, the amplitude bases lie in the fully symmetric $\boxed{1\,|\,2\,|\,3}$ representation of the permutation group S_3

- First, find $\boxed{1\,|\,2\,|\,3}$ representation matrix in the space of amplitude bases $\{\mathcal{O}\}$

$$\underbrace{\mathcal{Y}_{\boxed{1\,|\,2\,|\,3}}}_{\text{Young operator}}\{\mathcal{O}\} = \underbrace{M_{\boxed{1\,|\,2\,|\,3}}}_{\text{Representation}}\{\mathcal{O}\}$$

- Apply the symmetrizer to the candidate amplitudes and keep a linearly independent set of nonzero results.

Application: SMEFT to HEFT matching

- On-shell matching maps SMEFT bases to HEFT bases and makes coefficient relations explicit

SMEFT bases $$\mathcal{A}(H^i H_k^\dagger H^j H_l^\dagger) \supset c^+_{HHHH} \frac{s_{13}}{\Lambda^2} T^{+\,ij}_{\ \ kl} + c^-_{HHHH} \frac{s_{12} - s_{14}}{\Lambda^2} T^{-\,ij}_{\ \ kl}$$

HEFT bases $$\boldsymbol{\mathcal{M}}(W^+W^-hh) = C^{00}_{6,WWhh} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

Just bold the spinors

$$\mathcal{A}(G^+G^-hh) = \frac{1}{2}\left(\mathcal{A}(H^1 H_1^\dagger H^2 H_2^\dagger) + \mathcal{A}(H^1 H_1^\dagger H_2^\dagger H^2)\right) = -\frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2} \frac{s_{12}}{2\Lambda^2}$$

$$-\frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2} \frac{s_{12}}{2\Lambda^2} \longrightarrow \frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

An EFT hunter's guide to two-to-two scattering: HEFT and SMEFT on-shell amplitudes, H. Liu, **T. Ma**, Y. Shadmi, M. Waterbury, arXiv:2301.11349.

Heavy-baryon chiral EFT

Goal: construct the baryon EFT through $O(Q^{\nu})$.

$$P^{\mu} = Mv^{\mu} + k^{\mu}$$

Hard mass

$Mv^{\mu}, \quad M \sim \Lambda_{\chi}$

No soft power

Soft recoil

$k^{\mu} \sim Q, \quad Q \ll M$

Count powers of Q

More derivatives do not always mean higher chiral order.

Separate hard mass from soft recoil *before* counting.

Q: soft momentum or pion mass. Recoil: k/M; chiral expansion: Q/Λ_{χ}.

Z. Dong, **T. Ma**, C. Yang, and Z. Zhou, arXiv:2607.12037, Secs. 2–4.

Angular momentum separates scales

Heavy spinors scale as $\sqrt{M}$; hard relative momentum as M.

Resolved tensor content fixes the hard mass powers.

$$\mathcal{A} = \sum_{\text{channels}} \underbrace{\mathcal{H}}_{\text{heavy-pair tensor}} \odot \underbrace{\mathcal{S}}_{\text{soft remainder}}$$

Factor out hard powers; count the soft remainder.

J organizes the separation; it is *not* the chiral order. $\odot$: tensor contraction.

Z. Dong, **T. Ma**, C. Yang, and Z. Zhou, arXiv:2607.12037, Sec. 4.3.

Keep the target soft orders

After removing hard factors:

$$\text{EFT through } O(Q^{\nu}) : \quad \boxed{d_\chi \le \nu}.$$

Example: $\bar{B}Buf_+$ **through** $O(Q^3)$

Static baryon tensor	$\xrightarrow{\times k/M}$	Recoil correction
$O(1) \times Q \times Q^2$		$O(Q^3) \times k/M$
$O(Q^3)$: keep		$O(Q^4)$: omit

$u_\mu \sim Q$: Goldstone derivative; $f_+^{\mu\nu} \sim Q^2$: source field strength.

Selected amplitudes → **Flavor & statistics** → **Independent operators**

Collect every contributing canonical dimension; remove linear redundancies.

Z. Dong, **T. Ma**, C. Yang, and Z. Zhou, arXiv:2607.12037, Secs. 4–5.

On-shell matching: three routes

$$\sum_i C_i^{(1)} B_i = \left[\mathcal{A}_{\text{UV}}^{(1)} - \mathcal{A}_{\text{EFT}}^{(1)}[C^{(0)}]\right]_{\text{low energy, local}}$$

Direct matching [1]	Expand UV amplitudes and subtract EFT terms. Project analytically or solve at on-shell points.
Unitarity cuts [2]	Trees → loop masters → hard expansion. D-dimensional sewing retains rational contributions.
Dispersion [3]	Scale $s_I \to z s_I$; extract a chosen low-energy order from heavy poles, cuts, and any boundary term.

UV on-shell data ⟶ **Local hard remainder** ⟶ **Physical basis: C_i**

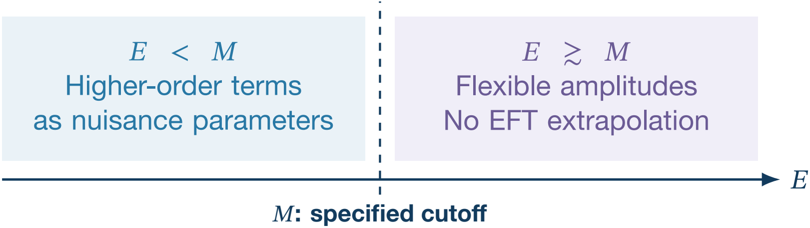
Same inputs, residues and IR scheme; retain rational/evanescent terms. Cuts alone do not fix all local terms.

Review: *Scattering-Amplitude Methods in EFT: Bases, Matching, Running, and Projections*, Sec. 7 (7 Sep 2026).
[1] M. Chala et al., SciPost Phys. 18 (2025) 185. [2] Z. Dong, C. Li, **T. Ma** et al., arXiv:2507.17829v3 (2026). [3] S. De Angelis and G. Durieux, SciPost Phys. 16 (2024) 071.

Quantifying EFT uncertainties

Motivation: reliable new-physics limits from LHC data.
High-energy tails are sensitive, but omitted EFT terms grow with E/M.

Ignoring this uncertainty can produce overly strong exclusions.



Solution: fit the EFT coefficient together with its uncertainty.

$$G \longrightarrow G\,\mathcal{R}_\eta(\hat{s}, \hat{t}; M), \qquad \mathcal{R}_\eta = 1 + \eta_s \frac{\hat{s}}{M^2} + \eta_t \frac{\hat{t}}{M^2} + \cdots \quad (E \ll M).$$

G: coefficient being tested; η: unknown higher-order corrections.

Profile η with power-counting priors; report limits on G versus M.

S. Chang, M. A. Luty, **T. Ma**, F. Montagno and A. Wulzer, *Quantifying EFT Uncertainties in LHC Searches*, JHEP 01 (2026) 094, arXiv:2507.15954.

Summary

- **Construct independent interactions.**
  On-shell bases handle massless and massive fields systematically.
  Applications: HEFT, dark-photon, DM and CCWZ EFTs.
- **Separate scales before counting.**
  Heavy-baryon χEFT: remove hard M, then count soft recoil.
- **Match UV physics onto the basis.**
  Direct amplitudes, cuts and dispersion determine Wilson coefficients.
- **Include EFT uncertainty in LHC limits.**
  Profile higher-order effects; state the cutoff and prior assumptions.

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[3] **T. Ma**, J. Shu and M.-L. Xiao, *Standard Model Effective Field Theory from On-shell Amplitudes*, Chin. Phys. C 47 (2023) 023105.

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[6] Z. Dong, **T. Ma**, C. Yang and Z. Zhou, *Dark photons and high spin particles: complete EFT operator basis*, JHEP 07 (2025) 104.

[7] Z.-Y. Dong, **T. Ma**, J. Shu and Z.-Z. Zhou, *Constructing generic effective field theory for all masses and spins*, Phys. Rev. D 106 (2022) 116010.

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[10] Z. Dong, C. Li, **T. Ma**, J. Shu and Z. Zhou, *An Efficient On-shell Framework for EFT Matching*, arXiv:2507.17829.

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Thank you