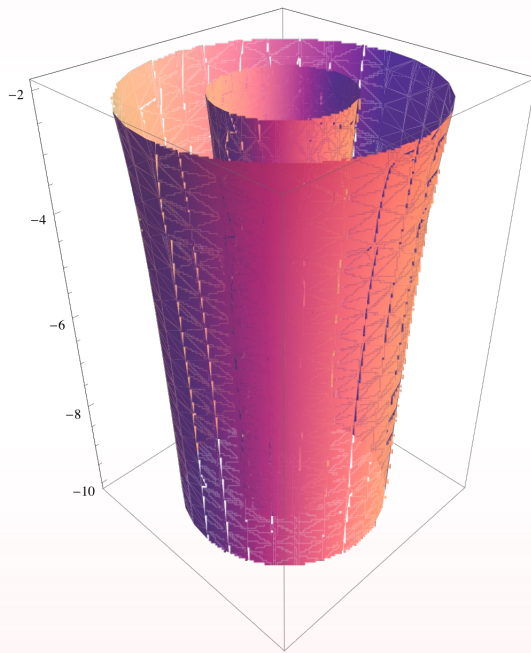


PRE PRIMORDIAL BLACK HOLES



RUTH GREGORY



Mainz PBH, 23 July 2026

*RG, KASTOR, TRASCHEN: JHEP 10 (2017) 118 & CQG35 (2018) 155008
CRONEY, RG, PATRICK: JCAP 01 (2025) 096; COVIELLO, RG: 2512.01898*

BLACK HOLES & SCALARS

Scalar fields are ubiquitous in cosmology as a mechanism for inflation, dark energy, or even dark matter. And in the early universe, gravity could have scalar components.

Λ enters in extended black hole thermodynamics, dark energy provides a concrete way to vary Λ , so how does this affect the black hole, and how far can we get analytically?

Understanding how scalars interact with black holes from first principles allows a proper understanding of what can or cannot happen.

STRUCTURE

➤ Slow roll with a black hole

- Describing the scalar roll
- Backreaction

➤ Black holes & Ultra Slow Roll

- The role of Quasi-Normal Modes

➤ Creating a black hole

- Hawking-Moss, Hawking-Bousso & singular instantons

SLOW ROLL DONE FAST

In slow roll inflation, Λ varies gradually, and our universe is quasi-de Sitter.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2} \left[\cancel{\frac{\dot{\varphi}^2}{2}} + V(\varphi) \right]$$

$$\cancel{\ddot{\varphi}} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi}$$

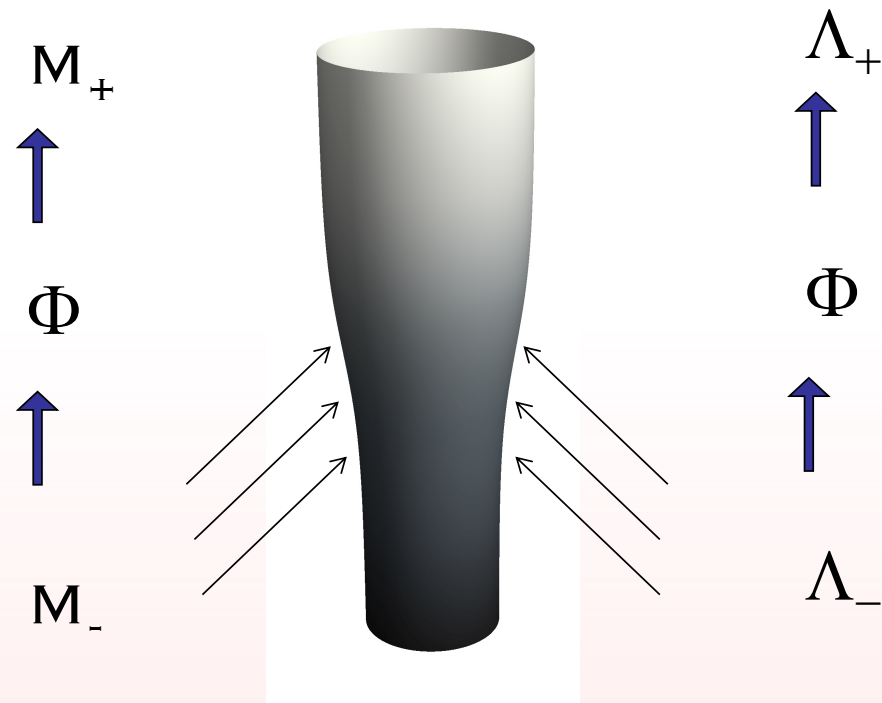
Small slow-roll parameters ensure that inflation is maintained:

$$\varepsilon = \frac{M_p^2}{2} \frac{V'^2}{V^2} \quad , \quad \eta = M_p^2 \frac{V''}{V}$$

Want to derive a similar picture with a black hole.

SCALARS AROUND BLACK HOLES

Originally, no hair theorems seemed to rule out nontrivial scalars around a black hole – but these had restrictive assumptions.



One of the clearest ways of evading “no-hair” is to have time-dependence – e.g. a rolling scalar.

The black hole should absorb scalar energy and grow, while the cosmological constant drops.

BLACK HOLE & LAMBDA

A black hole with cosmological constant is given by the Schwarzschild de Sitter solution:

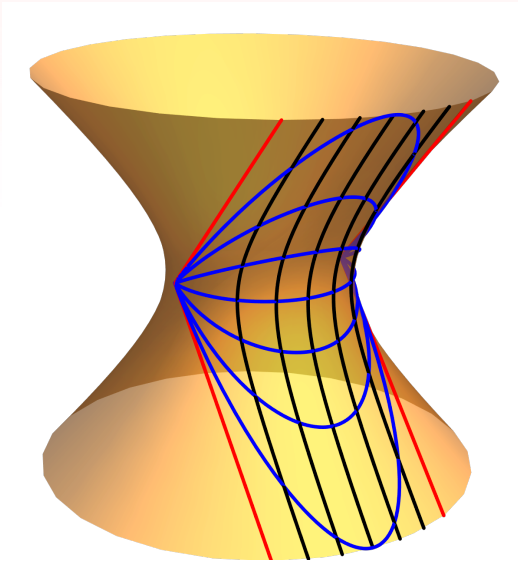
$$ds^2 = f dt^2 - \frac{dr^2}{f} - r^2 d\Omega_{II}^2$$

Where

$$f = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

But this is not explicitly time dependent – so first challenge is to understand how “slow roll” proceeds

STATIC PATCHES:



Without the black hole, have a “static patch”
de Sitter potential:

$$f = 1 - H^2 r^2$$

– not the familiar (flat) cosmological
coordinates

$$d\tau^2 = e^{2H\tau} d\mathbf{x}^2$$

The transformation to cosmological time is nontrivial:

$$\tau_{cos} = t_s + \frac{1}{2H} \log(1 - H^2 r_s^2) \quad \rho_{cos} = \frac{r_s e^{-Ht_s}}{\sqrt{1 - H^2 r_s^2}}$$

And with black hole, even this relative simplicity is lost.

BLACK HOLE-SLOW ROLL APPROXIMATION

With a black hole, intuition is that the geometry is approximately SDS, the scalar still slow-rolls, but that this produces a sub-leading effect on the background black hole geometry. The spacetime slides from one Λ to a lower one, and the black hole accretes a little mass.

$$\begin{array}{c} \phi = \phi_0 + \delta\phi_{SR} \\ g_{\mu\nu} = g_{0\mu\nu} + \delta g_{SR\mu\nu} \\ \begin{array}{cc} \uparrow & \uparrow \\ \text{SDS} & \mathcal{O}(\delta\phi_{SR})^2 \end{array} \end{array}$$

SCALAR FIELD EQN

Idea is to turn e.o.m for Φ

$$\frac{\varphi_{,tt}}{f} - \frac{1}{r^2} (r^2 f \varphi_{,r})_{,r} = -\frac{\partial V}{\partial \varphi}$$

into something like a slow roll equation by taking $\phi = \phi(T)$, then writing

$$T = t + \xi(r)$$

T is constructed so that ϕ is regular at both horizons.

Substitute in: $\varphi = \varphi(t + \xi(r))$

$$\frac{1}{r^2} (r^2 f \xi')' \dot{\varphi} - \frac{\ddot{\varphi}}{f} (1 - \cancel{f^2 \xi'^2}) = \frac{\partial V}{\partial \varphi}$$

Dropping second term, and remember $\phi = \phi(T)$, we must have

$$\frac{1}{r^2} (r^2 f \xi')' = -3\gamma$$

γ constant, and hence

$$\xi' = \frac{1}{f} \left(-\gamma r + \frac{\beta}{r^2} \right)$$

Find γ and β by regularity: $\gamma = \frac{r_c^2 + r_b^2}{r_c^3 - r_b^3}$, $\beta = \frac{r_c^2 r_b^2 (r_c + r_b)}{r_c^3 - r_b^3}$

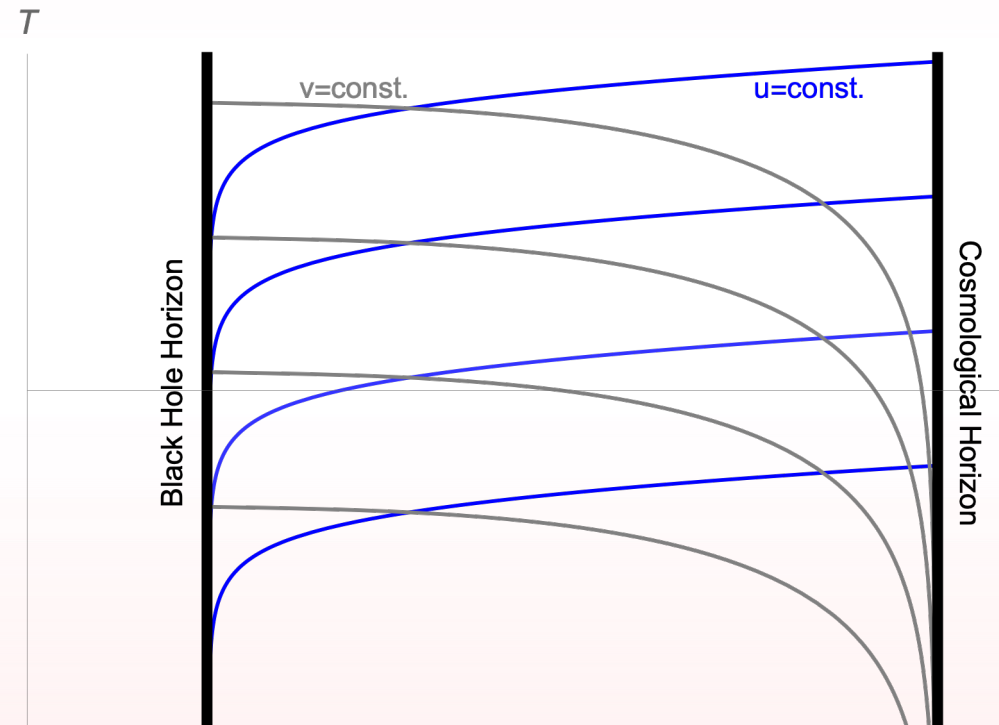
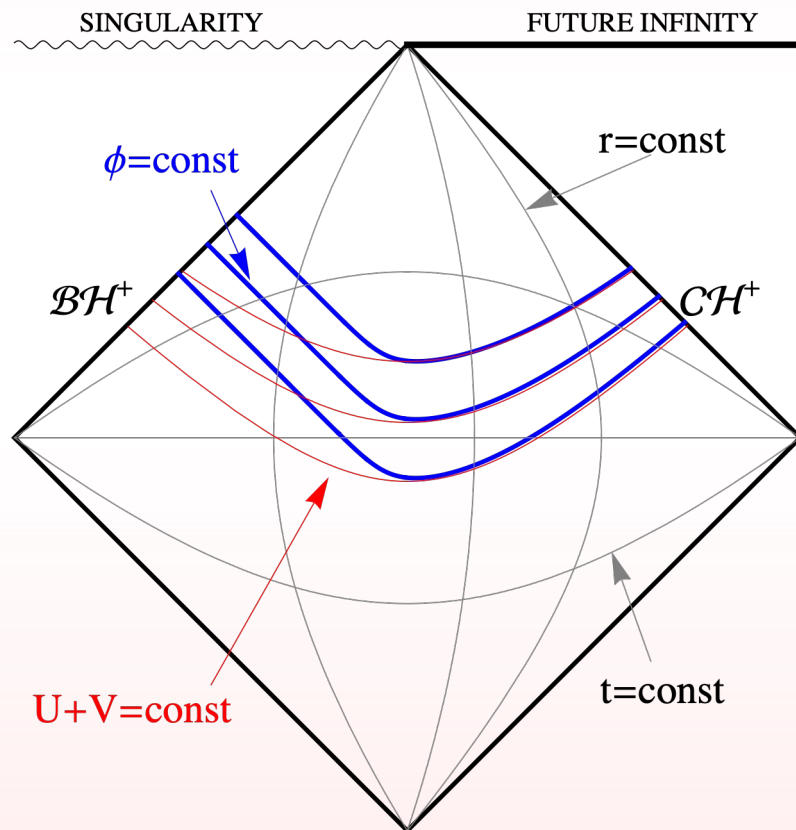
Φ must be ingoing on event horizon and outgoing on cosmological horizon.
Gives T in terms of logs

$$T = t - \frac{1}{2\kappa_c} \log \left| \frac{r - r_c}{r_c} \right| + \frac{1}{2\kappa_b} \log \left| \frac{r - r_b}{r_b} \right| \\ + \frac{r_b r_c}{r_c - r_b} \log \frac{r}{r_0} + \left(\frac{r_c}{4\kappa_b r_b} - \frac{r_b}{4\kappa_c r_c} \right) \log \left| \frac{r - r_n}{r_n} \right|$$

T looks like Kruskal V at the black hole horizon (r_b) and Kruskal U at the cosmological horizon (r_c)

THE T COORDINATE

T looks like Eddington-Finkelstein on each horizon, at r_h a fn of v , and at r_c a fn of U .



QUICK CHECK

Let the black hole mass tend to zero: $\gamma = \frac{r_c^2 + r_b^2}{r_c^3 - r_b^3} \rightarrow \frac{1}{r_c} = H$

$$\begin{aligned} T &\rightarrow t - \frac{1}{2\kappa_c} \log \left| \frac{r - r_c}{r_c} \right| + \frac{r_c}{2} \log \left| \frac{r - r_n}{r_n} \right| \\ &= t + \frac{1}{2} \log(1 - H^2 r^2) = \tau_{cos} \end{aligned}$$

T becomes cosmological time as required.

Φ EQUATION

The Φ equation is now a standard slow-roll type $3\gamma\dot{\varphi}(T) = -\frac{\partial V}{\partial \varphi}$

but with friction parameter modified from H: $\gamma = \frac{r_c^2 + r_h^2}{r_c^3 - r_h^3} = \frac{A_{TOT}}{3V}$

Physical effect of black hole is (apparently) to add friction to roll, or to slow down the scalar, however extending across the cosmological horizon shows that this is a local patch effect: T is proportional to cosmological time – the black hole “drags” the scalar.

$$\tau_{\text{cos}} = \frac{HT}{\gamma}$$

BACK-REACTION

We looked at back-reaction using the (T,r) coords:

$$ds^2 = f(r, T) dT^2 - 2h(r, T) dT dr - \frac{dr^2}{f} (1 - h^2) - r^2 d\Omega^2$$

The energy momentum of the scalar has 2 independent cpts:

$$T_{TT} = \left(W(\phi) + \frac{1 + h^2}{2f} \dot{\phi}^2 \right) |g_{TT}| ,$$

$$T_{ab} = \left(-W(\phi) + \frac{1 - h^2}{2f} \dot{\phi}^2 \right) g_{ab}$$

$$h = f\xi'$$

Which relate to the Einstein tensor:

$$G_{TT} = \left[\frac{1}{r^2} (1 - f - r f') - \frac{h \dot{f}}{r f} \right] |g_{TT}|$$

$$G_{rr} = \left[-\frac{1}{r^2} (1 - f - r f') + \frac{h \dot{f}}{r f} + \frac{2 \dot{h}}{r (1 - h^2)} \right] g_{rr}$$

$$G_{rT} = \left[-\frac{1}{r^2} (1 - f - r f') - \frac{(1 - h^2) \dot{f}}{r h f} \right] g_{rT}$$

$$G_{\theta\theta} = \left[\frac{f''}{2} + \frac{f'}{r} - \frac{h' \dot{f}}{2f} + \frac{\dot{h} f'}{2f} + \frac{\dot{h}}{r} + \dot{h}' + \frac{1}{2} \left(\frac{(h^2 - 1)}{f} \right)'' \right] g_{\theta\theta} = \frac{G_{\phi\phi}}{\sin^2 \theta}$$

SLOW ROLL WITH A BLACK HOLE

Need to have control of the slow-roll approximation to identify the key dependences in these equations:

➤ Scalar:

$$\frac{1 - h^2}{f} \ddot{\phi} - \frac{(r^2 h)'}{r^2} \dot{\phi} = -W'(\phi)$$

➤ Einstein:

Recall: $(r^2 h)' = -3\gamma r^2$

$$\left[\frac{1}{r^2} (1 - f - r f') + \frac{(1 - h^2) \dot{f}}{r h f} \right] = \frac{1}{M_p^2} \left(W(\phi) - \frac{1 - h^2}{2f} \dot{\phi}^2 \right)$$

SLOW ROLL CONDITIONS:

Now depend on position:

Geometry should be dominated by the potential energy:

$$\frac{1 - h^2}{f} \dot{\phi}^2 \ll W \Rightarrow \frac{1 - (rf)'}{r^2} = 3H^2 = \frac{W}{M_p^2}$$

And the scalar motion by friction:

$$\frac{1 - h^2}{f} \ddot{\phi} \ll 3\gamma \dot{\phi} \Rightarrow -3\gamma \dot{\phi} = W'(\phi)$$

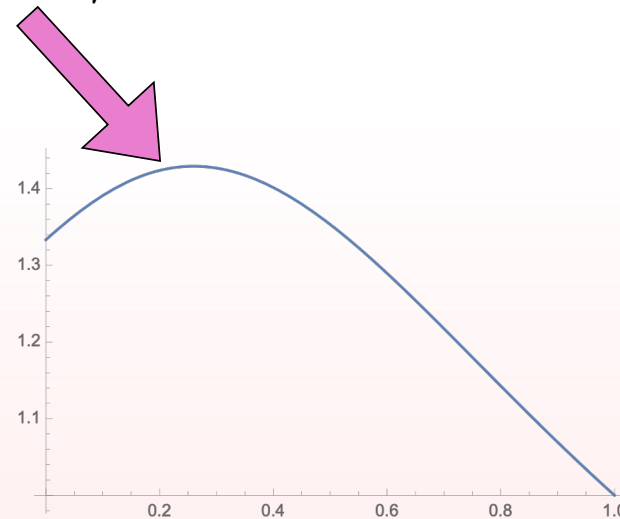
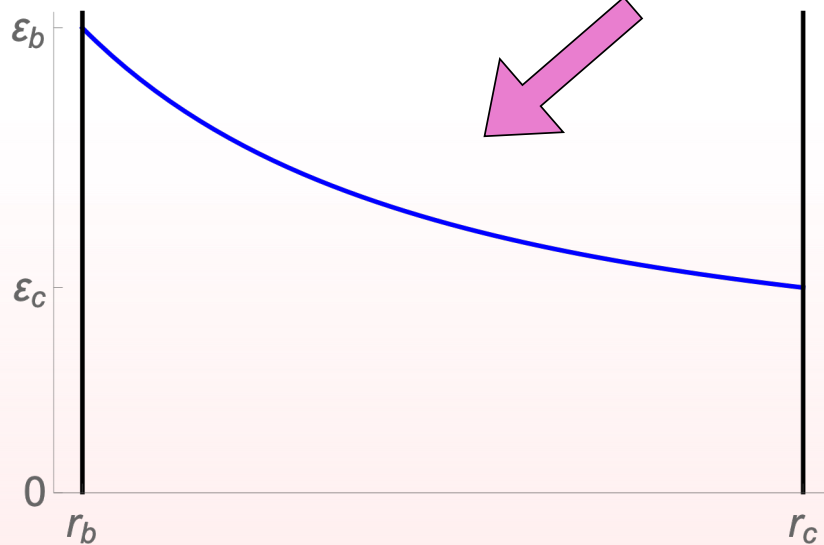
Take each in turn.

GEOMETRY

$$\frac{1 - h^2}{f} \dot{\phi}^2 \ll W$$

Geometry dominated by potential energy gives a radially dependent condition, but dependence is $O(1)$

$$M_p^2 \frac{W'^2}{W^2} \frac{(1 - h^2)}{f} \frac{H^2}{3\gamma^2} \ll 1$$



SCALAR

$$\frac{1-h^2}{f}\ddot{\phi} \ll 3\gamma\dot{\phi}$$

Use scalar equation to find

$$M_p^2 \frac{W''}{W} \frac{(1-h^2)}{f} \frac{H^2}{3\gamma^2} \ll 1$$

..the same spatial dependence as before, giving a bound

$$\eta = M_p^2 \frac{W''}{W} \ll 1$$

The η parameter.

This allows us to solve the Einstein equations to leading order in the slow-roll parameters.

➤ TT + Tr:
$$\dot{f} = -rh \frac{\dot{\phi}^2}{M_p^2}$$

implies

$$f(r, T) = f_0(r) + \delta f(r, T) - rh_0 \int \frac{\dot{\phi}^2}{M_p^2}$$

Where δf is order $\epsilon\eta$, but slowly varying. We can then integrate the ϕ kinetic energy:

$$\int_{T_0}^T \dot{\phi}^2 dT' = -\frac{1}{3\gamma} \{W(\phi(T)) - W(\phi(T_0))\} = -\frac{\delta W}{3\gamma}$$

And end up with a familiar expression:

$$f(r, T) = 1 - \frac{\Lambda(T)}{3M_p^2} r^2 - \frac{2GM(T)}{r} + \delta f$$

With

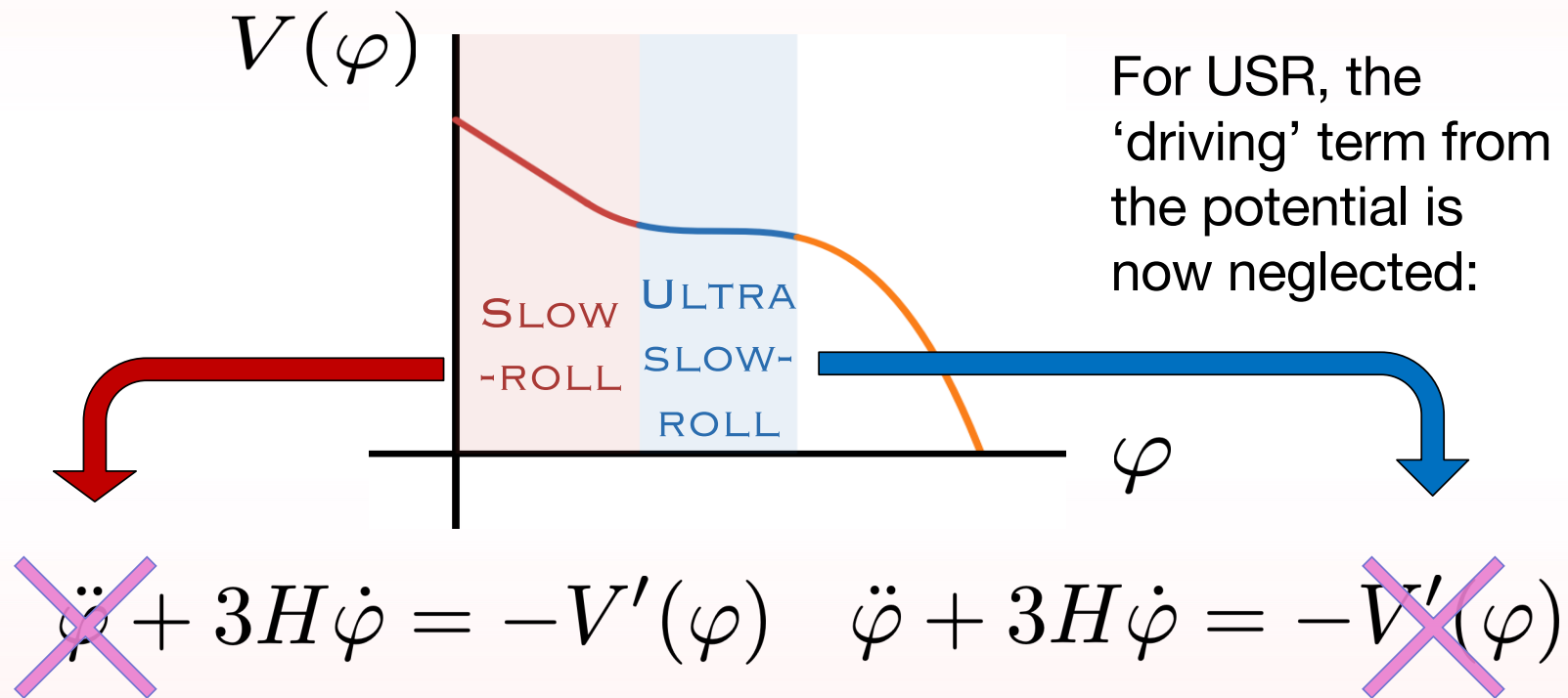
$$\Lambda(T) = W[\phi(T)] \qquad M(T) = M_0 - 4\pi\beta \frac{\delta W}{3\gamma}$$

A somewhat finicky argument shows that δf is transient, and of sub-leading order ($\varepsilon\eta$) to the changes in Λ and M .

SLOW ROLL LESSONS

- Scalar rolls with a radial dependence
- Scalar lags behind near the black hole / the black hole “drags” the scalar
- Backreaction causes both horizons to grow – the scalar accretes onto the black hole.

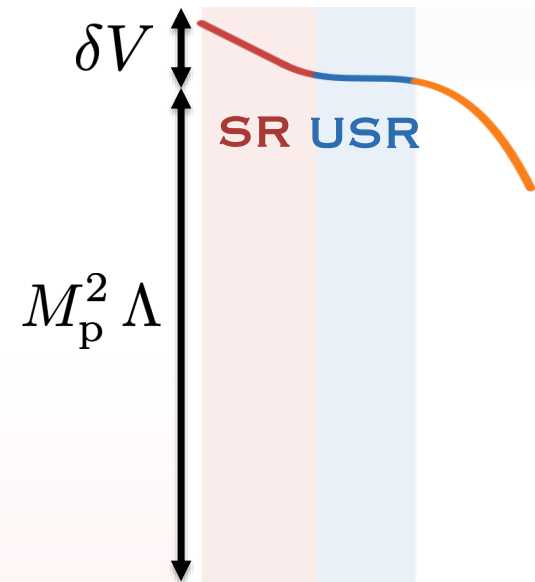
SLOW TO ULTRA-SLOW



SLOW TO ULTRA-SLOW

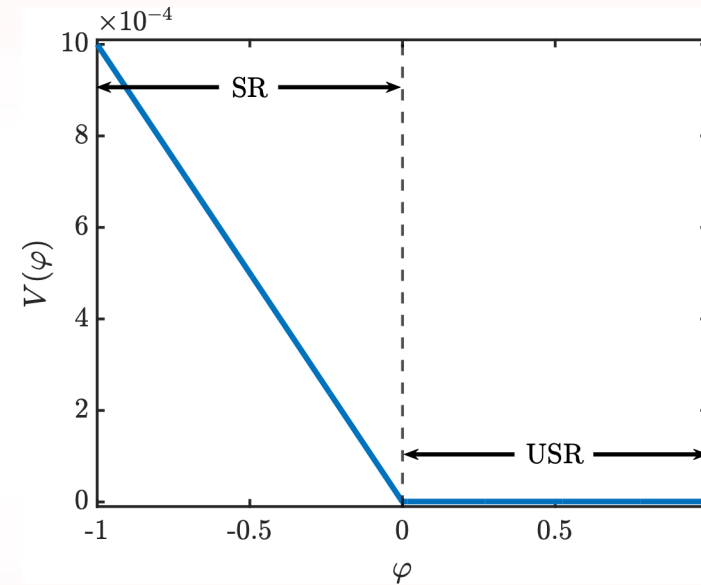
For USR, the vacuum energy is not changing much, so study in “probe” approximation, i.e. without scalar backreaction.

$$H^2 = \frac{\Lambda}{3} + \frac{1}{3M_{\text{p}}^2} \left(\cancel{\delta V(\varphi)} + \cancel{\frac{1}{2}\dot{\varphi}^2} \right)$$



COSMOLOGICAL SOLN

As a reminder, an exact (leading order) solution transitioning from SR toUSR (at time τ_0) would be a function of cosmological time



$$\varphi = \begin{cases} \varphi_i + \frac{|V_1|}{3H} \tau & \text{SR, } \tau < \tau_0 \\ \varphi_f - \frac{|V_1|}{9H^2} e^{-3H(\tau - \tau_0)} & \text{USR, } \tau > \tau_0 \end{cases}$$

ULTRA-SLOW + BH

With the black hole, the cosmological ODE becomes a PDE

$$\frac{\varphi_{,tt}}{f} - \frac{1}{r^2} (r^2 f \varphi_{,r})_{,r} = -\cancel{\frac{\partial V}{\partial \varphi}}$$

But what remains is just the massless scalar wave equation – a well understood problem.

As well as scattering wave solutions, we also have quasi-normal modes (QNM's).

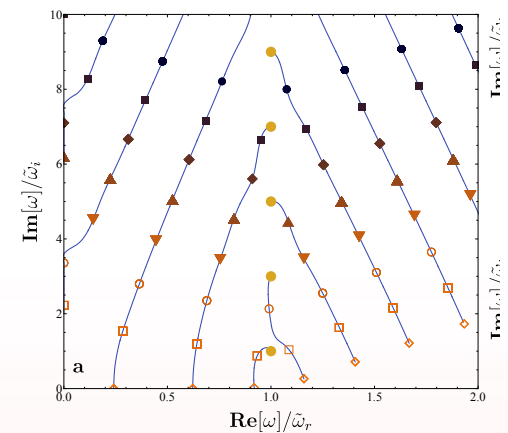
QUASI-NORMAL MODES

QNM's arise as characteristic resonances in dissipative systems. They typically have a characteristic tone with a decaying profile, i.e. a complex frequency eigenvalue solution to a wave equation:

$$\omega = \omega_R + i \omega_I$$

Very well studied due to black hole ringdown.

$$\varphi \sim e^{-i\omega t} R(r) Y_{\ell m}(\theta, \phi)$$



QNMS IN SDS

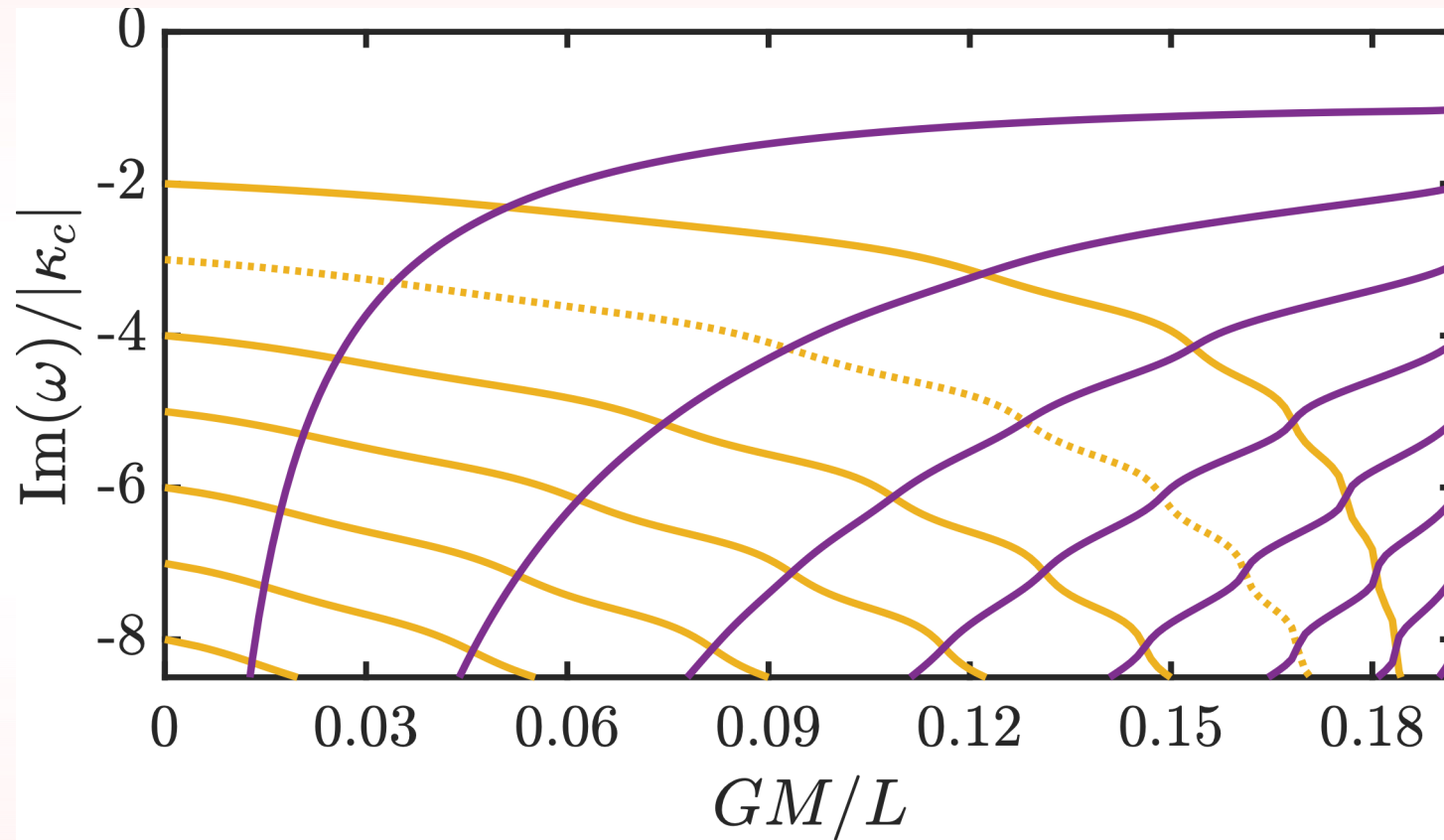
Expect solutions with time and radial dependence in the black hole spacetime. Angular dependence will arise at the perturbative level.

$$\varphi \sim R(r)e^{-i\omega t}$$

Solution for Φ must be ingoing at the black hole horizon and outgoing at the cosmological horizon.

$$R(r) \sim \begin{cases} -\frac{i\omega}{2\kappa_c} \log(r_c - r) & r \rightarrow r_c \\ \frac{i\omega}{2\kappa_b} \log(r - r_b) & r \rightarrow r_b \end{cases}$$

SDS QNM's



[PRODUCED USING CODE FROM JANSEN, 1709.09178]

COSMOLOGICAL QNMs

In pure dS, can find these QNM's analytically, in cosmological coordinates these are:

$$\varphi_n = \frac{(H\rho e^{H\tau} - n)(1 + H\rho e^{H\tau})^n + (H\rho e^{H\tau} + n)(1 - H\rho e^{H\tau})^n}{(1 - n)H\rho e^{(n+1)H\tau}}$$

however without any radial background dependence, there is nothing to give spatial dependence.

The usual decaying USR solution is the $n=3$ (second) mode.

$$\delta\varphi \propto -e^{-3H(\tau-\tau_0)} \quad \text{or} \quad \delta\varphi \propto -e^{-3Ht}(1 - H^2r^2)^{-3/2}$$

SDS QNMs

For pure DS, the “time” for the slow-roll scalar is identical to cosmological time and the pure QNM mode. With the black hole, the radial dependence (now found numerically) is not aligned with the slow-roll T.

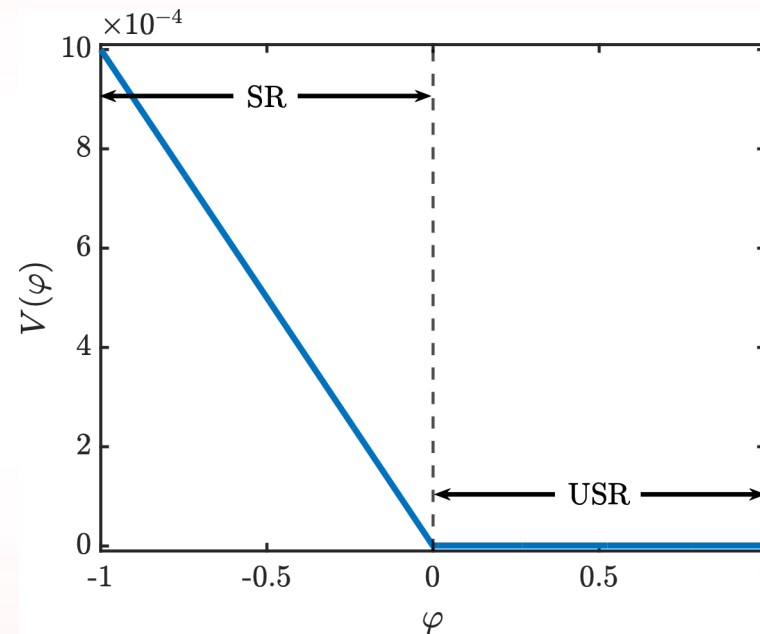
The transition from SR to USR is therefore uneven and several QNM's might be expected to be excited.

SR $\rightarrow$ USR MODELLING

To see which QNM's are most relevant, focus in on the SR to USR transition.

Solve scalar wave equation numerically.

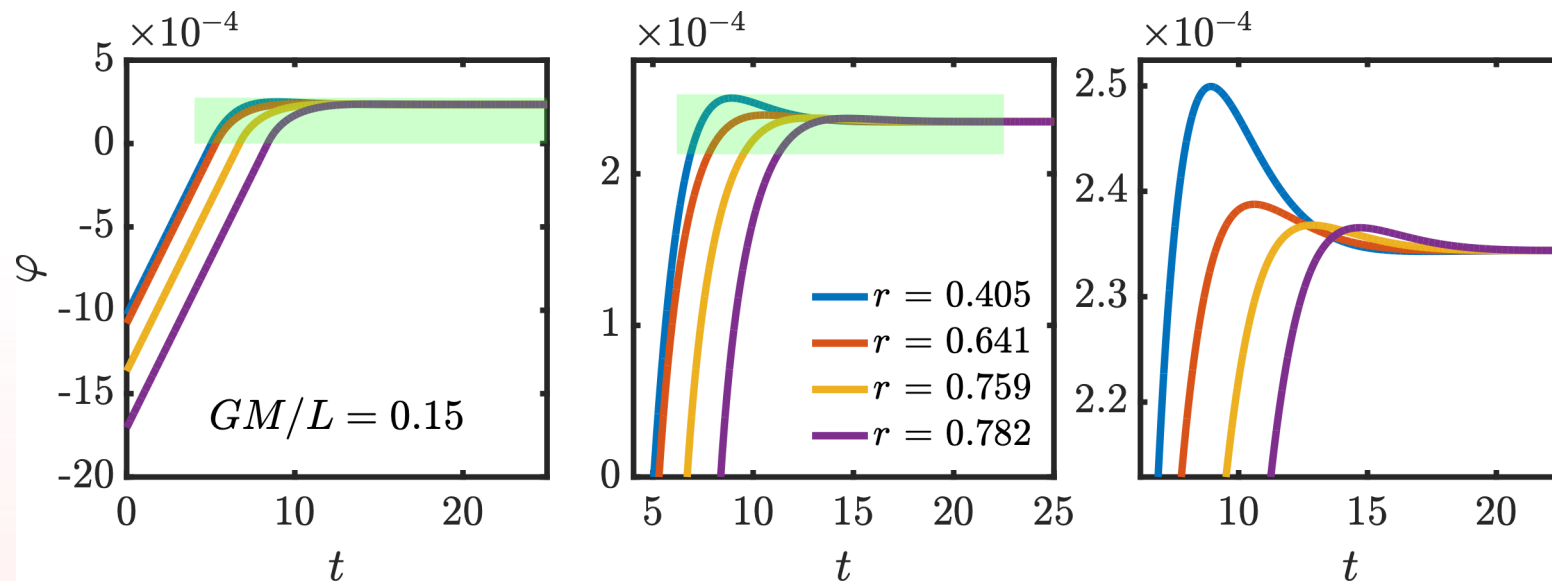
Fit solution to a “basis” of QNM's.

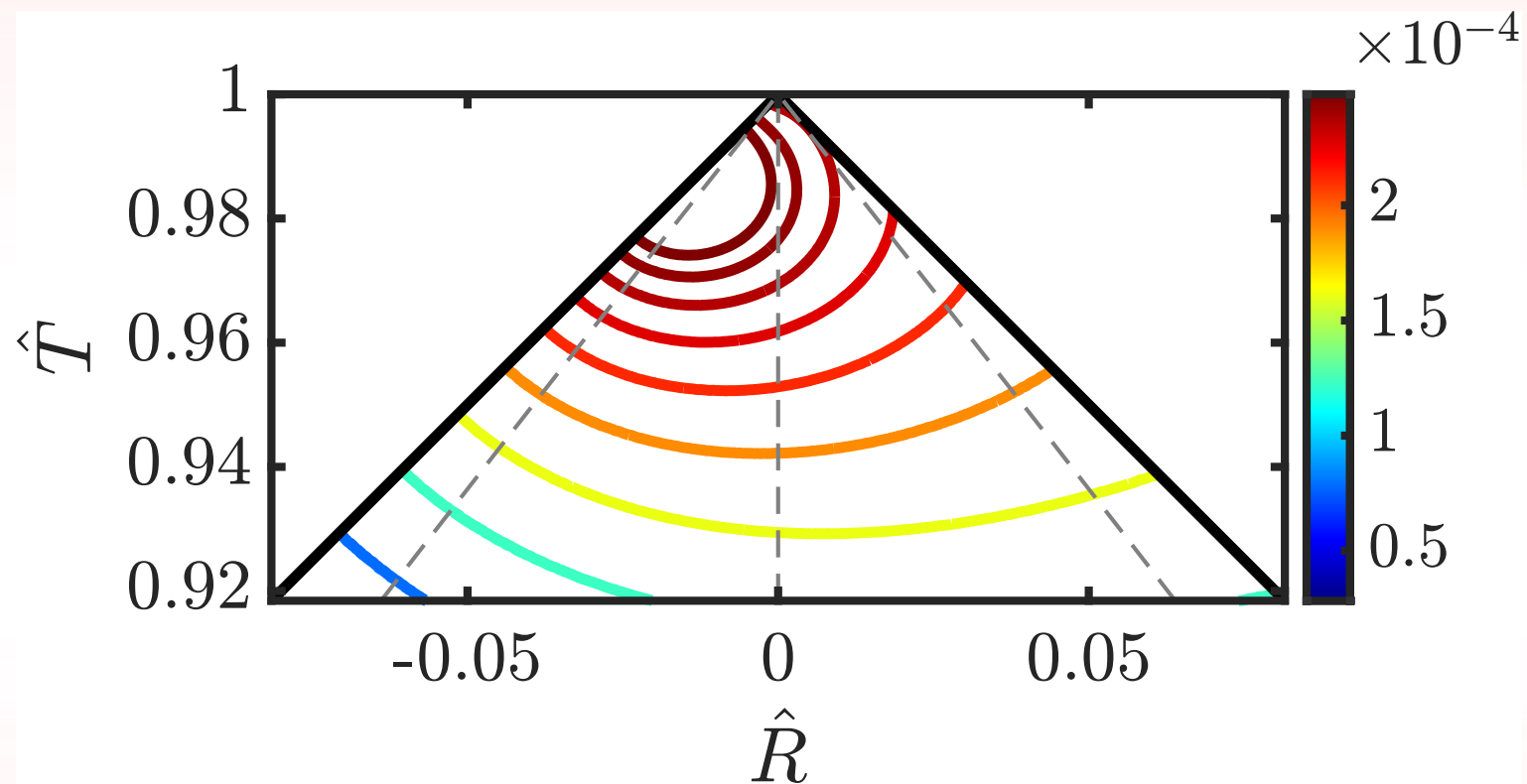


$$\begin{aligned} V(\varphi) &= V_0' \varphi, & \varphi < 0 \\ &= 0, & \varphi \geq 0 \end{aligned}$$

SOLVING SCALAR

Numerically solving through the transition and plotting at various radii looks largely similar to the cosmological case, but the scalar overshoots and rings down.



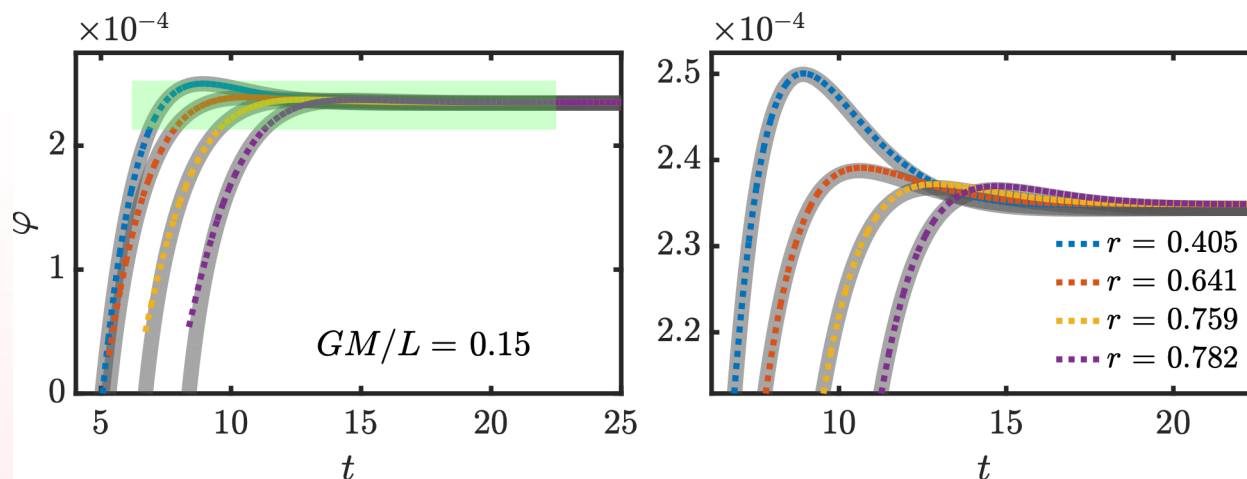


Plotting constant Φ contours on a Penrose diagram shows the ringing behaviour – here for $M/L = 0.15$

EXTRACT QNM TONES

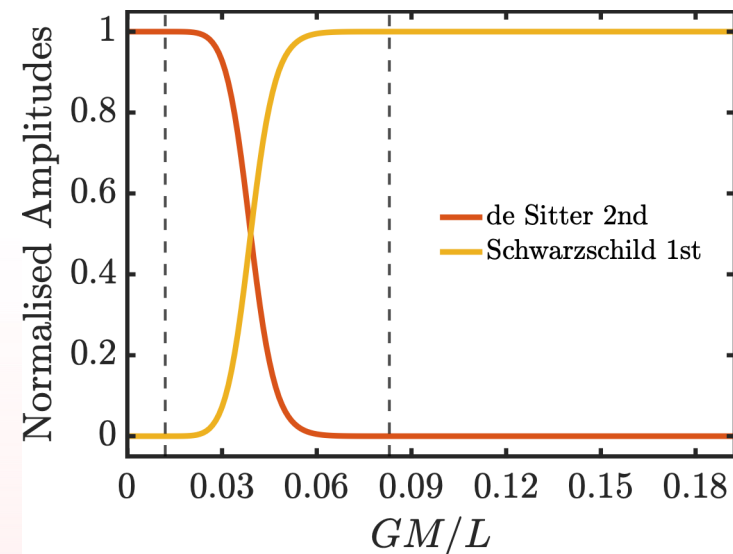
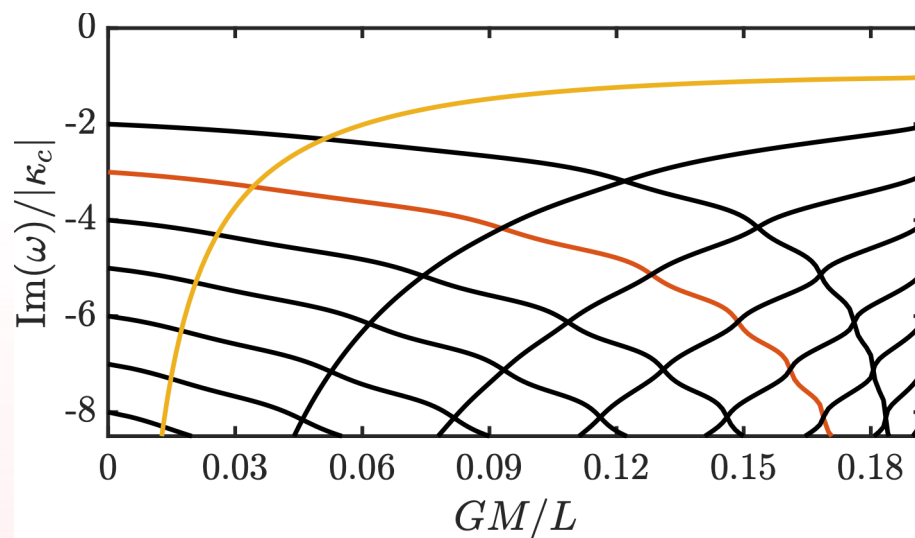
Fit the solution to the QNM overtones (in practice to the first few)

$$\varphi = C + \text{Re} \left(\sum_n A_n z_n(r) e^{-i\omega_n(t-t_{\text{USR}})} \right)$$



DOMINANT MODES

The scalar solution is dominated by the (continuation of) dS mode for small mass black holes, but by the ringing mode for larger masses.



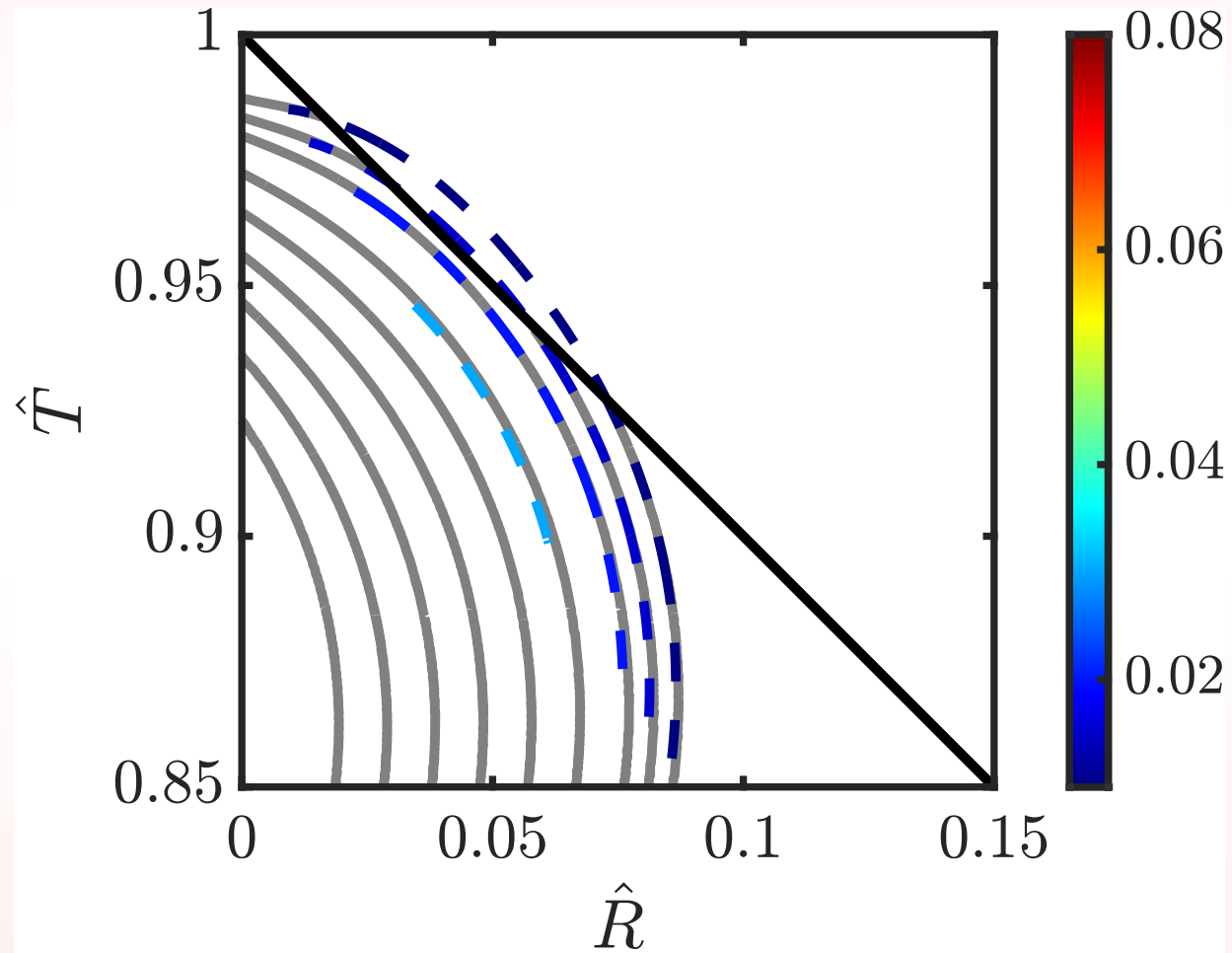
USR – WHAT CHANGES?

Usually, spatial dependence and any ringing happens in cosmological perturbation theory.

Here the black hole causes ringing at the level of the *background* scalar solution.

Expect additional peak enhancement, perhaps seeding more black holes.

Ringing persists across the horizon.



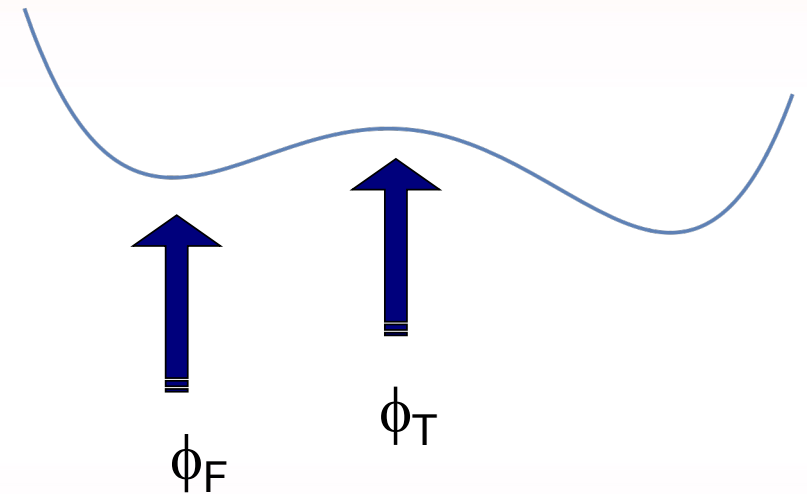
MAKING BLACK HOLES

How can we get black holes in the first place?

1. They are there as a random start-up event.
2. Tunnelling!

HAWKING-MOSS

The Hawking Moss instanton is a transition from a false vacuum to a higher vacuum energy (from which we can roll to a lower energy). This is best understood via a “Quantum Cosmology” interpretation. Since a higher Λ de Sitter has lower entropy, this process is Boltzmann suppressed.

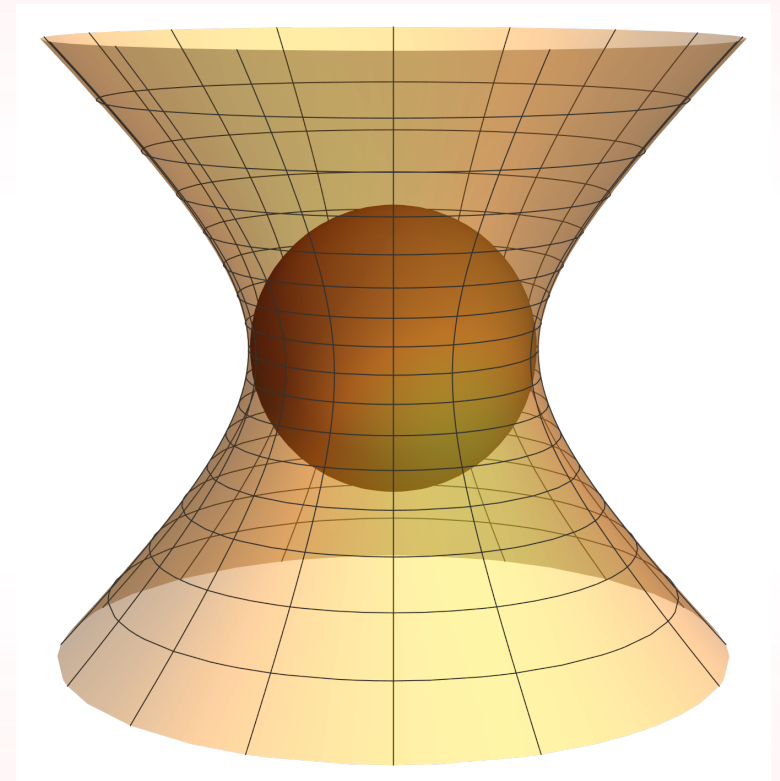


$$\Gamma \sim e^{-\delta\mathcal{A}/4} \sim e^{-\pi(\ell_F^2 - \ell_T^2)}$$

BOUSSO-HAWKING

Bousso and Hawking instead explored the transition from de Sitter to Schwarzschild de Sitter / Reissner Nordstrom de Sitter as a means for spontaneous creation of black holes.

But the computation of these actions involves Euclidean analytic continuation, and the standard (Gibbons-Hawking) lore is that Euclidean sections must be regular.



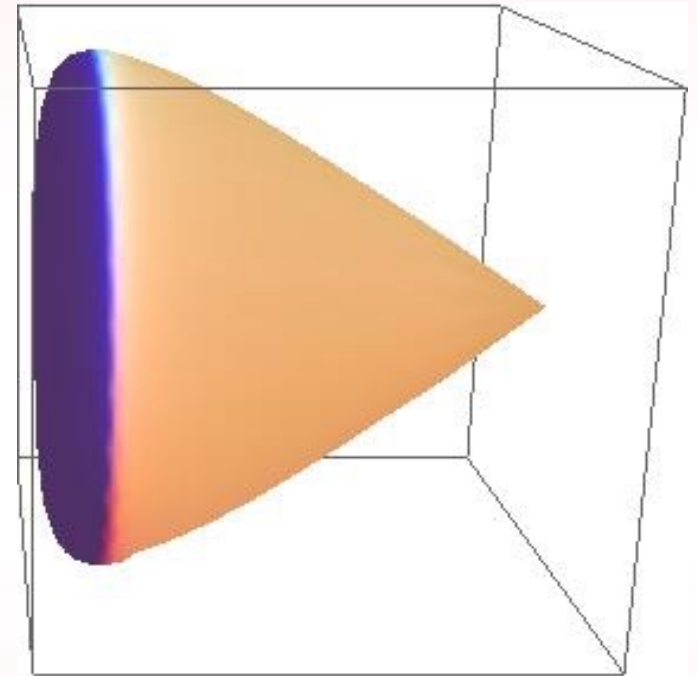
BOUSSO-HAWKING

Euclidean de Sitter is a sphere, but SdS has conical deficits.

This is because the cosmological and black hole horizons have different natural temperatures.

B-H demanded regularity, so had to take Nariai limit, or a “lukewarm” instanton with near extremal charge (charge drops T).

Found very low probabilities.



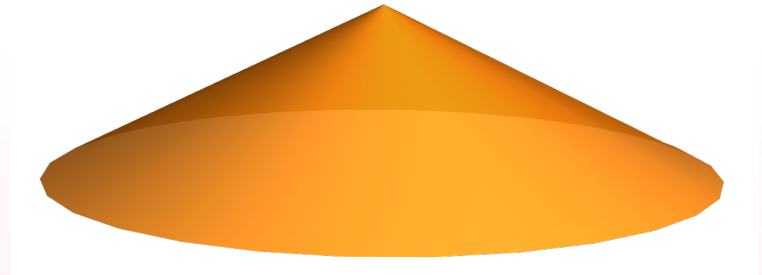
SINGULAR INSTANTONS

But we don't have to do this!

The action of a conical singularity is finite, and proportional to the deficit angle and area of the transverse space.

We can calculate the action of SdS (or RNdS) at any periodicity – i.e. temperature.

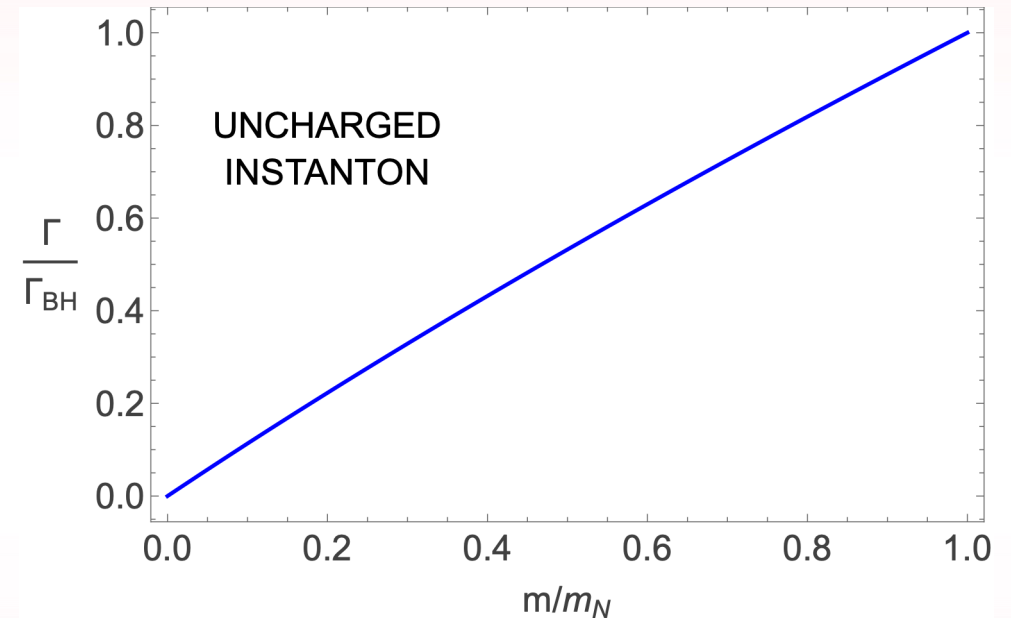
$$\lim_{\epsilon \rightarrow 0} \int_{r-r_i \leq \epsilon} \sqrt{g} R d^4 x = 2\pi \delta_i \mathcal{A}_i,$$



SINGULAR INSTANTONS

And a little exact solution magic shows that this arbitrary periodicity cancels out, and the action is simply the Boltzmann entropy drop.

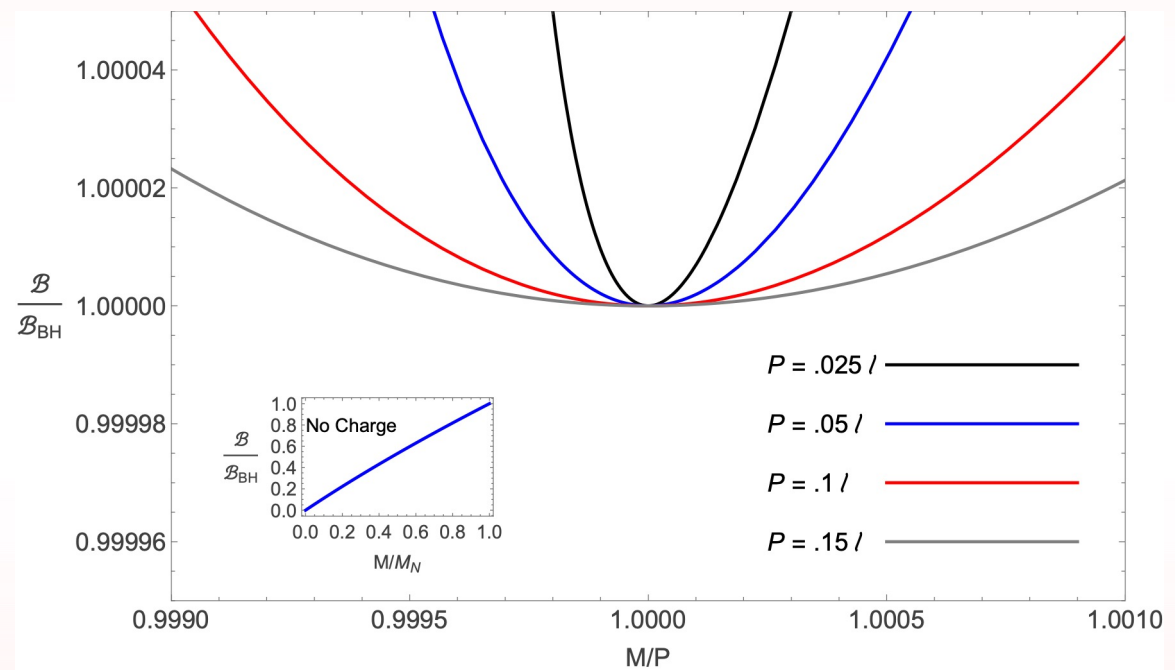
The out of equilibrium black hole is more likely to be produced.



$$\begin{aligned} I_E &= -\frac{\beta}{2\ell^2}(r_c^3 - r_h^3) - \frac{1}{4}(\mathcal{A}_h + \mathcal{A}_c) + \frac{\beta}{4} \left(\frac{\mathcal{A}_h}{\beta_h} + \frac{\mathcal{A}_c}{\beta_c} \right) - \frac{P^2\beta}{2} \left(\frac{1}{r_c} - \frac{1}{r_h} \right) \\ &= -\frac{1}{4}(\mathcal{A}_h + \mathcal{A}_c) \end{aligned}$$

LUKEWARM INSTANTONS

Interestingly, the same is not true for the lukewarm instantons. However, the framework of the calculation is not obviously accurate.



SUMMARY

- Can generalise slow-roll description to non-homogeneous black hole background
- Backreaction causes accretion of mass and shifting of effective Λ
- Transition to USR is fundamentally spatially inhomogeneous when BH present
- QNM's provide key behaviour for USR and ringing transmits across cosmological horizon.
- Black holes can be spontaneously created via Bousso-Hawking-Moss type instantons, tunnelling of uncharged black holes has been underestimated