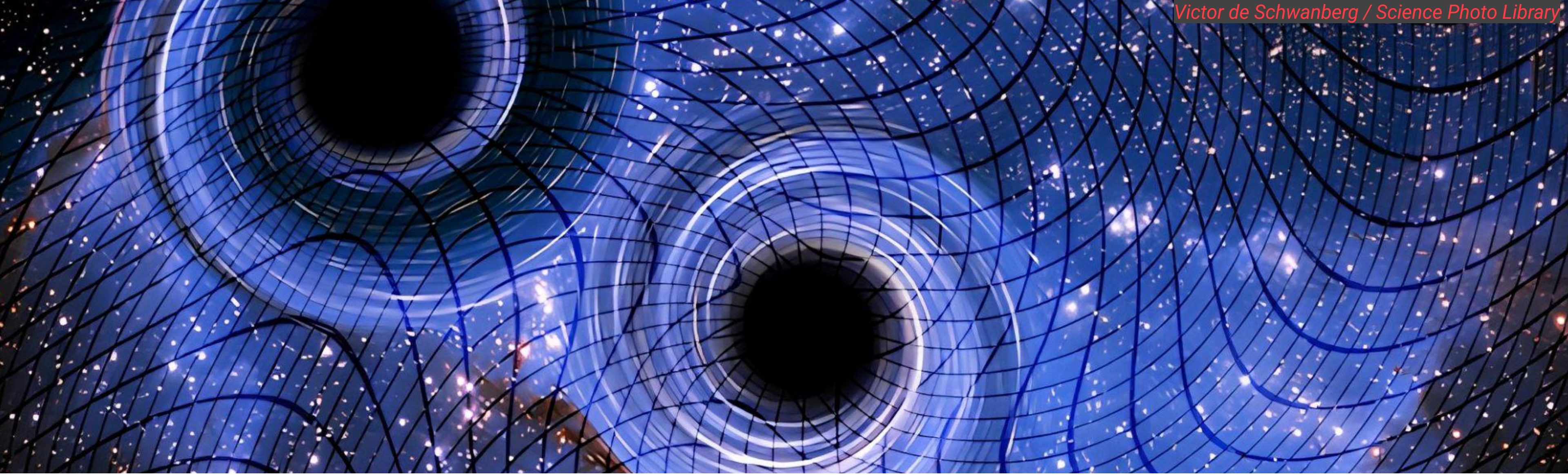

On formal aspects of black holes: from scalar test fields to moduli space

Quasinormal modes, Strong Cosmic Censorship, and moduli excursions in near-extremal throats

Anna Chrysostomou

PBH2026 @ MITP
23 July – Mainz

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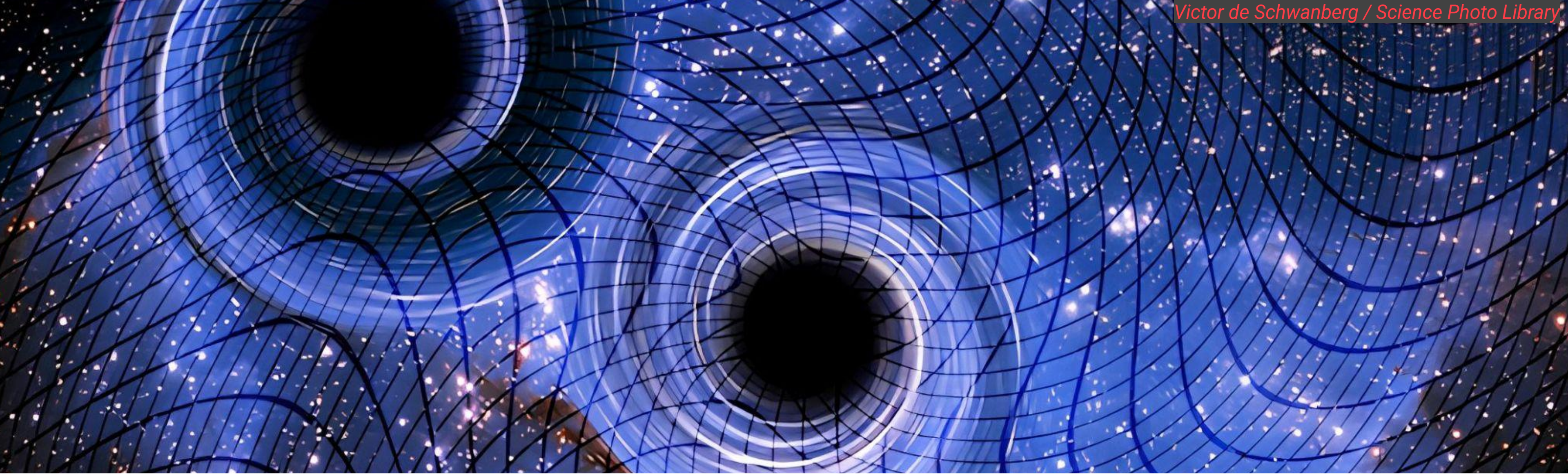
THEORIQUE ET HAUTES ENERGIES



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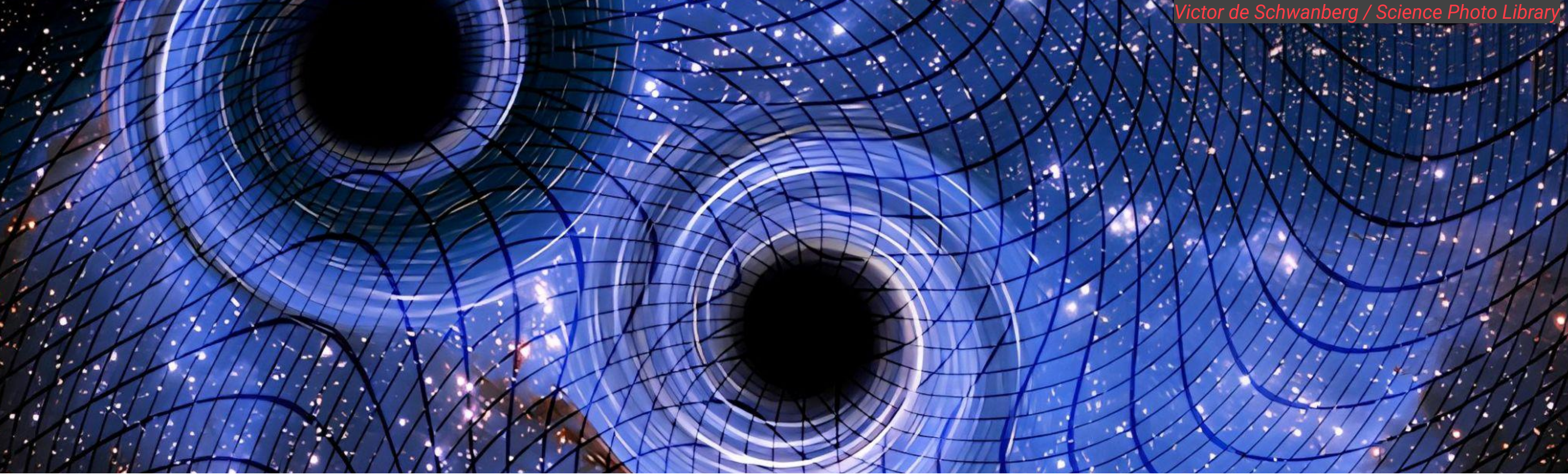

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exterior perturbations ⇢ interior determinism

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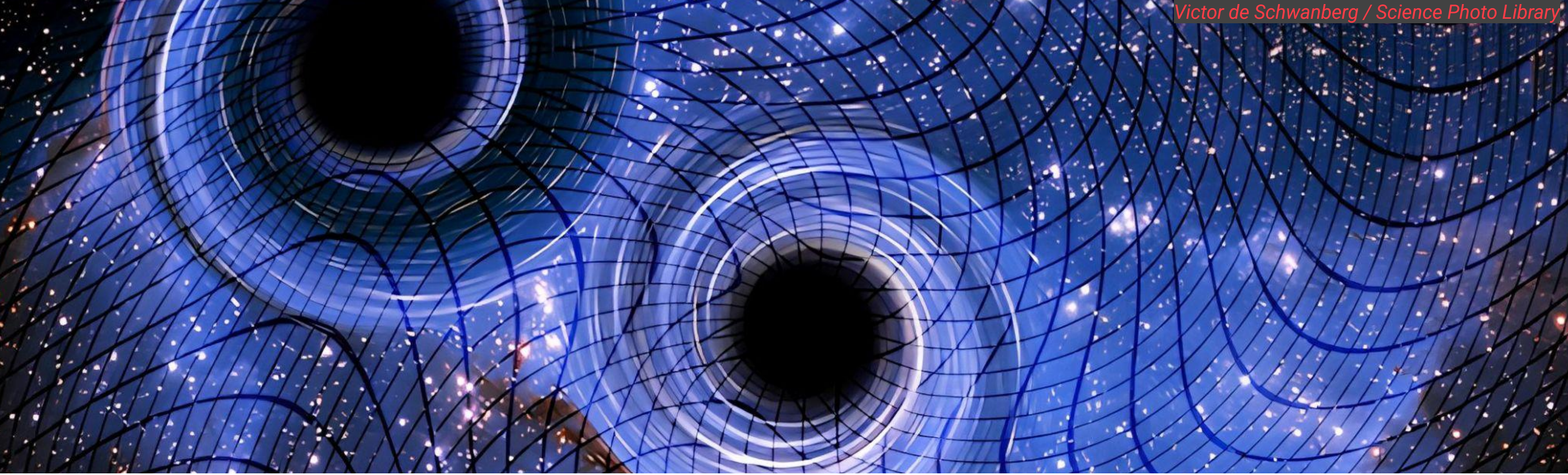

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exterior perturbations ⇢ interior determinism near-horizon effective theory

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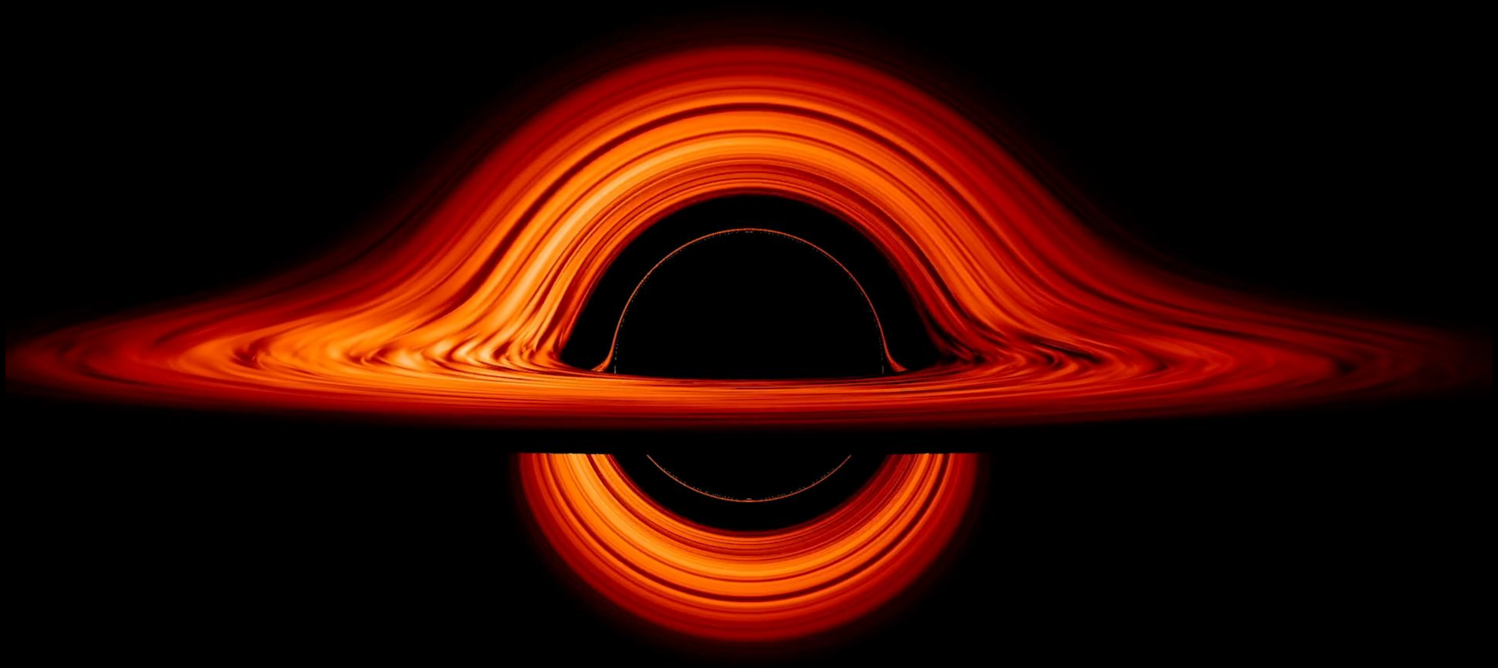
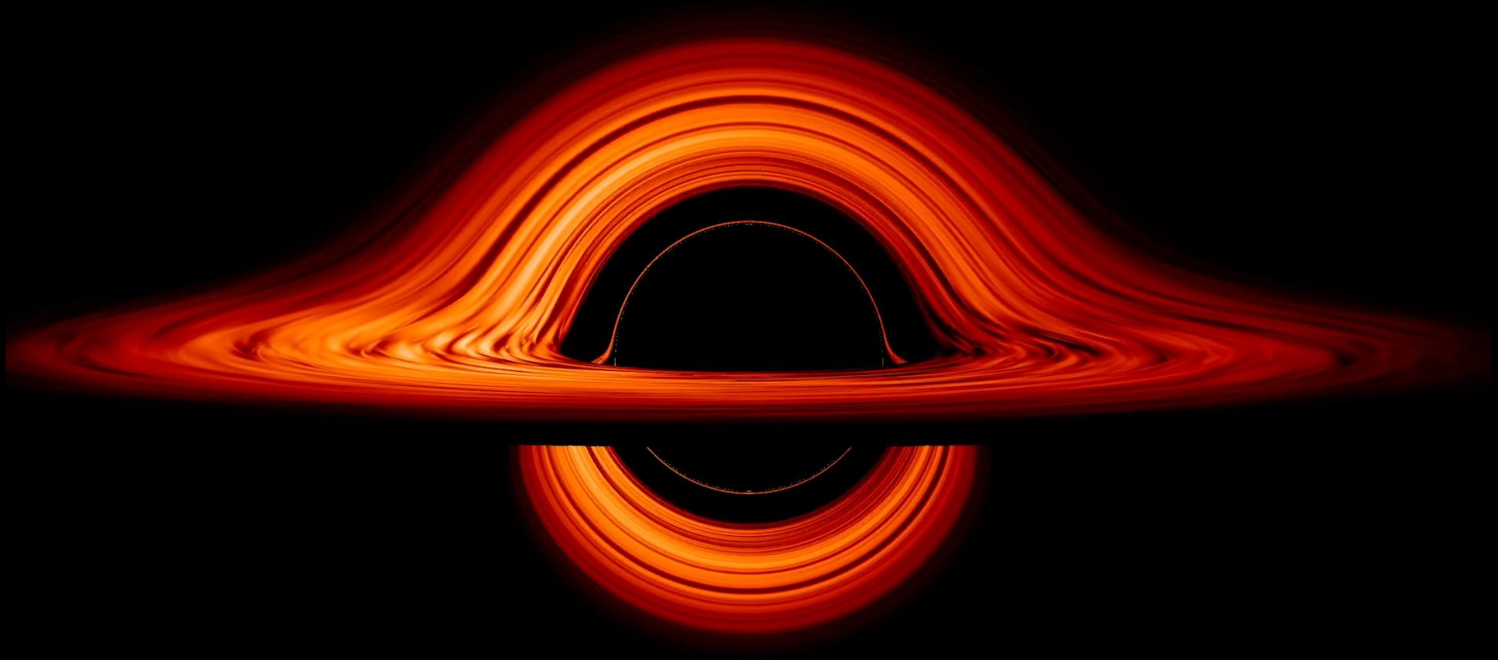
Credit: NASA's Goddard Space Flight Center/J. Schnittman & B. Powell



Credit: *NASA's Goddard Space Flight Center/J. Schnittman & B. Powell*

First, let us consider *astrophysical* black hole "anatomy"



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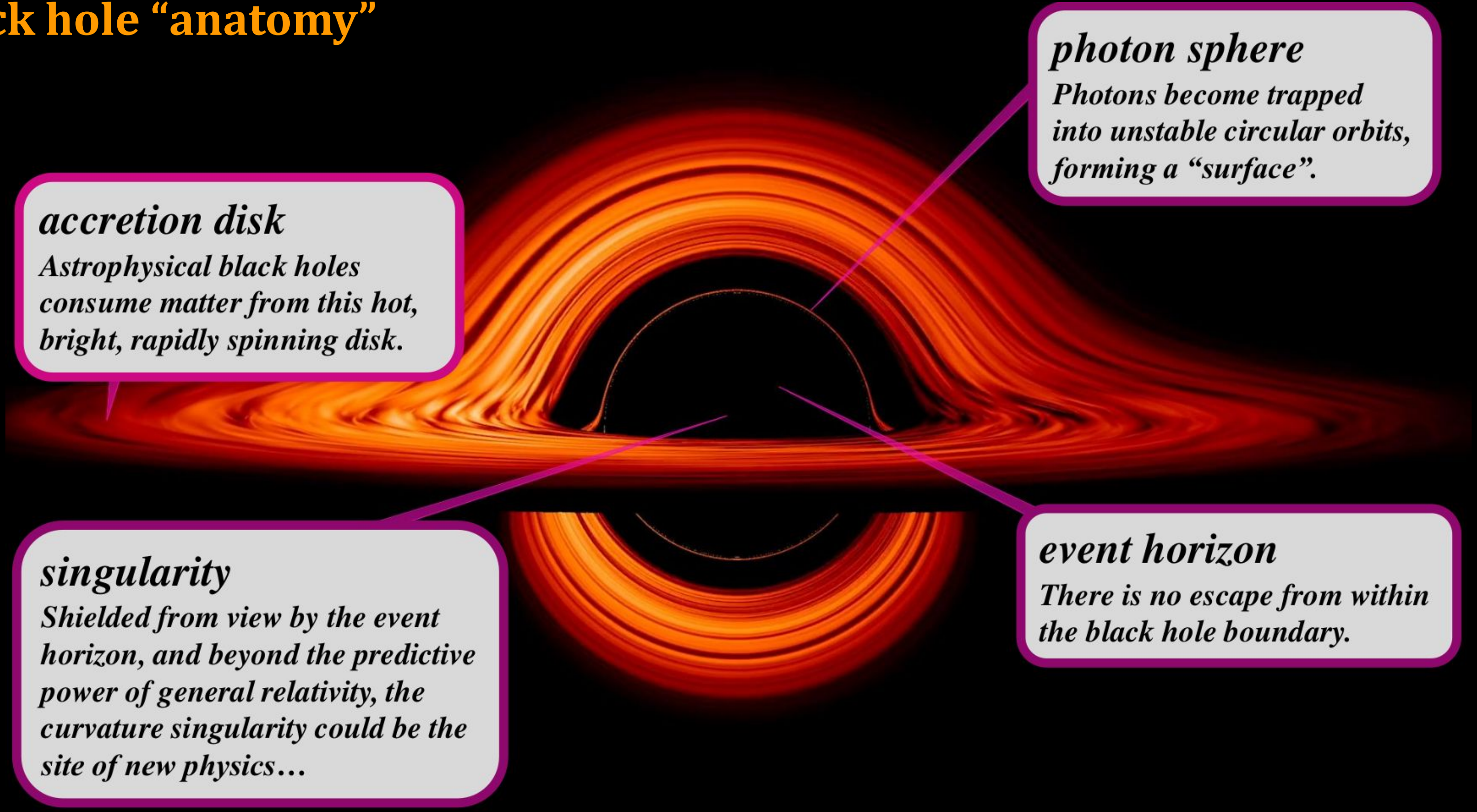
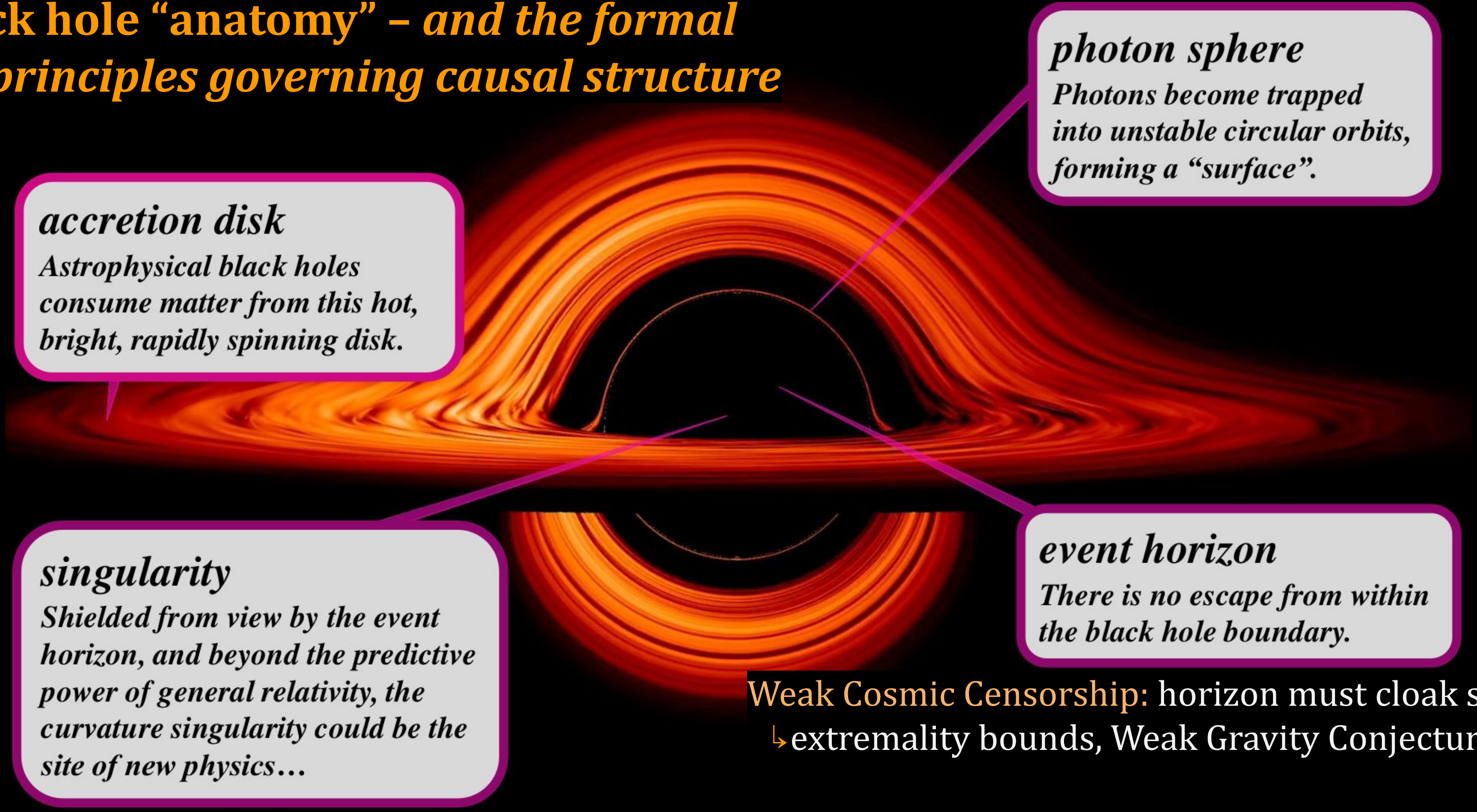


Credit: NASA's Goddard Space Flight Center/J. Schnittman & B. Powell

First, let us consider *astrophysical* black hole "anatomy" – *and the formal GR principles governing causal structure*

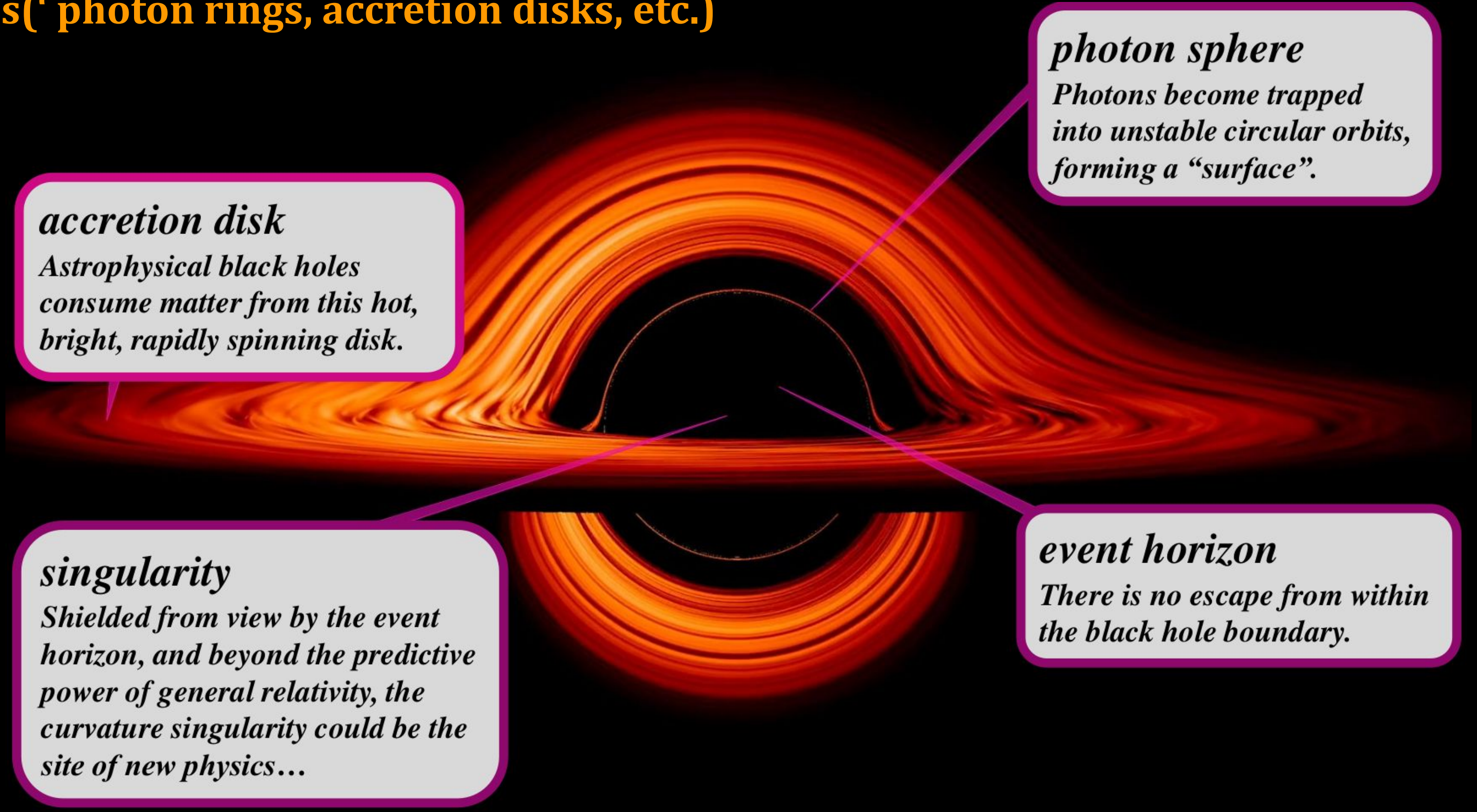


Weak Cosmic Censorship: horizon must cloak singularity
↳extremality bounds, Weak Gravity Conjecture, etc.

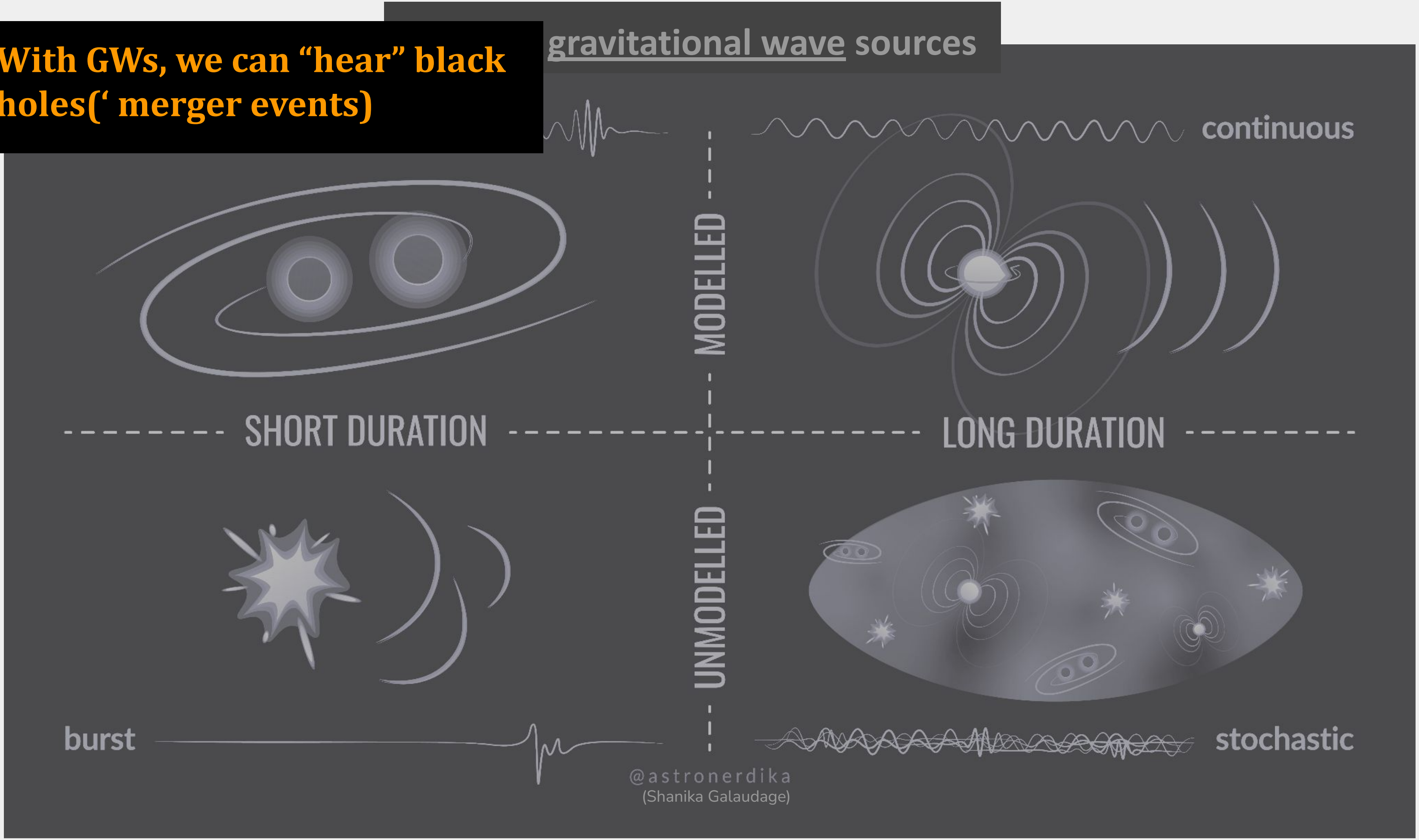
Strong Cosmic Censorship: generic initial data should determine a unique spacetime evolution
↳preserves determinism in GR; singularity is ~~coordinate artefact~~ a genuine inextendible boundary

Credit: NASA's Goddard Space Flight Center/J. Schnittman & B. Powell

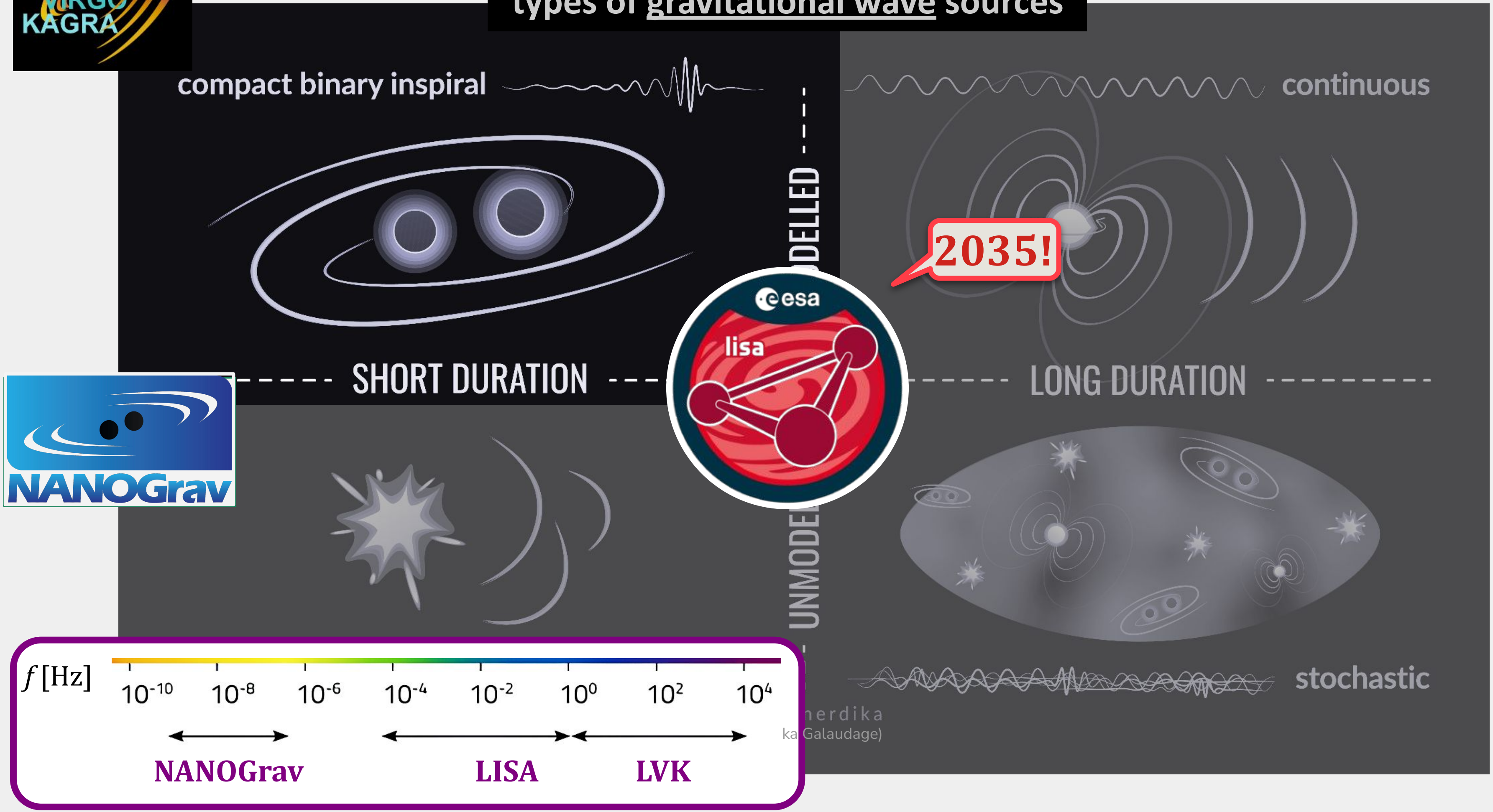
With EHT, we can "see" black holes(' photon rings, accretion disks, etc.)

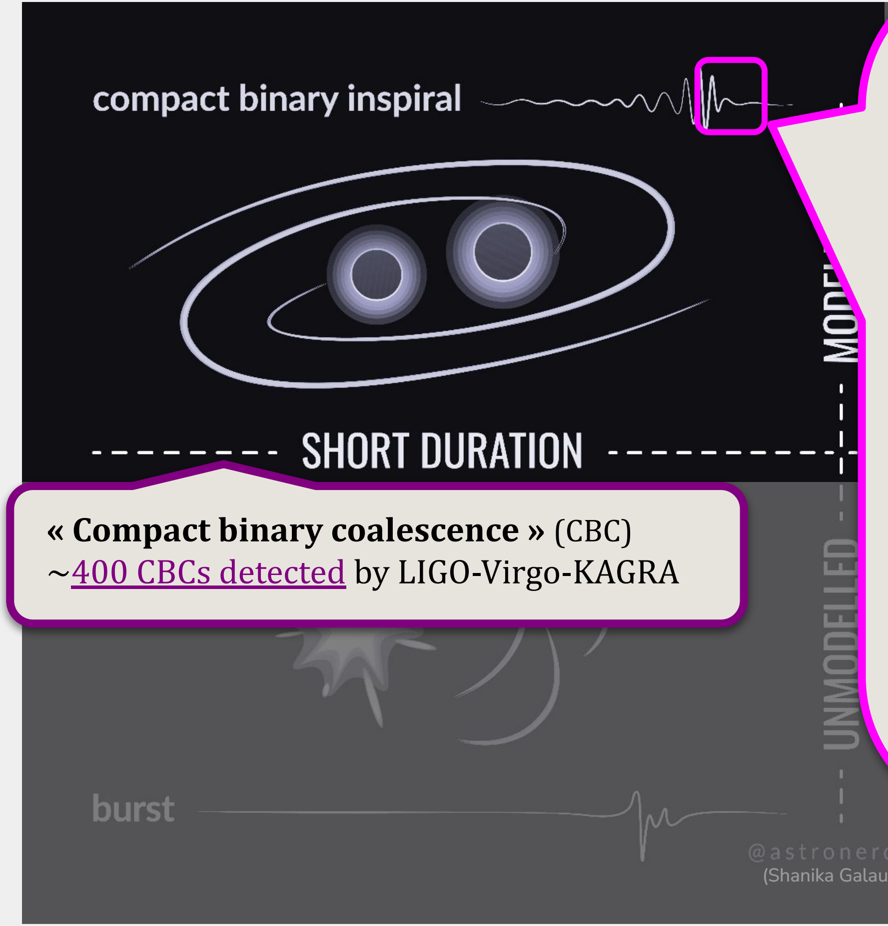


With GWs, we can “hear” black holes(‘ merger events)

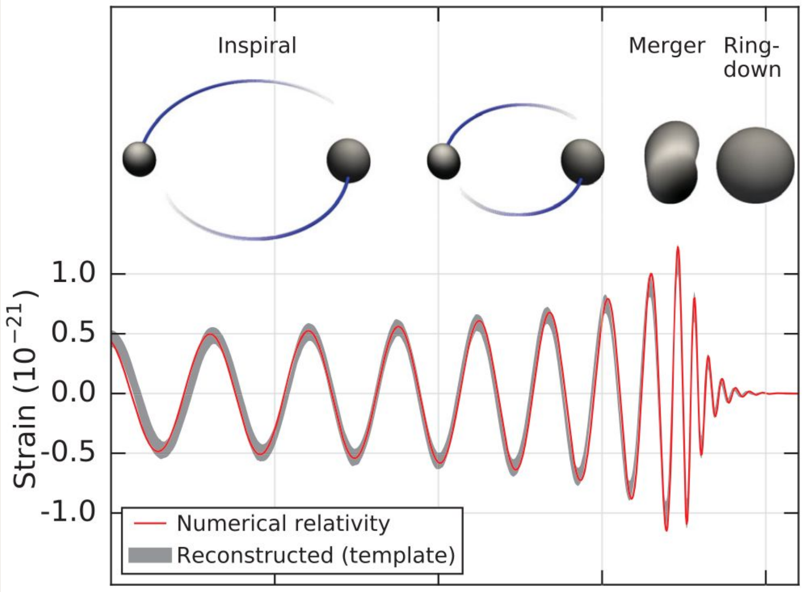


types of gravitational wave sources

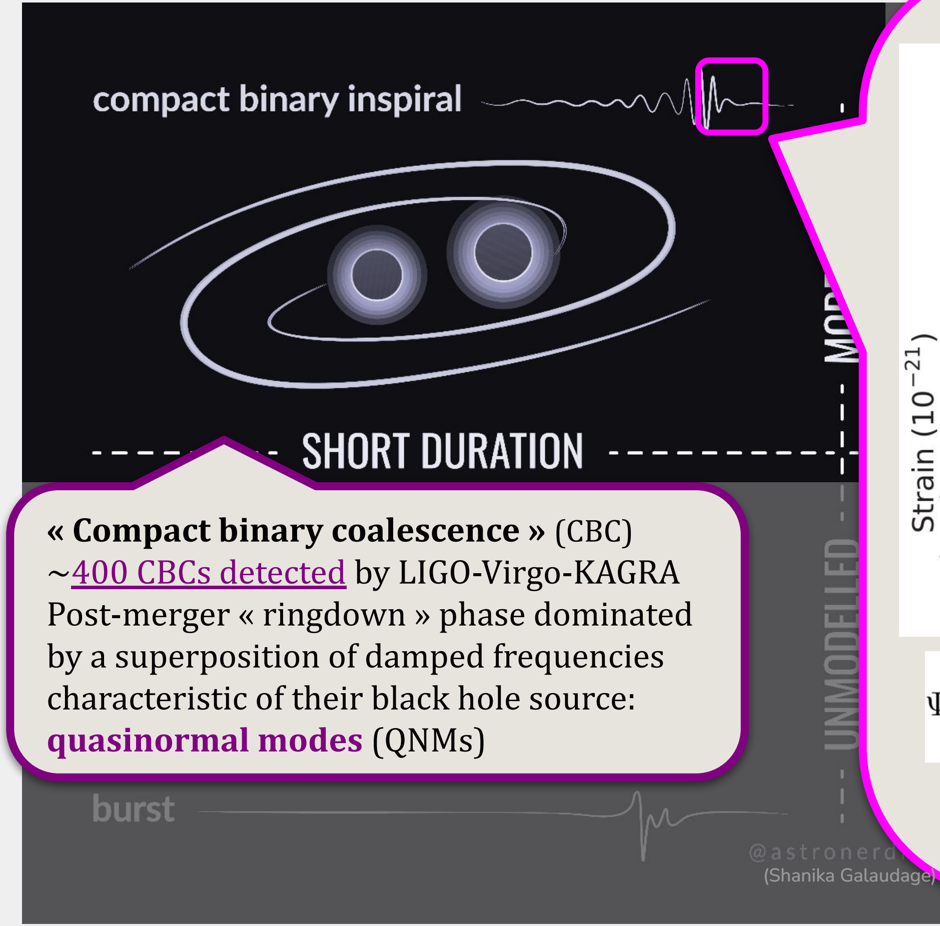




« Compact binary coalescence » (CBC)
~400 CBCs detected by LIGO-Virgo-KAGRA



LIGO and Virgo Collaborations, PRL 116 (2016)



« Compact binary coalescence » (CBC)
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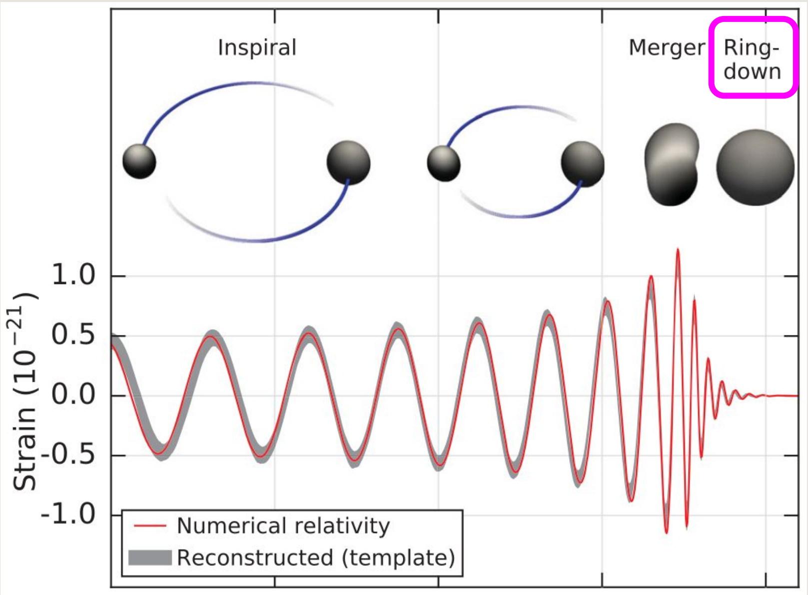
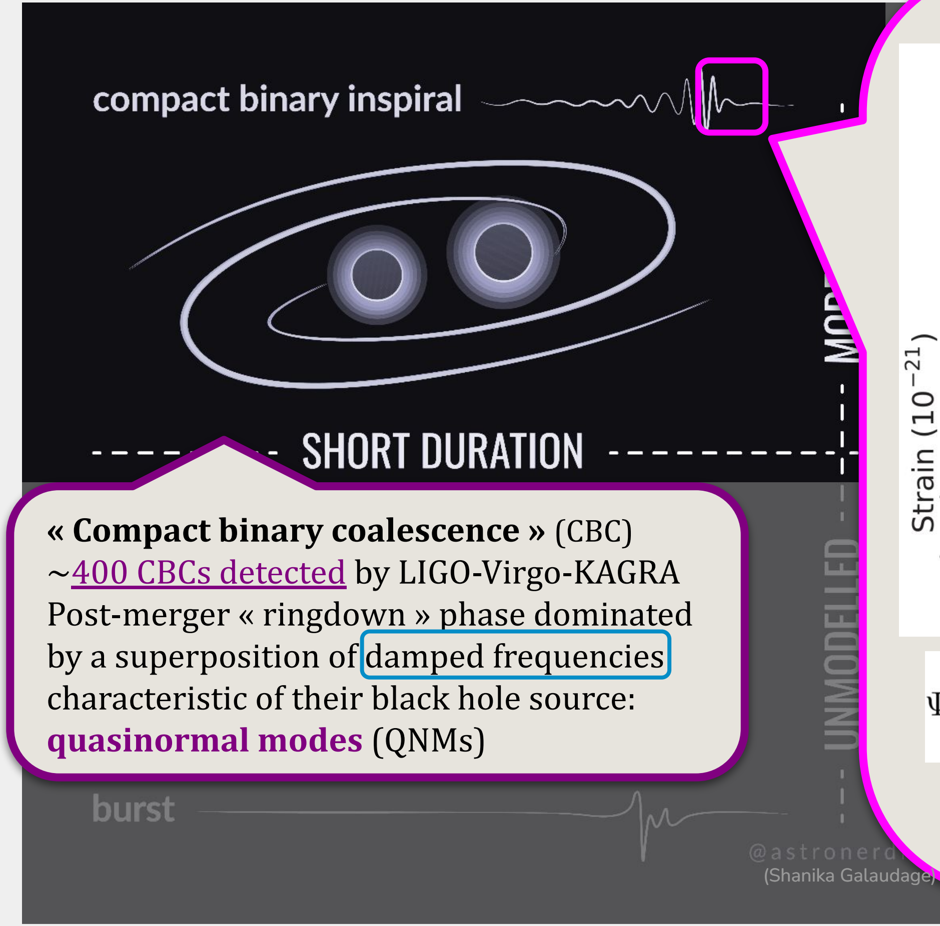
Post-merger « ringdown » phase dominated by a superposition of damped frequencies characteristic of their black hole source: **quasinormal modes** (QNMs)



$$\Psi = \frac{1}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{\infty} A_{\ell mn}\, e^{-i\omega_{\ell mn} t}\, S_{\ell mn}(\theta, \omega)\, e^{im\phi}$$

$$\omega_{\ell mn} = \omega^{\mathrm{R}}_{\ell mn} + i\omega^{\mathrm{I}}_{\ell mn}\ , \ \omega^{\mathrm{I}}_{\ell mn} < 0$$



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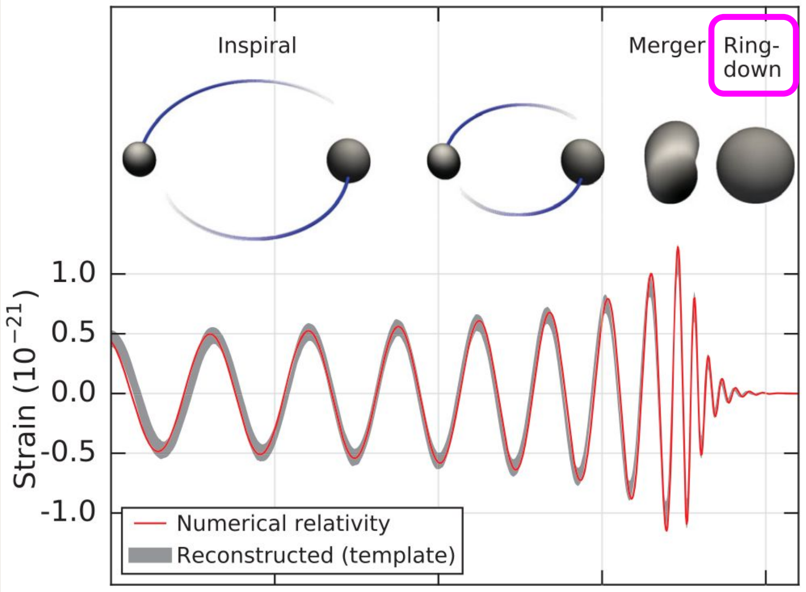

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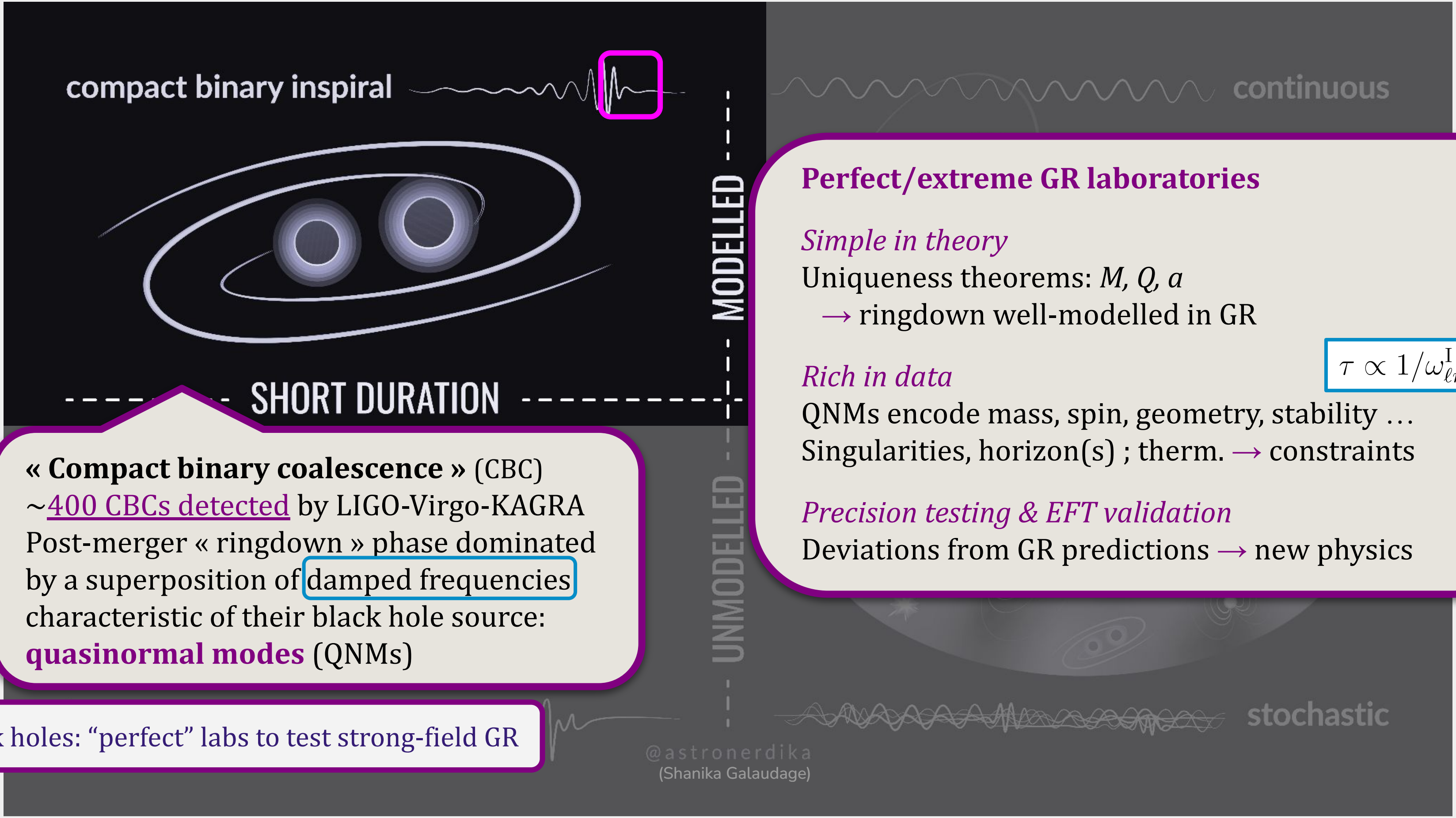

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Black holes: "perfect" labs to test strong-field GR

Perfect/extreme GR laboratories

Simple in theory
Uniqueness theorems: M, Q, a
→ ringdown well-modelled in GR

Rich in data

$\tau \propto 1/\omega^{\mathrm{I}}_{\ell mn}$

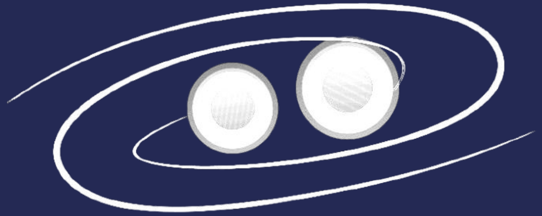
QNMs encode mass, spin, geometry, stability …
Singularities, horizon(s) ; therm. → constraints

Precision testing & EFT validation
Deviations from GR predictions → new physics

Black holes & their perturbations

Quasinormal modes + their excitations : a Schwarzschild example

HT Cho, CH Chen, AC, AS Cornell, Phys. Rev. D 111 (2025)



A quick GR primer: the Schwarzschild black hole

Stationary, neutral, spherically-symmetric black hole

$$\boxed{g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)}$$

Einstein field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \cancel{\Lambda g_{\mu\nu}} = \cancel{16\pi T_{\mu\nu}}}$$

for *flat space-time*, *in vacuum*

$[G] = M^{-1}L^3T^{-2}$, $[c] = LT^{-1}$

$[m_{BH}] = M$

The "no-hair" conjecture (Schwarz.):

Isolated, stationary BH in GR is fully characterised by M. All other info is lost

$$f(r) = 1 - \frac{2M}{r}$$

event horizon: $r_+ = 2M$

length scale: $M = Gm_{\rm BH}c^{-2}$ $(G{=}c{=}1)$

all isolated, stationary black hole solutions of the ***Einstein-Maxwell*** *equations in GR are fully characterised by their mass (**M**), electric charge (**Q**), and angular momentum (**a**) [Israel, Carter]*

Black hole perturbation theory

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

Black hole quasinormal modes

Quasinormal mode (QNM)

$$\boxed{\Psi(x^\mu) = \sum_{n=0}^{\infty} \sum_{\ell,m} \frac{\psi_{sn\ell}(r)}{r} e^{-i\omega t} Y_{\ell m}(\theta,\phi)}$$

s: spin of perturbing field
m: azimuthal number for spherical harmonic decomposition in θ, ϕ
ℓ: angular/multipolar number for spherical harmonic decomposition in θ, ϕ
n: overtone number labels ω by a monotonically increasing $|\Im m\{\omega\}|$

Quasinormal frequency (QNF)

$$\boxed{\omega_{sn\ell} = \omega_R - i\omega_I}$$

$\Re e\{\omega\}$: physical oscillation frequency $\rightarrow$ $\Re e\{\omega\} \propto \ell$
$\Im m\{\omega\}$: damping $\rightarrow$ dissipative, "quasi" $\rightarrow$ $\lim_{\ell\to\infty} |\Im m\{\omega\}| = constant$

Quasinormal mode eigenvalue problem: boundary conditions

Black hole wave equation

$$\boxed{\frac{d^2}{dr_*^2}\varphi + \left[\omega^2 - V(r)\right]\varphi = 0\ , \qquad \frac{dr}{dr_*} = f(r)}$$

$$e.g.\quad V_s(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\left(1 - s^2\right)\right]\ , \qquad s = 0, 1, 2$$

Subjected to **QNM boundary conditions**:

$$\text{purely ingoing:} \quad \varphi(r) \sim e^{-i\omega r} \qquad r \to r_+$$

$$\text{purely outgoing:} \quad \varphi(r) \sim e^{+i\omega r} \qquad r \to +\infty$$

Waves escape domain of study at the boundaries $\Rightarrow$ dissipative

How can we identify individual QNMs in the superposition?

S. Detweiler, Proc. R. Soc. A 352 (1977)
E. W. Leaver, Phys. Rev. D 34 (1986)
N. Andersson, Phys. Rev. D 51 (1995)

In the GW context, strain is a function of the « excitation coefficient »
Oshita, Phys. Rev. D 104 (2021)

$$h_+ + ih_\times = \sum_{\ell mn} \mathcal{C}_{\ell n} \, Y_{\ell m}(\theta, \phi) \, \frac{\psi_{\ell n}}{r} \, e^{-i\omega_{\ell n} t} \; ,$$

which is a product of a source factor (initial data) × an independent « quasinormal excitation factor »

Formally, we model the QNM contribution to the black hole response through a Green's function analysis. This requires explicit expressions for the wavefunction, evaluated at the QNF.

$$\mathcal{C}_{\ell n} = \mathcal{B}_{\ell n} \times \mathcal{T}_{\ell n}$$

excitation coefficient
= QNEF × initial data

QNFs = poles of *f*-domain retarded Green function
QNEF = residue at each pole, measuring intrinsic weight with which each mode contributes to black hole response

Near a simple QNM pole

$$A^-_{\ell\omega} \simeq (\omega - \omega_{\ell n}) \left. \frac{\partial A^-_{\ell\omega}}{\partial \omega} \right|_{\omega=\omega_{\ell n}}$$

$$\boxed{\mathcal{B}_{\ell n} \equiv \left[\frac{A^+_{\ell\omega}}{2\omega} \left(\frac{\partial A^-_{\ell\omega}}{\partial \omega} \right)^{-1} \right]_{\omega=\omega_{\ell n}}}$$

How do we calculate these QNM frequencies, wavefunctions, excitation factors?

Challenges:

★ Non-Hermitian Operator: complex eigenvalues; QNMs do not form a complete set of eigenfunctions

★ Boundary conditions: dissipative system requiring semi-classical/ numerical treatment

★ Spectral instability: highly sensitive - small perturbations alter, even destabilise QNM spectrum

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Dolan & Ottewill

Class. Quant. Grav. 26 (2009), Phys. Rev. D 84 (2011)

A new computation method for BH QNMs through a novel ansatz based on **null geodesics** + expansion of the QNF in inverse powers of $L = \ell + 1/2$

$$\boxed{\Phi(r) = e^{i\omega z(r_*)} v(r) \ , \quad \omega = \sum_{k=-1}^{\infty} \omega_k L^{-k}}$$

- ⋆ iterative procedure best performed in the eikonal limit
- ⋆ more efficient means of calculating detectable QNMs?
- ⋆ can extend to compute QNM wavefunctions [♦ rare find!]

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$$r_c = \left. \frac{2f(r)}{\partial_r f(r)} \right|_{r=r_c}\ , \quad b_c = \left. \sqrt{\frac{r^2}{f(r)}} \right|_{r=r_c}\ , \quad k_c(r)^2 = \frac{1}{b^2} - \frac{f(r)}{r^2}$$

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$$f(r)\frac{d}{dr}\left(f(r)\frac{dv}{dr}\right) + 2i\omega\rho(r)\frac{dv}{dr} + \left[i\omega f(r)\frac{d\rho}{dr} + \left(1-\rho(r)^2\right)\omega^2 - V(r)\right] v(r) = 0$$

We solve iteratively for ω_k and $S_k'(r)$ and sub into ω

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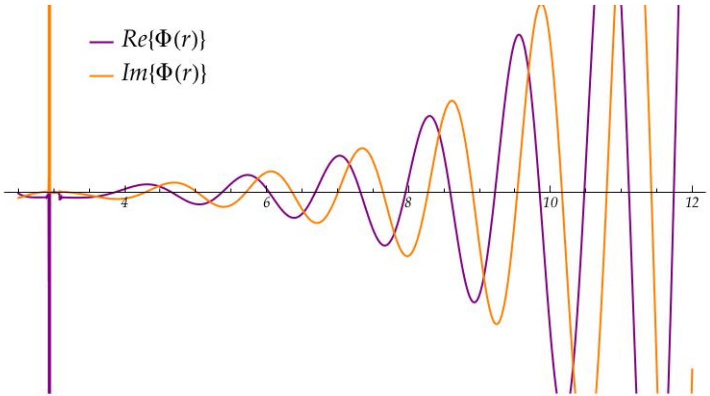
$$\boxed{r_*^{peak} \approx 3}$$



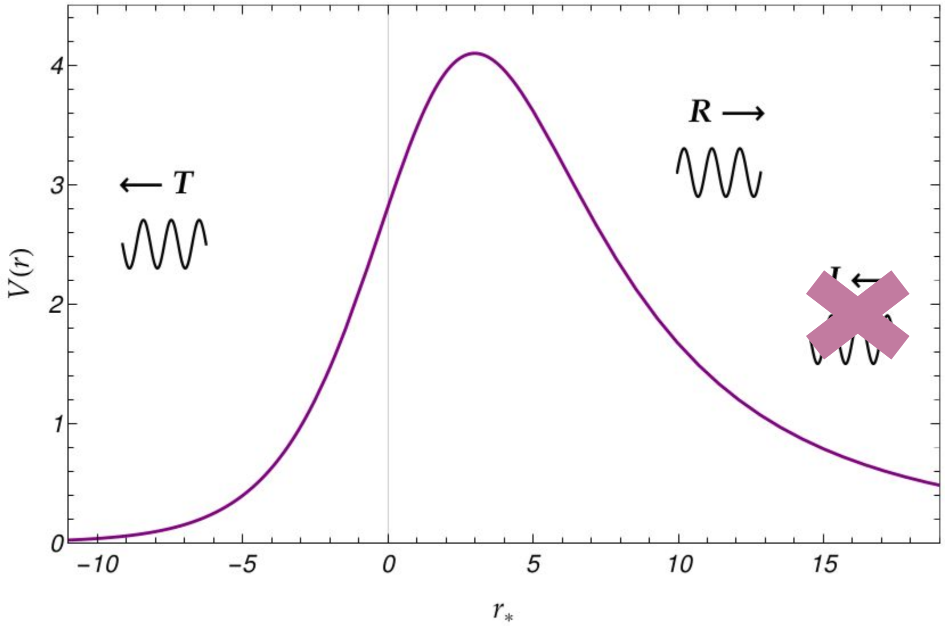
breakdown at photon orbit radius

Reminiscent of a scattering problem, subjected to QNM boundary conditions

$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\right]$$

QNM boundary conditions at the event horizon & spatial infinity

$$\psi_{r_+} \sim \begin{cases} e^{-i\omega r_\star} & r_\star \to -\infty \\ \cancel{A^-_{\ell\omega} e^{-i\omega r_\star}} + A^+_{\ell\omega} e^{+i\omega r_\star} & r_\star \to +\infty \end{cases}$$



Managing breakdowns in the DO method: QNM solutions

$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\right]$$

$$r_*^{peak} \approx 3 + \sqrt{3} z L^{-1/2} + 12\sqrt{3}\omega_0 L^{-1} + 36 z \omega_0 L^{-3/2} + \left[\frac{11}{6} + 342\omega_0^2 + 12\sqrt{3}\omega_1\right] L^{-2} + \ldots$$

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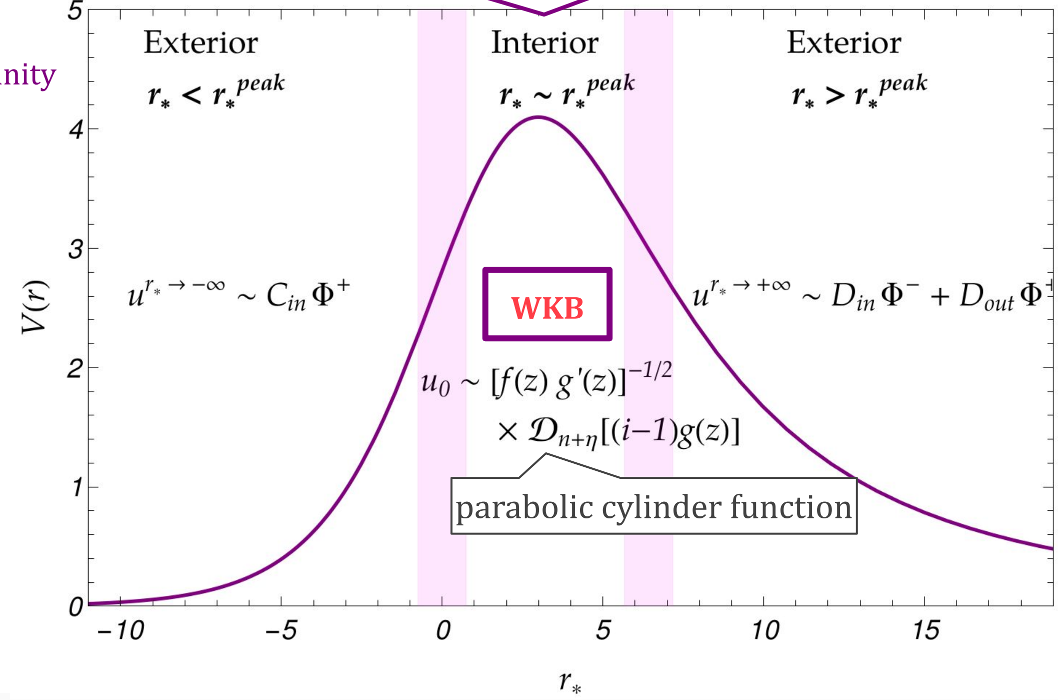


exterior: Dolan-Ottewill
interior: WKB (DO breaks down at $r \sim r^{peak}$)

Iyer and Will, Phys. Rev. D 35, 3621 (1987)

Managing breakdowns in the DO method: QNM solutions

$$\lim_{r_*\to+\infty} u_0(e^{\pm iz^2/2}) \sim \lim_{r_*\to r_*^{peak}} \left[D_{in}\Phi^-(e^{-iz^2/2}) + D_{out}\Phi^+(e^{+iz^2/2}) \right]$$

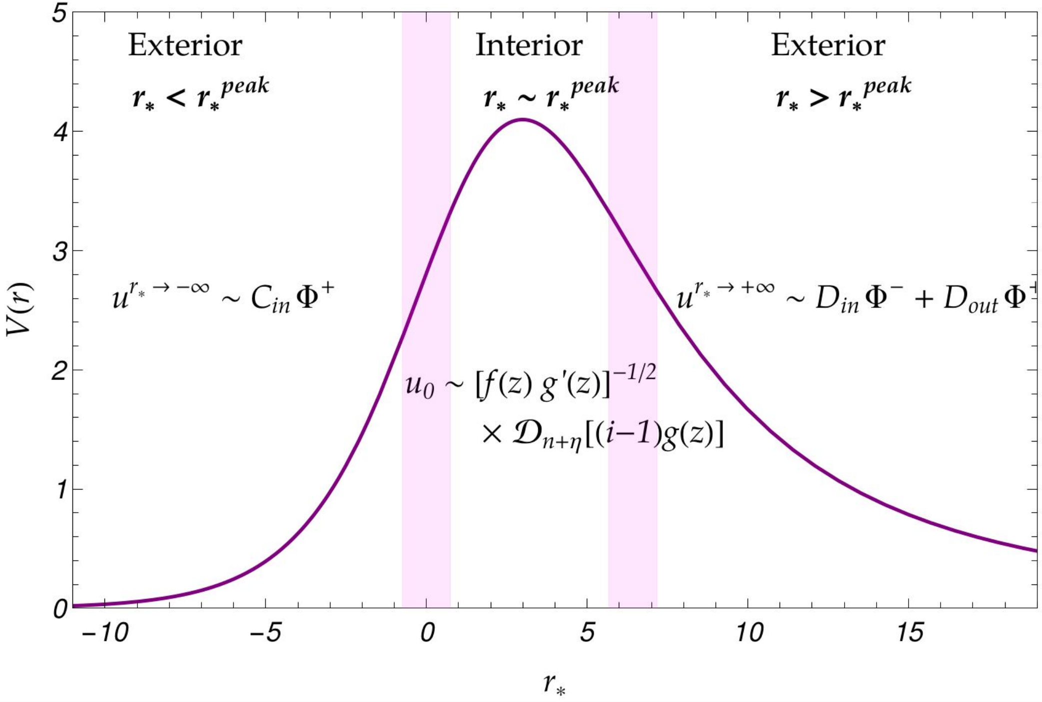
$$\lim_{r_*\to-\infty} u_0(e^{+iz^2/2}) \sim \lim_{r_*\to r_*^{peak}} C_{in}\Phi^+(e^{+iz^2/2})$$

$$A^+_{\ell\omega} \propto \frac{D_{out}}{C_{in}}$$

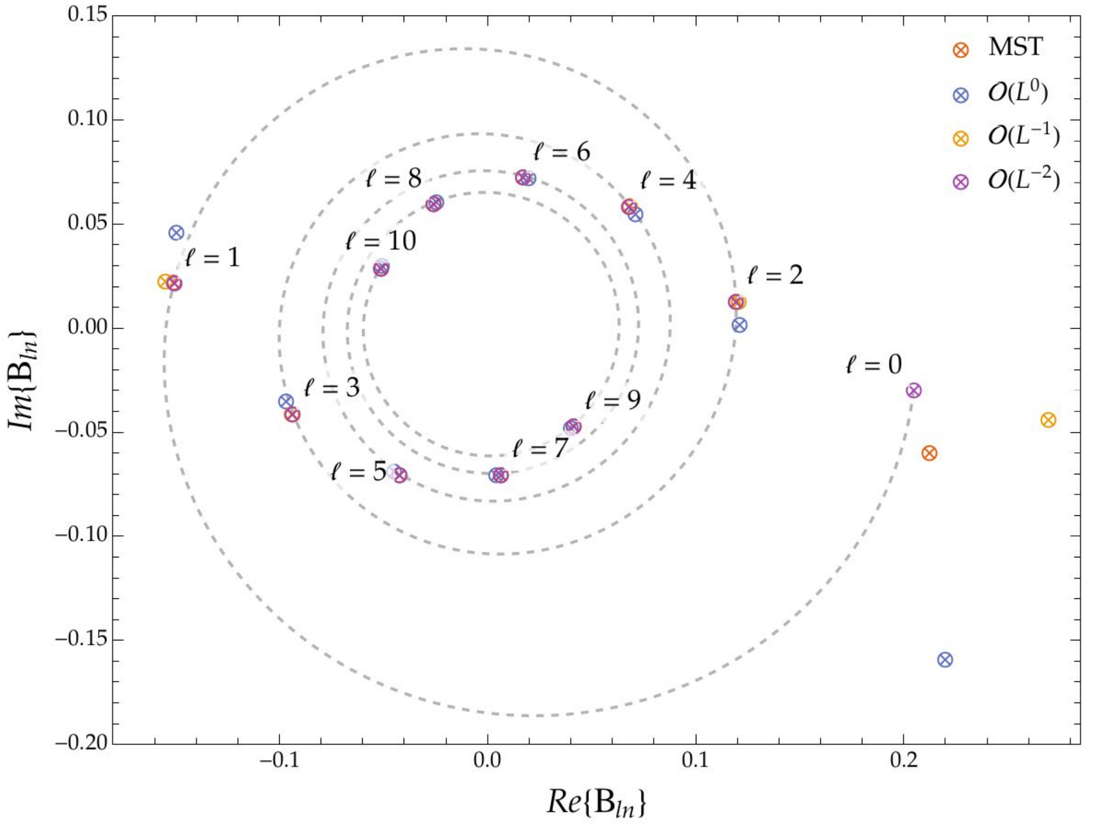
$$A^-_{\ell\omega} \propto \left.\frac{D_{in}}{C_{in}}\right|_{\omega\to\tilde\omega,\ \epsilon\to 0} = 0$$

$$\boxed{\mathcal{B}_{\ell n} \equiv \left[\frac{A^+_{\ell\omega}}{2\omega} \left(\frac{\partial A^-_{\ell\omega}}{\partial\omega} \right)^{-1} \right]_{\omega=\omega_{\ell n}}}$$

with the perturbation $\tilde\omega \approx \sum_{k=-1} \omega_k L^{-k} + \epsilon$



Schwarzschild QNEFs for n = 0 and increasing ℓ

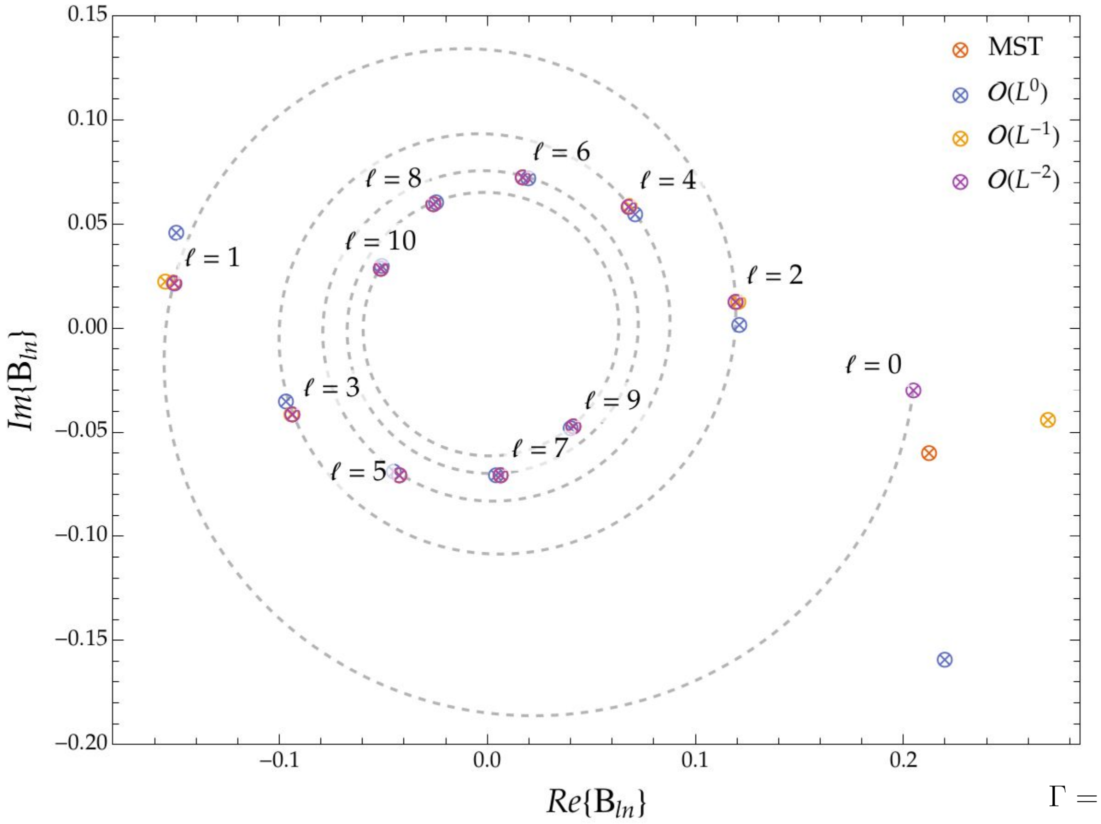


Results:

- ★ For n = 0, larger excitation for lower harmonics: $|B| \propto L^{-½}$ $_{L=ℓ+½}$
- ★ Excellent agreement with analytical formalism developed by Mano, Suzuki and Takasugi at NNLO
- ★ As a "no-hair" quantity, QNEFs to be used as a new test for mod-GR?

ℓ-dependent scattering & matching phase accumulated by wave around photon-sphere potential barrier
Recall: QNEF = residue of Green function at QNM pole
=> naturally contains amplitude + phase

Schwarzschild QNEFs for n = 0 and increasing ℓ



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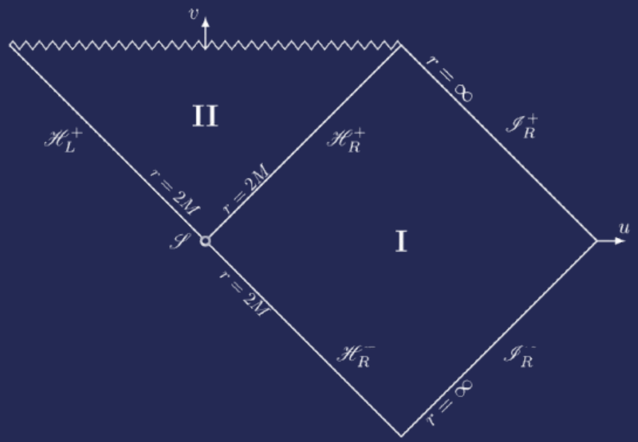
NB: *closely related to greybody factors that contribute to Hawking radiation*

$$\Gamma = \left|\frac{\mathcal{T}}{\mathcal{I}}\right|^2 \qquad \frac{d^2E}{dt\,d\omega} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{\hbar\omega}{2\pi}\frac{N_\ell\,\Gamma}{e^{\hbar\omega/k_BT}\mp 1}$$ bosons / fermions

Black holes & their interiors

Strong Cosmic Censorship, probed using quasinormal frequencies for Q, Λ >0

AC, AS Cornell, A Deandrea, SC Park, Class. Quantum Grav. 42 (2025)



A quick GR primer: the Reissner-Nordström black hole

Stationary, *charged*, spherically-symmetric black hole

$$\boxed{g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)}$$

Einstein–Maxwell field equations

$$\boxed{\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \cancel{\Lambda g_{\mu\nu}} \\ = 16\pi\left[F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right] \\ \nabla^\nu F_{\mu\nu} = 0\,,\ \nabla_{[\mu}F_{\nu\rho]=0} \end{aligned}}$$

The "no-hair" conjecture

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

horizons: $r_-,\ r_+$

length scale: $M = Gm_{BH}c^{-2}$

$Q = G^{1/2}q_{BH}c^{-2}$

Black hole perturbation theory

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

$$r_\pm = M^2 \pm \sqrt{M^2 - Q^2}$$

A quick GR primer: the Reissner-Nordström de Sitter black hole

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in de Sitter space-time

The "no-hair" conjecture

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L_{dS}^2}$$

horizons: $r_-,\ r_+,\ r_c$

length scale: $M = Gm_{BH}c^{-2}$

$Q = G^{1/2}q_{BH}c^{-2}$

$L_{dS} = (3/\Lambda)^{1/2} = 1$

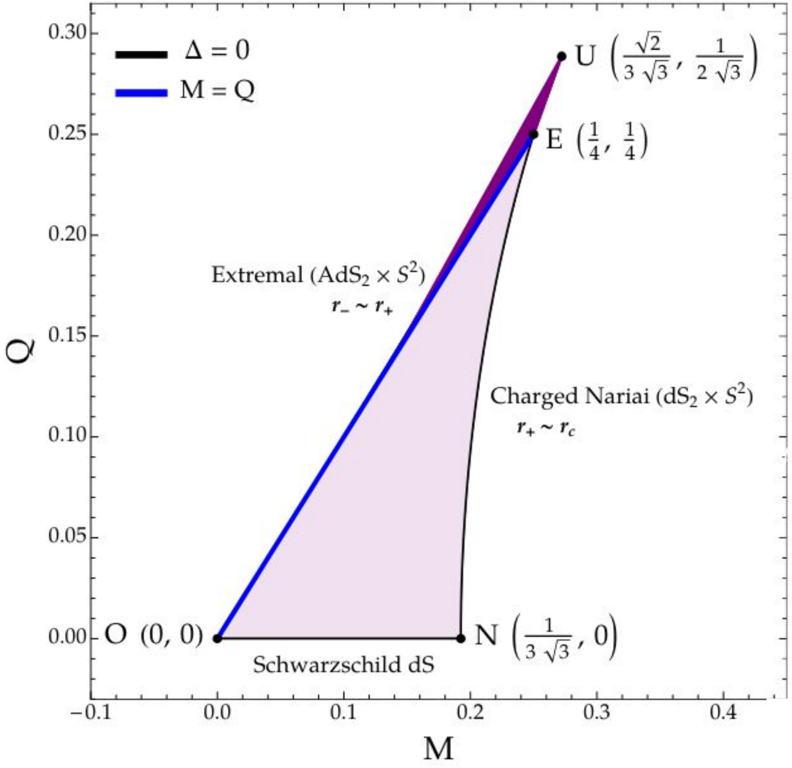
$$r_\pm \neq M^2 \pm \sqrt{M^2 - Q^2}$$

Black hole perturbation theory

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

A quick GR primer: the Reissner-Nordström de Sitter black hole

The RNdS "sharkfin" phase space



$$f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

The "no-hair" conjecture

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L_{dS}^2}$$

horizons: r_-, r_+, r_c

length scale: $M = Gm_{BH}c^{-2}$

$Q = G^{1/2}q_{BH}c^{-2}$

$L_{dS} = (3/\Lambda)^{1/2} = 1$

$$r_\pm \neq M^2 \pm \sqrt{M^2 - Q^2}$$

$$\frac{Q^2}{M^2} \lesssim 1 + \frac{1}{3}(M^2\Lambda) + \ldots$$

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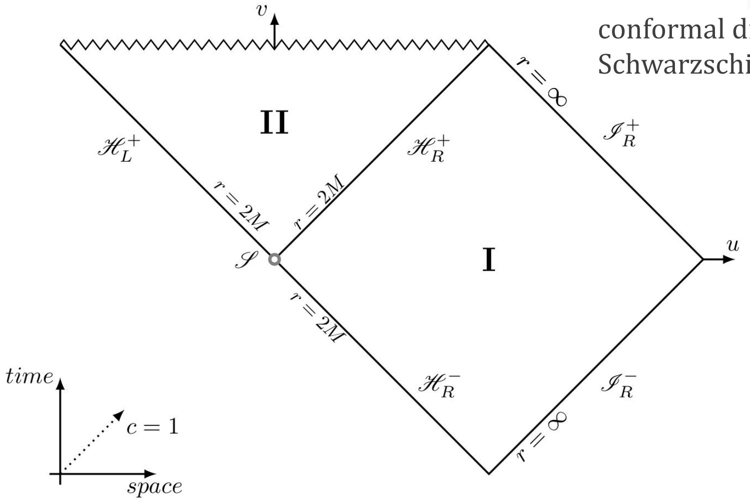
Formal GR theory: massive scalar QNMs in a Reissner-Nordström de Sitter black hole



conformal diagram:
Schwarzschild black hole

Symbol	*Coordinates* Schwarzschild	*Coordinates* Eddington-Finkelstein
$\mathscr{H}^-$	$t \to -\infty,\ r_* \to -\infty$	$v \to -\infty,$ finite u
$\mathscr{H}^+$	$t \to +\infty,\ r_* \to -\infty$	finite $v,\ u \to +\infty$
$\mathscr{I}^-$	$t \to -\infty,\ r_* \to +\infty$	finite $v,\ u \to -\infty$
$\mathscr{I}^+$	$t \to +\infty,\ r_* \to +\infty$	$v \to +\infty,$ finite u

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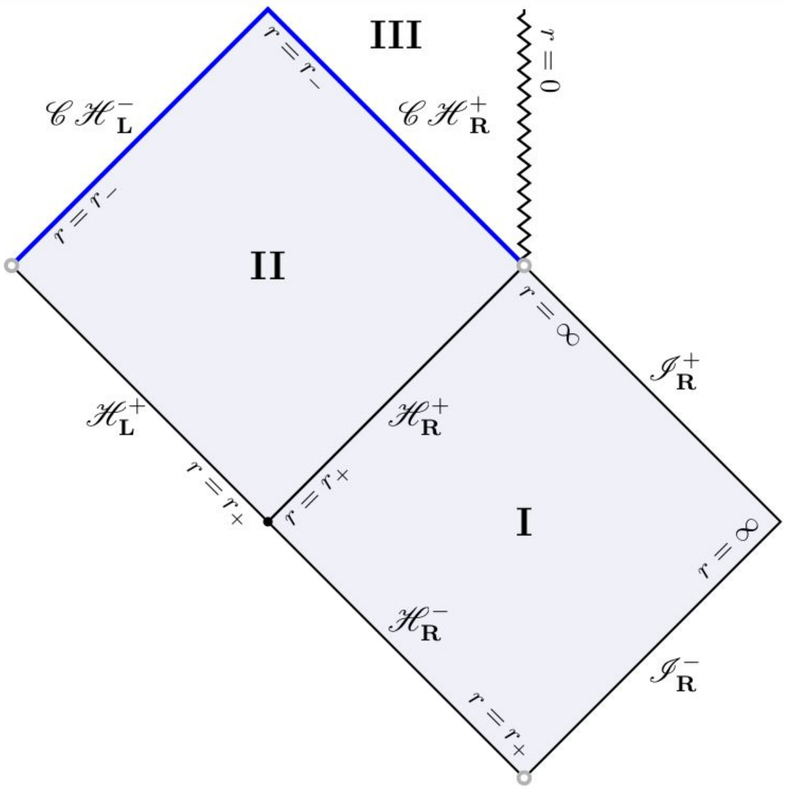

<u>Problem:</u> Cauchy horizon = boundary of domain of dependence of an initial data.
Before reaching it, every event is fixed by initial data
Beyond it, causal curves can enter from regions not controlled by those data, so **different continuations of the metric may all satisfy Einstein's equations** while agreeing on everything before the horizon.

Strong Cosmic Censorship ensures that for generic initial data, the maximal future Cauchy development is inextendible, at the specified regularity, past the singularity

→ rigorously approached from 2003 [Dafermos] (!)

→ no formal universal proof , many revisions

e.g. Sbierski (2018): proved C^0–inextendability of the metric beyond the r = 0 singularity in Schwarzschild

⇒ SCC preservation under strongest criteria

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Strong Cosmic Censorship (SCC) in Reissner-Nordström de Sitter black holes

For our purposes: SCC requires that the metric *(M, g)*, as a well-defined solution to the vacuum Einstein field equations, cannot be extended past the Cauchy horizon (CH)

Different SCC versions distinguished by smoothness level required at CH

C^0 version (strongest SCC): generically no continuous extension exists across CH
$\Rightarrow$ singularity exists before Cauchy horizon forms

Christodoulou version: generically no weak extension (with loc. sq. integrable Christoffel sym.)
$\Rightarrow$ Cauchy surface is a weak null singularity

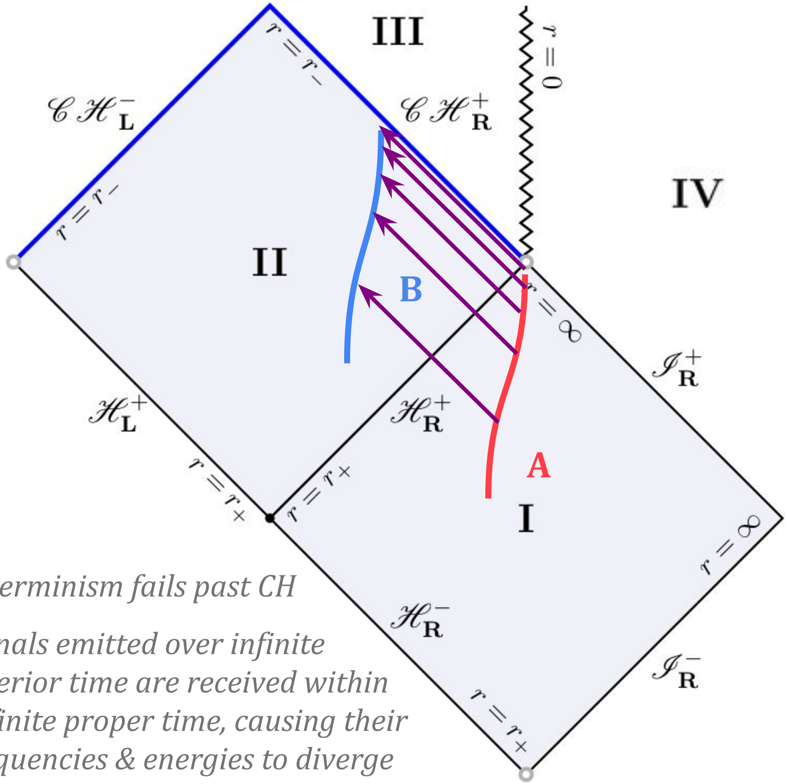
C^2 version: generically no extension in C^2

strength

For Einstein-Maxwell(-massless scalar) theory, with $\Lambda = 0$:
C^0 version is false [*Christodoulou*; *Dafermos-Luk*]
Christodoulou version believed to be true [*Dafermos-Rothman*; *Luk-Oh*]

Scalar field is used as a proxy for the full nonlinear solution

Reissner-Nordström de Sitter black holes: blueshift vs redshift



determinism fails past CH

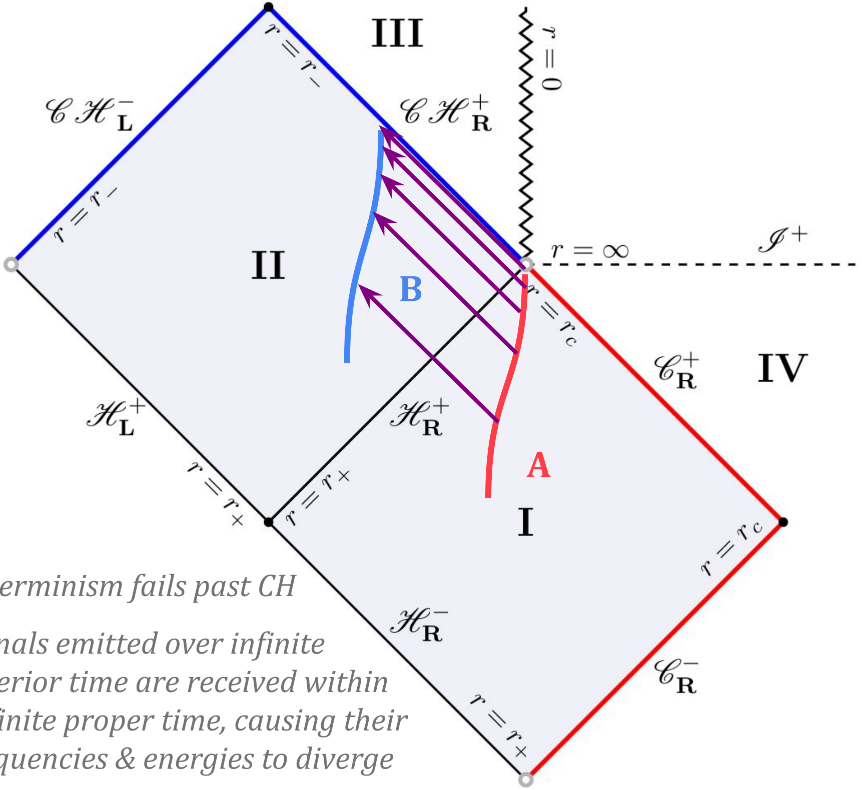
signals emitted over infinite exterior time are received within a finite proper time, causing their frequencies & energies to diverge

For $\Lambda = 0$, consider two time-like observers:
A: remains in I $\longrightarrow$ reaches ∞ at infinite proper time
B: falls into II $\longrightarrow$ reaches CH at finite proper time

If observer A sends periodic (in A's time) signals to observer B, observer B will perceive them as incoming with greater frequency
$\Rightarrow$ energy of an oscillating Φ entering the black hole will increase infinitely as it approaches $r = r_-$

Infinite energy at Cauchy horizon $\Rightarrow$ curvature divergence
***"Blueshift instability" rescues SCC** for $\Lambda = 0$ [Penrose 1978]*

Reissner-Nordström de Sitter black holes: blueshift vs redshift



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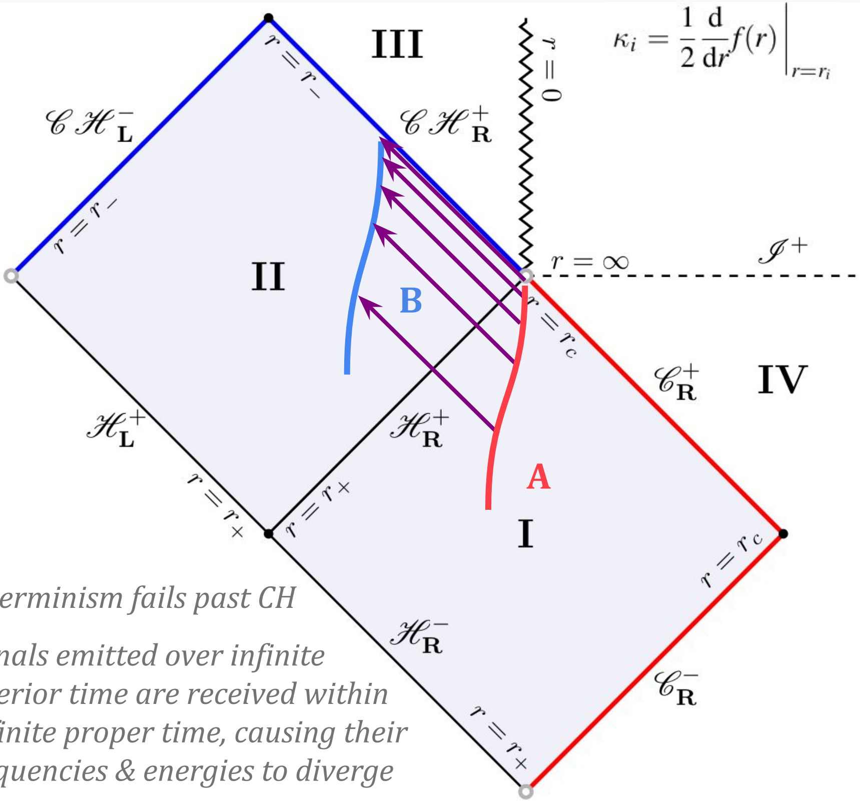
Infinite energy at Cauchy horizon $\Rightarrow$ curvature divergence
***"Blueshift instability" rescues SCC** for $\Lambda = 0$ [Penrose 1978]*

For $\Lambda > 0$, perturbations decay rapidly (exponentially).
Is there enough energy to lead to blueshift instability?

Formal GR theory: massive scalar QNMs in a Reissner-Nordström de Sitter black holes



$$\kappa_i = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}r}f(r)\bigg|_{r=r_i}$$

determinism fails past CH

signals emitted over infinite exterior time are received within a finite proper time, causing their frequencies & energies to diverge

Two competing behaviours:

redshift at $\mathscr{H}^+_{\mathbf{R}}$ $\quad \psi \sim e^{-i\omega t}$ $\quad$ [quasinormal mode]

Blueshift at $\mathscr{CH}^+_{\mathbf{R}}$ $\quad \psi \sim e^{\kappa_- t}$

A blueshift amplification phenomenon characterises the dynamics of infalling fields as they accumulate along the inner Cauchy horizon

SCC preservation:
[Hintz & Vasy 2017]

$$\beta \equiv -\frac{\Im m\left\{\omega^{n=0}\right\}}{|\kappa_-|} < \frac{1}{2}$$

where $\Im m\{\omega\} < 0$ is a necessary condition for black hole stability

an upper bound on the damping rate of the incoming scalar field Φ above which the red-shift effect will overcome the blue-shift

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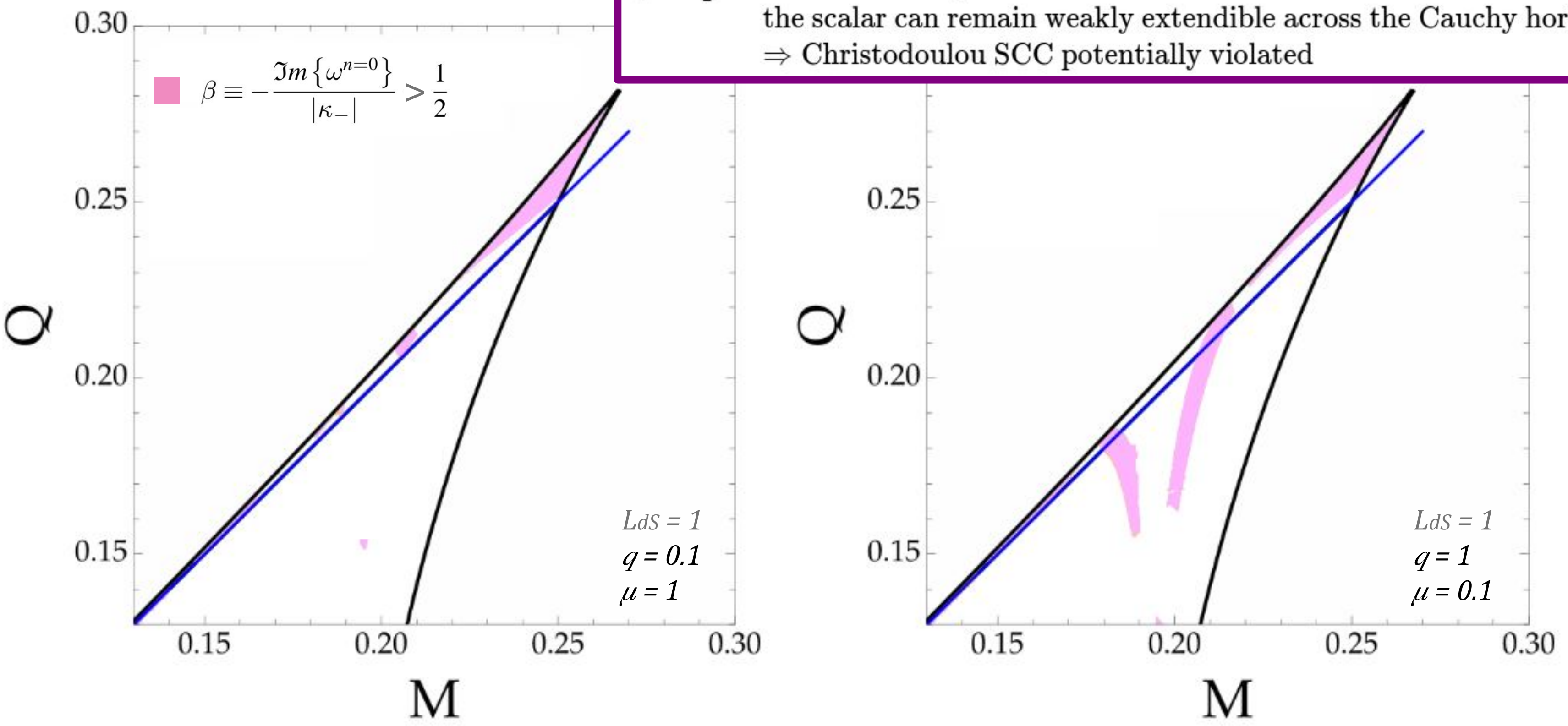
SCC violations in Reissner-Nordstr

$\beta < \frac{1}{2}$: **blueshift dominates**
local energy diverges at the Cauchy horizon
$\Rightarrow$ Christodoulou SCC preserved

$\beta > \frac{1}{2}$: **exterior decay dominates**
the scalar can remain weakly extendible across the Cauchy horizon
$\Rightarrow$ Christodoulou SCC potentially violated

too damped to trigger blueshift instability



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Reissner-Nordström de Sitter black holes: violations in SCC

To summarise:

linearised massive + charged Klein-Gordon eq.

$$\mathcal{L}_{\text{sc.}} = -\frac{1}{2}(\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}^\mu\Phi) - \frac{1}{2}\mu^2\Phi^\dagger\Phi \qquad \mathcal{D}_\mu = (\partial_\mu - \mathrm{i}qA_\mu)$$

$$\nabla_\mu\nabla^\mu\Phi - \mu^2\Phi = 0\,, \quad G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

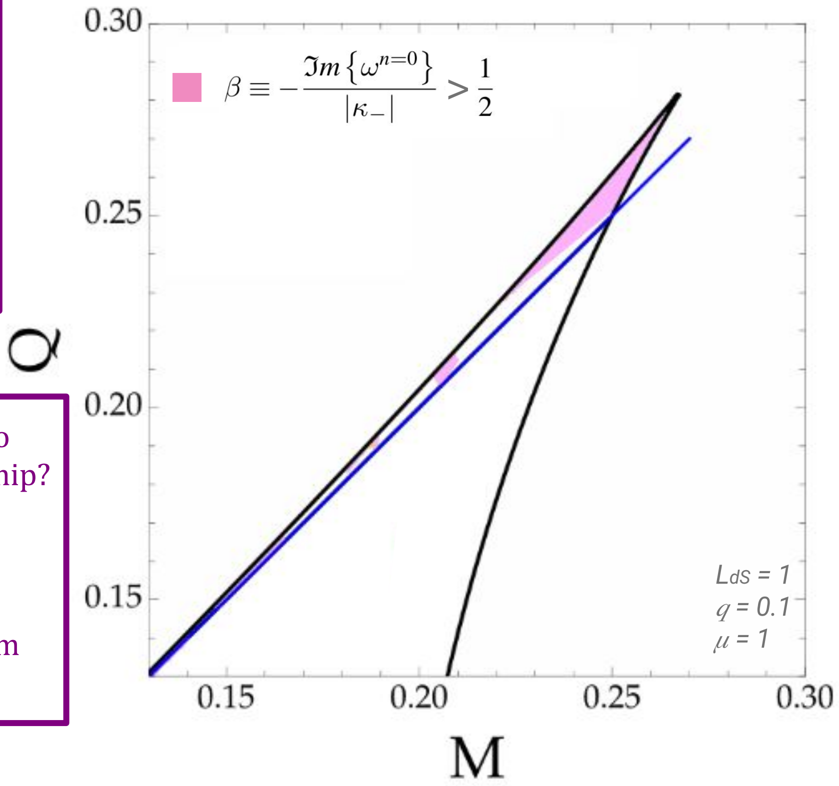
Result:

$\beta > \frac{1}{2}$ *consistently for* $Q \gtrsim M$

increasing ℓ, q *leads to further failures of SCC within the RNdS solution space*

Do large, cold BHs fail to respect cosmic censorship?

Can non-linear/quantum effects save SCC?



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Black holes & their throat deformations

Succumbing to non-vanishing scalar potentials in Einstein-Maxwell-dilaton theory

K Benakli, AC, arXiv:2607.05488 [hep-th]

Black holes as probes of field space

Instead of scalar test-fields as probes of black hole structure, we now consider black holes as probes of moduli space

Can a black hole create a macroscopic region in which the local effective theory differs from the theory at infinity?

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Sen 2025:

$$\boxed{\mathcal{R}\,,\quad F^2\,,\quad (\nabla\phi)^2 \ll 1}$$

$$\boxed{\Delta\phi \sim \mathcal{O}(1)}$$

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adding a potential changes horizon structure,
modifies extremal limit,
& casts doubt on whether infinite near-horizon throat of the massless solution survives

$$\boxed{\Box\phi = \underbrace{\frac{1}{4}B_{,\phi}F^2}_{\text{black hole gauge source}} + \underbrace{V_{,\phi}}_{\text{stabilising potential}}}$$

For example:

$$V = \frac{1}{2}m_\phi^2\left(\phi - \phi_0\right)^2$$

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$$m_\phi r_{\rm BH} \sim \frac{r_{\rm BH}}{\lambda_{\rm C}}$$

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Considered by Harvey&Gregory in 1993!

$$\boxed{r_{\rm BH} \ll m_\phi^{-1} \Rightarrow \text{dilatonic/GHS-like}}$$

$$\boxed{r_{\rm BH} \gg m_\phi^{-1} \Rightarrow \text{pinned/RN-like}}$$

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Here:
> quadratic stabilising
> shifted exponentials
> exponential runaways
> inverse-power runaways
> racetrack barriers
> axion-like periodic
> supergravity-inspired corrections

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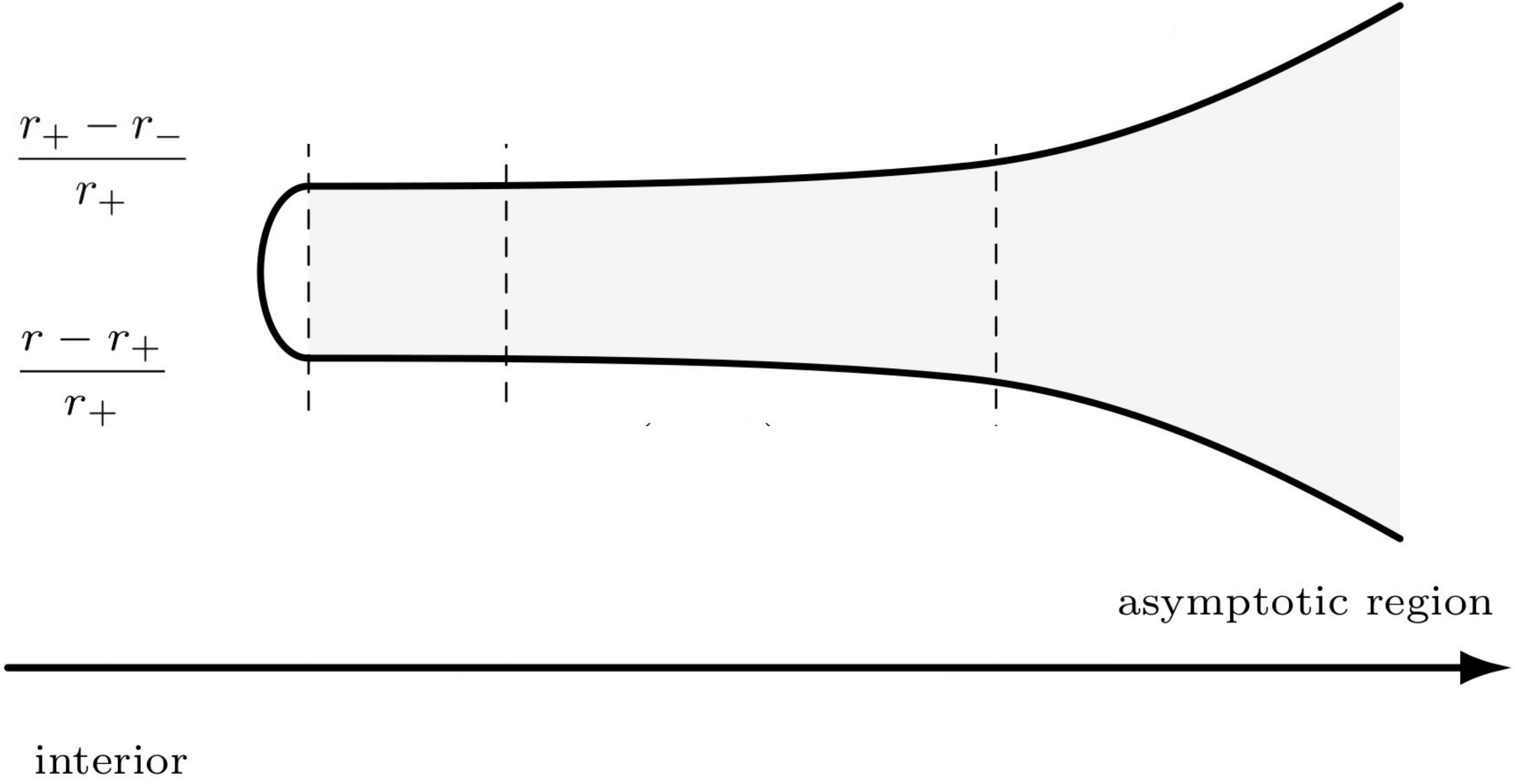
Gauge-induced & potential-induced forces are in competition.
What determines whether the black hole-induced modulus excursion survives?

Can a black hole change the local effective theory?

$$a = \frac{r_+ - r_-}{r_+}$$

$$b = \frac{r - r_+}{r_+}$$



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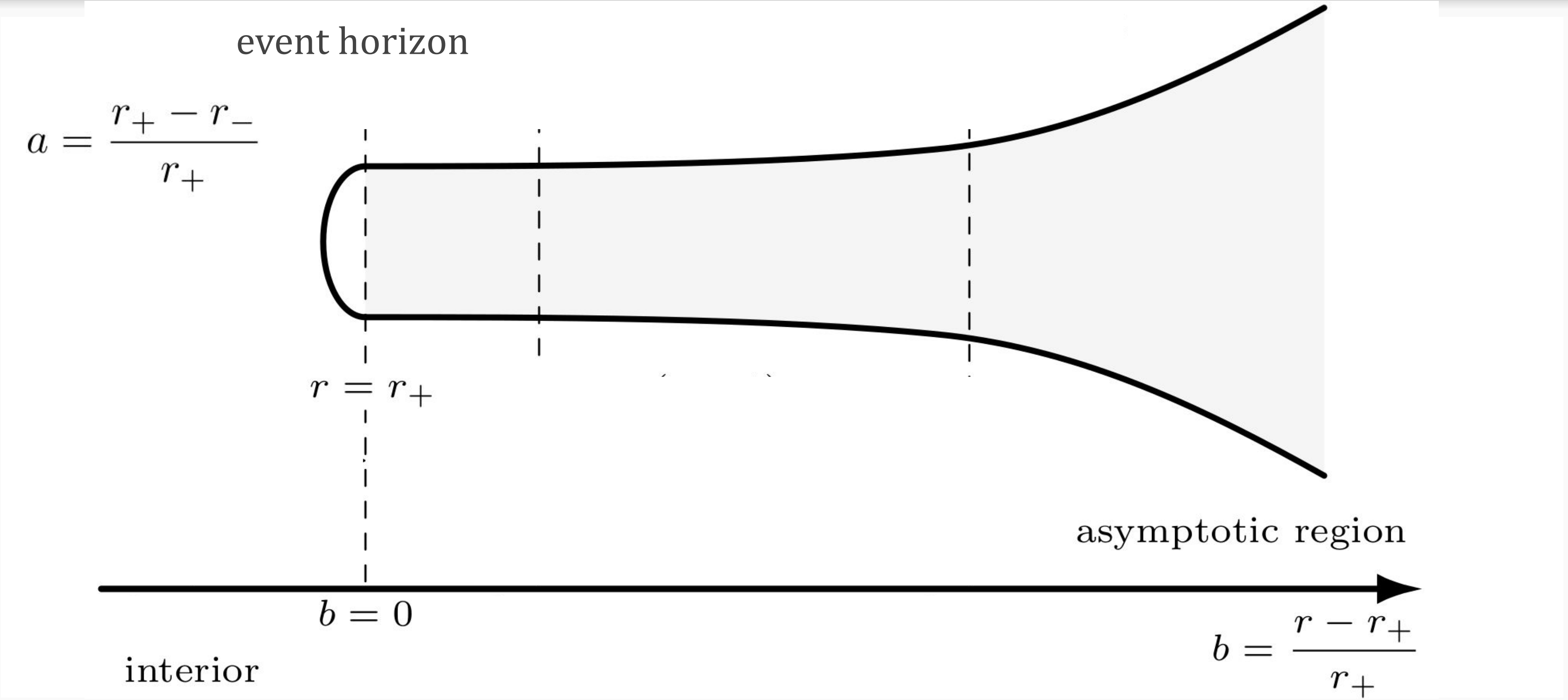

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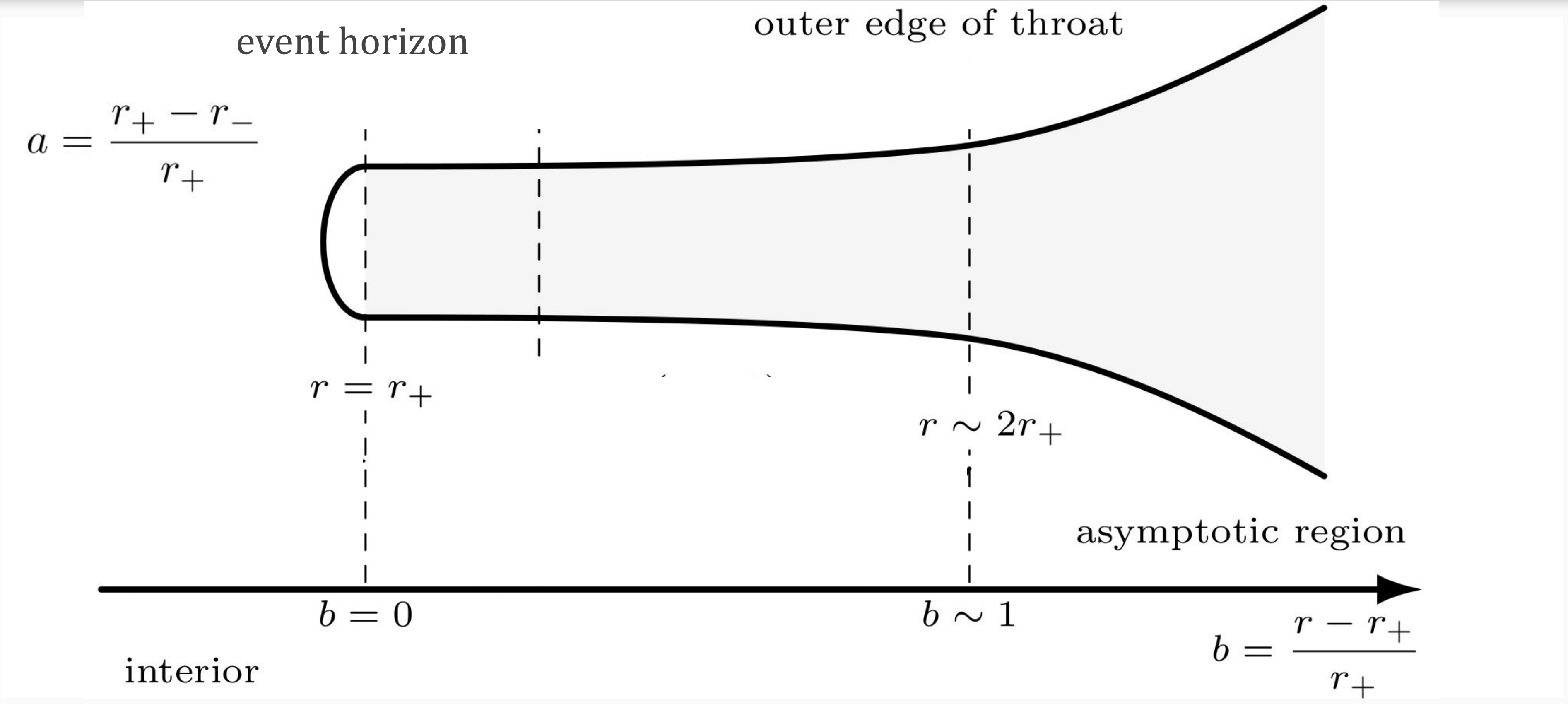

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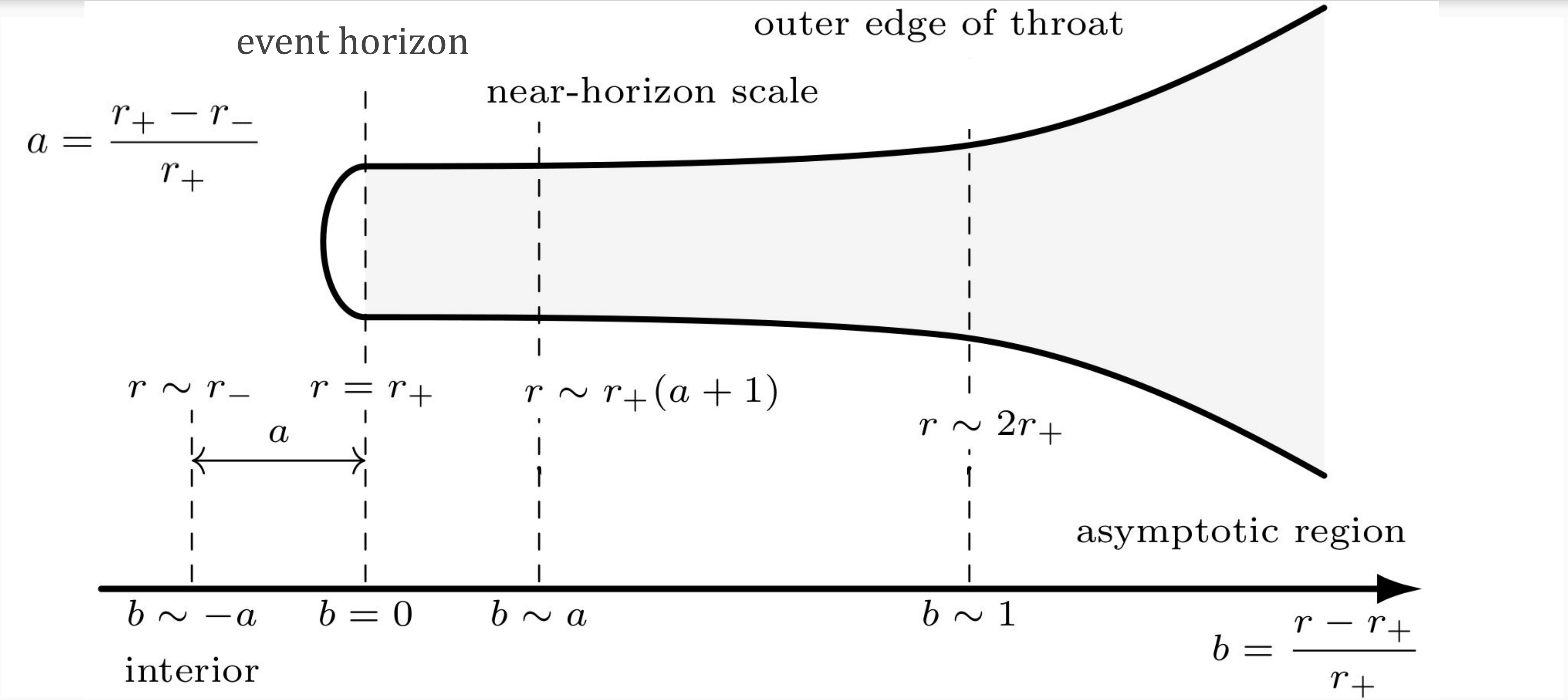
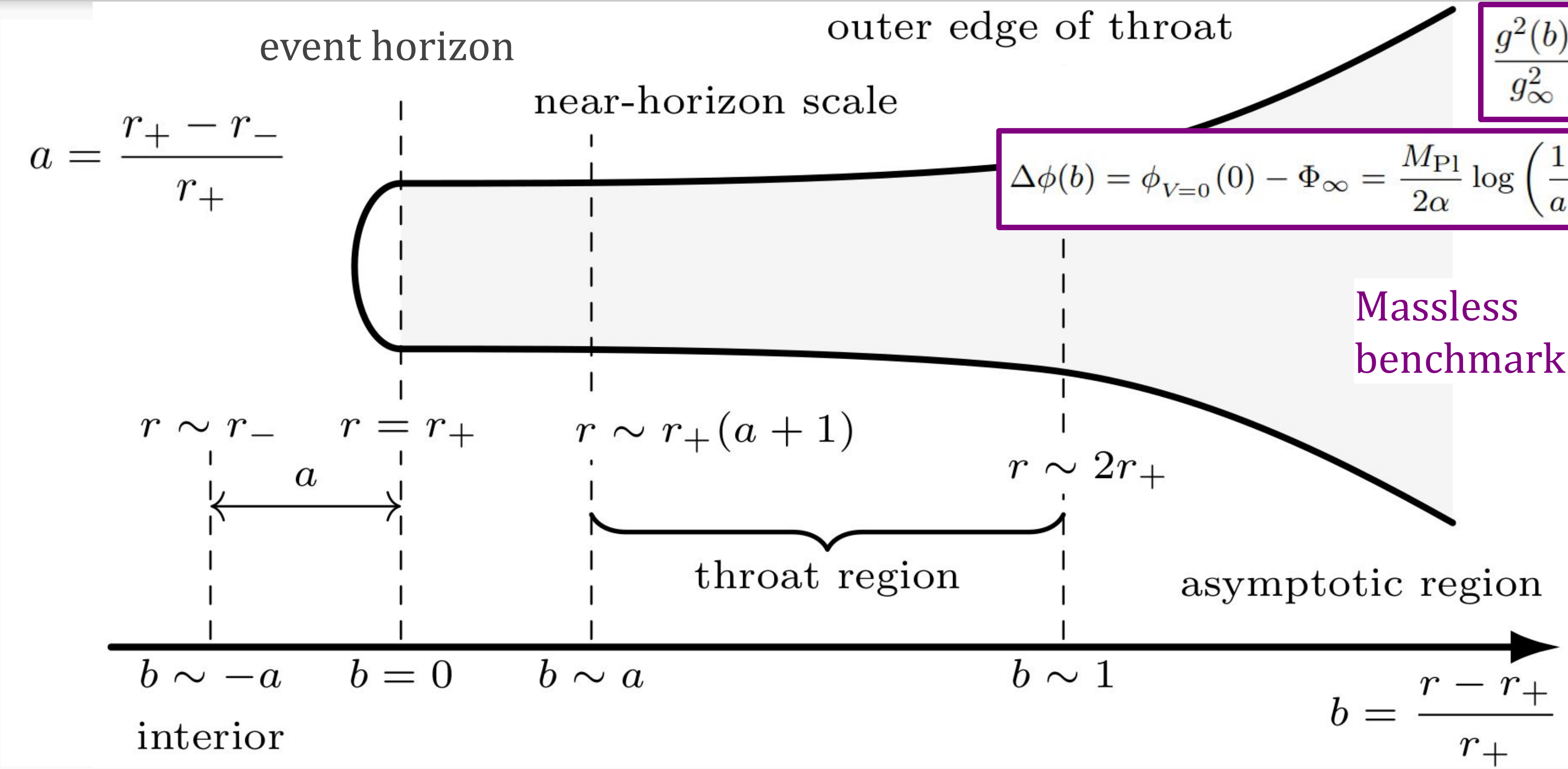


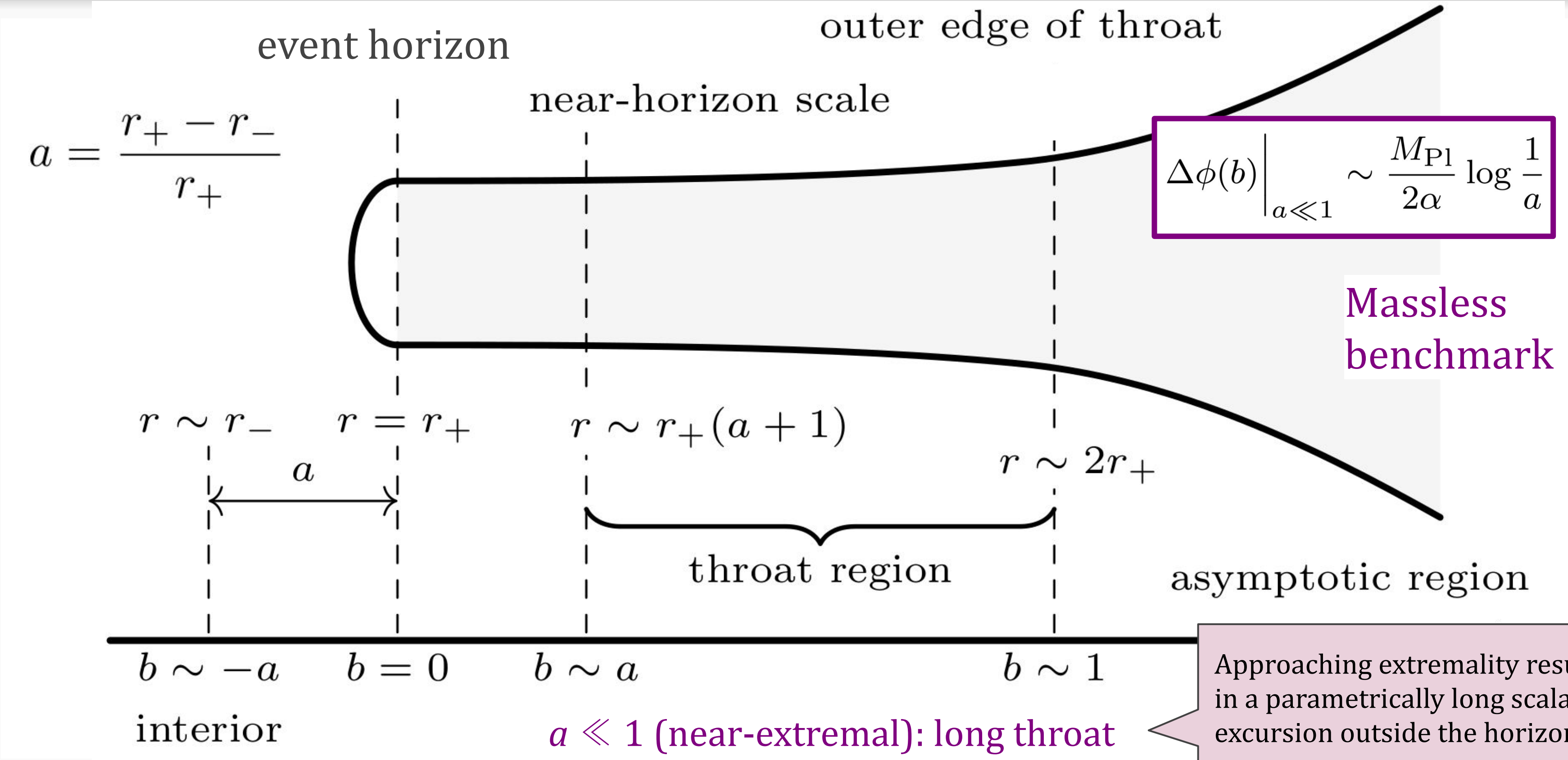


$$\frac{g^2(b)}{g^2_\infty} = \frac{1+b}{a+b}$$

$$\Delta\phi(b) = \phi_{V=0}(0) - \Phi_\infty = \frac{M_{\rm Pl}}{2\alpha}\log\left(\frac{1+b}{a+b}\right)$$

Massless benchmark

Can a black hole change the local effective theory?



$$\Delta\phi(b)\Big|_{a\ll 1} \sim \frac{M_{\rm Pl}}{2\alpha}\log\frac{1}{a}$$

Massless benchmark

$a \ll 1$ (near-extremal): long throat

Approaching extremality results in a parametrically long scalar excursion outside the horizon

Quantifying survival

radial operator takes the Sturm–Liouville form

$$\frac{1}{w(b)}\frac{d}{db}\underbrace{\boxed{\left[p(b)\frac{d\phi}{db}\right]}}_{\textbf{Flux}} = r_+^2\left[V_{,\phi}(\phi)+\frac{1}{4}B_{,\phi}(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

$$w(b) = (1+b)(a+b) \qquad p(b) = b(a+b)$$

For each case, we proceed in two stages:

1. With V=0 geometry & scalar profile fixed, analyse deformation via set diagnostics:
   pointwise; local; cumulative
2. Compare against corresponding back-reacted check

e.g. pointwise diagnostic

$$\eta_{\rm src}(b) \equiv \frac{r_+^2\, w(b)\, |V_{,\phi}(\phi_{\rm V\,=\,0}\ (b))|}{\left|\frac{d}{db}\left[p(b)\frac{d\phi_{\rm V\,=\,0}}{db}\right]\right|}$$

threshold

$$\eta_{\rm src}^{\max} \sim 1$$

Example: racetrack potential

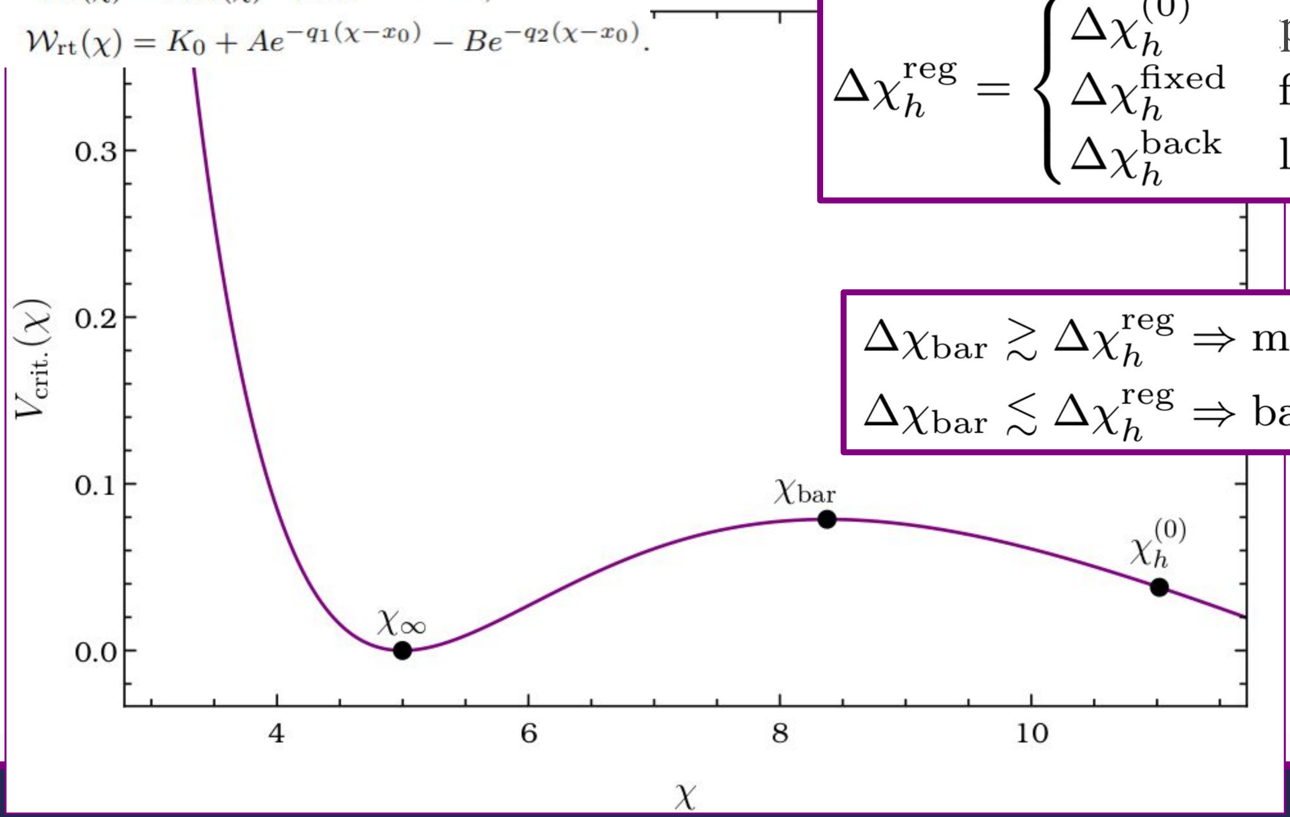
$$\Delta\chi_{\rm bar} = \chi_{\rm bar} - \chi_\infty$$

$$\Delta\chi_h = \chi_h - \chi_\infty$$

$$V_{\rm rt}(\chi;T) = T\,\widehat{V}_{\rm rt}(\chi),$$
$$\widehat{V}_{\rm rt}(\chi) = \mathcal{W}_{\rm rt}(\chi)^2 + U e^{-q(\chi - x_0)},$$
$$\mathcal{W}_{\rm rt}(\chi) = K_0 + A e^{-q_1(\chi - x_0)} - B e^{-q_2(\chi - x_0)}.$$

$$\Delta\chi_h^{\rm reg} = \begin{cases} \Delta\chi_h^{(0)} & \text{pure } V = 0 \text{ estimate} \\ \Delta\chi_h^{\rm fixed} & \text{fixed-throat solution} \\ \Delta\chi_h^{\rm back} & \text{local back-reacted solution} \end{cases}$$

$$\Delta\chi_{\rm bar} \gtrsim \Delta\chi_h^{\rm reg} \Rightarrow \text{metastable vacuum survives}$$
$$\Delta\chi_{\rm bar} \lesssim \Delta\chi_h^{\rm reg} \Rightarrow \text{barrier crossing/runaway}$$



Summary of results

Potential	Physics tested	Main result
Quadratic $(\mu = mr_+)$	Benchmark stabilisation	Mild throat deformation for $\mu \lesssim O(0.1-1)$; modulus pinned for $\mu \gtrsim O(1)$
Shifted exponential $(\mu = mr_+)$	Saturating restoring force	Same order-one crossover; $\mu \simeq 0.3$ gives a $\sim$10% gauge-coupling deformation
Exponential runaway $(q = \lambda/2\alpha)$	Monotonic force along throat	Favourable roll protects throat. For dangerous wall, local source enhanced for $q > 1$, cumulative bottom impulse enhanced for $q > 2$; global UV matching can fail
Inverse-power runaway (q, Λ)	Force weakening toward IR	No deep-throat instability: critical amplitude approaches $O(1)$ as $a \to 0$ Large amplitudes can still deform outer matching region
Racetrack (L)	Trapping behind finite barrier	Survival is essentially geometric: $L_{\mathrm{crit}} \simeq \Delta\chi_h^{(0)}/\Delta\chi_{\mathrm{bar}}$ Potential force & back-reaction give only per-mille corrections
Axion-like periodic $(A_a = m_a^2 f_a)$	Sign-changing, oscillatory force	$A_{a,\mathrm{crit}}^{\mathrm{throat}} \sim 10^{-1}$; for $f_a \ll M_{\mathrm{Pl}}$, oscillatory cancellation suppresses deformation

Summary & conclusions

Black hole perturbations

QNMs encode the geometry & stability of the remnant;
their excitation factors retain the amplitude & phase info associated with the photon-sphere potential barrier

Black hole interiors

In RNdS, the exterior decay rate competes with blueshift at the Cauchy horizon.
For sufficiently rapid decay, $\beta > \frac{1}{2}$, linear perturbations can remain extendible, threatening SCC

Black holes as probes of field space

Near-extremal magnetically-charged black holes generate long throats in which the gauge-field source can compete with moduli stabilisation, producing an O(1) scalar displacement while curvature & gradients remain parametrically controlled

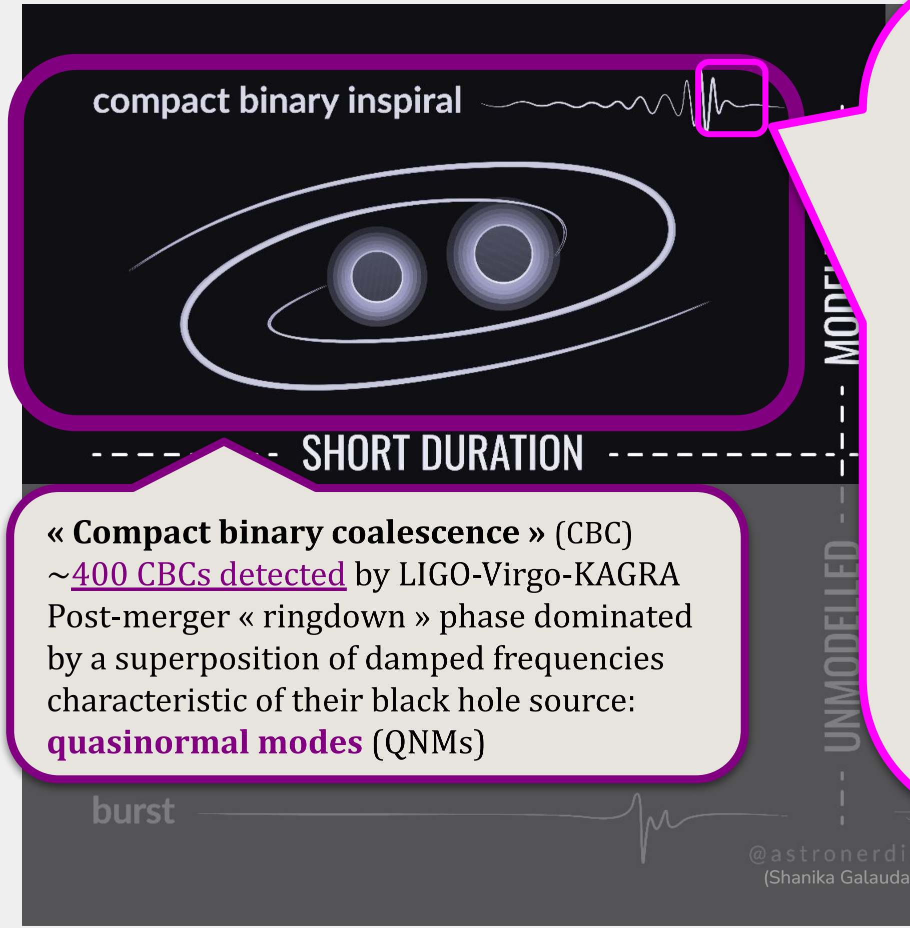
Exterior: QNMs & their excitation factors probe black hole geometry, stability, & scattering
Interior: QNM decay rates determine whether blueshift destabilises the Cauchy horizon & preserves SCC
Field space: near-extremal throats can support controlled, macroscopic excursions of stabilised moduli

Thank you for your time & attention!

Questions ? I'm around !

Or write to me at
chrysostomou@lpthe.jussieu.fr


Victor de Schwanberg / Science Photo Library



« Compact binary coalescence » (CBC)
~400 CBCs detected by LIGO-Virgo-KAGRA
Post-merger « ringdown » phase dominated by a superposition of damped frequencies characteristic of their black hole source: **quasinormal modes** (QNMs)

$$\Psi = \frac{1}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{\infty} A_{\ell mn}\, e^{-i\omega_{\ell mn} t}\, S_{\ell mn}(\theta,\omega)\, e^{im\phi}$$

$$\omega_{\ell mn} = \omega^{\mathrm{R}}_{\ell mn} + i\omega^{\mathrm{I}}_{\ell mn} \,,\quad \omega^{\mathrm{I}}_{\ell mn} < 0$$

(spherically-symmetric)
QNM eigenvalue problem

$$\frac{dr_*}{dr} = \frac{1}{f(r)}$$

$$\frac{d^2\psi(r_*)}{dr_*^2} + \left(\omega^2 - V\right)\psi(r_*) = 0$$

$$V = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}(1-s^2)\right] \qquad s = 0,1,2$$

QNM boundary conditions : purely ingoing/outgoing

$$\psi(r_*) \sim e^{-i\omega r_*} \,,\quad r_* \to -\infty$$

$$\psi(r_*) \sim e^{+i\omega r_*} \,,\quad r_* \to +\infty$$

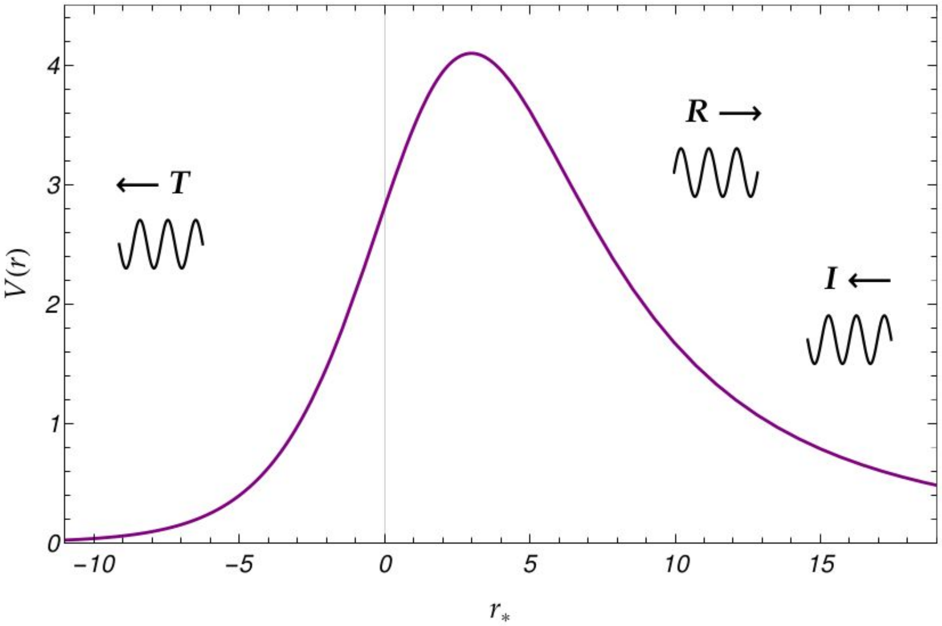
Reminiscent of a scattering problem…

symmetries: $\omega \to -\overline{\omega}$

$\Rightarrow \; \psi, \overline{\psi}$ solve the wave eq.

$$\psi \sim \begin{cases} \mathcal{T} e^{-ik_+ r_*} & r_* \to -\infty \\ \mathcal{I} e^{-ik_+ r_*} + \mathcal{R} e^{+ik_+ r_*} & r_* \to +\infty \end{cases}$$

$$W\big|_{r_* \to -\infty} = -2ik_+ |\mathcal{T}|^2$$
$$W\big|_{r_* \to +\infty} = 2ik_\infty \left[|\mathcal{R}|^2 - |\mathcal{I}|^2 \right]$$



DO method: details

Components of the ansatz

$$r_c = 3\ ,\ b_c = \sqrt{27} \quad \Rightarrow \quad \rho(r) = \left(1 - \frac{3}{r}\right)\sqrt{1 + \frac{6}{r}}$$

For a scalar field $s = 0$, $N = n + 1/2$, *and* $L = \ell + 1/2$,

$$\begin{aligned}\sqrt{27}\omega_{\ell n} \quad = \quad & L - iN + \left[\frac{1}{3} - \frac{5N^2}{36} - \frac{15}{432}\right]L^{-1} - iN\left[\frac{1}{9} + \frac{235N^2}{3888} - \frac{1415}{15552}\right]L^{-2} \\ & + \left[-\frac{1}{27} + \frac{204N^2 + 211}{3888} + \frac{854160N^4 - 1664760N^2 - 776939}{40310784}\right]L^{-3} \\ & + iN\left[\frac{1}{27} + \frac{1100N^2 - 2719}{46656} + \frac{11273136N^4 - 52753800N^2 + 66480535}{2902376448}\right] \\ + \ldots \end{aligned}$$

From black hole theory to ringdown observables

$$\Psi = \frac{1}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{\infty} A_{\ell mn}\, e^{-i\omega_{\ell mn} t}\, S_{\ell mn}(\theta, \omega)\, e^{im\phi}$$

High-SNR black hole ringdown detections ⇒ measure subdominant harmonics & overtones

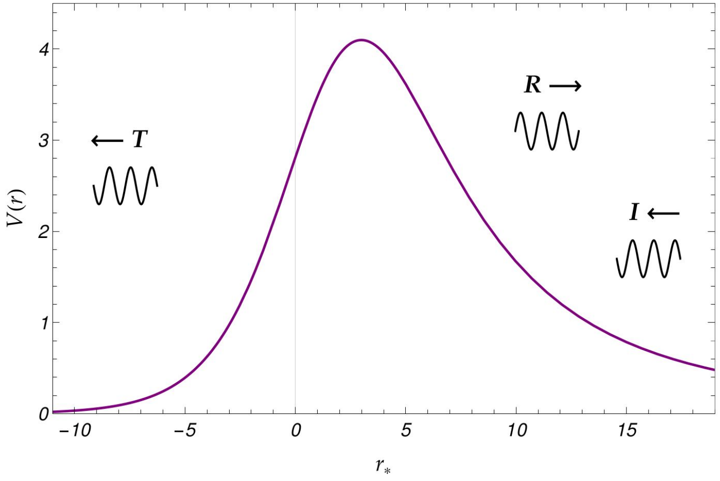
What this means for phenomenologists:

spacetime structure → ringdown spectrum → detector tests

excitation factors as detector-facing observable [direct]

greybody factors & Hawking radiation [indirect] → test semiclassical limits

$$\Gamma = \left| \frac{\mathcal{T}}{\mathcal{I}} \right|^2 \qquad \frac{d^2E}{dt\, d\omega} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{\hbar\omega}{2\pi} \frac{N_\ell\, \Gamma}{e^{\hbar\omega/k_B T} \mp 1} \quad \begin{matrix}\text{bosons}\\ \text{fermions}\end{matrix}$$



Beyond-GR compact objects:

e.g. black holes coupled to additional DoF

greybody factor boundary conditions

$$\psi \sim \begin{cases} \mathcal{T} e^{-i\omega r_\star} & r_* \to -\infty \\ \mathcal{I} e^{-i\omega r_\star} + \mathcal{R} e^{+i\omega r_\star} & r_* \to +\infty \end{cases}$$

QNM frequencies tell us what modes exist;
excitation factors tell us what detectors can measure. Both can be used to test GR.

Quasinormal excitation factor via semi-classical methods – spherically-symmetric case

GW strain is a function of the excitation coefficient

$$h_+ + ih_\times = \sum \mathcal{C}_{\ell n}\, Y_{\ell m}(\theta,\phi)\, \frac{\psi_{\ell n}}{r}\, e^{-i\omega_{\ell n} t}$$

$$\mathcal{C}_{\ell n} = \mathcal{B}_{\ell n} \times \mathcal{T}_{\ell n}$$

excitation coefficient
= QNEF × initial data

$$\boxed{\mathcal{B}_{\ell n} \equiv \left[\frac{A^+_{\ell\omega}}{2\omega}\left(\frac{\partial A^-_{\ell\omega}}{\partial\omega}\right)^{-1}\right]_{\omega=\omega_{\ell n}}}$$

QNM boundary conditions at the event horizon & spatial infinity

$$\psi_{r+} \sim \begin{cases} e^{-i\omega r_\star} & r_\star \to -\infty \\ \cancel{A^-_{\ell\omega} e^{-i\omega r_\star}} + A^+_{\ell\omega} e^{+i\omega r_\star} & r_\star \to +\infty \end{cases}$$

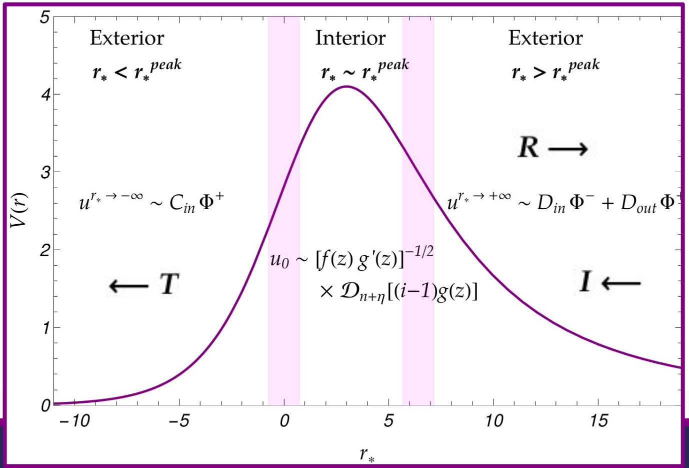
$$\psi_\infty \sim e^{+i\omega r_\star} \qquad r_\star \to +\infty$$

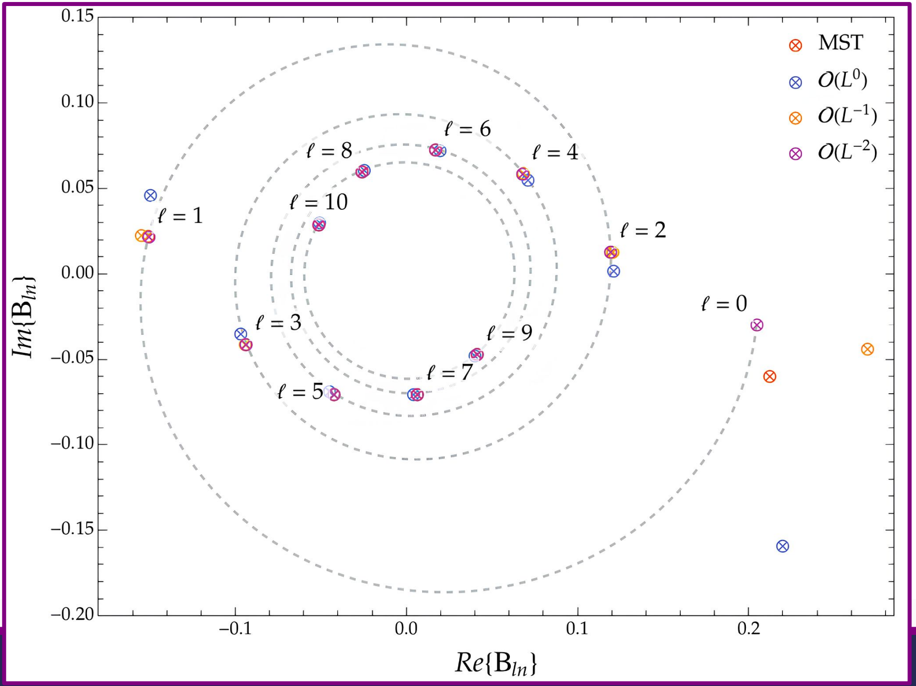
$$A^+_{\ell\omega} \propto \frac{D_{out}}{C_{in}}$$

$$A^-_{\ell\omega} \propto \left.\frac{D_{in}}{C_{in}}\right|_{\omega\to\tilde{\omega},\ \epsilon\to 0} = 0$$

$$\tilde{\omega} \approx \sum_{k=-1}^{2} \omega_k L^{-k} + \epsilon$$

exterior: Dolan-Ottewill
interior: WKB (DO breaks down at r~r$^{\text{peak}}$)





Higher harmonics and overtones

“the *fundamental* $(\ell, m, n) = (2, 2, 0)$ *mode* dominates ringdown"

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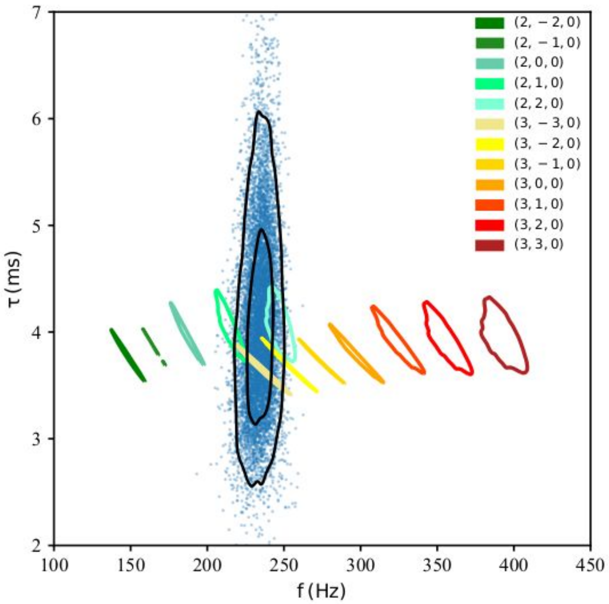
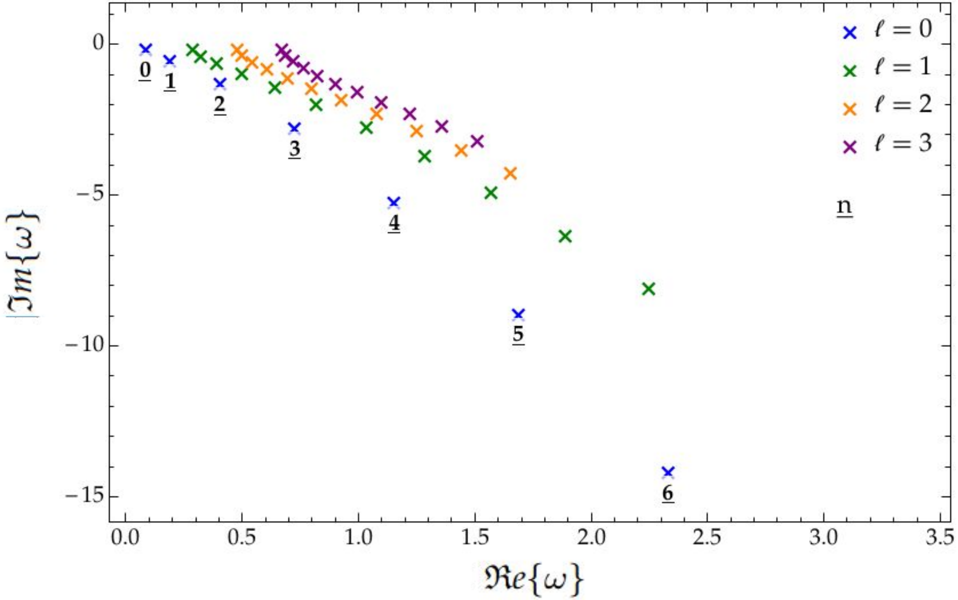


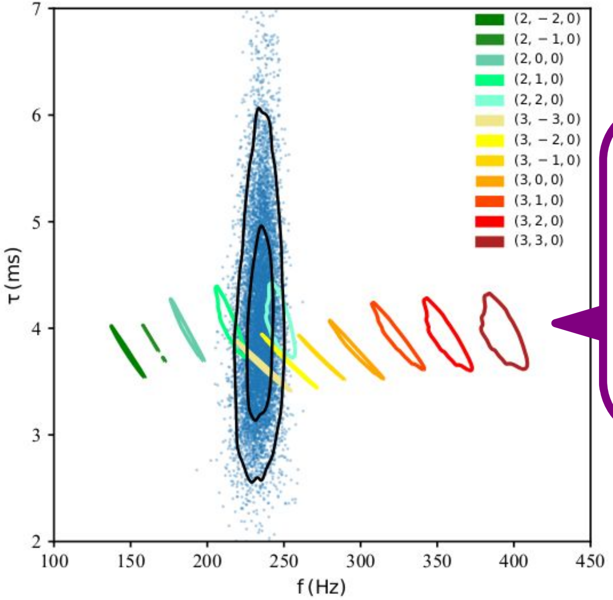
a multimodal analysis of the GW150914 data using PYRING, see Carullo et al.



first 10 overtones for s = 0 QNFs for increasing n

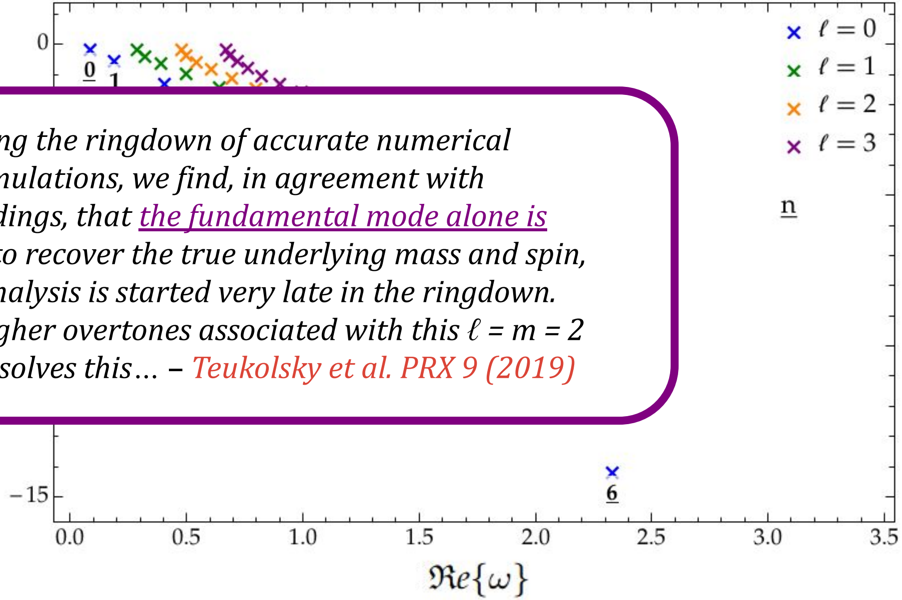
Higher harmonics and overtones

"the *fundamental* $(\ell, m, n) = (2, 2, 0)$ *mode* dominates ringdown"



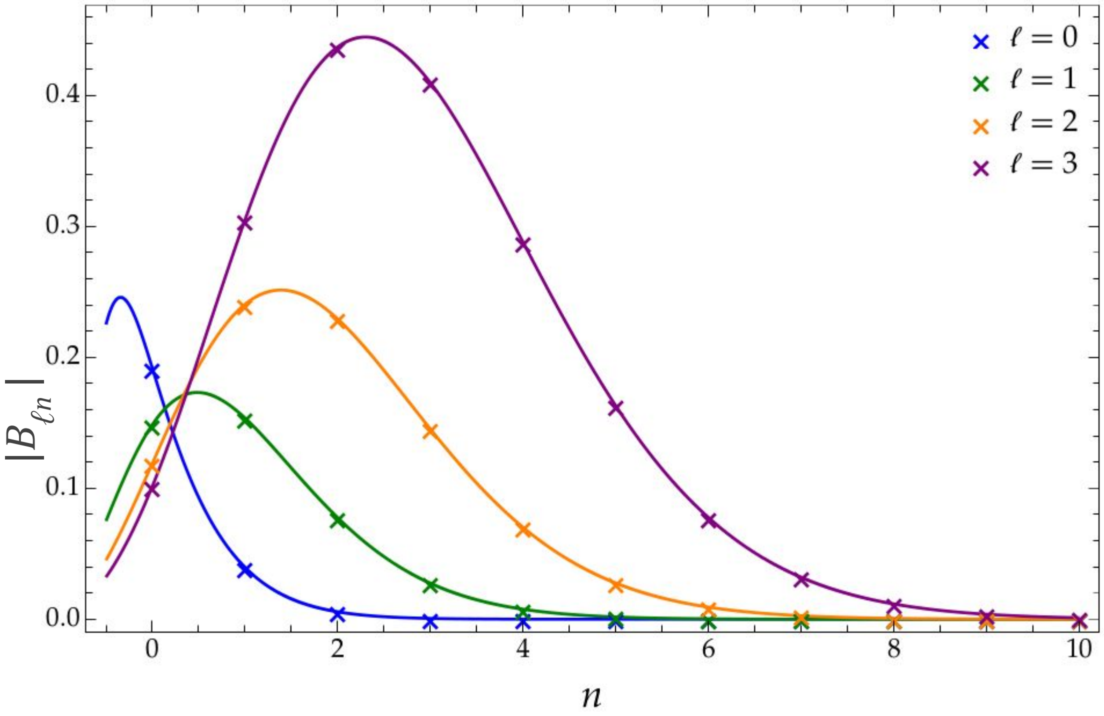
a multimodal analysis of the GW150914 data using pyRing, see Carullo et al.

…By modelling the ringdown of accurate numerical relativity simulations, we find, in agreement with previous findings, that the fundamental mode alone is insufficient to recover the true underlying mass and spin, unless the analysis is started very late in the ringdown. Including higher overtones associated with this $\ell = m = 2$ harmonic resolves this… – Teukolsky et al. PRX 9 (2019)



first 10 overtones for s = 0 QNFs for increasing n

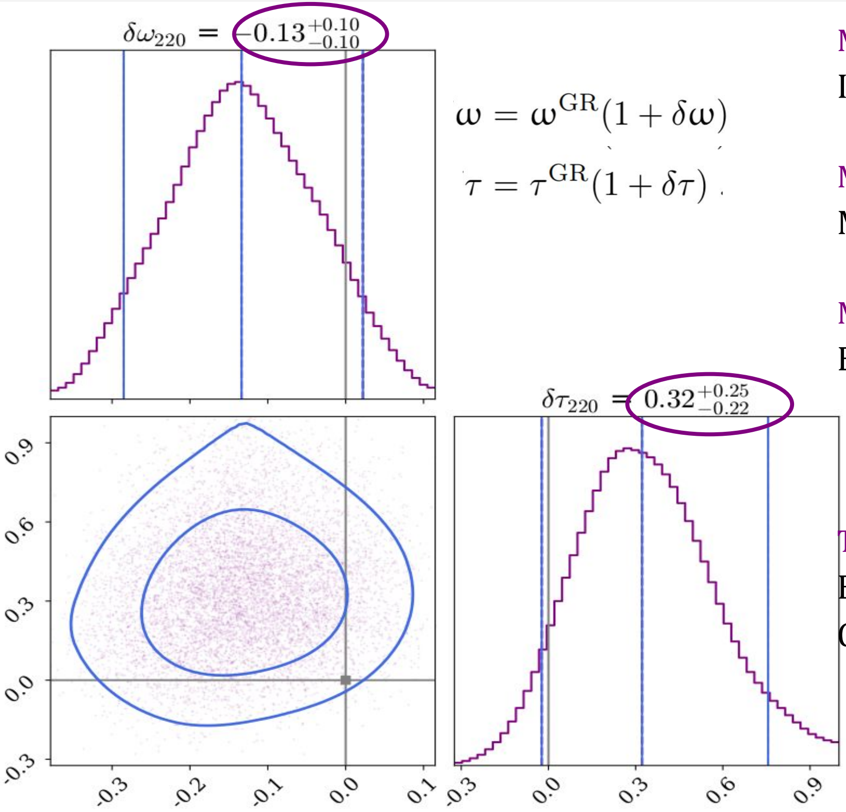
Schwarzschild QNEFs for various ℓ and increasing n



Results:

- ★ Larger excitation for higher harmonics/overtones, beyond "fundamental mode"
- ★ For $n = 0$, excellent agreement with analytical formalism developed by Mano, Suzuki and Takasugi at NNLO with rel. ease
- ★ As a "no-hair" quantity, QNEFs to be used as a new test for mod-GR?

Leveraging ringdown data in the search for new physics – e.g. compact extra dimensions



$$\omega = \omega^{\mathrm{GR}}(1 + \delta\omega)$$
$$\tau = \tau^{\mathrm{GR}}(1 + \delta\tau)\ .$$

Chrysostomou *et al. EPJC 83* (2023)

Motivation:

If gravity propagates beyond 4D $\longrightarrow$ GWs appear suppressed ("leak")

My questions:

Measurable imprints in ringdown? Is LVK sensitive?

Model:

Black hole embedded in 4D × compact negatively curved space*

$\longrightarrow$ numerics: $V_{\mathrm{eff}}(\mu)$, shifted spectra

$$\omega = \omega^{\mu=0}\,(1 + \delta\omega) \qquad \tau = \tau^{\mu=0}\,(1 + \delta\tau)$$

Test GR:

Bayesian ringdown analysis [*PyRing*] on public *GW150914* LVK data:

GR = null hypothesis vs GR + parametric deviations

* D. Andriot, A. Cornell, A. Deandrea, F. Dogliotti, D. Tsimpis, *JHEP* 05 (2020)

Quasinormal mode eigenvalue problem: charged, massive scalar on RNdS

Black hole wave equation:

$$\boxed{\frac{d^2}{dr_*^2}\varphi + \left[\left(\omega - \frac{qQ}{r}\right)^2 - V(r)\right]\varphi = 0\,, \qquad \frac{dr}{dr_*} = f(r)}$$

$$e.g.\;\; V_{s=0} = f(r)\left(\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r} + \mu^2\right)$$

$$\mu = \frac{m_s}{\hbar},\; q \propto e \;\; ([\mu] = L^{-1},\; [q] = 1)$$

Subjected to **QNM boundary conditions**:

purely ingoing: $\varphi(r) \sim e^{-i\left(\omega - \frac{qQ}{r_+}\right)r} \qquad r \to r_+$

purely outgoing: $\varphi(r) \sim e^{+i\left(\omega - \frac{qQ}{r_c}\right)r} \qquad r \to r_c$

Waves escape domain of study at the boundaries $\Rightarrow$ dissipative

Semi-classical analysis of the RNdS QNFs

For $L = \sqrt{\ell(\ell+1)}$, as a series expansion $\omega = \sum_{k=-1} \omega_k L^{-k}$

$$\omega_k = \sqrt{V(r_*^{max}) - 2iU}\,, \quad U \equiv U(V^{(2)}, V^{(3)}, V^{(4)}, V^{(5)}, V^{(6)})$$

$$V^j = \frac{d^j V(r_*^{max})}{dr^j} = f(r)\frac{d}{dr}\left[f(r)\frac{d}{dr}\left[\ldots\left[f(r)\frac{dV(r)}{dr}\right]\ldots\right]\right]_{r\to r_*^{max}}\,.$$

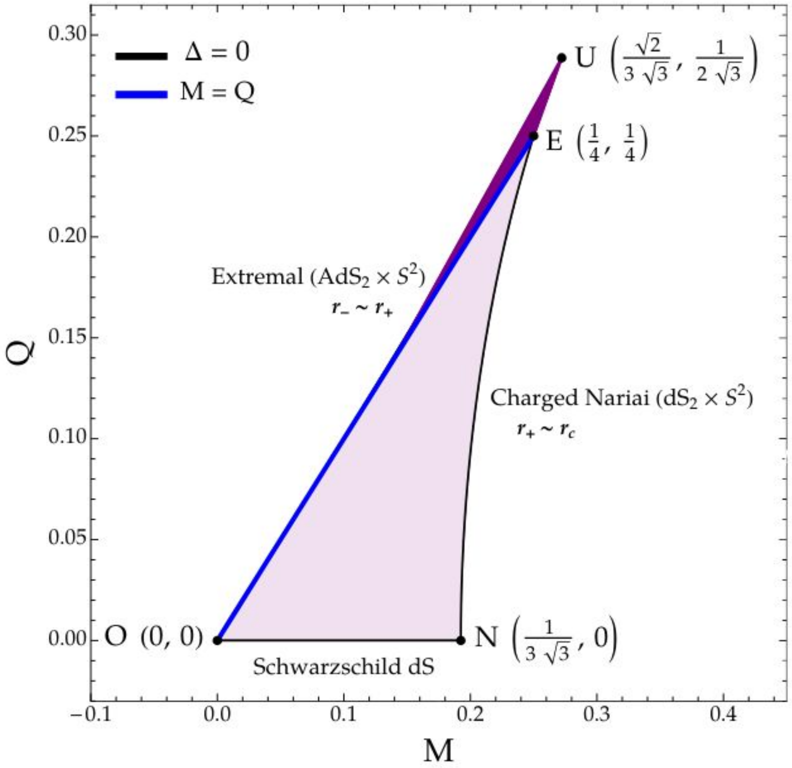
$$\begin{aligned} r_*^{max} &\approx r_0 + r_1 L^{-2} + \ldots\,, \\ V(r_*^{max}) &\approx V_0 + V_1 L^{-2} + \ldots \end{aligned}$$

$q \neq 0 \Rightarrow$ two sets of solutions:

ω_+ (black-hole family) & ω_c (cosmological-horizon family)

For a fixed q, $|\mathfrak{Re}\{\omega_+\}| < |\mathfrak{Re}\{\omega_c\}|$ & $|\mathfrak{Im}\{\omega_+\}| < |\mathfrak{Im}\{\omega_c\}|$

Reissner-Nordström de Sitter black hole



Schwarzschild modes (I) & de Sitter modes (II):

(I.i) photon-sphere modes: beneath line $M = Q$, approaching NU large $\mathfrak{Im}\{\omega\}, \mathfrak{Re}\{\omega\} \sim \mathcal{O}(0.01)$ for $Q < 0.1$

$$\mathfrak{Im}\{\omega_{PS}\} \approx -i\left(n + \frac{1}{2}\right)\kappa_+ \quad \text{on } NU$$

(I.ii) near-extremal modes: branch OU $(r_- \sim r_+)$

$$\omega_{NE} \approx -i(\ell + n + 1)\kappa_- = -i(\ell + n + 1)\kappa_+ \ ;$$

(II) dS modes: along branch OU $(\kappa_c \sim 1/L_{dS})$

$$\omega_{dS_{n=0}} \approx -i\ell\kappa_c \ , \quad \omega_{dS_{n\neq 0}} \approx -i(\ell + n + 1)\kappa_c \ ;$$

$$\frac{Q^2}{M^2} \lesssim 1 + \frac{1}{3}(M^2\Lambda) + \ldots$$

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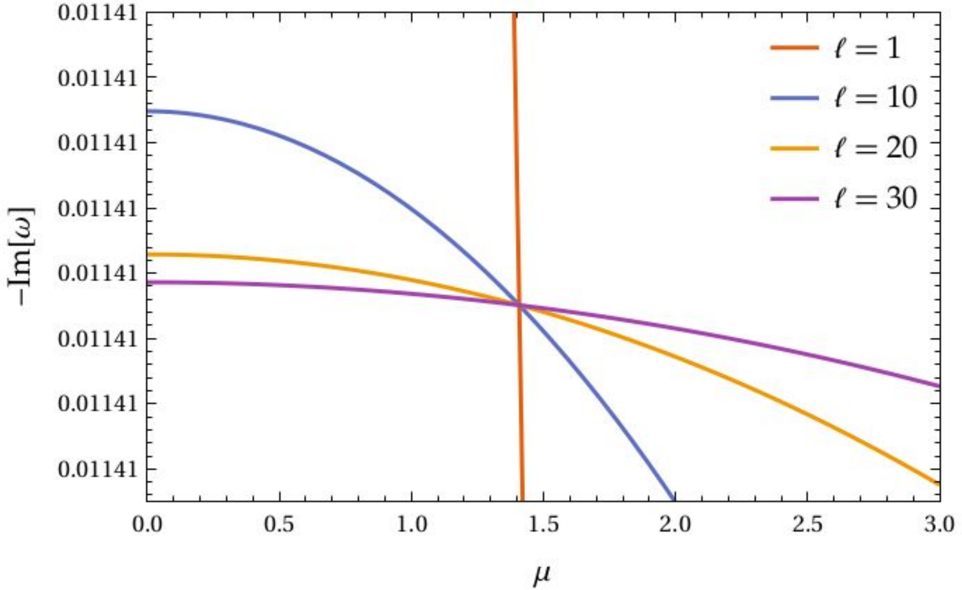
Anomalous decay rates for massive, charged scalar fields in charged de Sitter black holes

e.g. for the Nariai case $(M, Q) = \left(\frac{1}{\sqrt{27}}, 0\right)$,



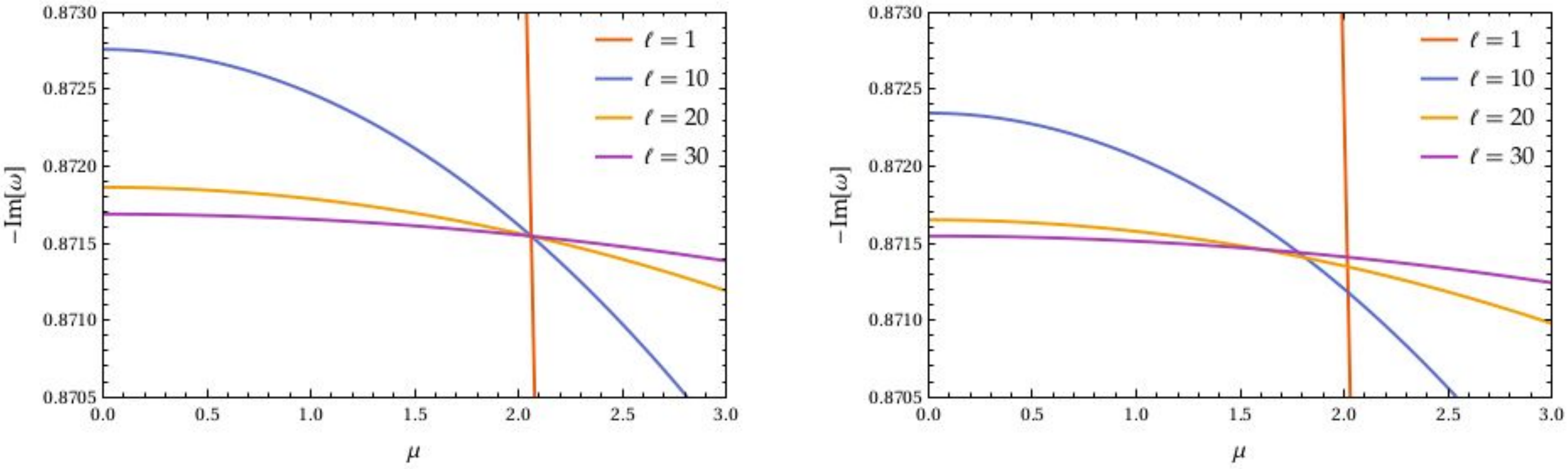
anomalous decay precedes "critical mass" – intersection
higher multipoles decaying slower than lower multipoles

Recall: $\mu \uparrow \Rightarrow |\mathfrak{Im}\{\omega\}| \downarrow$ & $\ell \uparrow \Rightarrow |\mathfrak{Im}\{\omega\}| \uparrow$

$\hookrightarrow$ i.e. *only* for $\mu > \mu_{crit}$ is $|\mathfrak{Im}\{\omega_{\ell=10}\}| < |\mathfrak{Im}\{\omega_{\ell=30}\}|$

Anomalous decay rates for massive, charged scalar fields in charged de Sitter black holes

e.g. for $M = Q = 0.104$,



Increasing the charge from $q = 0$ *to* $q = 0.1$ *displaces the intersection*
$\hookrightarrow$ μ_{crit} *can only be found in the* $\mu M \gg qQ$ *regime.*

Strong Cosmic Censorship (SCC) in RNdS

For our purposes: SCC requires that the metric *($\mathscr{M}$, g)*, as **a well-defined solution** to the vacuum Einstein field equations, cannot be extended past the Cauchy horizon ($\mathscr{CH}$)

Theorem A.5 (Maximal future Cauchy development). *Let Σ be a three-dimensional C^∞ manifold, let h_{ab} be a smooth Riemannian metric on Σ, and let K_{ab} be a smooth symmetric tensor field on Σ. Suppose that the initial data (Σ, h_{ab}, K_{ab}) satisfies the vacuum constraint equations,*

$$0 = D_b K^b{}_a - D_a K^b{}_b \,, \tag{A.6}$$

$$0 = \frac{1}{2}\left({}^{(3)}R + (K^a{}_a)^2 - K_{ab}K^{ab}\right) \,, \tag{A.7}$$

where D_a is the derivative operator associated with the metric h_{ab}. Then there exists a unique C^∞ space-time $(\mathcal{M}, g_{ab})$ known as the maximal future Cauchy development of (Σ, h_{ab}, K_{ab}) that satisfies the following properties:

(i) $(\mathcal{M}, g_{ab})$ is a solution to the Einstein field equations.
(ii) $(\mathcal{M}, g_{ab})$ is globally hyperbolic[10] with a Cauchy surface[11] Σ.
(iii) h_{ab} and K_{ab} are the induced metric and extrinsic curvature, respectively, of Σ.
(iv) A space-time $(\mathcal{M}', g'_{ab})$ satisfying properties (i) – (iii) can be mapped isometrically into a subset of $(\mathcal{M}, g_{ab})$. To elaborate, suppose (Σ, h_{ab}, K_{ab}) and $(\Sigma', h'_{ab}, K'_{ab})$ are initial data sets with maximal developments $(\mathcal{M}, g_{ab})$ and $(\mathcal{M}', g'_{ab})$, respectively. Suppose also that there is a diffeomorphism between $S \subset \Sigma$ and $S' \subset \Sigma'$ that carries (h_{ab}, K_{ab}) on S into (h'_{ab}, K'_{ab}) on S'. Then $D(S)$ in $(\mathcal{M}, g_{ab})$ is isometric to $D(S')$ in $(\mathcal{M}', g'_{ab})$. Finally, the solution g_{ab} on $\mathcal{M}$ depends continuously on the initial data (h_{ab}, K_{ab}) on Σ.

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‖A set $\mathcal{N}$ is said to be globally hyperbolic if for any two points $p, q \in \mathcal{N}$, $J^+(p) \cap J^-(q)$ is compact and contained in $\mathcal{N}$. In other words, $J^+(p) \cap J^-(q)$ does not contain any points on the edge of space-time i.e. at spatial infinity or at a singularity [25].

**A Cauchy surface Σ in $\mathcal{M}, g_{ab})$ is a surface that is intersected exactly once by every inextendible null and time-like curve in $(\mathcal{M}, g_{ab})$ [3].

Definition A.3 (Weak partial derivative) *Let us suppose that u and v are locally summable on the Lebesgue space L^1 (i.e. $u, v \in L^1_{loc}(U)$). Suppose also that α is a multi-index such that $\alpha = (\alpha_1, ..., \alpha_n)$ and $|\alpha| = \alpha_1 + ... + \alpha_n$. Then we can say that v is the α^{th}-weak partial derivative of u,*

$$D^\alpha u = v \ , \tag{A.2}$$

provided that for all test functions $\varphi \in C_c^\infty(U)$,

$$\int_U dx\ u D^\alpha \varphi = (-1)^{|\alpha|} \int_U dx\ v\varphi \ . \tag{A.3}$$

The "strong" solution to a (partial) differential equation is expected to be smooth, and to hold point-wise for every point in U. The "weak" solution, on the other hand, satisfies the equation without meeting the same strict criteria (e.g. the solution can only be integrable rather than continuously differentiable). In this way, we generalise or broaden the class of functions that can serve as solutions to the equation.

It becomes useful to define a space in which these weak (partial) derivatives are well-defined. This is the "Sobolev" space [57]:

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Definition A.4 (Sobolev space) *Let us consider the open set $U \subset \mathbb{R}^n$, the Sobolev space $W^{k,p}(U)$ then consists of all locally summable functions $u\colon U \to \mathbb{R}$ such that for each multi-index $|\alpha| \leq k$, the weak derivative $D^\alpha u$ exists and belongs to the space $L^p(U)$ for a fixed $1 \leq p \leq \infty$ and $k \in \mathbb{Z}^+$.*

Upon introducing the norm to $W^{k,p}(U)$,

$$||u||_{W^{k,p}(U)} \equiv \sum_{|\alpha|\leq k} ||D^\alpha u||_{L^p(U)} \,, \tag{A.4}$$

and $W^{k,p}(U)$ becomes complete and Eq. (A.4) represents a Banach space. If we impose $p = 2$, $k \in \mathbb{N}_0$, and $U = \mathbb{R}^n$,

$$H^k(U) = W^{k,2}(U) \,; \tag{A.5}$$

$H^k(U)$ is then a Hilbert space as well as a Banach space, and $H^0(U) = L^2(U)$. Throughout this note, we discuss functions belonging to the space $H^1_{loc}(U)$. This implies that both the function and its first weak derivatives are square-integrable (i.e. they are in $L^2(U)$). The subscript *loc* conveys that the function need not be globally regular, but retains local regularity properties.

The Sobolev space is ideal for defining (less regular) functions and their weak

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Strong Cosmic Censorship (SCC) in RNdS

For a scalar field, one finds near the Cauchy horizon

$$\partial_V \Phi \sim |V|^{\beta-1},$$

where V is a Kruskal-like coordinate that vanishes on the Cauchy horizon. The field belongs to the Sobolev space $H^1_{\rm loc}$ (i.e. has finite energy) provided

- $\int |\partial_V \Phi|^2 \, dV < \infty$.

Substituting the above behavior gives

$$\int |V|^{2\beta-2} \, dV.$$

This integral converges only if

$$2\beta - 2 > -1,$$

which implies

$$\beta > \frac{1}{2}.$$

Therefore:

- If $\beta < 1/2$: the energy diverges at the Cauchy horizon, and SCC is respected.
- If $\beta > 1/2$: the energy remains finite, and weak solutions may be extended beyond the Cauchy horizon.

So the $1/2$ is **not** the blueshift factor itself. The blueshift is governed by κ_-. The $1/2$ arises when you ask whether the blueshifted perturbation remains square integrable, i.e. whether it lies in $H^1_{\rm loc}$. It is fundamentally a consequence of the convergence condition

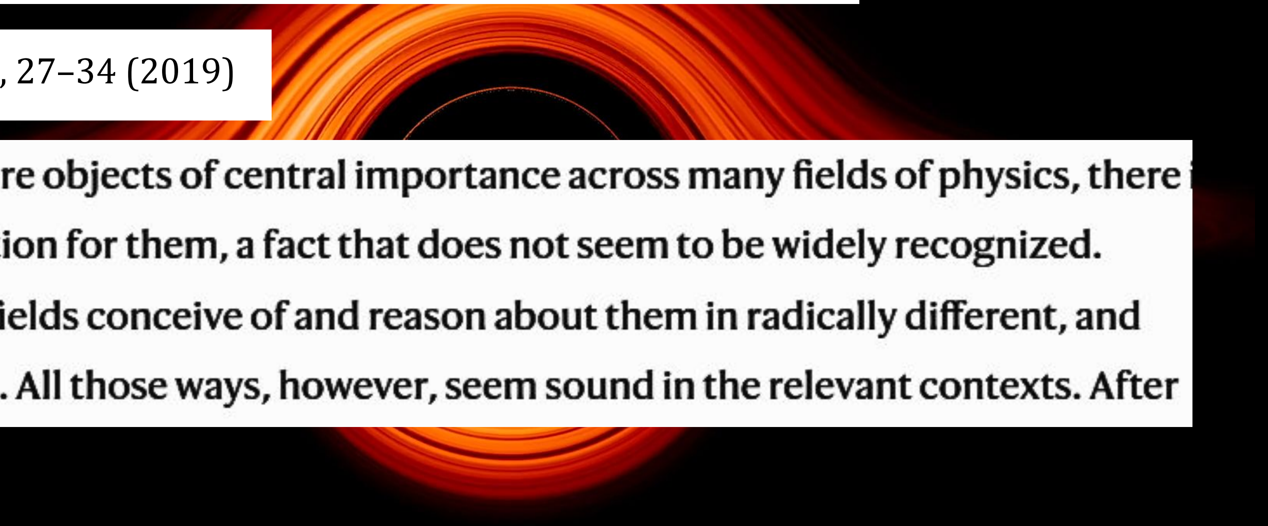
Credit: NASA's Goddard Space Flight Center/J. Schnittman & B. Powell

nature astronomy

Perspective | Published: 08 January 2019

The many definitions of a black hole

Erik Curiel. *Nat Astron* **3**, 27–34 (2019)



Although black holes are objects of central importance across many fields of physics, there i
no agreed upon definition for them, a fact that does not seem to be widely recognized.
Physicists in different fields conceive of and reason about them in radically different, and
often conflicting, ways. All those ways, however, seem sound in the relevant contexts. After