$B \to K\,a$ in QCD factorization

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Based on the ongoing work with: Cristhian Calderon, Matthias Neubert

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Alexander von HUMBOLDT STIFTUNG

Outline of the talk

- Axion and axion-like particles
- Effective Lagrangian for an ALP coupled to the Standard Model
- Probe of ALP couplings through flavour observables
- ALP effective Lagrangian and weak effective theory
- $B \to K\, a$ with QCD factorization
- Side remark: generation of new operators for ALP Lagrangian in the non-derivative basis

Axions and axion-like particles

- Theoretically: Peccei-Quinn solution to strong CP problem.

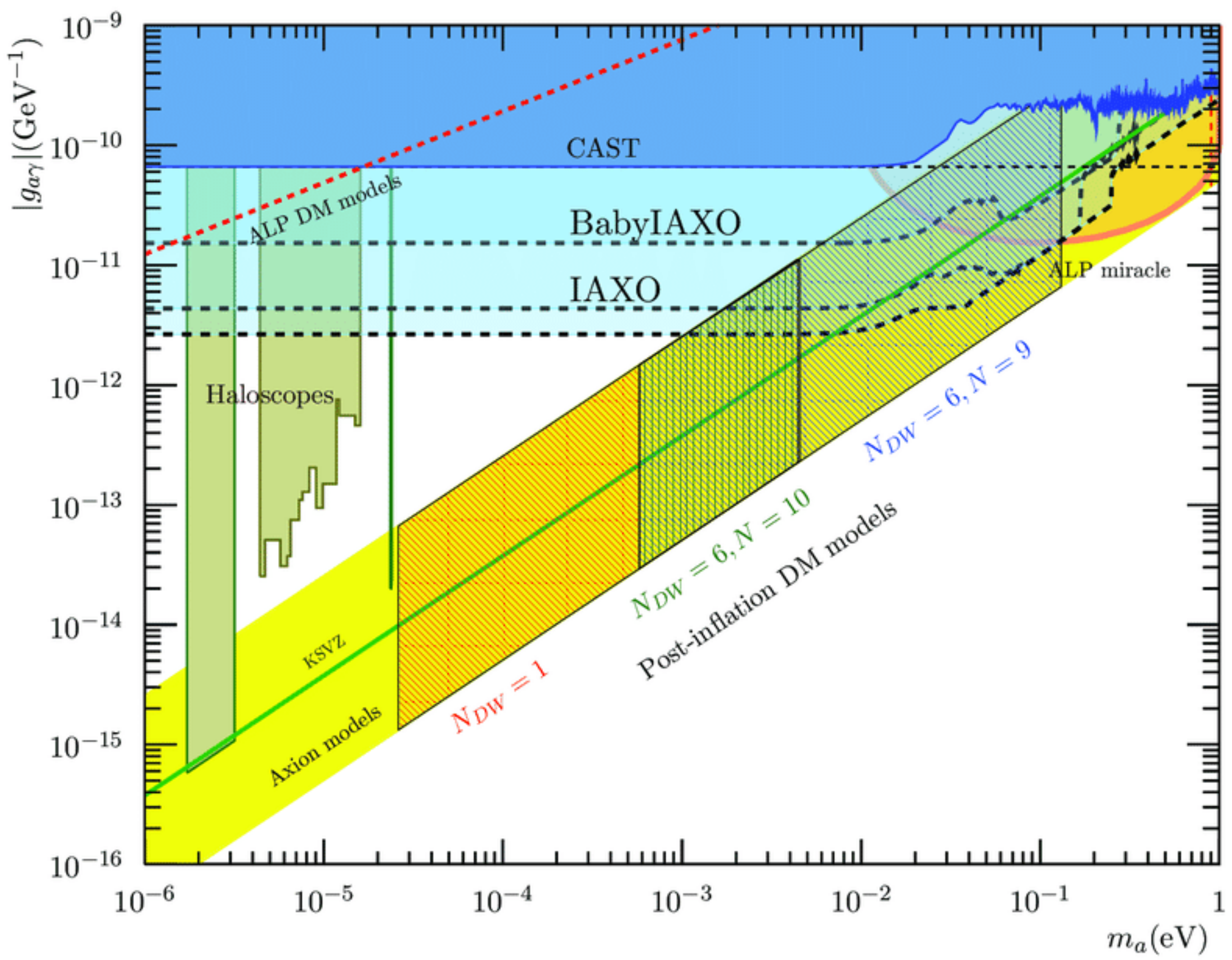
$$\mathscr{L}_{\text{QCD}} \supset \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu,\,a}$$

- Axion mass and couplings to Standard Model are inversely related to the scale f_a

$$m_a^2 f_a^2 = \frac{m_\pi^2 f_\pi^2}{2} \frac{m_u m_d}{(m_u + m_d)^2}$$

- More generally, axion-like particles (ALPs) arise as pseudo Nambu- Goldstone bosons of spontaneously broken global $U(1)$ symmetry → That's what makes it light & weakly coupled

- Well-motivated experimentally as well, as we do not have conclusive evidence for new states at the TeV scale. → New Physics could be light & weakly coupled



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Effective Lagrangian in the UV

- Most general effective Lagrangian for a pseudo-scalar boson a coupled to SM via classically shift-invariant interactions: [Georgi, Kaplan, Randall (1986)]

$$\mathscr{L}_{\text{ALP,eff}}^{D\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,o}^2}{2}a^2 + \frac{\partial^\mu a}{f}\sum_F \bar{\Psi}_F \boldsymbol{c}_F \gamma^\mu \Psi_F + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}^A\tilde{G}^{A,\mu\nu} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f}W_{\mu\nu}^I\tilde{W}^{I,\mu\nu} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- Below the electroweak scale, the ALP Lagrangian reduces to:

$$\mathscr{L}_{\text{eff}}^{D\leq 5}(\mu < \mu_w) = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \mathscr{L}'_{\text{ferm}}(\mu) + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}^A\tilde{G}^{A,\mu\nu} + c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

With $c_{\gamma\gamma} = C_{WW} + C_{BB}$, and the couplings to SM fermions:

$$\mathscr{L}'_{\text{ferm}}(\mu) = \frac{\partial^\mu a}{f}\left[\bar{u}_L\,\boldsymbol{k}_U(\mu)\gamma_\mu\,u_L + \bar{u}_R\,\boldsymbol{k}_u(\mu)\gamma_\mu\,u_R + \bar{d}_L\,\boldsymbol{k}_D(\mu)\gamma_\mu\,d_L + \bar{d}_R\,\boldsymbol{k}_d(\mu)\gamma_\mu\,d_R + \bar{\nu}_L\,\boldsymbol{k}_\nu(\mu)\gamma_\mu\,\nu_L + \bar{e}_L\,\boldsymbol{k}_E(\mu)\gamma_\mu\,e_L + \bar{e}_R\,\boldsymbol{k}_e(\mu)\gamma_\mu\,e_R\right]$$

In the up-aligned flavour basis:

$$\boldsymbol{k}_U = \boldsymbol{c}_Q, \quad \boldsymbol{k}_E = \boldsymbol{k}_\nu = \boldsymbol{c}_L, \quad \boldsymbol{k}_{u,d,e} = \boldsymbol{c}_{u,d,e}, \quad \boldsymbol{k}_D = \boldsymbol{V}_{CKM}^\dagger\,\boldsymbol{c}_Q\boldsymbol{V}_{CKM}$$

$$\boxed{\begin{aligned} c_{q_iq_i}^a &= [\mathbf{k}_q]_{ii} - [\mathbf{k}_Q]_{ii} \\ c_{q_iq_i}^v &= [\mathbf{k}_q]_{ii} + [\mathbf{k}_Q]_{ii} \end{aligned}}$$

Alternative form of the Lagrangian

- An alternative form of the Lagrangian which involves the non-derivative ALP-fermion couplings: [Bauer, Neubert, Renner, Schnubel, Thamm (2020)]

$$\mathcal{L}^{D\le 5}_{\mathrm{ALP,eff}} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,o}^2}{2}a^2 - \frac{a}{f}(\bar{Q}\phi\tilde{\mathbf{Y}}_d d_R + \bar{Q}\tilde{\phi}\tilde{\mathbf{Y}}_u u_R + \bar{L}\phi\tilde{\mathbf{Y}}_e e_R + \mathrm{h.c.}) + \tilde{c}_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + \tilde{c}_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f}W^a_{\mu\nu}\tilde{W}^{a,\mu\nu} + \tilde{c}_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- Relating the couplings of two Lagrangians:

$$\tilde{\boldsymbol{Y}}_d = i(\boldsymbol{Y}_d\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d), \quad \tilde{\boldsymbol{Y}}_u = i(\boldsymbol{Y}_u\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u), \quad \tilde{\boldsymbol{Y}}_e = i(\boldsymbol{Y}_d\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e),$$

$$\tilde{c}_{GG} = c_{GG} + \frac{1}{2}\,\mathrm{Tr}(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\boldsymbol{c}_Q)$$

$$\tilde{c}_{WW} = c_{WW} - \frac{1}{2}\,\mathrm{Tr}(N_c\boldsymbol{c}_Q + \boldsymbol{c}_L)$$

$$\tilde{c}_{BB} = c_{BB} + \mathrm{Tr}(\frac{4}{3}\boldsymbol{c}_u + \frac{1}{3}\boldsymbol{c}_d - \frac{1}{6}\boldsymbol{c}_Q + \boldsymbol{c}_e - \boldsymbol{c}_L)$$

Flavour probes of ALP couplings

Observable	Mass range [MeV]	ALP decay mode	Constrained coupling c_{ij}	Limit (95% CL) on $c_{ij} \cdot \left(\frac{\text{TeV}}{f}\right) \cdot \sqrt{\mathcal{B}}$	Limit (95% CL) on $c_{ij}/\lvert V_{ti}^* V_{tj}\rvert \cdot \left(\frac{\text{TeV}}{f}\right) \cdot \sqrt{\mathcal{B}}$
$\mathrm{Br}(K^- \to \pi^- a(\mathrm{inv}))$	$0 < m_a < 261^{(*)}$	long-lived	$\lvert k_D + k_d\rvert_{12}$	1.2×10^{-9}	3.9×10^{-6}
$\mathrm{Br}(K_L \to \pi^0 a(\mathrm{inv}))$	$0 < m_a < 261$	long-lived	$\lvert \mathrm{Im}[[k_D + k_d]_{12}\rvert$	8.1×10^{-9}	7.0×10^{-5}
$\mathrm{Br}(K^- \to \pi^- \gamma\gamma)$	$m_a < 108$	$\gamma\gamma$	$\lvert k_D + k_d\rvert_{12}$	2.1×10^{-8}	6.9×10^{-5}
$\mathrm{Br}(K^- \to \pi^- \gamma\gamma)$	$220 < m_a < 354$	$\gamma\gamma$	$\lvert k_D + k_d\rvert_{12}$	2.0×10^{-7}	6.5×10^{-4}
$\mathrm{Br}(K_L \to \pi^0 \gamma\gamma)$	$m_a < 110$	$\gamma\gamma$	$\lvert \mathrm{Im}[[k_D + k_d]_{12}]\rvert$	1.3×10^{-8}	1.1×10^{-4}
$\mathrm{Br}(K_L \to \pi^0 \gamma\gamma)$	$m_a < 363^{(\maltese\maltese)}$	$\gamma\gamma$	$\lvert \mathrm{Im}[[k_D + k_d]_{12}]\rvert$	1.3×10^{-7}	1.1×10^{-3}
$\mathrm{Br}(K^+ \to \pi^+ a(e^+e^-))$	$1 < m_a < 100$	e^+e^-	$\lvert k_D + k_d\rvert_{12}$	3.4×10^{-7}	1.1×10^{-3}
$\mathrm{Br}(K_L \to \pi^0 e^+e^-)$	$140 < m_a < 362$	e^+e^-	$\lvert \mathrm{Im}[[k_D + k_d]_{12}]\rvert$	3.1×10^{-9}	2.6×10^{-5}
$\mathrm{Br}(K_L \to \pi^0 \mu^+\mu^-)$	$210 < m_a < 350$	$\mu^+\mu^-$	$\lvert \mathrm{Im}[[k_D + k_d]_{12}]\rvert$	4.0×10^{-9}	3.4×10^{-5}
$\mathrm{Br}(B^+ \to \pi^+ e^+e^-)$	$140 < m_a < 5140$	e^+e^-	$\lvert k_D + k_d\rvert_{13}$	7.0×10^{-7}	8.7×10^{-5}
$\mathrm{Br}(B^+ \to \pi^+ \mu^+\mu^-)$	$211 < m_a < 5140^{(\ddagger\ddagger)}$	$\mu^+\mu^-$	$\lvert k_D + k_d\rvert_{13}$	1.2×10^{-7}	1.4×10^{-5}
$\mathrm{Br}(B^- \to K^- \nu\bar{\nu})$	$0 < m_a < 4785$	long-lived	$\lvert k_D + k_d\rvert_{23}$	6.2×10^{-6}	1.6×10^{-4}
$\mathrm{Br}(B \to K^* \nu\bar{\nu})$	$0 < m_a < 4387$	long-lived	$\lvert k_D - k_d\rvert_{23}$	4.1×10^{-6}	1.1×10^{-4}
$d\mathrm{Br}/dq^2(B^0 \to K^{*0} e^+e^-)_{[0.0,0.05]}$	$1 < m_a < 224$	e^+e^-	$\lvert k_D - k_d\rvert_{23}$	6.4×10^{-7}	1.6×10^{-5}
$d\mathrm{Br}/dq^2(B^0 \to K^{*0} e^+e^-)_{[0.05,0.15]}$	$224 < m_a < 387$	e^+e^-	$\lvert k_D - k_d\rvert_{23}$	9.3×10^{-7}	2.4×10^{-5}
$\mathrm{Br}\big(B^- \to K^- a(\mu^+\mu^-)\big)$	$250 < m_a < 4700^{(\dagger)}$	$\mu^+\mu^-$	$\lvert k_D + k_d\rvert_{23}$	4.4×10^{-8}	1.1×10^{-6}
$\mathrm{Br}\big(B^0 \to K^{*0} a(\mu^+\mu^-)\big)$	$214 < m_a < 4350^{(\dagger)}$	$\mu^+\mu^-$	$\lvert k_D - k_d\rvert_{23}$	5.1×10^{-8}	1.3×10^{-6}
$\mathrm{Br}(B^- \to K^- \tau^+\tau^-)$	$3552 < m_a < 4785$	$\tau^+\tau^-$	$\lvert k_D + k_d\rvert_{23}$	8.2×10^{-5}	2.1×10^{-3}
$\mathrm{Br}(D^0 \to \pi^0 e^+e^-)$	$1 < m_a < 1730^{(\ddagger)}$	e^+e^-	$\lvert k_U + k_u\rvert_{12}$	2.8×10^{-5}	–
$\mathrm{Br}(D^+ \to \pi^+ e^+e^-)$	$200 < m_a < 1730^{(\dagger\dagger)}$	e^+e^-	$\lvert k_U + k_u\rvert_{12}$	8.4×10^{-6}	–
$\mathrm{Br}(D_s^+ \to K^+ e^+e^-)$	$200 < m_a < 1475^{(\maltese)}$	e^+e^-	$\lvert k_U + k_u\rvert_{12}$	2.4×10^{-5}	–
$\mathrm{Br}(D^+ \to \pi^+ \mu^+\mu^-)$	$250 < m_a < 1730^{(**)}$	$\mu^+\mu^-$	$\lvert k_U + k_u\rvert_{12}$	2.1×10^{-6}	–
$\mathrm{Br}(D_s^+ \to K^+ \mu^+\mu^-)$	$200 < m_a < 1475^{(***)}$	$\mu^+\mu^-$	$\lvert k_U + k_u\rvert_{12}$	5.7×10^{-5}	–

[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

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$\text{Br}(K_L \to \pi^0 \gamma\gamma)$	$m_a < 363^{(✠✠)}$	$\gamma\gamma$	$\lvert\text{Im}[[k_D + k_d]_{12}]\rvert$	1.3×10^{-7}	1.1×10^{-3}
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$\text{Br}(K_L \to \pi^0 e^+e^-)$	$140 < m_a < 362$	e^+e^-	$\lvert\text{Im}[[k_D + k_d]_{12}]\rvert$	3.1×10^{-9}	2.6×10^{-5}
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$\text{Br}(B^- \to K^- \nu\bar\nu)$	$0 < m_a < 4785$	long-lived	$\lvert k_D + k_d\rvert_{23}$	6.2×10^{-6}	1.6×10^{-4}
$\text{Br}(B \to K^* \nu\bar\nu)$	$0 < m_a < 4387$	long-lived	$\lvert k_D - k_d\rvert_{23}$	4.1×10^{-6}	1.1×10^{-4}
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[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

- Comprehensive ALP Searches in Meson Decays
- ALPaca: the ALP Automatic Computing Algorithm

[Alda, Fuentes Zamora, Merlo, Ponce Diaz, Rigolin (2025)]

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Observable	Mass range [MeV]	ALP decay mode	Constrained coupling c_{ij}	Limit (95% CL) on $c_{ij} \cdot \left(\frac{\text{TeV}}{f}\right) \cdot \sqrt{\mathcal{B}}$	Limit (95% CL) on $c_{ij}/\lvert V_{ti}^* V_{tj}\rvert \cdot \left(\frac{\text{TeV}}{f}\right) \cdot \sqrt{\mathcal{B}}$
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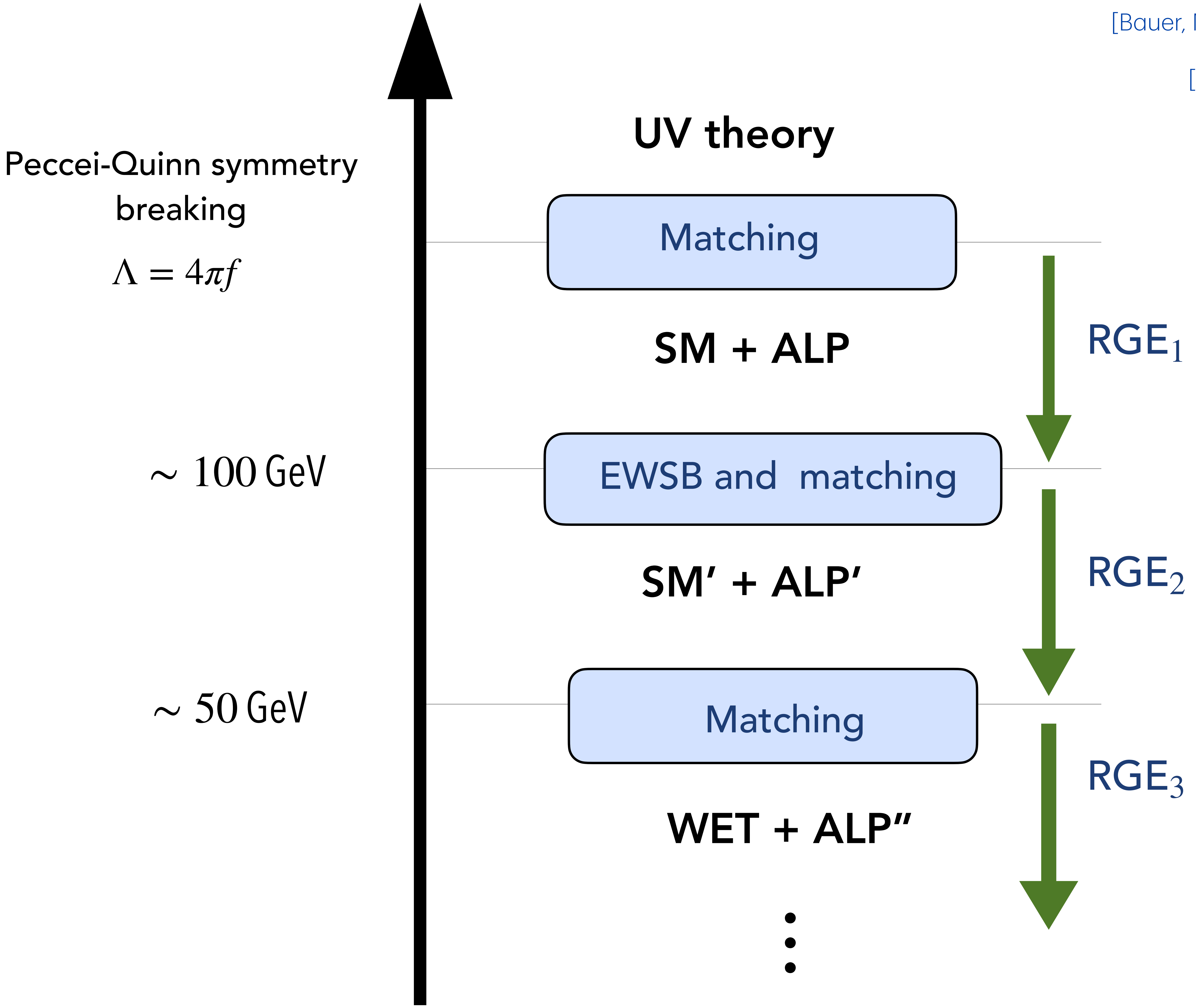
[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

RG evolution from UV down to low-energy scale

[Bauer, Neubert, Renner, Schnubel, Thamm (2020)]

[Chala, Guedes, Ramos, Santiago (2020)]



Weak effective interactions

- For B-meson decays mediated by $b \to s$ flavor-changing neutral current transitions in the SM, the effective weak Lagrangian obtained after integrating out the top quark and electroweak gauge bosons consists of local four-quarks operators and dipole operators:

[Beneke, Buchalla, Neubert, Sachrajda (2000)]

$$Q_1^p = (\bar{s}_i\, p_i)_{(V-A)}(\bar{p}_j\, b_j)_{(V-A)} \qquad Q_2^p = (\bar{s}_i\, p_j)_{(V-A)}(\bar{p}_j\, b_i)_{(V-A)}$$

$$Q_3 = (\bar{s}_i\, b_i)_{(V-A)} \sum_q (\bar{q}_j\, q_j)_{(V-A)} \qquad Q_4 = (\bar{s}_i\, b_j)_{(V-A)} \sum_q (\bar{q}_j\, q_i)_{(V-A)}$$

$$Q_5 = (\bar{s}_i\, b_i)_{(V-A)} \sum_q (\bar{q}_j\, q_j)_{(V+A)} \qquad Q_6 = (\bar{s}_i\, b_j)_{(V-A)} \sum_q (\bar{q}_j\, q_i)_{(V+A)}$$

$$Q_{7\gamma} = -\frac{e m_b}{8\pi^2}\bar{s}_i\, \sigma^{\mu\nu}(1+\gamma_5)\, b_j\, F_{\mu\nu} \qquad Q_{8g} = -\frac{g_s m_b}{8\pi^2}\bar{s}_i\, \sigma^{\mu\nu}(1+\gamma_5) t^a_{ij}\, b_j\, G^a_{\mu\nu}$$

$$\boxed{\begin{aligned} p &= u, c \\ q &= u, d, c, s, b \end{aligned}}$$

- Weak effective Lagrangian:

[Buchalla, Buras, Lautenbacher (1995)]

$$\mathscr{L}^{\text{eff}}_{\text{weak}} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1(\mu)\, Q_1^p(\mu) + C_2(\mu)\, Q_2^p(\mu) + \sum_{i=3,\ldots,6} C_i(\mu)\, Q_i(\mu) + C_{7\gamma}(\mu)\, Q_{7\gamma}(\mu) + C_{8g}(\mu)\, Q_{8g}(\mu) \right] + \text{h.c.}$$

$B \to K\,a$ in QCD factorization

- The QCD factorization approach allows one to systematically separate the short-distance contributions to the hadronic matrix elements from long-distance hadronic effects:

[Becher, Hill, Neubert (2005)]

$$< K\,a\,|\,Q_i^p\,|\,B > = T_i^I F_0^{B\to K}(m_a^2) + \int_0^1 du \int_0^\infty \frac{d\omega}{\omega}\, T_i^{II}(u,\omega) f_K\, \Phi^K(u) f_B\, \Phi_+^B(\omega) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

- Hard-scattering kernels: $T_i^{I,II}$ are associated with short-distance dynamics.

- Form factor:

$$< K\,|\,\bar{s}\gamma^\mu b\,|\,B > = \left[(p_B+p_K)^\mu - \frac{m_B^2-m_K^2}{q^2}\,q^\mu\right] F_+^{B\to K}(q^2) + \frac{m_B^2-m_K^2}{q^2}\,q^\mu F_0^{B\to K}(q^2)$$

$$< K\,|\,\bar{s}\,b\,|\,\bar{B} > = \frac{m_B^2-m_K^2}{m_b-m_s}\,F_0^{B\to K}(q^2)$$

- Meson-distribution amplitude:

$$M_{\alpha\beta}^K = \left[\frac{if_K}{4}\left\{\not{p}_K\gamma_5\Phi^K(u) - \mu_K\gamma_5\left(\Phi_p^K - i\sigma_{\mu\nu}\frac{p_K^\mu\bar{n}^\nu}{p_K\cdot\bar{n}}\frac{\Phi_\sigma^{K'}(u)}{6} + i\sigma_{\mu\nu}p_K^\mu\frac{\Phi_\sigma^K}{6}\frac{\partial}{\partial k_{\perp\nu}}\right)\right\}\right]_{\alpha\beta}$$

$$M_{\alpha\beta}^B = -\left[\frac{if_B m_B}{4}\left(\frac{1+\not{v}}{2}\right)\left\{\Phi_+^B(\omega)\,\not{n} + \Phi_-^B(\omega)\left(\not{\bar{n}} - \omega\gamma^\mu\frac{\partial}{\partial l_{\mu\perp}}\right)\right\}\gamma_5\right]_{\alpha\beta}$$

are associated with long-distance, non-perturbative dynamics.

$$n^\mu = (1,0,0,+1)$$
$$\bar{n}^\mu = (1,0,0,-1)$$

$B \to K\, a$: Kinematics

- Four vectors of B-meson, Kaon and the ALP momenta:

$$p_B^\mu = m_B\, v^\mu = m_B \frac{n^\mu}{2} + m_B \frac{\bar{n}^\mu}{2} \qquad p_K^\mu \approx 2E_K \frac{n^\mu}{2} + \frac{m_K^2}{2E_K} \frac{\bar{n}^\mu}{2} \qquad p_a^\mu \approx \frac{m_a^2}{m_B} \frac{n^\mu}{2} + m_B \frac{\bar{n}^\mu}{2}$$

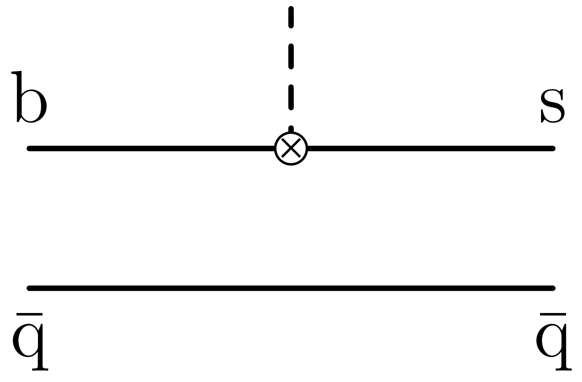
$$\boxed{\begin{aligned} n^\mu &= (1,0,0,+1) \\ \bar{n}^\mu &= (1,0,0,-1) \end{aligned}}$$

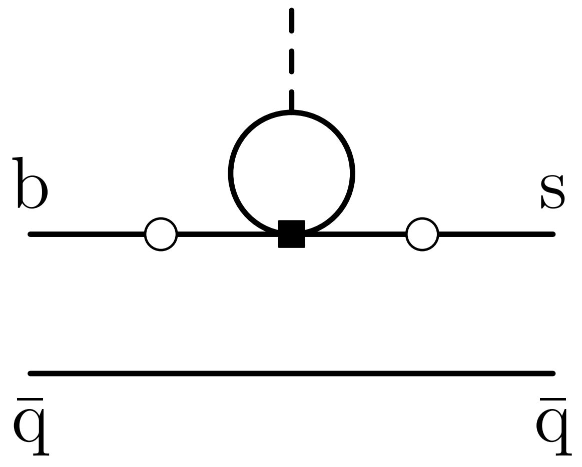
- The momentum carried by the partons inside the mesons:
  - B-meson: $p_b^\mu = m_b v^\mu + k^\mu\ ,\qquad p_{\bar{u}}^\mu = l^\mu = l_+ \frac{\bar{n}^\mu}{2} + l_- \frac{n^\mu}{2} + l_\perp^\mu$
  - Kaon: $p_s^\mu = u E_K n^\mu + k_\perp^\mu,\qquad p_{\bar{u}}^\mu = (1-u) E_K n^\mu - k_\perp^\mu$

- Kaon is collinear, depending on the mass of the ALP, the ALP becomes either anti-collinear or hard anti-collinear.

$B \to K\,a$: Diagrams and scaling relations

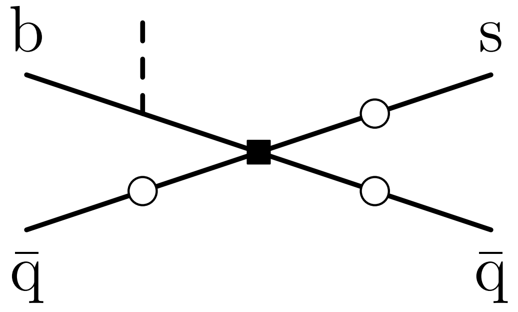
- Form factor correction:





$$\mathscr{F} \sim \frac{\lambda^{3/2}\, m_b^2}{f} \left\{ (m_c/m_b)^2\, c_{cc}^a,\, c_{bb}^a,\, (m_s/m_b)\, c_{ss}^a \right\}$$

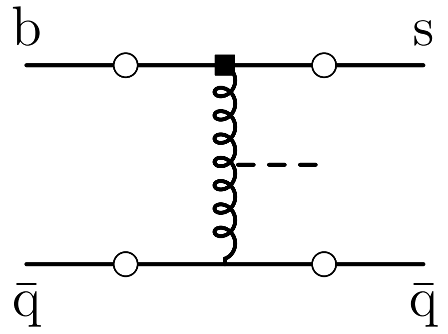
- Weak annihilation:

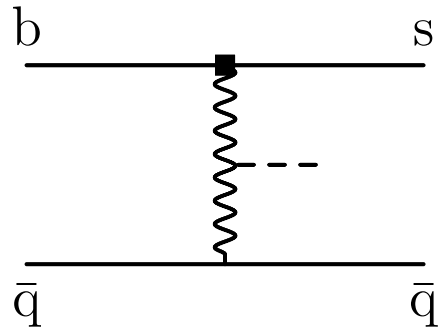


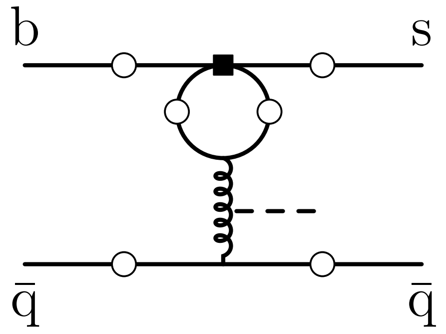
$$\mathscr{W} \sim \frac{\lambda^{5/2}\, m_b^2}{f} \left\{ c_{ss}^a,\, c_{bb}^a,\, (c_{bb}^v - c_{ss}^v) \right\}$$

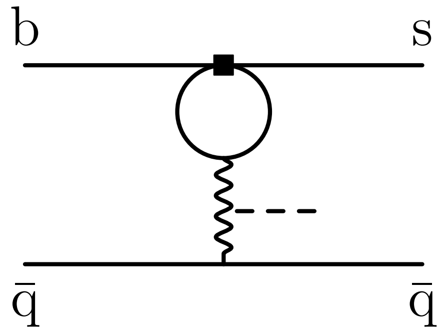
$$\boxed{\lambda = \frac{\Lambda_{\text{QCD}}}{m_b}}$$

- Hard spectator scattering:





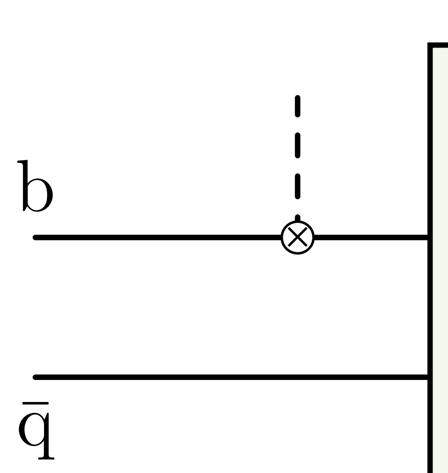




$$\mathscr{H} \sim \frac{\lambda^{3/2}\, m_b^2}{f} \left\{ c_{GG},\, c_{\gamma\gamma} \right\} + \frac{\lambda^{5/2}\, m_b^2}{f} \left\{ c_{ss}^a,\, (m_c/m_b)^2\, c_{cc}^a,\, c_{bb}^a,\, (c_{bb}^v - c_{ss}^v) \right\}$$

$B \to K\,a$: Diagrams and scaling relations

- Form factor correction:



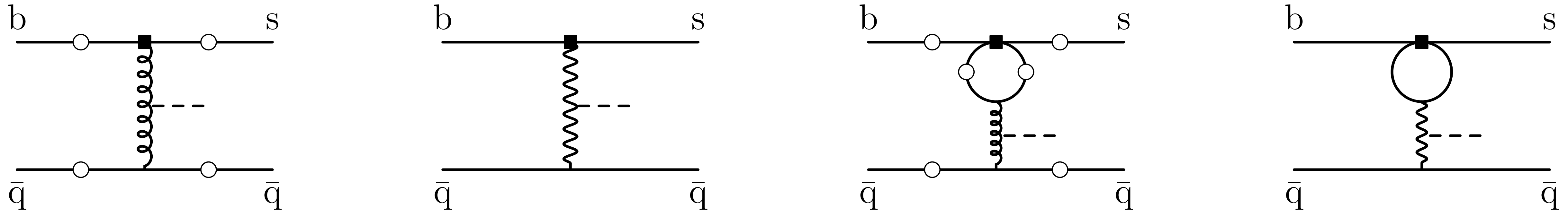
$m_b)\, c_{ss}^a\}$

$$i\mathcal{A}(B^+ \to K^+ a) = -\,4.76653\,\frac{[\mathbf{k}_D + \mathbf{k}_d]_{23}}{f} + (7.3838\times10^{-8} - 1.14459\times10^{-9} i)\,\frac{c_{bb}^a}{f}$$
$$+(4.32518\times10^{-8})\frac{c_{cc}^a}{f} - (2.65284\times10^{-7} + 1.0981\times10^{-9} i)\frac{c_{bb}^v}{f}$$
$$+(2.65284\times10^{-7} + 1.0981\times10^{-9} i)\frac{c_{ss}^v}{f} + (2.65284\times10^{-7} + 1.0981\times10^{-9} i)\frac{c_{ss}^a}{f}$$
$$-(7.15689\times10^{-10} - 1.10249\times10^{-10} i)\frac{c_{GG}}{f} + (1.35574\times10^{-12} - 9.19518\times10^{-16} i)\frac{c_{\gamma\gamma}}{f}$$

- Weak annihilation:

$$\boxed{\lambda = \frac{\Lambda_{\mathrm{QCD}}}{m_b}}$$

- Hard spectator scatteri



$$\mathcal{H} \sim \frac{\lambda^{3/2}\, m_b^2}{f}\left\{c_{GG}, c_{\gamma\gamma}\right\} + \frac{\lambda^{5/2}\, m_b^2}{f}\left\{c_{ss}^a, (m_c/m_b)^2\, c_{cc}^a, c_{bb}^a, (c_{bb}^v - c_{ss}^v)\right\}$$

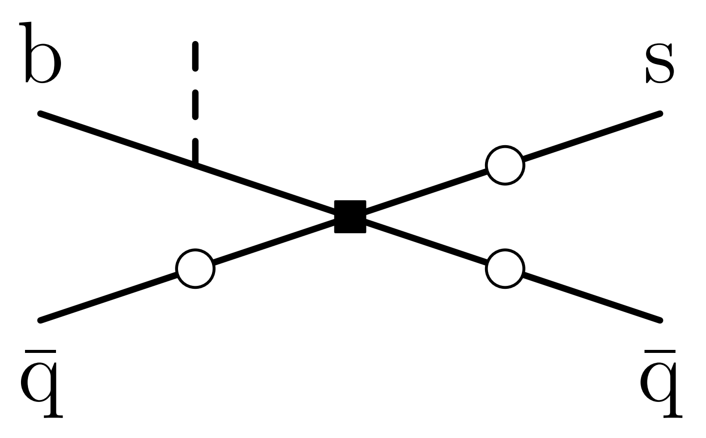
Appearance of new operators in non-derivative basis

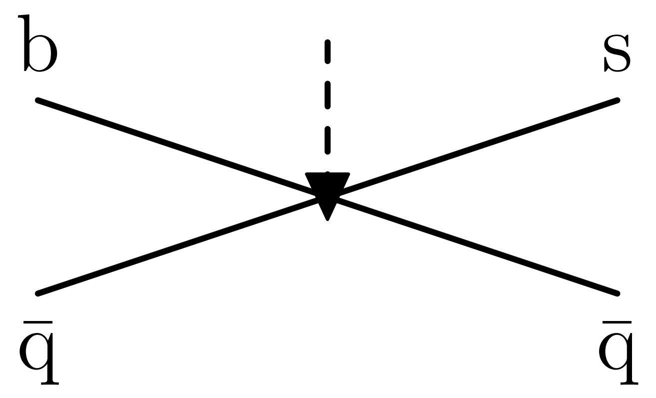
- The presence of the weak operators modifies the usual SM equations of motion for the fermions, and when we use integration by parts on the derivative-involved ALP–fermion operator together with these modified equations, new higher-dimensional operators appear:

$$\mathscr{L}_{\text{ALP}}^{\text{eff (7)}} = \frac{ia}{f}\left([\mathbf{k}_D]_{22} - [\mathbf{k}_D]_{33}\right)\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\left[C_1(\mu)\,Q_1^p(\mu) + C_2(\mu)\,Q_2^p(\mu) + \sum_{i=3,\ldots,6} C_i(\mu)\,Q_i(\mu)\right]$$

$$-\frac{ia}{f}\left([\mathbf{k}_D]_{22} - [\mathbf{k}_d]_{33}\right)\frac{G_F}{\sqrt{2}}\lambda_t\left[C_{7\gamma}(\mu)\,Q_{7\gamma}(\mu) + C_{8g}(\mu)\,Q_{8g}(\mu)\right] + \text{h.c.}\,.$$

- Weak annihilation diagrams to consider in the non-derivative basis





Conclusions

- Axions and ALPs belong to a class of well-motivated light BSM particles with weak couplings to the Standard Model.

- They can be probed across a wide range of experiments: from astrophysical/ cosmological, flavor, high energy colliders to precision measurements, depending on the mass window we are looking into. Rare meson decays like $B \to K\, a$, provide strong bounds for ALPs of few GeV mass range.

- We separate short-distance physics from long-distance, non-perturbative physics through QCD factorization approach.

- Apart from direct flavor-violation from the ALP-fermion couplings, we consider flavor-violation through the weak effective operators while the ALP-fermion couplings are flavor-conserving. Numerically these effects are much suppressed and can be neglected in a dedicated phone analysis.

- If we work in the non-derivative basis of the ALP operators along with weak effective operators, we would need to add new higher dimensional operators in our calculation as well, to be able to get the complete result.

Thank you for your attention!