Dynamics of Primordial Black Hole formation in a matter dominated universe

Chulmoon Yoo (Nagoya Univ. / KMI)

PBH formation in early MD

PBH in MD

◎Early **MD** era (e.g., oscillation of inflaton soon after the inflation ~ effectively no pressure)

◎PBH formation would be easier than radiation dominated era

◎What prevents PBH formation?

- ➢ Ellipticity [1609.01588 T. Harada, CY, K. Kohri, K. Nakao, S. Jhingan]
- ➢ Inhomogeneity [1810.03490 T. Kokubu, K. Kkyutoku, K. Kohri, T. Harada]
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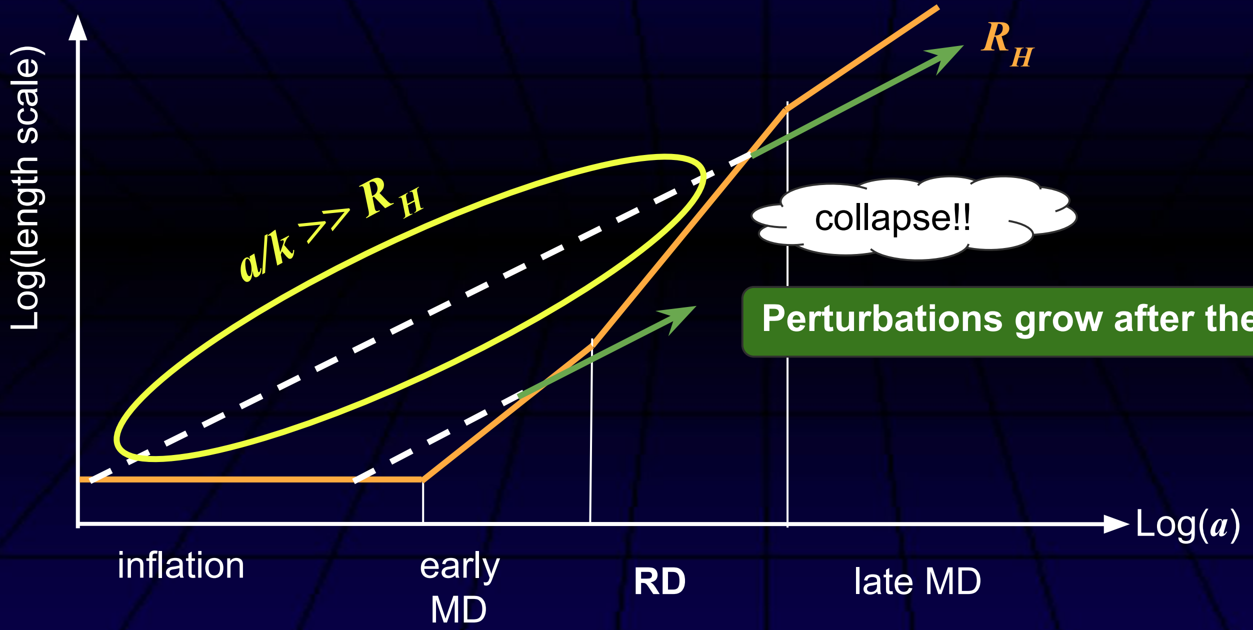
- ➢ Velocity dispersion [2211.13950 T. Harada, K. Kohri, M. Sasaki, T. Terada, CY]

PBH formation

◎Relevant perturbation: initially long-wavelength mode ($a/k >> R_H$)



◎Dominant mode is **non-decaying mode in the long-wavelength limit**

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Ellipticity

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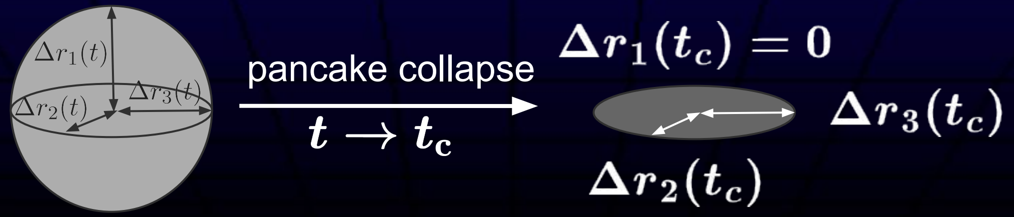
Effects of ellipticity on PBH formation in MD

◎Spherical comoving region with the comoving radius q



$M = \frac{4\pi}{3}\rho_b a^3 q^3$

◎Zel'dovich approximation ~ extrapolation of the linear growth

- The displacement vector $\boldsymbol{D}$ from the background comoving motion

$$\vec{r}(t,\vec{x}) = a(t)\left(\vec{x} + \vec{D}(t,\vec{x})\right) \qquad \vec{D}(t,\vec{x}) \propto a(t)\nabla\Psi(\vec{x})$$

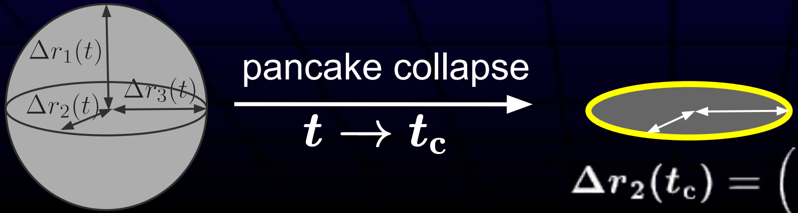
- Evolution of the ball radius

$$\Delta\vec{r}(t,\Delta\vec{x}) = a(t)\Delta\vec{x}\left(I + \nabla\vec{D}\right) \qquad -\nabla\vec{D} = \mathrm{diag}\{\alpha,\beta,\gamma\} \sim \frac{1}{a^2H^2}\partial_i\partial_j\Psi(\vec{x}=0)\ \ (\alpha \geq \beta \geq \gamma)$$

PBH formation criterion from hoop conjecture

◎Collapsing time

$$\Delta r_1(t_c) = 0$$



$$M = \frac{4\pi}{3}\rho_b a^3 q^3$$

$$\Delta r_3(t_c) = \left(1 - \frac{\gamma}{\alpha}\right) a_{\rm col} q$$

$$\Delta r_2(t_c) = \left(1 - \frac{\beta}{\alpha}\right) a_{\rm col} q$$

$$\mathcal{C} = 4\left(1 - \frac{\gamma}{\alpha}\right) E\left(\sqrt{1 - \left(\frac{\alpha-\beta}{\alpha-\gamma}\right)}\right) a_{\rm col} q$$

◎Hoop conjecture

$$\mathcal{C} \lesssim 4\pi M = 2\pi H_{\rm ent} \sim 2 a_{\rm ent} q$$

$$\Rightarrow h(\alpha,\beta,\gamma) := \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2} E\left(\sqrt{1 - \left(\frac{\alpha-\beta}{\alpha-\gamma}\right)}\right) \lesssim 1$$ @ horizon entry

PBH abundance

◎Formation criterion

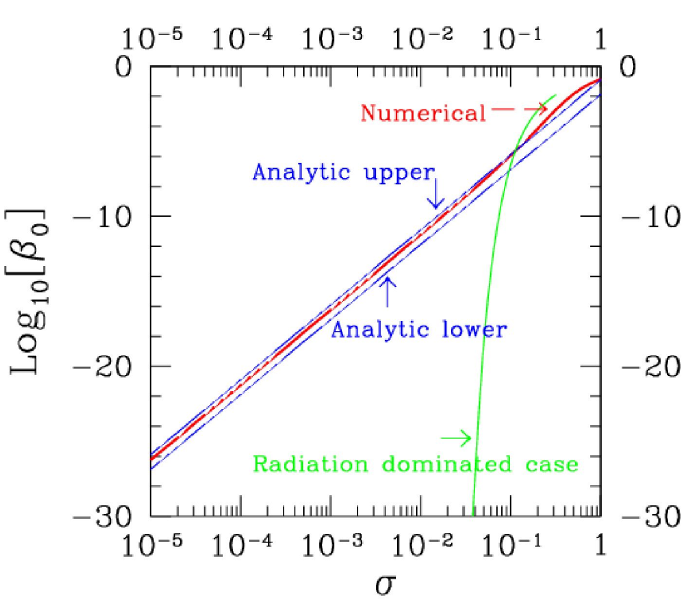
$$h(\alpha,\beta,\gamma) := \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2}E\left(\sqrt{1-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)}\right) \lesssim 1$$

◎Doroshkevich probability distribution

$$w(\alpha,\beta,\gamma)d\alpha d\beta d\gamma$$
$$= -\frac{3^3 5^3}{8\sqrt{5}\pi\sigma^6}\exp\left[-\frac{3}{5\sigma^2}\left\{(\alpha^2+\beta^2+\gamma^2)-\frac{1}{2}(\alpha\beta+\beta\gamma+\gamma\alpha)\right\}\right]$$
$$\cdot(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma$$

◎Abundance

$$\beta_0 \simeq 0.056\sigma^5$$



[1609.01588 T. Harada, CY, K. Kohri, K. Nakao, S. Jhingan]

Inhomogeneity

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Spherically symmetric profile

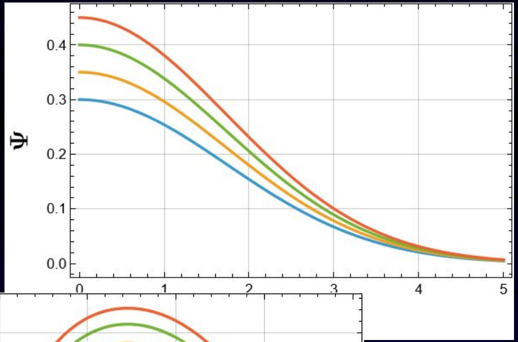
◎Metric at the leading order of the gradient expansion

$$ds^2 = -dt^2 + \Psi^4(dX^2 + dY^2 + dZ^2) \qquad r^2 = X^2 + Y^2 + Z^2$$

◎We focus on the following specific profile of the spatial conformal factor ψ^4

$$\ln \Psi = \frac{\mu}{2} \exp\left[-\frac{1}{6}k^2 r^2\right]$$

$$\mu : \text{amplitude}, k : \text{wave number}$$



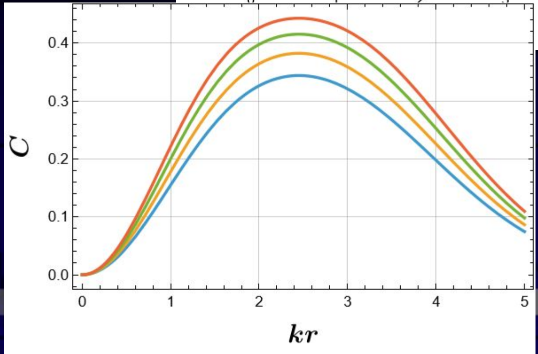
◎Characteristic radius

- ➢ Shibata-Sasaki compaction function

$$\frac{\delta M}{R} \sim \mathcal{C}_{\rm SS} := \frac{1}{2}\left[1 - (1 + 2r\partial_r \ln \Psi)^2\right]$$

- ➢ The radius of the maximum compaction function

$$\left(\partial_r + r\partial_r^2\right) \ln \Psi\big|_{r=r_{\rm m}} = 0 \Leftrightarrow k^2 r_{\rm m}^2 = 6$$



Lemaitre-Tolman-Bondi Solution

◎Lemaitre-Tolman-Bondi solution with arbitrary functions $m(r)$ and $k(r)$

$$ds^2 = -dt^2 + \frac{(\partial_r R)^2}{1-k(r)r^2}dr^2 + R(t,r)^2 d\Omega^2$$

$$(\partial_t R)^2 = -kr^2 + \frac{m(r)r^3}{3R} \qquad \rho = \frac{\partial_r m r^3 + 3mr^2}{24\pi \partial_r R R^2}$$

- ➢ Solution with uniform bigbang (no decaying modes)

$$R = rm^{1/3}t^{2/3}S(x) \quad \text{with} \quad x = k\left(\frac{t}{m}\right)^{2/3}$$

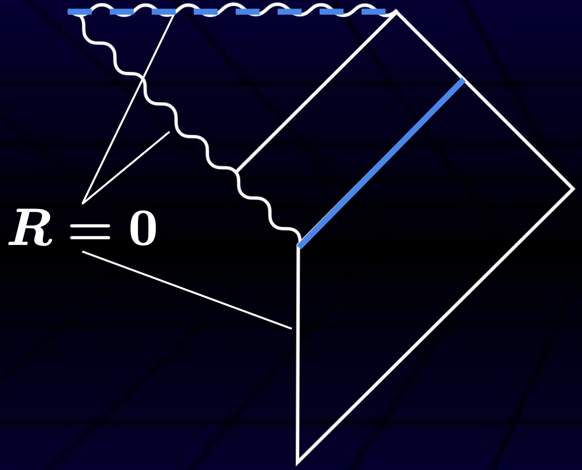
$$S(x) = \frac{1-\cos\sqrt{\eta}}{6^{1/3}(\sqrt{\eta}-\sin\sqrt{\eta})^{2/3}} \;\&\; x = \frac{(\sqrt{\eta}-\sin\sqrt{\eta})^{2/3}}{6^{2/3}} \text{ for } x > 0$$

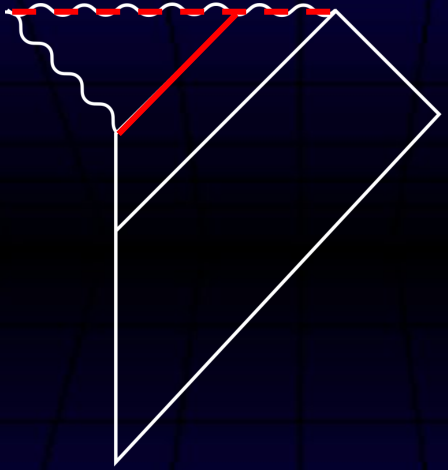
◎Comparing the LTB at the leading order of the gradient expansion

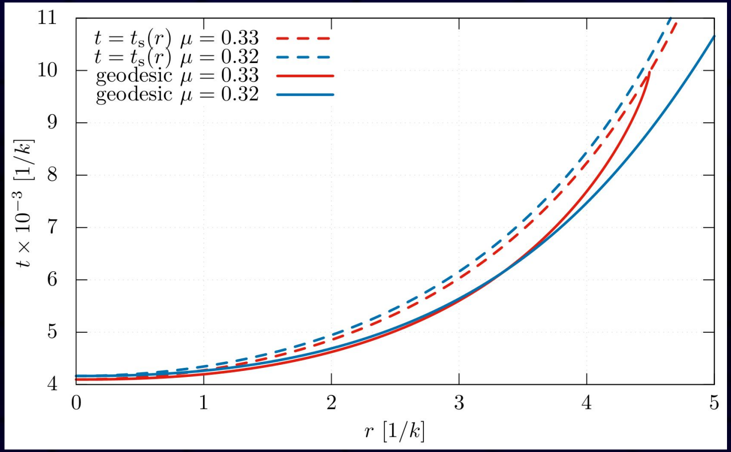
$$m(r) = \frac{4\Psi^6}{3t_\mathrm{i}^2} \qquad k(r) = \frac{1}{r^2}\left[1-(1+2r\partial_r \ln\Psi)^2\right]$$

Spacetime Structure of the LTB Solution

◎Globally naked or Locally naked







$$\mu \leq 0.32$$

"Globally naked"
⇒No **PBH** formation

$$\mu \geq 0.33$$

"Locally naked"
⇒**PBH** formation

[1810.03490
T. Kokubu, K. Kyutoku, K. Kohri, T. Harada]

Comparison of analytic criteria and numerical simulation

- Spherically symmetric case w/ dust fluid
- Anisotropic (elliptic) collapse w/ dust fluid
- Anisotropic (elliptic) collapse w/ collisionless particles

Ongoing work

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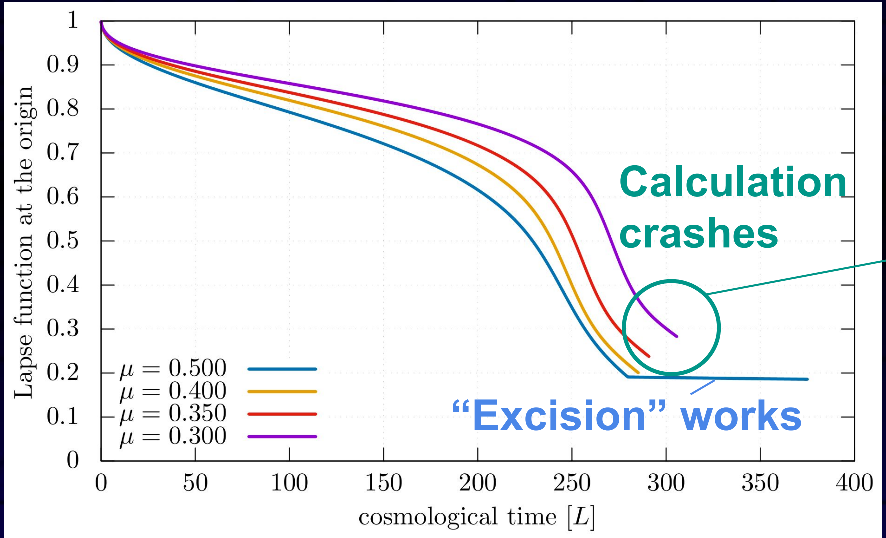
Ongoing work

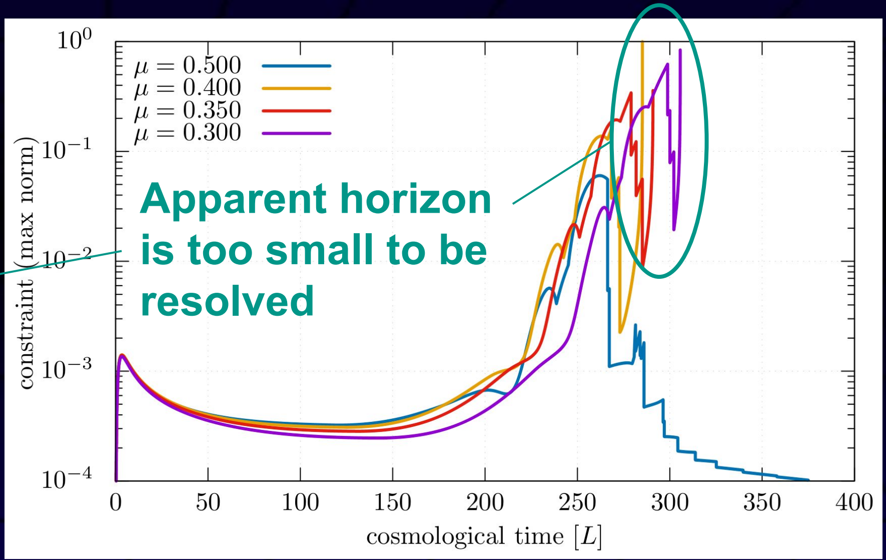
Comparison with numerical relativity simulation

◎Full 3+1 dim GR + fluid with P=0 COSMOS code (https://sites.google.com/view/cosmoscode/)

◎Time evolution of the lapse at the origin



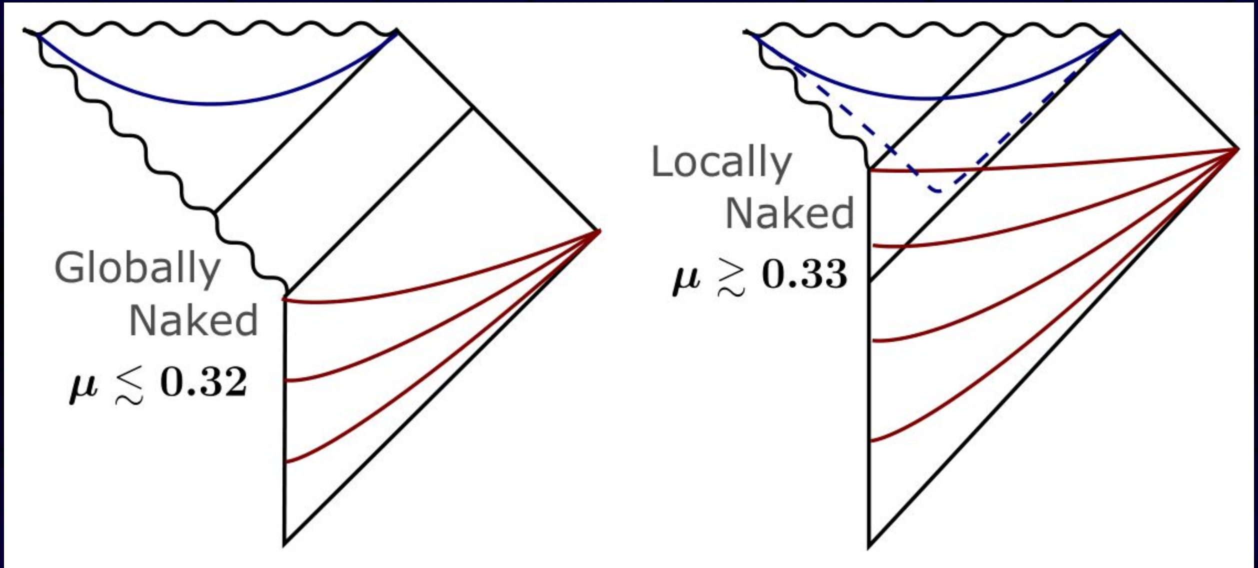
◎Time evolution of the constraint violation



◎Horizon formation without crash for $\mu \geq 0.5$ while we cannot resolve the horizon for $\mu \lesssim 0.5$

Implication of the numerical results

◎What causes the crash of the computation? Singularity!



◎It may not be avoided by simply increasing the resolution

◎Numerical simulation imply the sufficient condition $\mu \gtrsim 0.5$ (consistent w/ $\mu \gtrsim 0.33$)

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Ongoing work

A specific profile

◎Metric at the leading order of the gradient expansion

$$ds^2 = -dt^2 + \Psi^4(dX^2 + dY^2 + dZ^2)$$

◎We focus on the following specific profile of Ψ

$$\ln\Psi = \frac{\mu}{2}\exp\left[-\frac{1}{6}k^2r^2\right]\left[1 + \frac{k^2}{6}\left(p(2X^2 - Y^2 - Z^2) + 3e(Y^2 - Z^2)\right)\right]$$

μ : amplitude, k : wave number p : prolateness, e : ellipticity

$$r^2 = X^2 + Y^2 + Z^2$$

◎Initial data for simulation can be generated from Ψ

◎Full 3+1 dim GR + fluid with P=0 COSMOS code (https://sites.google.com/view/cosmoscode/)

◎At the linear order, deformation tensor $\vec{D} \simeq \frac{4}{5}\frac{1}{a^2H^2}\nabla\ln\Psi$

PBH formation criterion

◎Diagonalized deformation tensor

$$\nabla \vec{D} = \frac{4}{5}\frac{1}{a^2 H^2}\nabla\nabla \ln \Psi = -\mathrm{diag}\{\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\} \quad \text{with} \quad \tilde{\alpha} \geq \tilde{\beta} \geq \tilde{\gamma}$$

◎PBH formation criterion [1609.01588 T. Harada, CY, K. Kohri, K. Nakao, S. Jhingan]

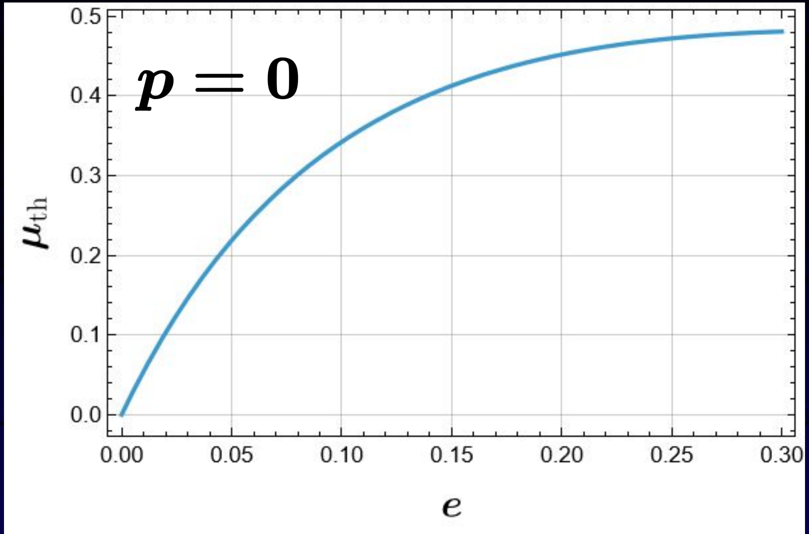
$$h(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) := \frac{2}{\pi}\frac{\tilde{\alpha}-\tilde{\gamma}}{\tilde{\alpha}^2} E\left(\sqrt{1-\left(\frac{\tilde{\alpha}-\tilde{\beta}}{\tilde{\alpha}-\tilde{\gamma}}\right)}\right)$$

$$h(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})|_{\text{horizon entry}} < 1$$

◎For the specific profile

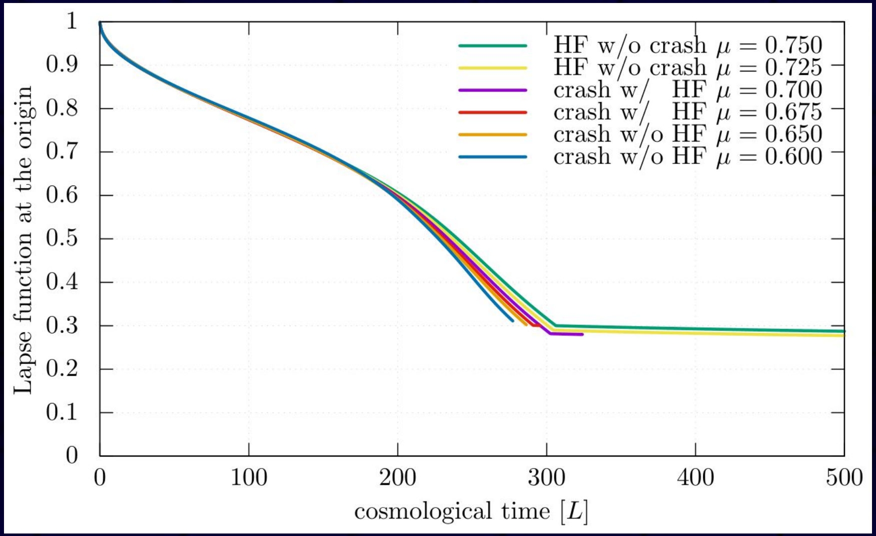
$$\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right) = \frac{2\mu}{15}\frac{k^2}{a^2 H^2}(1+3e+p, 1-2p, 1-3e+p)$$

$$\Rightarrow \mu_{\mathrm{th}} = \frac{15}{\pi}\frac{e}{(1+3e+p)^2} E\left(\sqrt{1-\frac{(e+p)^2}{4e^2}}\right)\frac{6a^2H^2}{k^2}$$

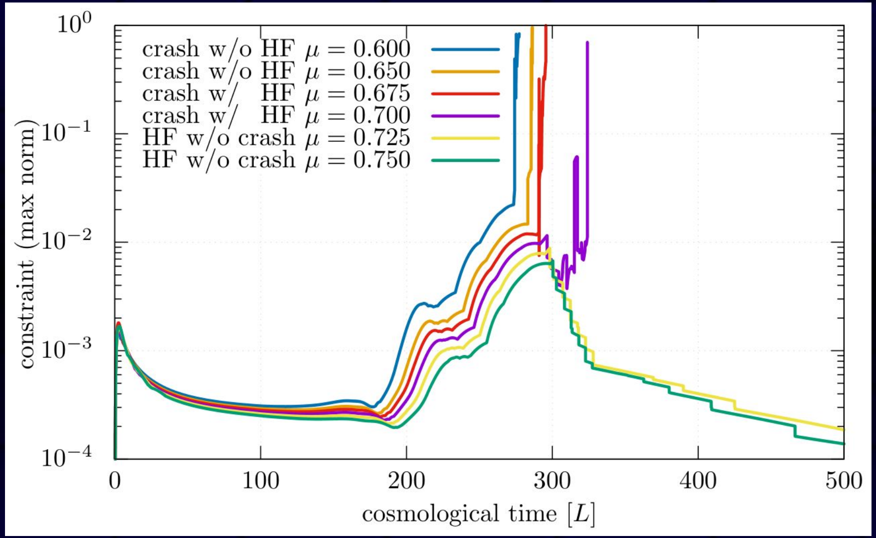


Non-spherical simulation (e=0.1)

◎Time evolution of the lapse at the origin



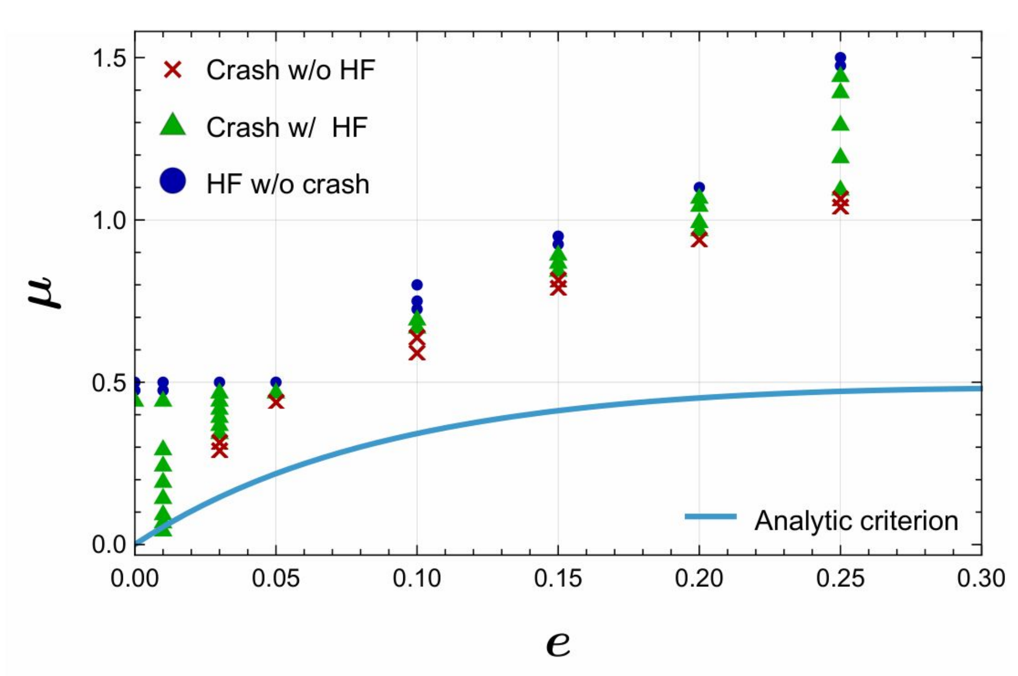
◎Time evolution of the constraint violation



◎The results can be classified to, in order of μ,

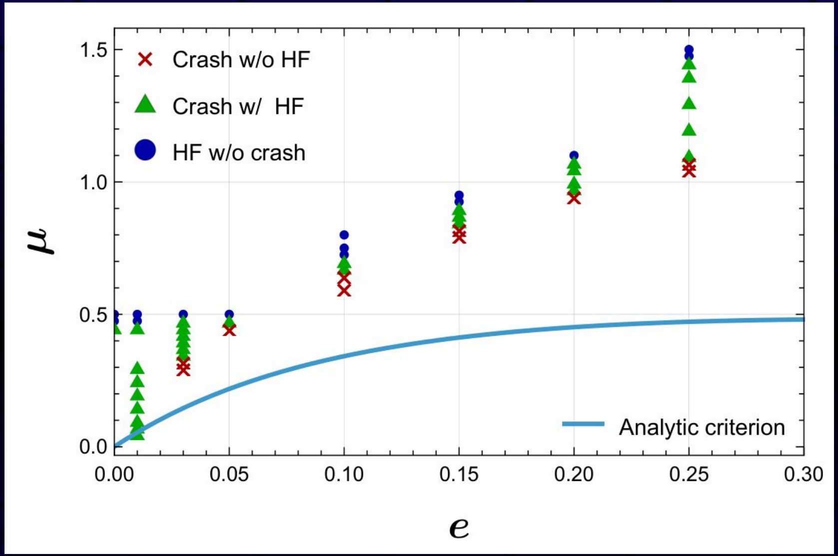
1. Crash without Horizon Formation
2. Crash after Horizon Formation
3. Horizon Formation without Crash

Comparison with the analytic criterion



◎"HF w/o crash" may give sufficient conditions for **PBH** formation
(consistent with the analytic criterion)

Comparison with the analytic criterion



◎The threshold value might be greater within the range of fluid approximation is valid

⇒**PBH** formation might be harder

◎Caveat: Does the failure of the fluid description mean non **PBH** formation?

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Ongoing work

Numerical Procedure

◎Particle method (full GR Particle-In-Cell) [Shibata(1999),CY-Okawa-Ikeda(2017)]

- 2nd order leap frog with BSSN (with time filtering)
- (modified) 1+log slice for lapse
- Time evolution

1. Evolve geometrical variables
2. Evolve particle variables solving geodesic eqs.
3. Set energy momentum tensor from the particle distribution
4. Return to 1

Particle treatment

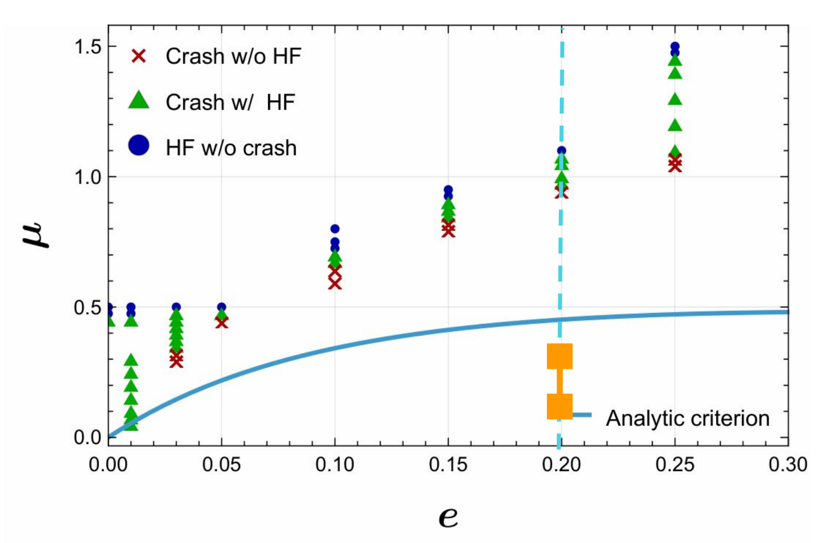
◎Specific setting for the current purpose

- Properly aligned cubic particles ⇒ initially no space no overlap to reduce noises
- Particle masses are set to reproduce the inhomogeneity
  (each particle mass is time independent but not identical)

Settings

◎We focus on the following specific profile of ψ

$$\ln\Psi = \frac{\mu}{2}\exp\left[-\frac{1}{6}k^2r^2\right]\left[1+\frac{k^2}{6}\left(p(2X^2-Y^2-Z^2)+3e(Y^2-Z^2)\right)\right]$$



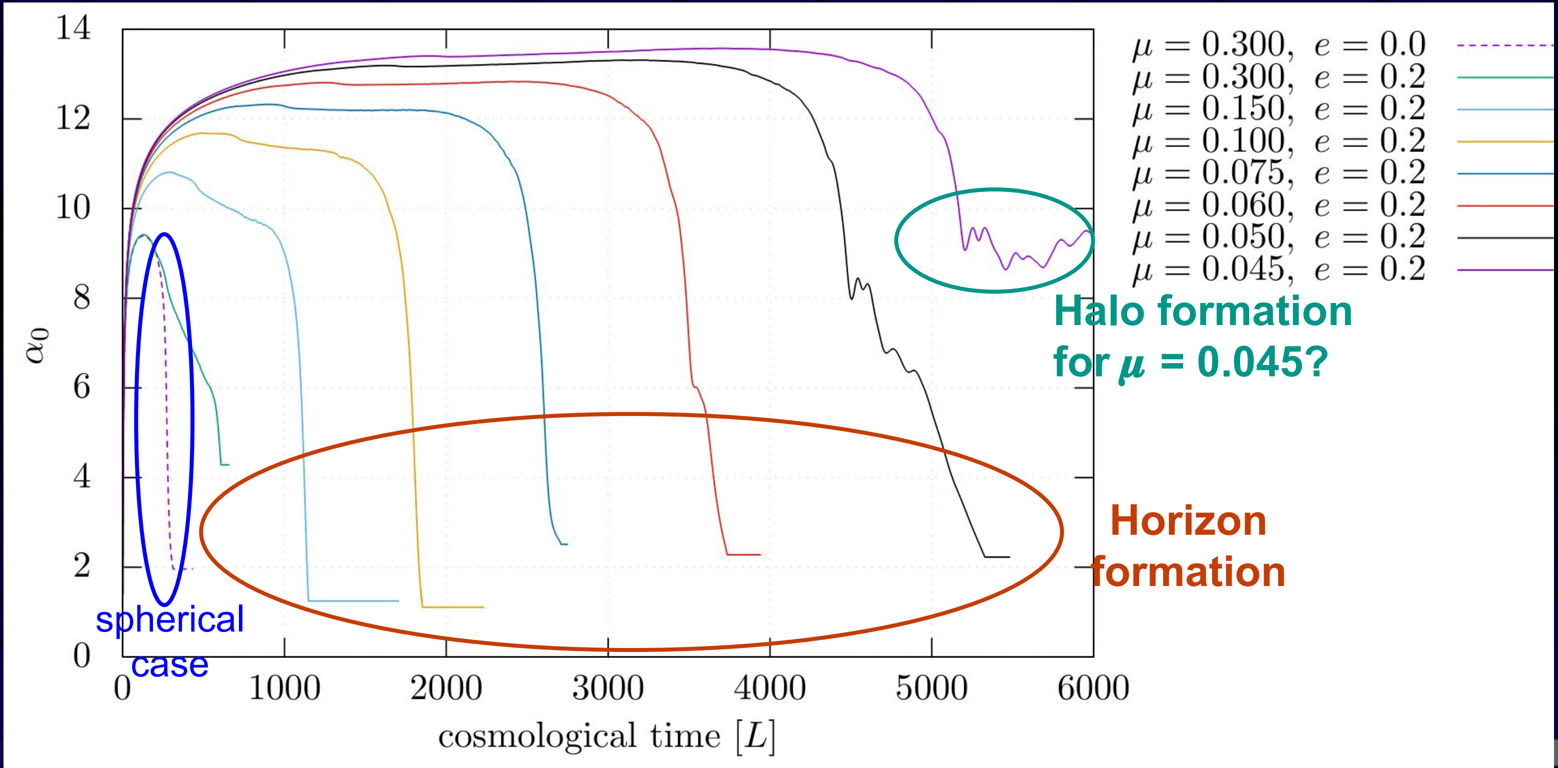
◎Parameters

p = 0, e = 0.2

0.045 ≤ μ ≤ 0.03

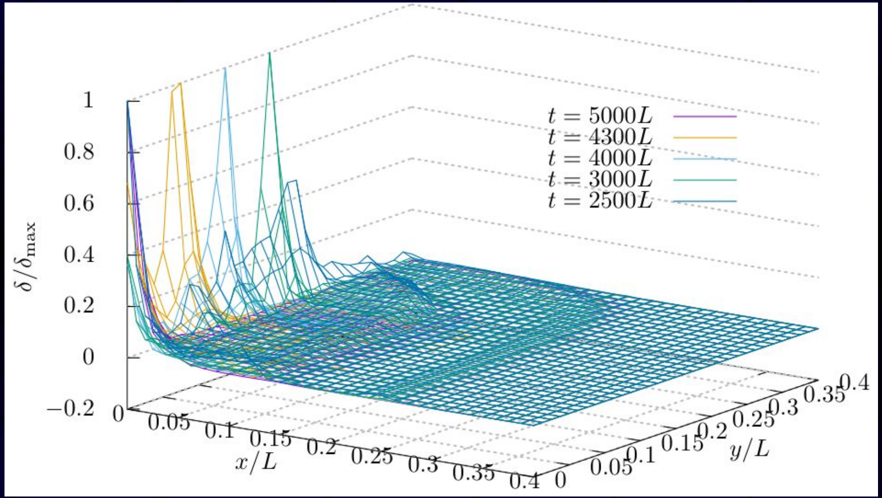
Evolution of the lapse function at origin

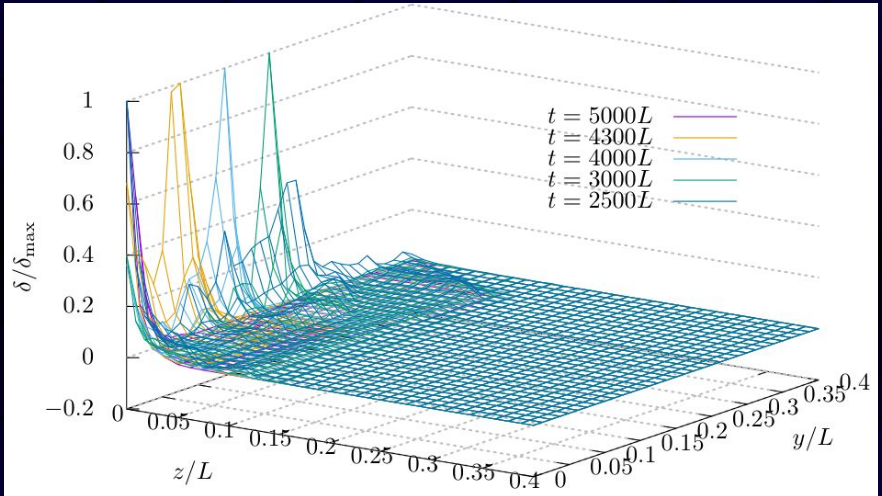
◎Calculations do not crash even for $\mu \lesssim 0.3$, horizons form for $\mu \lesssim 0.05$



Evolution of density distribution

◎μ=0.05

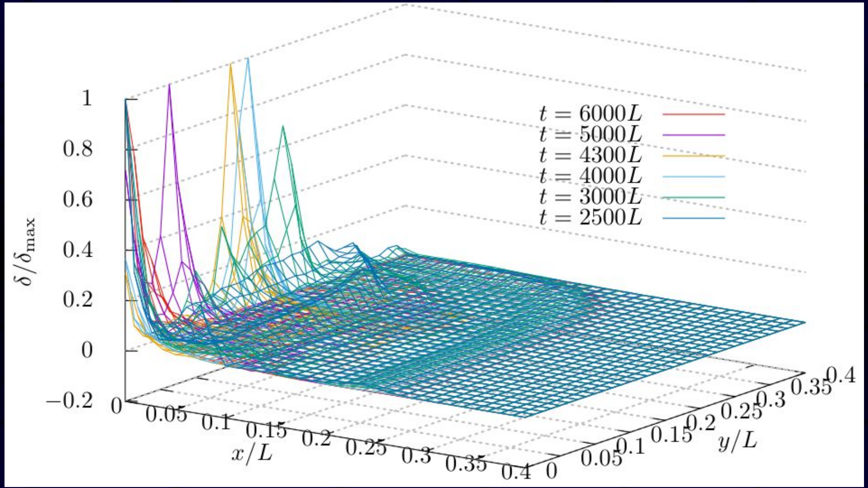


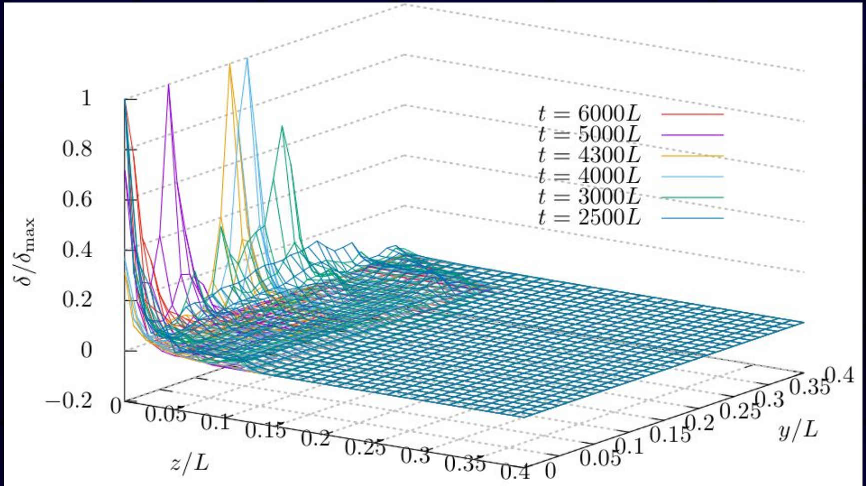


◎A density peak appears at the tip of the longest direction of the distribution
→Shrinks towards center→BH forms

Evolution of density distribution

◎μ=0.045

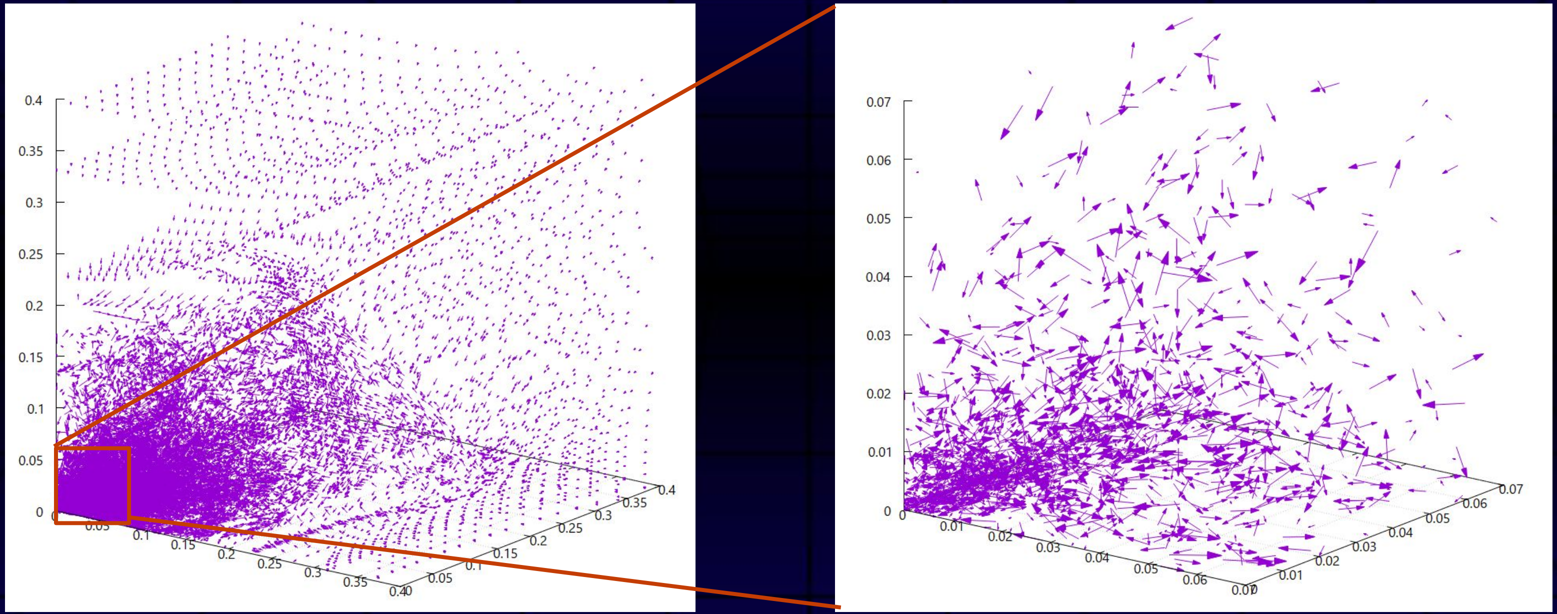




◎A density peak appears at the tip of the longest direction of the distribution
→Shrinks towards center→Halo formation (?)

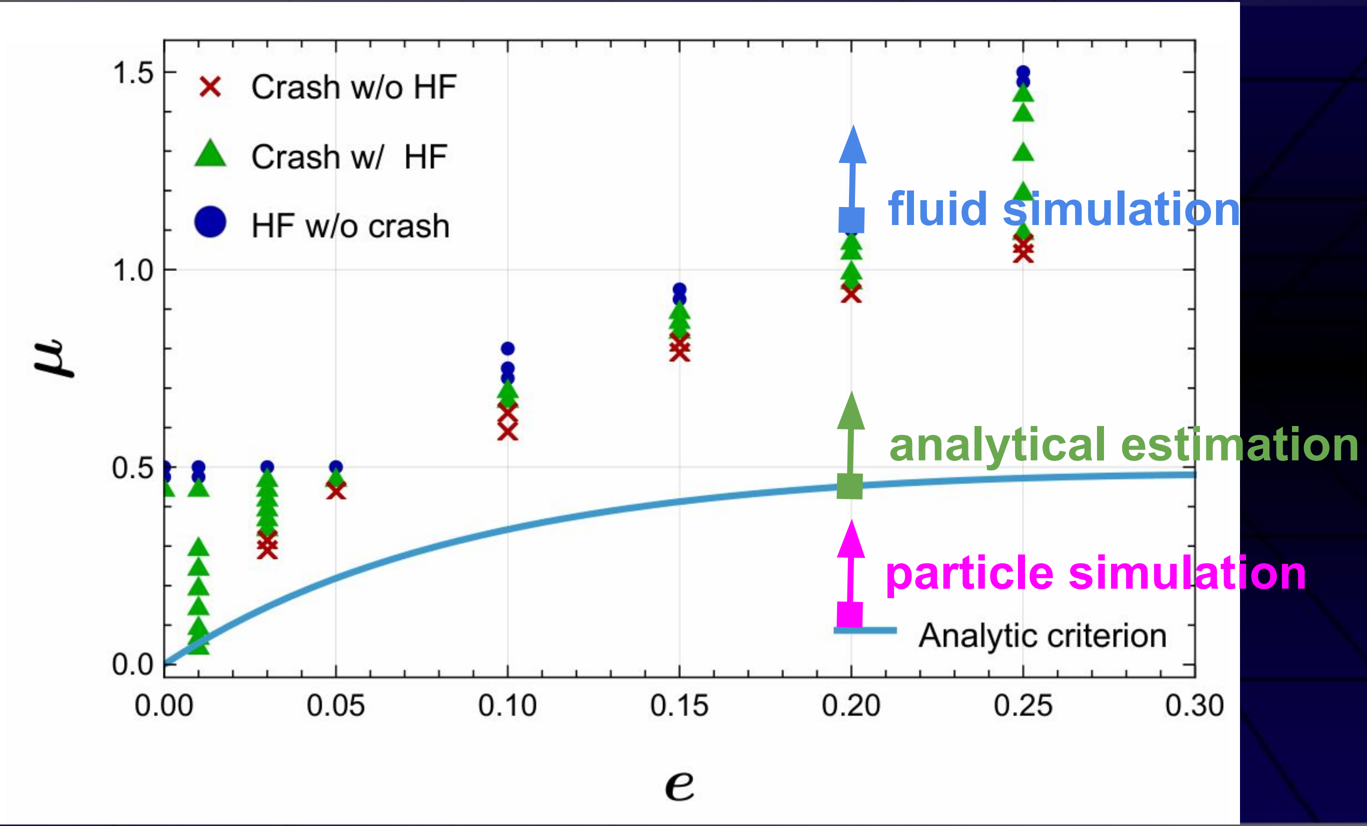
Hale supported by the velocity dispersion?

◎μ=0.045



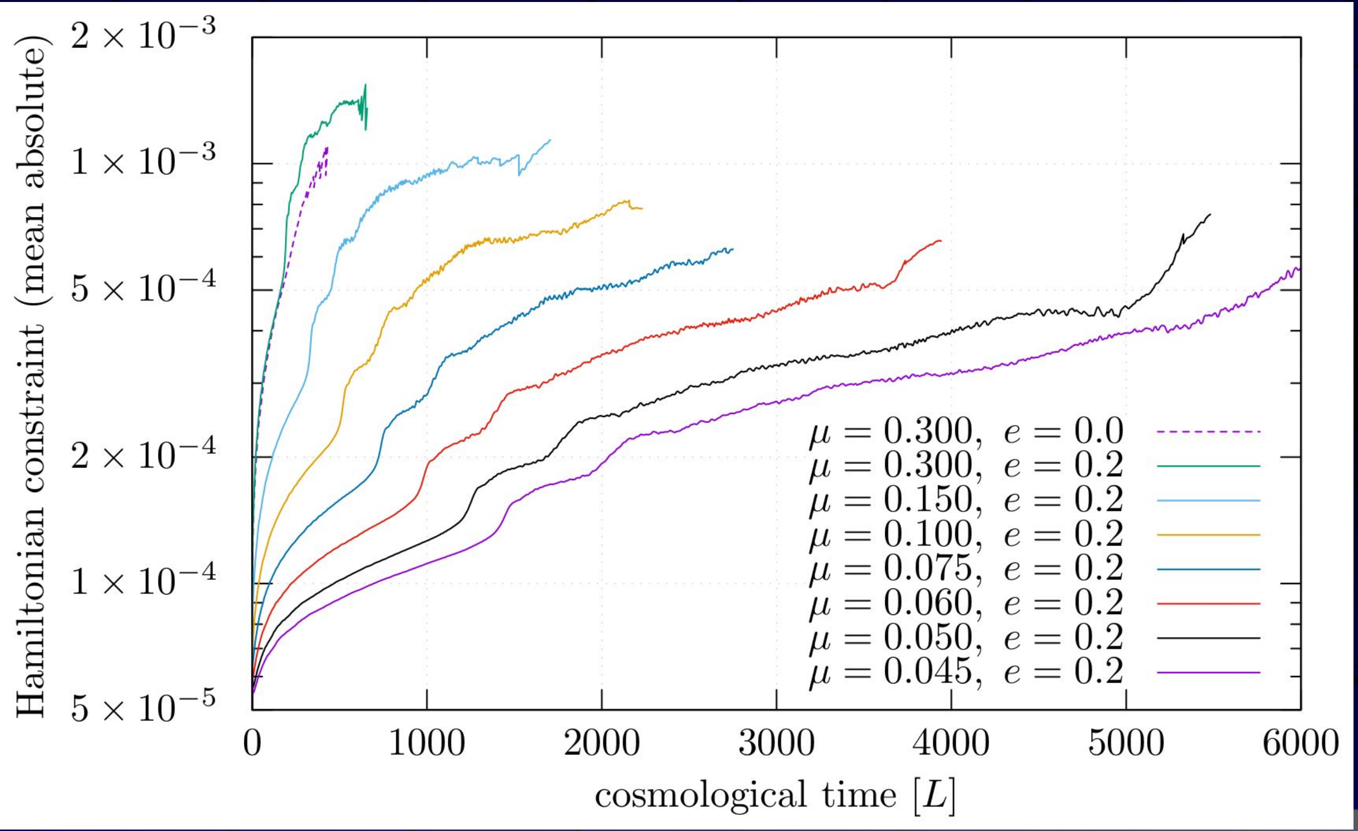
◎Velocity dispersion is certainly generated

Comparison

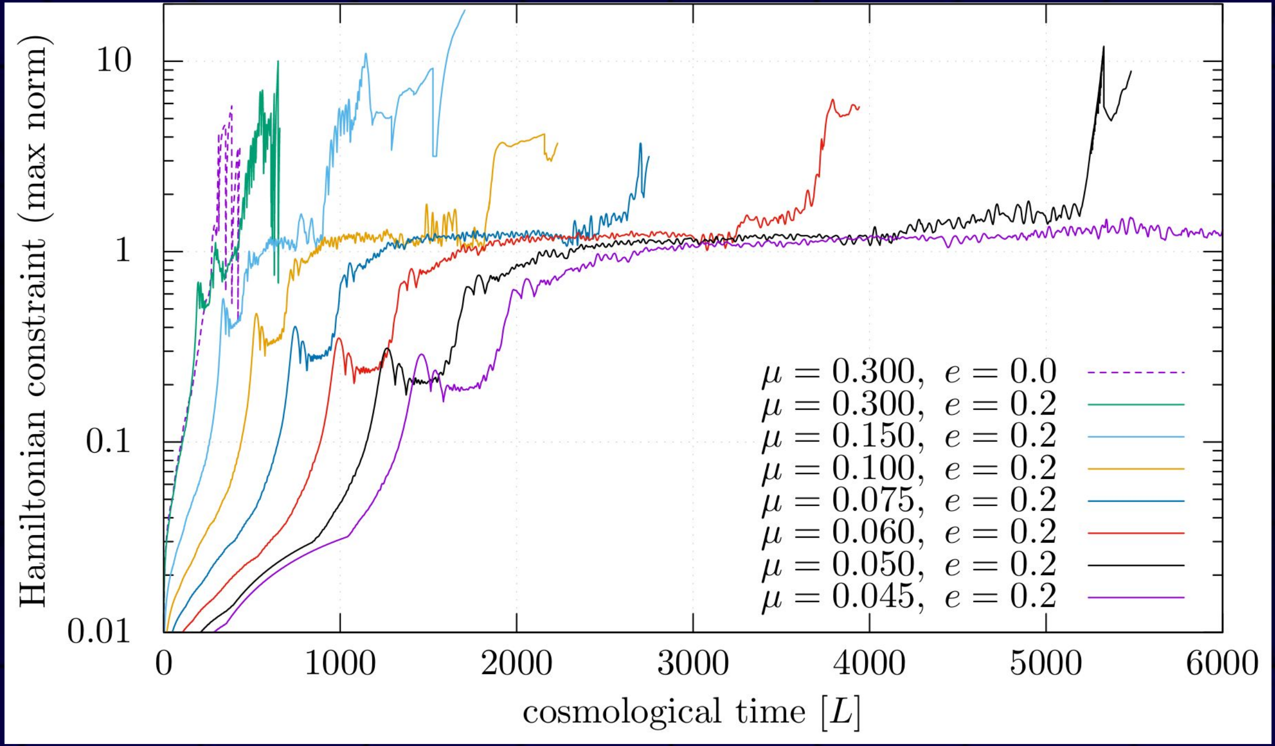


1.5
1.0
0.5
0.0
μ
Crash w/o HF
Crash w/ HF
HF w/o crash
fluid simulation
analytical estimation
particle simulation
Analytic criterion
0.00
0.05
0.10
0.15
0.20
0.25
0.30
e

Constraint violation - mean absolute value



Caveat: Constraint violation -max-norm



Significant constraint violation! (locally)

Summary

◎PBH formation criterion in non-spherical collapse in a matter dominated universe is investigated through numerical simulations

◎Sufficient condition given by fluid simulation is consistent with the analytic estimation. But the indicated threshold is much higher ~ PBH formation harder.

◎Particle system calculation indicates that PBH may form through subsequent dynamical processes ~ PBH formation easier.

⇒Effects of ellipticity might be much weaker
(⇒Spin effects might be more important)

◎Caveat: constraints are significantly violated for the simulation with collisionless particles

◎We still have a huge uncertainty…

Thank you for your attention

End

Angular momentum
(skip details)

PBH in MD

◎Early **MD** era (e.g., oscillation of inflaton soon after the inflation ~ effectively no pressure)

◎PBH formation would be easier than radiation dominated era

◎What prevents PBH formation?

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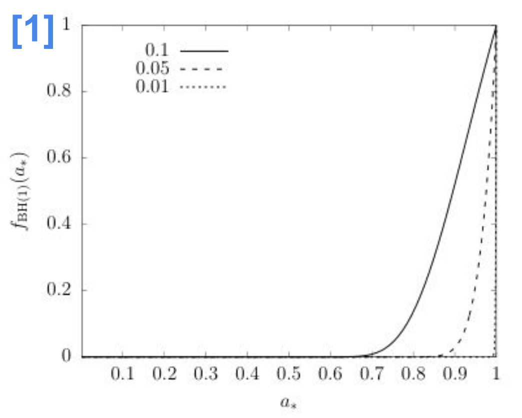
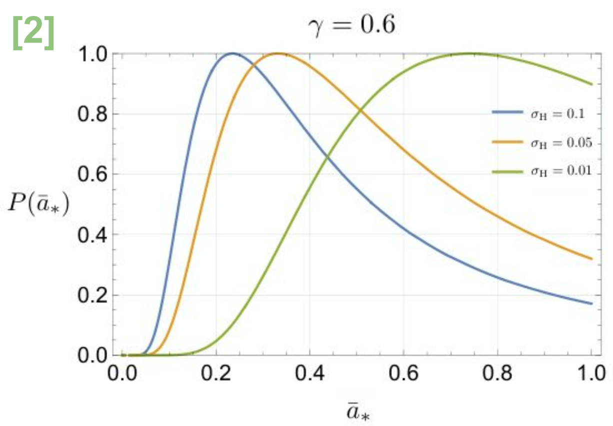
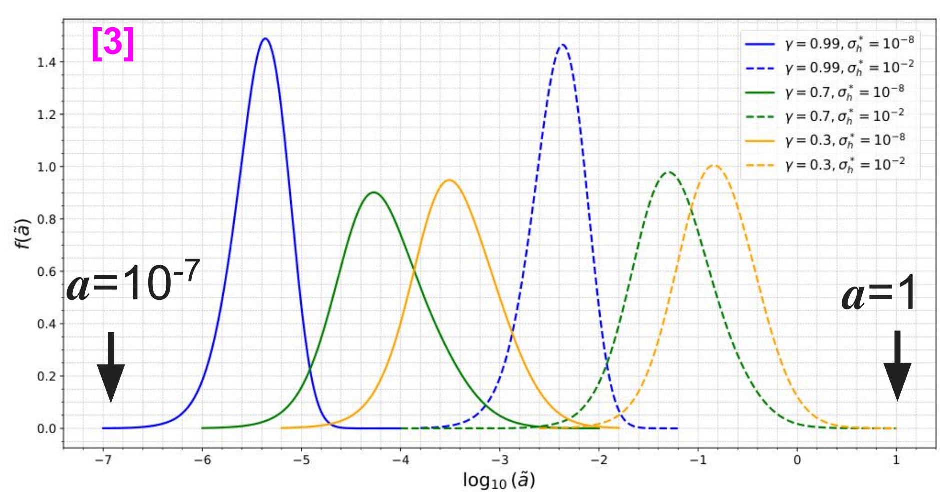
Spin distribution

[2508.10070 W. Ye, Y. Gong, T. Harada, Z. Kang, K. Kohri, D. Saito, CY]



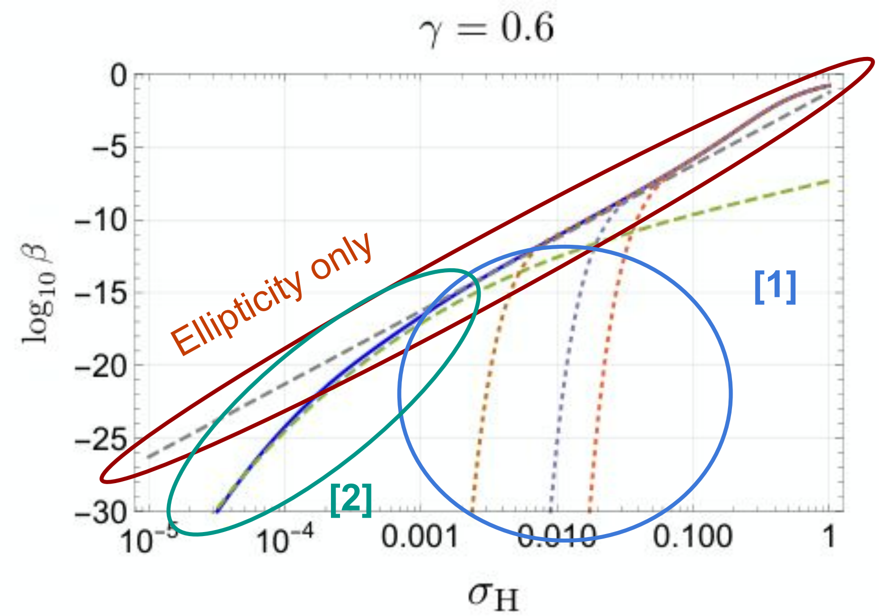


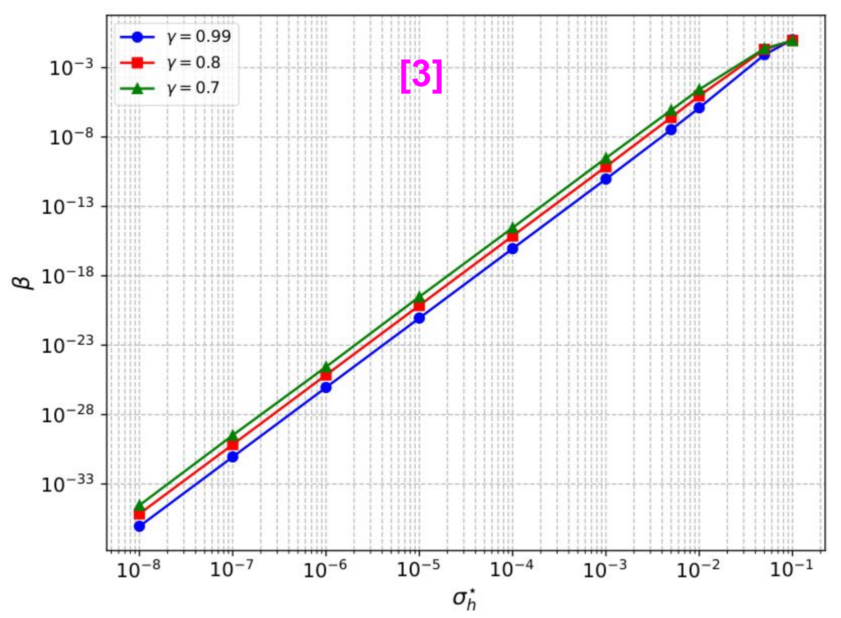


◎PBH spin is expected to be negligibly small
(with Zeldovich approx, hoop conjecture and analytic spin estimation)

Spin effect on abundance

[2508.10070 W. Ye, Y. Gong, T. Harada, Z. Kang, K. Kohri, D. Saito, CY]

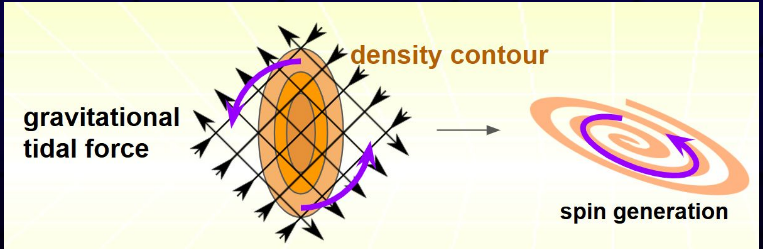




◎Negligible with Zeldovich approx, hoop conjecture and analytic spin estimation

Spin generation due to tidal torque

◎Angular momentum inside an ellipsoid Γ



$$\vec{S} \simeq \rho_{\rm b} a^5 \int_\Gamma d^3\vec{x} \underbrace{\left(\vec{x} \times \frac{D}{dt}\vec{D}\right)}_{\sim (\vec{r} \times \vec{v})/a^2}$$

$$S^i = \rho_{\rm b} a^3 t \epsilon_{ijk} \mathcal{D}_{jl} V^{kl}$$

Deformation tensor $\mathcal{D}_{ij} = \partial_i \partial_j \Psi$

Quadrupole moment $V^{ij} = \int_\Gamma d^3\vec{x} x^i x^l$

Principal direction of V $\downarrow$

$$S^1 \propto D_{23}(V^{33} - V^{22})$$
$$S^2 \propto D_{31}(V^{11} - V^{33})$$
$$S^3 \propto D_{12}(V^{22} - V^{11})$$

⇒ No spin generation for aligned cases

Mean square of the spin -1

◎Dimension less spin

$$\frac{(S^i)^2}{M^4} = \left(\frac{t^2}{\Gamma^2 \rho_b a^3}\right)^2 \epsilon_{ijk}\epsilon_{imn} V^{kl} V^{ns} \mathcal{D}_{jl} \mathcal{D}_{ms}$$

No correlation as probability variables (assumption or derived with peak theory with Gaussian assumption)

◎Ensemble average

$$\langle \mathcal{D}_{ij} \mathcal{D}_{ms} \rangle$$

[1]
simple ensemble average in the background

$$\propto \frac{1}{15}(\delta_{ij}\delta_{ms} + \delta_{im}\delta_{js} + \delta_{is}\delta_{jm})\sigma_0^2/a^2$$

Time independent

[2]
ensemble average through peaks with peak theory

$$= \frac{1-\gamma^2}{15}(4\pi\rho_b a^2 \sigma_0)^2 (\delta_{ij}\delta_{ms} + \delta_{im}\delta_{js} + \delta_{is}\delta_{jm})\sigma_0^2/a^2 \quad \text{with } \gamma = \frac{\sigma_1^2}{\sigma_0\sigma_2}$$

$$\Rightarrow \langle a^2 \rangle_{\mathcal{D}} := \left\langle \frac{(S^i)^2}{M^4} \right\rangle_{\mathcal{D}} = \frac{2^2 \cdot 3}{5^3}(1-\gamma^2)\sigma_H^2 \left(\frac{t}{t_H}\right)^2 Q^2$$

Q: non-dimensional quadrupole moment

Mean square of the spin -2

◎Partial ensemble average of the dimension less spin

$$\langle a^2\rangle_{\mathcal{D}} := \left\langle \frac{(S^i)^2}{M^4} \right\rangle_{\mathcal{D}} = \frac{2^2\cdot 3}{5^3}(1-\gamma^2)\sigma_{\mathrm{H}}^2 \left(\frac{t}{t_{\mathrm{H}}}\right)^2 Q^2$$

[1]

- $t = \langle t_{\text{turn around}} \rangle = t_{\mathrm{H}}\sigma_{\mathrm{H}}^{-3/2}$
- Q: treated as a free parameter

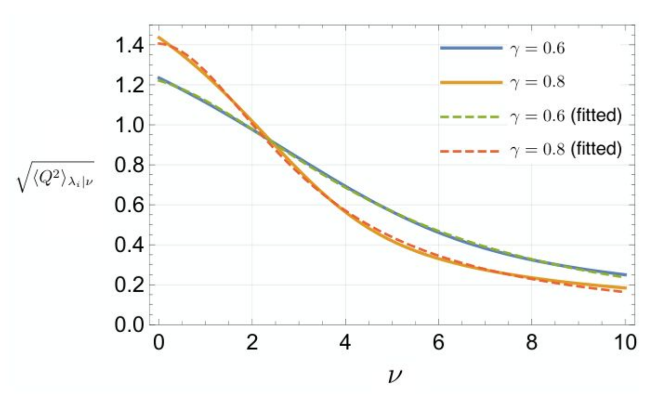
[2]

- $t = t_{\text{turn around}} = \frac{3\pi}{4}\left(\frac{5}{3}\nu\sigma_{\mathrm{H}}\right)^{-3/2}$

$$\nu = \delta_{\text{peak}}/\sigma_0$$

- $Q^2 \to \langle Q \rangle$ with ν fixed through peak theory

  Q is treated as a stochastic variable

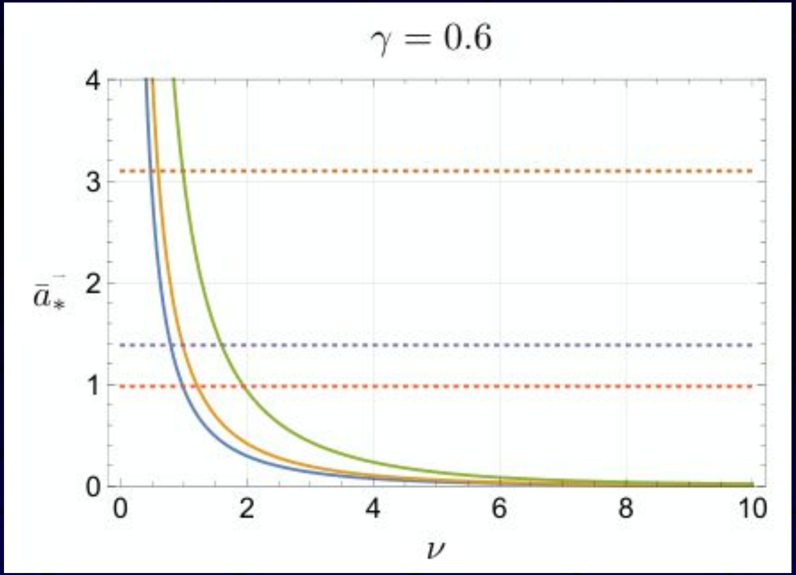


$$\Rightarrow \bar{a}(\nu;\sigma_{\mathrm{H}}) := \sqrt{\langle a^2\rangle} = \frac{2\cdot 3^2}{5^3}\frac{3\pi}{4}\sqrt{1-\gamma^2}\sqrt{\langle Q^2\rangle}\nu^{-3/2}\sigma_{\mathrm{H}}^{-1/2} \propto \sigma_{\mathrm{H}}^{-1/2}$$

PBH formation threshold from spin

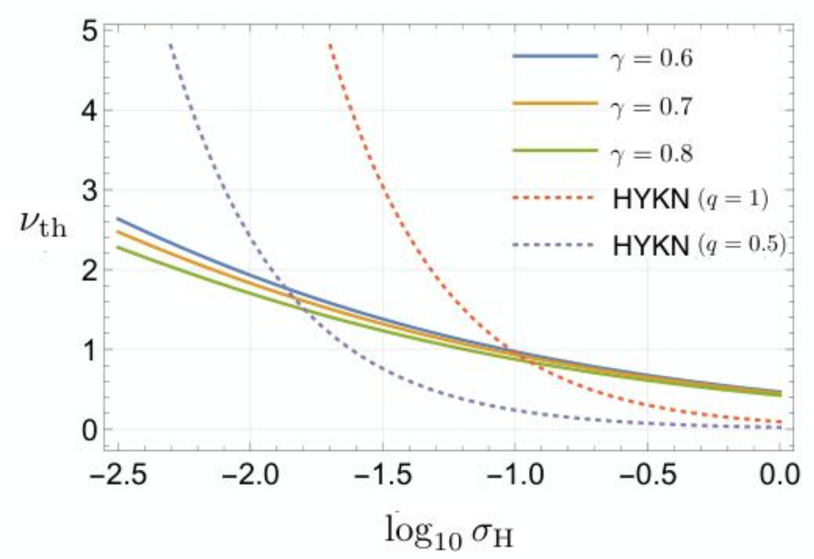
◎Dimensionless spin as a function of ν

$$\bar{a}(\nu;\sigma_{\rm H}) = \frac{2\cdot 3^2}{5^3}\frac{3\pi}{4}\sqrt{1-\gamma^2}\sqrt{\langle Q^2\rangle}\nu^{-3/2}\sigma_{\rm H}^{-1/2} \propto \sigma_{\rm H}^{-1/2}$$



◎Threshold from a<1

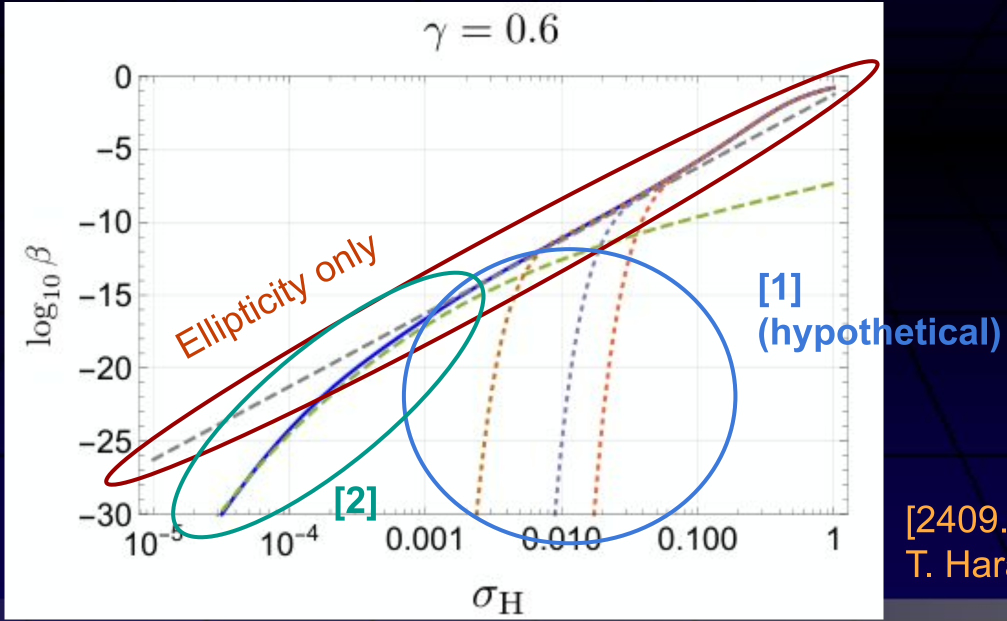
$$\bar{a}(\nu_{\rm th};\sigma_{\rm H}) = 1$$



PBH abundance

◎Abundance based on the Doroshkevich probability distribution

$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma \theta \underbrace{\left[\delta_{\rm H}(\alpha,\beta,\gamma) - \delta_{\rm H,th}\right]}_{\text{From spin}} \underbrace{\theta\left[1 - h(\alpha,\beta,\gamma)\right]}_{\text{From ellipticity}} w(\alpha,\beta,\gamma)$$



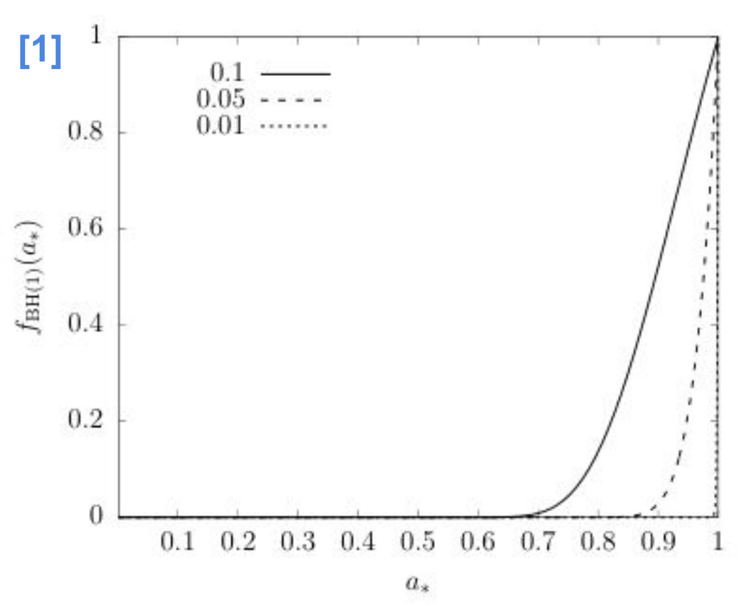
[2409.00435 D. Saito, T. Harada, Y. Koga, CY]

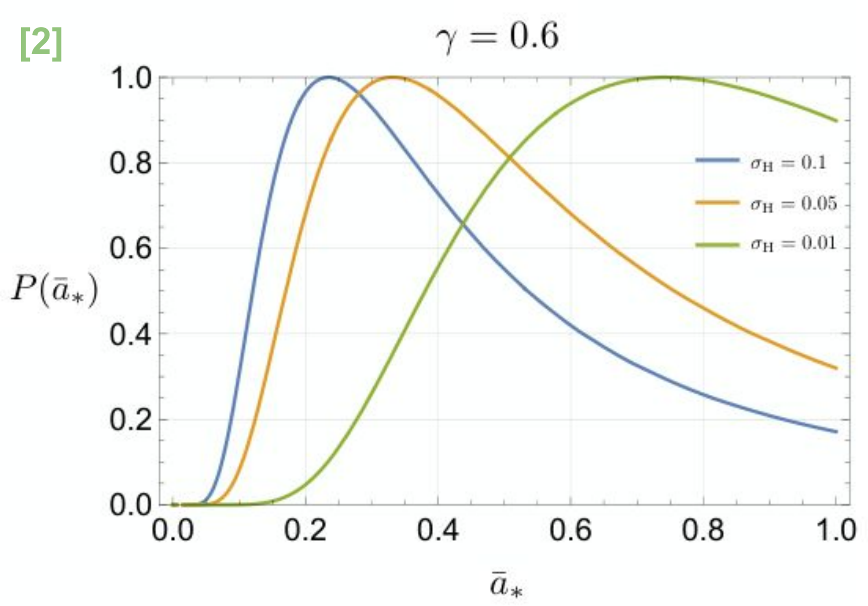
Spin distribution

◎Simple estimation of the spin distribution

$$P(a)da \propto N_{\rm peak}(\nu(a))\frac{d\nu}{da}da$$

[2409.00435 D. Saito, T. Harada, Y. Koga, CY]





Selection effect due to the ellipticity constraint

◎In [2], effects of the ellipticity on PBH spin generation have not been taken into account

- ➢ For realization of large spin, large non-sphericity is needed
  ⇒ Large ellipticity is needed ⇒ Large spin generation would be suppressed

◎In [3], more dedicated analyses have been performed (mainly by Weitao Ye)

- ➢ Due to the ellipticity constraint, $\mathcal{D}_{ij} \sim \sigma_{\mathrm{H}}$ ⇒ spin is not effective for $\sigma_{\mathrm{H}} \ll 1$

$$\frac{(S^i)^2}{M^4} = \left(\frac{t^2}{\Gamma^2 \rho_{\mathrm{b}} a^3}\right)^2 \epsilon_{ijk}\epsilon_{imn} V^{kl} V^{ns} \mathcal{D}_{jl} \mathcal{D}_{ms}$$

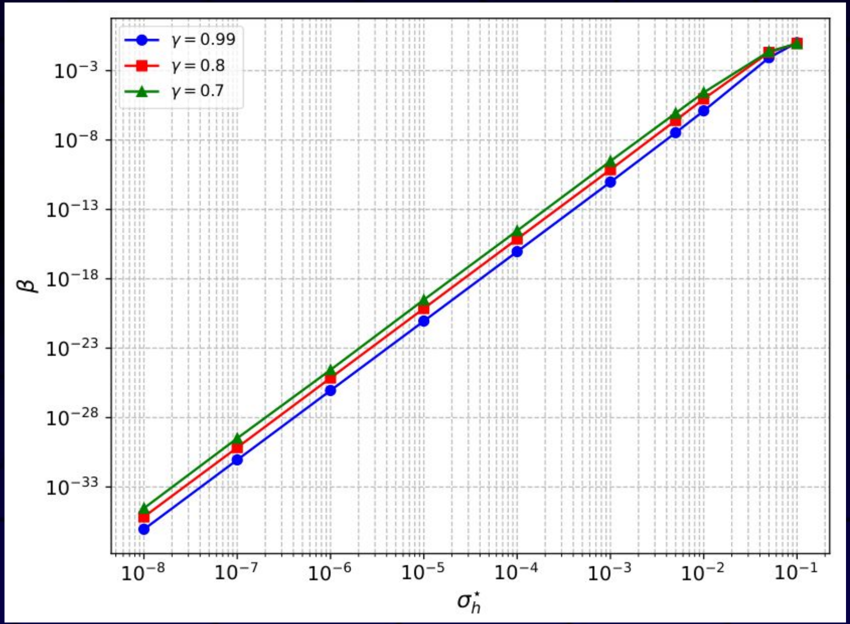
[2] $$\sqrt{\langle a_{\mathrm{PBH}}^2 \rangle} \propto \sigma_{\mathrm{H}}^{-1/2}$$ ⇒ [3] $$\sqrt{\langle a_{\mathrm{PBH}}^2 \rangle} \propto \sigma_{\mathrm{H}}^{1/2}$$

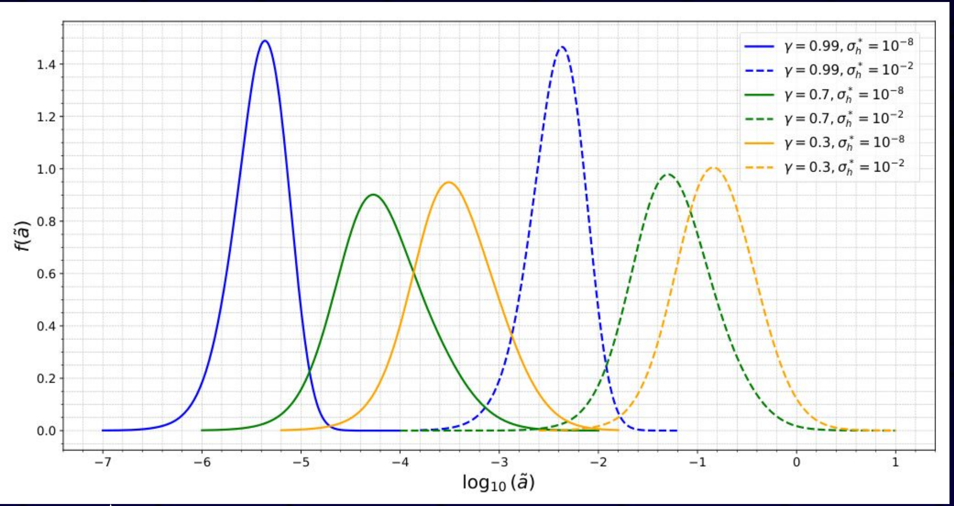
- ➢ 9-dimensional integration with the Monte-Carlo method

Latest results of abundance and spin distribution

◎Suppression due to spin disappeared, and much smaller spin value is predicted [3]

[2508.10070 W. Ye, Y. Gong, T. Harada, Z. Kang, K. Kohri, D. Saito, CY]





a=10^{-7}

a=1

Comparison with LTB solution

◎Spherically symmetric profile

$$\ln \Psi = \frac{\mu}{2}\exp\left[-\frac{1}{6}k^2 r^2\right]$$

◎Method

- ➢ Perform the simulation with the functional form of ψ
- ➢ Extract the value of trK as a function of the proper length as K(l) from the origin for a time slice
- ➢ Identify the time slice by solving the ODEs derived from trK=K(l) in the LTB spacetime
- ➢ Compare the set of geometrical variables: areal radius R vs proper length l