NUCLEAR MATTER PROPERTIES FROM FERMI LIQUID THEORY USING MBPT

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MITP Workshop 2026: Many-Body Theory: Nuclear Physics Meets Quantum Chemistry

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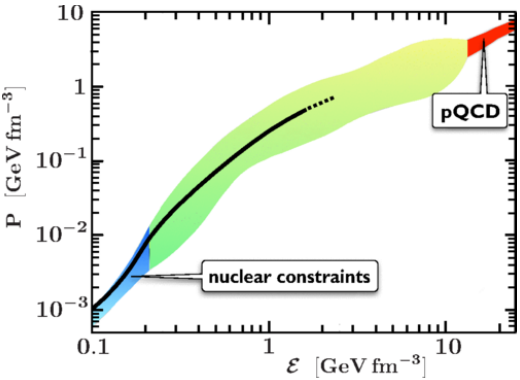
Nuclear equation of state



from Friman and Weise, Phys. Rev. C **100**, 065807 (2019)

- Equation of state (EOS) relates pressure *P* and energy density $\mathcal{E}$ at temperature T
- EOS spans large density region $\rightarrow$ change in degrees of freedom
- Low-density regime well constrained from chiral effective field theories (chiral EFT)
- High-density regime constrained by perturbative QCD
- Astrophysical observations probe intermediate densities

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Challenge of describing dense matter

Starting from:

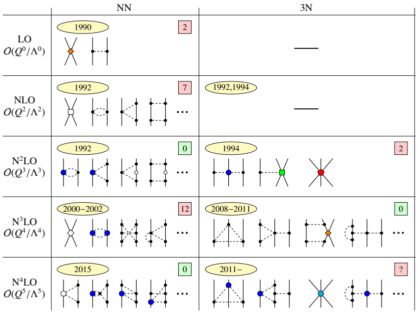

Figure from Hebeler, Phys. Rep. **890**, 1 (2021)

$\longrightarrow$? $\longrightarrow$

We want: **Properties of dense nuclear matter** e.g.

- Equation of state (EOS)
- Finite temperature and asymmetry
- Effective masses
- Transport properties

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Introduction to Fermi liquid theory (FLT)

- Developed by Lev Landau in 1956

  The Theory of a Fermi Liquid., Sov. Phys. JETP **3**, 920 (1956)

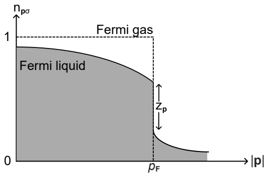
- Describe interacting Fermi system at low-excitation energies
- Free Fermi gas forms reference state for interacting normal Fermi liquid
  $\rightarrow$ **adiabatic connection**
- Connects macroscopic observables with microscopic parameters
- Experimentally well studied in liquid ^{3}He and routinely used in solid state physics
- Constitutes an effective field theory
- Degrees of freedom are long-lived quasiparticles near Fermi surface

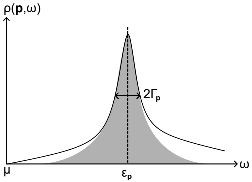
From Fermi gas to Fermi liquid

Goal: Describe interacting Fermi system at low temperatures

Idea: Use free Fermi gas as reference state and adiabatically turn on interactions

- From $\varepsilon^{(\mathrm{FG})} = \frac{\mathbf{p}^2}{2m}$ to quasiparticles with energy $\varepsilon_{\mathbf{p}\sigma}[n_{\mathbf{p}\sigma}]$
- Consider change from ground-state distribution: $\delta n_{\mathbf{p}\sigma} = n_{\mathbf{p}\sigma} - n^0_{\mathbf{p}\sigma}$
- Change in energy density: $\delta\mathcal{E}^{(\mathrm{FL})} = \sum_{\mathbf{p}\sigma} \varepsilon_{\mathbf{p}\sigma} \delta n_{\mathbf{p}\sigma} = T\delta s + \mu\delta n$





Quasiparticle lifetime

$$\tau_{\mathbf{p}} \propto (|\mathbf{p}| - p_F)^{-2}$$

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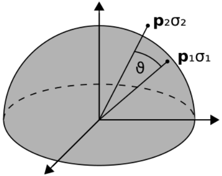
Quasiparticle interaction

- Energy density expanded around $n^0_{\mathbf{p}\sigma}$

$$\mathcal{E}[n_{\mathbf{p}\sigma}] = \mathcal{E}_0 + \sum_1 \varepsilon_1 \delta n_1 + \frac{1}{2}\sum_{1,2} \mathcal{F}_{(1,2)} \delta n_1 \delta n_2 + \dots$$



Decomposing interaction:

$$\mathcal{F} = \frac{\delta^2 \mathcal{E}}{\delta n_1 \delta n_2} = f + g\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \dots$$

Expanding in Legendre polynomials:

$$x(\cos\vartheta) = \sum_{L=0}^{\infty} x_L P_L(\cos\vartheta)$$

Landau parameters:

$$X_L = N(0) \cdot x_L$$

Stability criteria:

$$X_L > -(2L+1)$$

Decomposition for pure neutron matter (PNM)

$$\mathcal{F}^{\text{PNM}} = f\,(\mathbb{1}_1 \otimes \mathbb{1}_2) + g\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + h\,S_{12}(\hat{q}) + k\,S_{12}(\hat{P}) + l\,(\vec{\sigma}_1 \times \vec{\sigma}_2)(\hat{q} \times \hat{P})$$

$\langle s_1, s_2 \| \mathcal{F}^{\text{PNM}} \| s_3, s_4 \rangle$	$\|\uparrow\uparrow\rangle$	$\|\uparrow\downarrow\rangle$	$\|\downarrow\uparrow\rangle$	$\|\downarrow\downarrow\rangle$
$\langle\uparrow\uparrow\|$	$f+g+2h-k$	l	$-l$	$3k$
$\langle\uparrow\downarrow\|$	l	$f-g-2h+k$	$2g-2h+k$	l
$\langle\downarrow\uparrow\|$	$-l$	$2g-2h+k$	$f-g-2h+k$	$-l$
$\langle\downarrow\downarrow\|$	$3k$	l	$-l$	$f+g+2h-k$

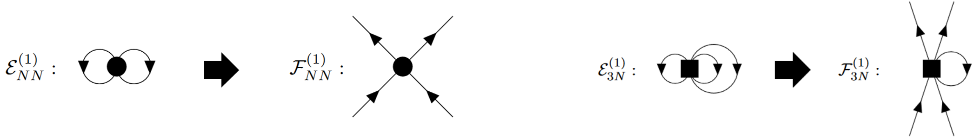
We focus on:

- Speed of sound
  $$c_s^2 = \frac{\partial P}{\partial \mathcal{E}} = \frac{p_F^2}{3\mu m*}(1+F_0)$$
- Effective mass
  $$\frac{m*}{m} = 1 + \frac{F_1}{3}$$

Diagrammatic calculation

- MBPT procedure for quasiparticle interaction:

$$\boxed{\mathcal{E}^{(n)} \longrightarrow \mathcal{F}^{(n)}_{(1,2)} = \frac{\delta^2 \mathcal{E}^{(n)}}{\delta n_1 \delta n_2} \longrightarrow f_L^{(n)}(p_F) \longrightarrow m^*, c_s^2}$$



Friman, Hebeler, and Schwenk, Lect. Notes Phys. **852**, 245 (2012)

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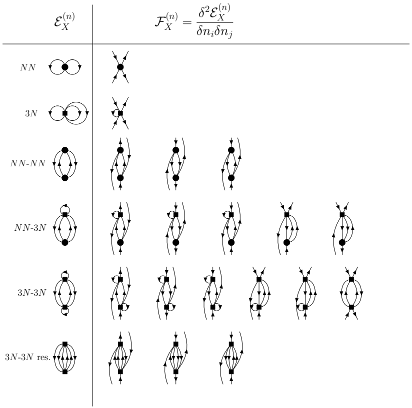
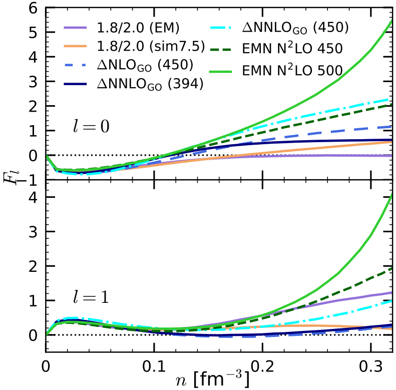
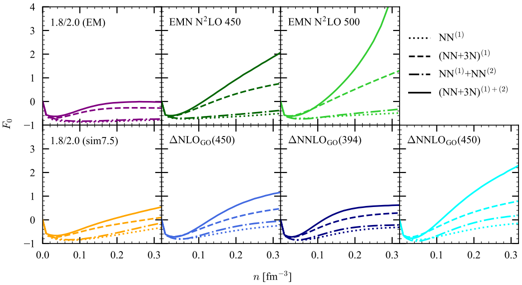
Landau parameters for different interactions

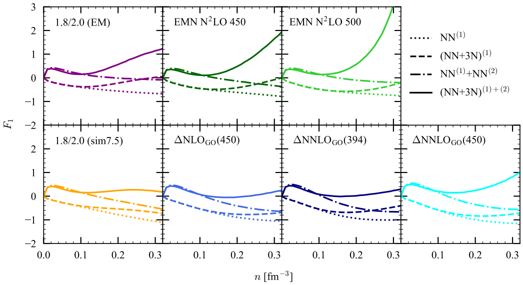
FA, Dietz, Hebeler & Schwenk, Phys. Rev. C **112** (2025)





MBPT behavior





- 3N forces (dashed) yield repulsive contribution
- Landau parameters sensitive to interaction and cutoff at larger densities
  $\rightarrow$ not well converged for large Λ_{3N}
- Need MBPT(3) for better convergence assessment

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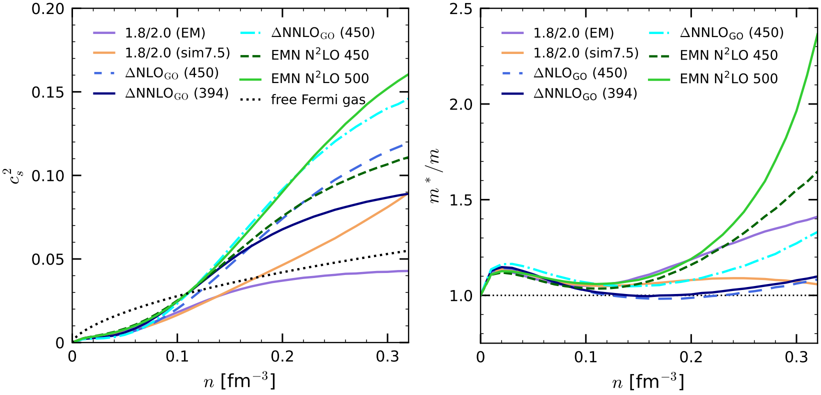
Speed of sound and effective mass



- From Landau parameters

$$c_s^2 = \frac{1}{3}\frac{k_F^2}{m^*\mu}(1+F_0)$$

and

$$\frac{m^*}{m} = 1 + \frac{F_1}{3}$$

- Similar trend as Landau parameters

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Effective mass from self energy

- Idea is to calculate effective mass from single-particle energy

$$\frac{m^*}{m} = \left[\frac{m}{k}\frac{d\varepsilon}{dk}\right]^{-1}_{k=k_F}$$

- Single-particle energy given by

$$\varepsilon(k) = \frac{k^2}{2m} + \Sigma(k, \omega = \varepsilon(k))$$

- With the self energy

$$\Sigma(k, \omega) = G_0^{-1} - G^{-1}$$

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MBPT for self energy

- First-order self energy

$$\Sigma^{(1)}(k_1) = \sum_{2} \langle 12 | V_{NN} | 12 \rangle n_2 + \frac{1}{2} \sum_{23} \langle 123 | V_{3N} | 123 \rangle n_2 n_3,$$

- Second-order self energy requires iterative solving

$$\Sigma^{(2)}(k, \varepsilon_k) = \Sigma^{(2)}_{\mathrm{2p1h}} + \Sigma^{(2)}_{\mathrm{1p2h}}$$

$$= \frac{1}{2} \sum_{234} \frac{|\langle 12 | V_{NN} | 34 \rangle + \sum_5 \langle 125 | V_{3N} | 345 \rangle \mathrm{n}_5 |^2}{\varepsilon_k + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + i\eta} n_2 \bar{n}_3 \bar{n}_4 + \Sigma^{(2)}_{\mathrm{1p2h}}$$

- For following investigation we pick EMN450 NN+3N interaction at N^{2}LO

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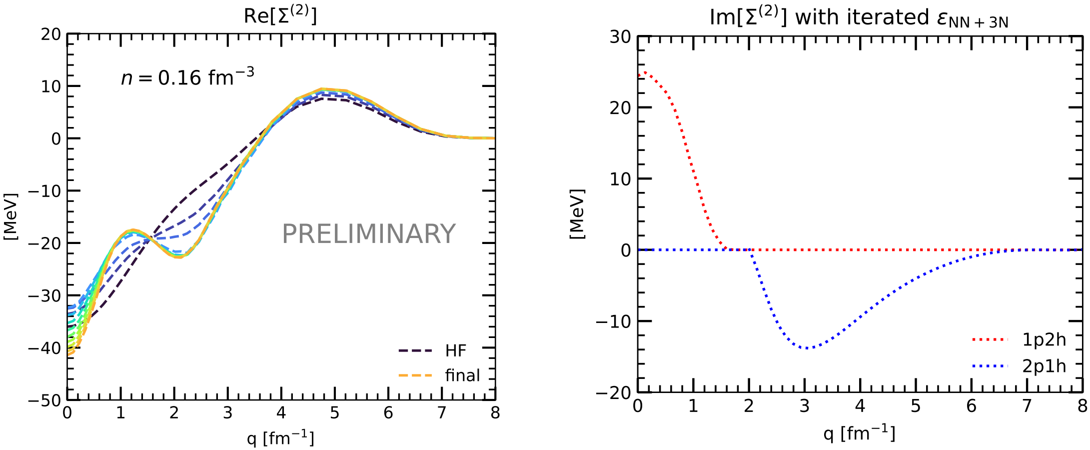
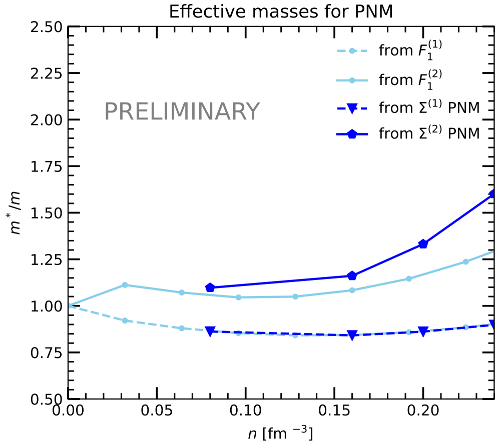
Iteration solution to self energy



Resulting effective masses



- Comparing effective mass from self-energy and FLT calculations
- HF calculations are equivalent
- Effective mass from self-energy contribution picks up larger repulsive contribution from iterative solving
- Reminder: FLT calculations are performed with HF reference state

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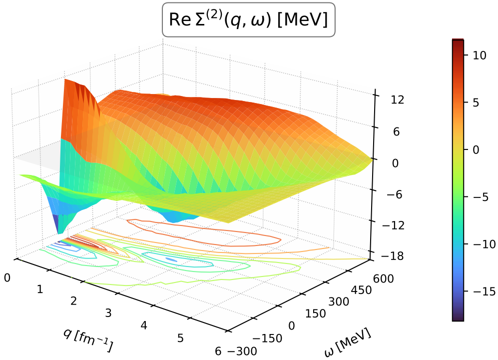
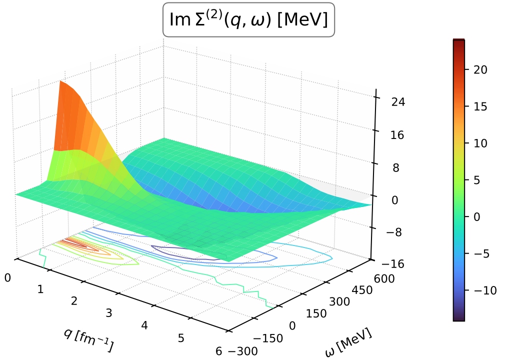
Investigation for complete structure of Σ





- Access to $\Sigma(q, \omega)$ allows for construction of spectral function

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Conclusion & Outlook

- Using chiral EFT and MBPT provides useful tool for determining nuclear matter properties
- FLT can provide access to additional properties such as c_s^2 and m^*
- Investigations of N^3LO contributions (including 3N)
- Currently working on extending this analysis to symmetric nuclear matter
- Future work on two-component FLT to access arbitrary proton fractions