

Puzzles in $B \rightarrow VV$ decays and measurements of optimised L observables

MITP - Flavourful Opportunities in Semileptonic B Decays

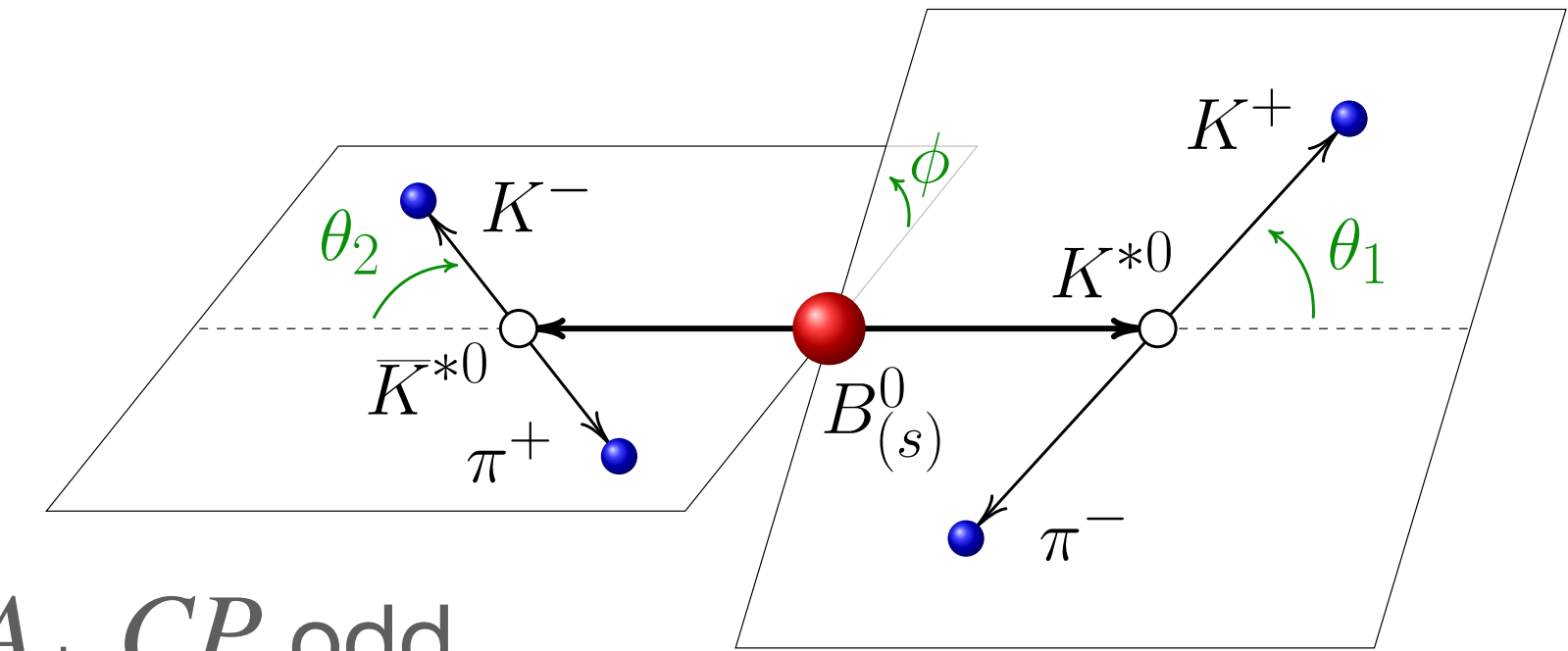
Angular analyses of $B \rightarrow VV$ decays

- In a $B \rightarrow VV$ decay the total amplitude is decomposed in three independent helicity states A_0, A_+, A_-

- Recast in transversity basis where:

$$A_0, A_{\perp} = \frac{A_+ - A_-}{\sqrt{2}}, A_{\parallel} = \frac{A_+ + A_-}{\sqrt{2}}$$

- Transversity amplitudes are CP eigenstates with $A_0, A_{\parallel}$ CP even and $A_{\perp}$ CP odd



$$f_{L,\parallel,\perp} = \frac{|A_{0,\parallel,\perp}|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}$$

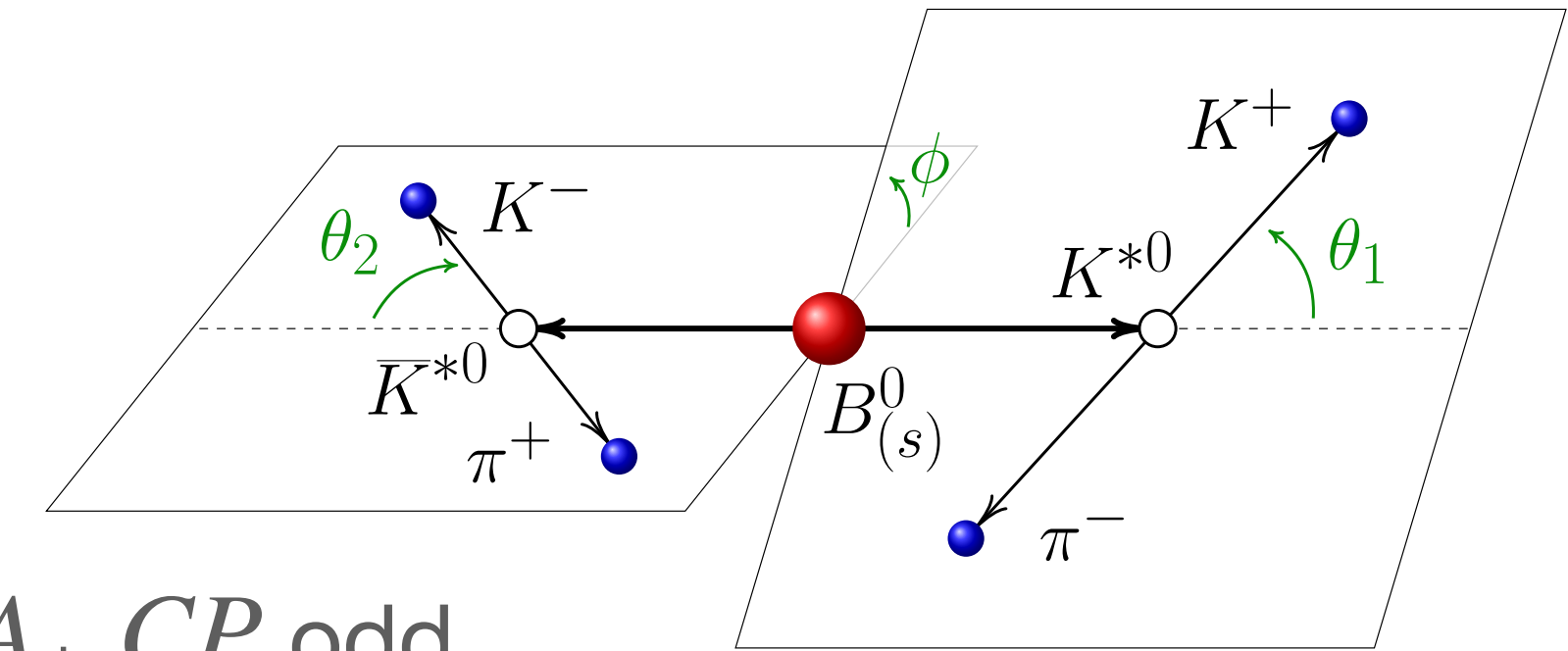
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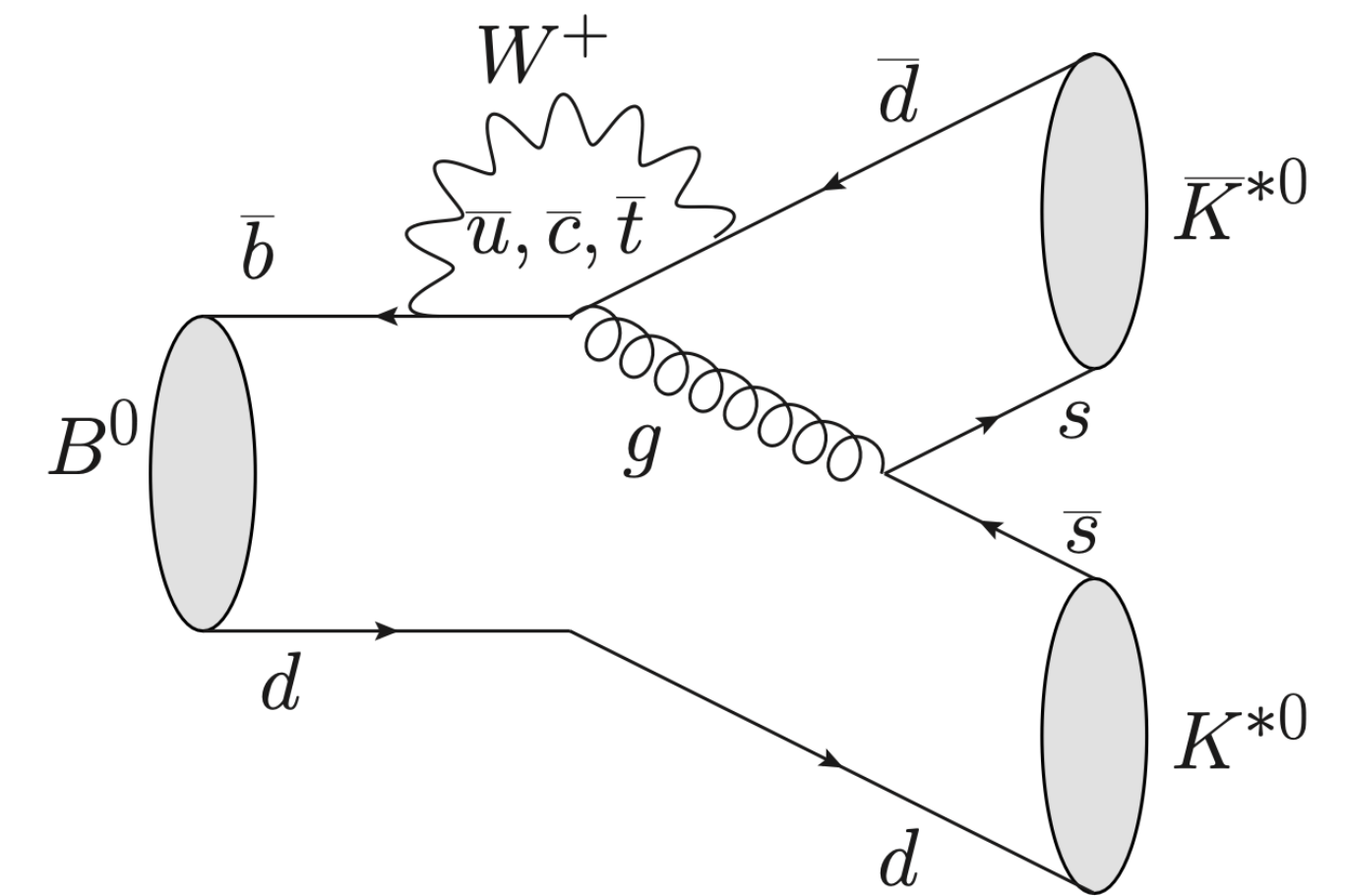
$$f_{L,\parallel,\perp} = \frac{|A_{0,\parallel,\perp}|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}$$

- Expectation from heavy quark symmetry is that in for a $V - A$ coupling, A_0 should prevail over the other two:

$$A_0 : A_- : A_+ \sim 1 : \left(\frac{\Lambda_{\text{QCD}}}{m_B} \right) : \left(\frac{\Lambda_{\text{QCD}}}{m_B} \right)^2 \Rightarrow f_T = f_{\parallel} + f_{\perp} = 1 - f_L = O(m_V^2/m_B^2), \quad f_{\parallel}/f_{\perp} = 1 + O(m_V/m_B)$$

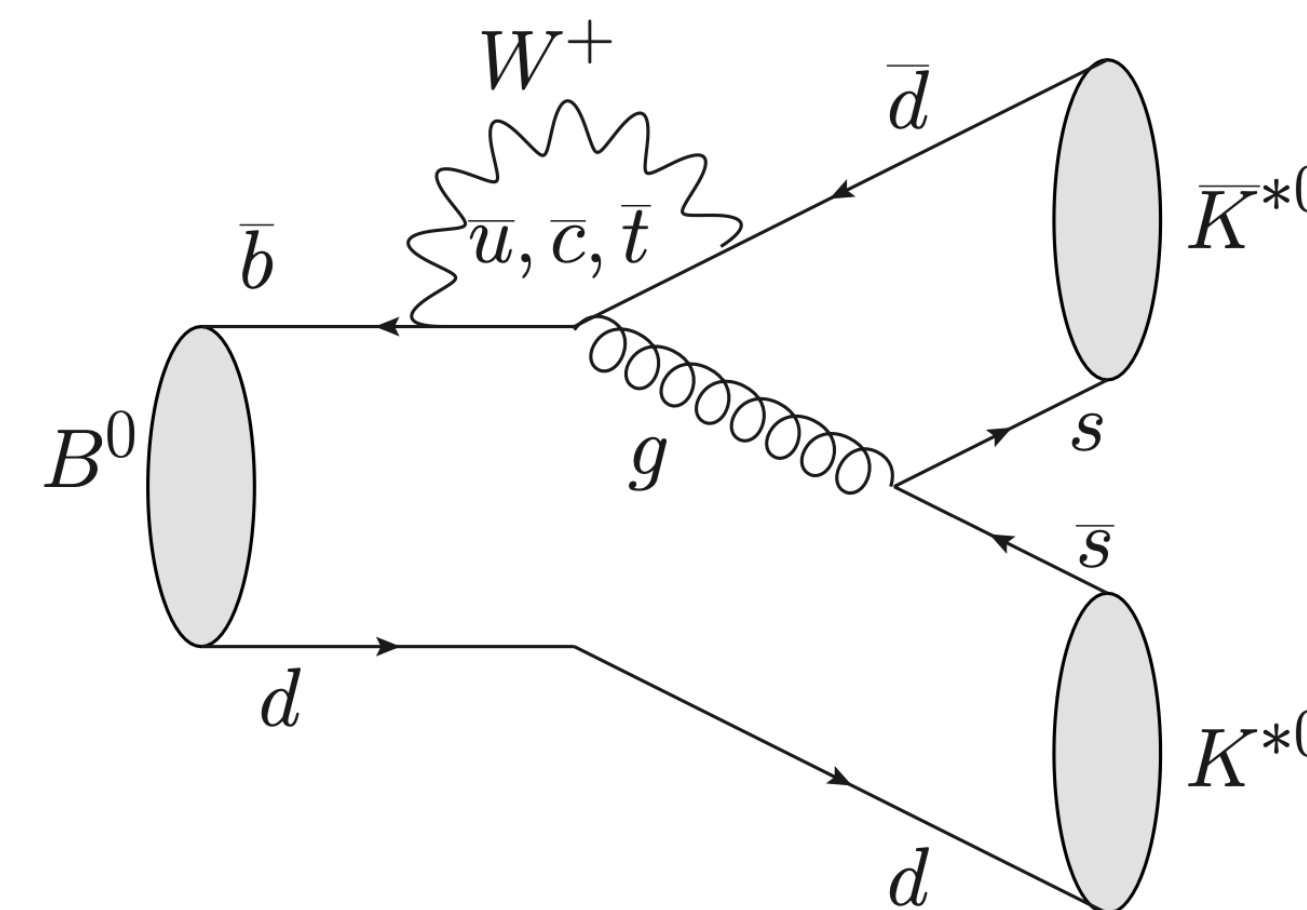
The polarisation puzzle

- This amplitude hierarchy seems to be respected in tree level decays but not in loop induced $B \rightarrow VV$ decays
- This puzzle predates LHCb, and was initiated by the B-factories unexpectedly low f_L in $B^0 \rightarrow \phi \bar{K}^{*0}$ decays
[\[Nucl.Phys.B774:64-101,2007\]](#)
- Gluonic penguin mediated $B_s^0 \rightarrow K^{*0} \bar{K}^{*0}$ decays have exhibited breaking of this hierarchy since when first measured at LHCb



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[JHEP 03 (2018) 140]

- TD angular analysis determined $f_L^{B_s}$ with 3fb^{-1} of data

$$f_L^{B_s} = 0.208 \pm 0.032(\text{stat}) \pm 0.046(\text{syst})$$

$$\phi_s^{d\bar{d}} = -0.10 \pm 0.13(\text{stat}) \pm 0.14(\text{syst})$$

[JHEP 07 (2019) 032]

- TI angular analysis determined f_L in both B^0 and B_s with 3fb^{-1} of data

$$f_L^{B_s^0} = 0.240 \pm 0.031(\text{stat}) \pm 0.025(\text{syst})$$

$$f_L^{B^0} = 0.724 \pm 0.051(\text{stat}) \pm 0.016(\text{syst})$$

Optimised L observables

- Recently, a series of optimised observables was introduced, involving ratios of (longitudinal) branching fractions of U-spin related $B \rightarrow VV$ decays [\[JHEP 09 \(2025\) 188\]](#)
- If we consider $B_{(s)}^0 \rightarrow K^{*0} \bar{K}^{*0}$, by decomposing of the amplitude as

$$\bar{A}_f \equiv A(\bar{B}_q \rightarrow K^{*0} \bar{K}^{*0}) = \lambda_u^{(q)} T_q + \lambda_c^{(q)} P_q = \lambda_u^{(q)} \Delta_q - \lambda_t^{(q)} P_q$$

with $\lambda_U^q = V_{Ub} V_{Uq}^*$, one can construct the following observable:

$$L_{K^* \bar{K}^*} = \frac{|A_0^s|^2 + |\bar{A}_0^s|^2}{|A_0^d|^2 + |\bar{A}_0^d|^2} = \kappa \left| \frac{P_s}{P_d} \right|^2 \left[\frac{1 + |\alpha^s|^2 \left| \frac{\Delta_s}{P_s} \right|^2 + 2 \text{Re} \left(\frac{\Delta_s}{P_s} \right) \text{Re}(\alpha^s)}{1 + |\alpha^d|^2 \left| \frac{\Delta_d}{P_d} \right|^2 + 2 \text{Re} \left(\frac{\Delta_d}{P_d} \right) \text{Re}(\alpha^d)} \right]$$

- α_d is Cabibbo allowed, α_s is $O(\lambda^2)$ suppressed, Δ_q/P_q expt. to be small in case of penguin mediated decays
- L observables are directly related to $|P_s/P_d|^2$, which can be predicted with good accuracy within QCDF and is protected by U-spin symmetry from large $1/m_b$ power corrections

Optimised L observables

- Experimentally, this translates to the measurement of:

$$L_{K^* \bar{K}^*} = \rho(m_{K^{*0}}, m_{\bar{K}^{*0}}) \frac{\mathcal{B}(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0})}{\mathcal{B}(\bar{B}_d \rightarrow K^{*0} \bar{K}^{*0})} \frac{f_L^{B_s}}{f_L^{B_d}}.$$

$$L_{K^* \bar{K}^*}^{I, \text{SM}} = 18.34_{-5.83}^{+7.47}, \quad L_{K^* \bar{K}^*}^{II, \text{SM}} = 26.08_{-4.72}^{+5.70}$$

$$L_{K^* \bar{K}^*}^{III, \text{SM}} = 22.88_{-8.46}^{+15.35}.$$

$$L_{K^* \bar{K}^*}^{\text{exp}} = 4.43 \pm 0.92$$

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Measurement of $L_{K^*\bar{K}^*}$ using
Run 1+Run 2 data

Covariant tensor formalism: motivation

$$\frac{d\Gamma}{d\Phi_4} \propto \left| \sum_i A_i(\Phi_4) \right|^2$$

- Terms in the amplitude can be decomposed as:

$$A(\Phi_4) \equiv B_{(K^+\pi^-)(K^-\pi^+)}(\Phi_4) \left[B_{(K^+\pi^-)}(\Phi_4) T_{(K^+\pi^-)}(\Phi_4) \right] \left[B_{(K^-\pi^+)}(\Phi_4) T_{(K^-\pi^+)}(\Phi_4) \right] S(\Phi_4)$$

- Where $B(\Phi_4)$ are Blatt-Weisskopf barrier factors, $T(\Phi_4)$ mass propagators (e.g. Breit-Wigner, LASS, dispersive scattering model..) and $S_i(\Phi_4)$ angular terms

$$B(q, L) \equiv \begin{cases} 1 & \text{if } L = 0, \\ \frac{1}{\sqrt{1 + (qr)^2}} & \text{if } L = 1, \\ \frac{1}{\sqrt{9 + 3(qr)^2 + (qr)^4}} & \text{if } L = 2. \end{cases}$$

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- The angular model in helicity formalism is fine, the **problem arises in the mass variables:**

- A_0 and $A_{||}$ amplitudes are not eigenstates of angular momentum but rather a **superposition of $\ell = 0, 2$**

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- Set $\ell = 0$ as baseline for Blatt-Weisskopf barrier factors, trial $\ell = 2$ as systematic
 - Leads to sizeable systematics, **switch to covariant tensor formalism where angular amplitudes are eigenstates of L**

Covariant tensor formalism

[PhysRevD.57.431] [PhysRev.140.B97]

- For a J -spin particle with momentum p polarisation λ define the projection operator P as

$$P_{\mu_1 \dots \mu_J, \nu_1 \dots \nu_J}(p) = \sum_{\lambda} \epsilon_{\mu_1 \dots \mu_J}(p, \lambda) \epsilon_{\nu_1 \dots \nu_J}^*(p, \lambda)$$

Where $\epsilon_{\mu_1 \dots \mu_J}(p, \lambda)$ rank- J tensor with Lorentz indices μ_i satisfying the Rarita-Schwinger conditions:

$\epsilon_{\mu_1 \dots \mu_J}(p, \lambda)$ is symmetric traceless and orthogonal to p

- If that state is undergoing 2-body decay with relative momentum $q \equiv p_1 - p_2$ between decay products the relative orbital angular momentum tensor is given by

$$L_{\mu_1 \dots \mu_J}(p, q) = P_{\mu_1 \dots \mu_J, \nu_1 \dots \nu_J} q^{\nu_1 \dots \nu_J}$$

Surfing the S-wave

- Within the $K\pi$ system:
 - P-wave: relative $\ell = 1$ between K^+ and π^- , e.g. $K^*(892)^0 \rightarrow K^+\pi^-$
 - S-wave: relative $\ell = 0$ between K^+ and π^- , e.g. $K^*(700)^0 \rightarrow K^+\pi^-$, $K^*(1430)^0 \rightarrow K^+\pi^-$
- S-wave is the largest contribution in the $K^*(892)^0$ region (amounts to 50 – 70 %)

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- Increase the lever arm into S-wave constraint:

$$((K^\pm\pi^\mp) \text{ threshold} < m(K^\pm\pi^\mp) < 1042) \text{ MeV}/c^2$$

- Upper limit is chosen to remain in the elastic region (i.e. below $K\eta^{(\prime)}$ production) where the S-wave parametrisation is simpler
- First time at LHCb fitting the $K^\pm\pi^\mp$ S-wave down to threshold

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- $S_{K\pi}$ mass propagator model built on top of Peláez-Rodas parametrisation [\[PHYS. REP. 969 \(2022\) 1\]](#)
- Lineshapes obtained in χ -PT framework fulfilling unitarity, causality and crossing symmetry
- Improve on description of lineshape by including QM1 description of direct production of resonances from B decay

Result recasting to helicity basis

- Measured quantities are *CP*-averaged fit fractions, and strong phase differences

$$\mathcal{F}_i \equiv \frac{\int_m [|a_i A_i(\Phi_4)|^2 + |a_i \bar{A}_i(\Phi_4)|^2] \, d\Phi_4}{\int_m [|\sum_i a_i A_i(\Phi_4)|^2 + |\sum_i a_i \bar{A}_i(\Phi_4)|^2] \, d\Phi_4}$$

Parameter		Value	Parameter		Value
B^0	$\mathcal{F}_{VV}^{S+D} (\%)$	$37 \pm 2 \pm 2$	$\mathcal{F}_{VV}^P (\%)$		$14 \pm 1 \pm 0.7$
B_s^0	$\mathcal{F}_{VV}^{S+D} (\%)$	$15.6 \pm 0.63 \pm 0.26$	$\mathcal{F}_{VV}^P (\%)$		$15.55 \pm 0.56 \pm 0.38$

- To recast results in transversity basis generate pseudo-experiments from the fit result:
- Fit back with angular amplitude in trasversity basis in 3D

$$A_{V\bar{V}}(\theta_{K^+\pi^-}, \theta_{K^-\pi^+}, \varphi) \propto A_L \cos \theta_{K^+\pi^-} \cos \theta_{K^-\pi^+} + \frac{A_{\parallel}}{\sqrt{2}} \sin \theta_{K^+\pi^-} \sin \theta_{K^-\pi^+} \cos \varphi + \frac{A_{\perp}}{\sqrt{2}} \sin \theta_{K^+\pi^-} \sin \theta_{K^-\pi^+} \sin \varphi$$

- Only *P*-odd amplitudes are $A_{\perp}$ in helicity and A_P in covariant formalism: important cross check of consistency

Prospects for L_{total} measurement in Run 3

Thanks for your attention

Spare slides

