Towards efficient 4th-order MBPT calculations of nuclear matter

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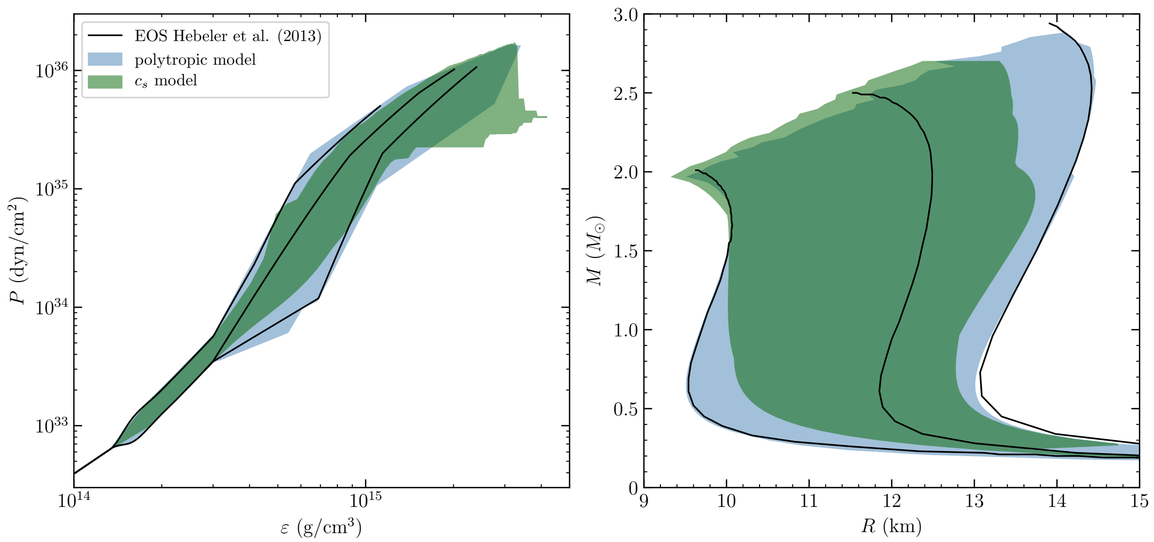
Nuclear Equation of State



Greif *et al.*, MNRAS (2019)

- Equation of state (EOS) relates pressure P and energy density ε (or here: $\frac{E}{A}$ with n)
- Understanding astrophysical phenomena requires knowledge about EOS
- This talk: calculating low-density EOS with many-body perturbation theory

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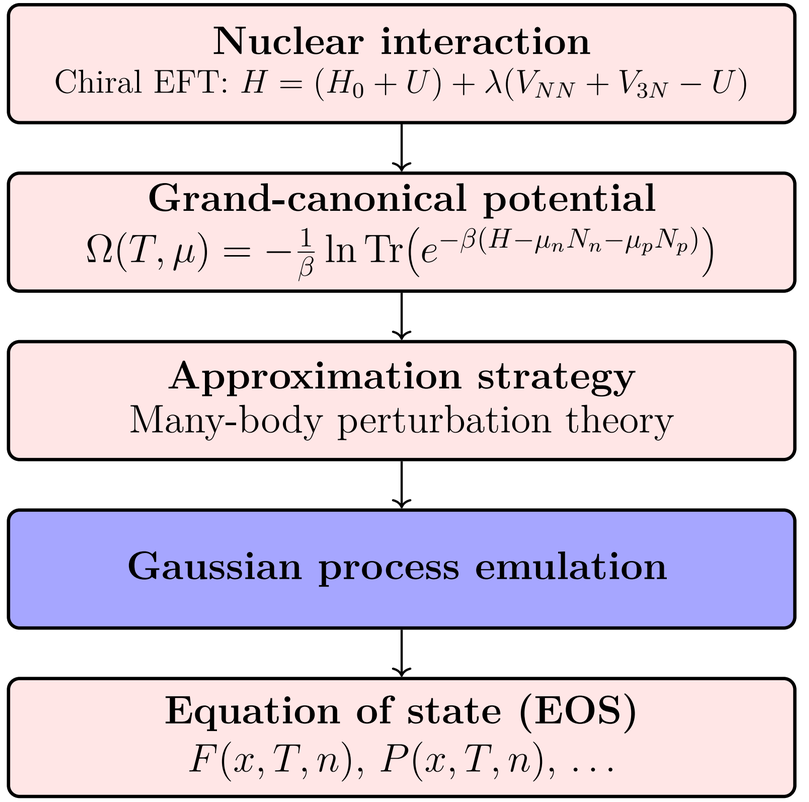
Low-density EOS from chiral EFT interactions

Drischler *et al*., PRL (2019), Keller *et al*., PRC (2021), Keller *et al*., PRL (2023)

- Use Chiral EFT as low-density effective theory of strong interaction
- Describe infinite, uniform, isotropic nuclear matter with grand-canonical potential $\Omega(T, \mu_n, \mu_p)$
- Perturbative expansion of Ω using MBPT
- Obtain free energy through Legendre transformation, pressure through derivative,…



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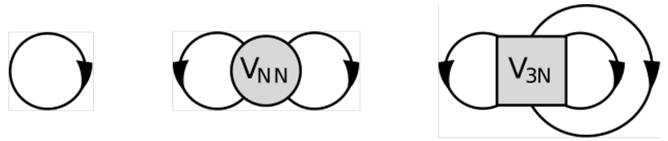
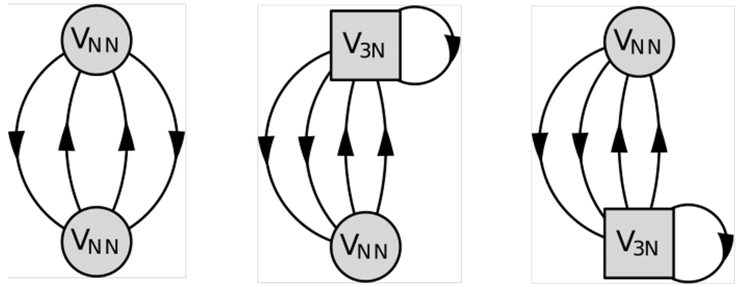
MBPT framework

Drischler *et al*., PRL (2019), Keller *et al*. PRC (2021), Keller *et al*., PRL (2023)



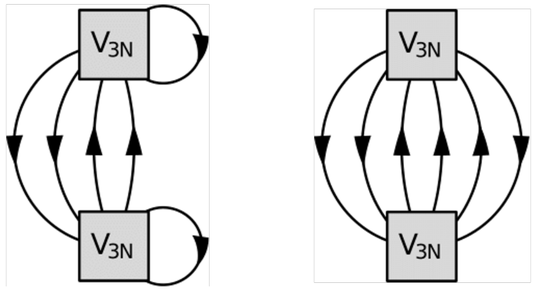
- Many-body method using expansion of grand-canonical potential

$$\Omega(T, \mu_n, \mu_p) = \sum_{l=0}^{\infty} \lambda^l \Omega_l$$



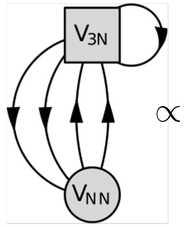
- Calculations conducted up to MBPT(3)

- Number and dimensionality of higher-order diagrams rises very fast
  → Monte-Carlo integration and importance sampling



Hebeler, Schwenk, PRC (2010)

e.g.:



$$\propto \mathrm{Tr}_{\sigma,\tau}\Pi_i \int \frac{d\mathbf{k_i}}{(2\pi)^3} \langle 12|V_{\mathrm{NN}}|34\rangle\langle 345|V_{\mathrm{3N}}|125\rangle f_R(p,q) f_R(p',q') \frac{n_{\mathbf{k_1}} n_{\mathbf{k_2}} (1-n_{\mathbf{k_3}})(1-n_{\mathbf{k_4}})}{\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4} n_{\mathbf{k_5}}$$

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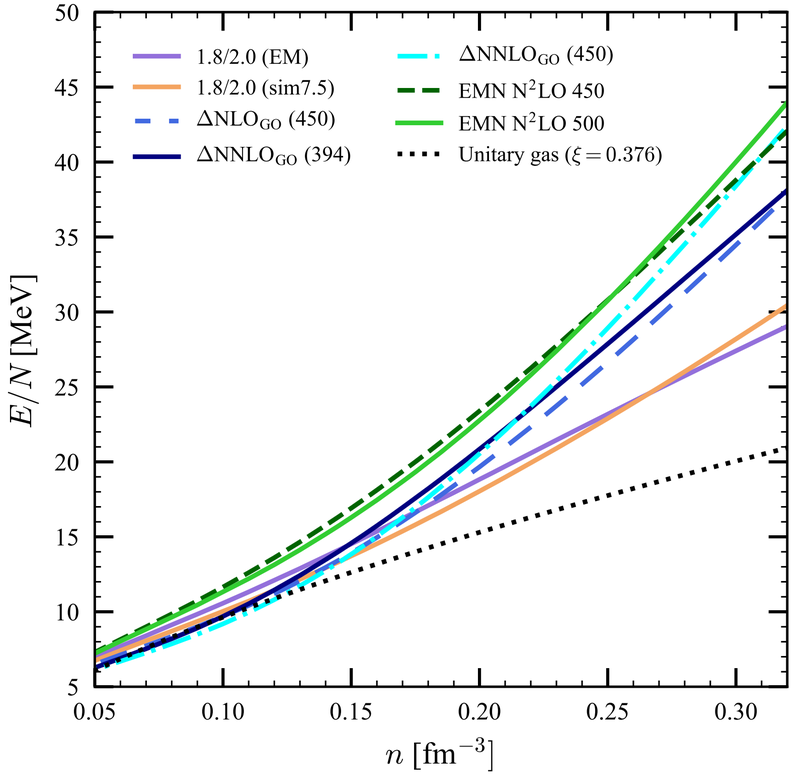
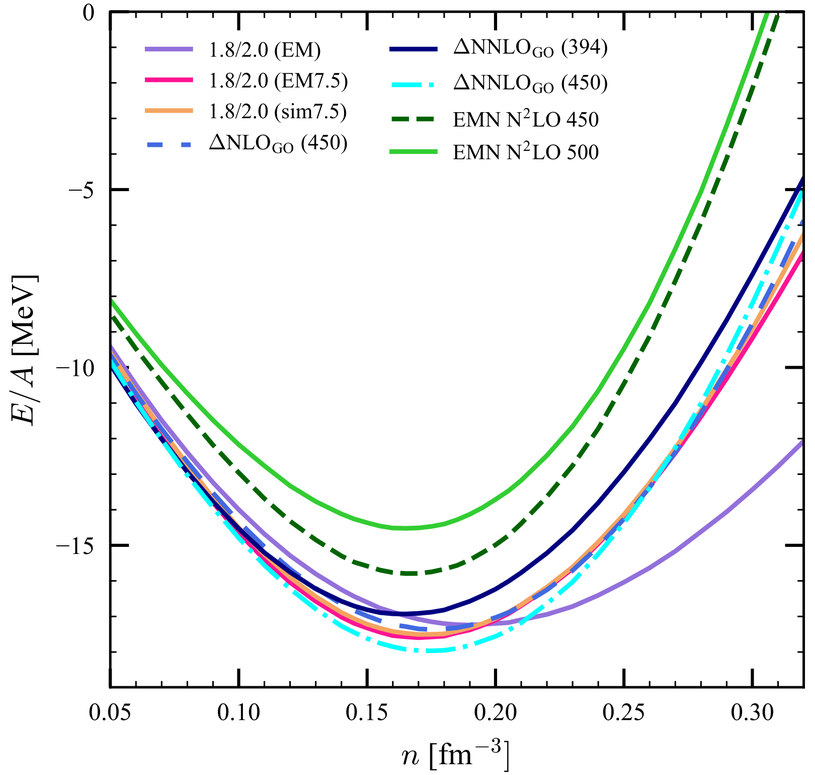
Low-density EOS from chiral EFT interactions

F.Alp, YD *et al*. PRC (2025)

- Calculated EOS for different chiral potentials at $T = 0$



PNM



SNM

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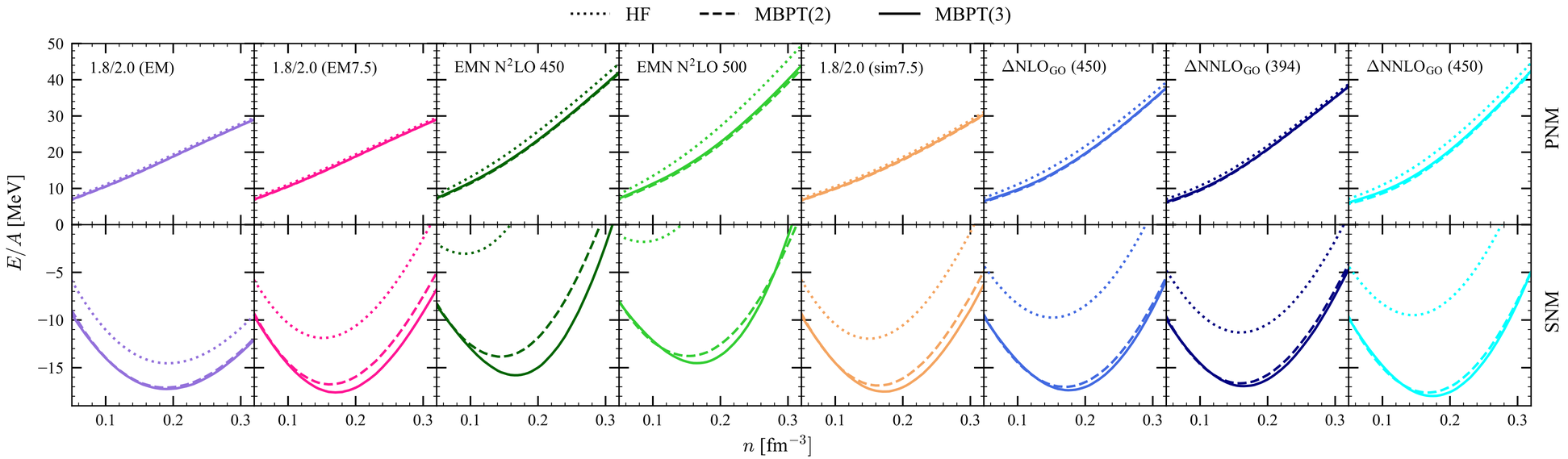
Many-body convergence

- Limited thorough tools to assess many-body uncertainties at the time
  → estimate by MBPT(3) contribution
- SRG-evolved and deltafull potentials show better MBPT convergence
- MBPT(3) contribution dominated by chiral uncertainty (where applicable)



F.Alp, YD *et al*. PRC (2025)

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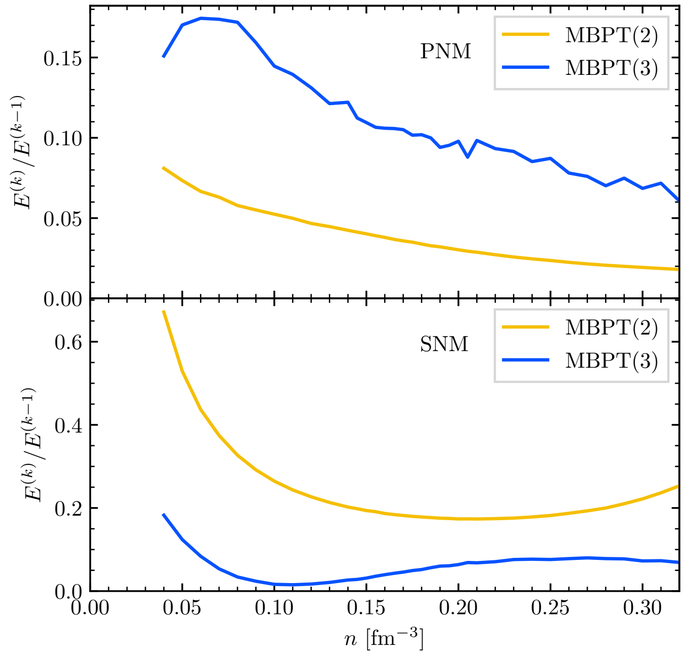
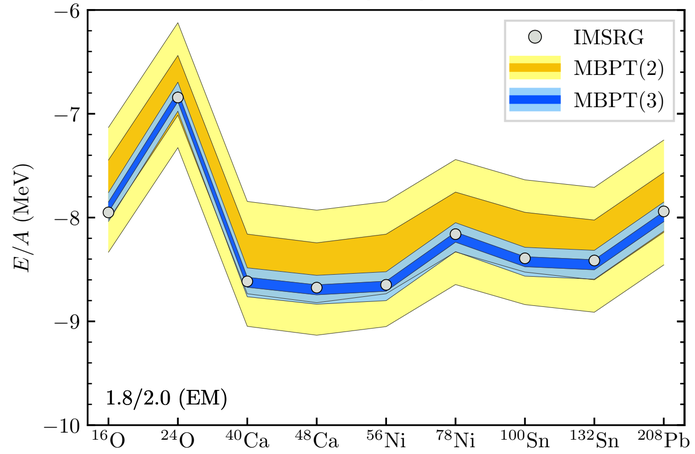
UQ for MBPT

Svensson *et al.*, PRC (2026)

- Goal: Estimate missing contribution of higher-order MBPT terms
- Bayesian framework to learn expansion coefficients of MBPT series
  - Fourth-order can give further insights into convergence behavior (50% more data)





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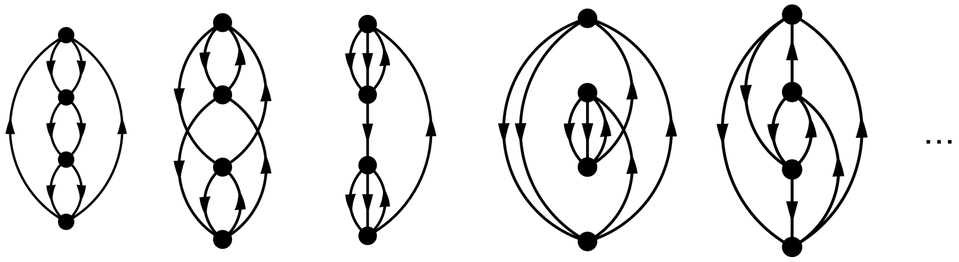
MBPT(4) calculations of nuclear matter

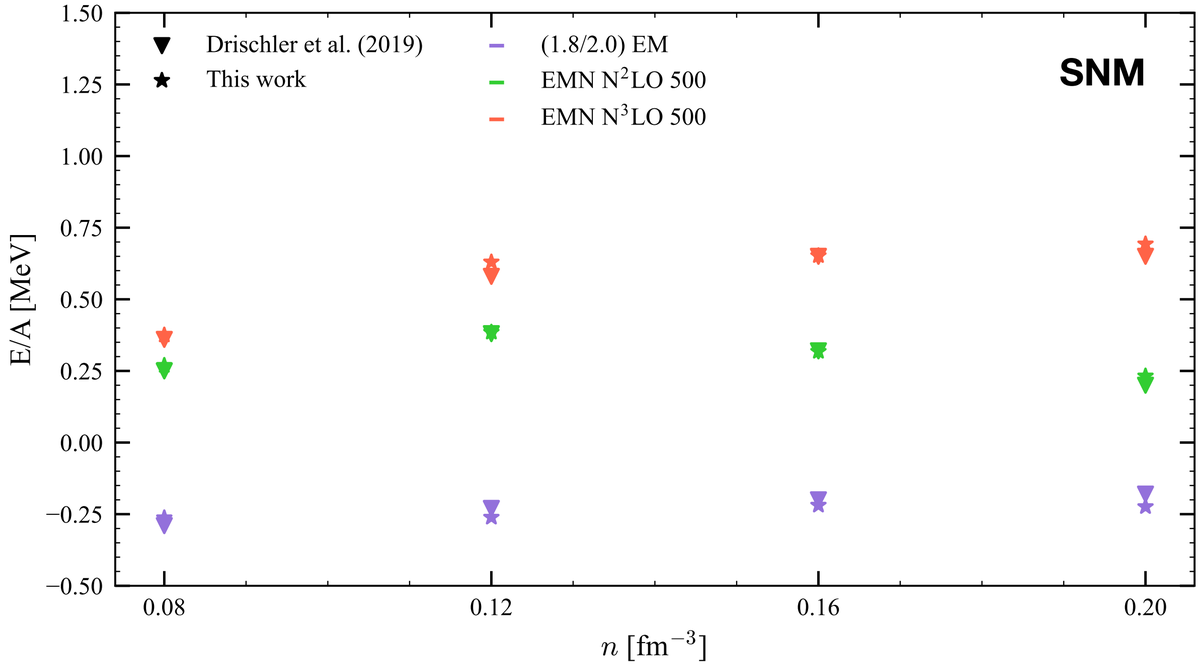
Arthuis *et al.*, Comput.Phys.Commun. (2019) also, see Zhen Li's talk

- MBPT(4) NN diagrams with help of ADG code



- Successfully tested NN contributions

  Drischler *et al.*, PRL (2019)



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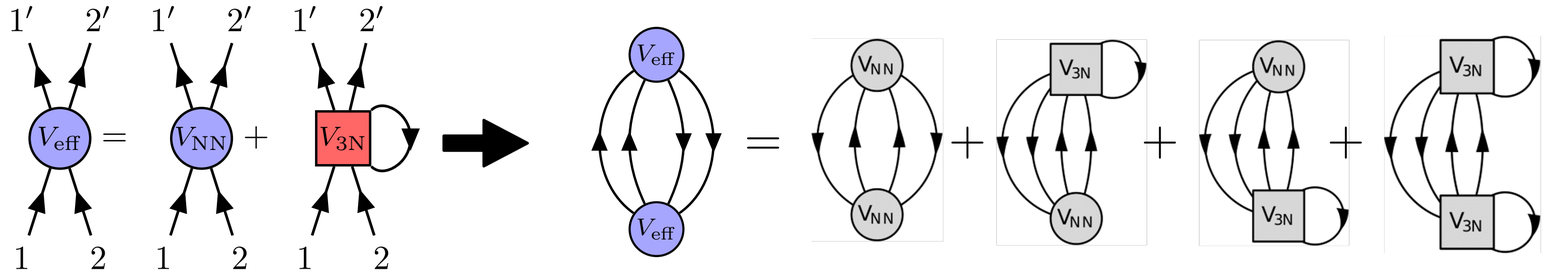
Including 3N: Normal-ordered two-body approx.

- Problem: Already hundreds of 3N diagrams at MBPT(4)→ costly!
- Many diagrams involve self-closing loops
- Construct effective two-body vertex corresponding to NO2B approx.:



where: $\overline{V}_{3\mathrm{N}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_1', \mathbf{k}_2') = \mathrm{Tr}_{\sigma_3,\tau_3} \int \frac{d\mathbf{k}_3}{(2\pi)^3} n_{\mathbf{k}_3} f_R(p,q) f_R(p',q') \langle 1'2'3|V_{3\mathrm{N}}|123\rangle$

- Benefit: Reduce number of diagrams at high MBPT orders

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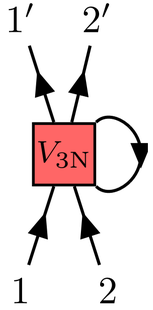
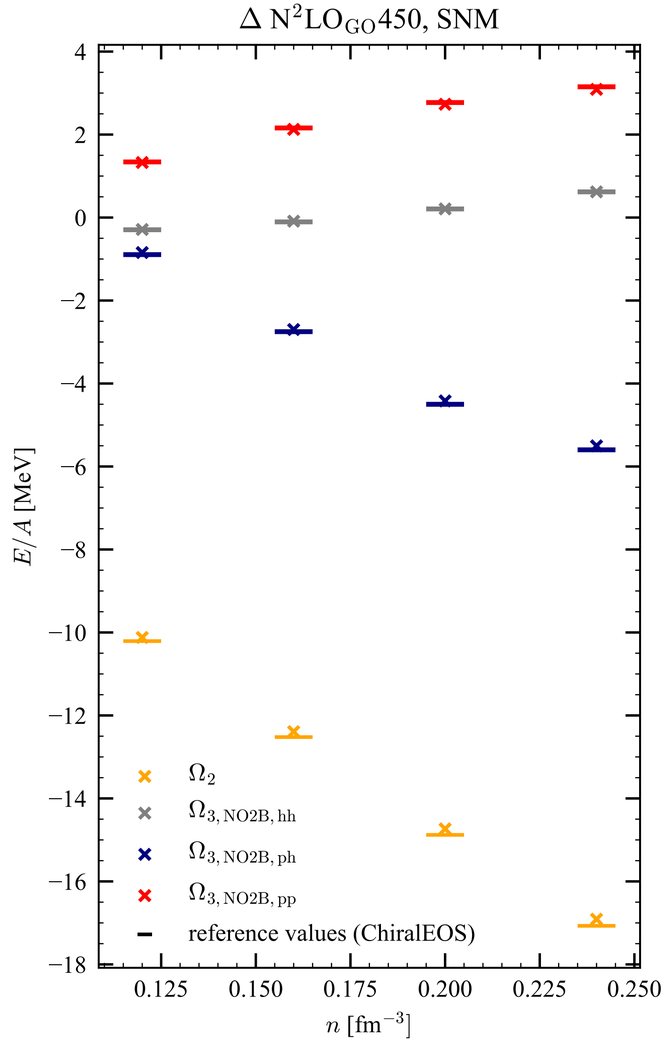
NO2B-vertex: Benchmarks

- Precalculate eff. 2B-vertex on grid in $\mathbf{P}_{\text{com}}, \mathbf{p}_{\text{rel}}, \mathbf{p}'_{\text{rel}}$ and interpolate when evaluating diagrams

  ➡ 6-dimensional object to prestore

- Eff. 2B-vertex within ~1% of full results in MBPT(2)/(3)





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MBPT(4) calculations of nuclear matter

- Effective 2B-vertex permits calculation of NN+3N interaction at complexity of NN level
  - Renders MBPT(4) accessible at NN+3N level
  - Increase overall precision of MBPT calculations

Preliminary

SNM, EMN N2LO 450, n = $0.1661\,\mathrm{fm}^{-3}$:

MBPT order	HF	MBPT(2)	MBPT(3)	MBPT(4)
Contribution (MeV)	-22.02	-14.17	-2.28	0.07

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Summary

- Chiral EFT and pert. theory for nuclear matter properties at low densities
- Calculated PNM and SNM for different chiral interactions
- Recently: Implementation of effective 2B-vertex / MBPT(4)
- Outlook: Improved UQ for MBPT in nuclear matter

Thanks to my collaborators:

Faruk Alp, Kai Hebeler, Isak Svensson, Achim Schwenk

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