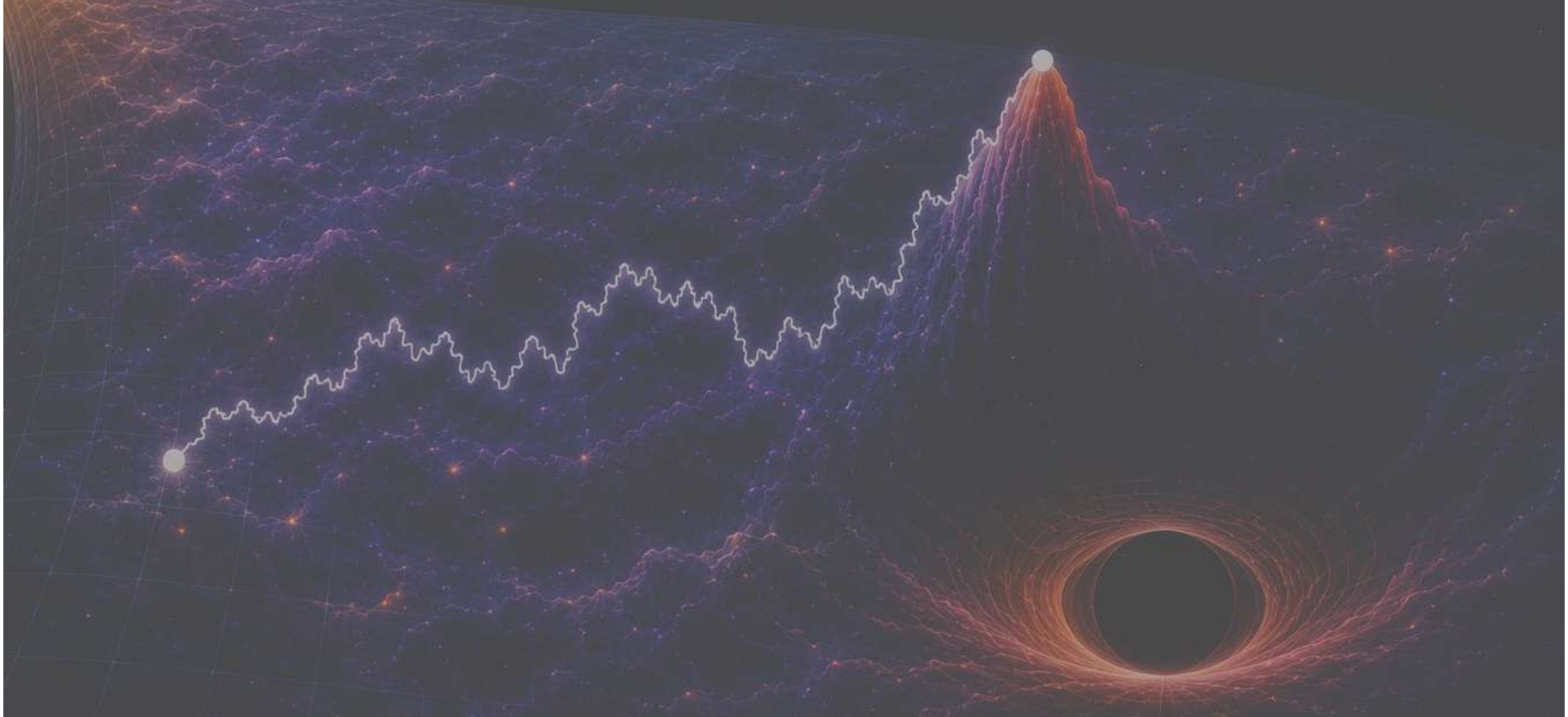
Harvesting primordial black holes from stochastic trees - part I



Vincent Vennin

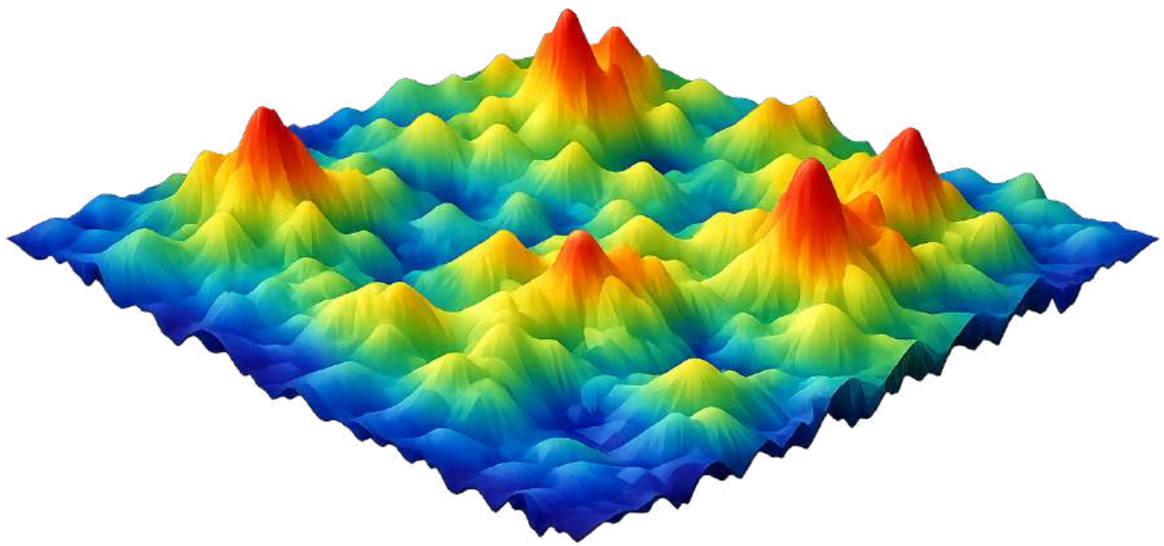
<u>The phenomenology of PBHs</u>, 21 July 2026, Mainz

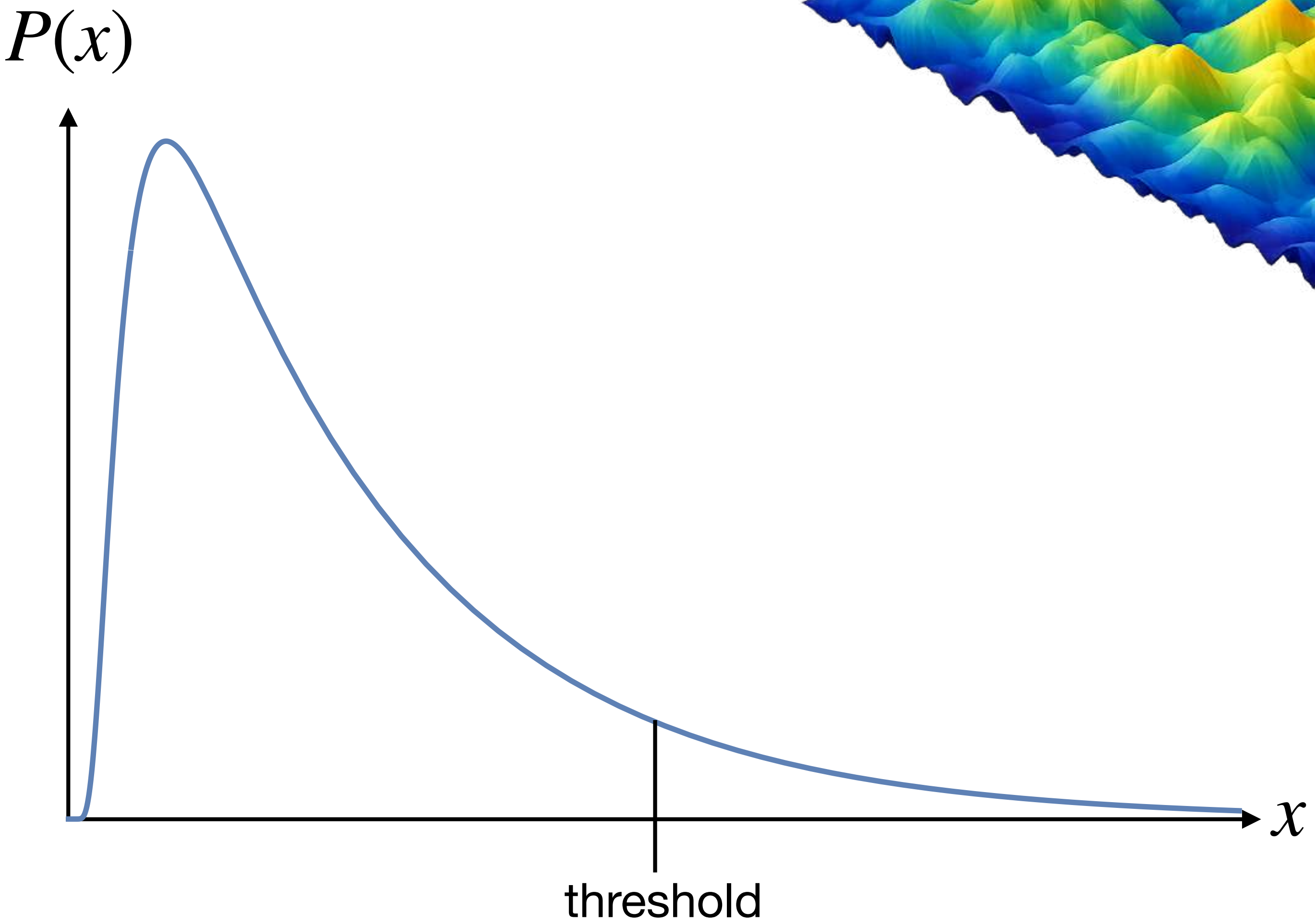
Harvesting primordial black holes from stochastic trees - part I

- Primordial black holes collapse from large density fluctuations in the primordial universe.
- They are rare and extreme events, hence they originate from the far tails of distribution functions.
- Perturbative methods are efficient to describe the bulk (near-maximum region) of distribution functions, which shape the n-point correlation functions.
- Primordial black holes require non-perturbative methods.

Black holes form in tails

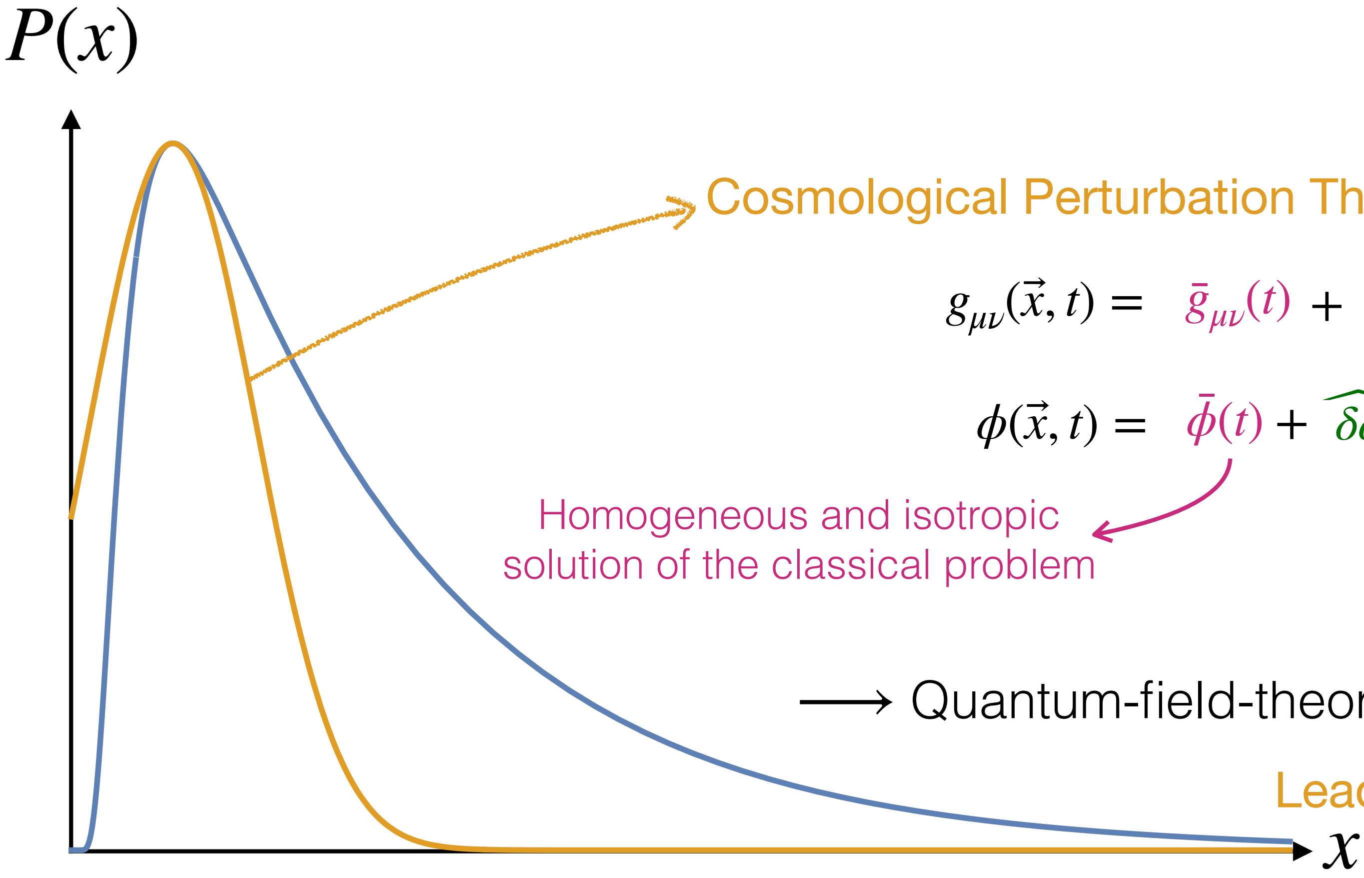
How likely is it to form a given cosmological structure?





- Local curvature
- energy density
- (average) compaction
- etc

Black holes form in tails



Cosmological Perturbation Theory, leading order

$$g_{\mu\nu}(\vec{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\vec{x}, t)$$

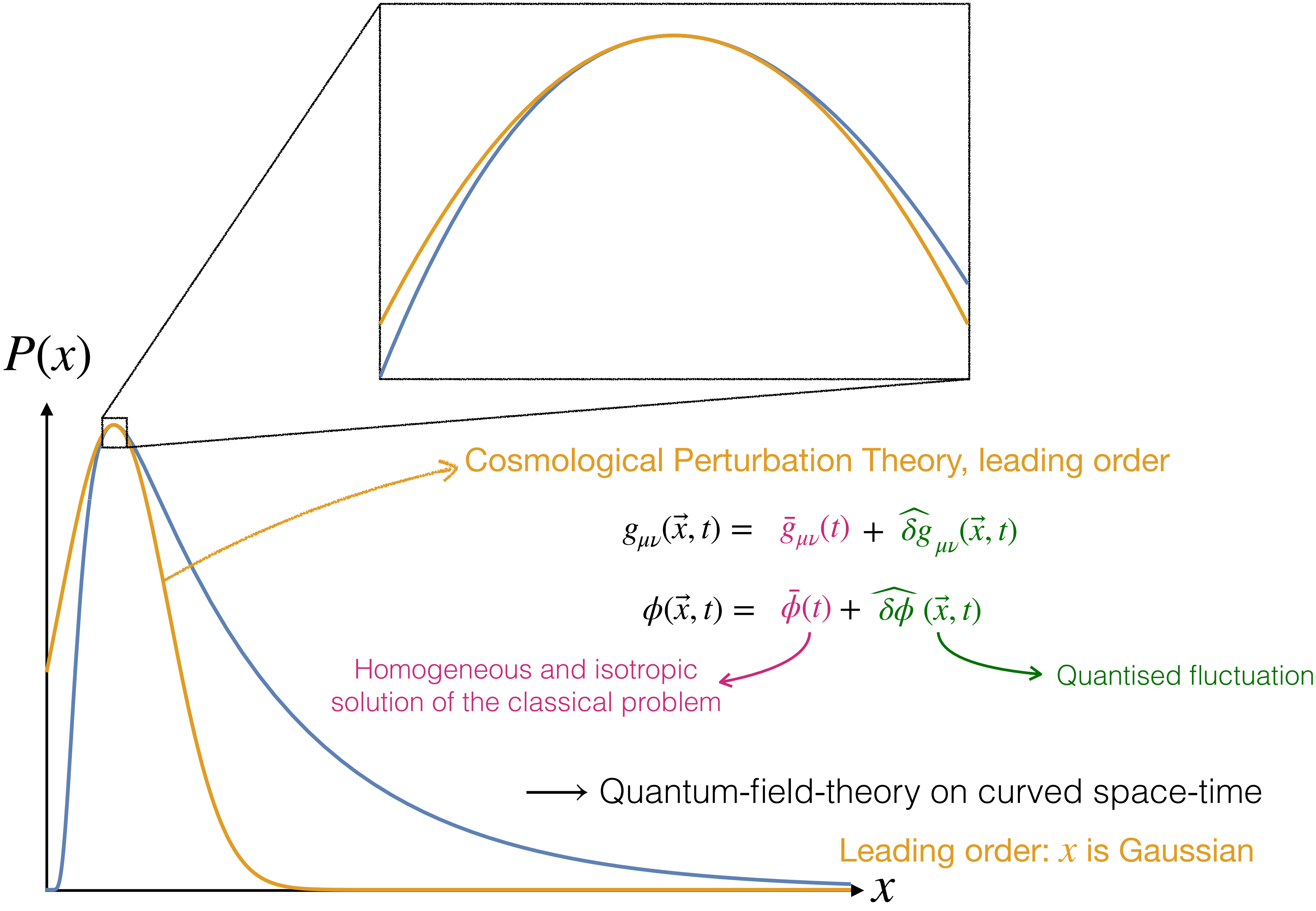
$$\phi(\vec{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\vec{x}, t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

$\longrightarrow$ Quantum-field-theory on curved space-time

Leading order: x is Gaussian



$P(x)$
Cosmological Perturbation Theory, leading order
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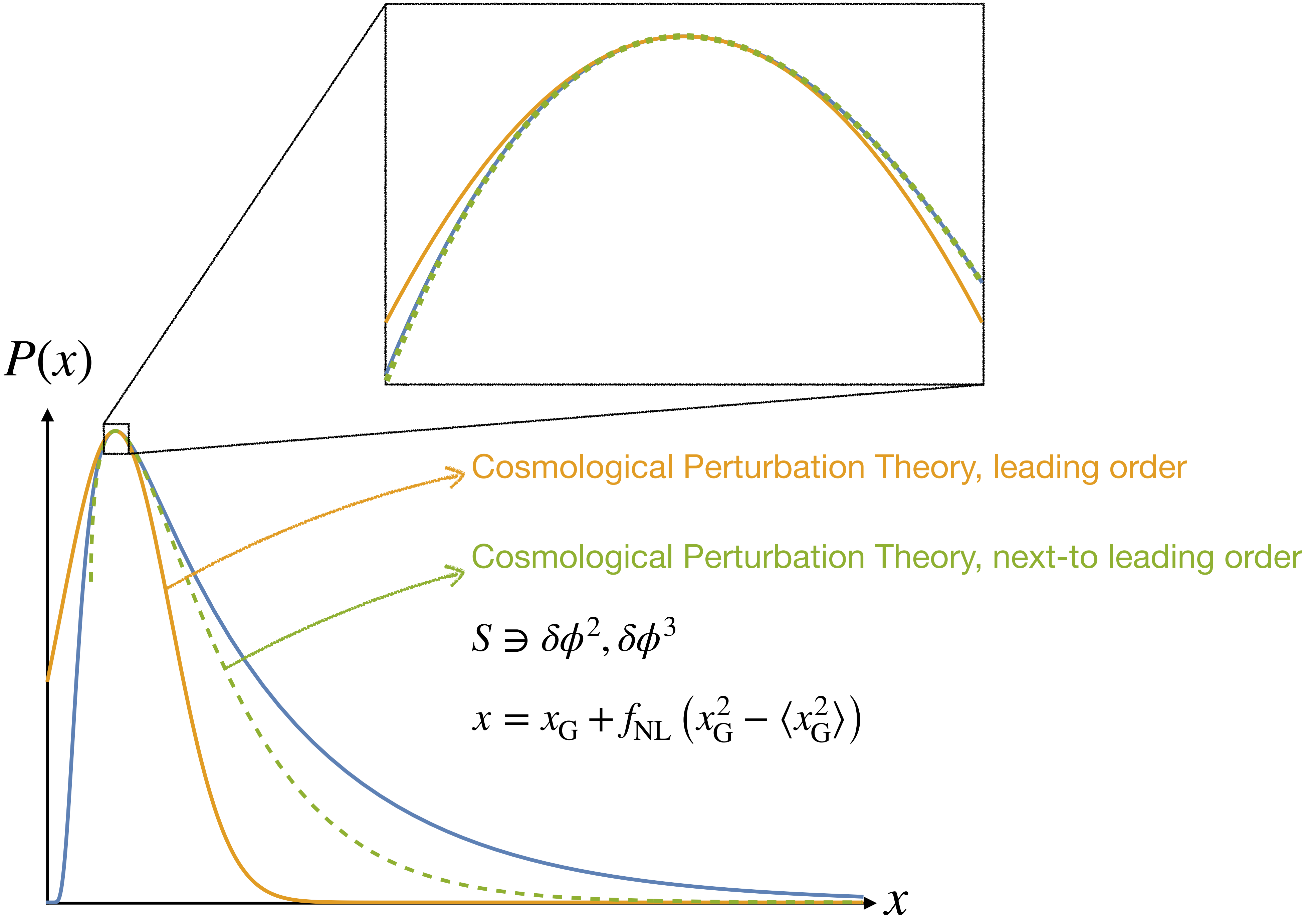
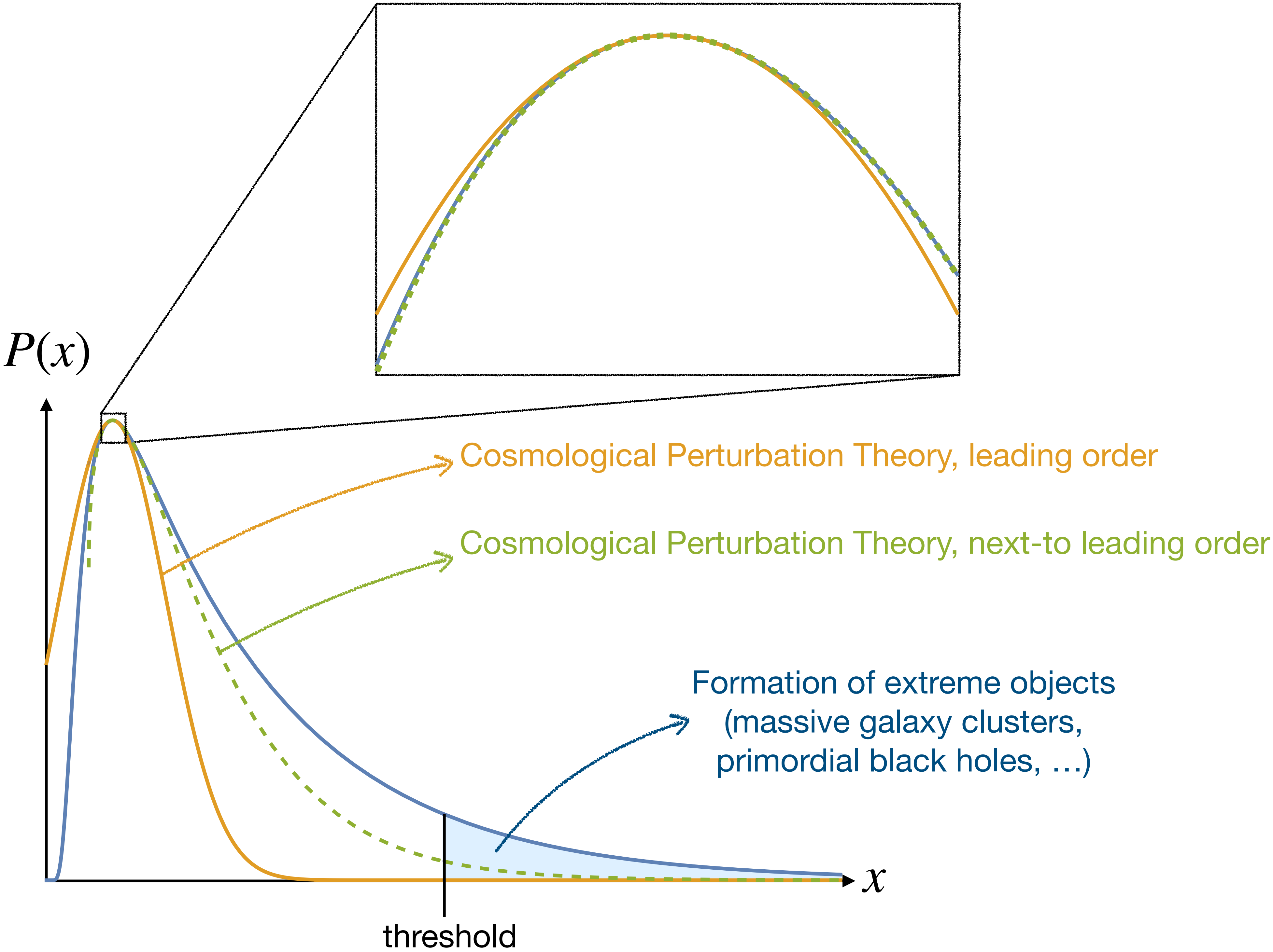
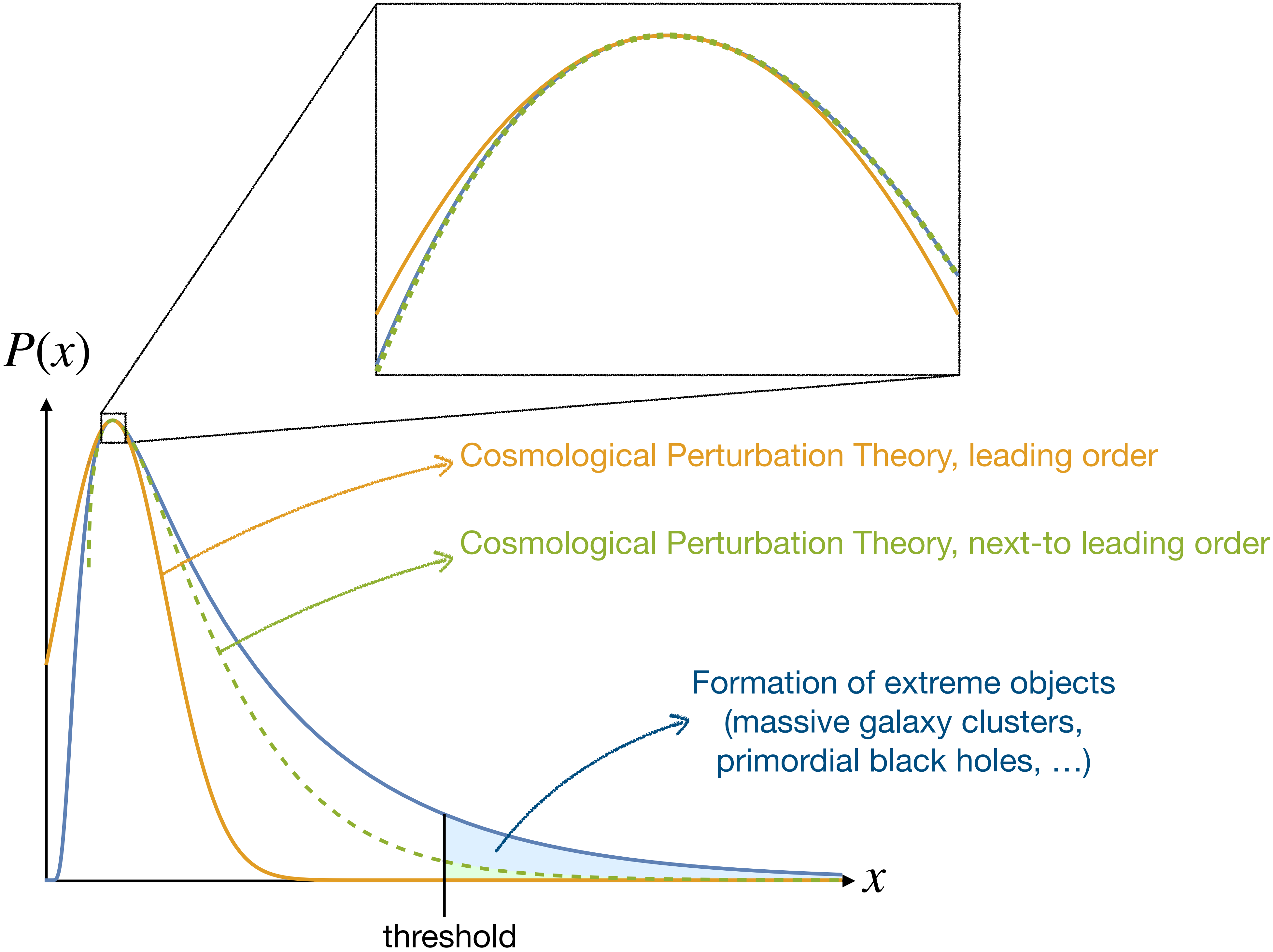
x



$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
$S \ni \delta\phi^2, \delta\phi^3$
$x = x_{\rm G} + f_{\rm NL}\left(x_{\rm G}^2 - \langle x_{\rm G}^2 \rangle\right)$
x



$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold



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Cosmological Perturbation Theory, leading order
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Formation of extreme objects
(massive galaxy clusters,
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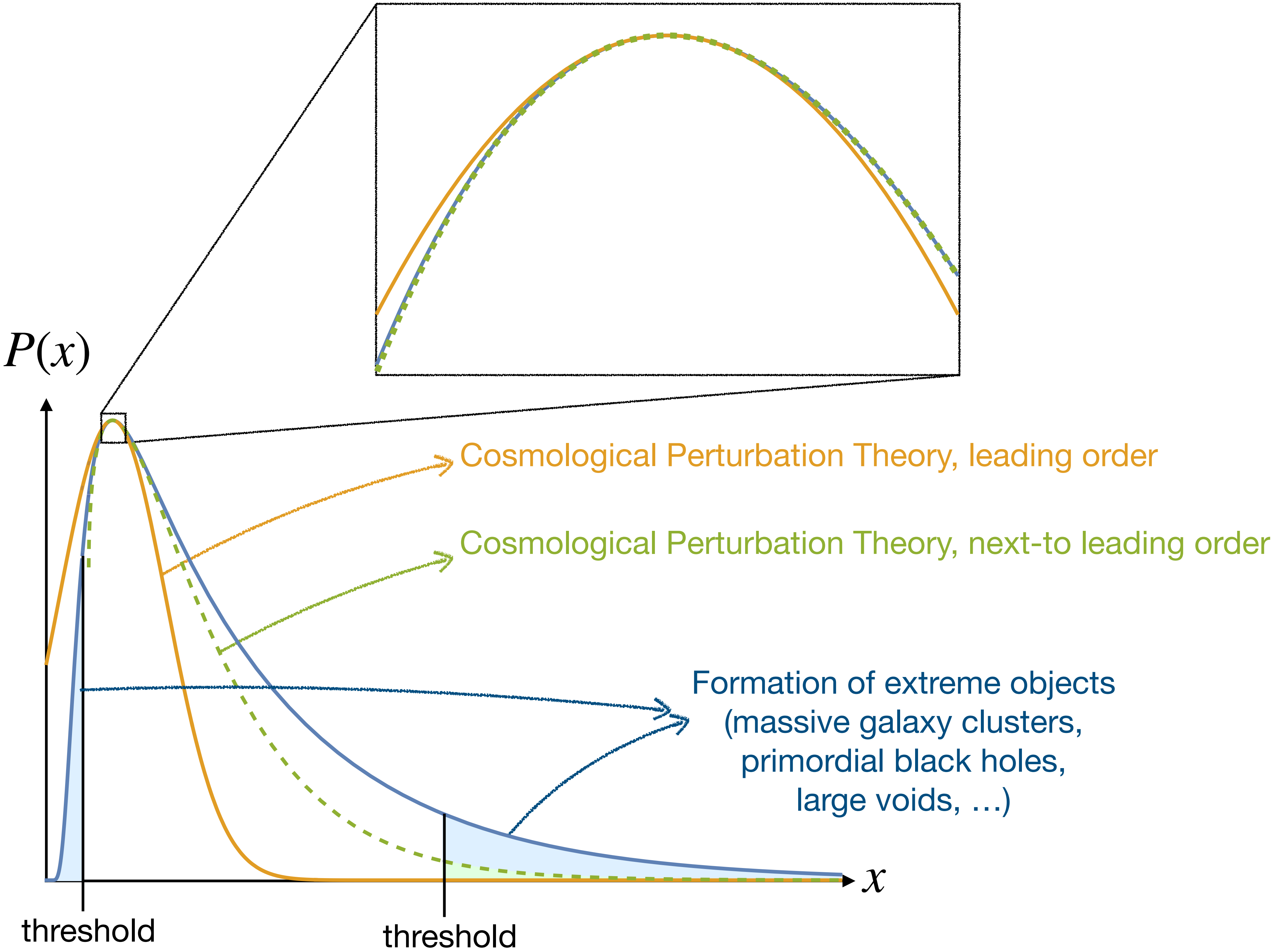


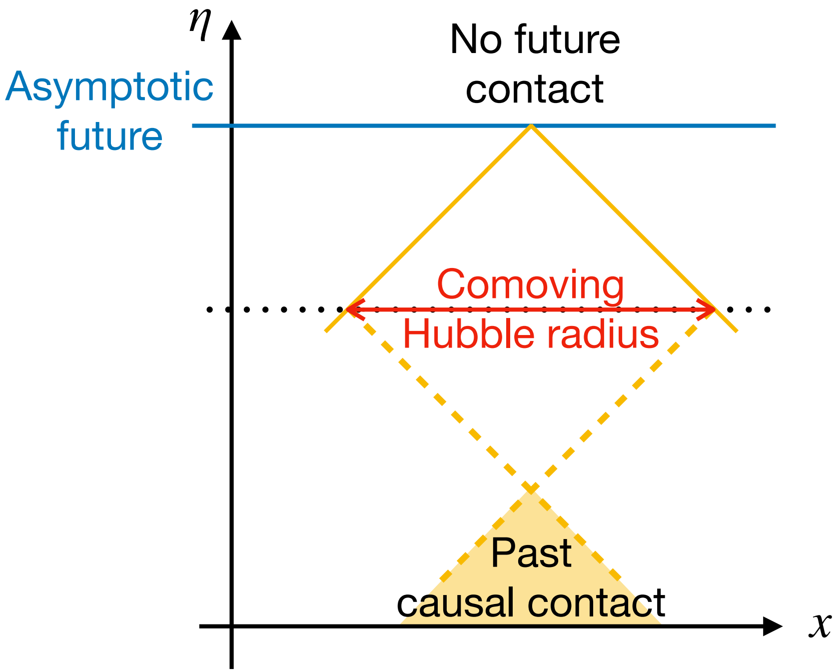
$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes,
large voids, …)
x
threshold
threshold

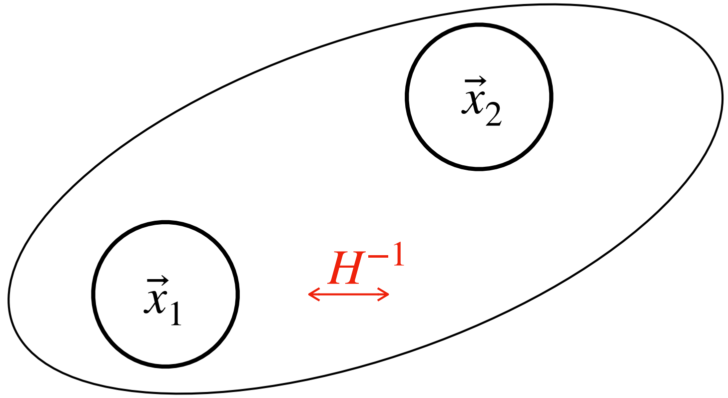
Separate universes

Salopek & Bond; Sasaki & Stewart; Wands, Malik, Lyth & Liddle (90's, 20's)

de-Sitter spacetime: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2 \quad \text{with} \quad a(t) = e^{Ht}, \ -\infty < t < \infty$

$$= a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right) \quad \text{with} \quad a(t) = -1/(H\eta), \ -\infty < \eta < 0$$





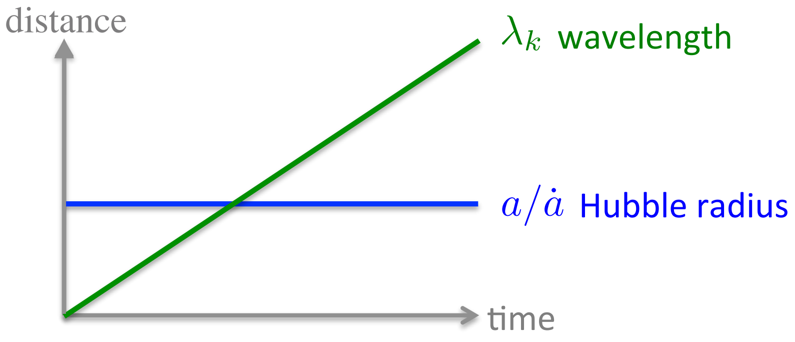
If a classical fluctuation develops at $\vec{x}_1$,
this cannot affect the local geometry at $\vec{x}_2$

"separate universes": on large scales, the universe can be described by an ensemble of independent, locally homogeneous and isotropic patches.

$\mathrm{d}s^2 = -\mathrm{d}t^2 + \underbrace{a^2(t)e^{2\zeta(t,\vec{x})}}_{\tilde{a}^2(t,\vec{x})}\mathrm{d}\vec{x}^2 \longrightarrow \zeta = \delta \ln \tilde{a} \equiv \delta N$ where the curvature perturbation $\zeta \propto \delta\rho/\rho, \delta T/T, \text{etc.}$

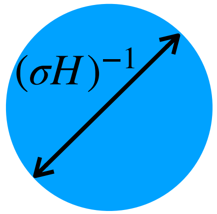
Stochastic inflation

Starobinsky, (1982) 1986



super-Hubble scales —> Stochastic Inflation

sub-Hubble scales —> Perturbation Theory



Coarse-grained field $\hat{\mathbf{\Phi}}_{\mathrm{cg}}(N,\vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k}\left[\mathbf{\Phi}_{\vec{k}}(N)e^{-i\vec{k}\cdot\vec{x}}\hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N)e^{i\vec{k}\cdot\vec{x}}\hat{a}^{\dagger}_{\vec{k}}\right]$

$N = \ln(a)$

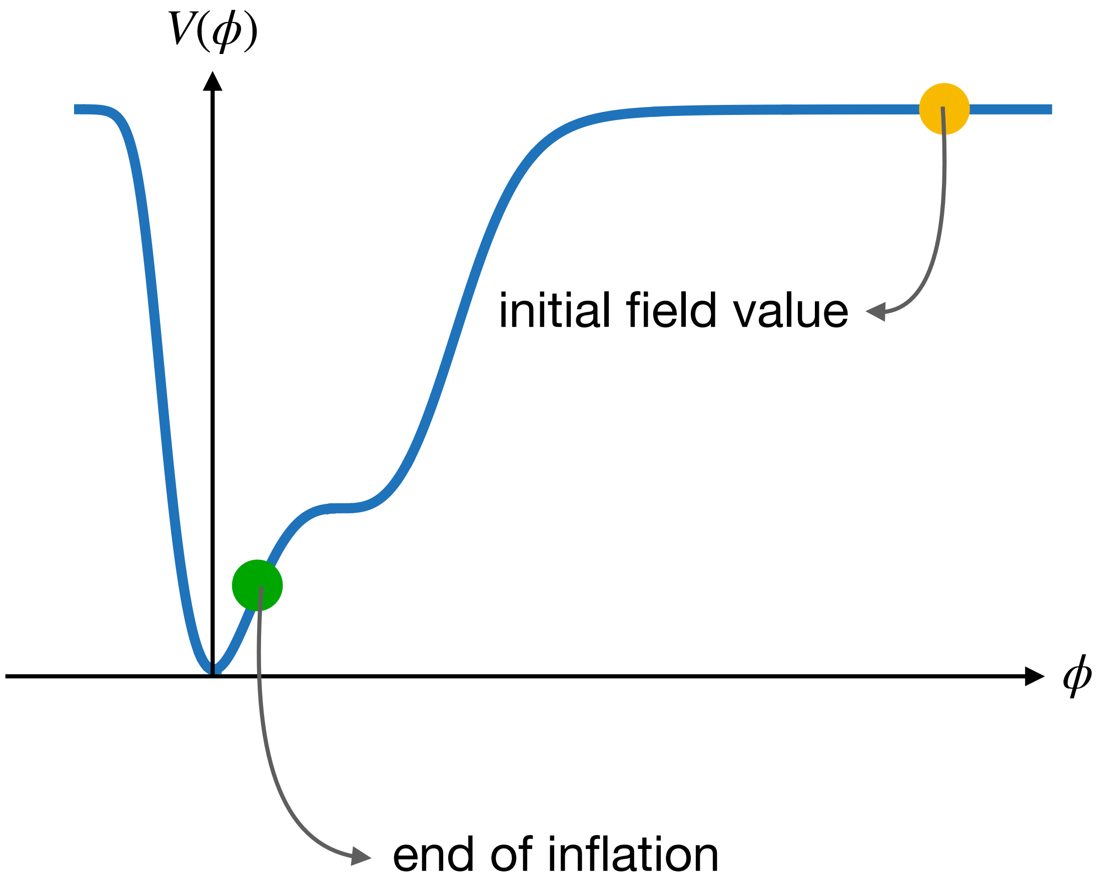
Quantum fluctuations
source the **background**

Equation of motion $$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\mathrm{cg}} = \mathscr{F}_{\text{H\&I background}}(\mathbf{\Phi}_{\mathrm{cg}}) + \mathscr{G}(\mathbf{\Phi}_{\mathrm{cg}})\cdot\boldsymbol{\xi}(N)$$

For a fully quantum derivation, see Christie, Joo, Kaplanek, Vennin, Wands 2026

Stochastic δN formalism

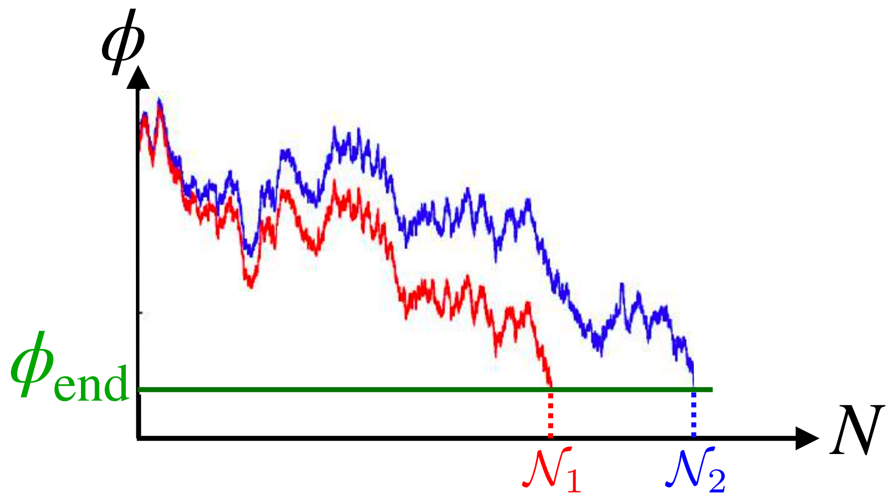
Vennin & Starobinsky (2015)



Langevin equation:

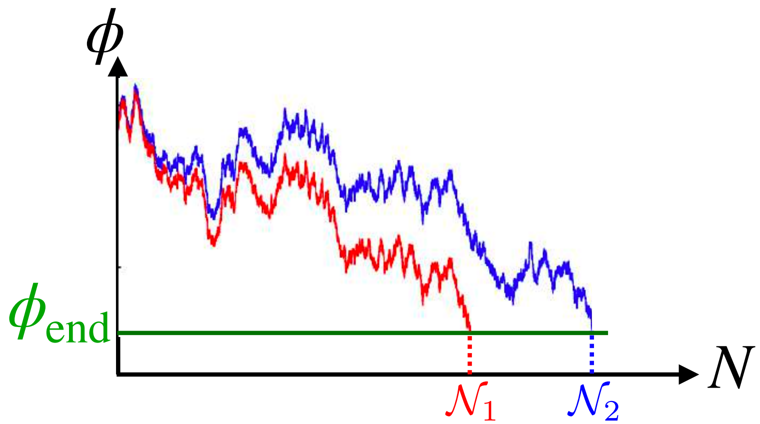
$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

where $\langle \xi(N)\xi(N') \rangle = \delta(N - N')$



$P(\zeta)$ is nothing but a first-passage-time distribution!

First-passage time analysis



- Stock prices and barrier options in finance
- Nucleation theory
- Neuronal firing
- Conformal changes of proteins
- Dynamics of epidemics
- etc

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right)$$

Langevin equation

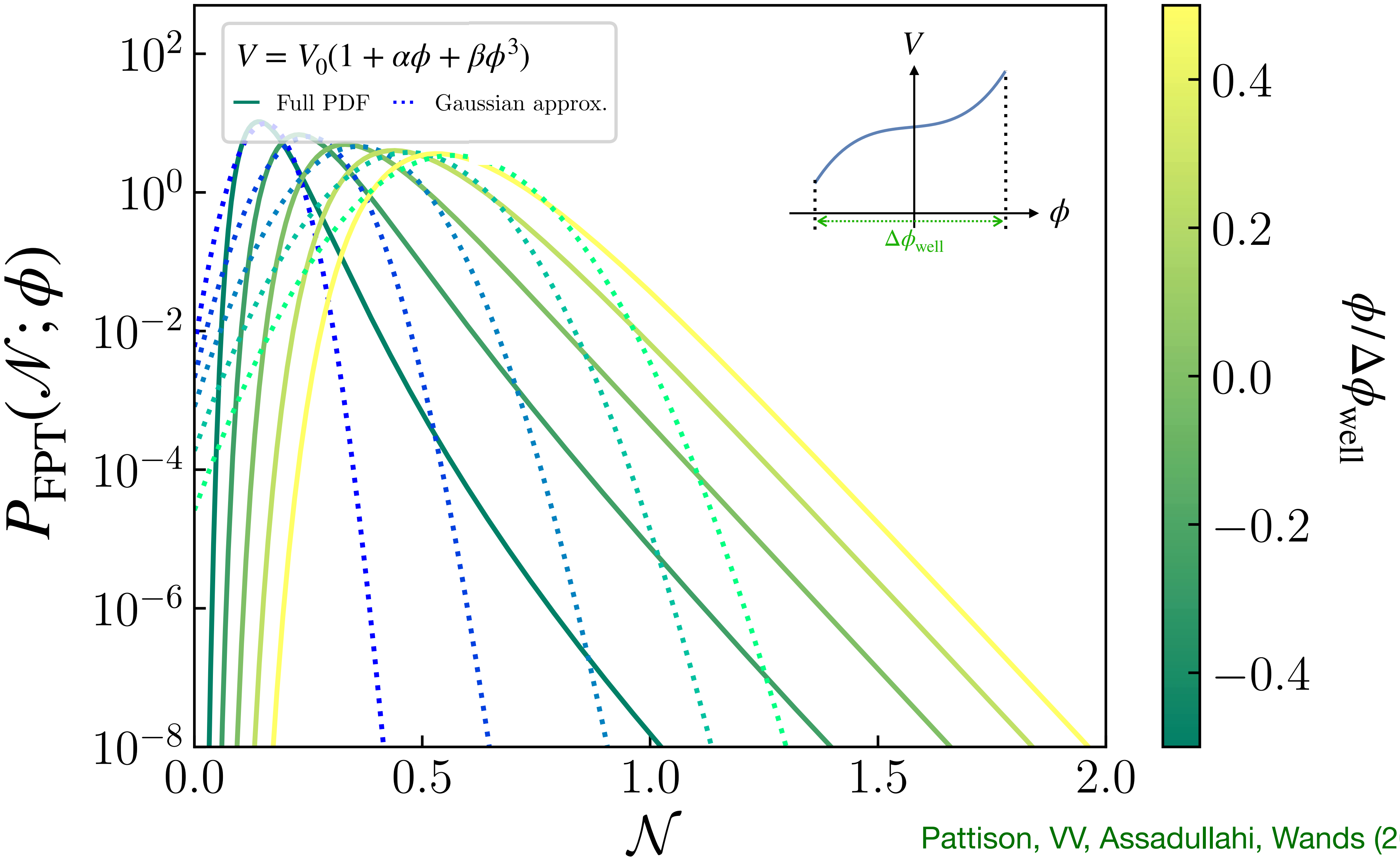
Fokker-Planck equation

$$\downarrow$$

$$P_{\rm FPT}(\mathcal{N};\phi) = \sum_n a_n(\phi)e^{-\Lambda_n\mathcal{N}} \longleftarrow \frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\rm FPT}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\rm FPT}$$

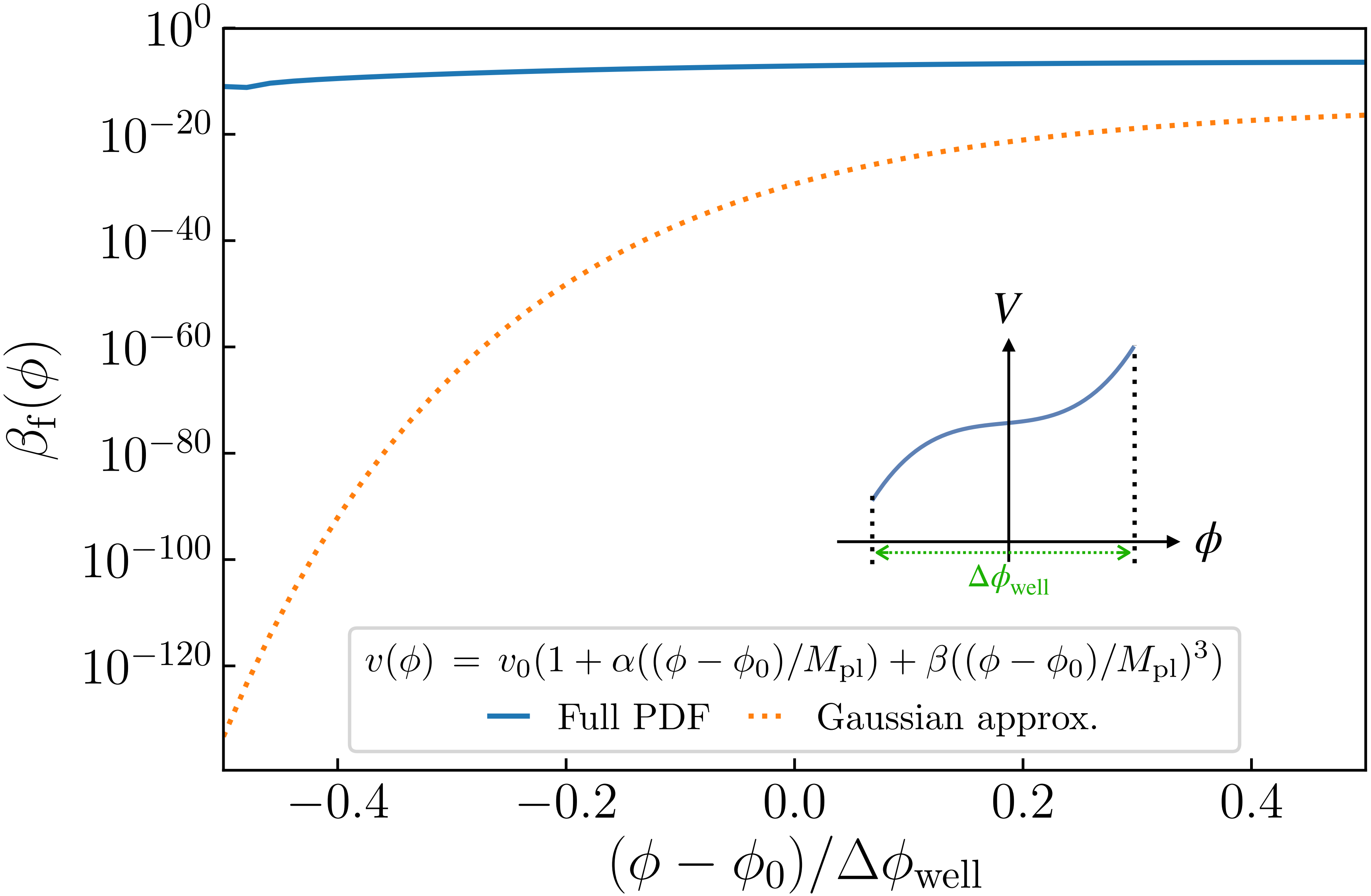
Adjoint FP for the first passage time

First-passage time analysis



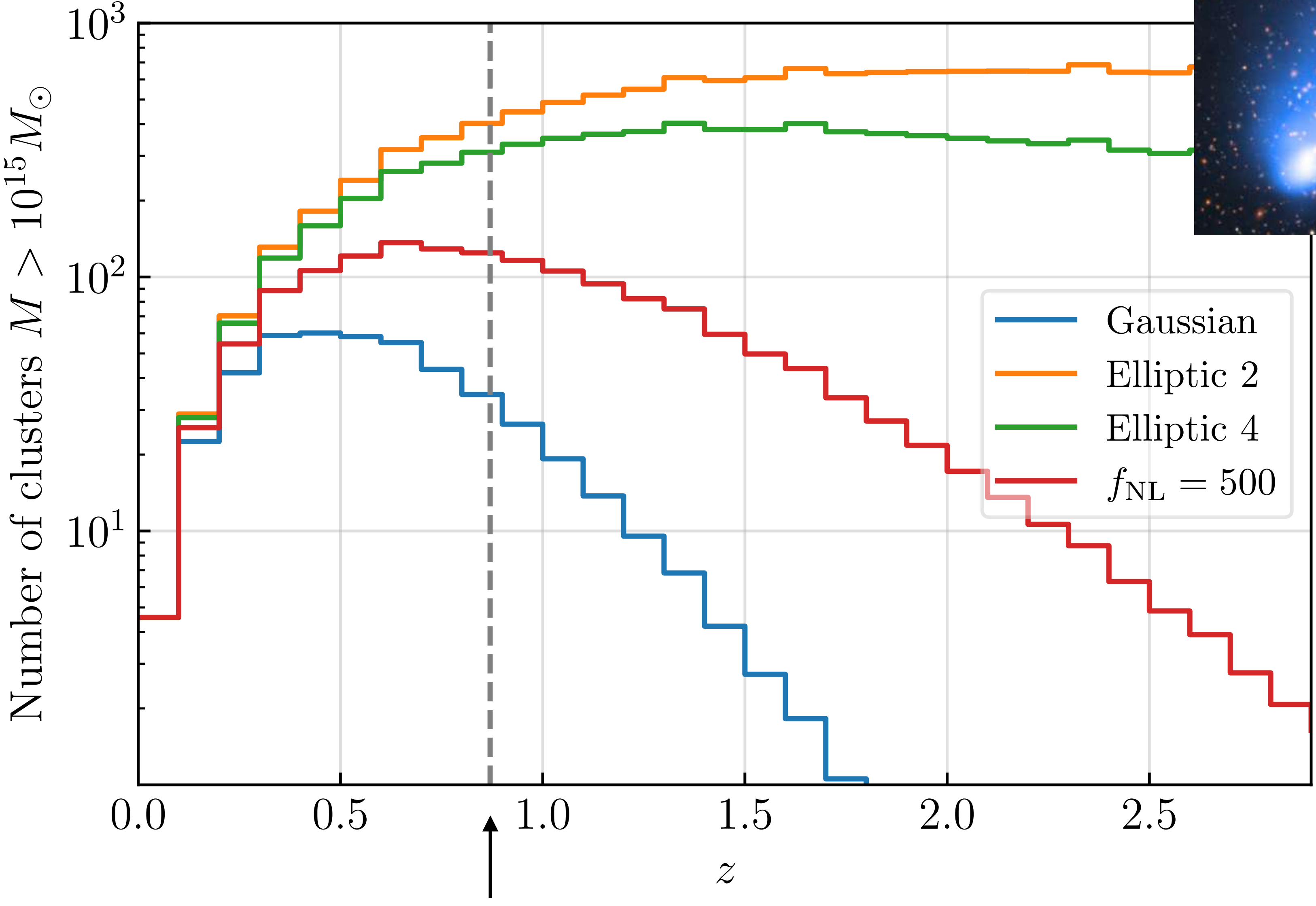
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

Impact on PBHs



Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

Impact on LSS

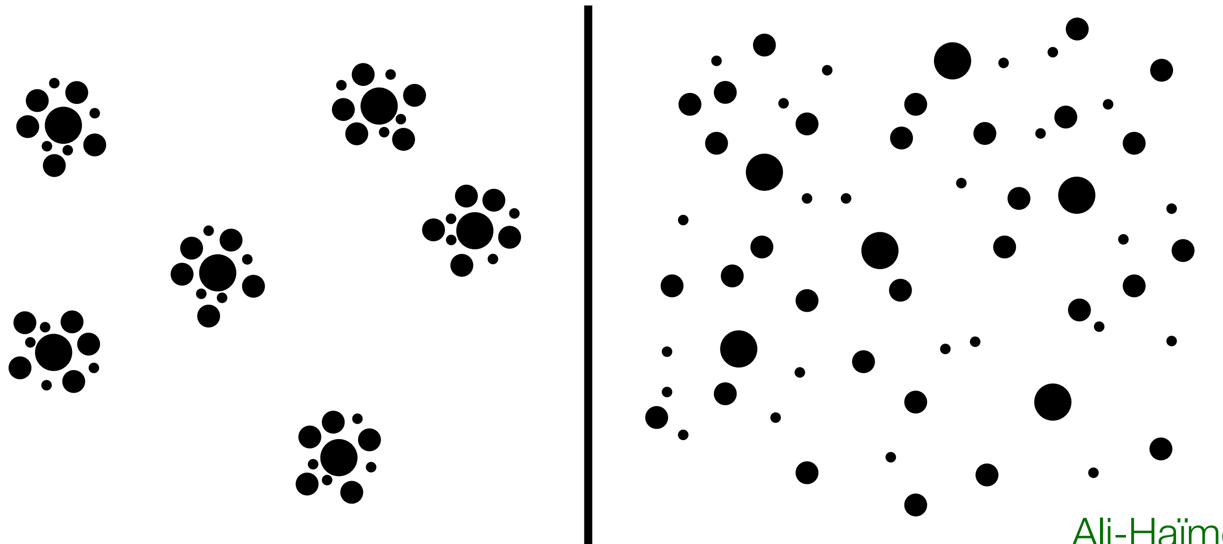


El gordo

Ezquiaga, Garcia-Bellido, VV (2022)

Reconstructing spatial information

- One realisation of the Langevin equation follows the worldline ending up in one Hubble patch at the end of inflation.
- By simulating many such realisations, we reconstruct the curvature perturbation $\zeta = \delta N$ in many such patches.
- However, information regarding the spatial arrangement of these patches is lost.
- In particular, can we determine whether PBHs are uniformly distributed in space or form in clusters?



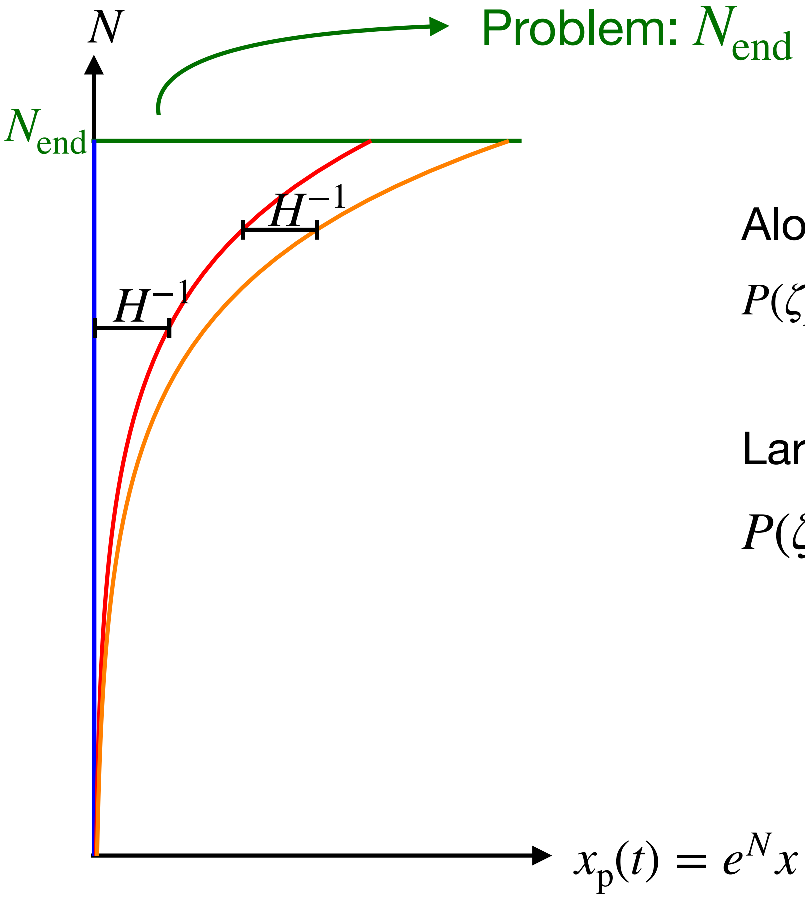
Ali-Haïmoud (2018)

Would lead to larger merger rates

Expected from large peaks of Gaussian random fields

Spatial reconstruction

Problem: $N_{\rm end}$ is not fixed!



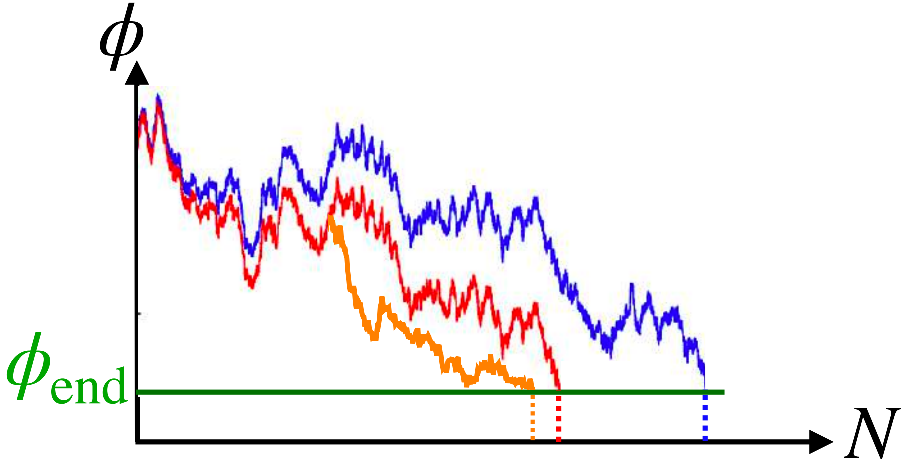
The final distance between two worldlines is encoded in the time at which they become stochastically independent.

Along a reference trajectory (Tada & VV 2021), at small distances

$$P(\zeta_R) = \int_\Omega \mathrm{d}\boldsymbol{\Phi}_* P_{\rm bw}\left[\boldsymbol{\Phi}_* \,|\, N_{\rm bw}(R)\right] P_{\mathrm{FPT},\boldsymbol{\Phi}_0\to\boldsymbol{\Phi}_*}\left[\zeta_R - \langle \mathcal{N}(\boldsymbol{\Phi}_*)\rangle + \langle \mathcal{N}(\boldsymbol{\Phi}_0)\rangle\right]$$

Large-volume limit (Animali & VV 2024), at large distances

$$P(\zeta_R) = \int_{\mathcal{S}_* = \{\boldsymbol{\Phi}_*, \langle V\rangle(\boldsymbol{\Phi}_*) = R^3\}} \mathrm{d}\boldsymbol{\Phi}_* P^{\rm V}_{\mathrm{FPTL},\boldsymbol{\Phi}_0\to\mathcal{S}_*}\left(\mathcal{N}_{\boldsymbol{\Phi}_0\to\mathcal{S}_*} = \zeta_R - \langle \mathcal{N}_{\boldsymbol{\Phi}_*}\rangle_{\rm V} + \langle \mathcal{N}_{\boldsymbol{\Phi}_0}\rangle_{\rm V}, \boldsymbol{\Phi}_*\right)$$

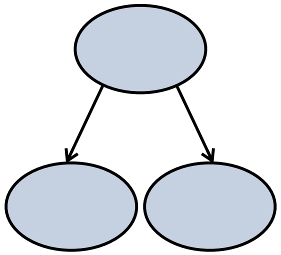


Stochastic trees

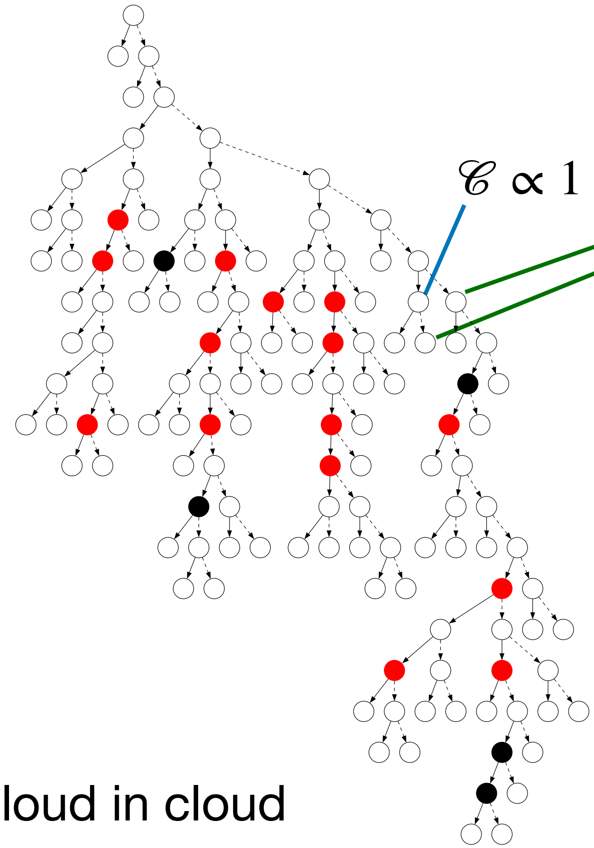
Animali, Auclair, Blachier, Vennin (2025)

Animali, Auclair, Blachier, Tomberg, Vennin (2026)

- As one Hubble patch expands during inflation, it gives rise to two Hubble patches.



- When this structure is repeated iteratively, this gives rise to a tree.



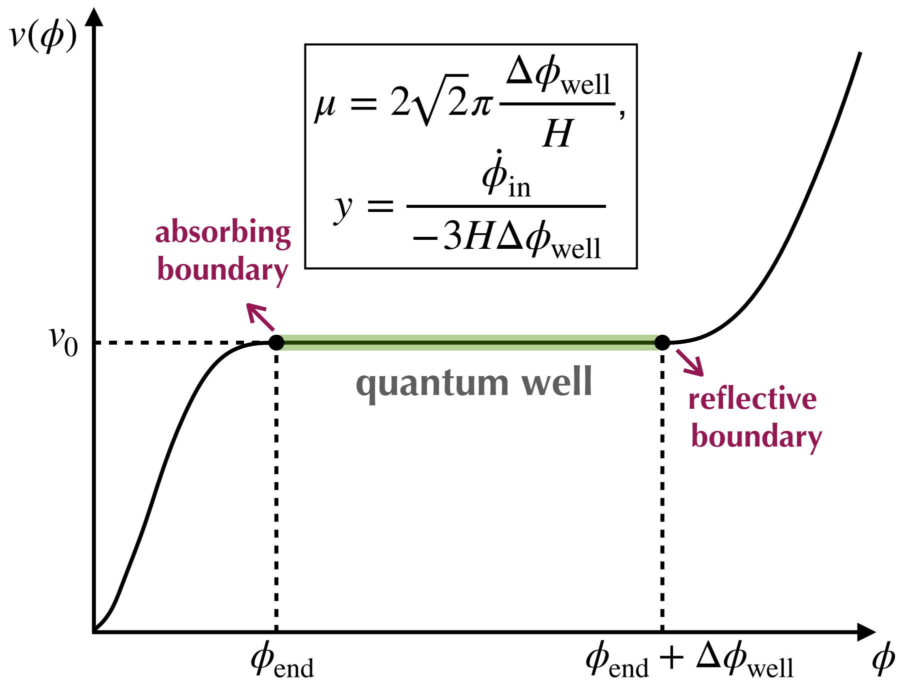
$\mathscr{C} \propto 1 - \ln_2(V_i/V_j)$

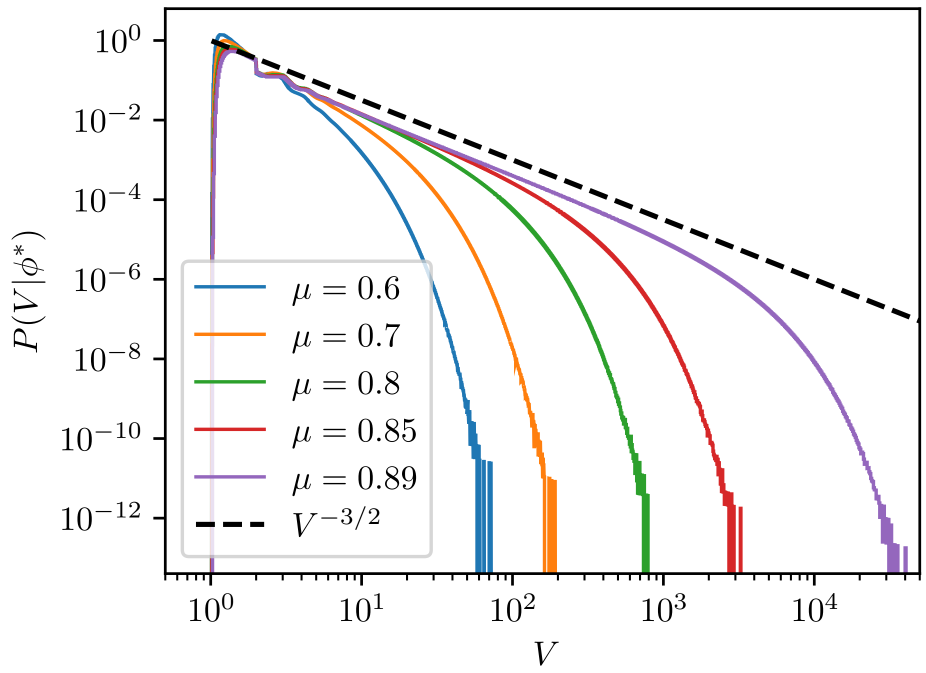
Why? c.f. part II!

- Automatically accounts for cloud in cloud

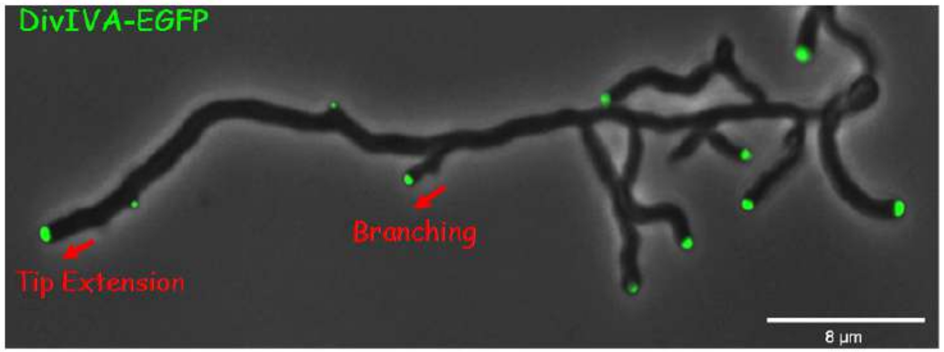
PBH mass distribution

Animali, Auclair, Blachier, Vennin (2025)





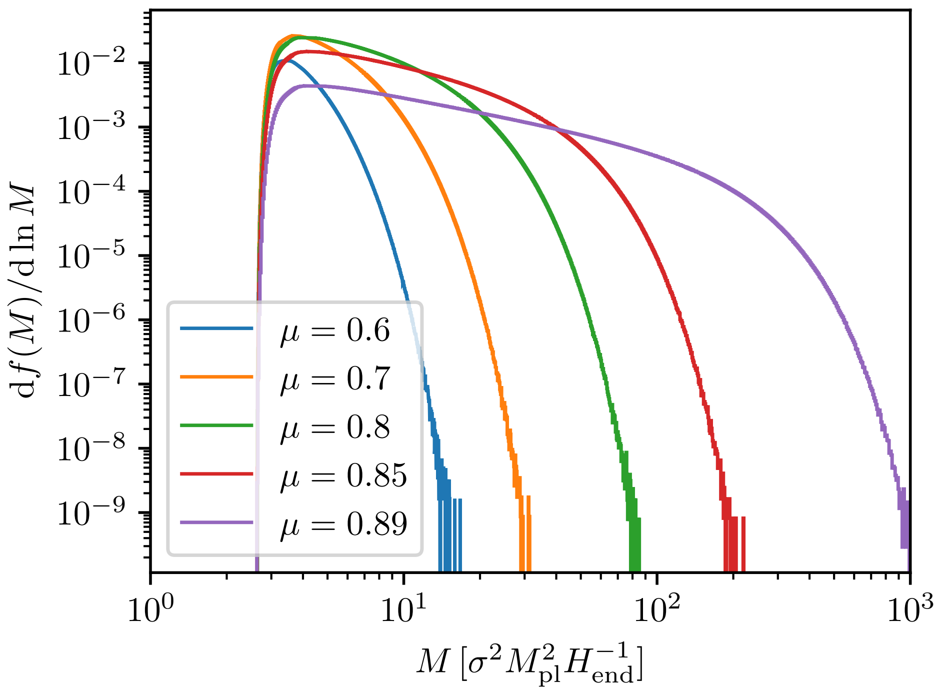
- μ controls the width of the field region dominated by quantum diffusion
- $P(V) \propto V^{-3/2}$ is expected from bacteria models (Galton-Watson processes)
- Bacteria non-extinction ↔ eternal inflation!

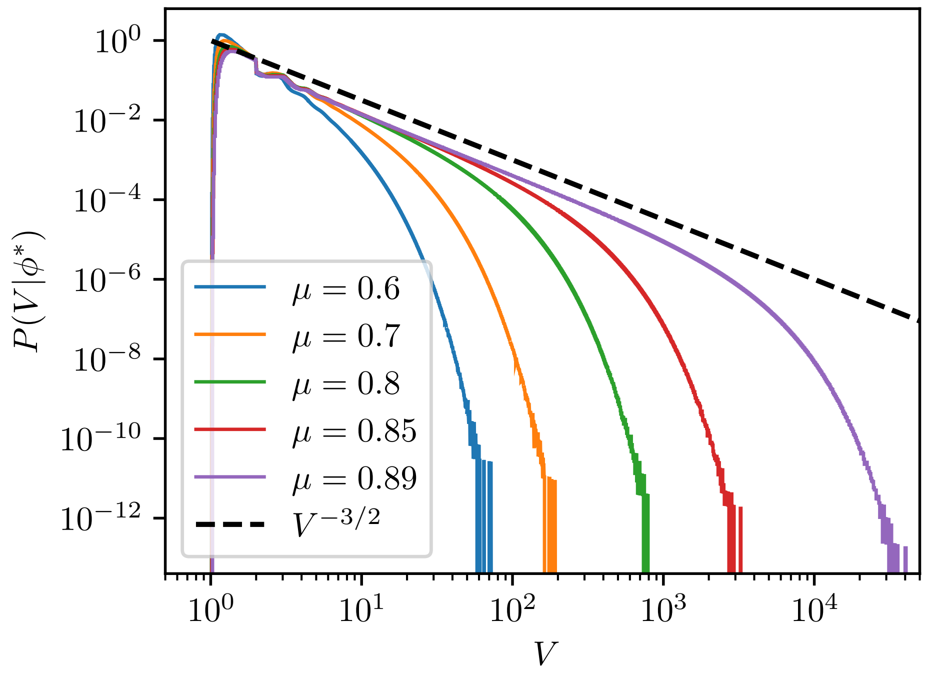


Streptomyces bacteria, Hempel *et al* 2005

PBH mass distribution

Animali, Auclair, Blachier, Vennin (2025)





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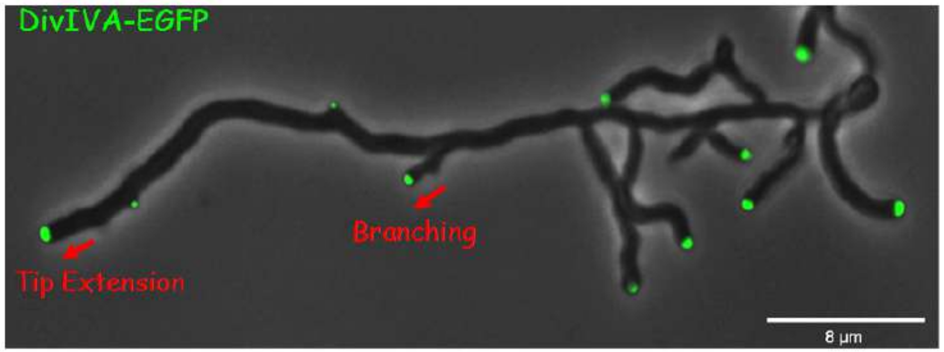
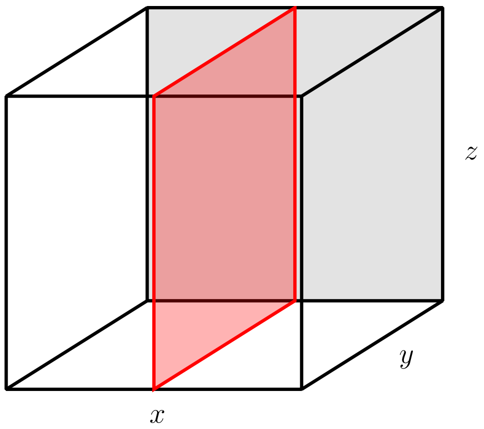
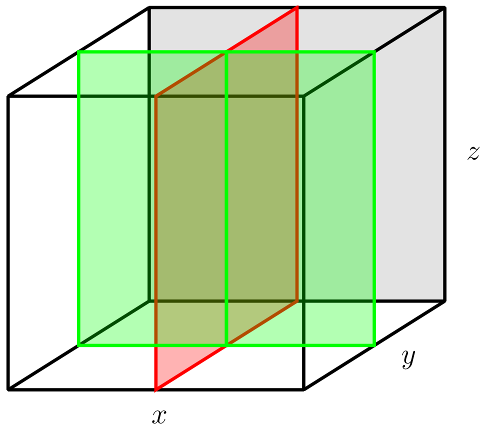


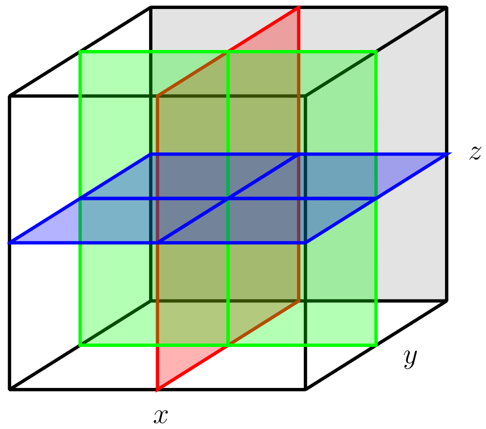
Streptomyces bacteria, Hempel *et al* 2005

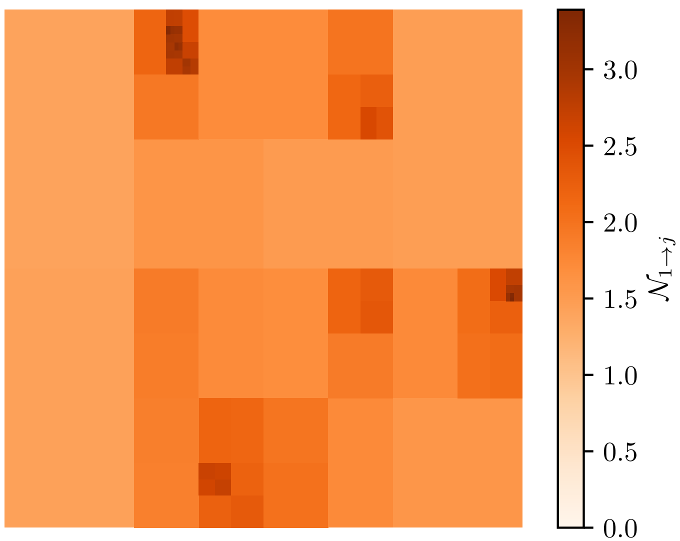
Creating maps

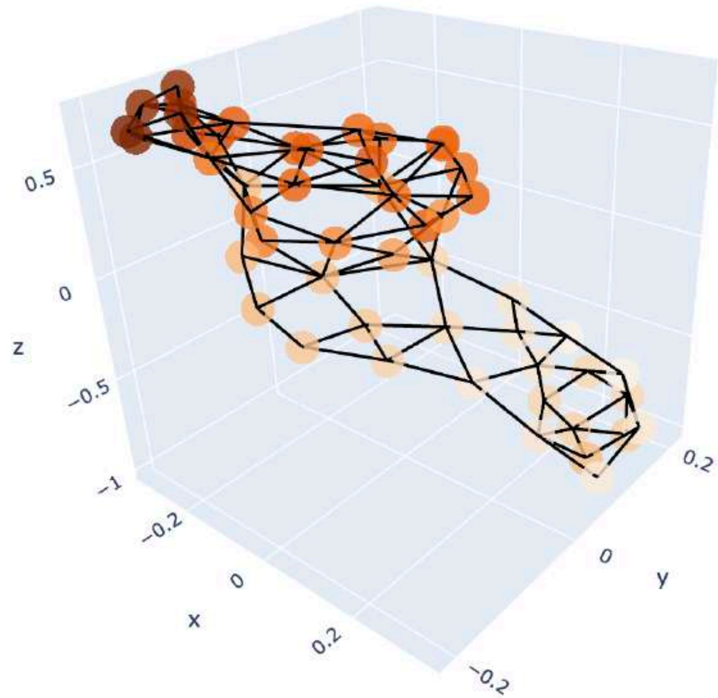
Animali, Auclair, Blachier, Tomberg, Vennin (in progress)











Conclusions

- The backreaction of quantum fluctuations on an inflating background can be incorporated within the formalism of stochastic inflation
- This is necessary to describe regimes leading to large fluctuations, such as those yielding primordial black holes
- Quantum diffusion leads to exponential tails: non-perturbative breakdown of Gaussian statistics
- Most cosmological observables can be reconstructed from first-passage time analysis in multi-type branching processes (power spectrum, mass functions, n-point functions, etc.)
- What about PBH clustering, type I versus type II fluctuations? Stay tuned for part II with Chiara Animali!