

Relativistic DFT Response Theory for Molecular Magnetic and Symmetry-Violating Properties

PhD student:
Ardhmeri Alija

Supervisor:
Dr.Konstantin Gaul



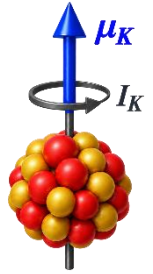
Many-Body Theory: Nuclear Physics Meets Quantum Chemistry

August 24, 2026 to September 4, 2026

From Nuclear Properties to Molecular Parity Violation

Nuclear properties

- Nuclear magnetic moment μ_K

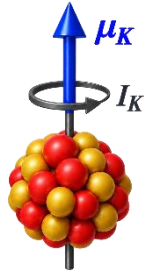


- Nuclear magnetic shielding σ
Chemical shift δ
$$\delta = \sigma - \sigma_{ref}$$
- Indirect nuclear spin-spin coupling (J-coupling)

From Nuclear Properties to Molecular Parity Violation

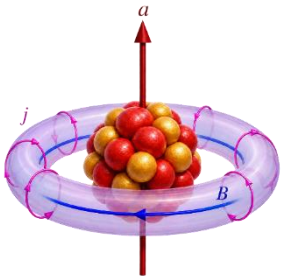
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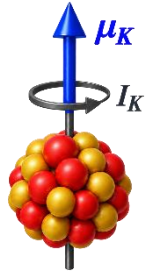


- Nuclear-spin-dependent PV interaction

From Nuclear Properties to Molecular Parity Violation

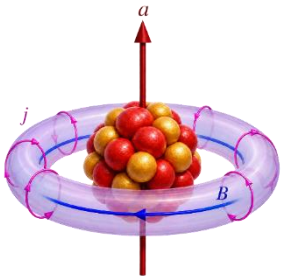
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Parity and chirality

Parity: inversion of all spatial coordinates

$$\hat{P}: \mathbf{r} \rightarrow -\mathbf{r}$$

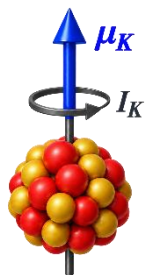
Electromagnetic and strong interactions **conserve P**

Weak interaction: **violates P**

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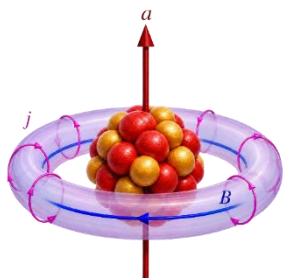
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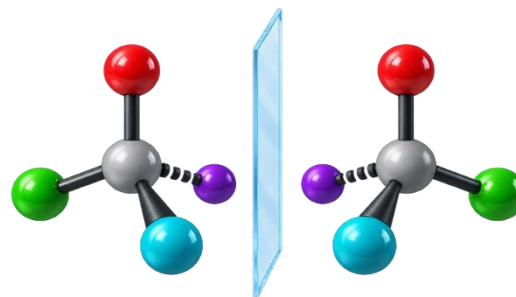
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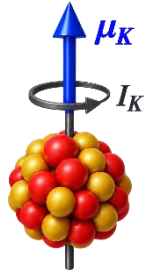
Enantiomers:
Non-superimposable mirror-image molecules

$$R \xrightarrow{\hat{P}} S$$

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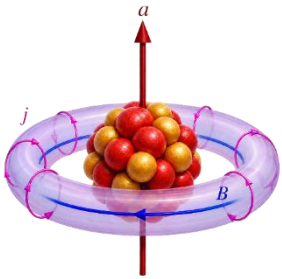
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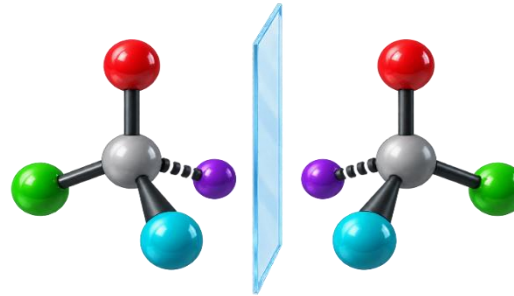
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Consequence for enantiomers

Without PV:

$$E_R^{(0)} = E_S^{(0)}$$

With PV:

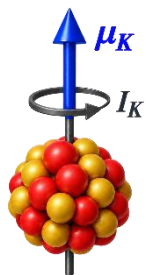
$$E_R^{(PV)} = -E_S^{(PV)}$$

$$\Rightarrow E_R \neq E_S$$

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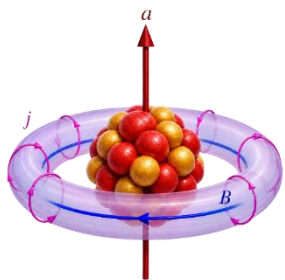
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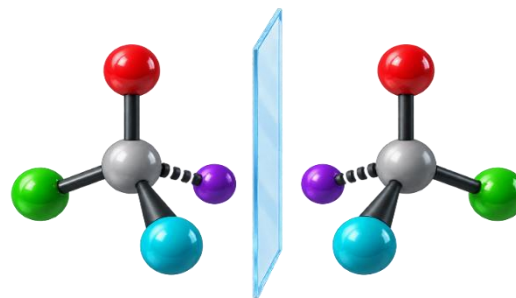
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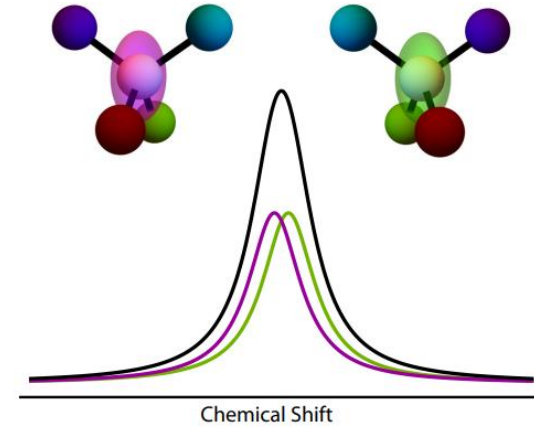
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Can this mirror-image difference be detected by NMR?

Detecting Molecular Parity Violation by NMR

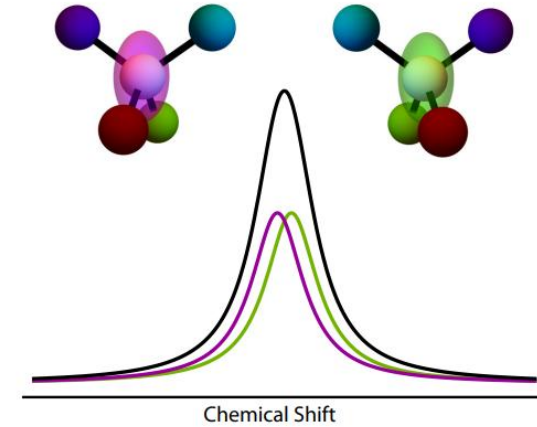
J. Eills et al., Phys. Rev. A (2017), 96, 042119



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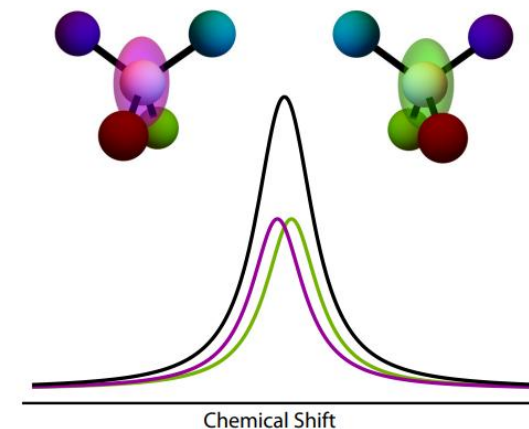


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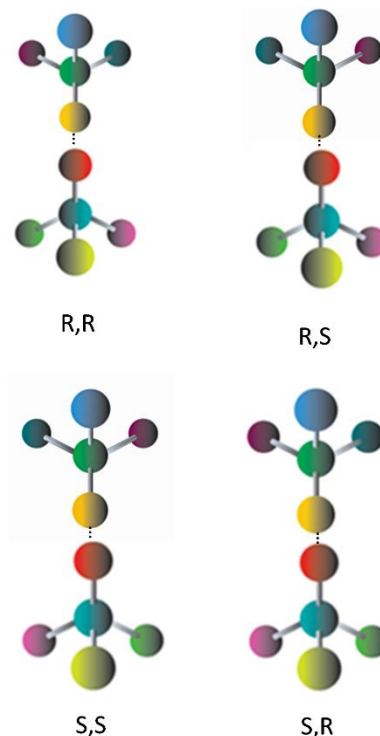
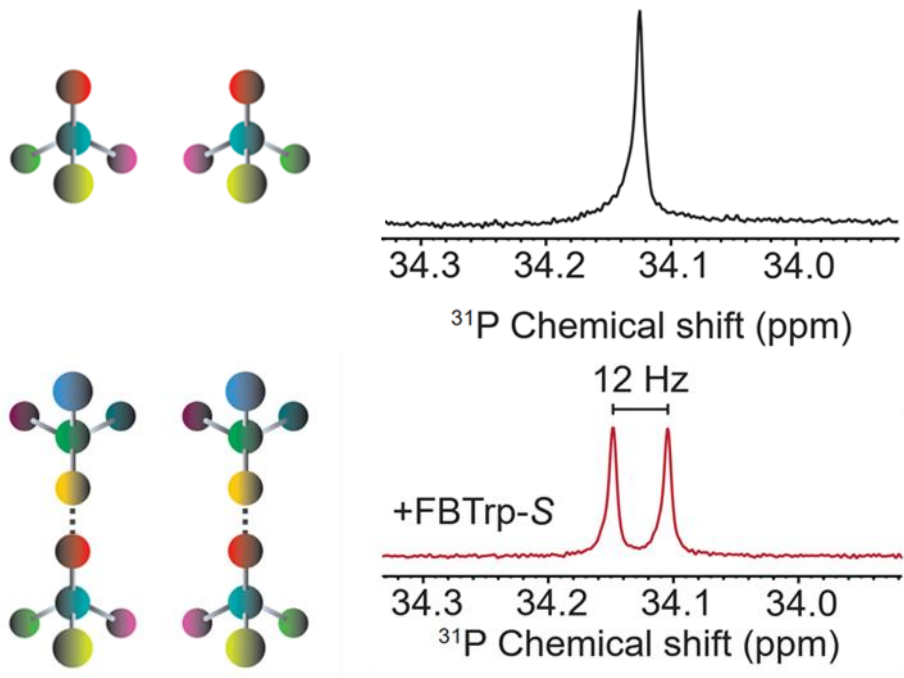
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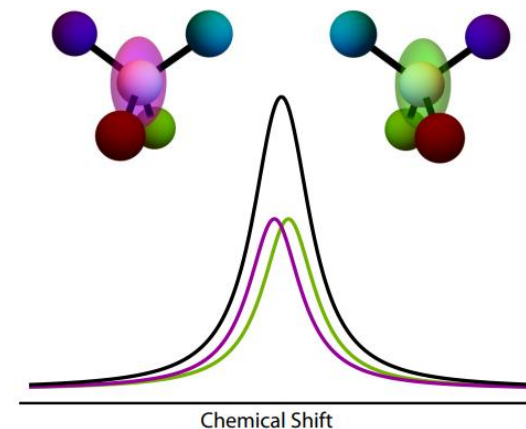
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Diastereomers

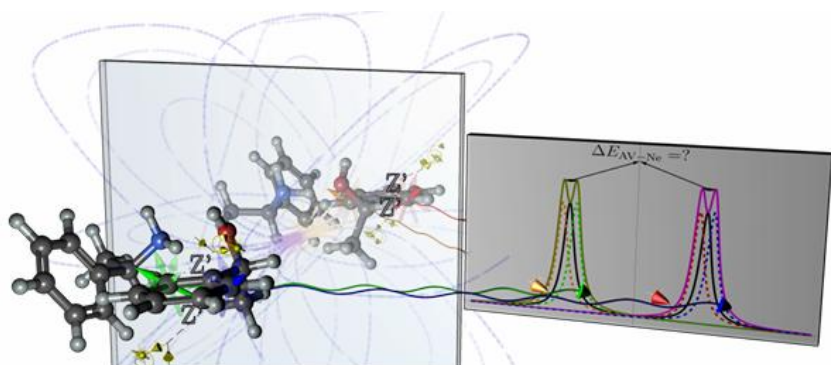
-stereoisomers that are not mirror images

$R,R \leftrightarrow R,S$
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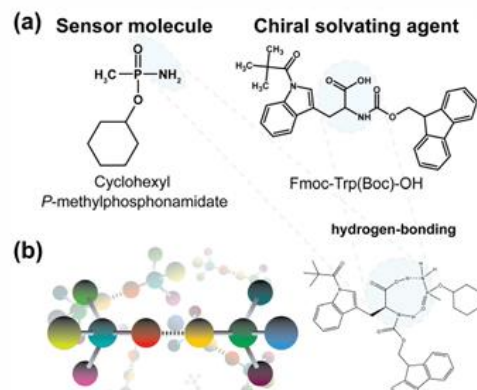
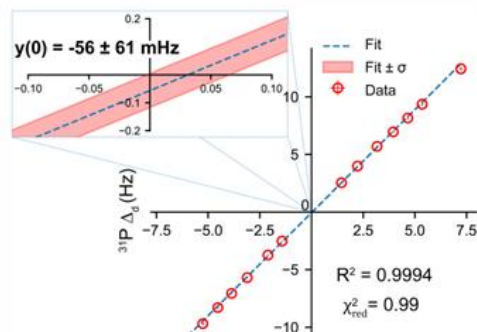
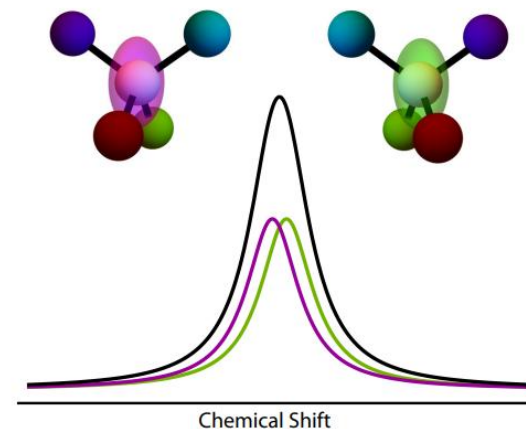
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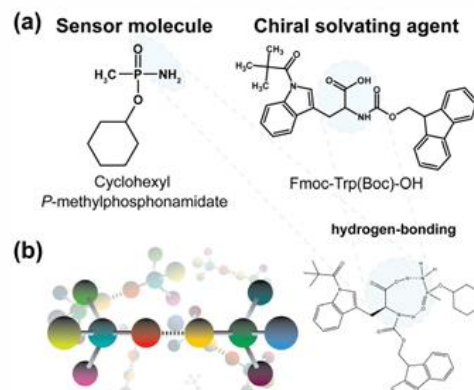
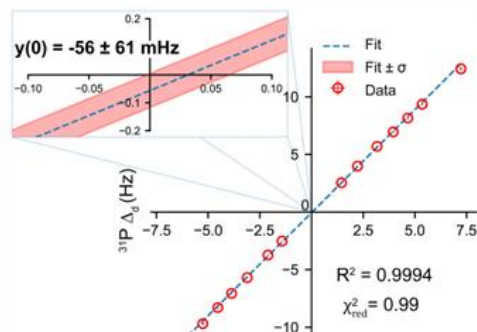
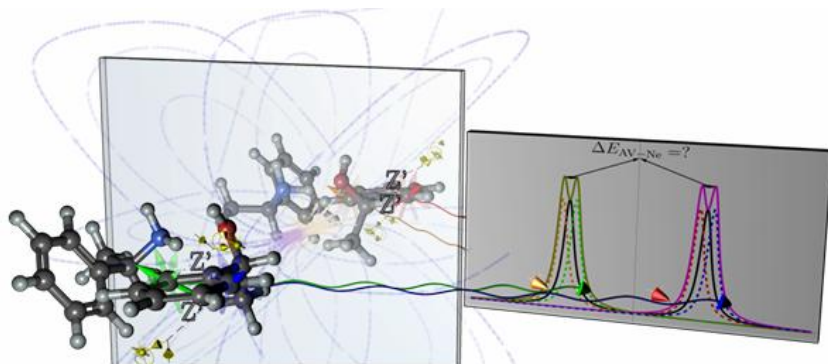
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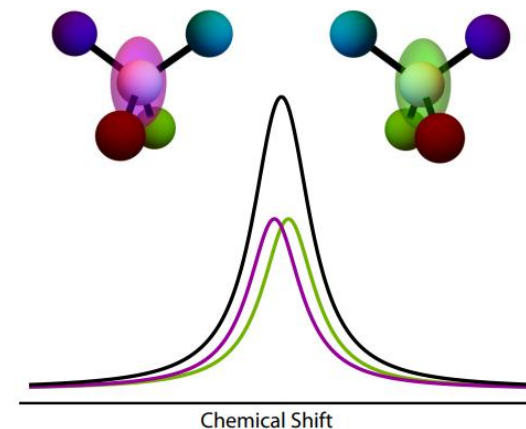
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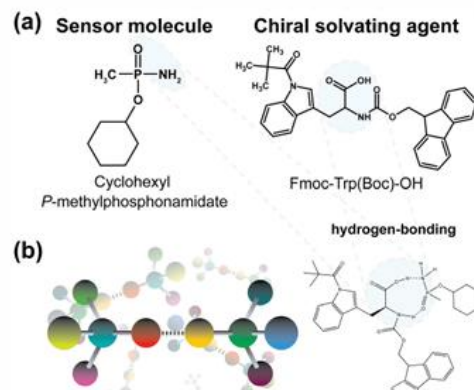
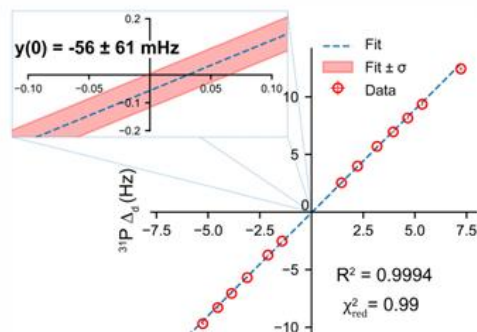
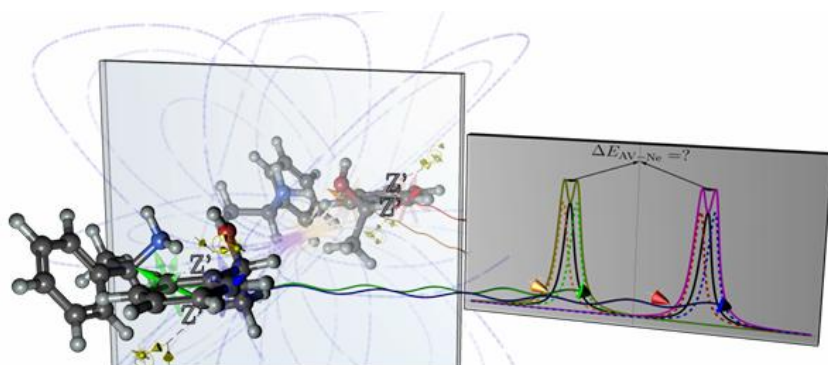
Requirements

- ✓ Soluble compound
- ✓ Few relevant conformers
- ✓ Heavy, NMR-sensitive nucleus
- ✓ High signal-to-noise ratio
- ✓ Narrow linewidth
- ✓ Resolvable diastereomeric interaction
- ✓ PV shifts (mHz)

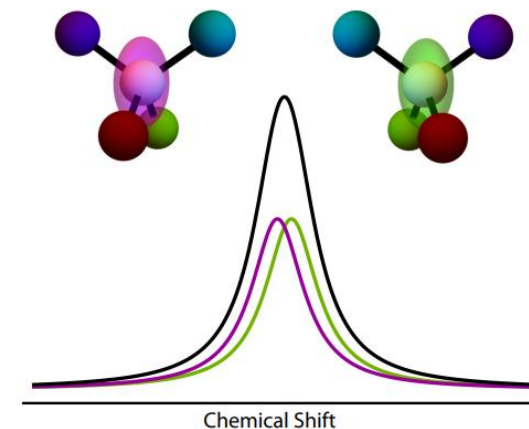
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PV effect

stronger relativistic effects

Relativistic two-component DFT framework

- **Four-component Dirac theory**

Fundamental relativistic reference

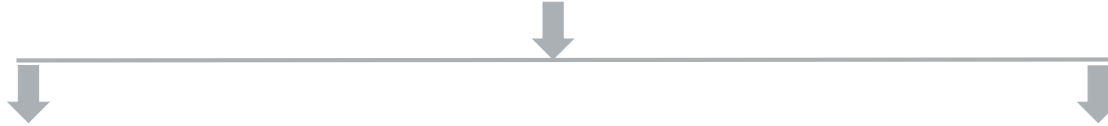
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Relativistic two-component DFT framework

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- **ZORA (Zeroth-Order Regular Approximation)**

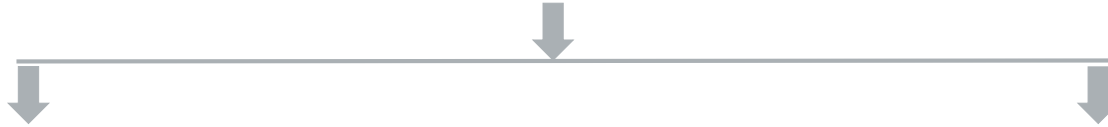
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● ZORA (Zeroth-Order Regular Approximation)

Eliminate the small component by expanding the energy dependence as a power series.

Exact relation:

$$\psi^S(\vec{r}) = \frac{c}{2m_e c^2 - V(\vec{r})} \left(1 + \frac{E}{2m_e c^2 - V(\vec{r})} \right)^{-1} (\vec{\sigma} \cdot \hat{p}) \psi^L(\vec{r})$$

Power series expansion:

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ZORA approximation (zeroth-order):

$$\psi^S(\vec{r}) \approx c\omega^{\text{ZORA}}(\vec{r}) (\vec{\sigma} \cdot \hat{p}) \psi^{\text{ZORA}}(\vec{r})$$

where

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- ω is a position-dependent factor that controls relativity locally

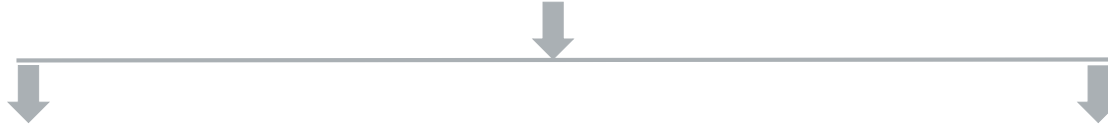
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Diagonalize the Dirac matrix:

$$\begin{pmatrix} H_{LL} & H_{Lp} \\ H_{pL} & H_{pp} \end{pmatrix} \begin{pmatrix} C_{L-} & C_{L+} \\ C_{p-} & C_{p+} \end{pmatrix} = \begin{pmatrix} S_{LL} & 0 \\ 0 & S_{pp} \end{pmatrix} \begin{pmatrix} C_{L-} & C_{L+} \\ C_{p-} & C_{p+} \end{pmatrix} \begin{pmatrix} E_{--} & 0 \\ 0 & E_{++} \end{pmatrix}$$

Define the transformation matrix X:

$$X = C_{p+} C_{L+}^{-1}$$

C_{L+} and C_{p+} are the large- and pseudo-large-component coefficient matrices for the positive-energy states

Unrenormalized X2C Hamiltonian:

$$H_{X2C} = H_{LL} + H_{Lp}X + X^\dagger H_{pL} + X^\dagger H_{pp}X$$

Renormalization matrix:

$$R = S_{LL}^{-1/2} \left[S_{LL}^{-1/2} (S_{LL} + X^\dagger S_{pp} X) S_{LL}^{-1/2} \right]^{-1/2} S_{LL}^{1/2}$$

Final renormalized Hamiltonian:

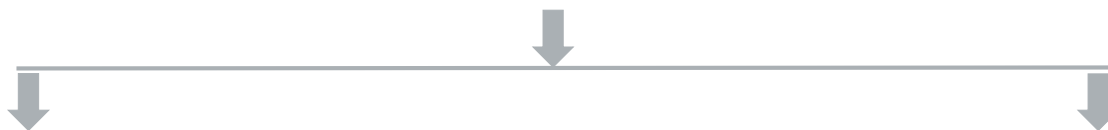
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Two- component Kohn–Sham DFT

$$F_{KS} C_i = \epsilon_i S C_i$$

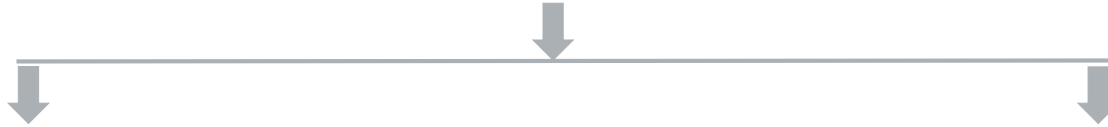
$$F_{KS} = H_{2c} + J[\rho] + V_{XC}[\rho, \vec{m}] + \vec{\sigma} \cdot \vec{B}_{XC}[\rho, \vec{m}]$$

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$$\begin{pmatrix} H_{LL} & H_{Lp} \\ H_{pL} & H_{pp} \end{pmatrix} \begin{pmatrix} C_{L-} & C_{L+} \\ C_{p-} & C_{p+} \end{pmatrix} = \begin{pmatrix} S_{LL} & 0 \\ 0 & S_{pp} \end{pmatrix} \begin{pmatrix} C_{L-} & C_{L+} \\ C_{p-} & C_{p+} \end{pmatrix} \begin{pmatrix} E_{--} & 0 \\ 0 & E_{++} \end{pmatrix}$$

Define the transformation matrix X:

$$X = C_{p+} C_{L+}^{-1}$$

C_{L+} and C_{p+} are the large- and pseudo-large-component coefficient matrices for the positive-energy states

Unrenormalized X2C Hamiltonian:

$$H_{X2C} = H_{LL} + H_{Lp}X + X^\dagger H_{pL} + X^\dagger H_{pp}X$$

Renormalization matrix:

$$R = S_{LL}^{-1/2} \left[S_{LL}^{-1/2} (S_{LL} + X^\dagger S_{pp} X) S_{LL}^{-1/2} \right]^{-1/2} S_{LL}^{1/2}$$

Final renormalized Hamiltonian:

$$H_{rX2C} = R^\dagger H_{X2C} R$$

Two- component Kohn–Sham DFT

$$F_{KS} C_i = \epsilon_i S C_i$$

$$F_{KS} = H_{2c} + J[\rho] + V_{XC}[\rho, \vec{m}] + \vec{\sigma} \cdot \vec{B}_{XC}[\rho, \vec{m}]$$

Molecular properties as response functions

$$\hat{H}(\lambda_1, \lambda_2) = \hat{H}^{(0)} + \lambda_1 \hat{H}_1 + \lambda_2 \hat{H}_2$$

$$\left. \frac{\partial^2 E}{\partial \lambda_1 \partial \lambda_2} \right|_{\lambda_1 = \lambda_2 = 0} = \langle \langle \hat{H}_1 ; \hat{H}_2 \rangle \rangle$$

- $\hat{H}_1$ and $\hat{H}_2$ are perturbation operators
- λ_1 and λ_2 control their strengths

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● NMR shielding

Perturbations

B_0, m_K

External magnetic field B_0 and magnetic moment of the nucleus K

$$\left. \frac{\partial^2 E}{\partial m_{K;\mu} \partial B_{0;\nu}} \right|_{m_{K;\mu}=B_{0;\nu}=0} = \left\langle \left\langle \hat{H}_{m_{K;\mu}} ; \hat{H}_{B_{0;\nu}} \right\rangle \right\rangle = \sigma_{K,\mu\nu}$$

Experimental meaning
The NMR chemical shift

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● Indirect spin-spin coupling

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$$m_K, m_L$$

Magnetic moments of nuclei K and L

$$\left. \frac{\partial^2 E}{\partial m_{K;\mu} \partial m_{L;\nu}} \right|_{m_{K;\mu}=m_{L;\nu}=0} = \left\langle \left\langle \hat{H}_{m_{K;\mu}} ; \hat{H}_{m_{L;\nu}} \right\rangle \right\rangle = K_{KL;\mu\nu}$$

Experimental meaning
Electron-mediated coupling between nuclei; observed as spectral splitting

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● PV-NMR response

Perturbations

$$B_0, m_K, \lambda_{PV}$$

External magnetic field, magnetic moment of nucleus K , and nuclear-spin-dependent weak interaction

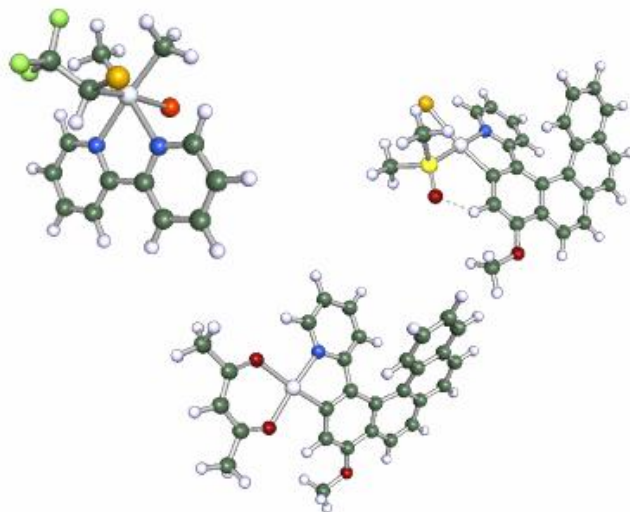
$$\left. \frac{\partial^3 E}{\partial \lambda_{PV} \partial m_{K;\mu} \partial B_{0;v}} \right|_{\lambda_{PV}=m_{K;\mu}=B_{0;v}=0} = \left\langle \left\langle \hat{H}_{B_{0;v}} ; \hat{H}_{\lambda_{PV};m_{K;\mu}} \right\rangle \right\rangle = v_{K;\mu v}^{PV}$$

Experimental meaning
 $\Delta v_{PV} = v_R - v_S$
Enantiomer-dependent NMR frequency difference

Theory-guided design of a molecular PV-NMR experiment

Candidate selection requires a large PV response and a narrow linewidth

PV effects increase strongly with nuclear charge



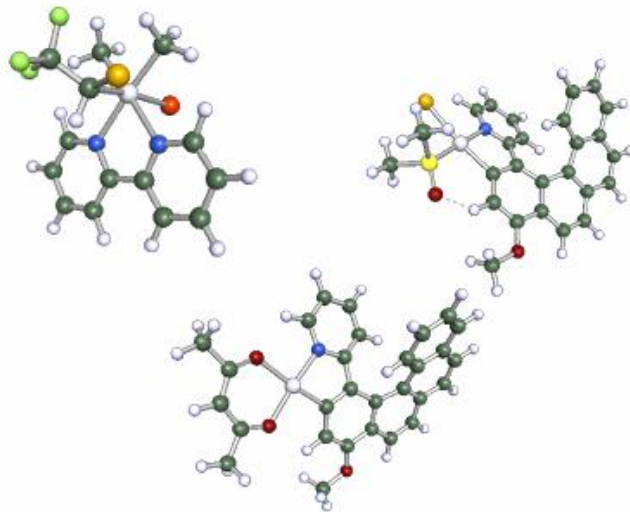
Why Platinum?

- ✓ Heavy element: $Z_{Pt} = 78$
- ✓ NMR-active ^{195}Pt
- ✓ Strong relativistic enhancement of the PV response
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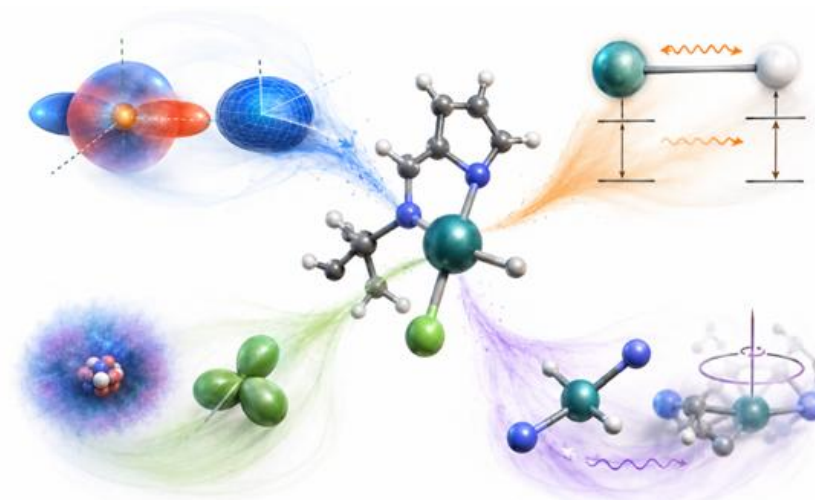


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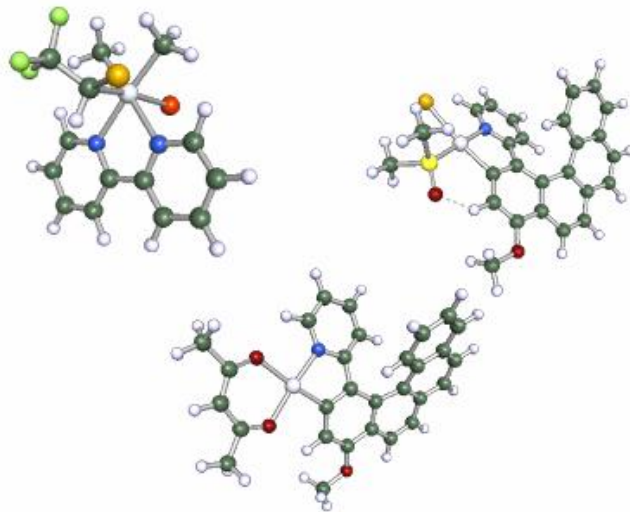
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First-principles calculation of relaxation-relevant properties



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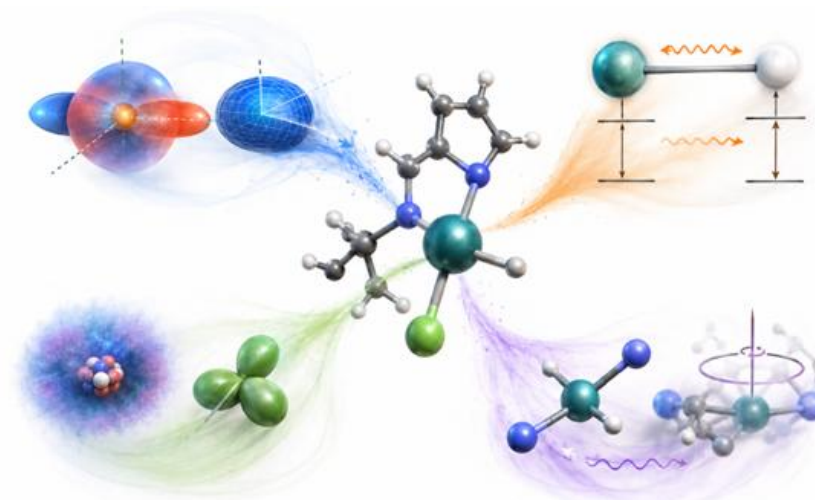


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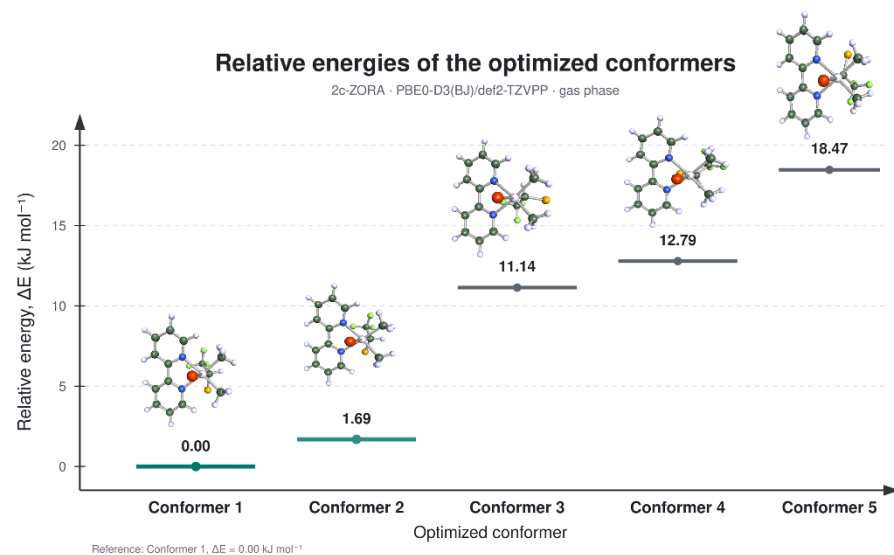
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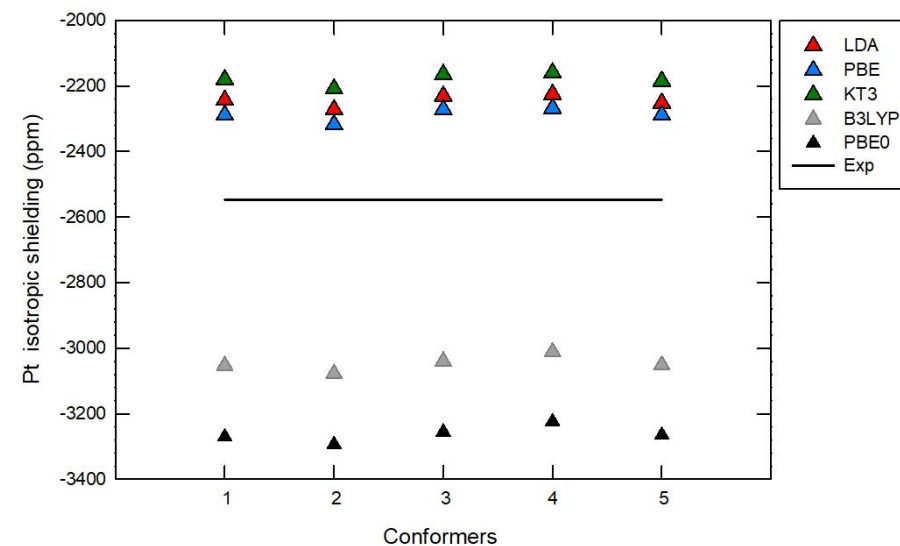


Is CSA the dominant source of line broadening?

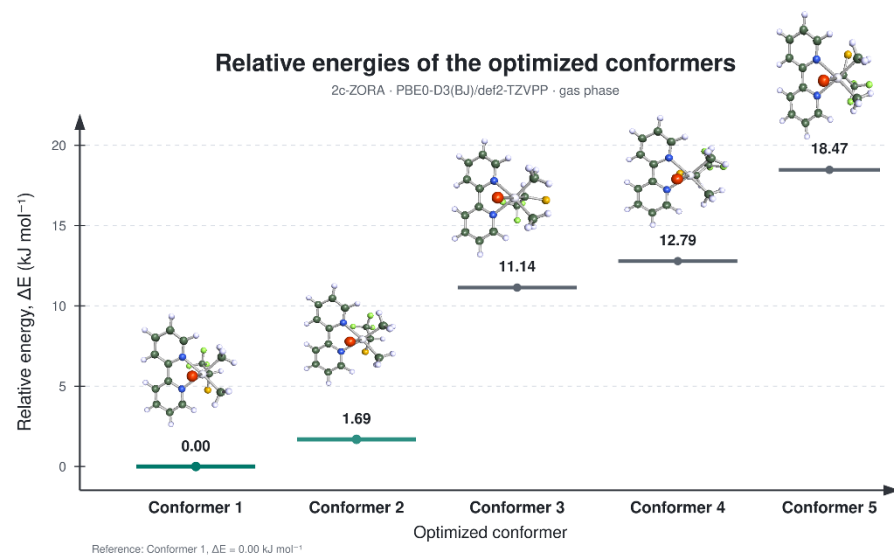
Representative case study: relativistic ^{195}Pt NMR and PV-NMR properties



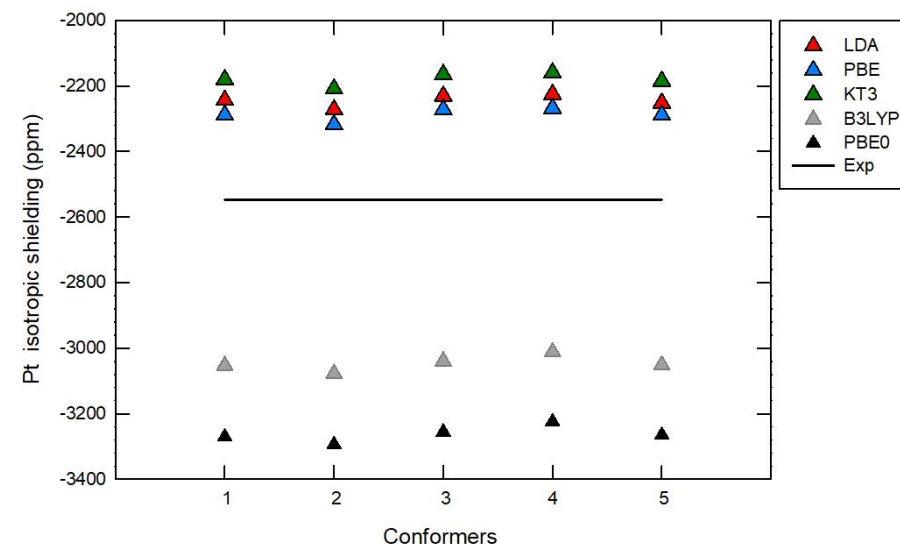
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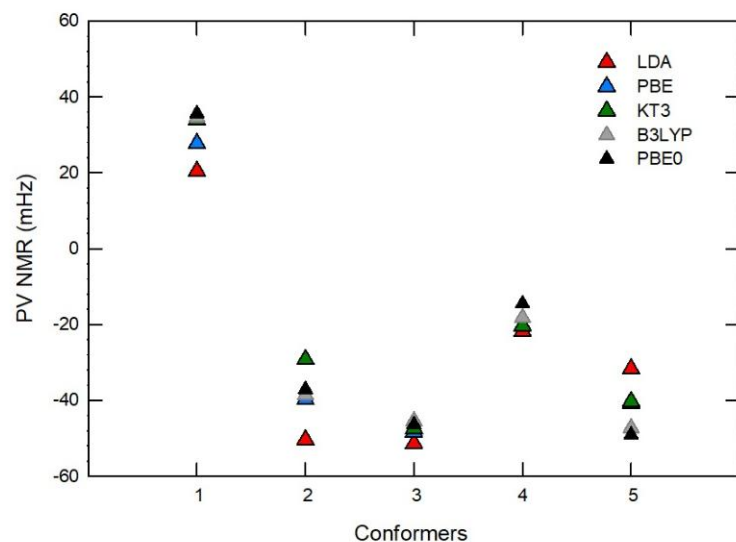
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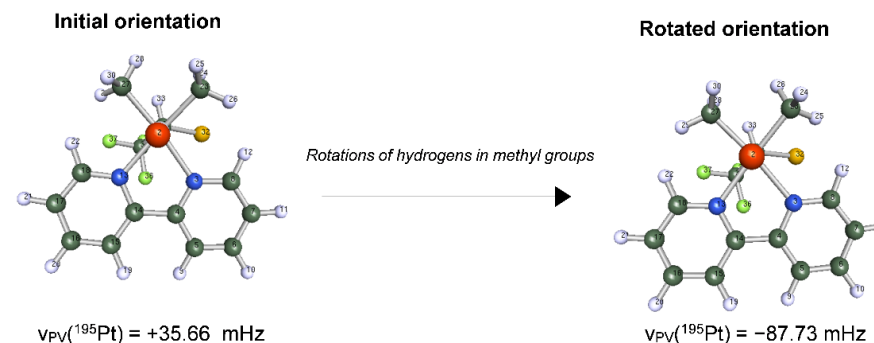
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^{195}Pt PV-NMR response



Methyl-Group Rotation in Conformer 1



Current Challenges for Reliable Magnetic Response

● Current-density treatment

$j_p(r)$ -paramagnetic current density

$J_{ia}(r)$ -spin-current density

$L(r)$ -orbital angular-momentum density

Existing approaches:

- Pauli-based framework
- Limited consistency under strong SOC

Goal

- First-principles relativistic SCDFT
- Consistent current treatment under strong SOC

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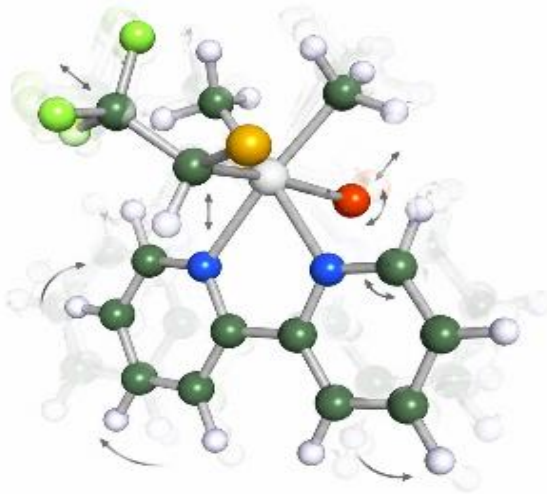
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● Rovibrational corrections

$$J_{vib} \approx J_{eq} + \Delta J_{ZPV}$$

$$|\Delta J_{ZPV}| \lesssim 10\%$$

→ Important for ZULF-NMR



- Numerical approach: $(9N^2)$
- Analytic gradients: $(3N)$ property calculations

Goal: Larger heavy-element systems

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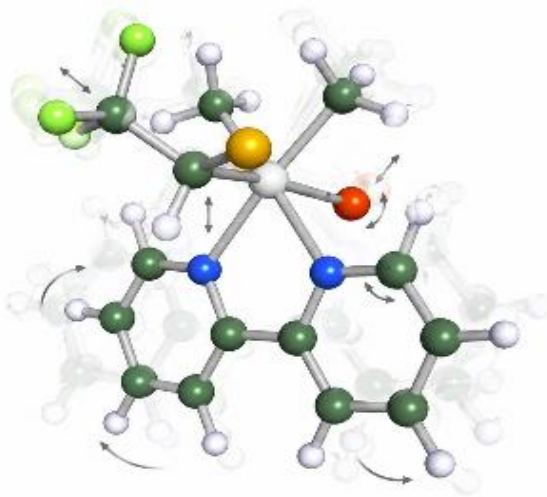
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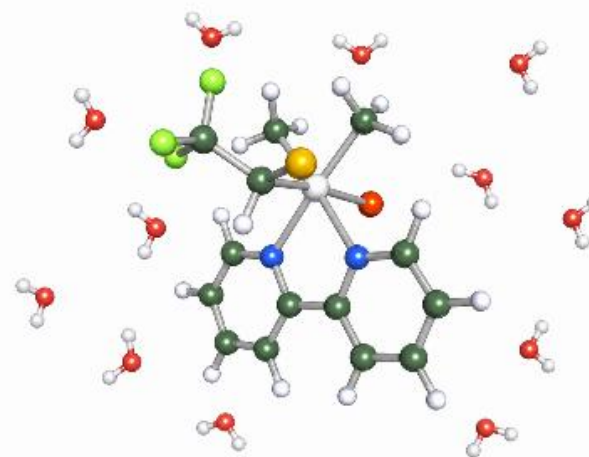


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Goal: Larger heavy-element systems

● Solvent effects

$$|\Delta J_{solv}| \sim \text{a few percent}$$



- Implicit solvent treatment using PCM
- Assess solvent screening of J- couplings
- Validation for ZULF-NMR systems

Goal: Realistic predictions in solution

Conclusions & Outlook

- Chiral Pt complexes: promising PV-NMR candidates
- Pt shielding: XC-functional dependence
- PV response: Conformer- sensitive
- Two-bond J -coupling: sensitive (ongoing)

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- Theory: Spin-Current DFT
 - Linewidths: first-principles relaxation calculations
 - Solvent and rovibrational corrections
 - Design PV-NMR going toward ZULF-NMR