Cluster of Excellence PRISMA++

Mainz Institute for Theoretical Physics

B-meson Di-Light-Cone Distribution Amplitudes (DLCDAs)

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PRISMA ++ Cluster of Excellence & MITP
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with

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MITP
SCIENTIFIC PROGRAM

Flavourful Opportunities in Semileptonic B decays : Connecting Theory and Experiment

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Based on arXiv:2606.20267 [hep-ph]

Motivations

- Light-cone distribution amplitudes (LCDAs) :

→ Universal key ingredient to parameterize non-perturbative dynamics in exclusive B decays !

- Describe long-distance dynamics at large recoil for exclusive decays (QCDF, LCSRs) :
  - Soft momentum distribution for light quark and gluons inside the B-meson along a single light-cone direction (n^μ), b quark described via HQET
  - Studied extensively over the past decades and classified using twist counting

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($\overline{q}(z_1 n) \to \overline{q}(z_1 n)[z_1 n, 0]$, $G_{\mu\nu}(z_2 n) \to [0, z_2 n] G_{\mu\nu}(z_2 n)[z_2 n, 0]$ for gauge invariance!)

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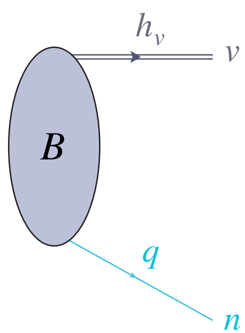

Two-particle LCDAs : $\phi_+^B(\omega;\mu), \phi_-^B$

$$\sim \int \frac{d\omega}{2\pi} e^{i\omega z} \langle 0 | \overline{q}(zn) \, \Gamma \, h_v(0) | \bar{B}(v) \rangle$$

[Grozin, Neubert 97], [Beneke, Feldman 01], [Lange, Neubert,03],…

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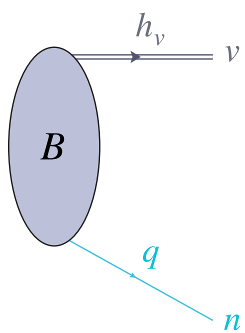

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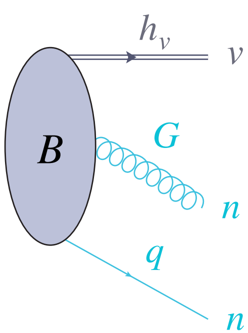

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Three-particle LCDAs : $\phi_3^B(\omega_1,\omega_2;\mu), \phi_4^B, \psi_4^B, \phi_5^B, \ldots$

$$\sim \int \frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi} e^{i(\omega_1 z_1+\omega_2 z_2)} \langle 0 | \bar{q}(z_1 n) g G_{\mu\nu}(z_2 n)\, \Gamma\, h_v(0) | \bar{B}(v) \rangle$$

[Kawamura, Kodaira,et al. 01], [Geyder,Witzel, 05], [Braun, Ji, et al. 17],...

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Why/When do we need a generalization to multi-light-cone functions ?

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- (QED) Two-particle DLCDAs :
  - QED corrections to $B_q \to \mu^+\mu^-$ [Beneke, Bobeth, et al. 19]
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  - Charm loop contributions in rare $B_{d,s} \to \gamma\gamma$ decays,... [Qin, Shen, et al. 23]
  - ..., and $B \to K^{(*)}\ell^-\ell^+$! [To be done !!!!]
  - Subleading shape function in inclusive $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\ell^+\ell^-$ [Bartocci , Böer, et al. 24]
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This talk !

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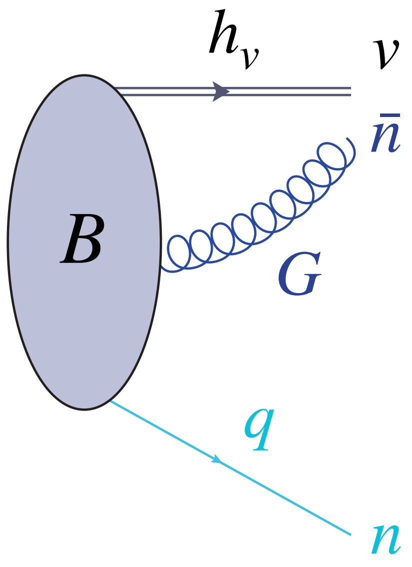
The plan : discovering those new DAs !

Goal : Systematic study of three-particle DLCDAs

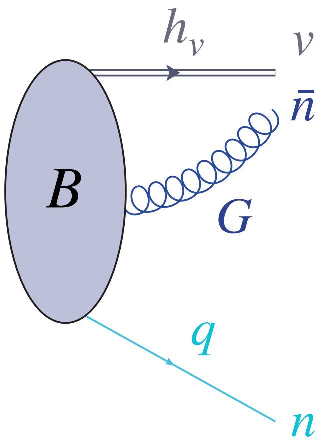
- Classification of the three-particle DLCDAs
- Models and constraints
- Radiative tail implementation



Classification

Definition

- Three-particle B-meson DLCDAs parameterises the trilocal hadronic matrix elements :



$$\langle 0 \,|\, \overline{q}(z_1 n)[z_1 n, 0]\, [0, z_2\bar{n}]\, g G_{\mu\nu}(z_2\bar{n})\, [z_2\bar{n}, 0]\, \Gamma\, h_v(0) \,|\, \bar{B}(v) \rangle$$

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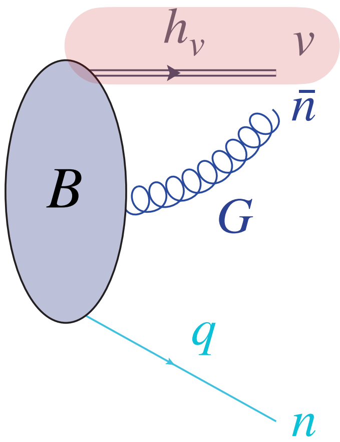

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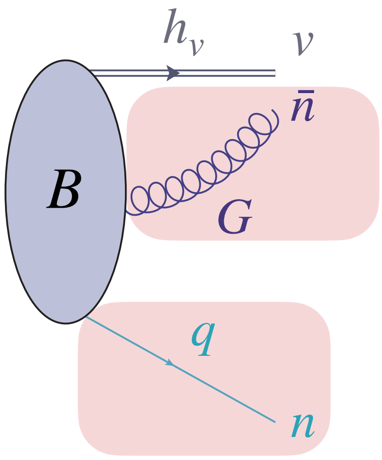

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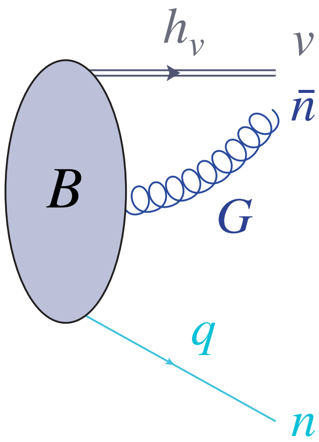

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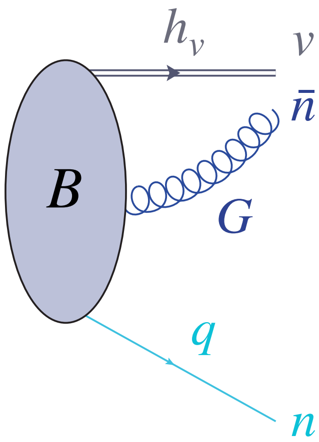

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- Γ spans the basis of Dirac matrices $\rightarrow$ defines linear combinaisons of DLCDAs

Parameterization of the DLCDAs

▸ DLCDAs basis : derived from the general decomposition the matrix elements for $\Gamma = \gamma_5 \gamma_\rho \sigma_{\alpha\beta}$ and imposing the Chilshom identity [Geyder,Witzel, 05]

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- Like the single light-cone case , we find a basis of 8 independent DLCDAs : [Braun, Ji, et al. 17]

$$
\begin{aligned}
&\langle 0 | \bar{q}(z_1 n)\, gG_{\mu\nu}(z_2\bar{n})\, \Gamma h_v(0) | \bar{B}(v)\rangle = \\
&\quad +\frac{1}{2} m_B F_B(\mu)\, \mathrm{Tr}\Big\{ \gamma_5 \Gamma P_+ \Big[\Big(n_\mu\gamma_\nu - n_\nu\gamma_\mu\Big)\, Y_x + \Big(\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu\Big)\, Y_y \\
&\quad + i\,\epsilon_{\mu\nu\alpha\beta}\, n^\alpha\gamma^\beta\gamma_5\, \widetilde{Y}_x + i\,\epsilon_{\mu\nu\alpha\beta}\, \bar{n}^\alpha\gamma^\beta\gamma_5\, \widetilde{Y}_y \\
&\quad + (n_\mu\gamma_\nu - n_\nu\gamma_\mu)\, \not{n}\, Z_x + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\, \not{\bar{n}}\, Z_y \\
&\quad - i\,\epsilon_{\mu\nu\alpha\beta}\, n^\alpha\bar{n}^\beta\gamma_5\, \widetilde{X}_{xy} - \Big(n_\mu\bar{n}_\nu - n_\nu\bar{n}_\mu\Big)\, \not{n}\, W_{xy} \Big] \Big\}(z_1, z_2; \mu)\,,
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$$+ (n_\mu\gamma_\nu - n_\nu\gamma_\mu)\, \not{\bar{n}}\, Z_x + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\, \not{n}\, Z_y$$

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\end{aligned}$$

$$P_+ := \frac{1 + \not{v}}{2}$$

- This analysis can be generalized for two arbitrary light-cone vector x^μ, y^μ

$\rightarrow$ 27 independent *generalised* DLCDAs $\Phi_i(v \cdot x, v \cdot y, x \cdot y)$ (still holds for $x^2, y^2 \neq 0$)

Twist counting

▸ One can construct DLCDAs of definite (anti)-collinear twist [Braun, Ji, et al. 17]

→ classification following the SCET power-counting rules

▸ Light-quark have leading and subleading projections

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$\tau = d - s$ d : Dimension
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$\bar{q}(z_1 n)\frac{\not{n}\,\not{\bar{n}}}{4}$	$\bar{q}(z_1 n)\frac{\not{\bar{n}}\,\not{n}}{4}$	$h_v(0)$
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- Similar for the gluon field-strenght tensor

$$\bar{n}^\mu G_{\mu\nu_\perp}(z_2\bar{n}) \qquad \bar{n}^\mu n^\nu G_{\mu\nu}(z_2\bar{n}),\, G_{\mu_\perp\nu_\perp}(z_2\bar{n}) \qquad n^\mu G_{\mu\nu_\perp}(z_2\bar{n})$$

$$\tau_G = 1 \qquad \tau_G = 2 \qquad \tau_G = 3$$

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▸ At twist three, we find a single operator

$$2m_B F_B(\mu)\,\Phi_3^{(n\bar{n})}(z_1, z_2; \mu) = \langle 0 |\, \bar{q}(z_1 n)\not{n} \;\; \bar{n}^\nu\, g G_{\mu_\perp\nu}(z_2\bar{n}) \;\; \gamma_\perp^\mu \gamma_5 h_v(0)\, | \bar{B}(v)\rangle$$

$$\Phi_3^{(n\bar{n})} = 2(Y_x - \widetilde{Y}_x)$$

Twist counting

- Followed by 3 twist four operators

$$
\boxed{
\begin{aligned}
2m_B F_B(\mu)\,\Phi_4^{(n\bar{n})}(z_1,z_2;\mu) &= \langle 0|\,\bar{q}(z_1 n)\,\not{\bar{n}}\quad \bar{n}^\nu\, gG_{\mu_\perp \nu}(z_2\bar{n})\quad \gamma_\perp^\mu \gamma_5\, h_v(0)\,|\bar{B}(v)\rangle,\\
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- 3 twist five operators

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- 3 twist five operators

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- And a single twist six operator

$$2m_B F_B(\mu)\,\Phi_6^{(n\bar{n})}(z_1,z_2;\mu) = \langle 0|\,\bar{q}(z_1 n)\,\not{\bar{n}}\quad n^\nu\, gG_{\mu_\perp\nu}(z_2\bar{n})\quad \gamma_\perp^\mu\gamma_5\, h_v(0)\,|\bar{B}(v)\rangle,$$

Twist counting

$$\widetilde{G}_{\mu\nu} := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$$

- Followed by 3 twist four operators

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► We assume positive support for the DLCDAs :

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- Defining the Fourier transform,

$$F^{(n\bar{n})}(z_1, z_2) := \int_0^\infty d\omega_1 \int_0^\infty d\bar{\omega}_2\, e^{-i\omega_1 z_1 - i\bar{\omega}_2 z_2} f^{(n\bar{n})}(\omega_1, \bar{\omega}_2)$$

we model the DLCDAs by a factorized ansatz :

$$f^{(n\bar{n})}(\omega_1, \bar{\omega}_2) = N_{[F]}\, \varphi_1^{[f]}(\omega_1)\, \varphi_2^{[f]}(\bar{\omega}_2) \qquad F \in \{\Phi_3, \ldots, \Phi_6\}, f \in \{\phi_3, \ldots, \phi_6\}$$

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$$\boxed{\begin{aligned} \varphi_1^{[f]}(\omega_1) &= \frac{1}{k!}\left(\frac{\omega_1}{\omega_0}\right)^k \frac{e^{-\frac{\omega_1}{\omega_0}}}{\omega_0}\, \frac{1 + b_{[f]}\,\omega_1/\omega_0}{1 + (1+k)\, b_{[f]}}\, \theta(\omega_1) \\ \varphi_2^{[f]}(\bar{\omega}_2) &= \frac{1}{\bar{k}!}\left(\frac{\bar{\omega}_2}{\bar{\omega}_0}\right)^{\bar{k}} \frac{e^{-\frac{\bar{\omega}_2}{\bar{\omega}_0}}}{\bar{\omega}_0}\, \frac{1 + c_{[f]}\,\bar{\omega}_2/\bar{\omega}_0}{1 + (1+\bar{k})\, c_{[f]}}\, \theta(\bar{\omega}_2) \end{aligned}}$$

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→Local limit related to matrix elements with two covariant derivatives

[Grozin, Neubert 97],[Braun, Ji, et al. 17]

$$\langle 0 | \bar{q} \left[i\overleftarrow{D}_\nu, i\overleftarrow{D}_\mu \right] \Gamma\, h_v \,|\, \bar{B}(v)\rangle = -\frac{i}{6} m_B F_B(\mu)\,\mathrm{Tr}\left\{ \gamma_5 \Gamma P_+ \left[i\,\lambda_H^2 \sigma_{\mu\nu} + (\lambda_H^2 - \lambda_E^2)(v_\mu\gamma_\nu - v_\nu\gamma_\mu) \right] \right\}$$

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- $N_{[F]}$ completely parameterized by λ_E^2 and λ_H^2!

$$N_{\Phi_3} = \frac{1}{3}(\lambda_E^2 + \lambda_H^2), \qquad N_{\Phi_4} = \frac{1}{3}(\lambda_E^2 - \lambda_H^2), \qquad \ldots$$

Low momentum limit

$$\varphi_1^{[f]}(\omega_1) = \frac{1}{k!}\boxed{\left(\frac{\omega_1}{\omega_0}\right)^k} \frac{e^{-\frac{\omega_1}{\omega_0}}}{\omega_0} \frac{1 + b_{[f]}\,\omega_1/\omega_0}{1 + (1+k)\,b_{[f]}}\,\theta(\omega_1)$$

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$\bar{q}(z_1 n)\frac{\not{n}\,\bar{\not{n}}}{4}$	$\bar{q}(z_1 n)\frac{\bar{\not{n}}\,\not{n}}{4}$	$\bar{n}^\mu G_{\mu\nu_\perp}(z_2\bar{n})$	$\bar{n}^\mu n^\nu G_{\mu\nu}(z_2\bar{n})\,,\, G_{\mu_\perp\nu_\perp}(z_2\bar{n})$	$n^\mu G_{\mu\nu_\perp}(z_2\bar{n})$
$j_{\bar{q}} = 1$	$j_{\bar{q}} = \frac{1}{2}$	$j_G = \frac{3}{2}$	$j_G = 1$	$j_G = \frac{1}{2}$

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- For our twist definite basis of DLCDAs, we find

$$\phi_i \sim \omega_1^{2j_{\bar{q}}-1}\,\bar{\omega}_2^{\,2j_G-1} \qquad (k, \bar{k})$$

ϕ_3	ϕ_4	$\psi_4, \widetilde{\psi}_4$	ϕ_5	$\psi_5, \widetilde{\psi}_5$	ϕ_6
$(1,2)$	$(0,2)$	$(1,1)$	$(1,0)$	$(0,1)$	$(0,0)$

Shape parameters

$$\varphi_1^{[f]}(\omega_1) = \frac{1}{k!}\left(\frac{\omega_1}{\omega_0}\right)^k \frac{e^{-\frac{\omega_1}{\omega_0}}}{\omega_0} \boxed{\frac{1 + b_{[f]}\,\omega_1/\omega_0}{1 + (1+k)\,b_{[f]}}} \theta(\omega_1)$$

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- The moments of the DLCDAs $\langle \omega_1^m\,\bar{\omega}_2^n \rangle_{[F]} := \int_0^\infty d\omega_1 \int_0^\infty d\bar{\omega}_2\, \omega_1^m\,\bar{\omega}_2^n\, f^{(n\bar{n})}(\omega_1, \bar{\omega}_2)$

depend on those shape parameters, N_F and the reference scales ω_0 and $\bar{\omega}_0$!

$$\langle \omega_1 \rangle_{[F]} = \omega_0\, N_{[F]} \frac{(1+k)\big(1 + (2+k)\, b_{[f]}\big)}{1 + (1+k)\, b_{[f]}}\,, \quad \langle \bar{\omega}_2 \rangle_{[F]} = \bar{\omega}_0\, N_{[F]} \frac{(1+\bar{k})\big(1 + (2+\bar{k})\, c_{[f]}\big)}{1 + (1+\bar{k})\, c_{[f]}}$$

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→Our model parameters are not independent and follow EOMs constraints!

First moments constraints

$$\left[\left[i\overleftarrow{D}_\rho, i\overleftarrow{D}_\nu\right], i\overleftarrow{D}_\mu\right] := -\, g\, \mathscr{D}^{\mathrm{adj}}_\mu G_{\nu\rho}\,.$$

- First moments related to local matrix elements with three covariant derivatives

$$2iF_B\,\langle\omega_1\rangle_{[F]} = \langle 0\left|\,\bar{q}\; i\overleftarrow{D}_\rho\,\left[i\overleftarrow{D}_\nu, i\overleftarrow{D}_\mu\right]\,\Gamma_F^{\mu\nu}\,h_v\right|\bar{B}(v)\rangle\, n^\rho$$

$$2iF_B\,\langle\bar{\omega}_2\rangle_{[F]} = \langle 0\left|\,\bar{q}\,\left[\left[i\overleftarrow{D}_\rho, i\overleftarrow{D}_\nu\right], i\overleftarrow{D}_\mu\right]\,\Gamma_F^{\mu\nu}\,h_v\right|\bar{B}(v)\rangle\, \bar{n}^\mu$$

→ Parameterized by 14 independent structures !

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- 8 exact constraints from the quark EOMs $\rightarrow$ depend on $\bar{\Lambda} = m_B - m_b\,,\ \lambda_E^2,\ \lambda_H^2$

$$\boxed{\begin{aligned} \langle 0 | \bar{q}\, i\, \overleftarrow{\not{D}}\, i\overleftarrow{D}_\nu\, i\overleftarrow{D}_\mu\, \Gamma\, h_v | \bar{B}(v)\rangle &= 0 \\ \langle 0 | \bar{q}\, i\overleftarrow{D}_\nu\, i\overleftarrow{D}_\rho\, i(v\cdot \overleftarrow{D})\, \Gamma\, h_v | \bar{B}(v)\rangle &= \bar{\Lambda}\, \langle 0 | \bar{q}\, i\overleftarrow{D}_\rho\, i\overleftarrow{D}_\nu\, \Gamma\, h_v | \bar{B}(v)\rangle \end{aligned}}$$

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- 2 more approximate contraints from gluon EOM $\quad \boxed{i\mathscr{D}^{\mathrm{adj}}_\mu G^{\mu\rho} \approx 0}$

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$\rightarrow$All first moments parameterized by $\langle\omega_1\rangle_{[\phi_3],[\phi_4],[\psi_4],[\phi_5]}$ and $\bar\Lambda, \lambda_E^2, \lambda_H^2$!!!

Illustration for ϕ_3

$$\boxed{\phi_3^{(n\bar{n})}(\omega_1, \bar{\omega}_2) = N_{\phi_3}\, \varphi_1^{\phi_3}(\omega_1)\, \varphi_2^{\phi_3}(\bar{\omega}_2)}$$

$$\varphi_1^{\phi_3}(\omega_1) = \frac{\omega_1}{\omega_0^2}\, e^{-\frac{\omega_1}{\omega_0}}\, \frac{1 + b_{\phi_3}\, \omega_1/\omega_0}{1 + 2\, b_{\phi_3}}\, \theta(\omega_1)$$

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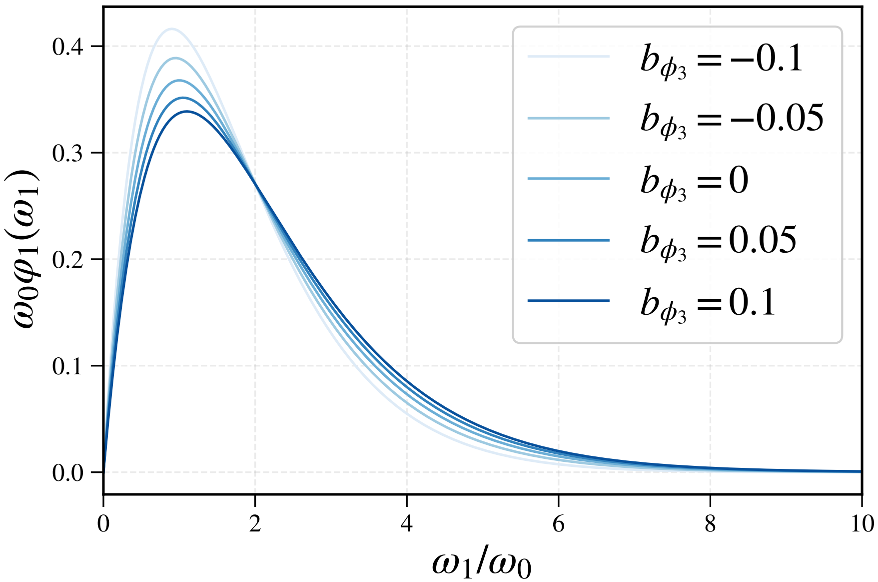

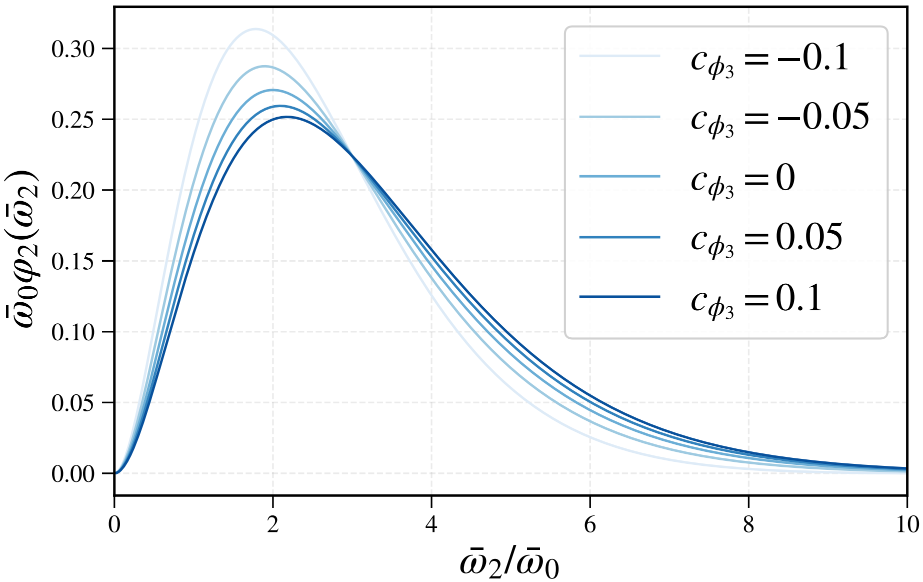

Radiative tails of ϕ_3

One-loop RGE for Φ_3

- The RGE of Φ_3 is know at one-loop in momentum-space [Huang , Ji, et al. 23],[Bartocci , Böer, et al. 24] and can be translated in position space [Beneke , Böer *to appear*]

$$\boxed{\frac{d}{d\ln\mu}\Phi_3(z_1,z_2;\mu) = \int_0^{1^+} dt\, G_1(t,z_1;\mu)\,\Phi_3(tz_1,z_2;\mu) + \int_0^{1^+} dt\, G_2(t,z_2;\mu)\,\Phi_3(z_1,tz_2;\mu) + g_3(z_1,z_2;\mu)\,\Phi_3(z_1,z_2;\mu)}$$

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$$+g_3(z_1,z_2;\mu)\,\Phi_3(z_1,z_2;\mu)$$

$$G_1(t,z_1;\mu)=-\frac{\alpha_s C_F}{\pi}\Gamma_{\mathrm{LN}}(t,z_1;\mu)\qquad G_2(t,z_2;\mu)=-\frac{\alpha_s C_A}{\pi}t\,\Gamma_{\mathrm{LN}}(t,z_2;\mu)$$

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$\rightarrow$ Independent Lange-Neubert evolution for quark and gluon part !

$$\Gamma_{\mathrm{LN}}(t,z;\mu) = \left(\ln\left(i(z-i0^+)\,\hat\mu\right) + \frac{1}{2}\right)\delta(1-t) - \left[\frac{t}{1-t}\right]_+ , \quad \hat\mu = \mu\, e^{\gamma_E}$$

[Braun , Ivanov 04]

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- (Partonic) final-state interactions generate (perturbatively) an imaginary part !
- We set $\theta(z_1)\,\theta(z_2) = 1$, reflected by assuming $\omega_1, \omega_2 \geq 0$ in momentum space !

$\longrightarrow$ Leads to a reduced RGE, preserving the « tree-level » factorized form

$$\boxed{\phi_3^{(n\bar{n})}(\omega_1,\bar{\omega}_2) = N_{\phi_3}\,\varphi_1^{\phi_3}(\omega_1)\,\varphi_2^{\phi_3}(\bar{\omega}_2)}$$

Iterative solution

$L_1 = \ln(i z_1 \hat{\mu})$ $\quad L_2 = \ln(i z_2 \hat{\mu})$ $\quad \hat{\mu} = \mu\, e^{\gamma_E}$

- This RGE can be solved iteratively : [Feldmann, Vladimirov 25]

$$\boxed{\Phi_3(z_1, z_2; \mu) = \Phi_3^{(0)}(z_1, z_2) + \frac{\alpha_s}{4\pi}\Phi_3^{(1)}(z_1, z_2; \mu) + \mathcal{O}(\alpha_s^2)}$$

Iterative solution

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- One-loop correction given by the RGE:

$$\Phi_3^{(1)}(z_1, z_2; \mu)\Big|_{G_1} = -C_F\left(\left(4L_1 - 2\ln\frac{\mu}{\mu_1(z_1)} + 2\right)\ln\frac{\mu}{\mu_1(z_1)} + c_1(z_1)\right)\Phi_3^{(0)}(z_1, z_2)$$
$$+ C_F\left(4\ln\frac{\mu}{\mu_1(z_1)} + c_2(z_1)\right)\int_0^1 dt\left[\frac{t}{1-t}\right]_+\Phi_3^{(0)}(tz_1, z_2),$$
$$\Phi_3^{(1)}(z_1, z_2; \mu)\Big|_{g_3} = C_A\frac{1-i\pi}{2}\left(4\ln\frac{\mu}{\mu_3(z_1, z_2)} + c_5(z_1, z_2)\right)\Phi_3^{(0)}(z_1, z_2).$$

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&\quad + C_F \left(4\ln\frac{\mu}{\mu_1(z_1)} + c_2(z_1) \right) \int_0^1 dt \left[\frac{t}{1-t} \right]_+ \Phi_3^{(0)}(t z_1, z_2)\,, \\
\Phi_3^{(1)}(z_1, z_2; \mu)\Big|_{g_3} &= C_A \frac{1 - i\pi}{2} \left(4\ln\frac{\mu}{\mu_3(z_1, z_2)} + c_5(z_1, z_2) \right) \Phi_3^{(0)}(z_1, z_2)\,.
\end{aligned}$$

- The choice of the integration constants μ_i, c_i defines a renormalization scheme for $\Phi_3^{(0)}$

Short distance limit

$L_1 = \ln(iz_1\hat{\mu})$ $L_2 = \ln(iz_2\hat{\mu})$ $\hat{\mu} = \mu\, e^{\gamma_E}$

- The short-distance expansion can be obtained from a partonic one-loop matching to local operators
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- G_1 : Standard 2-particle LCDAs kernel leading to [Feldmann, Vladimirov 25]

$$\begin{aligned}\Phi_3^{(1)}(z_1, z_2; \mu)\Big|_{G_1} = &- C_F \left((2L_1 + 2)\, L_1 + \frac{5\pi^2}{12} \right) \left(N_{\Phi_3} - iz_1 \langle \omega_1 \rangle_{\Phi_3} - iz_2 \langle \bar{\omega}_2 \rangle_{\Phi_3} + \ldots \right) \\ &+ C_F \left(4L_1 + c_2\right) \left(\frac{i}{2}\, z_1 \langle \omega_1 \rangle_{\Phi_3} + \ldots \right)\end{aligned}$$

$$\rightarrow \mu_1(z_1) = e^{-\gamma_E}/(iz_1), \quad c_1(z_1) = \frac{5\pi^2}{12} \quad c_2 = \text{cst} \ \ \text{(on general grounds)}$$

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- G_2 : Similar expression
- g_3 : Non-trivial analytic structure only involving the gluon separation z_2

$$\Phi_3^{(1)}(z_1, z_2; \mu)\Big|_{g_3} = C_A\, \frac{1 - i\pi}{2} \left(4L_2 + c_5\right) \left(N_{\Phi_3} - iz_1 \langle \omega_1 \rangle_{\Phi_3} - iz_2 \langle \bar{\omega}_2 \rangle_{\Phi_3} + \ldots \right)$$

$\rightarrow \mu_3(z_1, z_2) = \mu_2(z_2) \rightarrow e^{-\gamma_E}/(iz_2),$ $\quad c_5 = \text{cst}$ (structure of the partonic diagrams)

Interpolation

- To interpolate the short distance limit with a generic hadronic model for finite z_1, z_2, we modify the value of the integration constants μ_i, c_i [Feldmann, Vladimirov 25]

$$\hat{\mu}_1(z_1) = \frac{1 + i\hat{\mu}_F z_1}{i z_1}, \quad c_2(z_1) = \frac{c_2}{1 + i\hat{\mu}_F z_1}, \quad \hat{\mu} = \mu e^{\gamma_E}$$

→New auxiliary scale μ_F introduced to adjust to the shape of Φ_3^0 for large value of $|z_1|, |z_2|$

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$$\boxed{\begin{aligned} N_{\Phi_3}(\mu_F) &= N_{\Phi_3}(0) \\ \langle \omega_1 \rangle_{\Phi_3}(\mu_F) &= \langle \omega_1 \rangle_{\Phi_3}(0) + \frac{\alpha_s C_F}{2\pi}\, \hat{\mu}_F\, N_{\Phi_3}(0) + \ldots \\ \langle \bar{\omega}_2 \rangle_{\Phi_3}(\mu_F) &= \langle \bar{\omega}_2 \rangle_{\Phi_3}(0) + \frac{\alpha_s C_A}{2\pi} (1 + i\pi)\, \hat{\mu}_F\, N_{\Phi_3}(0) + \ldots \end{aligned}}$$

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\end{aligned}$$

→suggests $\bar{\omega}_0 > \omega_0$

→Induces an $\mathcal{O}(\alpha_s)$ shift in the shape parameters $b_{\phi_3}(\mu_F), c_{\phi_3}(\mu_F)$!

Radiative tails

- Our result can be translated in momentum space and takes the factorized form

$$\phi_3(\omega_1, \bar{\omega}_2) = N_{\Phi_3} \left(\varphi_1^{(0)}(\omega_1) + \frac{\alpha_s}{4\pi} \varphi_1^{(1)}(\omega_1) + \dots \right) \left(\varphi_2^{(0)}(\bar{\omega}_2) + \frac{\alpha_s}{4\pi} \varphi_2^{(1)}(\bar{\omega}_2) + \dots \right)$$

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- For large momenta $\omega_1,\bar\omega_2 \gg \mu_F$, ϕ_3 develops radiative tails in both directions :

$$\varphi_1(\omega_1)\Big|_{\text{tail}} = \frac{\alpha_s C_F}{4\pi}\left(4\ln\frac{\mu}{\omega_1}+2\right)\frac{1}{\omega_1}+\frac{\alpha_s C_F}{4\pi}\left(4\ln\frac{\mu}{\omega_1}+8\right)\frac{1}{\omega_1^2}\,\frac{\langle\omega_1\rangle_{\Phi_3}}{N_{\Phi_3}}+\dots\,.$$

$$\varphi_2(\bar\omega_2)\Big|_{\text{tail}} = \frac{\alpha_s C_A}{4\pi}\left(4\ln\frac{\mu}{\bar\omega_2}+(2+2i\pi)\right)\frac{1}{\bar\omega_2}+\frac{\alpha_s C_A}{4\pi}\left(4\ln\frac{\mu}{\bar\omega_2}+\left(\frac{22}{3}+2i\pi\right)\right)\frac{1}{\bar\omega_2^2}\,\frac{\langle\bar\omega_2\rangle_{\Phi_3}}{N_{\Phi_3}}+\dots$$

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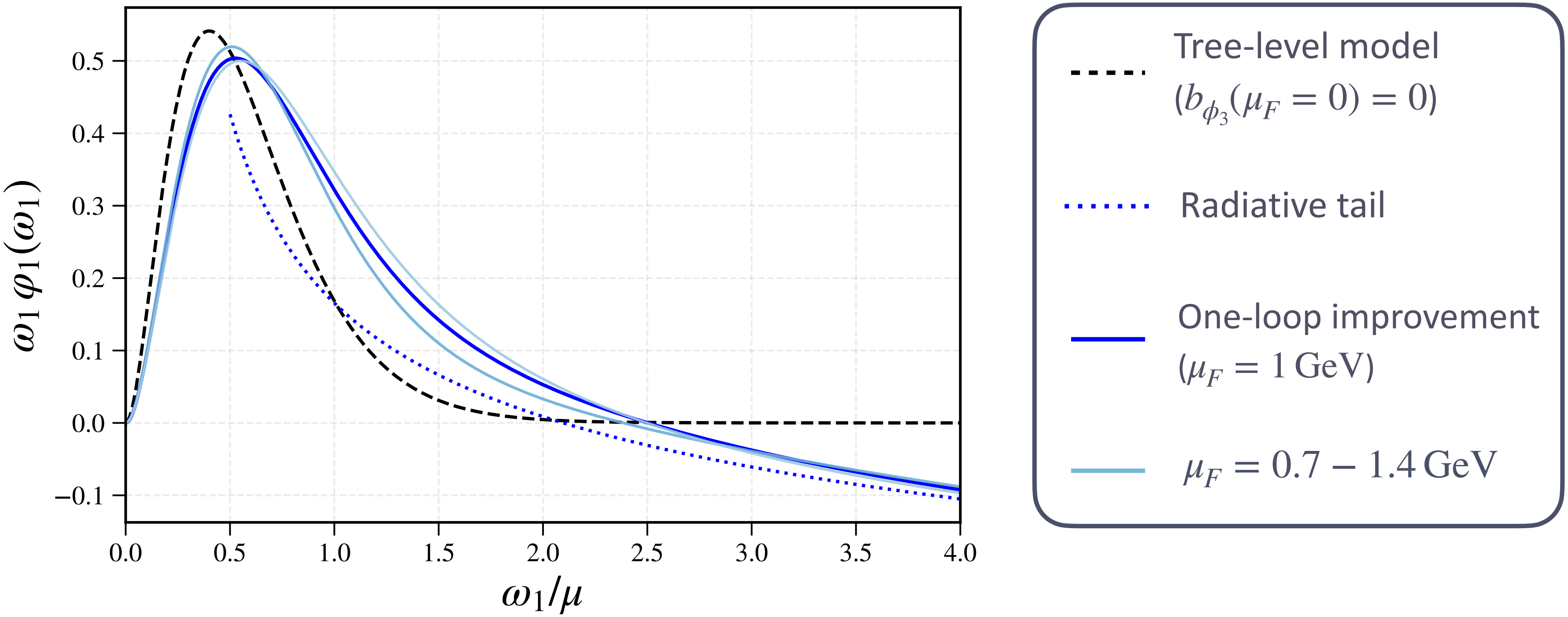
→Complex rescattering phases generated perturbatively!

Numerical illustration

$$\phi_3^{(n\bar{n})}(\omega_1, \bar{\omega}_2) = N_{\phi_3}\, \varphi_1^{\phi_3}(\omega_1)\, \varphi_2^{\phi_3}(\bar{\omega}_2)$$

- Input values : $\omega_0 = 0.3\,\text{GeV},\ \mu = 1.5\,\text{GeV},\ \alpha_s(\mu) = 0.3$ and $c_2 = 0$ (for simplicity)



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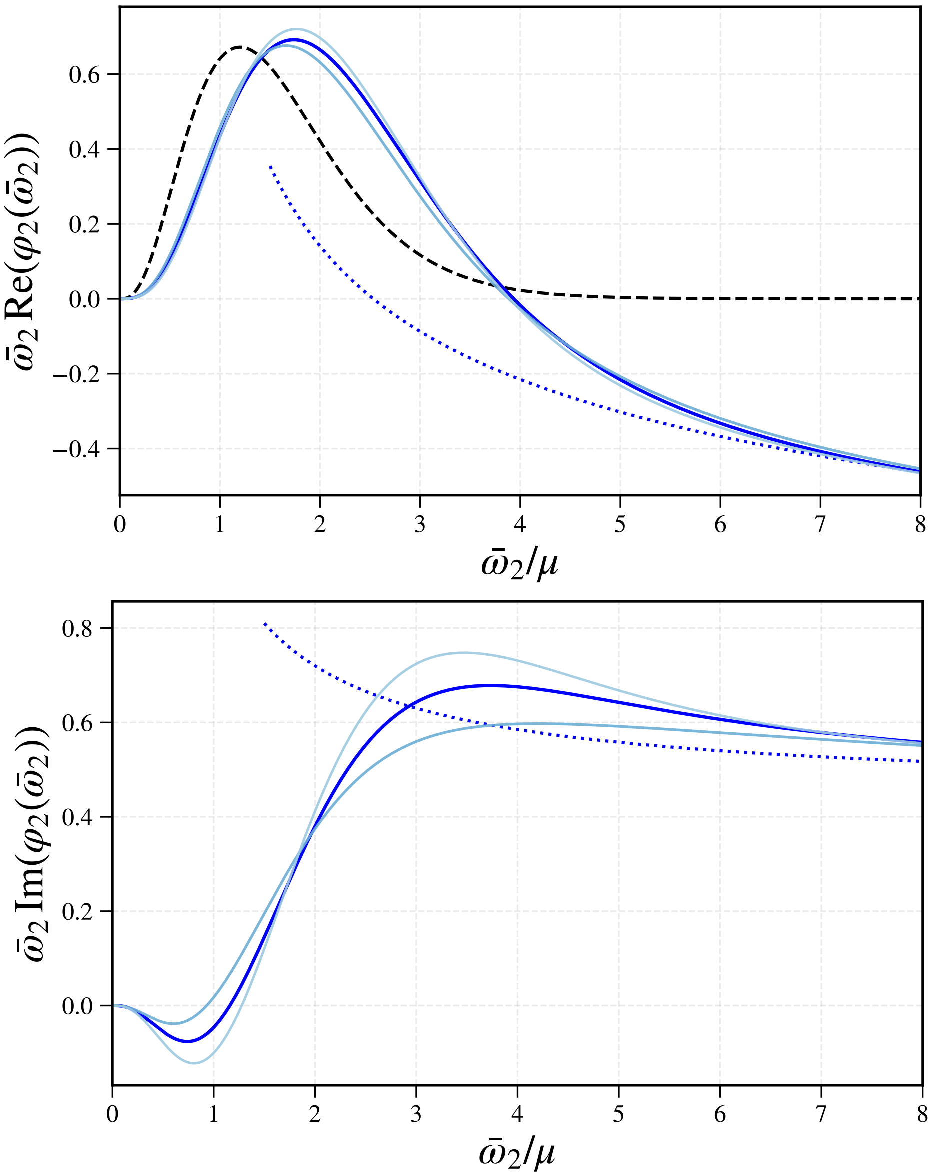

- Input values :

$\bar{\omega}_0 = 0.6\,\mathrm{GeV},\ \mu = 1.5\,\mathrm{GeV},\ \alpha_s(\mu) = 0.3$

and $c_4 = c_5 = 0$ (for simplicity)

Tree-level model ($c_{\phi_3}(\mu_F = 0) = 0$)

Radiative tail

One-loop improvement ($\mu_F = 1\,\mathrm{GeV}$)

$\mu_F = 0.7 - 1.4\,\mathrm{GeV}$

Conclusions

- We have initiated a systematic study of three-particle di-light-cone distribution amplitudes (DLCDAs) of the B meson in HQET

$\rightarrow$ Generalization of conventional LCDAs where the light-antiquark and the gluon are separated along two back-to-back light-like directions.

- B-meson DLCDAs can be expressed in terms of 8 independent distributions that can be organized in a basis of definite twist.

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- For ϕ_3 : radiative tails at large momenta modify this « tree-level » parameterization and generate (pertubative) complex rescattering phases

→New source of strong phases in exclusive B-decay observables!

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- B-meson DLCDAs can be expressed in terms of 8 independent distributions that can be organized in a basis of definite twist.

- Using constraints on the moments, we constructed phenomenological models for the DLCDAs consistent with their expected low momentum behaviors and parameterized in terms of $\langle \omega_1 \rangle_{[\phi_3],[\phi_4],[\psi_4],[\phi_5]}$,$\bar{\Lambda}$, λ_E^2, λ_H^2 and two reference scales ω_0, $\bar{\omega}_0$.

- For ϕ_3 : radiative tails at large momenta modify this « tree-level » parameterization and generate (pertubative) complex rescattering phases

$\rightarrow$New source of strong phases in exclusive B-decay observables!

- First step towards implementing DLCDAs into concrete factorization formulas$\rightarrow$ will allow us to estimate soft gluons attached to charm loops in radiative B decays or energetic quarks in non-leptonic two-body decays !

Thanks for listening !