GW probes of PBHs: lensing and binaries

Ville Vaskonen

M. Andrés-Carcasona et al, 2605.15749
M. Raidal, VV and H. Veermäe, 2404.08416
J. Urrutia, VV and H. Veermäe 2303.17601
J. Urrutia and VV, 2109.03213
O. Pujolas, VV and H. Veermäe, 2107.03379
VV and H. Veermäe, 1908.09752
M. Raidal, C. Spethmann, VV and H. Veermäe, 1812.01930

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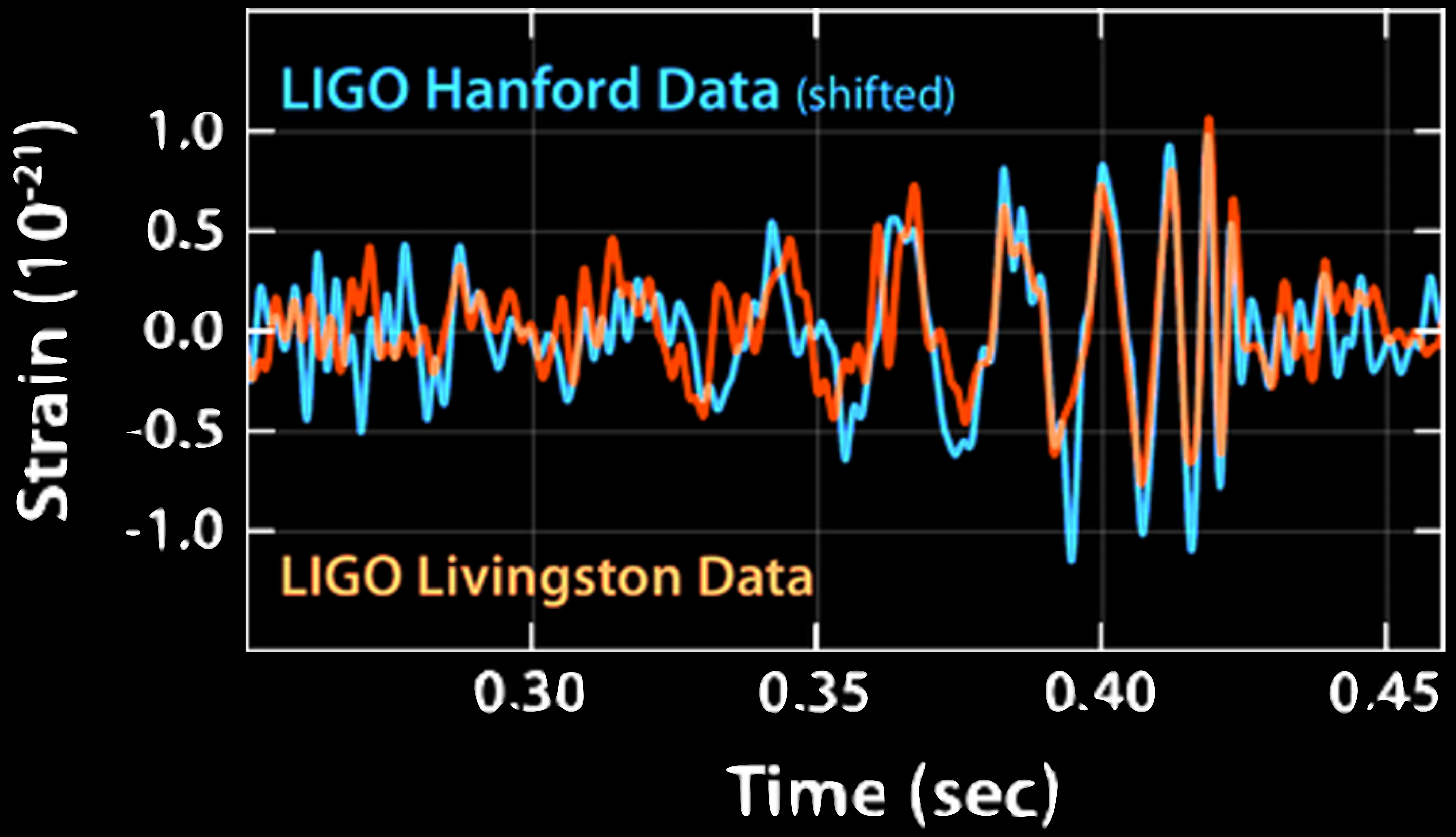
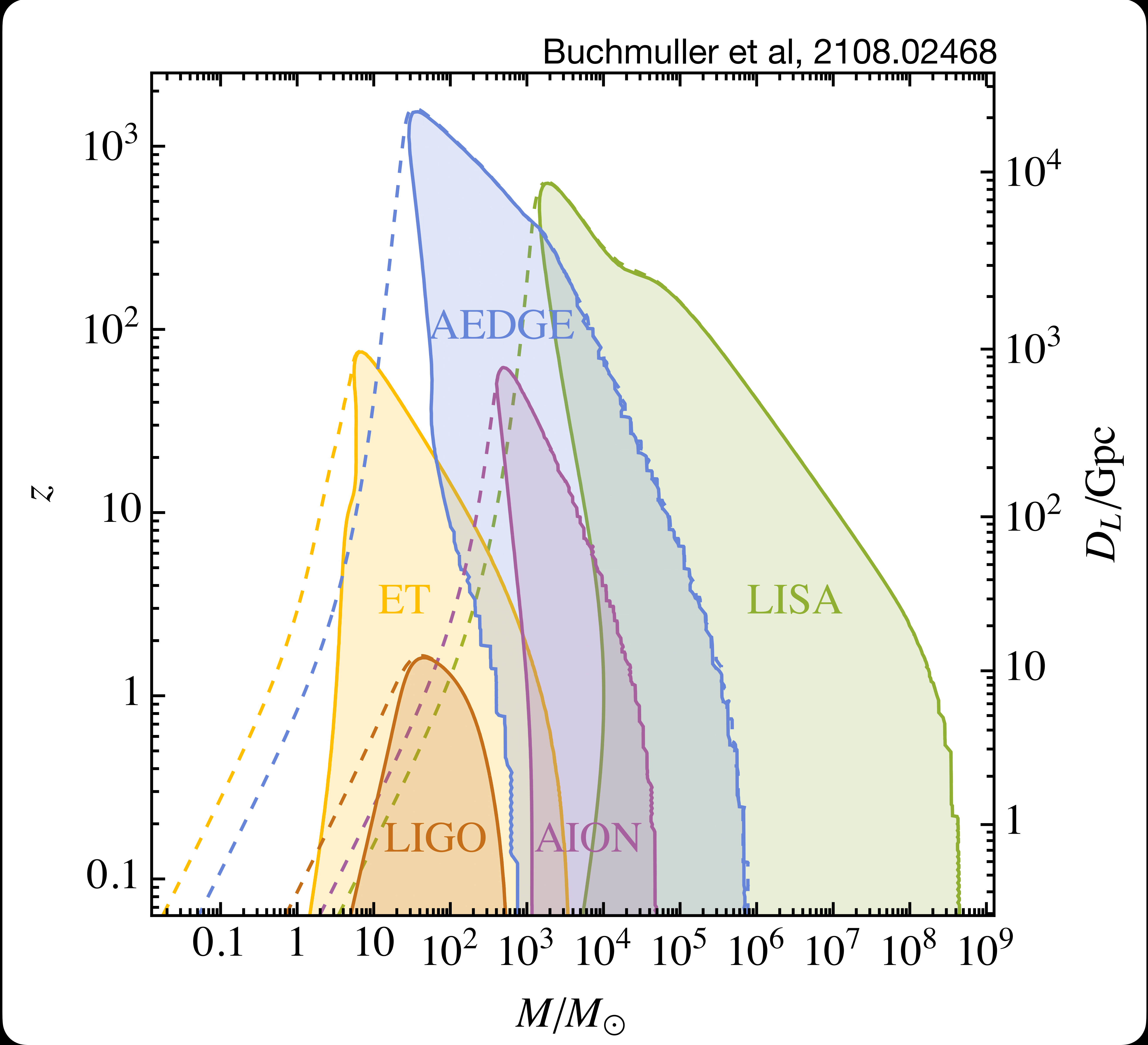
Mainz, 24 July 2026

Gravitational waves from binaries

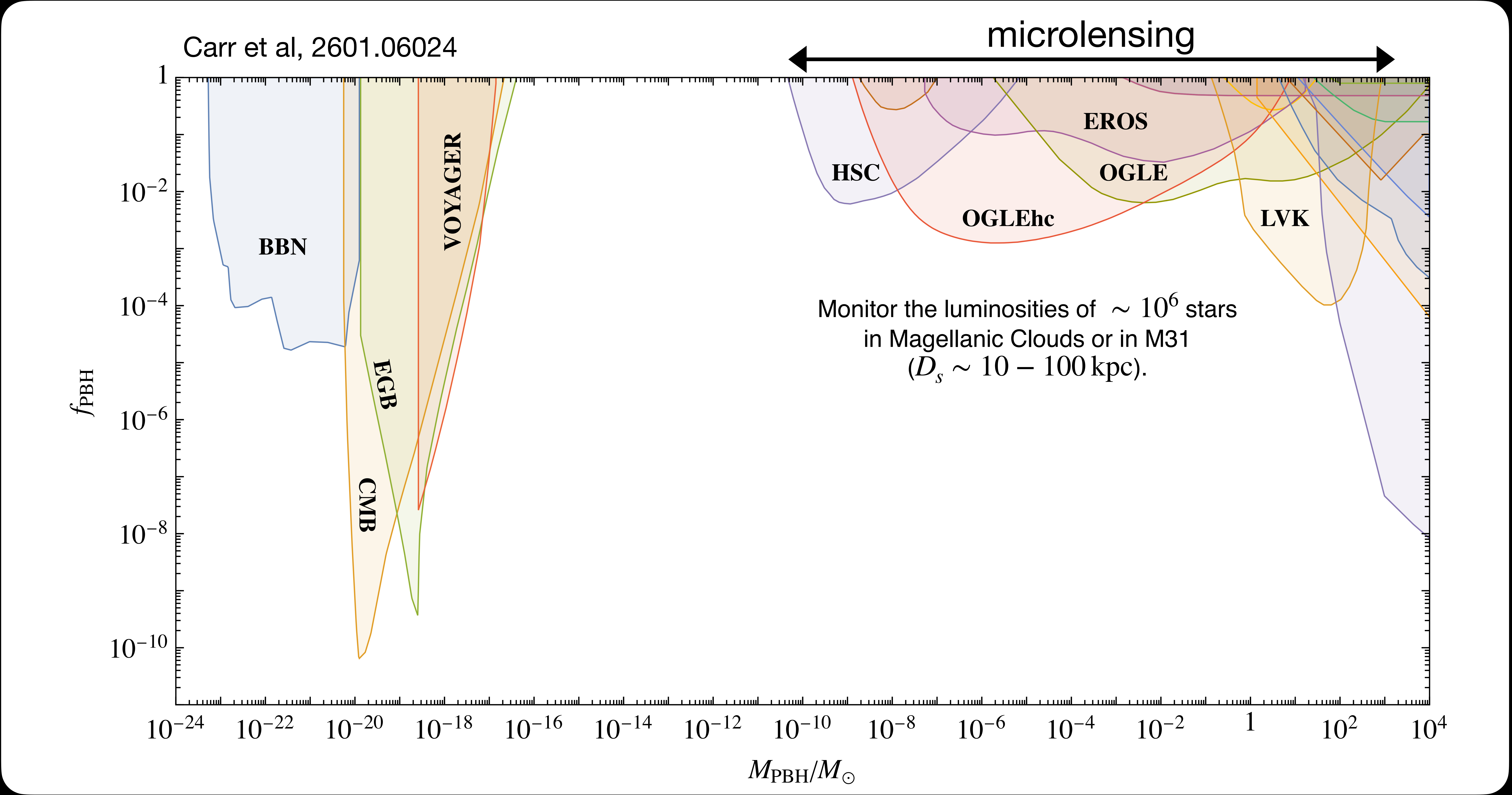
- LVK has observed almost 400 CBC events.
- Next-generation detectors (ET, CE) will observe $\mathcal{O}(10^3 - 10^4)$ CBC events per year.





GW lensing

Microlensing constraints



Lensing of gravitational waves

- distant sources
- transient signals
- coherent wave

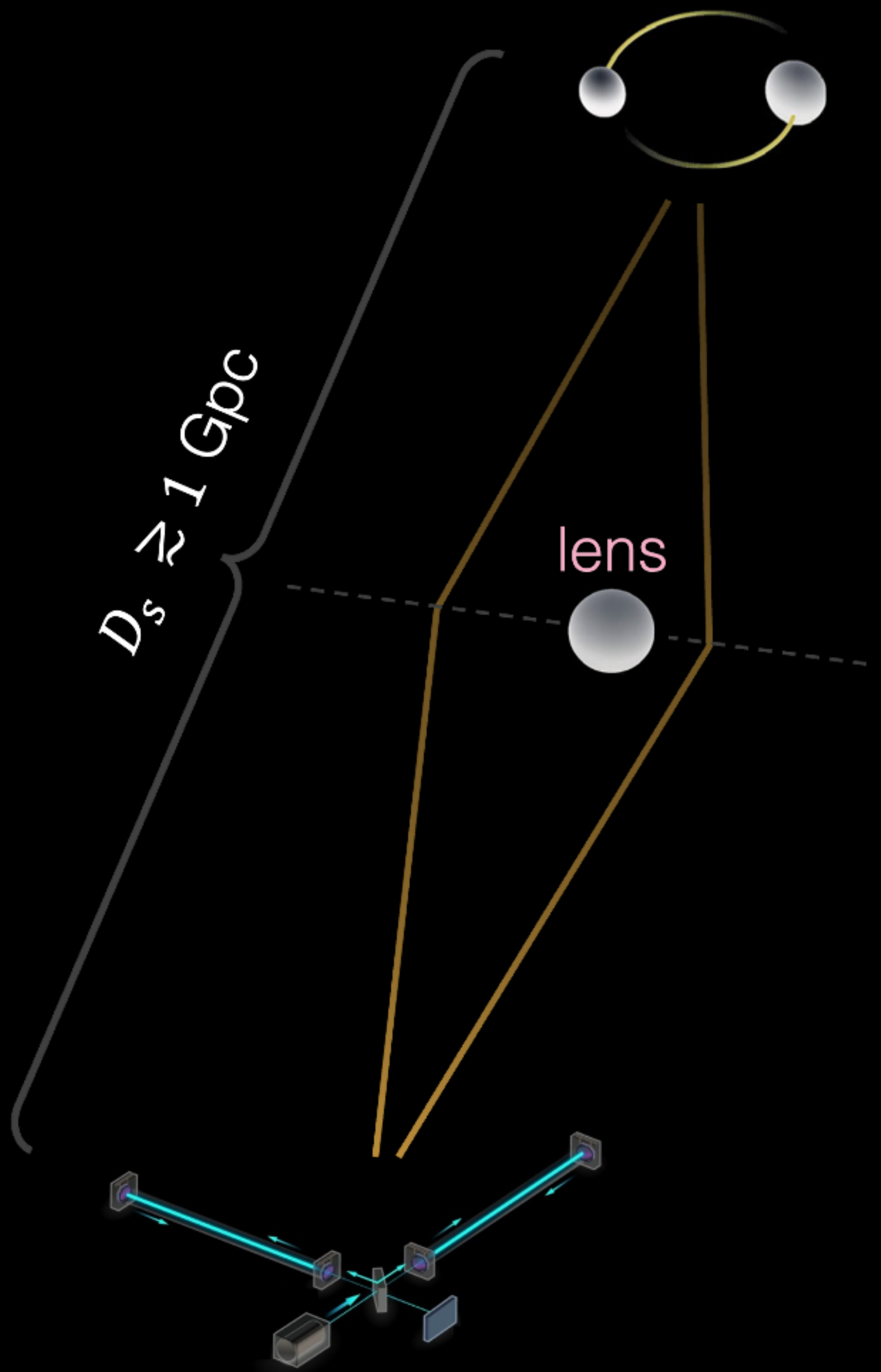
$$\tilde{h}(f) = F(f)\,\tilde{h}_0(f)$$

$$F(\omega, \vec{y}) = \frac{\omega}{2\pi i}\int \mathrm{d}^2\vec{x}\, e^{i\,\omega\, T(\vec{x},\vec{y})}$$

$$T(\vec{x}, \vec{y}) = \frac{1}{2}\left|\vec{x} - \vec{y}\right|^2 - \psi(\vec{x})$$

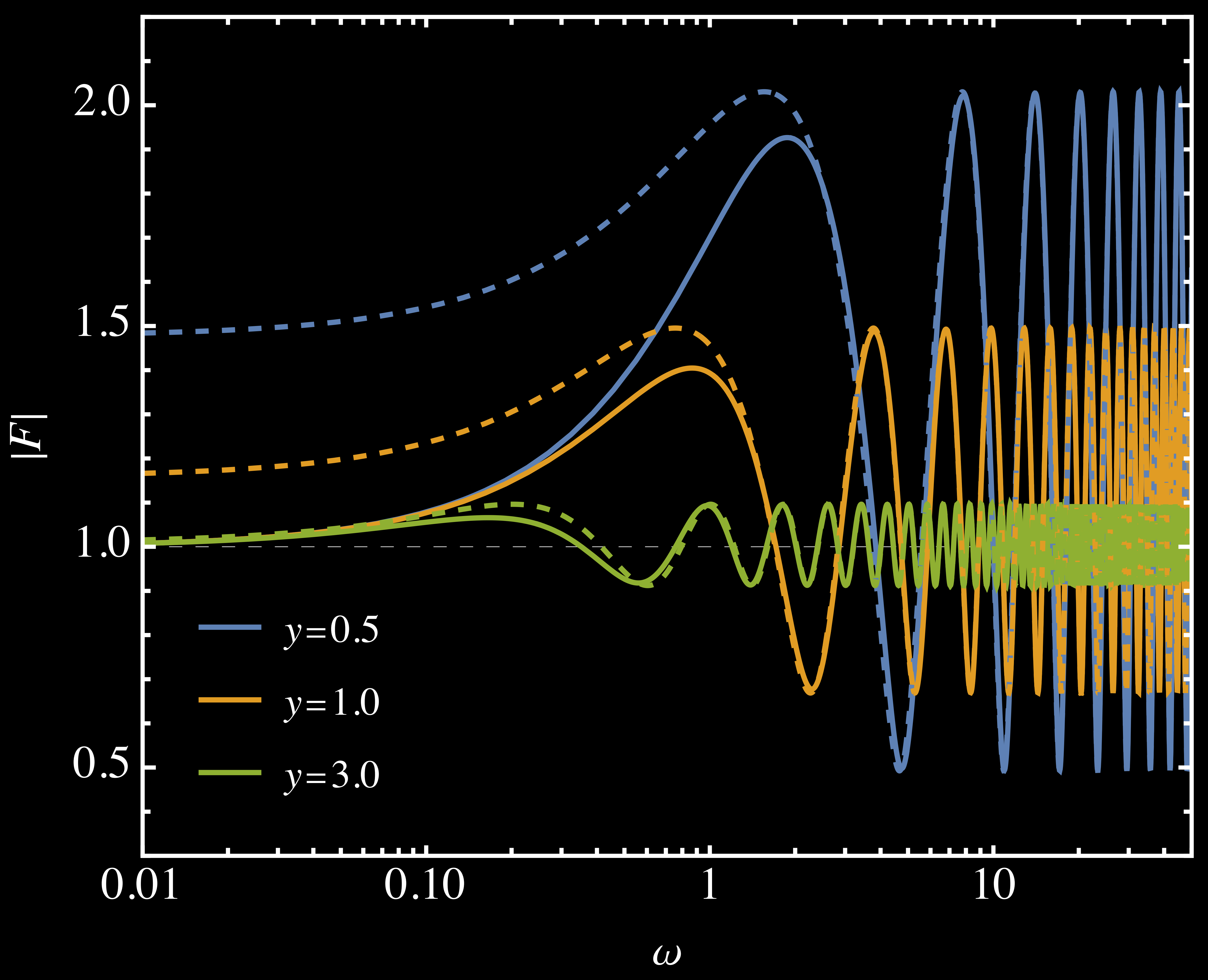
$y =$ impact parameter

point mass lens: $\psi(\vec{x}) = \ln x, \quad \omega = 8\pi\,(1 + z_l)\,M_l\, f$

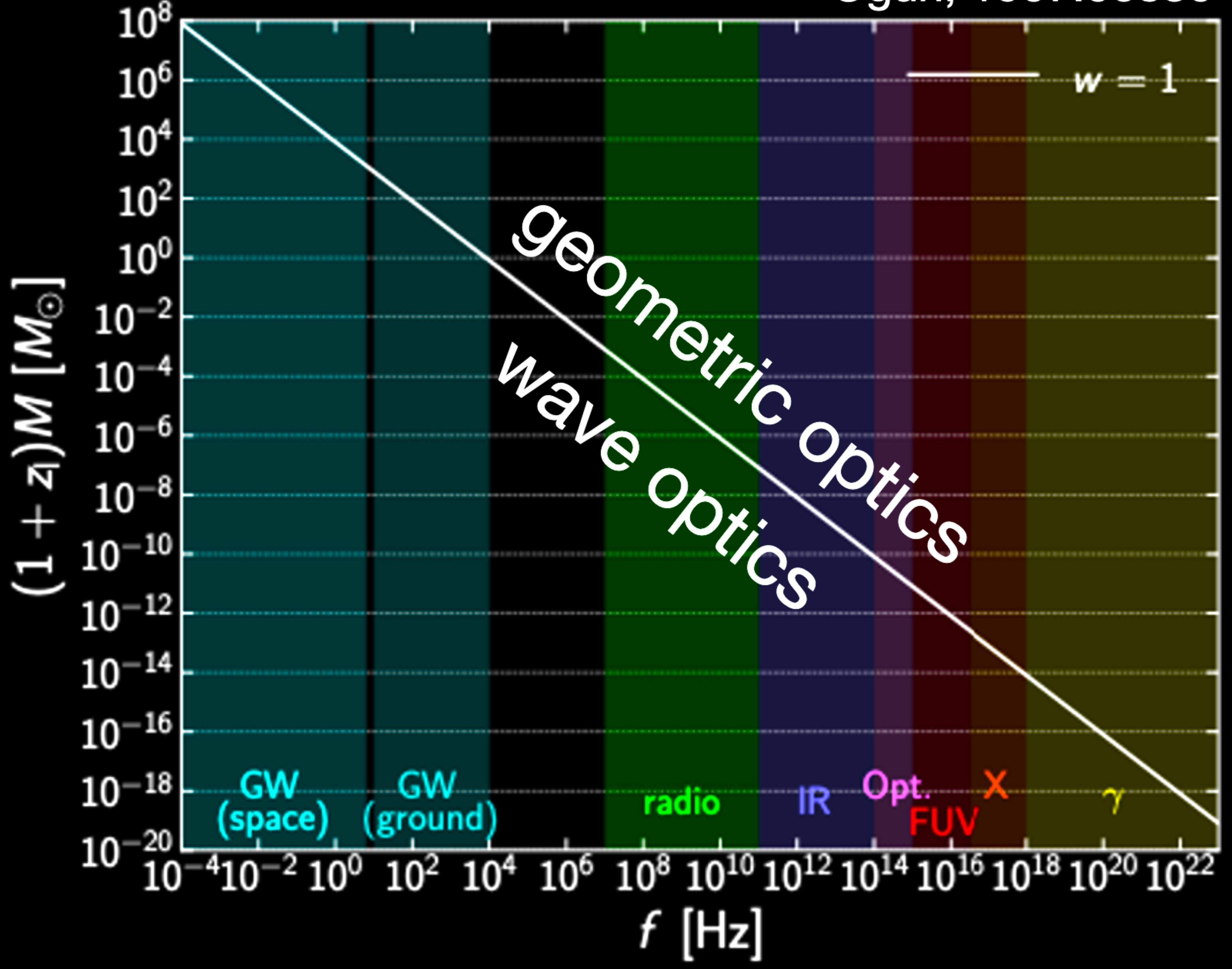


Wave optics and geometric optics

geom. optics: $F(f) = |\mu_+|^{\frac{1}{2}} - i|\mu_-|^{\frac{1}{2}} \exp[2\pi i f \Delta t]$



Oguri, 1907.06830



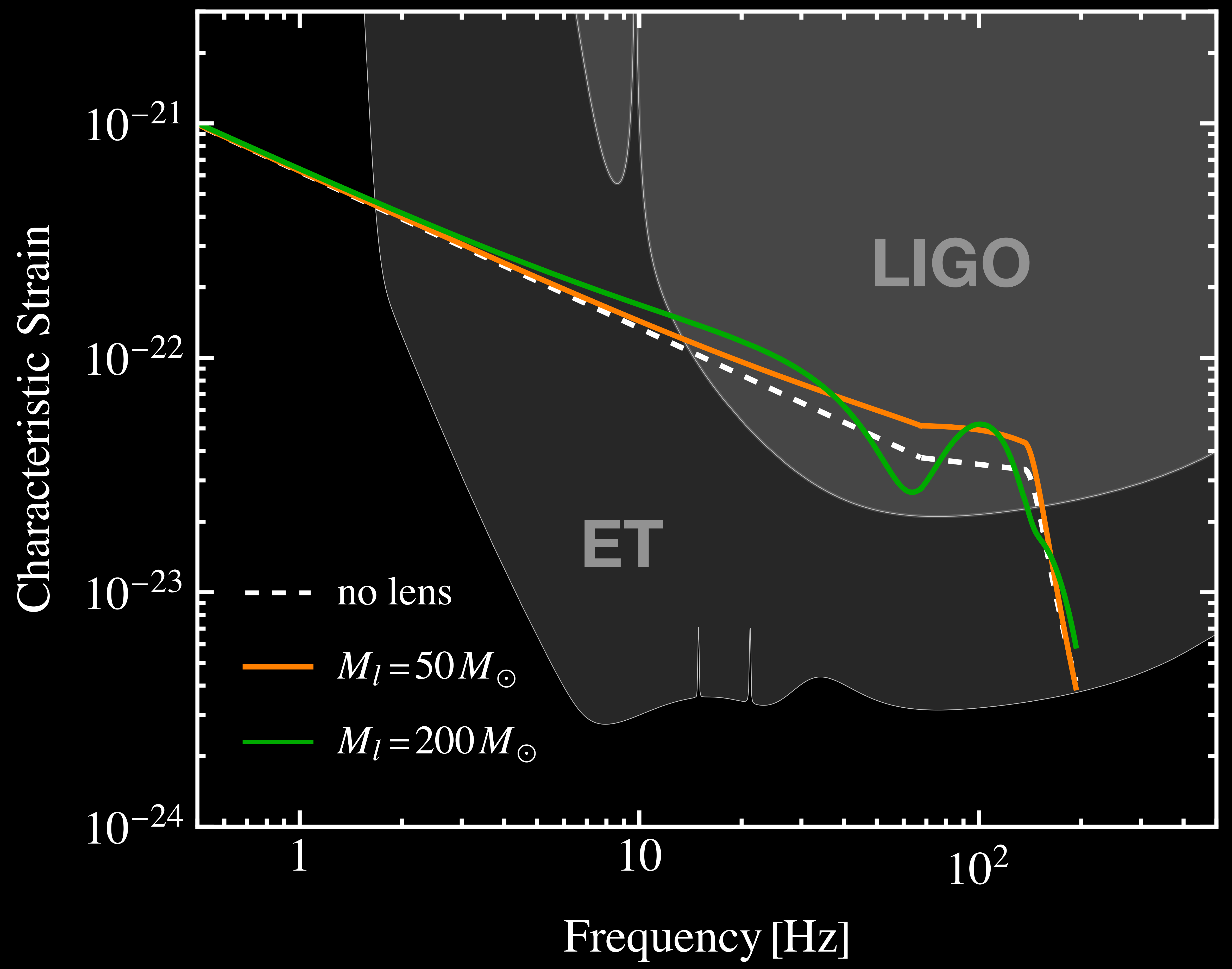
Detectability

lensed signal → unlensed template

$$\Delta\chi^2 = 4\min_{\vec\theta}\int \mathrm{d}f\,\frac{\left|\tilde h_{\rm L}(f) - \tilde h_{\rm T}(f,\vec\theta)\right|^2}{S_n(f)}$$

threshold $\Delta\chi^2$

$\Longrightarrow$ largest detectable y



Urrutia and Vaskonen, 2109.03213

	SNR	$\frac{\Delta\mathcal{M}_z}{\mathcal{M}_z}$ [%]	$\frac{\Delta\eta}{\eta}$ [%]	$\frac{\Delta D_L}{D_L}$ [%]	$\Delta\phi$	$\Delta t_c[s]$	$\Delta\sigma$	$\Delta\beta$	$\frac{\Delta M_{lz}}{M_{lz}}$ [%]	$\frac{\Delta y}{y}$ [%]
LIGO	58.49	0.40	1.32	3.50	2.13	0.00025	1.59	0.77	10.39	9.74
ET	453.86	0.0037	0.12	0.62	0.093	0.000019	0.053	0.019	1.89	1.79

$m_1 = m_2 = 20\,M_\odot$, source at 1.5 Gpc, lens at 1.0 Gpc, $M_{lz} = 100\,M_\odot$, $y = 1$

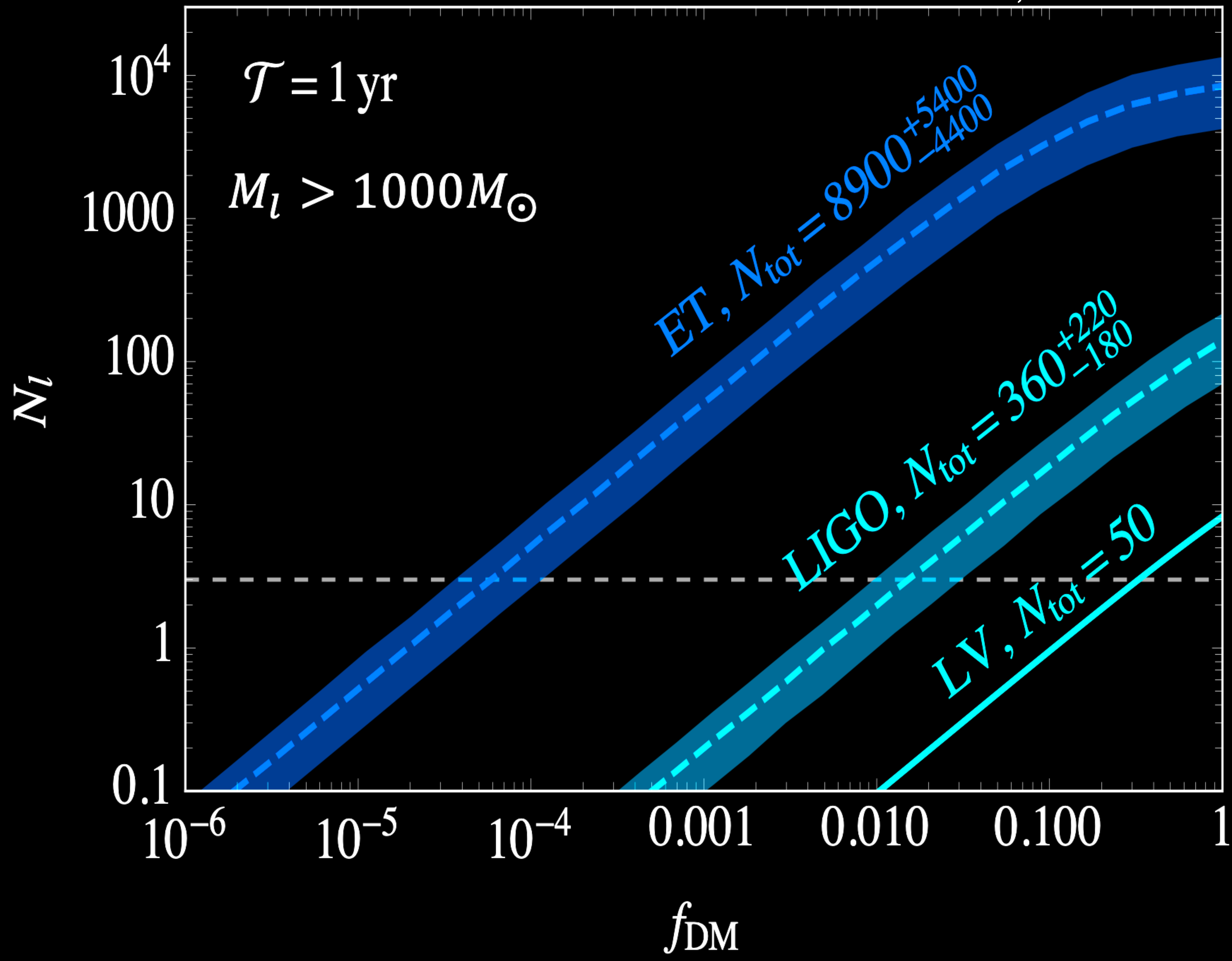
Lensing probability

$$P_l = 1 - e^{-\tau}$$

$$\tau = \int_0^{z_s} \mathrm{d}z_l \frac{\sigma(z_l)n(z_l)}{(1+z_l)H(z_l)} \propto f_{\mathrm{DM}}\, y_c^2$$

$$N_l = T \int \mathrm{d}\lambda(\vec{\theta}_s)\, P_l(\vec{\theta}_s)\, p_{\mathrm{det}}(\vec{\theta}_s)$$

Urrutia and Vaskonen, 2109.03213



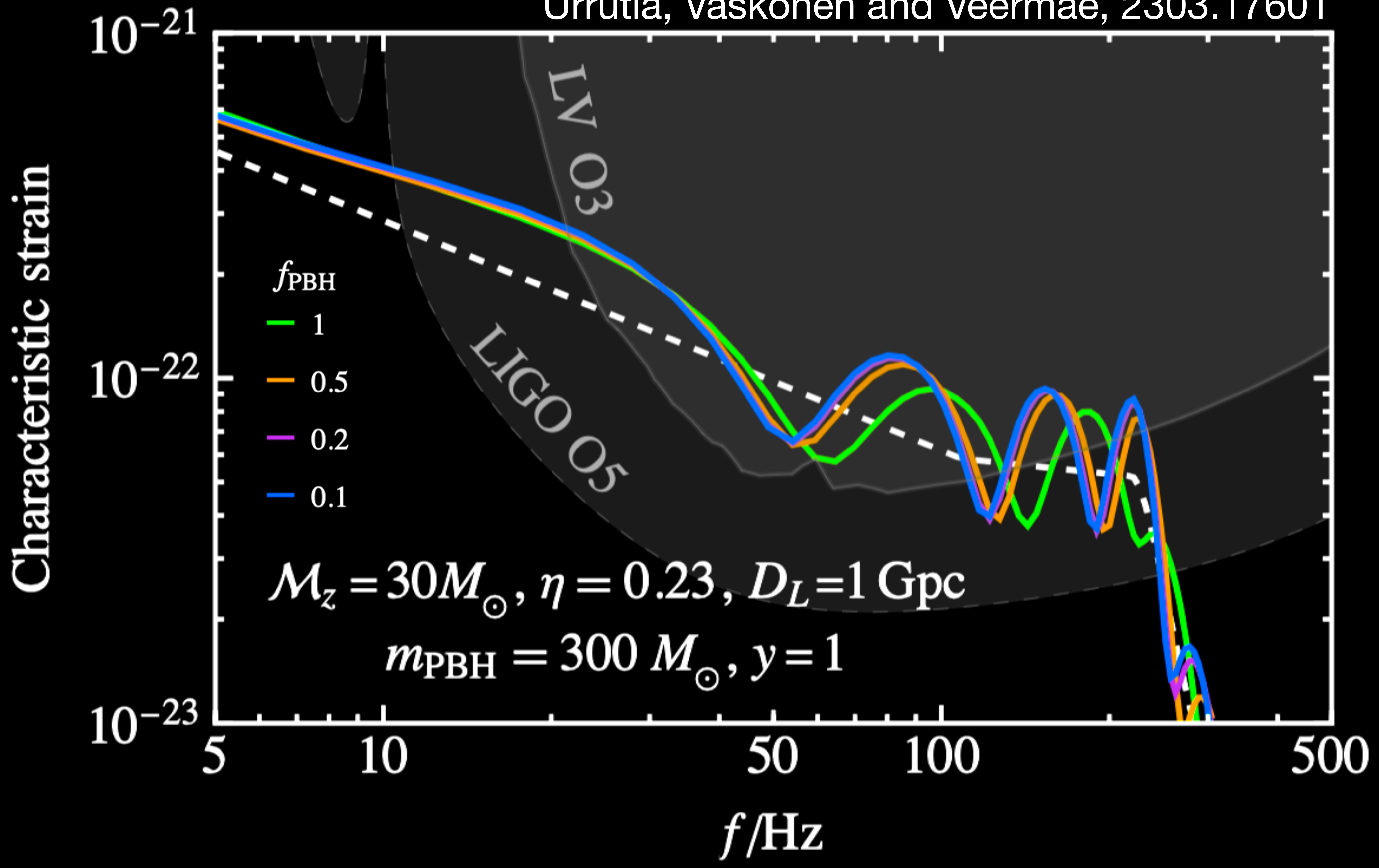
DM spikes around PBHs

$$\rho_h \propto r^{-9/4}\,\theta(R_h - r)$$

$$R_h = 14\,\mathrm{pc}\left[\frac{m_{\rm PBH}}{M_\odot}\right]^{1/3} \min(1, 10^{-3} f_{\rm PBH}^{-4/3})$$

$$M_h = (1 - f_{\rm PBH})\, m_{\rm PBH} \min(50, 0.28 f_{\rm PBH}^{-1})$$

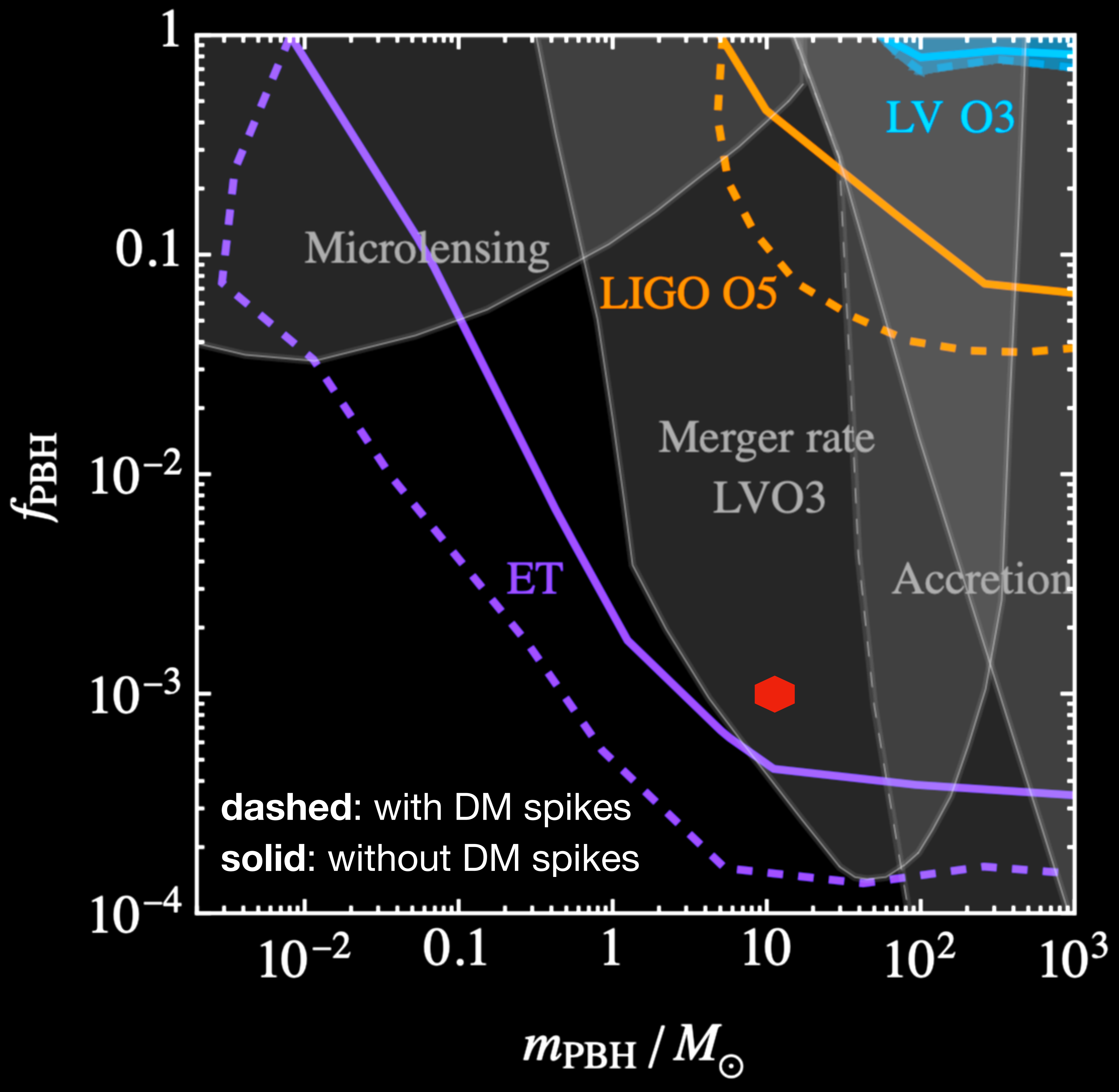
Urrutia, Vaskonen and Veermäe, 2303.17601

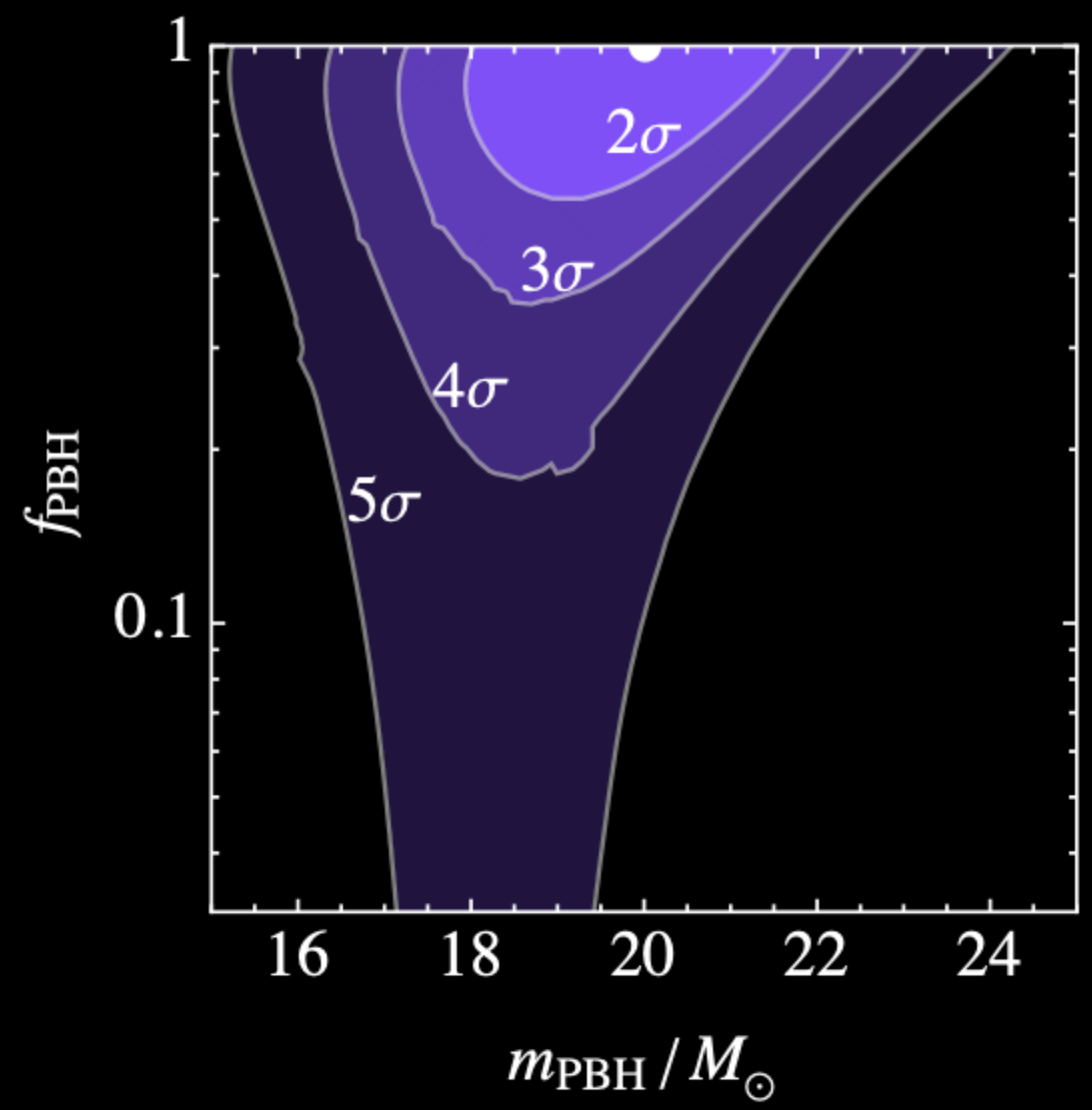


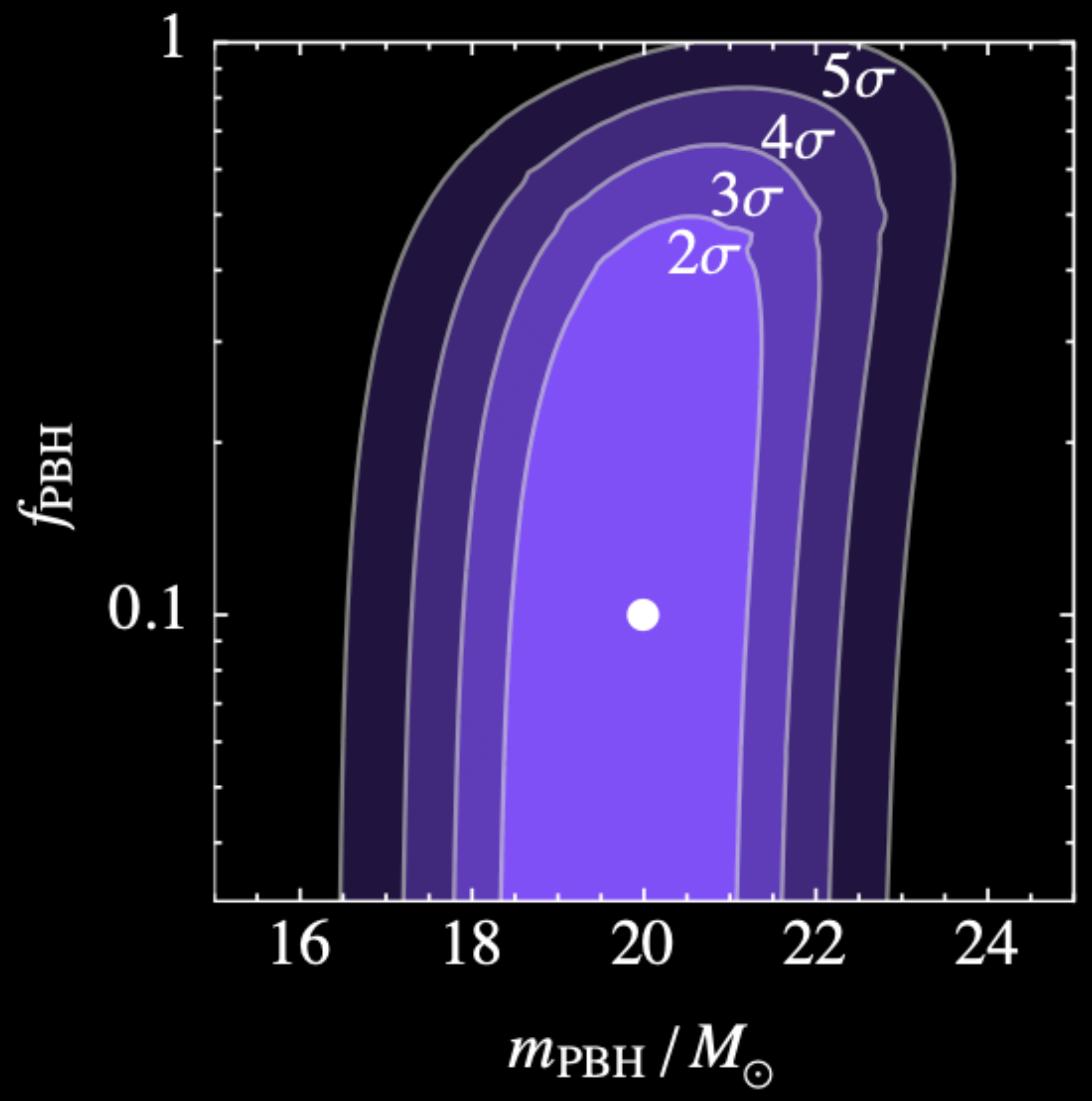
$$\psi(x) = \ln x + \psi_h(x)$$

Constraints and prospects

Urrutia, Vaskonen and Veermäe, 2303.17601



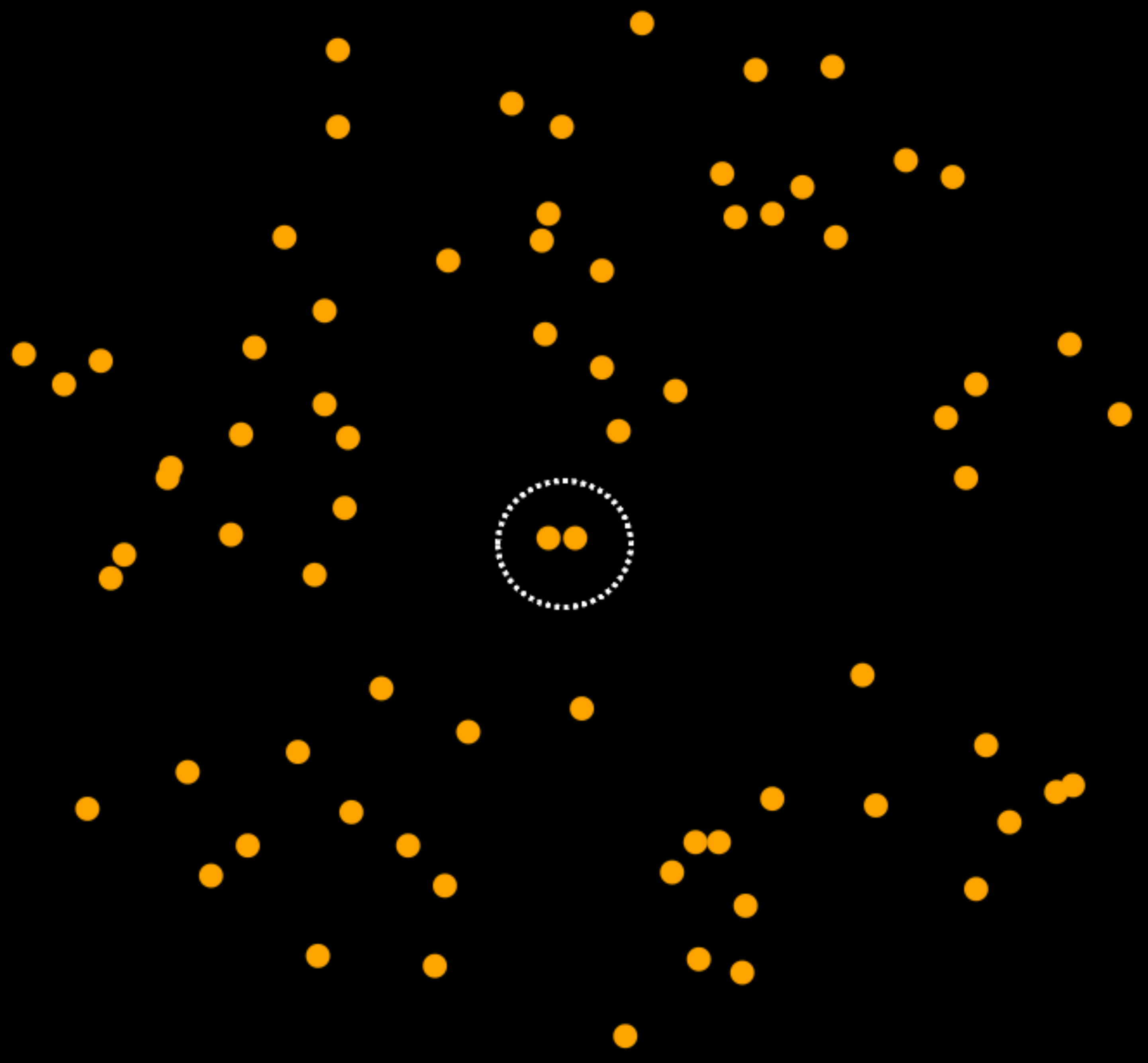




PBH binaries

PBH binary formation

Raidal, Spethmann, Vaskonen and Veermäe, 1812.01930

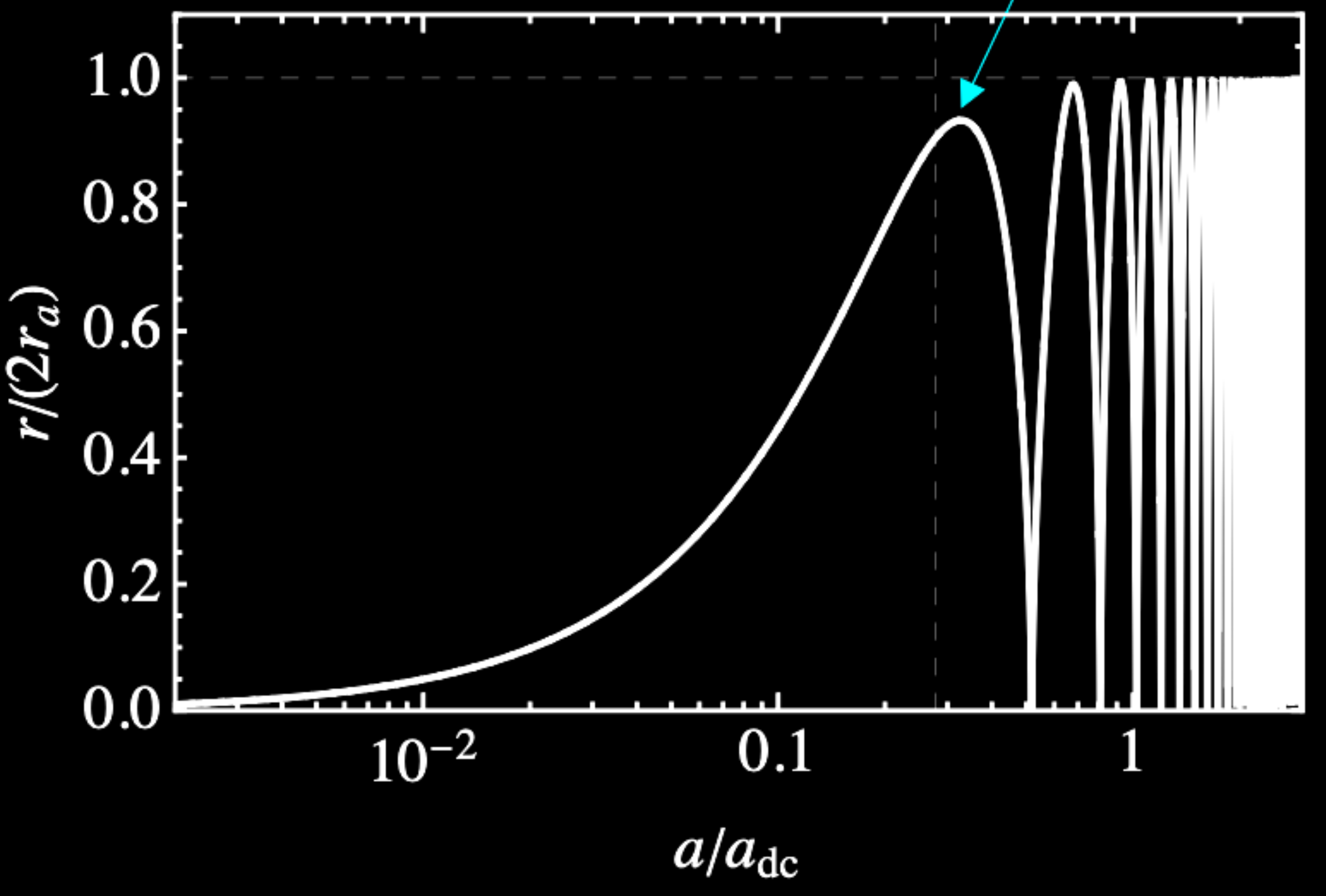


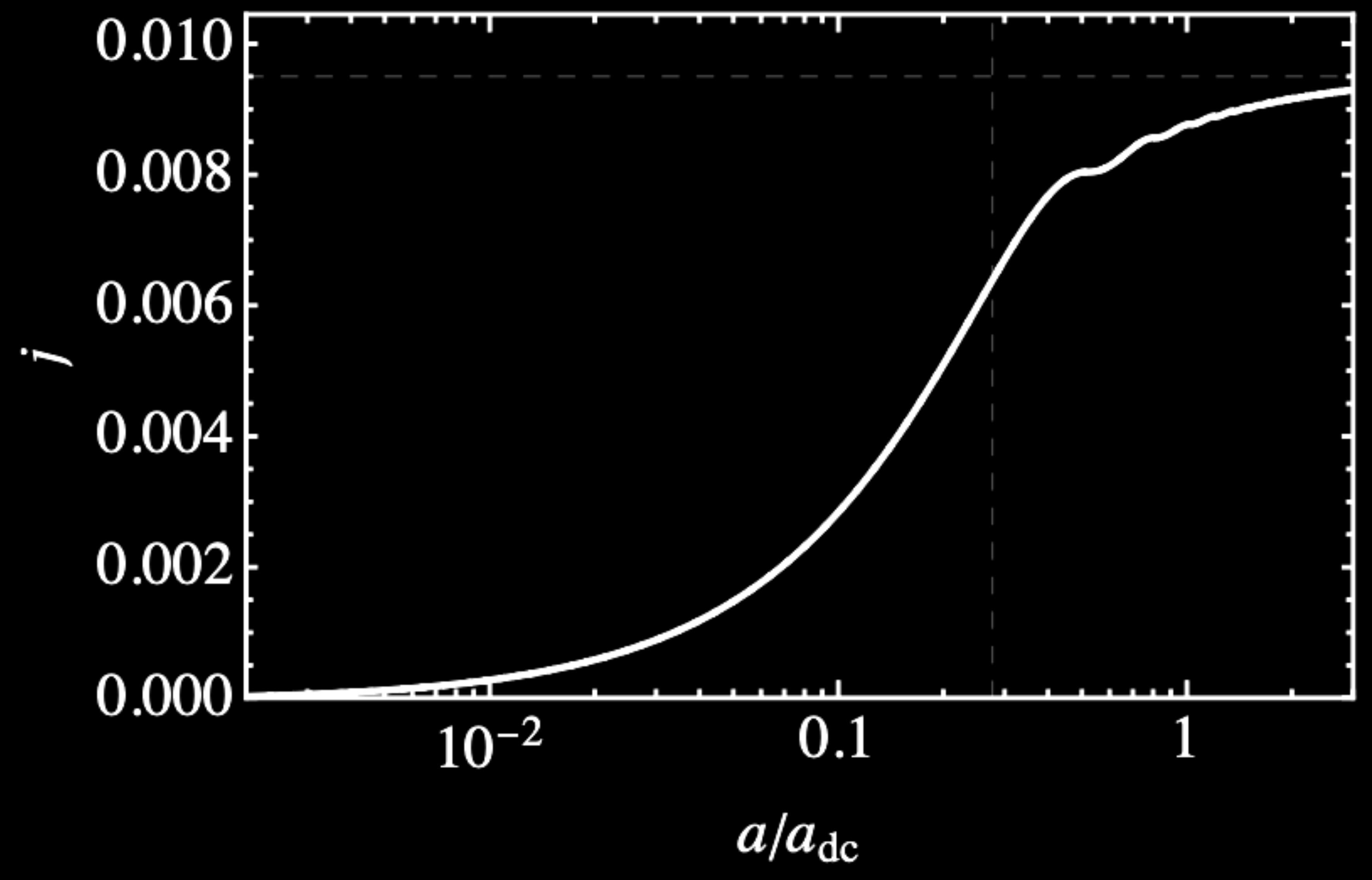
Force acting on a PBH pair:

$$\mathbf{F}/\mu = \underbrace{\mathbf{r}\ddot{a}/a}_{\text{Hubble flow}} - \underbrace{M\hat{\mathbf{r}}/r^2}_{\text{self-gravity}} + \underbrace{(\hat{\mathbf{r}}\cdot\mathbf{T}\cdot\mathbf{r})\hat{\mathbf{r}}}_{\text{radial tidal forces}} + \underbrace{(\mathbf{r}\times(\mathbf{T}\cdot\mathbf{r}))}_{\text{tidal torque}}\times(\hat{\mathbf{r}}/r)$$

decoupling

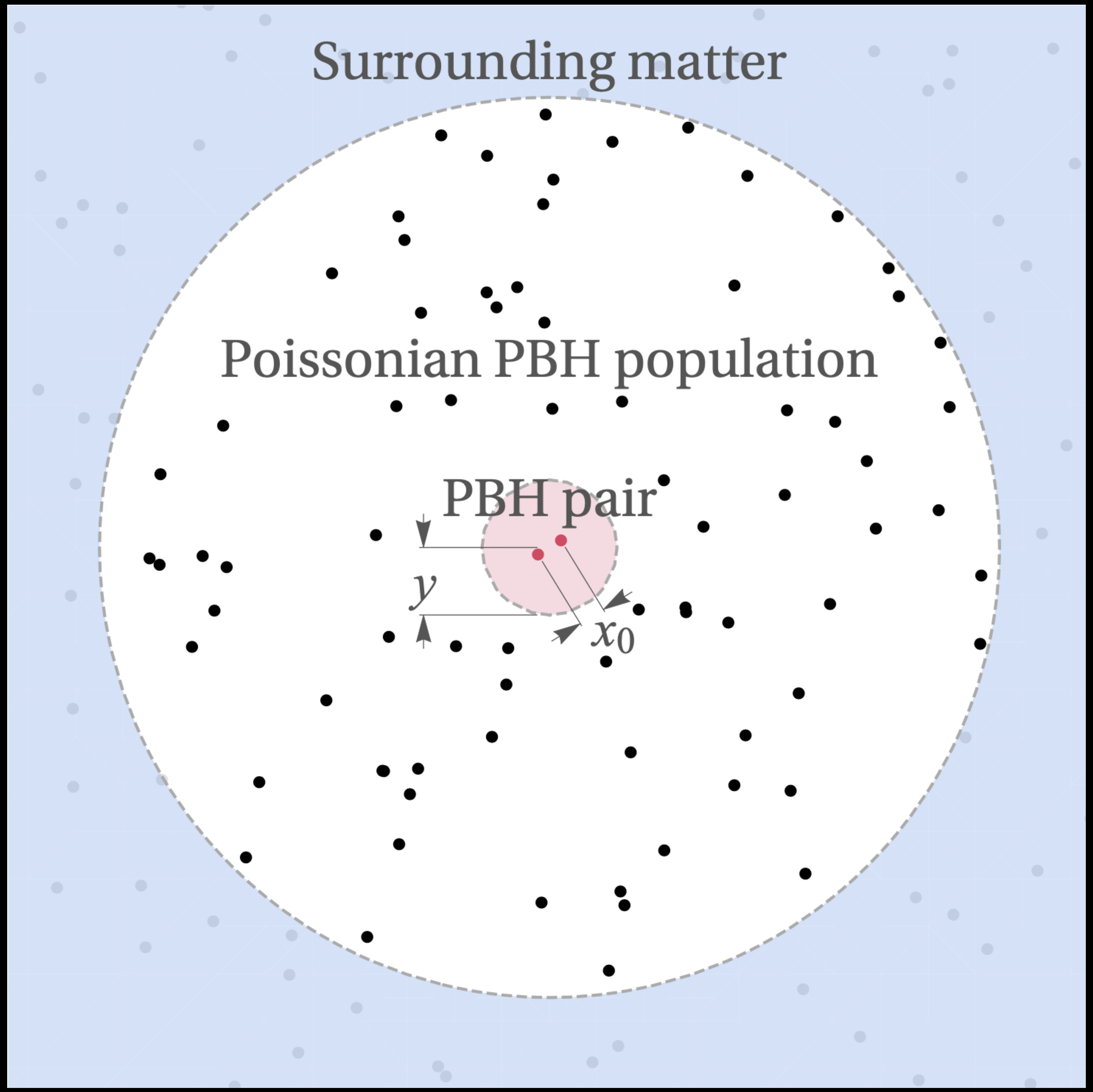
angular momentum





PBH binary formation

Raidal, Spethmann, Vaskonen and Veermäe, 1812.01930



$$\mathrm{d}n_b(x_0) = \frac{1}{2} e^{-\bar{N}(y)} \mathrm{d}n(m_1) \mathrm{d}n(m_2) \mathrm{d}V(x_0)$$

$$\tau(j, x_0) = \frac{3}{85} \frac{r_a(x_0)}{\eta M^3} j^7$$

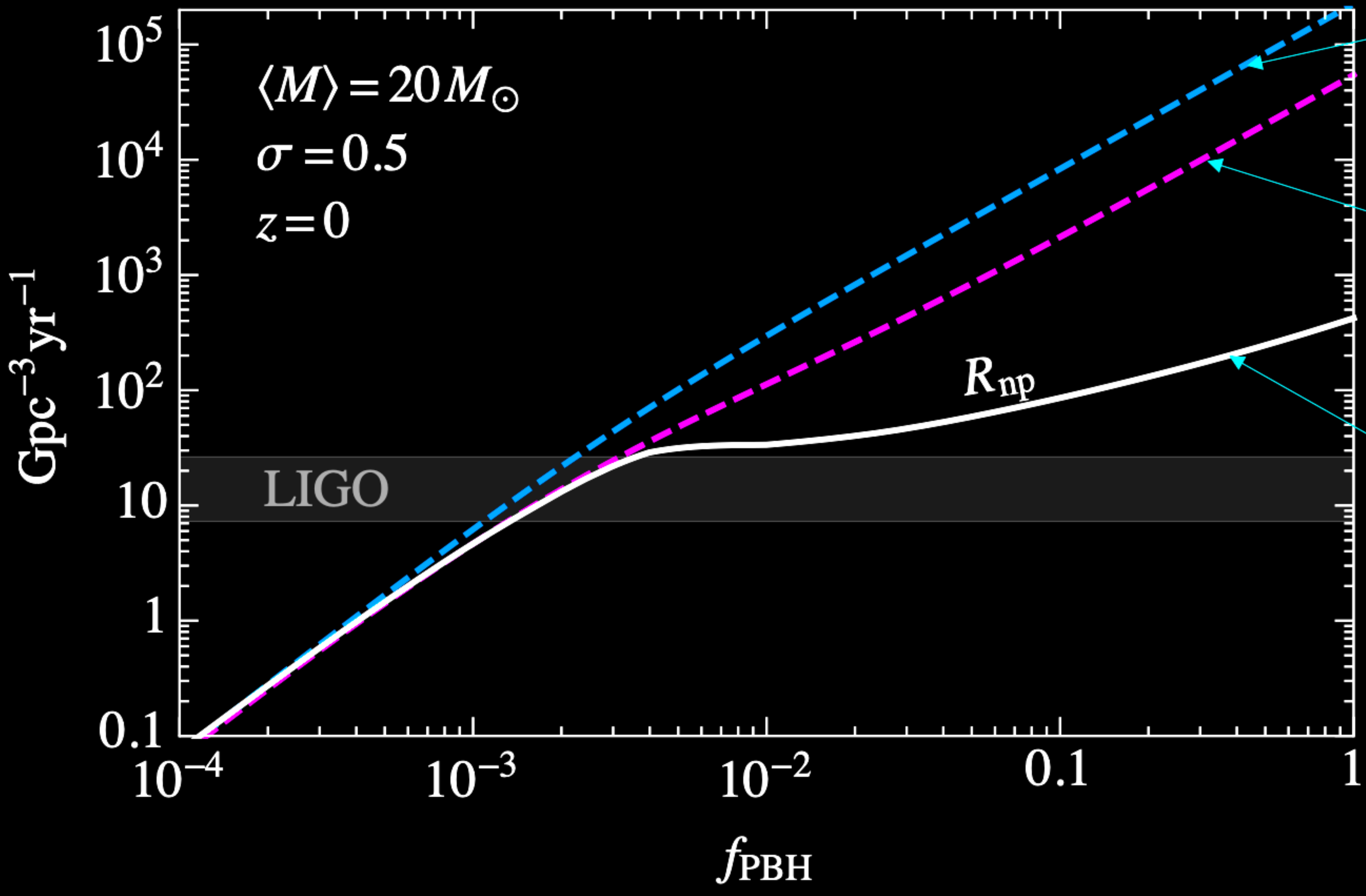
$$\mathrm{d}R = \int \mathrm{d}n_b(x_0)\, \mathrm{d}j\, \frac{\mathrm{d}P(j\,|\,y, x_0)}{\mathrm{d}j} \delta(\tau - \tau(j, x_0))$$

$$\frac{\mathrm{d}P(j\,|\,y, x_0)}{\mathrm{d}j} = \frac{1}{j} \int \mathrm{d}u\, u\, J_0(u)\, \exp\left[-\bar{N}(y)^2 \int \frac{\mathrm{d}n(m)}{n} F\left(\frac{u}{\bar{N}(y)} \frac{m}{\langle m \rangle} \frac{j_0}{j}\right) - \frac{3}{10} u^2 \frac{\sigma_M^2}{f_{\mathrm{PBH}}^2} \frac{j_0^2}{j^2}\right]$$

PBH binary evolution

Raidal, Spethmann, Vaskonen and Veermäe, 1812.01930

PBH merger rate



no disruption

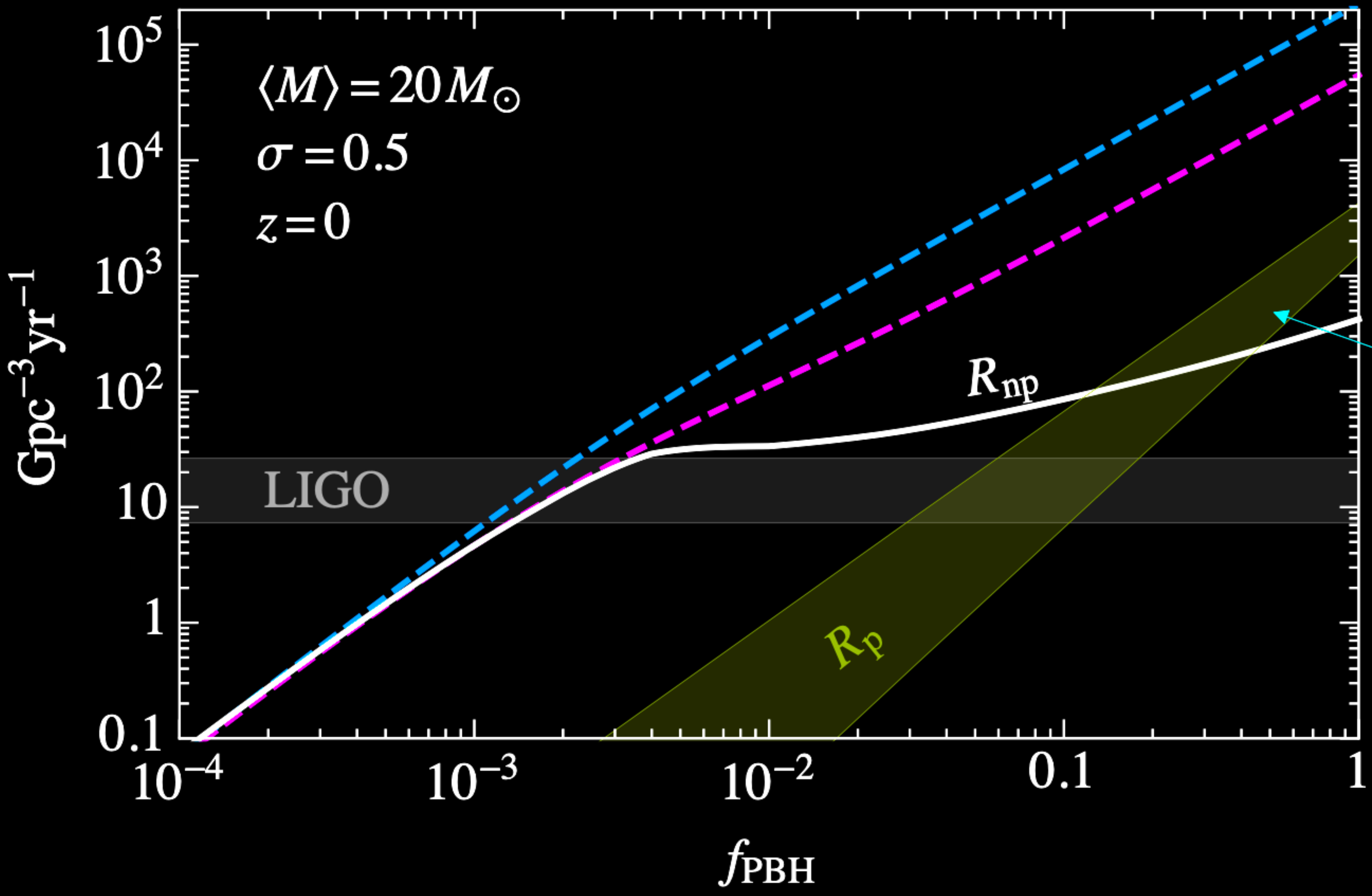
disruption by 3rd PBHs

Raidal et al, 1812.01930

disruption in small halos

Vaskonen and Veermäe, 1908.09752

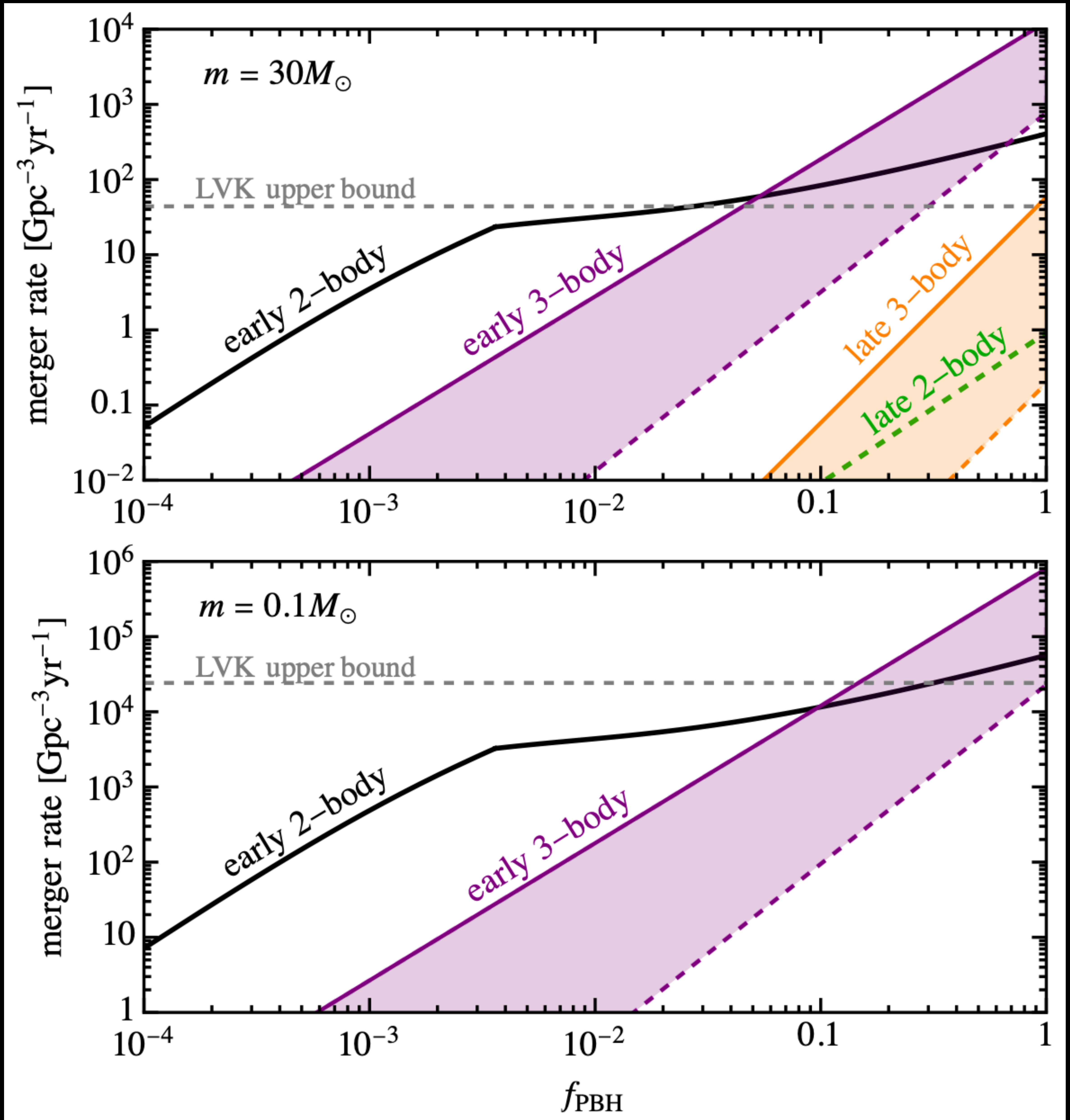
PBH merger rate



three-body channel

Vaskonen and Veermäe, 1908.09752

Other channels



Raidal, Vaskonen and Veermäe, 2404.08416

1. **Early two-body channel**

$$\frac{\mathrm{d}R_{E2}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} \approx \frac{1.6\times 10^6}{\mathrm{Gpc}^3\mathrm{yr}} f_{\rm PBH}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{M}{M_\odot}\right)^{-\frac{32}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} S_{\rm L} S_{\rm E}\, \psi(m_1)\psi(m_2)\,, \tag{101}$$

where $S_{\rm L}$ and $S_{\rm E}$ are suppression factors given in Eqs. (46) and (60).

2. **Early three-body channel**

$$\frac{\mathrm{d}R_{\rm E3}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} = \frac{7.9\times 10^4}{\mathrm{Gpc}^3\mathrm{yr}} f_{\rm PBH}^{\frac{144\gamma}{259}+\frac{47}{37}} \left[\frac{t}{t_0}\right]^{\frac{\gamma}{7}-1} \left[\frac{\langle m\rangle}{M_\odot}\right]^{\frac{5\gamma-32}{37}} \frac{e^{3.2(1-\gamma)}\gamma}{28/9-\gamma}\mathcal{K}$$
$$\times \left(\frac{M}{2\langle m\rangle}\right)^{\frac{179\gamma}{259}-\frac{2122}{333}} (4\eta)^{-\frac{3\gamma}{7}-1} \bar{\mathcal{F}}(m_1,m_2)\psi(m_1)\psi(m_2)\,, \tag{102}$$

where $\mathcal{K}$ and $\bar{\mathcal{F}}$ are given by Eqs. (81,80) and account for 3-body dynamics. Numerical simulations suggest that $\mathcal{K} \approx 4.0$. The second line contains the effect of extended mass functions.

3. **Late two-body channel**

$$\frac{\mathrm{d}R_{\rm L2}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} \approx \frac{3.4\times 10^{-6}}{\mathrm{Gpc}^3\mathrm{yr}} f_{\rm PBH}^2\, \delta_{\rm eff} \left(\frac{\sigma_v}{\mathrm{km/s}}\right)^{-\frac{11}{7}} \eta^{-\frac{5}{7}}\psi(m_1)\psi(m_2)\,, \tag{103}$$

where $\delta_{\rm eff}$ and σ_v characterize the PBH density contrast and velocity dispersion in the structures.

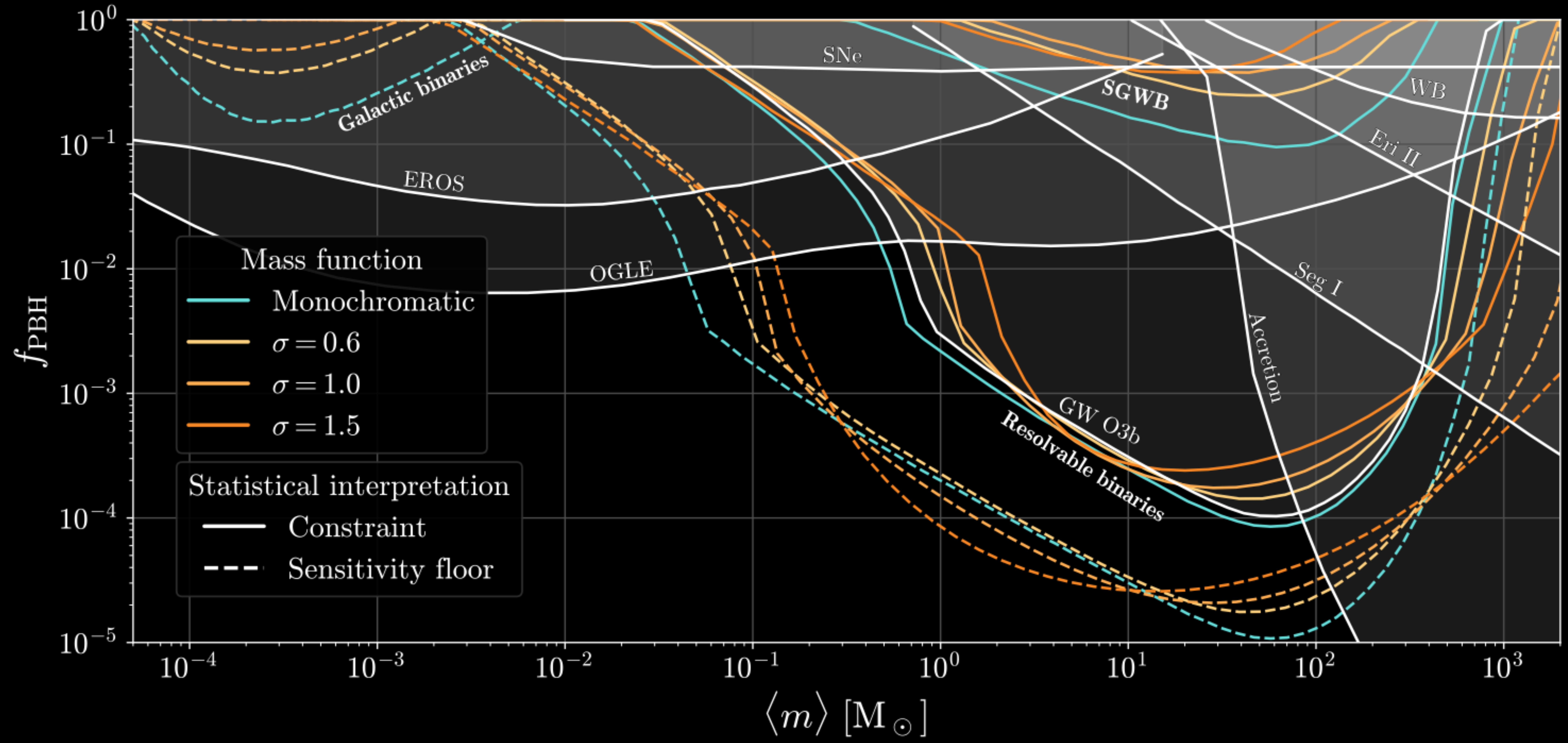
4. **Late three-body channel**

$$\frac{\mathrm{d}R_{\rm L3}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} \approx \frac{1.3\times 10^{-16} e^{-6.0(\gamma-1)}}{\mathrm{Gpc}^3\mathrm{yr}} f_{\rm PBH}^3 \delta_{\rm eff}^2 \left(\frac{\sigma_v}{\mathrm{km/s}}\right)^{-9+\frac{8\gamma}{7}}$$
$$\times \eta^{-1+\frac{\gamma}{7}} \left(\frac{M}{M_\odot}\right)^{3-\frac{\gamma}{7}} \left(\frac{t}{t_0}\right)^{\frac{\gamma}{7}} \mathcal{F}\left(\frac{\langle m\rangle}{2\eta M}\kappa_{\rm min}\right)\psi(m_1)\psi(m_2)\,, \tag{104}$$

Constraints from LVK

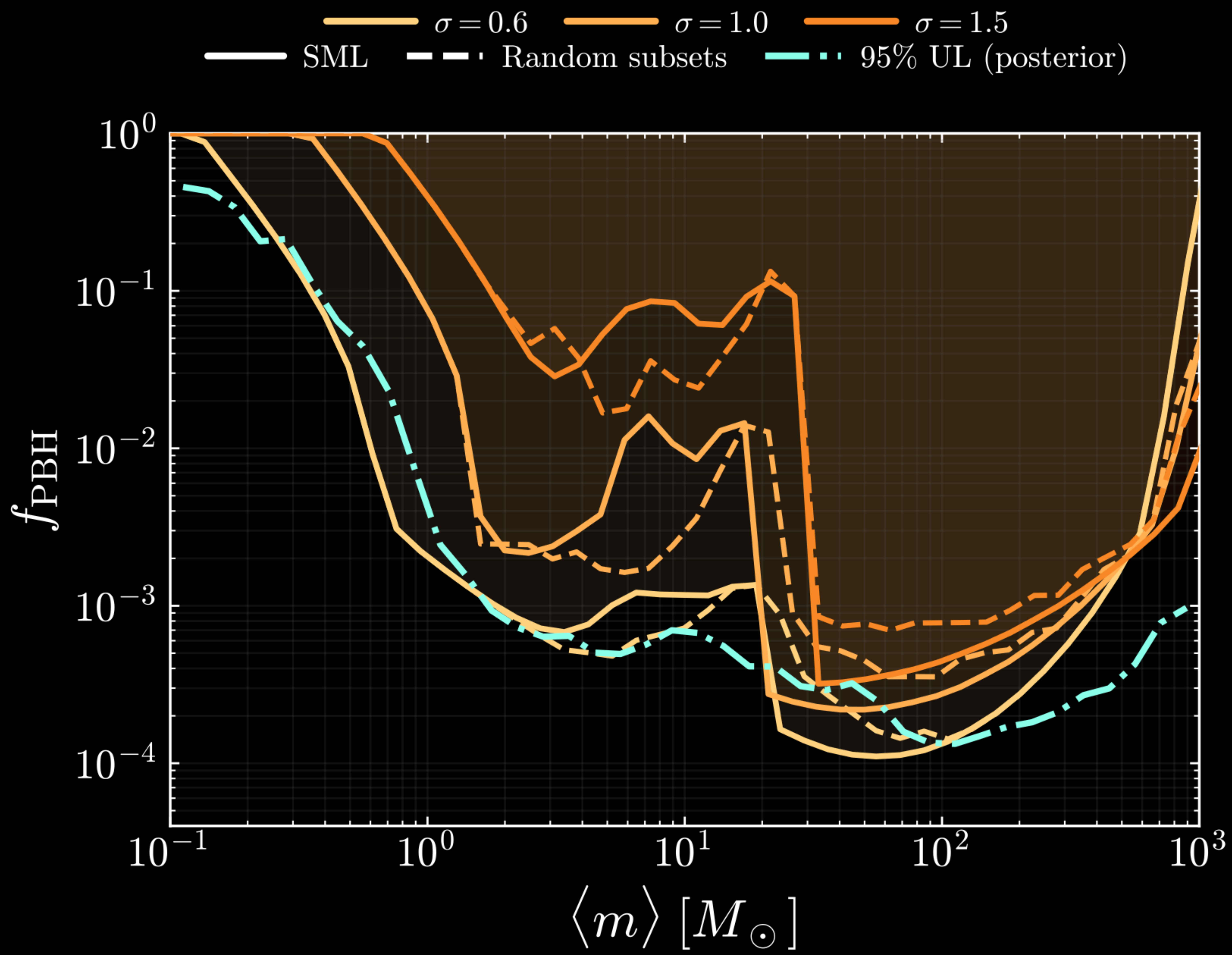
Andrés-Carcasona et al, 2605.15749

- 5% prob. for no events above exclusion curve
- 95% prob. for no events below the sensitivity floor



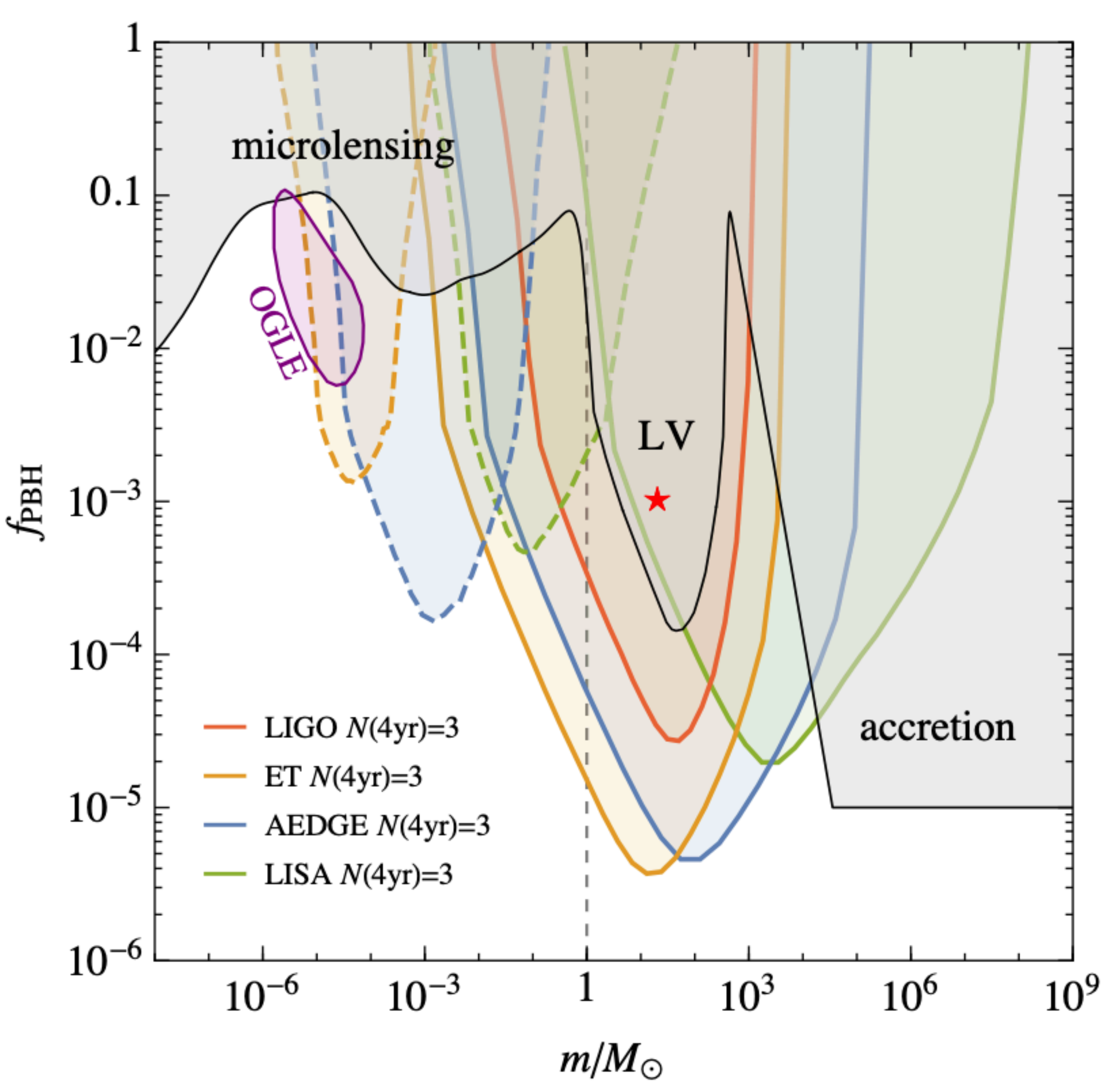
Constraints from LVK

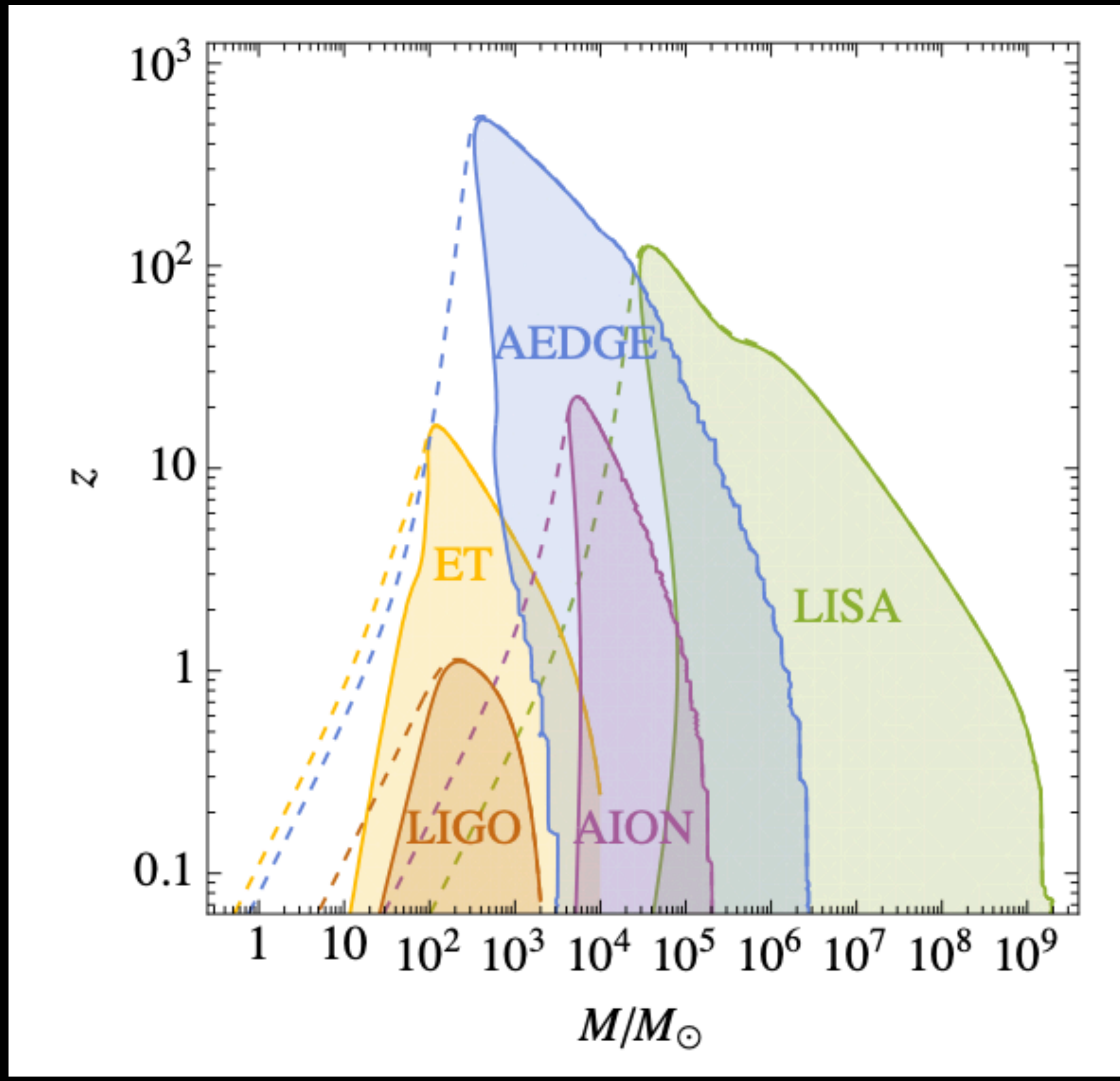
Andrés-Carcasona et al, 2605.15749

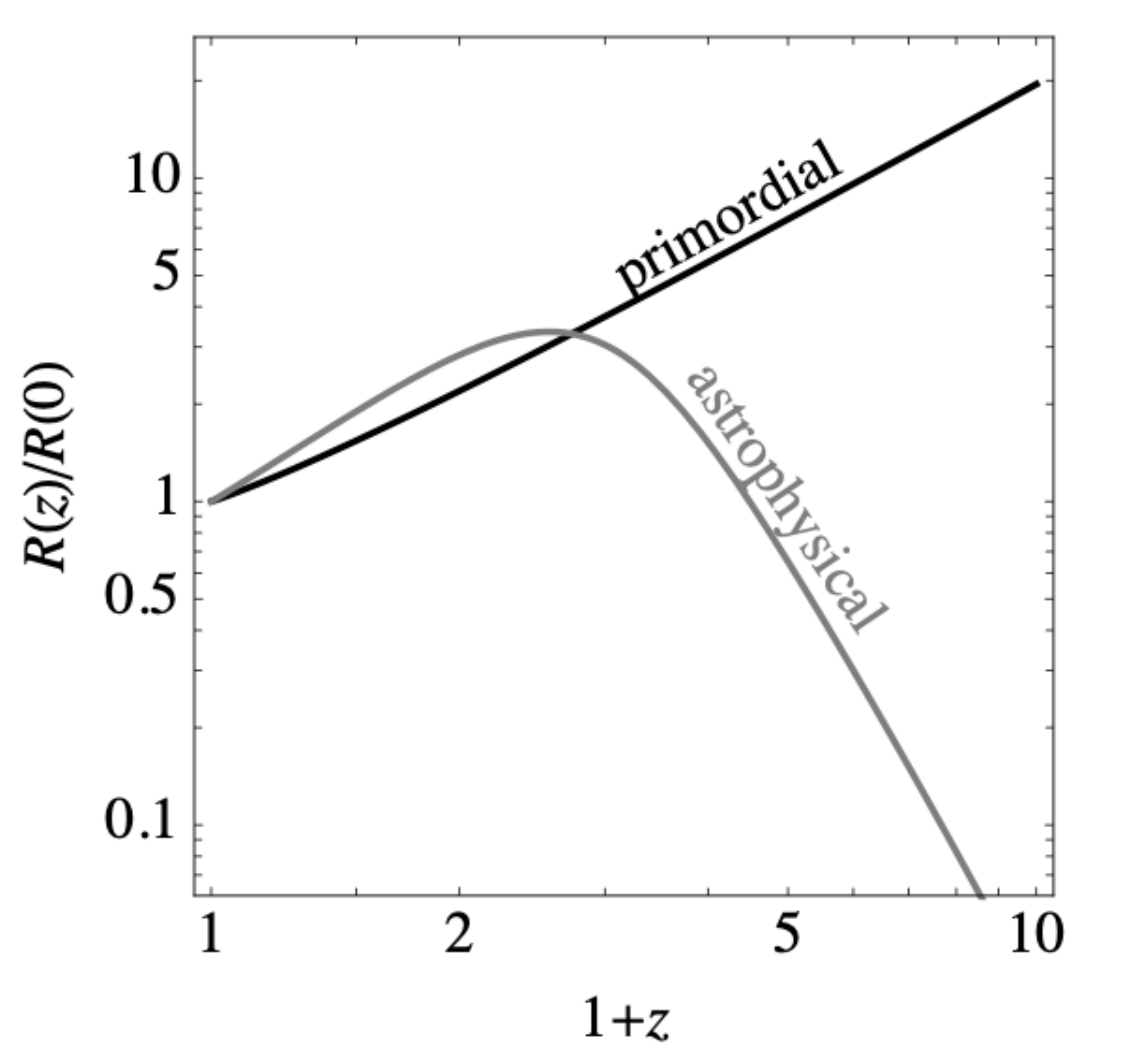


Prospects

Pujolas, Vaskonen and Veermäe, 2107.03379







Summary

- GW observations provide new direct probe of PBH populations in the ~ solar mass range.
- LVK binary searches give the strongest constraint in the $1 - 1000\,M_\odot$ range.
- GW lensing searches provide clean probe of massive PBH populations.