Leibniz Universität Hannover

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

Current issues in secondary gravitational waves

Guillem Domènech
(Leibniz University Hannover)

MITP, the phenomenology of PBHs
July 28th, 2026

GD, Pi & Wang, 2512.15704 (PRL)   Bari, Bartolo, GD & Matarrese, 2307.05404   GD, 2510.26883

My motivation

PBH formation always (?) accompanied by secondary GW background

The GW background is detectable by future GW detectors

So we must understand secondary GWs very well

But there are two issues:

1. Gauge ambiguities
2. "Wave-effects" not understood for GWBs

1. Standard picture

2. The gauge issue

3. Scalar-tensor induced GWs

Standard picture

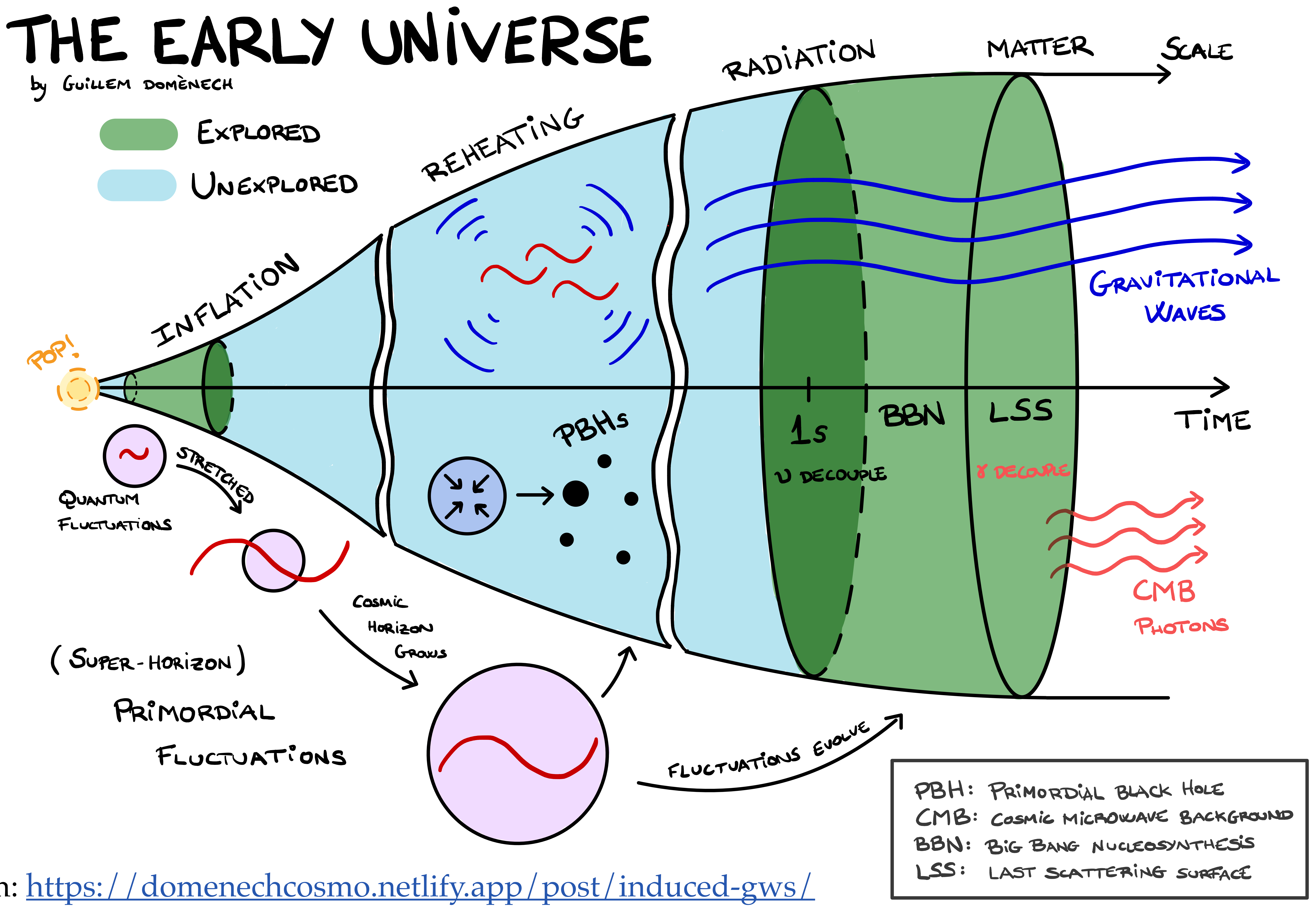
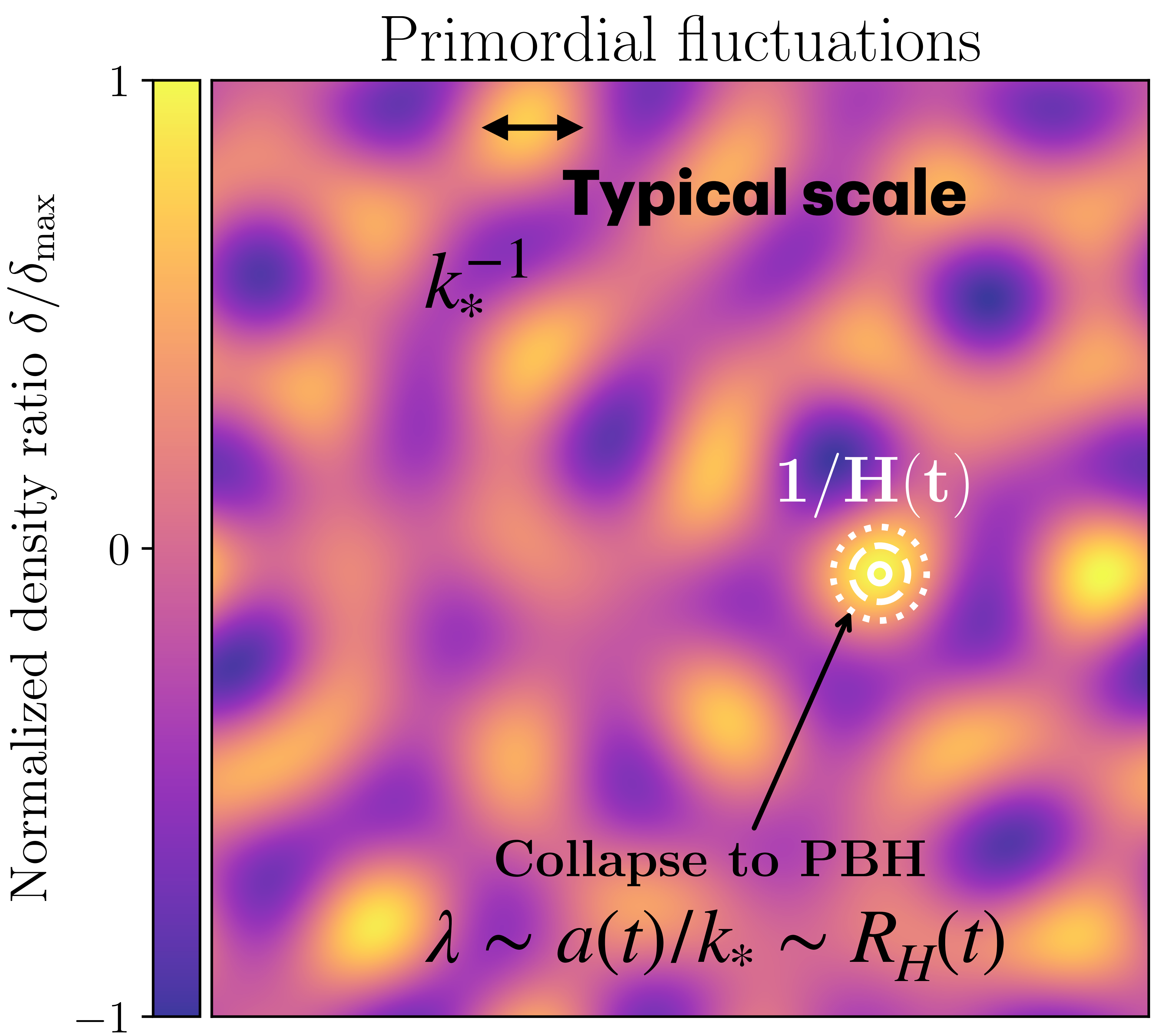


Image from: https://domenechcosmo.netlify.app/post/induced-gws/

Standard picture

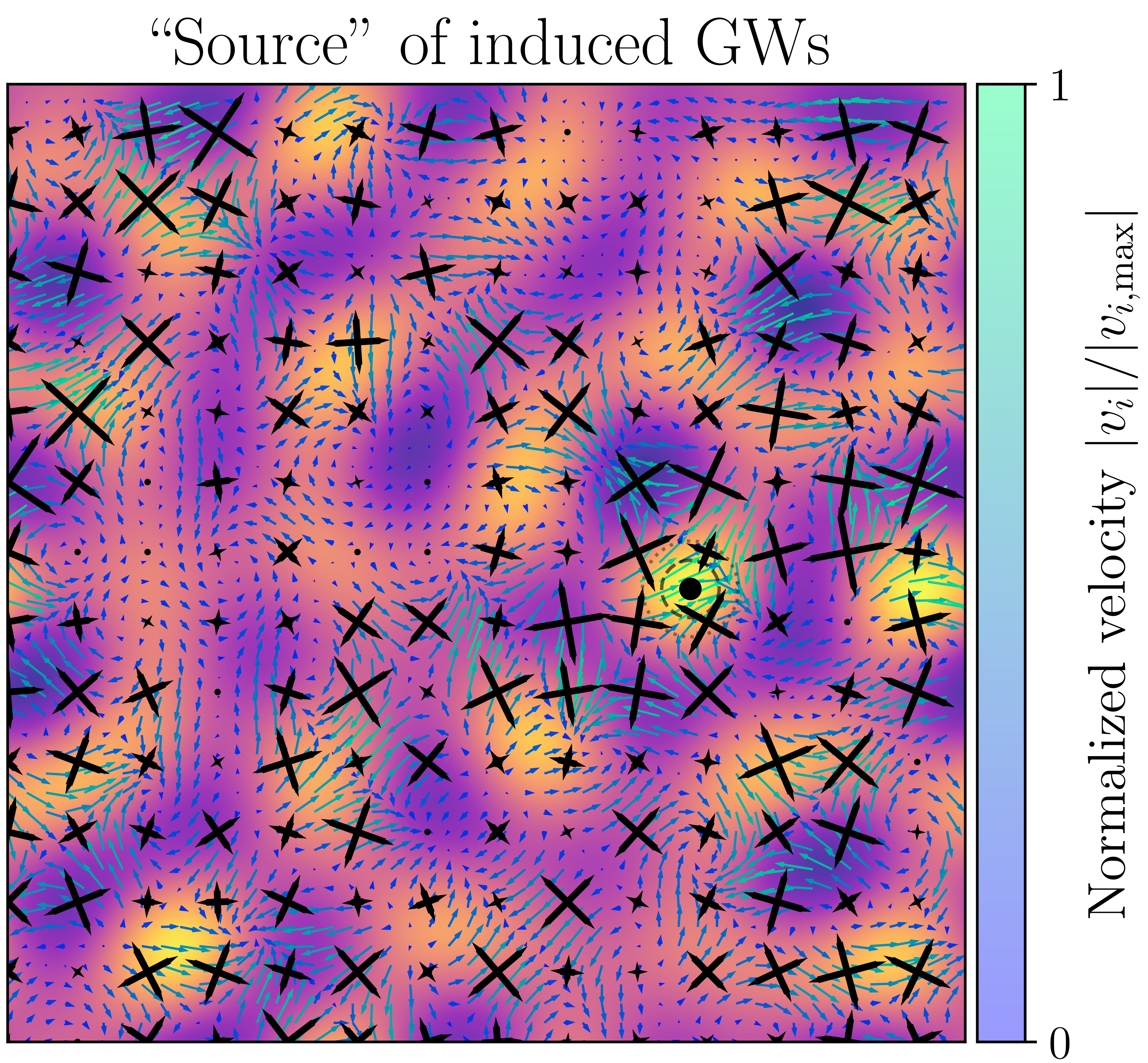


$\Box_s h_{ij} \sim (v_i v_j)^{TT}$

$v_i \propto \partial_i \delta$

Acoustic waves when

$\lambda \sim R_H(t)$



PBH fraction $\sim e^{-\frac{\delta_{\mathrm{th}}^2}{2\sigma^2}}$

Rare fluctuations

VS

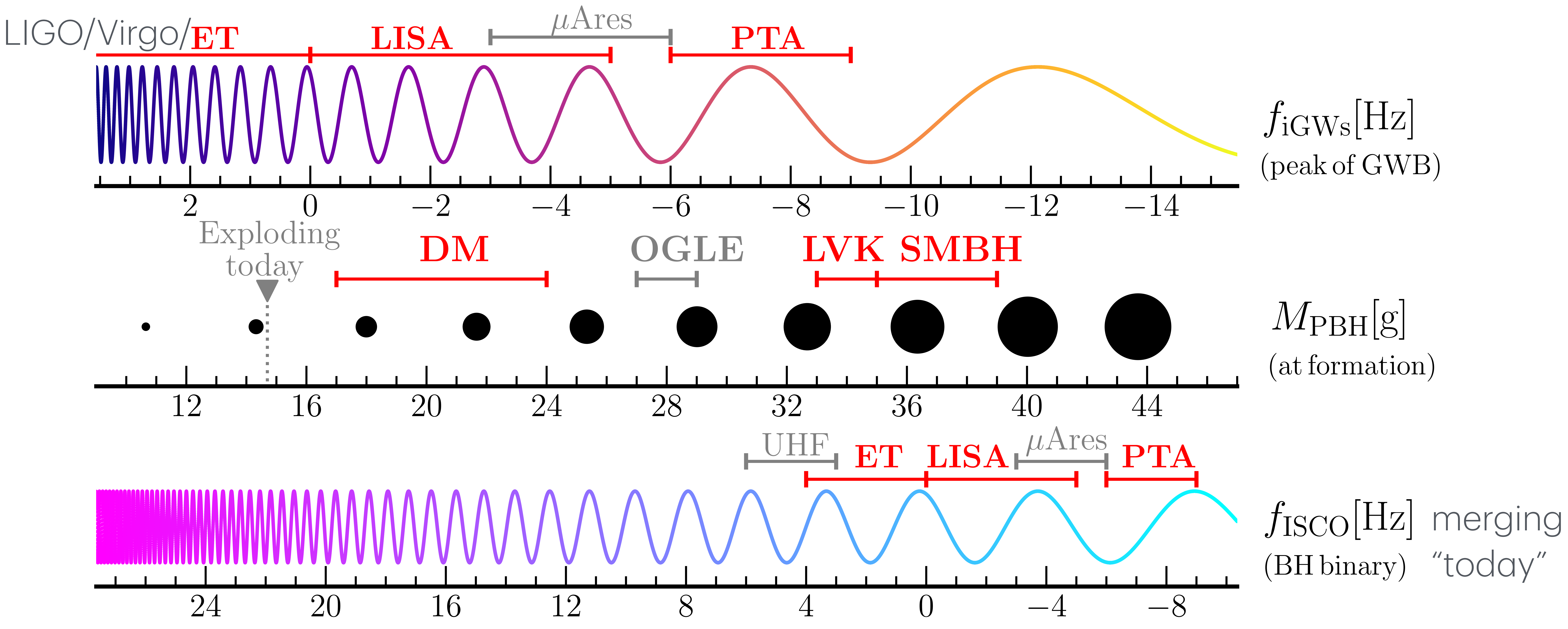
Strain variance: $\sigma_h \sim \sigma^2$

Typical fluctuations

PBH Mass <—> GW Frequency

$$\lambda \sim a(t)/k_* \sim a(t)/f_* \sim R_H(t) \sim 2GM_{\rm PBH}$$

GWs and PBHs from primordial curvature fluctuations – $\log_{10}$ scale



GW Forecasts

Peak GW amplitude:

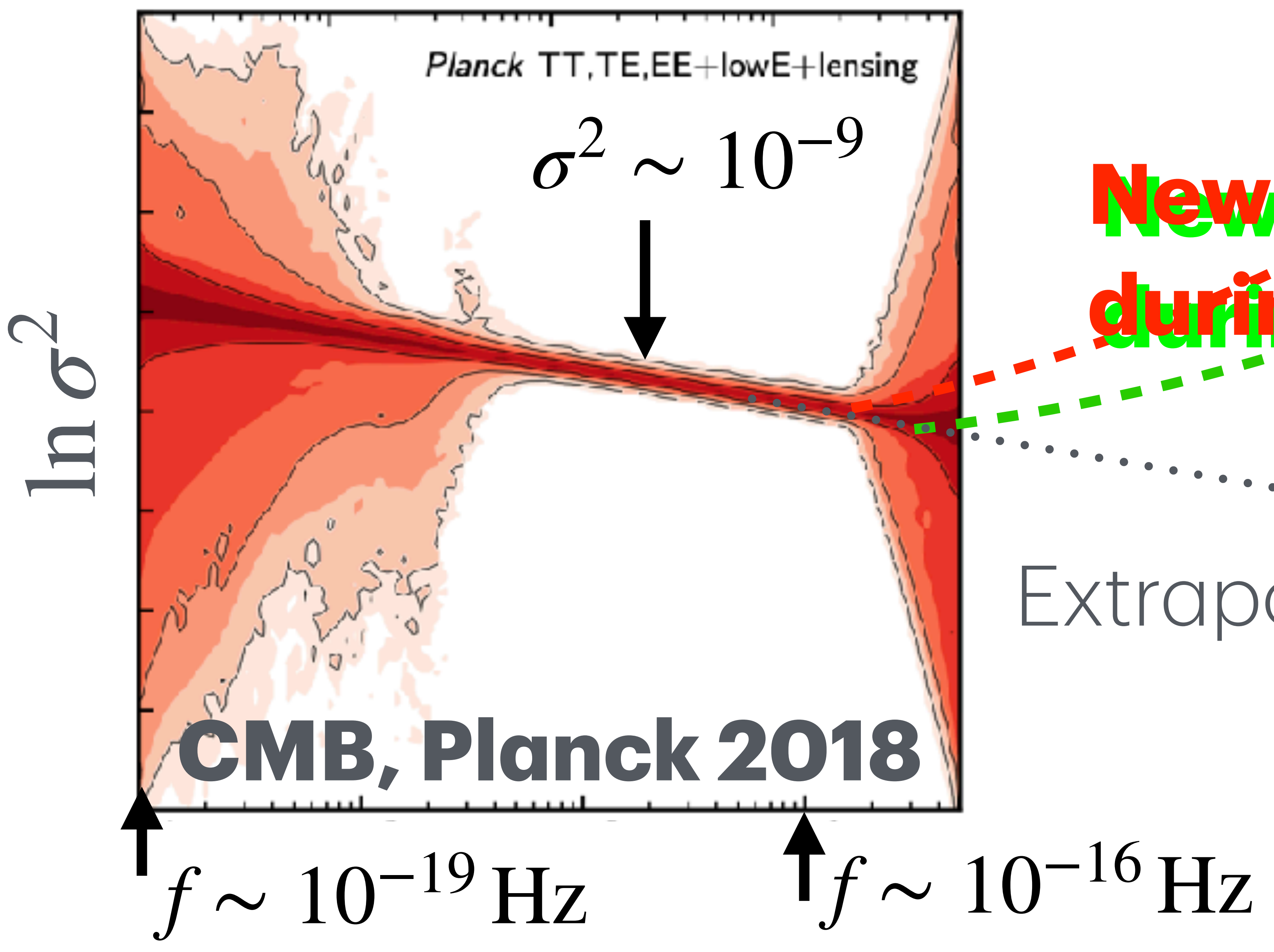
$$\Omega_{\mathrm{GW},0}^{\mathrm{induced}} \sim \frac{1}{12}\Omega_{r,0}\sigma_h^2 \sim 10^{-5}\sigma^4$$

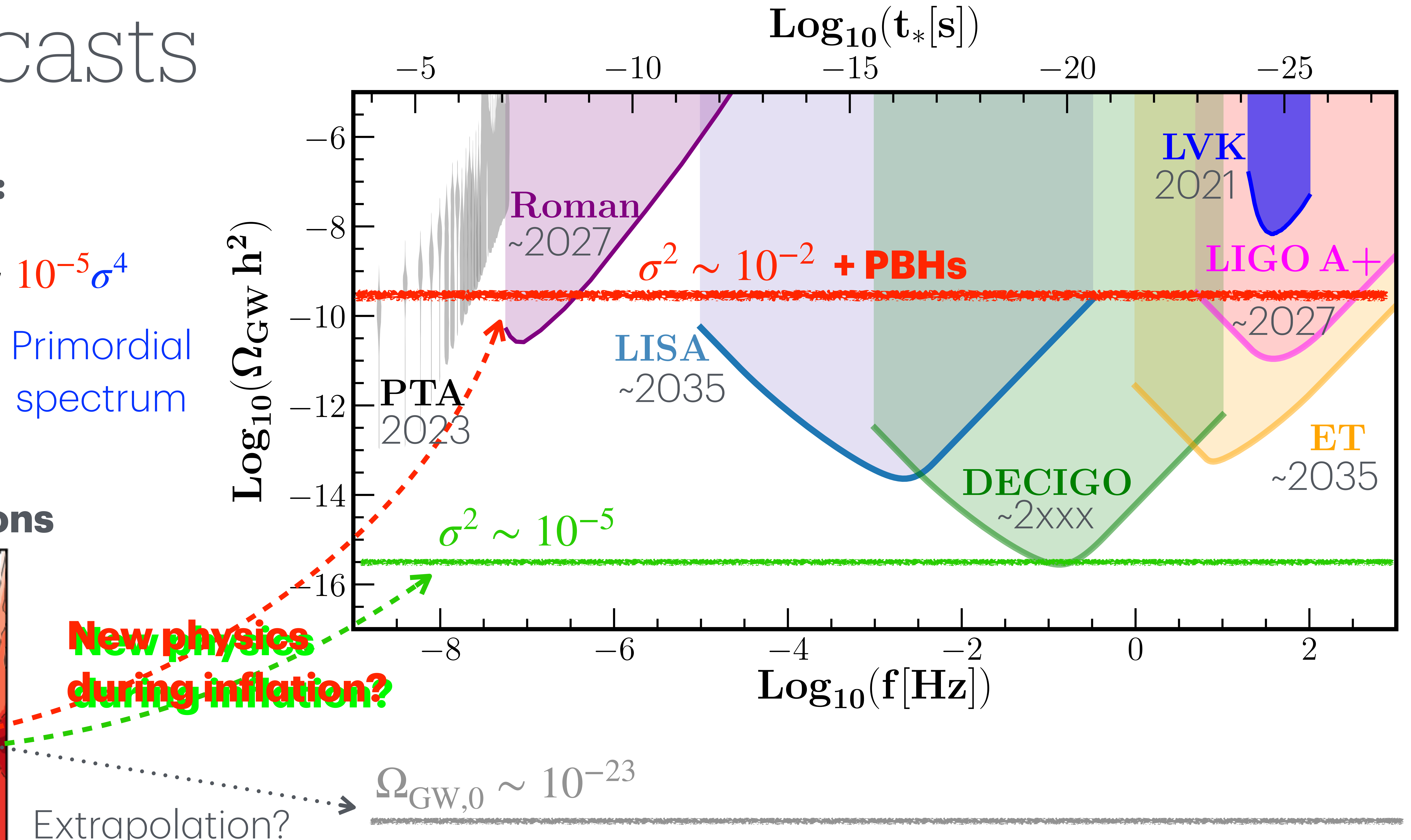
GW energy fraction per log f

Redshift

Primordial spectrum

Primordial fluctuations





The gauge issue

The gauge issue

So many wonderful cosmic sources of GWs:

(TT = Transverse-Traceless)

$$"\Box"\, h_{ij} \approx T_{ij}^{TT} = \boxed{(\rho + P)\left(v_i v_j\right)^{TT}}$$

Quadratic source

—> Second order fluctuations?

So many exciting prospects of detecting cosmic GWs:

$$\rho_{\mathrm{GWs}} = \frac{1}{32\pi G}\frac{1}{a^2}\boxed{\langle h'_{ij} h'_{ij}\rangle}$$

Quadratic in h_{ij}, quartic in v_i

—> Is ρ_{GWs} well-defined?

(Dramatic-mode) Are theoretical predictions even reliable?

$\tilde{t} = t + T(t, x)$ $\qquad$ $\tilde{h}_{ij} = h_{ij} + \left(\partial_i T \partial_j T\right)^{TT}$ $\qquad$ $\tilde{\rho}_{\mathrm{GWs}} \neq \rho_{\mathrm{GWs}}$

The gauge issue

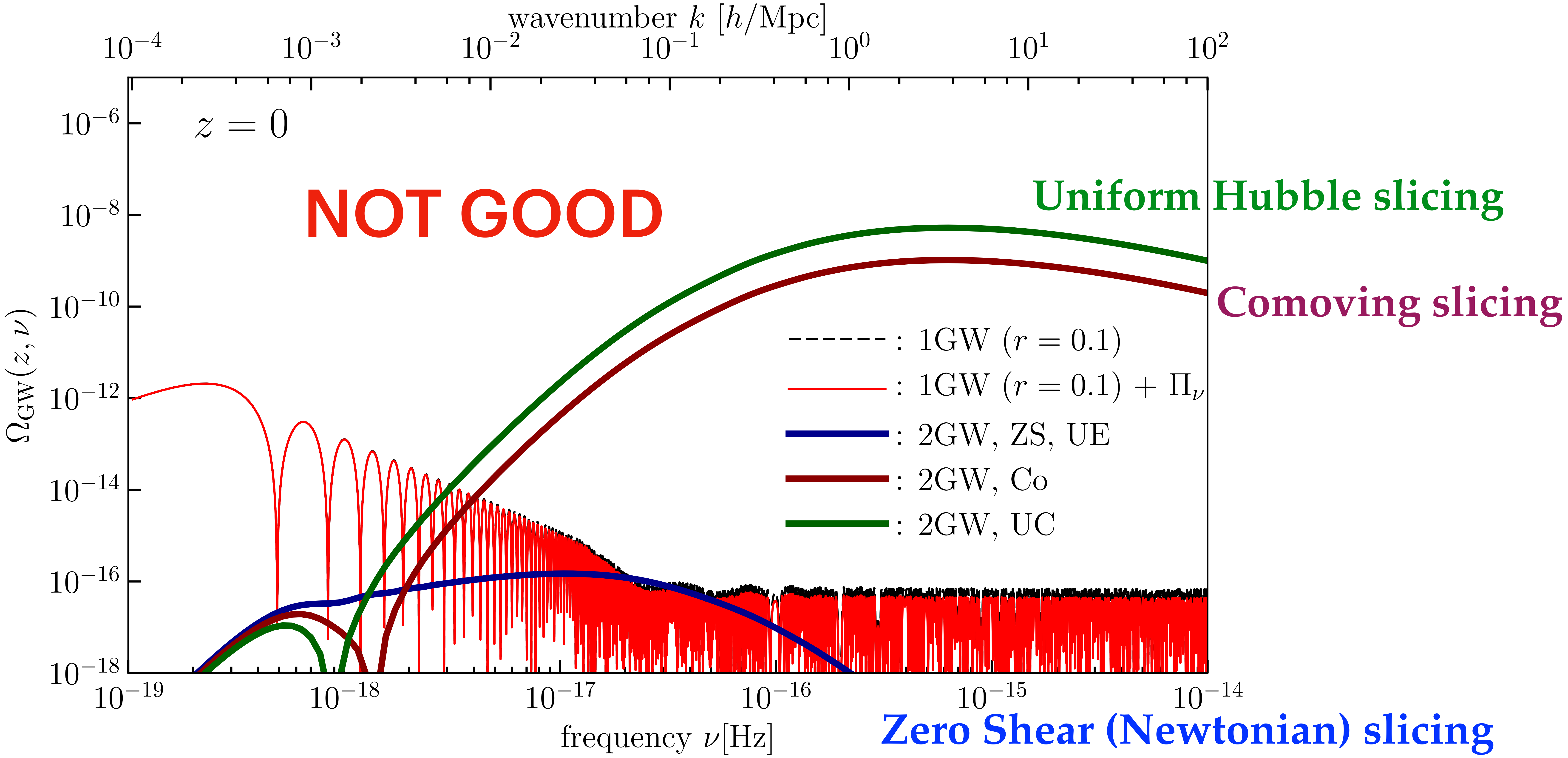
SIGWs Review:
GD, 2109.01398

Timeline: 2015, First GW detection —> PBH revival —> I got interested in SIGWs

2017:???

Hwang, Jeong & Noh
1704.03500
(in matter domination)

Also see Gong 1909.12708,
Tomikawa & Kobayashi: 1910.01880
And many more…



What IS the "gauge" issue?

The issue: (Observable) GWs = TT part in (which) spatial hypersurface?

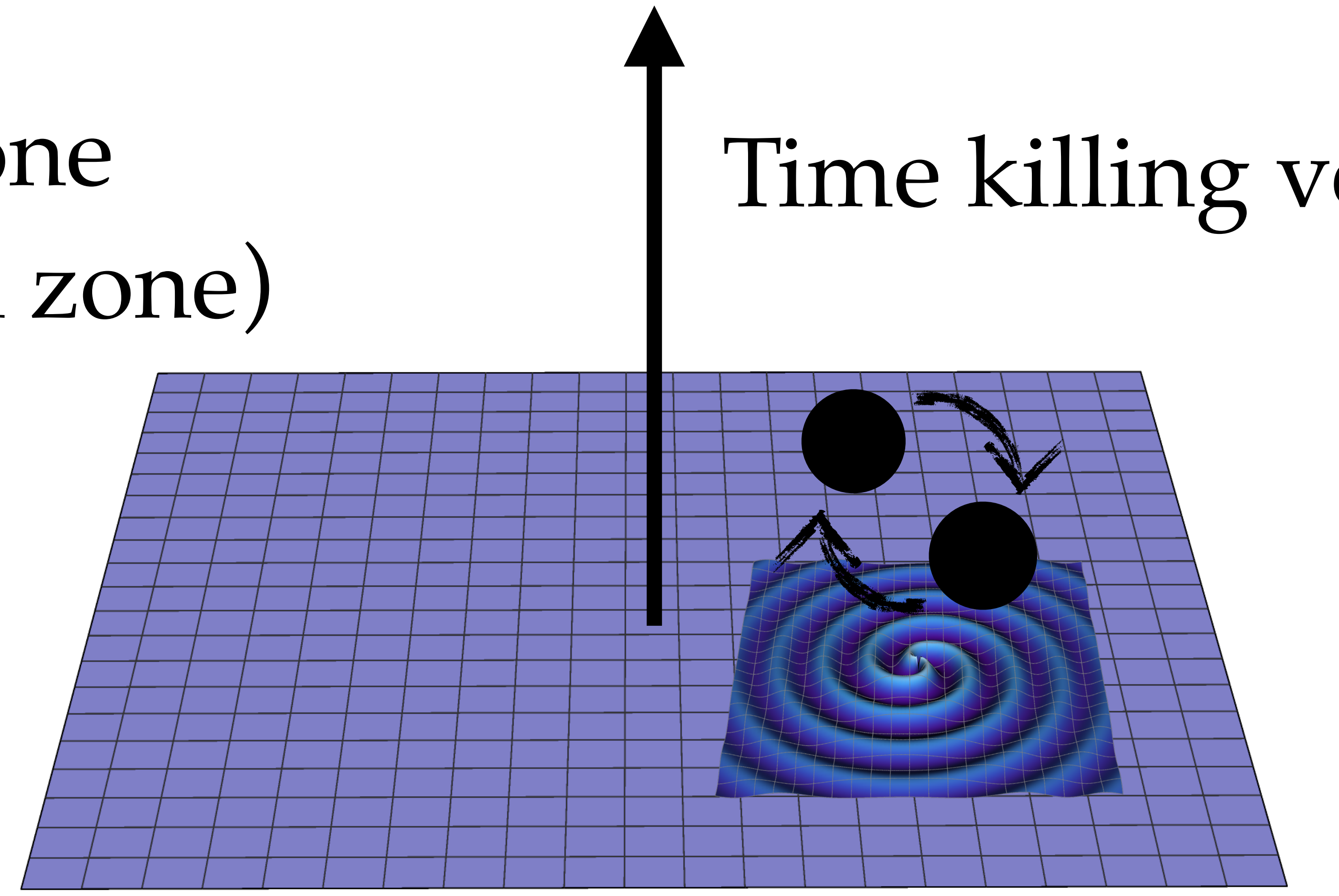
Flat spacetimes

Time direction

Far-zone
(radiation zone)

Time killing vector

Discrete
sources



Spatial hypersurface

Cosmological spacetimes

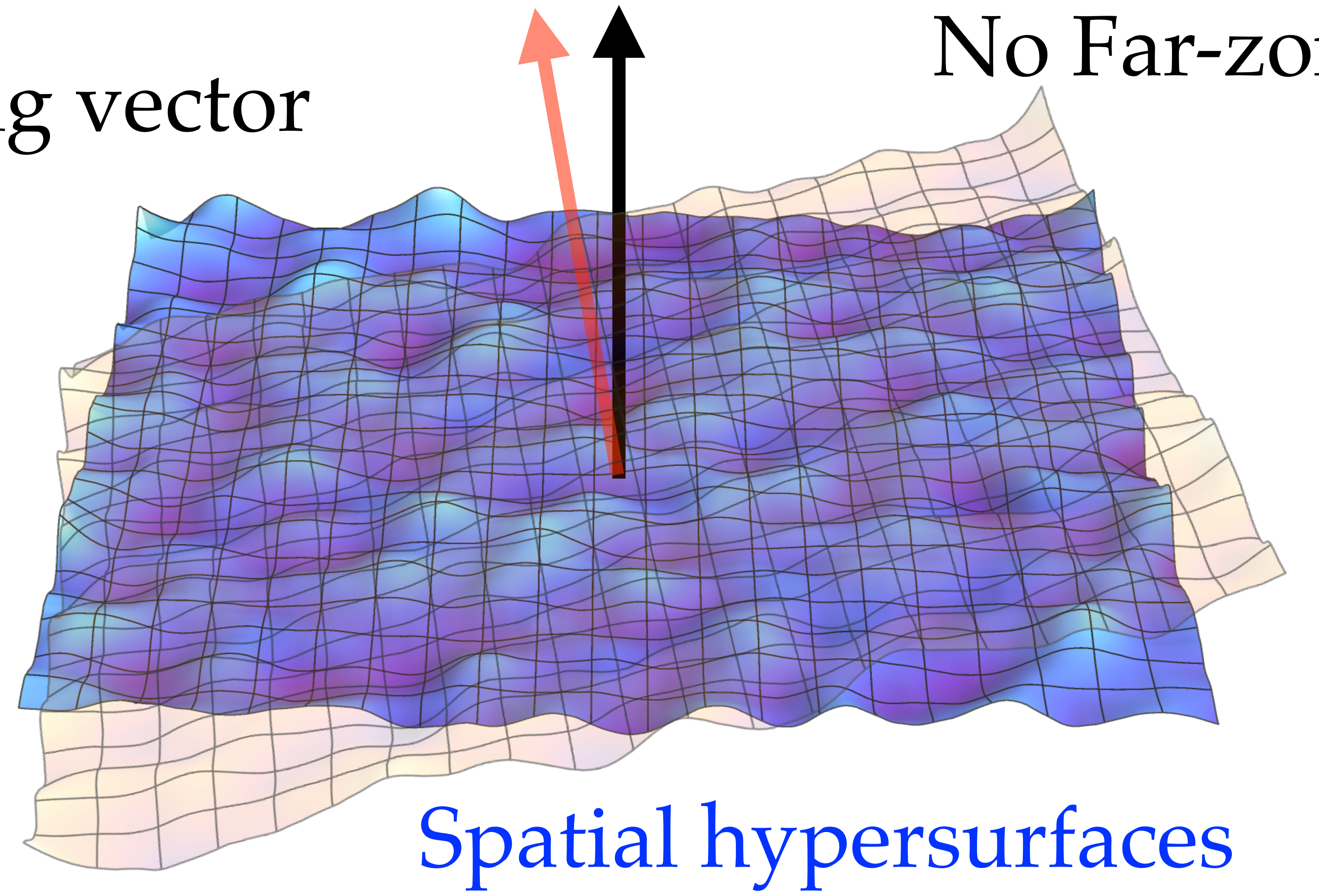
(intuition: Very short distances ~ Flat spacetime)

Time directions

No time killing vector

No Far-zone

Universe filled
with fluctuations



$$\tilde{t} = t + T(t, x)$$

Spatial hypersurfaces

GWs & density fluctuations non-linearly intertwined

$$\tilde{h}_{ij} = h_{ij} + \left(\partial_i T \partial_j T\right)^{TT}$$

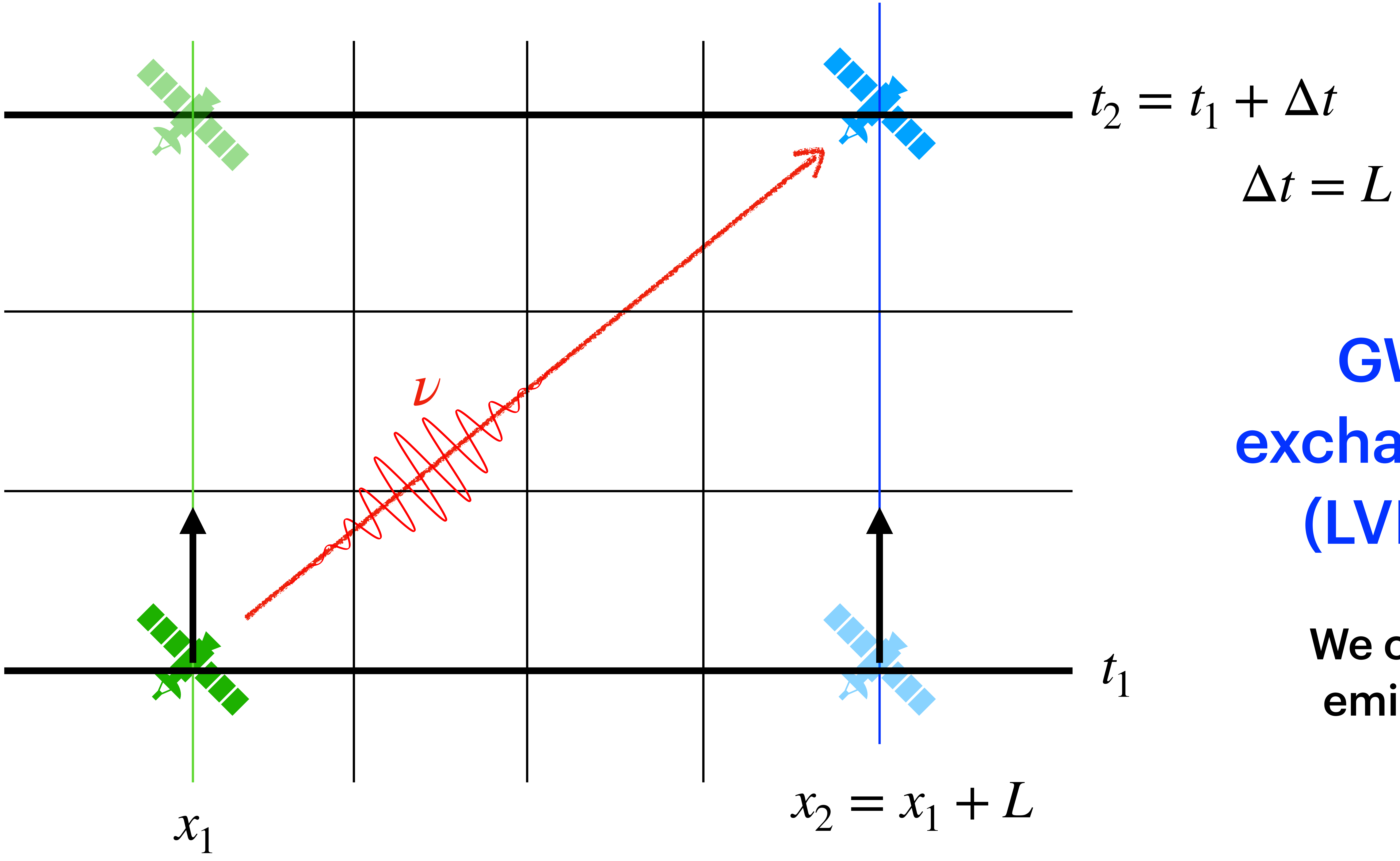
Our resolution:
compute observable GW strain at second order

GD, Pi & Wang, 2512.15704 (PRL, Editor's suggestion)

Ideal experiment

Flat Spacetime,
no GWs

$$ds^2 = -\,dt^2 + dx^2$$



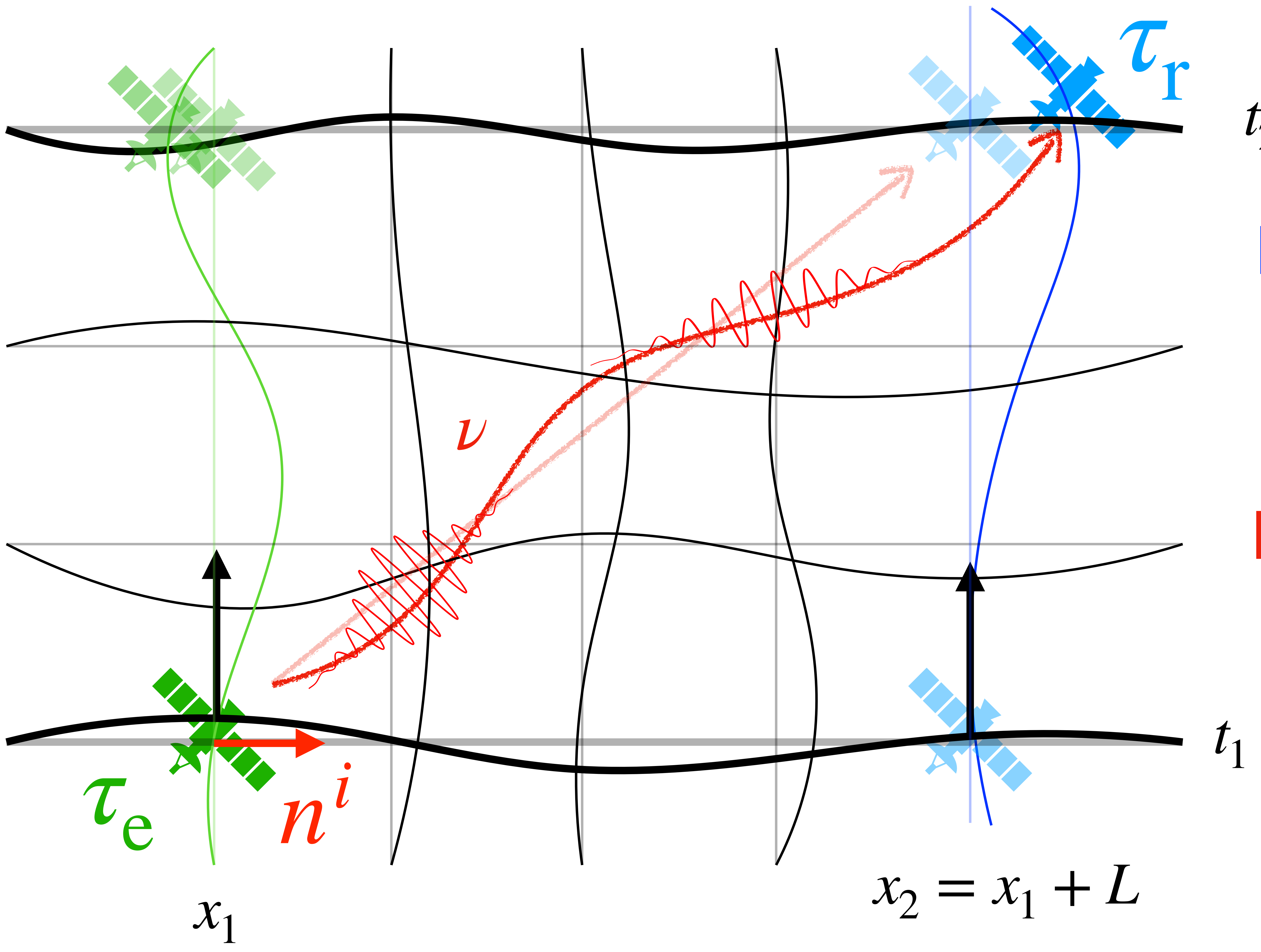
GW detectors
exchange EM signals
(LVK, PTAs, etc.)

We consider geodesic
emitter and detector

Ideal experiment

Flat Spacetime,
+ GWs

$$ds^2 = -dt^2 + dx^2 + h_{\mu\nu}dx^\mu dx^\nu$$



Proper Time Delay:

$$\Delta\tau_r = \Delta t = L + \frac{1}{2}\int_0^L d\lambda\, h_{ij}n^i n^j$$

Redshift:

$$\Delta\tau_r/\Delta\tau_e \to d\tau_{rf}(\tau_{ei})/d\tau_{ei} = 1 + z$$

$$\frac{\Delta\nu}{\nu} = \frac{\delta z}{1+z} = \frac{1}{2}\int_0^L d\lambda\, \dot{h}_{ij}n^i n^j$$

Synchronized geodesic clock: initially on constant proper time hypersurface

Cosmological Spacetime + Fluctuations

Take a general perturbed FLRW metric:

$$Y_{ij} = h_{ij} + 2\partial_i\partial_j E$$

$$ds^2 = a^2\left(-e^{2\phi}d\eta^2 + e^{2\psi}\left(e^Y\right)_{ij}(dx^i + \beta^i d\eta)(dx^j + \beta^j d\eta)\right)$$
$$= a^2\left(-(1+2\phi)d\eta^2 + 2\beta_i d\eta dx^i + (\delta_{ij} + 2\psi\delta_{ij} + 2\partial_i\partial_j E + h_{ij})dx^i dx^j\right) + \ldots$$

First order (proper) time delay: $\Delta\tau = \Delta\tau^{(0)} + \Delta\tau^{(1)} + \Delta\tau^{(2)} + \ldots$

First order redshift:

Cosmological Spacetime + Fluctuations

Take a general perturbed FLRW metric: $Y_{ij} = h_{ij} + 2\partial_i\partial_j E$

$$ds^2 = a^2\left(-e^{2\phi}d\eta^2 + e^{2\psi}\left(e^{Y}\right)_{ij}(dx^i + \beta^i d\eta)(dx^j + \beta^j d\eta)\right)$$
$$= a^2\left(-(1+2\phi)d\eta^2 + 2\beta_i d\eta dx^i + (\delta_{ij} + 2\psi\delta_{ij} + 2\partial_i\partial_j E + h_{ij})dx^i dx^j\right) + \ldots$$

First order (proper) time delay: $\Delta\tau = \Delta\tau^{(0)} + \Delta\tau^{(1)} + \Delta\tau^{(2)} + \ldots$

$$\Delta\tau^{(1)} = a_{\rm rf}\int_0^L \mathrm{d}\lambda\left(\underbrace{\frac{1}{2}n^i n^j h^{(1)}_{ij}}_{\text{GWs}} \underbrace{- \Phi^{(1)} - \Psi^{(1)}}_{\text{Shapiro time delay}}\right) + \underbrace{a_{\rm rf}\int_0^L d\eta\, n^i V^{(1)}_{{\rm r},i} + \int_0^L d\eta\, a\Phi^{(1)}}_{\text{Detector motion}}$$

First order redshift:

Cosmological Spacetime + Fluctuations

Take a general perturbed FLRW metric: $Y_{ij} = h_{ij} + 2\partial_i\partial_j E$

$$ds^2 = a^2\left(-e^{2\phi}d\eta^2 + e^{2\psi}\left(e^Y\right)_{ij}(dx^i + \beta^i d\eta)(dx^j + \beta^j d\eta)\right)$$
$$= a^2\left(-(1+2\phi)d\eta^2 + 2\beta_i d\eta dx^i + (\delta_{ij} + 2\psi\delta_{ij} + 2\partial_i\partial_j E + h_{ij})dx^i dx^j\right) + \ldots$$

First order (proper) time delay: $\Delta\tau = \Delta\tau^{(0)} + \Delta\tau^{(1)} + \Delta\tau^{(2)} + \ldots$

$$\Delta\tau^{(1)} = a_{\rm rf}\int_0^L \mathrm{d}\lambda\left(\frac{1}{2}n^i n^j h_{ij}^{(1)} - \Phi^{(1)} - \Psi^{(1)}\right) + a_{\rm rf}\int_0^L d\eta\, n^i V_{\mathrm{r},i}^{(1)} + \int_0^L d\eta\, a\Phi^{(1)}$$

GWs — Shapiro time delay — Detector motion

First order redshift:

$$\frac{z^{(1)}}{1+\tilde{z}} = \Phi_{\rm rf}^{(1)} - \Phi_{\rm ei}^{(1)} - \int_0^L \mathrm{d}\lambda\left(\Phi^{(1)'} + \Psi^{(1)'}\right) + n^i(V_{\mathrm{r},i}^{(1)} - V_{\mathrm{e},i}^{(1)}) + \int_0^L \mathrm{d}\lambda\, n^i n^j h_{ij}^{(1)'}$$

Sachs-Wolfe — Int. Sachs-Wolfe — Dipole — GWs

Observable GW strain at second order

Spacetime shear:

$$\sigma = \beta - E'$$

Second order (proper) time delay:

$$\Delta\tau^{(2)}_{\rm GWs} = a_{\rm rf}\, n^i n^j \frac{1}{2}\int_0^L {\rm d}\lambda \left(\boxed{h^{(2)}_{ij} + P^{lk}_{ij}\left(\sigma^{(1)}_l \sigma^{(1)}_k + E^{(1)}_{,lm}E^{(1)}_{,mk}\right)}\right)$$

Gauge invariant combination!

TT-component in Newton gauge:

(e.g., GD, Sasaki, 1709.09804)

$$h^{N(2)}_{ij} \equiv h^{(2)}_{ij} + P^{lk}_{ij}\left(\sigma^{(1)}_l \sigma^{(1)}_k + E^{(1)}_{,lm}E^{(1)}_{,mk}\right)$$

This is the observable quantity!

Second order redshift:

$$\frac{z^{(2)}_{\rm GW}}{1+z} = \frac{1}{2} n^i n^j \int_0^L {\rm d}\lambda\, h^{N(2)'}_{ij}$$

A question for future work

What about ambiguities in the energy density of GWs???

$$\rho_{\mathrm{GWs}} = \frac{1}{32\pi G}\frac{1}{a^2}\langle h'_{ij}h'_{ij}\rangle$$

Not important for GW detectors but important for BBN and CMB bounds

Scalar-tensor induced GWs?

GD, 2510.26883

Bari, Bartolo, GD & Matarrese, 2307.05404

and preliminary results (BBGM + Tasinato)

Scalar-tensor induced GWs

Metric:

$$ds^2 = -a^2 e^{2\Phi} d\tau^2 + a^2 e^{-2\Psi} (e^h)_{ij} dx^i dx^j$$

The standard (scalar-scalar) induced GWs:

$$(\partial_\tau^2 + 2\mathscr{H}\partial_\tau - \Delta) h_{ij} \sim \widehat{TT}_{ij}^{\ ab} (\partial_a \Phi \partial_b \Phi)$$

Scalar-tensor induced GWs:

$$h''_{ij} + 2\mathcal{H} h'_{ij} - \Delta h_{ij} = 4\Phi' h'_{ij} + 4\Phi\, \Delta h_{ij}$$

But very different physical interpretation, and STIGWs are weird

The “divergence” issue

Treat as source, solve with Green’s function

v: vector momentum
u: scalar momentum

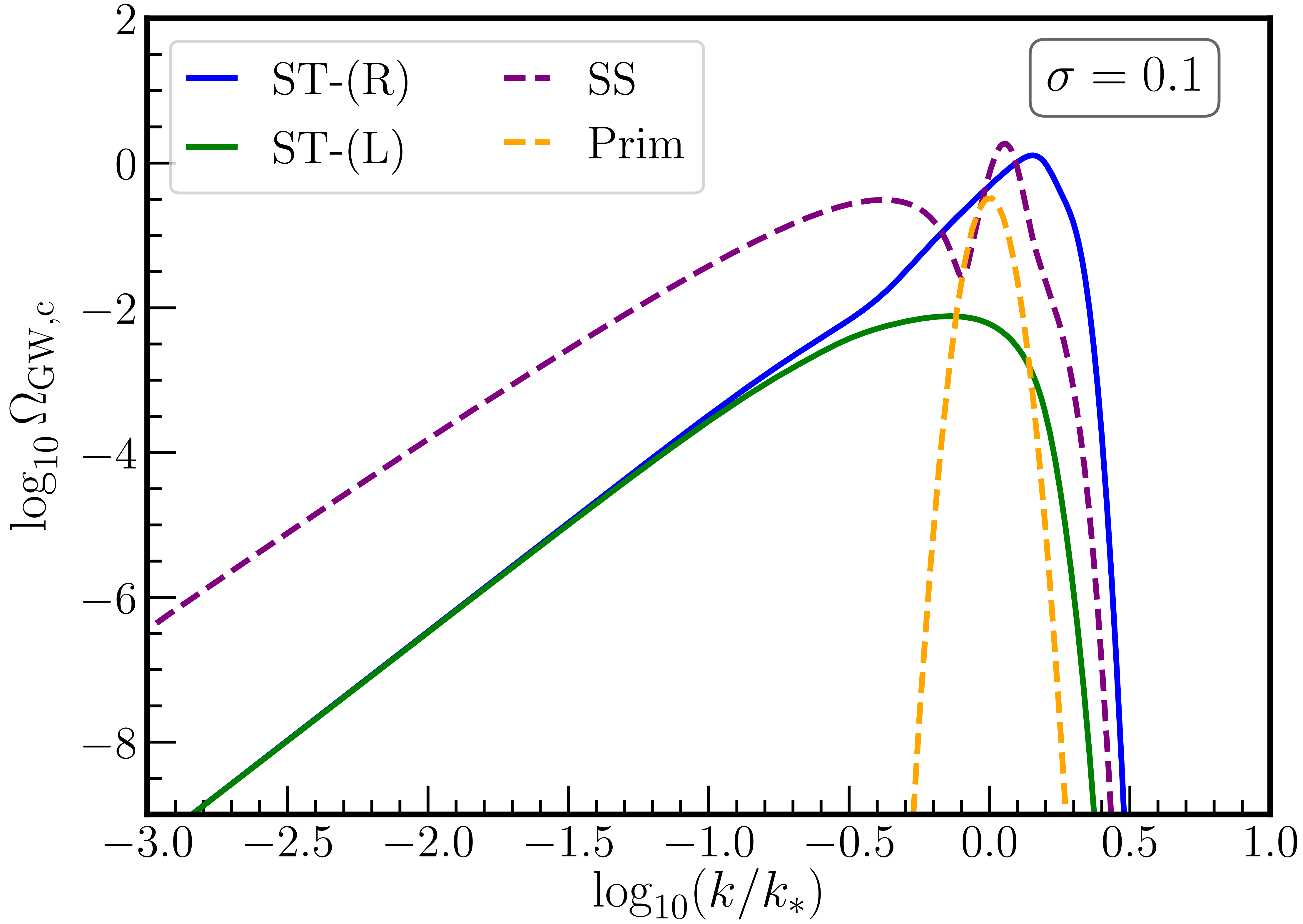
$$\Delta^2_{\gamma_1,\lambda}(k) = \frac{1}{16}\int_0^\infty dv \int_{|1-v|}^{1+v} du \, \frac{\Delta^2_\Phi(uk)\Delta^2_{\gamma_0,\lambda}(vk)}{v^6u^2} \times \overline{\mathcal{I}^2(u,v)}\left[\left((v+1)^2-u^2\right)^4 + \left((v-1)^2-u^2\right)^4\right],$$

But the kernel is divergent for long-wavelength scalar modes

$$\overline{\mathcal{I}^2(u,v)} \sim \frac{\pi^2}{u^4}, \qquad u \to 0, \qquad v \to 1.$$

The “divergence” issue

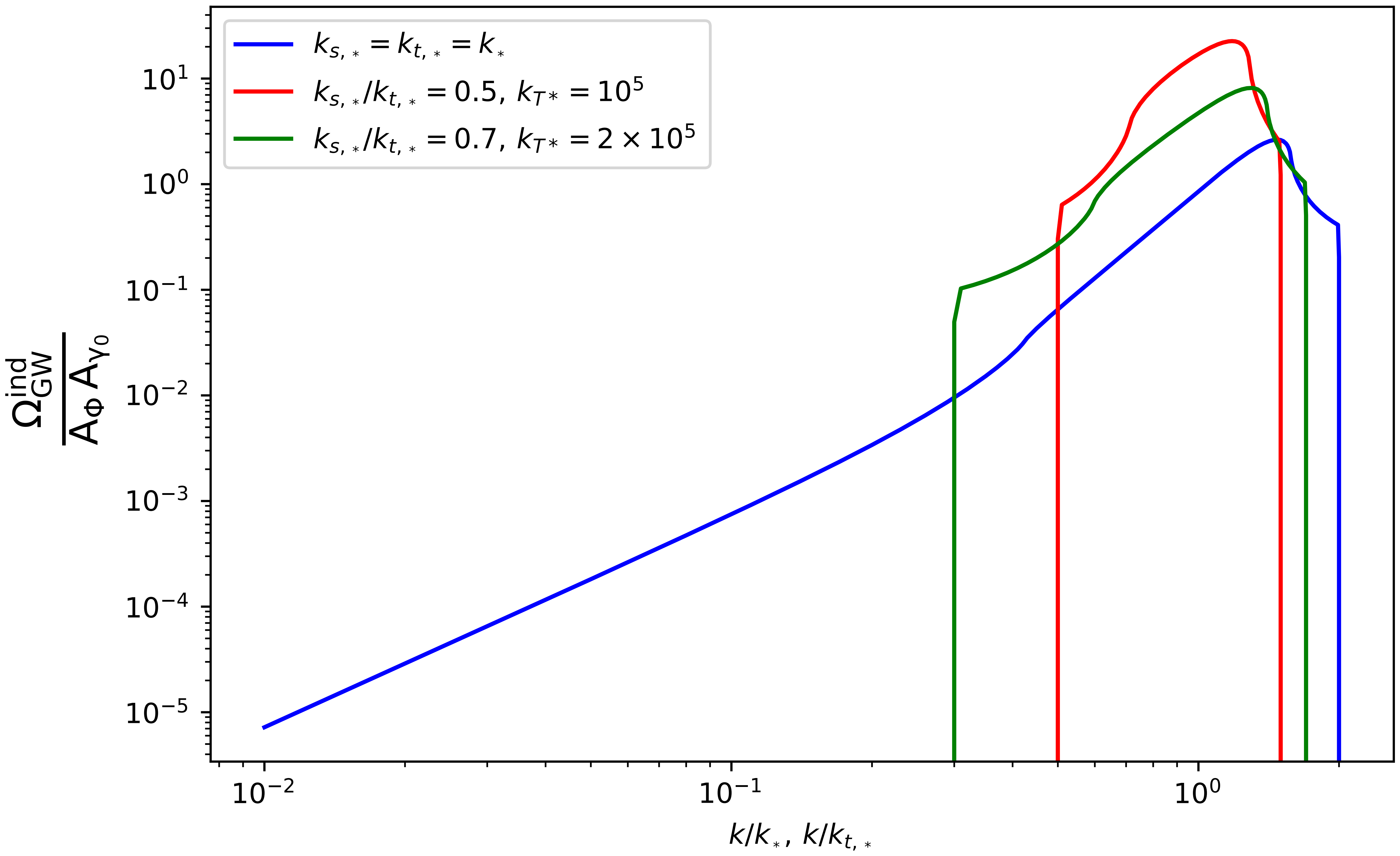
All fine for peaked scalar and tensor spectrum (same peak location)



Note the Universal unpolarized IR tail (paper in preparation)

The "divergence" issue

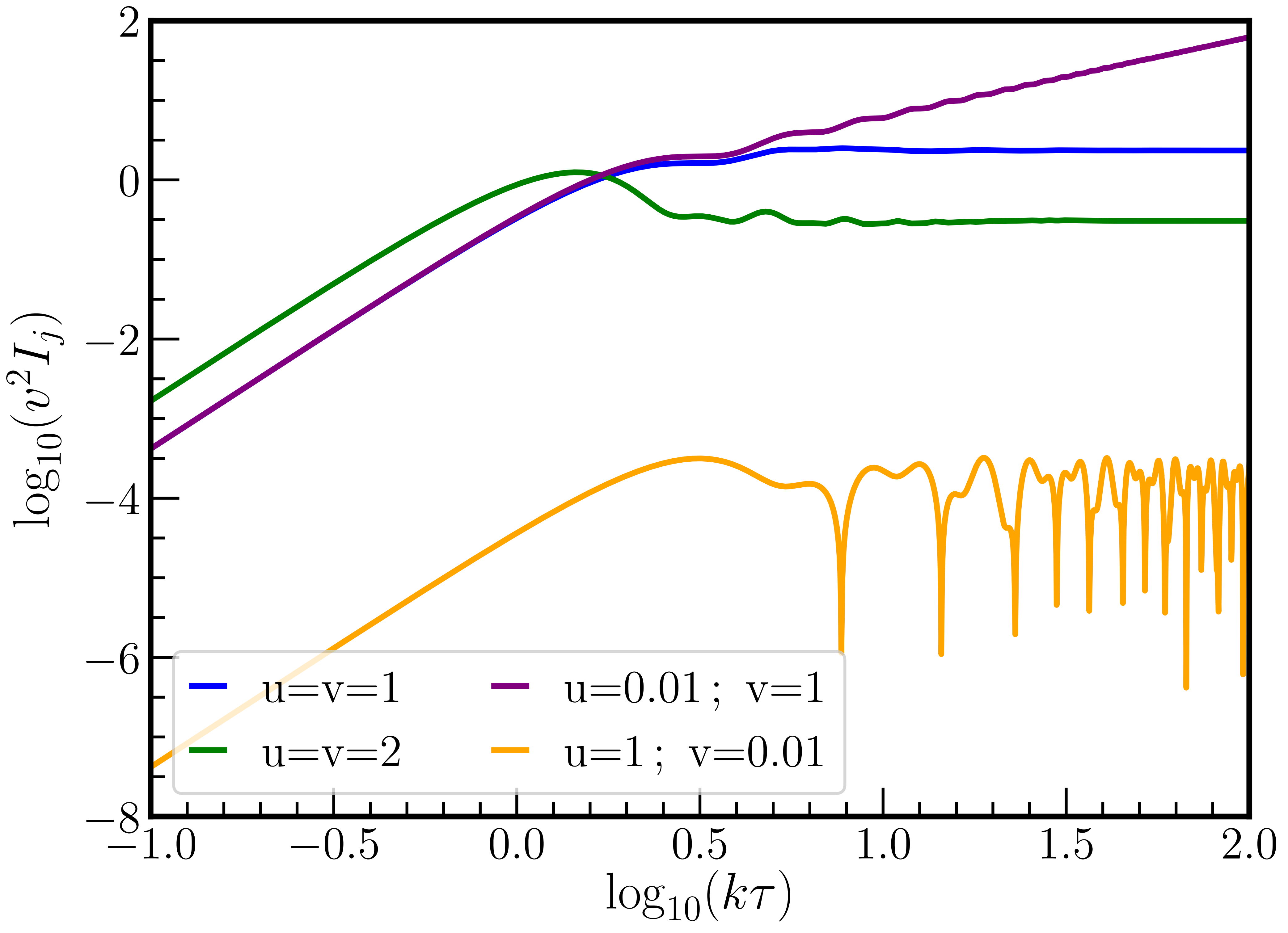
Strange for separate peak positions



The origin of the "divergence"

The long-wave length ("constant") scalar acts as a constant source

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = 4\Phi' h'_{ij} + 4\Phi\,\Delta h_{ij}$$



The kernel just grows

Apparently not a gauge issue

Suppression of the induced gravitational wave background due to third-order perturbations

Raphaël Picard,[a] Luis E. Padilla,[a] Karim A. Malik,[b] and David J. Mulryne[a]

Abstract. In this work, we revisit and evaluate new source terms which contribute to the induced gravitational wave background. We study their respective contributions to the stochastic gravitational wave background by computing their spectral densities in a radiation-dominated universe. These terms appear at third order in cosmological perturbation theory, however, their correlations with primordial gravitational waves are non-trivial and appear at the same order as so-called scalar induced and scalar-tensor induced gravitational waves. We find that these gravitational wave sources suppress the spectral density at the scales we consider. Furthermore, similarly to scalar-tensor source terms at second order, we find that some terms are enhanced when the input primordial power spectrum of scalar fluctuations is not sufficiently peaked. Hence, where possible, we show that under certain limits the integrands of these terms diverge in the UV sector.

We also agree...

Look at the action

The fully non-linear action is

$$S = \int d^3x d\tau \frac{a^2}{8} \left(e^{-4\Phi} h'_{ij} h'_{ij} + h_{ij} \Delta h_{ij} \right) + \dots ,$$

This is a wave in a random, spacetime dependent, refractive index.

Time-independent refractive index —> Preserves power

Also, in geometric optic limit:

$$h_{ij} = \mathcal{A}_{ij} \, e^{i\theta}, \qquad k_\mu = \nabla_\mu \theta,$$

$$e^{-4\Phi} \omega^2 - |\boldsymbol{k}|^2 = 0, \qquad \omega = -\nabla_\tau \theta. \qquad \omega = e^{2\Phi} |\boldsymbol{k}| \simeq |\boldsymbol{k}| (1 + 2\Phi).$$

Modulation of frequency, not amplitude.

Exact solution in Matter Domination

In MD, in Fourier space:

$$h''_{s,\boldsymbol{k}} + 2\mathcal{H}h'_{s,\boldsymbol{k}} + k^2 h_{s,\boldsymbol{k}} = -4\sum_r \int \frac{d^3q}{(2\pi)^3} \epsilon^{s*}_{ij}(\hat{\boldsymbol{k}})\epsilon^r_{ij}(\hat{\boldsymbol{q}}) q^2 h_{r,\boldsymbol{q}} \Phi_{\boldsymbol{k}-\boldsymbol{q}}.$$

Exact solution:

$$h^{(1)}_{s,\boldsymbol{k}} = 4\sum_r \int \frac{d^3q}{(2\pi)^3} \epsilon^{s*}_{ij}(\hat{\boldsymbol{k}})\epsilon^r_{ij}(\hat{\boldsymbol{q}}) \frac{q^2}{k^2 - q^2} h^p_{r,\boldsymbol{q}} \Phi_{\boldsymbol{k}-\boldsymbol{q}} \times \left[T_h(q\tau) - T_h(k\tau)\right] + \cdots .$$

Strange frequency of oscillations

Exact solution in Matter Domination

q~k limit:

$$h^{(1)}_{s,\boldsymbol{k}} \simeq -2k\tau\, T_h'(k\tau) h^p_{s,\boldsymbol{k}} \int_{\text{soft}} \frac{d^3p}{(2\pi)^3} \Phi_{\boldsymbol{p}},$$

which looks like

$$T_h[k(1-\delta_Z)\tau] = T_h(k\tau) - \delta_Z\, k\tau\, T_h'(k\tau) + \cdots, \qquad \delta_Z = 2\int_{\text{soft}} \frac{d^3p}{(2\pi)^3} \Phi_{\boldsymbol{p}}.$$

Again, a change of frequency (not in geometric optics now)

We don’t know how to solve it yet, but one can absorb the “divergence” in a redefinition of frequency for a scale invariant spectra

Namely, you remove all 1/u powers and get a finite reasonable number

But there must be a way to do better

Connection to gas of gravitons

GD, 2510.26883

Local distribution function:

$$dN = f(x^\mu, P_I) dx^1 dx^2 dx^3 dP_{A_1} dP_{A_2} dP_{A_3} \,,$$

Local vs Global momenta in EMT:

$$T^0{}_0 = -a^{-4} \int d\Omega \, dQ^{A_0} \, (Q^{A_0})^3 \, (f(Q^{A_0}) + \delta f(x^\alpha, Q^{A_0}, \hat{P}^{A_i})) \,,$$

Local

$$\eta^{AB} P_A P_B = -P^2_{A_0} + \delta^{A_i A_j} P_{A_i} P_{A_j} = 0 \,.$$

$$T^0{}_0 = -\frac{1}{a^4} e^{-2\Phi - 4\Psi} \int d^3p |\vec{p}| f(x^\sigma, p_\beta) \,.$$

Global

$$p^0 = e^{-(\Phi+\Psi)} |\vec{p}|$$

Connection to gas of gravitons

Non-adiabatic initial conditions?

$$\rho = a^{-4} \int d\Omega\, dQ^{A_0}\, (Q^{A_0})^3\, f(Q^{A_0}) \quad \text{and} \quad \delta\rho = a^{-4} \int d\Omega\, dQ^{A_0}\, (Q^{A_0})^3\, \delta f(x^\alpha, Q^{A_0}, \hat{P}^{A_i})\,.$$

Local

$$\rho = \frac{1}{a^4} \int d^3p |\vec{p}| f(|\vec{p}|) \quad \text{and} \quad \delta\rho = -(2\Phi + 4\Psi)\rho + \frac{1}{a^4} \int d^3p |\vec{p}| \delta f(x^\sigma, p_\beta)\,.$$

?

Global

Yes. Redefinition of frequency:

$$p^0 = e^{-(\Phi+\Psi)} |\vec{p}|$$

Future work

Green function method not suitable for
propagation in inhomogeneous background

How to solve $h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = 4\Phi' h'_{ij} + 4\Phi\,\Delta h_{ij}$ in general?

Probably not possible, but perhaps we can say something about power spectrum

Any feature in the power spectrum of any cosmic GWB will
probe the small-scale power of Φ

Conclusions

Secondary GWs are a crucial test of the PBH scenario

But there are some issues it their theoretical foundations

Gauge issue of strain —> observable GW strain

Gauge issue of energy density —> ?

Scalar-tensor induced or not?

Thank you!

MITP, the phenomenology of PBHs
July 28th, 2026

GD, Pi & Wang, 2512.15704 (PRL) Bari, Bartolo, GD & Matarrese, 2307.05404 GD, 2510.26883