Theory of (exclusive) $b \to c\ell^-\bar{\nu}$ decays

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Motivation / Contents

- *“No pain, no palm; no thorns, no throne; no gall, no glory; no cross, no crown.“* [W. Penn]
  - here: no theory predictions; no $|V_{cb}|$, no $R_{D^{(*)}}$ predictions

- most recently: three lattice QCD results with multiple data points away from the zero-recoil point [FNAL/MILC 2105.14019; HPQCD 2304.03137; JLQCD 2306.05657]

- reviewing basics behind parametrizations (BGL, HQE) and fits to lattice data

  [BGL fits: Bordone,Jüttner 2406.10074; HQE fits: Bordone,Gubernari,Jung,DvD 2507.03569]

- dispersive matrix (not a parametrization) left out; yields equivalent results to BGL

  [see Martinelli,Simula,Vittorio 2310.03680]

What is a Hadronic Form Factor?

Lorentz Decomposition (ex: $\bar{B} \to D\ell^-\bar{\nu}$ Form Factors)

$$\langle D(k)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = \sum_n S_n^\mu(p,k)\; F_n(q^2 \equiv (p-k)^2)$$

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- describe mismatch between partonic current and hadronic transition
- generally assume on-shell hadrons: $p^2 = M_B^2,\, k^2 = M_D^2$.

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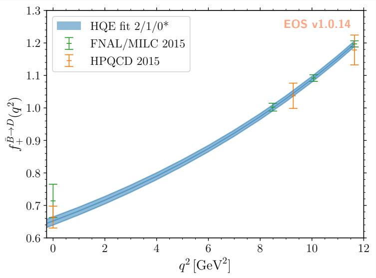

[Bordone,Gubernari,Jung,DvD '25]

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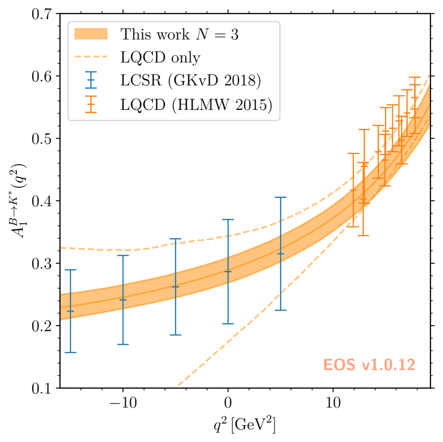

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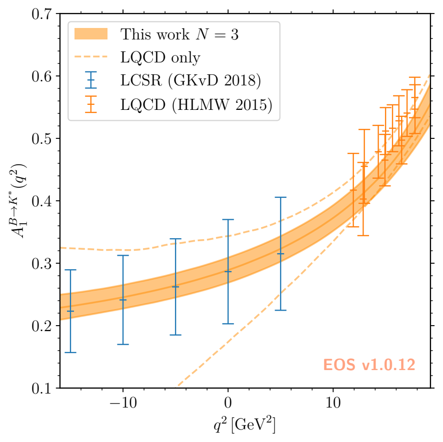

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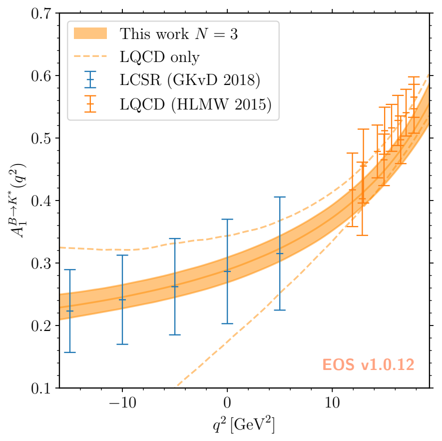


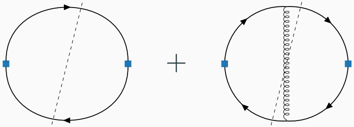
[Gubernari,Reboud,DvD,Virto ’23]

How to handle the systematic uncertainty inherent in the parametrization?

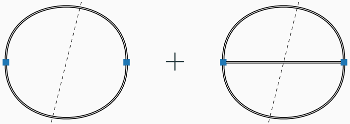
Unitarity Bounds / Dispersive Bounds

Enter Unitarity Bounds / Dispersive Bounds

$$\Pi(Q^2) \equiv i \int d^4x e^{iQ\cdot x} \, \langle 0|\mathcal{T}\{[\bar{c}\gamma^\mu b]^\dagger(x), [\bar{c}\gamma_\mu b](0)\}|0\rangle$$



$+ \ldots$ $\quad \mathrm{Disc}\,\Pi$



$+ \ldots$

Partonic 2-point Function

- discontinuity accessible in operator product expansion

Hadronic 2-point Function

- discontinuity expressible in terms of hadronic form factors

Global Quark-Hadron Duality

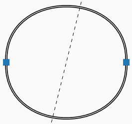
When summing & integrating over all on-shell intermediate states, the two discontinuities lead to the same result.

[Chibisov,Dikeman,Shifman,Uraltsev '96]

[Ling-Fong,Pagels '71] [Okubo '71] [Boyd,Grinstein,Lebed '94] [Caprini,Neubert '96]

[see also "Functional Analysis and Optimization Methods in Hadron Physics" (I. Caprini)]

Wait, what?



How does this relate to the form factors? $\langle D|[\bar{c}\gamma_\mu b]|\bar{B}\rangle$?

Crossing Symmetry

$$\langle D(k)|[\bar{c}\gamma_\mu b]|\bar{B}(+p)\rangle = \sum_n \mathcal{S}_n^\mu(+p,k)\ \mathcal{F}_n(q^2 \equiv (+p-k)^2)$$

$$\langle D(k)B(-p)|[\bar{c}\gamma_\mu b]|0\rangle = \sum_n \mathcal{S}_n^\mu(-p,k)\ \mathcal{F}_n(q^2 \equiv (-p-k)^2)$$

- the same function / form factor describes the semileptonic decay and the pair production for different regions of q^2
  - for $0 \le q^2 \le (M_B - M_D)^2$, it describes the semileptonic decay $\bar{B} \to D\ell^-\bar{\nu}$
  - for $q^2 \ge (M_B + M_D)^2$, it describes the pair production $\ell^+\nu \to BD$
- form factors appear “squared” in the discontinuity

Creating the Bound

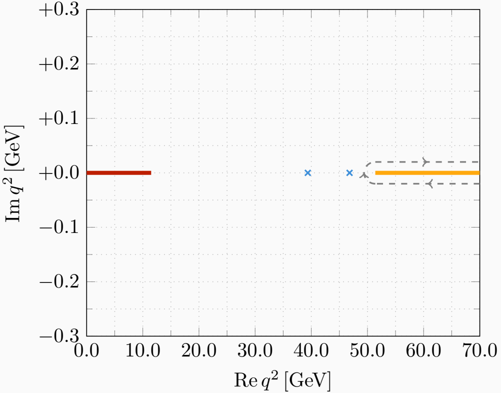
- Π is typically divergent
  - but: starting at some power of the derivative operator, that derivatives and those of higher order are finite
- define χ

$$\chi(Q^2) = \frac{d^n}{d(Q^2)^n}\Pi(Q^2) = \frac{1}{2\pi i}\frac{d^n}{d(Q^2)^n}\oint_C dt\,\frac{\Pi(t)}{t-Q^2}$$

  - select n as the smallest number of derivatives so that $\chi(Q^2)$ is finite
  - choose contour C to avoid singularities
- compute $\chi(Q^2=0)$ in an operator product expansion (OPE)
- small problem: so far we have related $\mathrm{Disc}\,\Pi$ to the form factors, but not yet related Π to the form factors!

Properties of Π in a Nutshell

$$\Pi(Q^2) \equiv i \int d^4x e^{iQ\cdot x} \; \langle 0|\mathcal{T}\{[\bar{c}\gamma^\mu b]^\dagger(x), [\bar{c}\gamma^\mu b](0)\}|0\rangle$$



relevant here:

- pair production branch cut
- integration contour
- $\dfrac{\Pi(t)}{t^{n+1}} \to 0$ as $|t| \to \infty$

contour shrinks to two rays

- below the cut (moving left)
- above the cut (moving right)

$$\Pi(t+i\varepsilon) - \Pi(t-i\varepsilon) \equiv \mathrm{Disc}\,\Pi(t+i\varepsilon)$$

Creating the Bound (2)

$$\chi^{\mathsf{FF}}(Q^2) = \frac{1}{2\pi i}\frac{d^n}{d(Q^2)^n}\oint_C dt\,\frac{\Pi^{\mathsf{FF}}(t)}{t-Q^2} = \frac{1}{2\pi i}\frac{d^n}{d(Q^2)^n}\int_{(M_B+M_D)^2}^{\infty} dt\,\frac{\mathrm{Disc}\,\Pi^{\mathsf{FF}}(t)}{t-Q^2}$$

- compute discontinuity of Π in terms of the hadronic form factors, single out one form factor F and its (positive) kinematic weight function

$$i\,\mathrm{Disc}\,\Pi^{\mathsf{FF}}(t) = \omega(t)|F(t)|^2 + \text{positive terms}$$

- drop positive terms to create inequality, the dispersive bound

$$\boxed{\chi^{\mathsf{OPE}}(0) \geq \frac{1}{2\pi}\frac{d^n}{d(Q^2)^n}\int dt\,\frac{\omega(t)|F(t)|^2}{t-Q^2}\Big|_{Q^2=0}}$$

Boyd-Grinstein-Lebed Idea

- create separate bounds for vector and axialvector currents
- can we create a parametrization that manifestly respects the bound?
- problematic quantity:

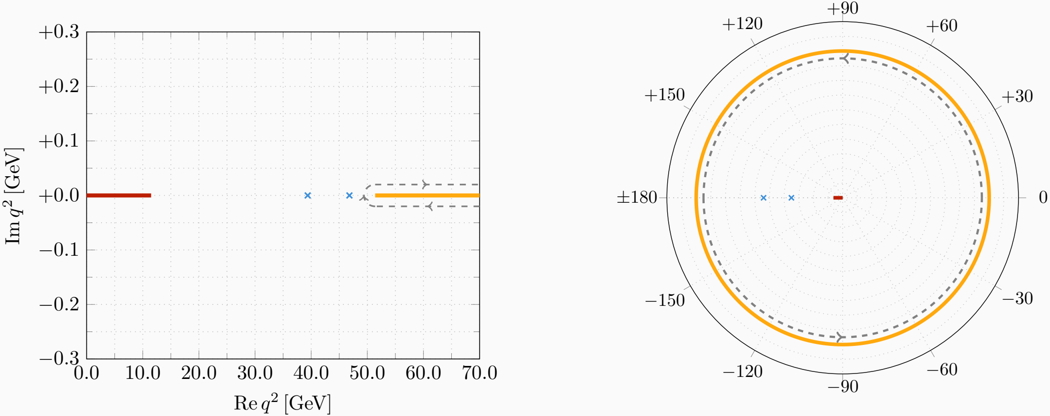
$$\frac{1}{2\pi}\frac{d^n}{d(Q^2)^n}\int_{(M_B+M_D)^2}^{\infty} dt\,\frac{\omega(t)|F(t)|^2}{t-Q^2}\bigg|_{Q^2=0} \xrightarrow{?} \int_{(M_B+M_D)^2}^{\infty} dt\,|\tilde{F}(t)|^2$$

- can we “diagonalise” the contribution to the integral by expanding $\tilde{F}$ in a suitable basis of functions?

[Boyd,Grinstein,Lebed '95]

Diagonalisation

$$t \mapsto z(t) \equiv \frac{\sqrt{(M_B+M_D)^2 - t} - \sqrt{(M_B+M_D)^2-(M_B-M_D)^2}}{\sqrt{(M_B+M_D)^2 - t} - \sqrt{(M_B+M_D)^2-(M_B-M_D)^2}}$$



$$\frac{1}{2\pi}\frac{d^n}{d(Q^2)^n}\int_{(M_B+M_D)^2}^{\infty} dt\, \frac{\omega(t)|F(t)|^2}{t-Q^2}\bigg|_{Q^2=0} = \int_{-\pi}^{+\pi}\frac{d\vartheta}{2\pi}\, z J_{z\to t}(z)\left[\frac{d^n}{d(Q^2)^n}\frac{\omega(t)|F(t)|^2}{t-Q^2}\right]_{z=e^{i\vartheta},Q^2=0}$$

Boyd-Grinstein-Lebed Parametrization

- absorb χ, weight factor ω, and kernel $1/(t - Q^2)$ into "outer function" ϕ[1]

$$1 \geq \frac{1}{\chi(0)} \int_{-\pi}^{+\pi} \frac{d\vartheta}{2\pi} z\, J_{z\to t}(z) \left[\frac{d^n}{d(Q^2)^n} \frac{\omega(t)|F(t(z))|^2}{t - Q^2} \right]_{z=e^{i\vartheta}, Q^2=0} \equiv \int_{-\pi}^{+\pi} \frac{d\vartheta}{2\pi} \left| \phi(z) F(t(z)) \right|^2 \Big|_{z=e^{i\vartheta}}$$

- use an orthonormal basis of analytic functions on the unit circle

$$\int_{-\pi}^{+\pi} \frac{d\vartheta}{2\pi} z^k z^{*,l} = \delta_{kl} \qquad \text{for } k, l \geq 0$$

- expand $\phi(z)F(t(z))$ into a series around $z = 0$

$$F(t) = \frac{1}{\phi(z(t))} \sum_{k=0}^{\infty} a_k [z(t)]^k \qquad \Rightarrow \qquad \sum_{k=0}^{\infty} |a_k|^2 \leq 1$$

- each expansion coefficient a_k is absolutely bounded to the interval $[-1, +1]$
  - stronger bounds apply when multiple form factors arise from the same quark current

[1] Omitting pole terms / Blaschke factors

- truncate series at order K

$$F_K(t) = \frac{1}{\phi(z(t))} \sum_{k=0}^{K} a_k [z(t)]^k$$

- what is the truncation error?

$$\epsilon_K(z) \equiv \left| \phi(z) \left[F(t(z)) - F_K(t(z)) \right] \right| = \left| \sum_{k=K+1}^{\infty} a_k z^k \right|$$

$$\leq \left\{ \left[\sum_{k=K+1}^{\infty} |a_k|^2 \right] \left[\sum_{k=K+1}^{\infty} |z_k|^2 \right] \right\}^{1/2}$$

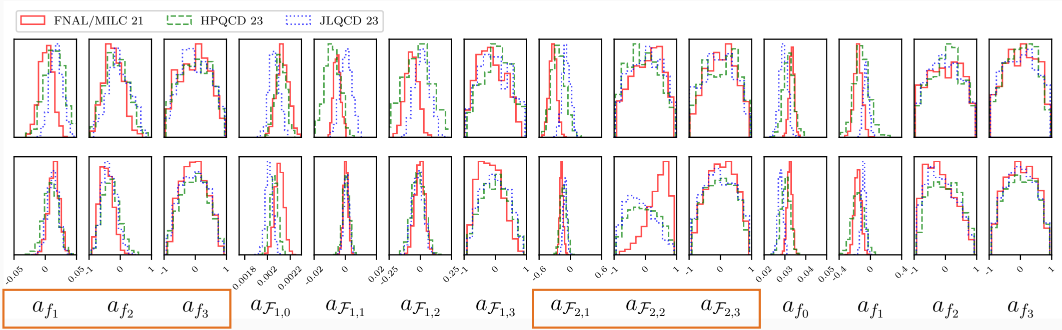
$$= \sqrt{A_K \cdot Z_K(z)}$$

- identify $A_K \leq 1 - \sum_{k=0}^{K} |a_k|^2 < 1$ and $Z_K(z) \leq |z^2|^{K+1} \sum_{k=0}^{\infty} |z^2|^k = \frac{|z^2|^{K+1}}{1-|z^2|}$

$$\epsilon_K(z) < \text{constant} \times |z|^{K+1} \qquad \text{if } |z| \ll 1$$

- most recent fit

[Bordone,Jüttner 2406.10074]



$\rightarrow |V_{cb}|$

$\rightarrow R_{D^*}$

BGL Fit Conclusions

- BGL + unitarity: model-independent predictions with strongly reduced residual truncation error
  - Bayesian: unitarity prior regulates higher coefficients $\Rightarrow$ truncation independent
  - frequentist: p value quantifies goodness of fit
- excellent agreement between LQCD results
- lattice covariances mix statistical & systematic errors
  - cannot be disentangled with available information $\rightarrow$ possibly over-optimistic p values
  - desirable: separate covariances for statistical and systematic effects

[Bordone,Jüttner 2406.10074]

Heavy Quark Expansion

What about the Heavy Quark Expansion (HQE)

$\bar{B} \to D^{(*)}\ell^- \bar{\nu}$ vector form factors

$$\langle D(k)|\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = f_+(q^2)\,(p+k)^\mu + \dots$$

$$\langle D^*(k,\eta)|\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = V(q^2)\,\frac{2}{M_B + M_D^*}\varepsilon^{\mu\alpha\beta\gamma}\eta^*_\alpha p_\beta k_\gamma$$

- physics intuition: the b quark makes up most of the mass of the $\bar{B}$ meson
  - average b momentum limited to $|p - m_b v| \approx \Lambda_{\text{had}} \ll m_b$
  - quark momentum largely aligned with meson momentum $p = M_B v$
  - introduces meson four-velocity v
  - similar for the c quark with velocity v'
- treat both quarks as heavy $\to$ apply Heavy Quark Effective Theory (HQET)
- heavy-quark spin symmetry connect $\bar{B} \to D$ and $\bar{B} \to D^*$ form factors

[Isgur,Wise Phys. Lett. B (232) 1989; review by Neubert SLAC-PUB-6283]

Heavy Quark Symmetry and Isgur-Wise Functions

$\bar{B} \to D\ell^- \bar{\nu}$ vector form factor

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- HQET implies simplification of Feynman rules, e.g. $\gamma^\mu \to v^\mu$ when sandwiched between to heavy quarks
  - introduces a *new symmetry* among matrix elements of different spin structures: Heavy Quark Spin Symmetry
  - in the limit $m_b = m_c \to \infty$, this implies

    $$f_+(q^2) \propto \xi(w) + \mathcal{O}(\alpha_s) \qquad\qquad V(q^) \propto \xi(w) + \mathcal{O}(\alpha_s)$$

    with ξ the *Isgur-Wise function* and $w \equiv v \cdot v' = (M_B^2 + M_{D^*}^2 - q^2)/(2M_B M_{D^*})$
  - corrections to the infinite-mass limit are systematically accessible: introduces further *subleading Isgur-Wise functions*
- truncate expansion in $1/m_b$ and $1/m_c$, fit Isgur-Wise functions

Fit Model

- z-expand Isgur-Wise functions up to order $1/m_b \sim 1/m_c^2 \sim \alpha_s/\pi$

  [Bordone,Gubernari,Jung,DvD 2507.03569 and earlier works]

  - at this order 10 functions arise, matching the total number of $\bar{B} \to D^{(*)}$ form factors
  - $3/2/1$ model:
    - expansion: $1/m^0$: up to z^3; $1/m$: up to z^2; $1/m^2$: up to z^1
    - reduces # of parameters of $1/m$-suppressed Isgur-Wise functions
  - good fit already with $2/1/0$ model; increase orders by tone to account for truncation uncertainty
- express saturation of the unitarity bound in terms of parameters of the Isgur-Wise functions

  [CLN: Caprini,Lellouch,Neubert hep-ph/9712417]

  - take into account 4 transitions: $\bar{B}^{(*)} \to D^{(*)}$
  - new: take into account unitarity bounds for vector currents with $J^P = 0^\pm, 1^\pm$ and (new!) tensor currents with $J^P = 1^\pm$

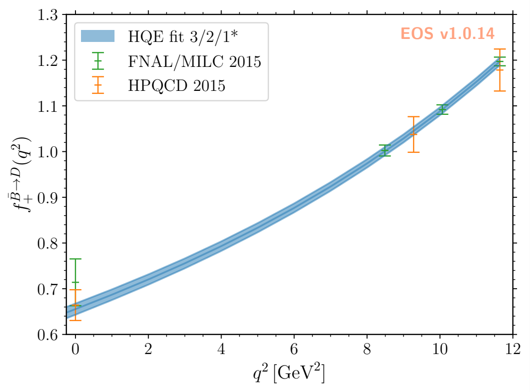
    [Bordone,Gubernari,Jung,DvD 2507.03569]

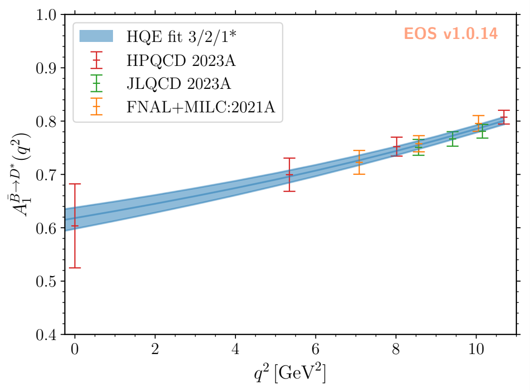
  - stronger constraint than accounting only for $\bar{B} \to D^{(*)}$ or in absence of tensor current

[also see Berlochner et al. 2206.11281 and earlier works]

Fit Results

- good fit overall: $\chi^2/\text{d.o.f.} = 43.33/69$





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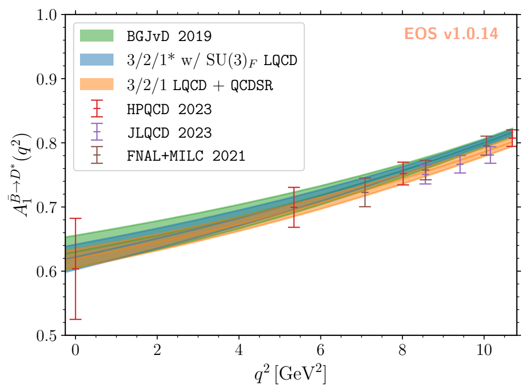

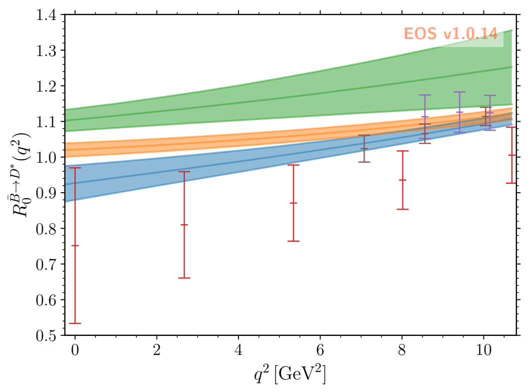

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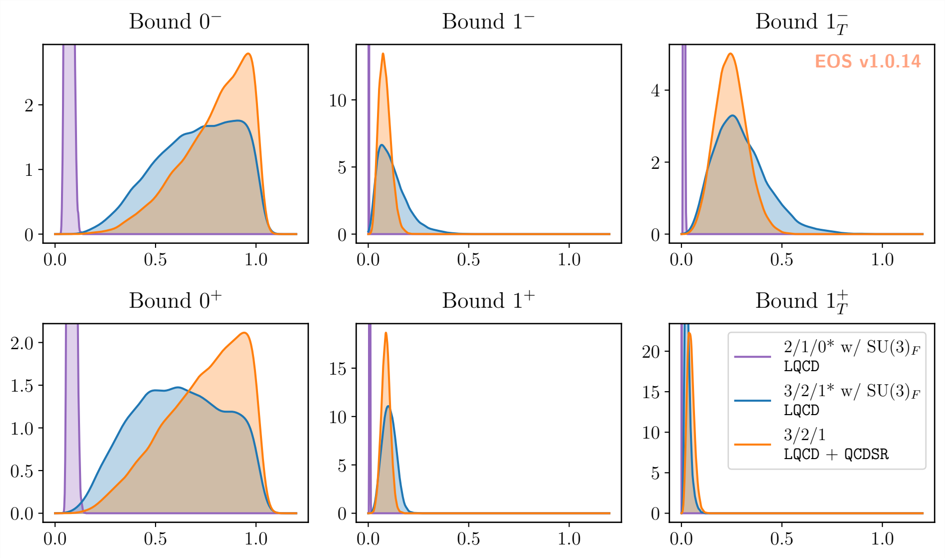
Bound 0^-
Bound 1^-
Bound 1^-_T
EOS v1.0.14
Bound 0^+
Bound 1^+
Bound 1^+_T
2/1/0* w/ $SU(3)_F$ LQCD
3/2/1* w/ $SU(3)_F$ LQCD
3/2/1 LQCD + QCDSR

HQE Conclusions

- lattice-only fit of HQE parametrization succeeds
  - removed sum-rule inputs used in previous HQE analyses
  - HQE well behaved: all power-suppressed parameters come out $\mathcal{O}(1)$
- subtlety: $\bar{B} \to D^{(*)}$ form factors do not constraint all direction in parameter space
  - identifying some of the subsubleading IW parameters removes them, hence: $3/2/1^*$ model
  - $\bar{B}^* \to D$ form factors could help to disentangle
- [shared with BGL] high variability in some form factors, despite excellent p value

Summary

Summary and Outlook

- theory description of exclusive $b \to c\ell^- \bar{\nu}$ decays in good state

- LQCD results statistically compatible with each other
  - still, high variability in between predictions e.g. for R_{D^*} in individualized analyses

- LQCD has improved by leaps and bounds over in the last half decade
  - looking forward to results beyond narrow-width approximation for the D^*
  - preparation ongoing

[see Gustafson et al. 2311.00864]