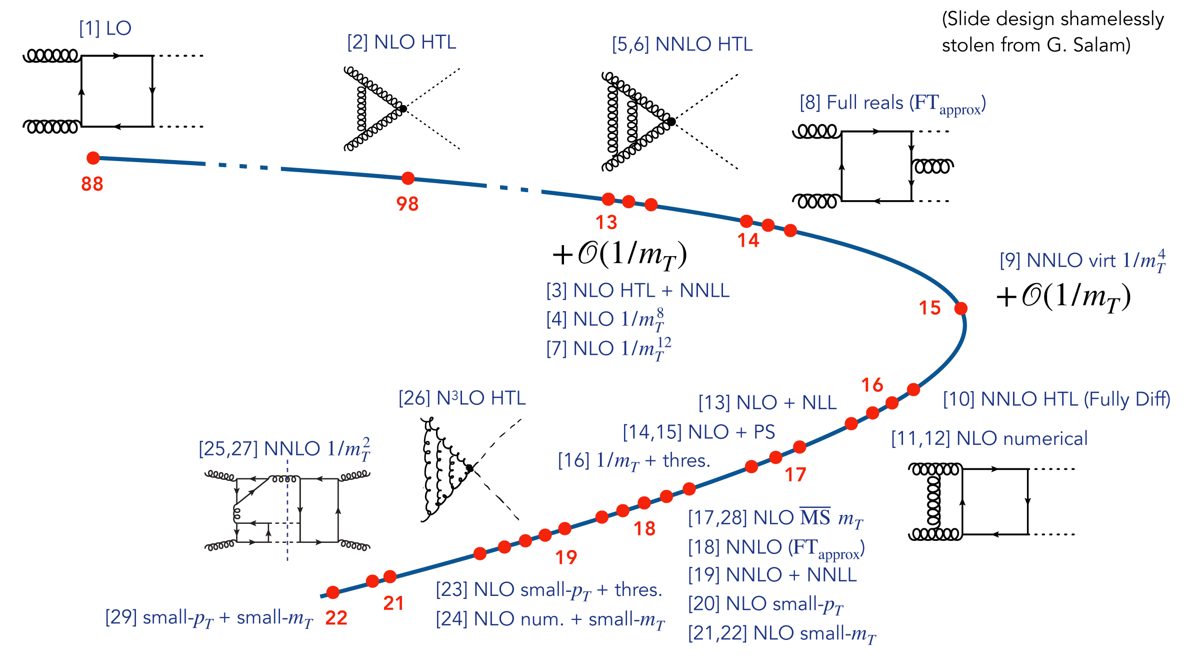

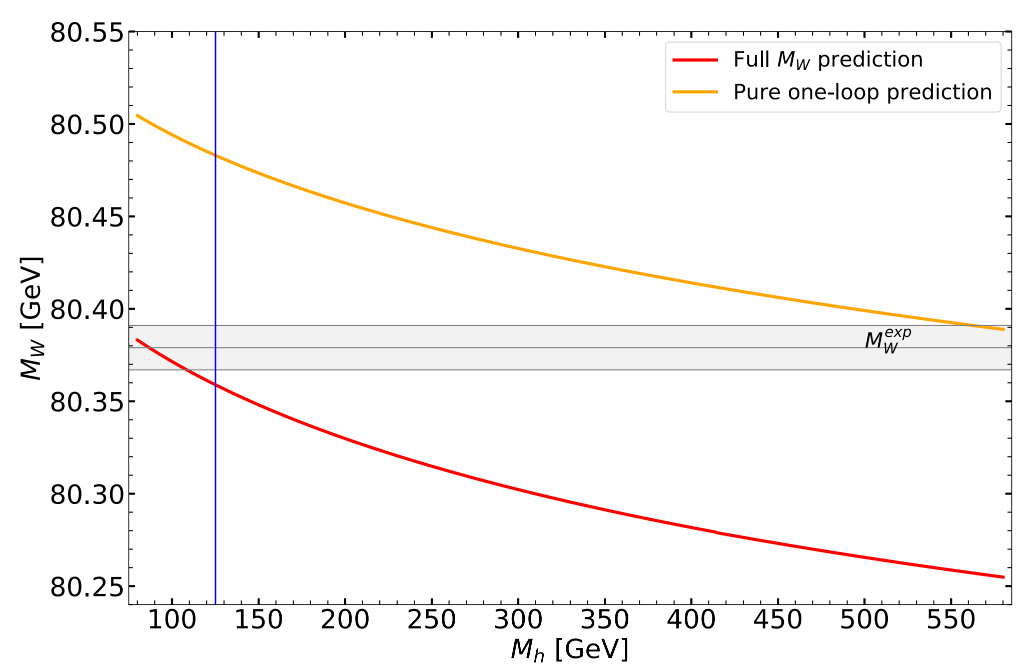

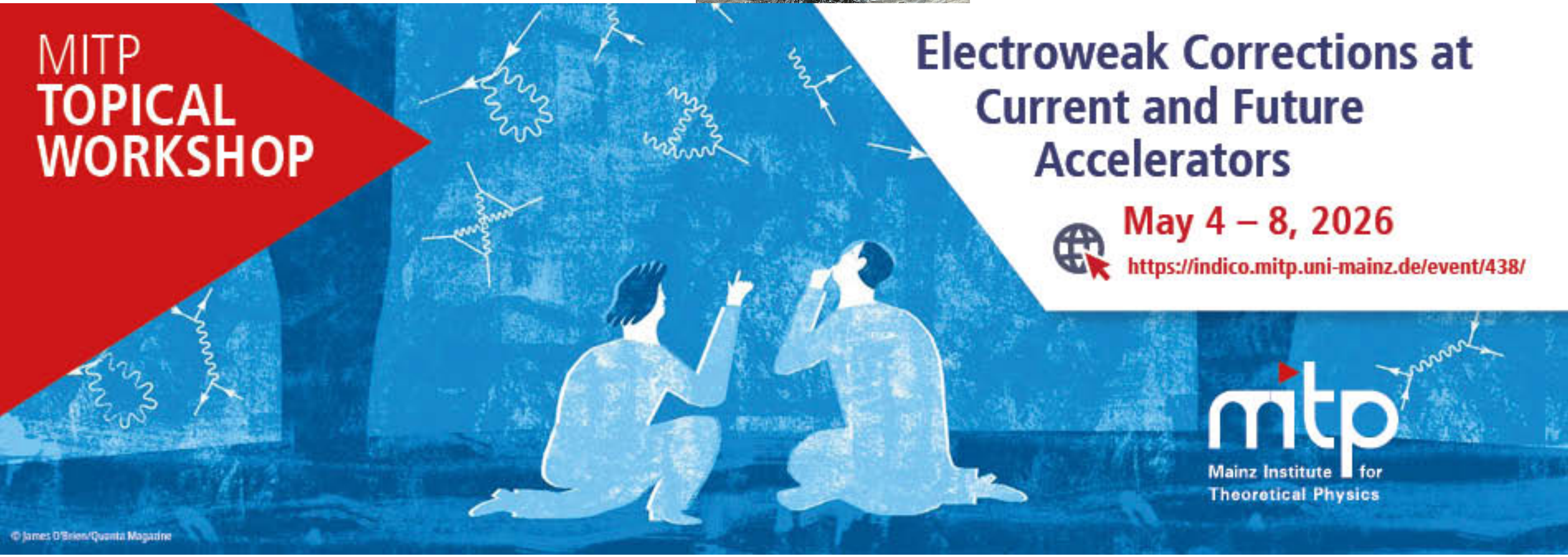

How to estimate theory uncertainties

Georg Weiglein, DESY & UHH

MITP WS, Mainz, 05 / 2026

Universität Hamburg
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Introduction

Confronting accurate theory predictions with precise experimental measurements:

[see talks by Matthias, Maarten, Alexander, Alessandro V., Fulvio, Jürgen, Giuseppe, Alessandro C., …]

- Extraction of ``pseudo-observables” (particle masses, branching ratios, total cross sections, …) from the experimental data requires procedures involving unfolding, deconvolution, extrapolation, …

- Comparison of the experimental results for pseudo-observables with theoretical predictions in different models

Both steps are affected by theoretical uncertainties

Instead of pseudo-observables, can perform comparison also at the level of fiducial cross sections, distributions, STXS, … : direct comparison of Monte Carlo prediction with the data

Example of pseudo-observables: masses of unstable particles

Masses of unstable particles are not directly physical observables (can only measure cross sections, branching ratios, kinematical distributions, …): pseudo-observables, determination involves a deconvolution procedure (unfolding)

Different parameterisations of the resonance: Breit-Wigner shape with running or constant width

The experimental mass parameter is obtained from comparison data — Monte Carlo prediction

⇒ The experimental mass parameters M_W, M_Z, m_t, … are not strictly model-independent

Example: model dependence of the Z-boson mass, $M_Z^{exp} = 91.1876 \pm 0.0021$ GeV, depends (slightly) on the Higgs-boson mass of the SM! $\delta M_Z^{exp} \approx \pm 0.2$ MeV for 100 GeV $< M_H <$ 1 TeV, corresponds to about 10% of the experimental error

Extraction of $\sin^2\theta_{eff}$ from data: when prediction reached full 2-loop level additional contributions were required for extraction from the data at LEP

Form factors implemented in *ZFITTER*:

[M. Awramik, M. Czakon, A. Freitas '06]

$$
\begin{aligned}
\mathcal{A}[e^+e^- \to f\bar{f}] = 4\pi i\,\alpha\,\frac{Q_\mathrm{e}Q_\mathrm{f}}{s}\,\gamma_\mu \otimes \gamma^\mu \\
+ i\frac{\sqrt{2}G_\mu M_\mathrm{Z}^2}{1 + i\Gamma_\mathrm{Z}/M_\mathrm{Z}}\, I_\mathrm{e}^{(3)}\, I_\mathrm{f}^{(3)}\,\frac{1}{s - \overline{M}_\mathrm{Z}^2 + i\overline{M}_\mathrm{Z}\overline{\Gamma}_\mathrm{Z}} \\
\times\,\rho_\mathrm{ef}\Big[\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu(1+\gamma_5) \\
- 4|Q_\mathrm{e}|s_\mathrm{W}^2\,\kappa_\mathrm{e}\,\gamma_\mu\otimes\gamma^\mu(1+\gamma_5) \\
- 4|Q_\mathrm{f}|s_\mathrm{W}^2\,\kappa_\mathrm{f}\,\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu \\
+ 16|Q_\mathrm{e}Q_\mathrm{f}|s_\mathrm{W}^4\,\kappa_\mathrm{ef}\,\gamma_\mu\otimes\gamma^\mu\Big]
\end{aligned}
$$

$$
\begin{aligned}
\kappa_\mathrm{ef}(s) &= \kappa_\mathrm{e}(s)\kappa_\mathrm{f}(s) - \frac{M_\mathrm{Z}^2 - s}{s}\,\frac{1}{(a_\mathrm{e}^{(0)} - v_\mathrm{e}^{(0)})(a_\mathrm{f}^{(0)} - v_\mathrm{f}^{(0)})} \\
&\quad\times\left[q_\mathrm{e}^{(1)}q_\mathrm{f}^{(0)} + q_\mathrm{f}^{(1)}q_\mathrm{e}^{(0)} - p_\mathrm{f}^{(1)}q_\mathrm{e}^{(0)}\frac{v_\mathrm{f}^{(0)}}{a_\mathrm{f}^{(0)}} - p_\mathrm{e}^{(1)}q_\mathrm{f}^{(0)}\frac{v_\mathrm{e}^{(0)}}{a_\mathrm{e}^{(0)}} - q_\mathrm{e}^{(0)}q_\mathrm{f}^{(0)}\frac{\Sigma_{\gamma\gamma}^{(1)}}{s} + \text{boxes}\right], \\
\kappa_\mathrm{e,f}(s) &= \kappa_\mathrm{Z}^\mathrm{e,f}(s) + \frac{M_\mathrm{Z}^2 - s}{s}\left[\frac{q_\mathrm{e,f}^{(0)}}{a_\mathrm{e,f}^{(0)} - v_\mathrm{e,f}^{(0)}}\,\frac{p_\mathrm{f,e}^{(1)}}{a_\mathrm{f,e}^{(0)}} + \text{boxes}\right], \\
\kappa_\mathrm{Z}^\mathrm{f}(s) &= \kappa_\mathrm{Z}^\mathrm{f}(M_\mathrm{Z}^2) + (s - M_\mathrm{Z}^2)\,\frac{\hat{a}_\mathrm{f}^{(1)\prime}(M_\mathrm{Z}^2)\,v_\mathrm{f}^{(0)} - \hat{v}_\mathrm{f}^{(1)\prime}(M_\mathrm{Z}^2)\,a_\mathrm{f}^{(0)}}{a_\mathrm{f}^{(0)}(a_\mathrm{f}^{(0)} - v_\mathrm{f}^{(0)})}.
\end{aligned}
$$

Relation between $\sin^2\theta_{eff}^f$ determined from expansion around the complex pole and the one defined in *ZFITTER*:

$$
\sin^2\theta_{\mathrm{eff,pole}}^f = \overline{s}_\mathrm{W}^2\,\mathrm{Re}\left\{\overline{\kappa}_\mathrm{Z}^\mathrm{f}(M_\mathrm{Z}^2)\right\} = \sin^2\theta_{\mathrm{eff,ZFITTER}}^f - \frac{\Gamma_\mathrm{Z}}{M_\mathrm{Z}}\,\frac{q_\mathrm{f}^{(0)}}{a_\mathrm{e}^{(0)}(a_\mathrm{f}^{(0)} - v_\mathrm{f}^{(0)})}\,\mathrm{Im}\{p_\mathrm{e}^{(1)}\}
$$

$$
\overline{s}_\mathrm{W}^2 = \left(1 - \frac{\overline{M}_\mathrm{W}^2}{\overline{M}_\mathrm{Z}^2}\right) = s_\mathrm{W}^2\left[1 + \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\left(\frac{\Gamma_\mathrm{W}^2}{M_\mathrm{W}^2} - \frac{\Gamma_\mathrm{Z}^2}{M_\mathrm{Z}^2}\right)\right]^{-1}.
$$

numerically small, but required at this order

Sources of theoretical uncertainties

- Parametric uncertainties from the experimental errors of the input parameters: can be treated in a systematic way in the theoretical predictions (note: the experimental error of the top-quark mass includes the uncertainty from relating the measured ``Monte Carlo mass" to a theoretically well-defined input parameter)

  [see talks by Matthias, Maarten, Giuseppe, …]

- PDF uncertainties

- ``Intrinsic" theoretical uncertainties from unknown higher-order contributions (perturbative / non-perturbative): non-trivial to estimate, there exists no algorithm that is guaranteed to provide the ``correct" result; ``try to quantify what we don't know"

In the following I will mainly focus on intrinsic theoretical uncertainties. I will not attempt to provide a recipe for obtaining the ``best" estimate of theory uncertainties, will discuss some aspects that are relevant in this context

How to estimate ``intrinsic” theoretical uncertainties?

- There exists no algorithm that is guaranteed to provide the ``correct” result: one can only find out how good the previous uncertainty estimate was once an improved calculation has been carried out
- People who have calculated a certain higher-order contribution have a tendency to estimate the remaining higher-order contributions beyond what they have calculated to be relatively small
- Projections for future theoretical uncertainties during the current update of the European strategy for particle physics: ``conservative” and ``aggressive” scenarios

Current status of electroweak precision observables

[see talks by Giuseppe, ...]

- From experimental errors of the input parameters

$\delta m_t = 0.7$ GeV, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$\delta M_W^{\text{para},m_t} = 4$ MeV, $\quad \delta M_W^{\text{para},\Delta\alpha_{\text{had}}} = 2$ MeV, $\quad \delta M_W^{\text{para},M_Z} = 2.5$ MeV

- From unknown higher-order corrections (“intrinsic”)

SM: Complete 2-loop result $+$ leading higher-order corrections known for $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$

$\Rightarrow$ Remaining uncertainties:

[*M. Awramik, M. Czakon, A. Freitas, G.W. ’03, ’04*]

[*M. Awramik, M. Czakon, A. Freitas ’06*]

update after latest results?

$$\Delta M_{\rm W}^{\rm intr} \approx 4\ {\rm MeV}, \quad \Delta\sin^2\theta_{\rm eff}^{\rm intr} \approx 5\times 10^{-5}$$

Projections for future theoretical uncertainties

[A. Freitas, Venice Open Symposium ’25]

Z-pole (SM predictions for POs)

18

- Comparison of POs extracted from data with SM predictions used to test SM and probe BSM physics
- EW SM corrections are relatively large ➔ need multi-loop contributions
- *Current:* NNLO + m_t-enhanced N3LO*
- *“Conservative”:* N3LO with $N_f \geq 2$ or $N(\alpha_S) \geq 1$ + m_t-enhanced N4LO
- *“Aggressive”:* N4LO with $N_f \geq 2$ or $N(\alpha_S) \geq 2$ + partial m_t-enhanced N5LO

* also $O(\alpha_t \alpha_S^3)$

For past work see: 1809.01830, 1906.05379

	current	**future (conservative)**	**future (aggressive)**
σ_{had} [pb]	6	1.6	0.3
R_ℓ [10^{-3}]	6	1.2	0.2
R_c [10^{-5}]	5	1	0.2
R_b [10^{-5}]	10	2	0.35
Γ_Z [MeV]	0.4	0.08	0.016
$\sin^2\theta_{eff,lept}$ [10^{-5}]	4.5	0.7	0.06
m_W [MeV]	4	1	0.1

Future prospects?

The question of how much the theoretical uncertainties of the electroweak precision observables can be reduced in the future was an important topic of the recent discussions of high-precision physics at future e^+e^- colliders:
experimental errors of the input parameters (m_b, m_t, $\alpha(M_Z)$, α_s, …),
unknown higher-order corrections, non-perturbative physics, …

Full exploration of Tera-Z and W-physics programme may be limited by conceptual obstacles of full electroweak 3-loop calculations:
massive 3-loop integrals (different scales), renormalisation, treatment of unstable particles, consistent treatment of γ_5, hadronic contributions to the vacuum polarisation, …

Achievable progress even on a time scale of 20 years or more is difficult to predict; from my perspective even the claimed ``conservative” estimate of future theory uncertainties does not look very conservative, not to speak of the ``aggressive” one …

Approaches for estimating intrinsic theoretical unc.

- Renormalisation / factorisation scale dependence: often used in calculations of QCD corrections; accounts for only a part of the higher-order corrections

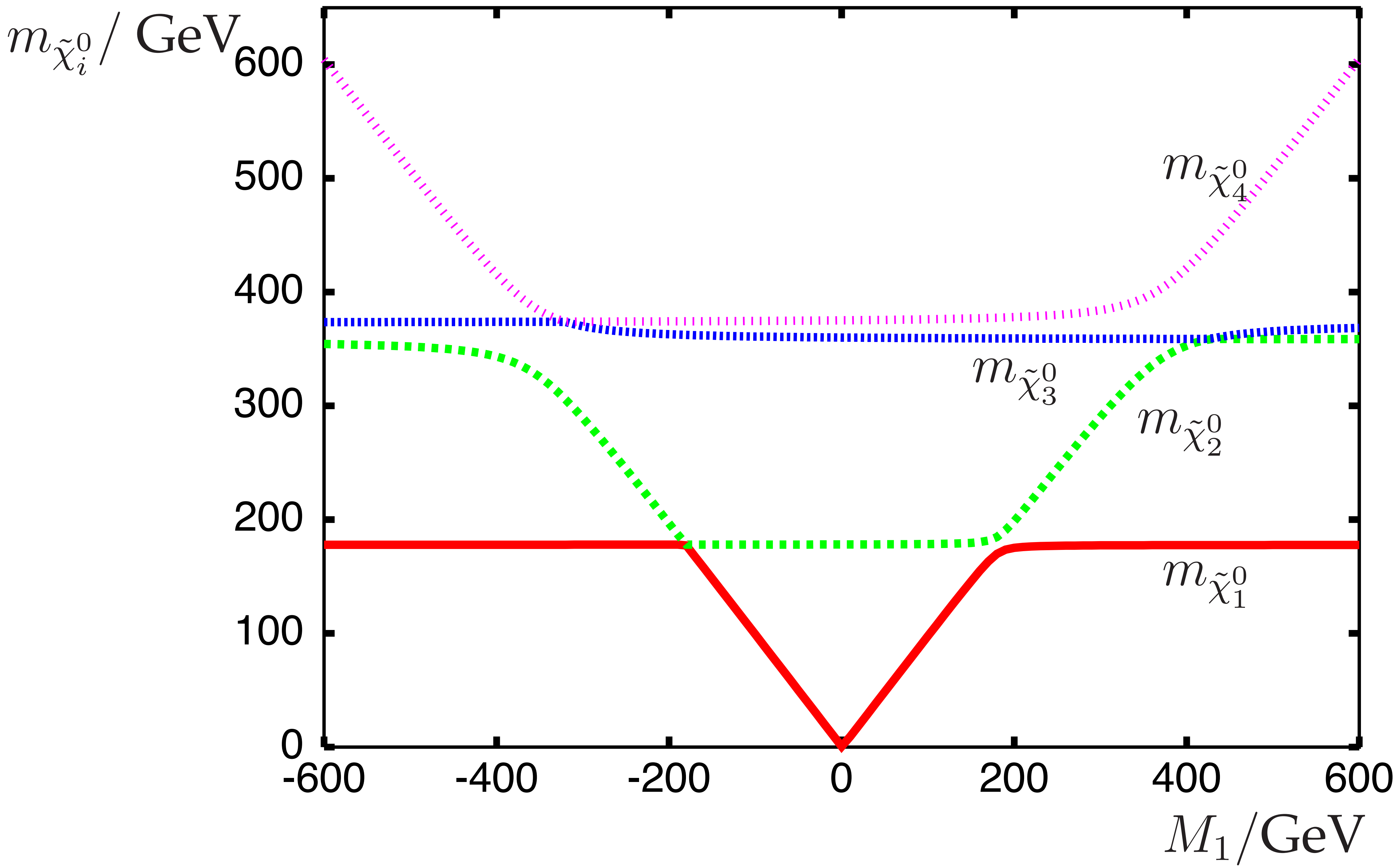
- The difference between the predictions in different renormalisation schemes / different parameterisations (e.g. G_μ or $\alpha(0)$ scheme) can be used to estimate the possible size of unknown higher-order contributions

  - Does not necessarily capture new sources of potentially large contributions at higher orders (e.g. new enhancement factors, new subprocesses, Sudakov logs, …)

  - Not every renormalisation scheme necessarily works equally well; for BSM scenarios it is in general not possible to find a single scheme that works for the whole parameter space!

Example: use of neutralino masses in SUSY models to determine parameters of neutralino sector

[K. Desch et al. ’03]



⇒Yields large uncertainties where the dependence on the model parameters is ``flat”

Remarks on ``hybrid” or ``mixed” schemes for SM, BSM

For the renormalisation of BSM models often ``mixed” schemes are proposed where some parameters are renormalised on-shell and others MSbar or DRbar. Schemes of this kind have also been put forward in the ``SPA Convention” *[J.A. Aguilar-Saavedra et al. ’06]*

However, it was realised that such schemes can lead to unphysical dependencies on the chosen tadpole scheme

Why unphysical? Because the dependence on the tadpole scheme drops out in relations between physical observables!

Even in relations between physical observables it may still be problematic to use the FJ tadpole scheme because of the numerically large contributions that it induces

All these problems are avoided if the free parameters (renormalised OS, MSbar, …) are determined from set of physical observables!

Extension of mixed schemes to the two-loop level?

[see talks by Giuseppe, Lisong, …]

Examples: M_W and MSbar $\sin^2\theta_{eff}$, OS M_Z (SM), tanβ (BSM)
2-loop MSbar/DRbar quantity in a mixed scheme will contain one-loop on-shell counterterms, e.g. for the top or Z mass, as subloop

This can be handled by starting with a pure scheme where all parameters are renomalised MSbar/DRbar and then perform a finite reparametrisation from m_t^{MSbar} to m_t^{OS}

If instead one naively applies the mixed scheme, results involving the two-loop MSbar/DRbar quantity will have a dependence on an unphysical contribution, in this case the (D–4) part (evanescent) of the top (Z) mass counterterm (cannot simply convert between schemes by a finite reparametrisation)

[H. Bahl, D. Meuser, G. W. '23]

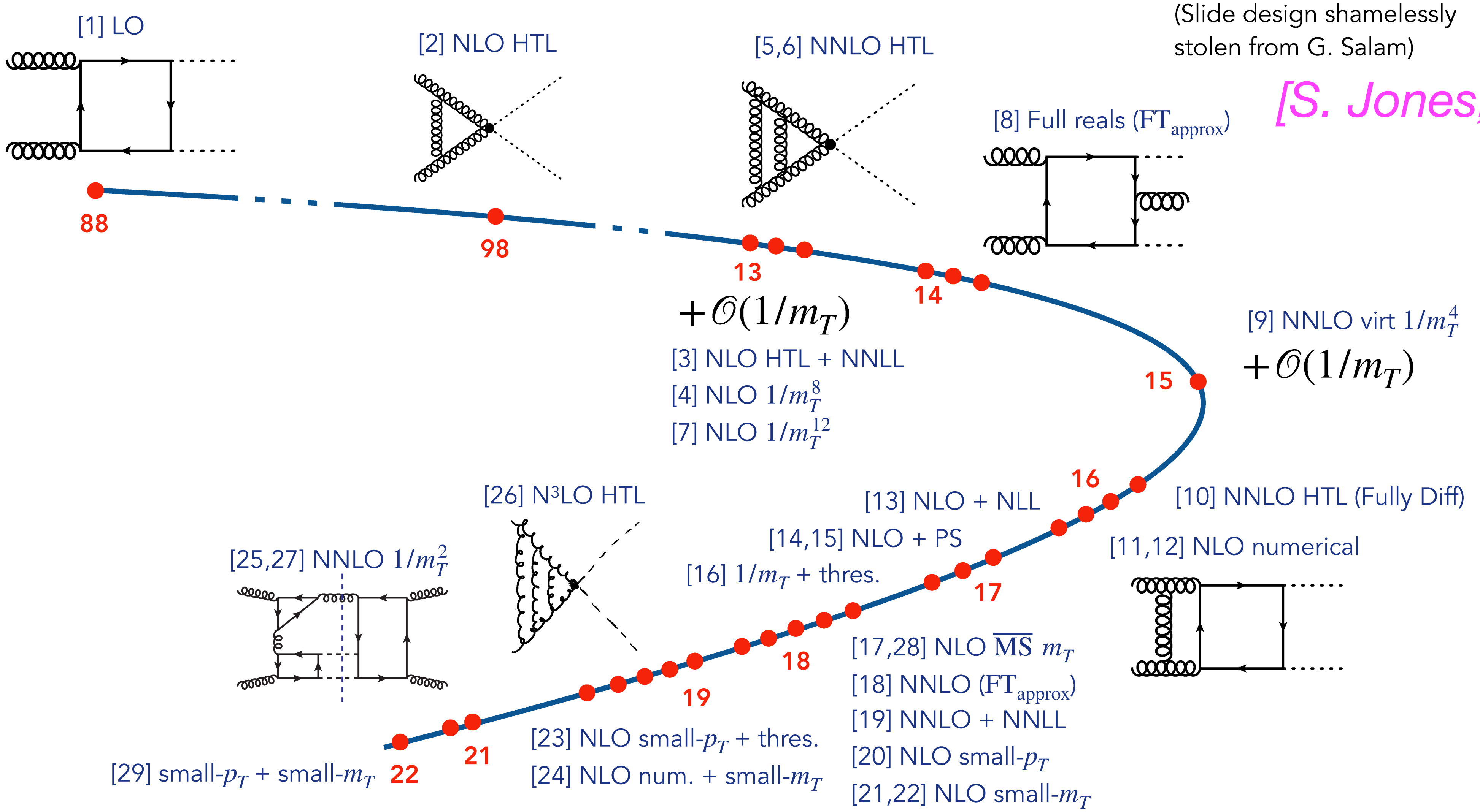
No problems of this kind occur for an OS renormalisation of tanβ at the 2-loop level (using a physical observable, A → ττ decay)

Scheme dep.: Higgs pair production at the LHC

An approximate history (30 years in 30 seconds)

(Slide design shamelessly stolen from G. Salam)

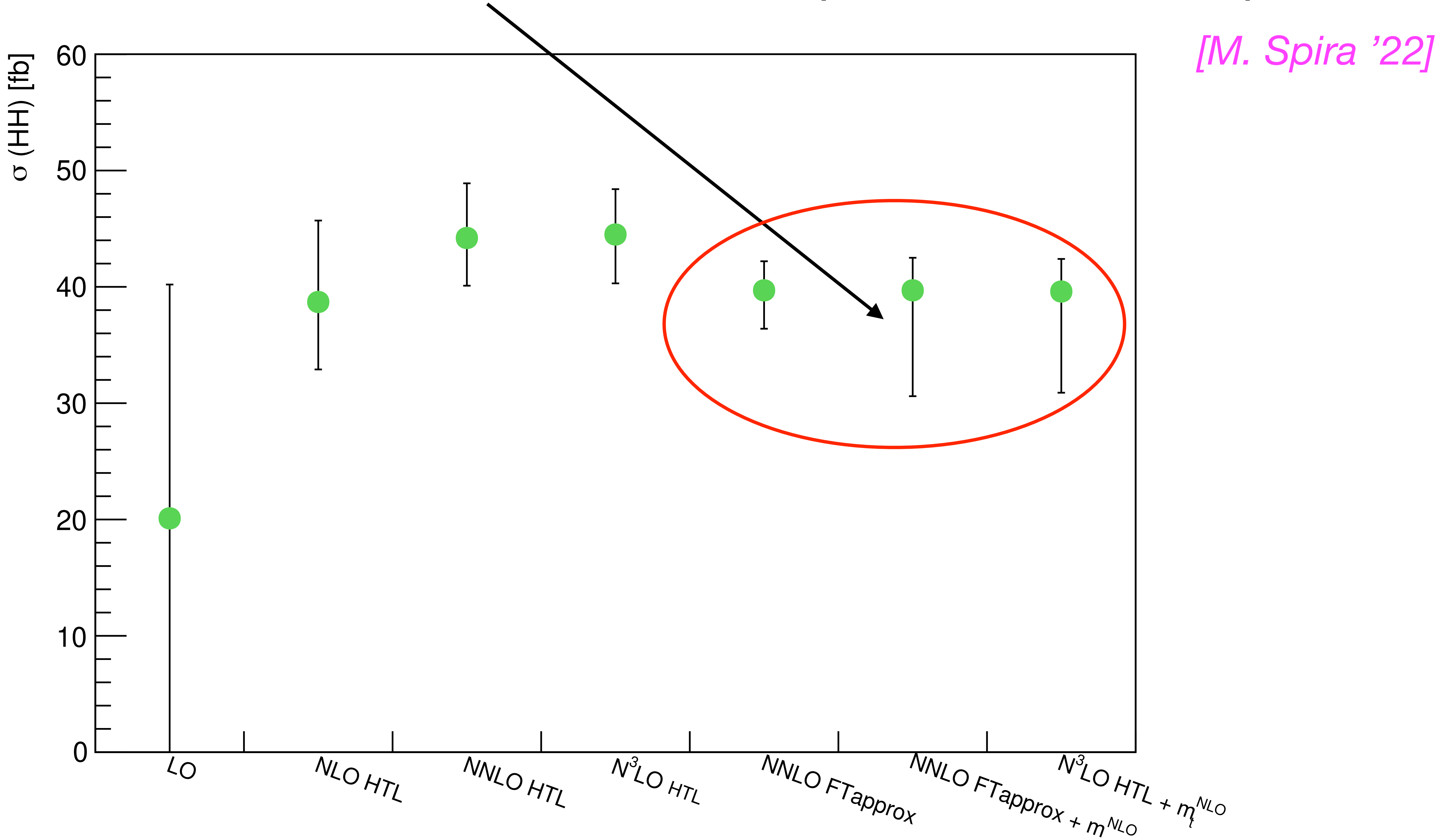
[S. Jones, G. Salam]



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22;

Higgs pair production, prediction and uncertainties

Impact of the renormalisation-scheme dependence of the top mass:



⇒Parameterisations in terms of pole vs. MSbar top quark mass give rise to shifts that are larger than previous uncertainty estimates

Approaches for estimating intrinsic theoretical unc.

- Coupling and enhancement factors (see examples below)

- Progression of relative size of known corrections (infer relative size of 3-loop vs. 2-loop contribution from relative size of 2-loop vs. 1-loop contribution, etc.)

- Different ``options" of state-of-the-art codes
  Example:

[D. Bardin et al. '97]

Electroweak Working Group Report

D. Bardin[ab], W. Beenakker[c], M. Bilenky[ad], W. Hollik[e], M. Martinez[f],
G. Montagna[gl], O. Nicrosini[h*], V. Novikov[i], L. Okun[i], A. Olshevsky[d],
G. Passarino[jk], F. Piccinini[gl], S. Riemann[a], T. Riemann[a], A. Rozanov[im],
F. Teubert[f], M. Vysotsky[i]

Possible sources for large higher-order contributions

Examples for new types of contributions:

- LHC processes where other initial state contributes (e.g. bbH production, 5-flavour vs. 4-flavour scheme, …)

- Enhancement factors (EWPO, Higgs mass in SUSY, trilinear Higgs couplings in extended Higgs sectors. …)
  Example: Higgs mass in the MSSM, $M_h^{\text{tree}} \leqq M_Z$ (gauge sector)
  Large radiative corrections (Yukawa sector, . . .):

  Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}, \frac{e\, m_t^2}{M_W s_W}, \ldots$

  $\Rightarrow$ Dominant one-loop corrections: $G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right), \quad \mathcal{O}(100\%)$!

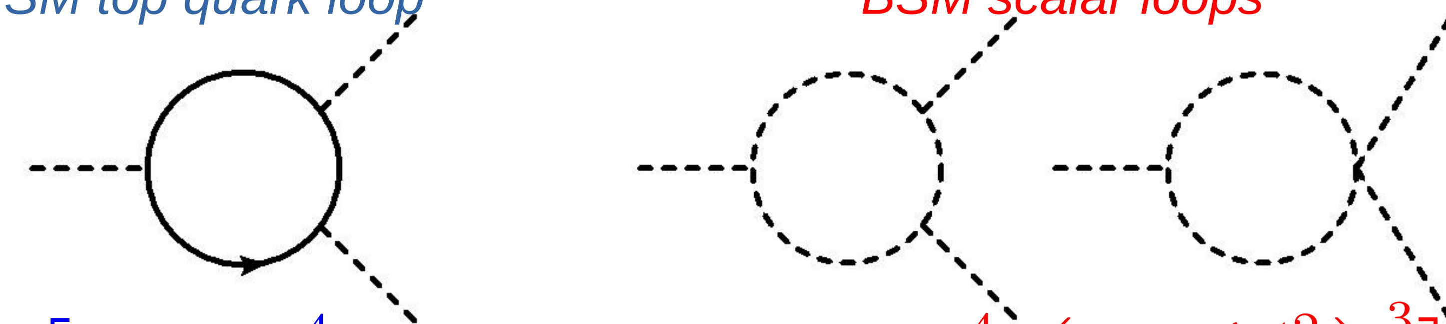
- Suppressed leading-order contribution (e.g. Higgs production at the LHC, …)

Effects of BSM particles on the trilinear Higgs coupling

Trilinear Higgs coupling in extended Higgs sectors: potentially large (several 100%) loop contributions

- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):

SM top quark loop — *BSM scalar loops*



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48 m_t^4}{v^3} + \sum_{\Phi} \frac{4 n_\Phi m_\Phi^4}{v^3}\left(1 - \frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \right]$$

First found in 2HDM: [Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda} v^2$

⇒ Large effects possible for sizeable splitting between m_Φ and $\mathcal{M}$

λ$_{hhh}$: very large deviations from the SM value possible!

[see talks by Sally, …]

EFT analysis:

[M. McCullough, ICHEP 2024]

Self-Coupling Dominance

No obstruction to having Higgs self-coupling modifications a "loop factor" greater than **all** other couplings. Could have

$$\left|\frac{\delta_{h^3}}{\delta_{VV}}\right| \lesssim \min\left[\left(\frac{4\pi v}{m_h}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

without fine-tuning any parameters, as big as,

$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

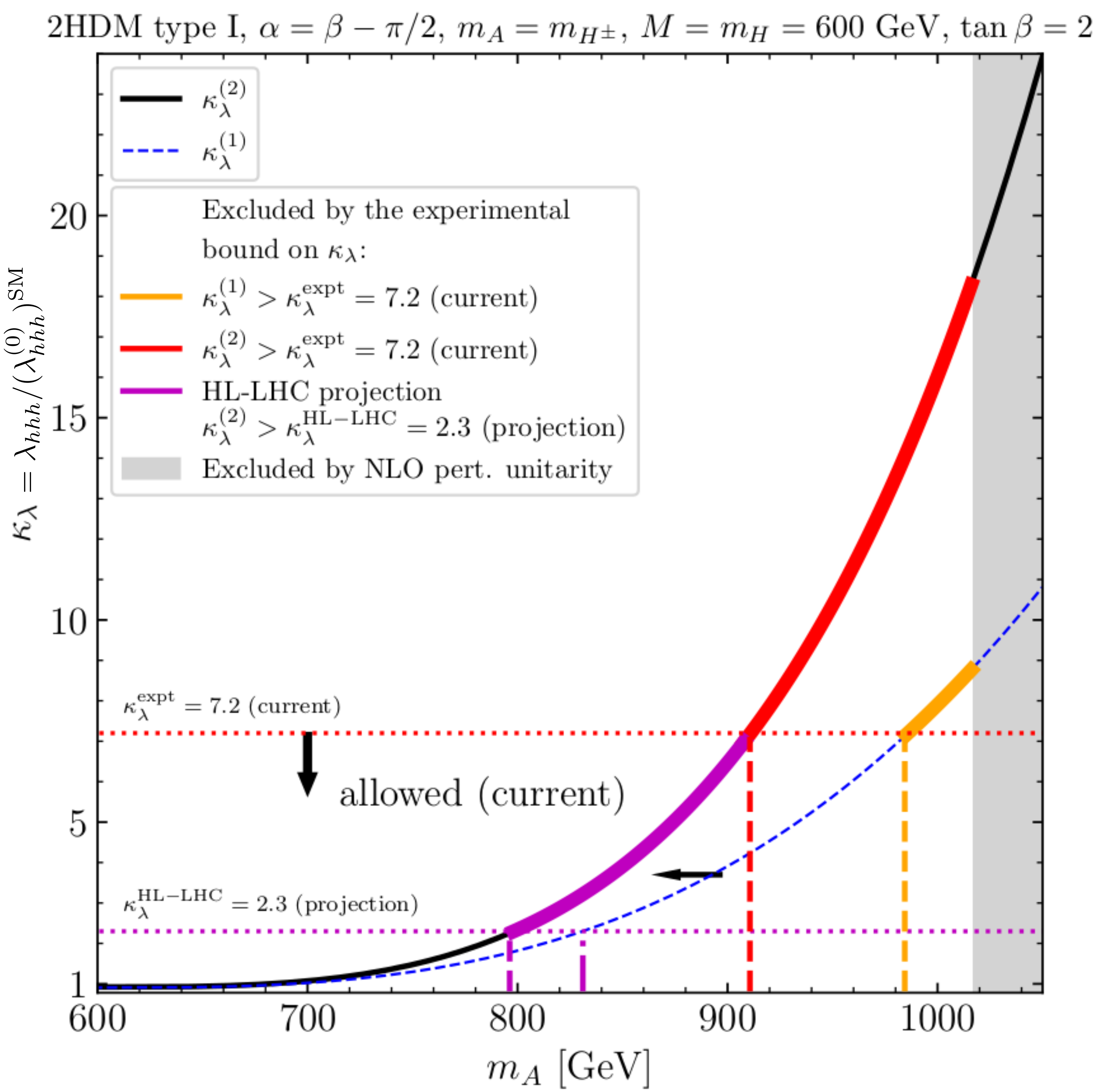
Durieux, MM, Salvioni. 2022

``Higgs self-coupling, … arguably the most important of them all!"

Trilinear Higgs coupling: current experimental limit vs. prediction from extended Higgs sector (2HDM)

Prediction for $\varkappa_\lambda$ up to the two-loop level:

[H. Bahl, J. Braathen, G. W. '22, '24
Phys. Rev. Lett. 129 (2022) 23, 231802]



⇒ Current experimental limit excludes important parameter region that would be allowed by all other constraints!

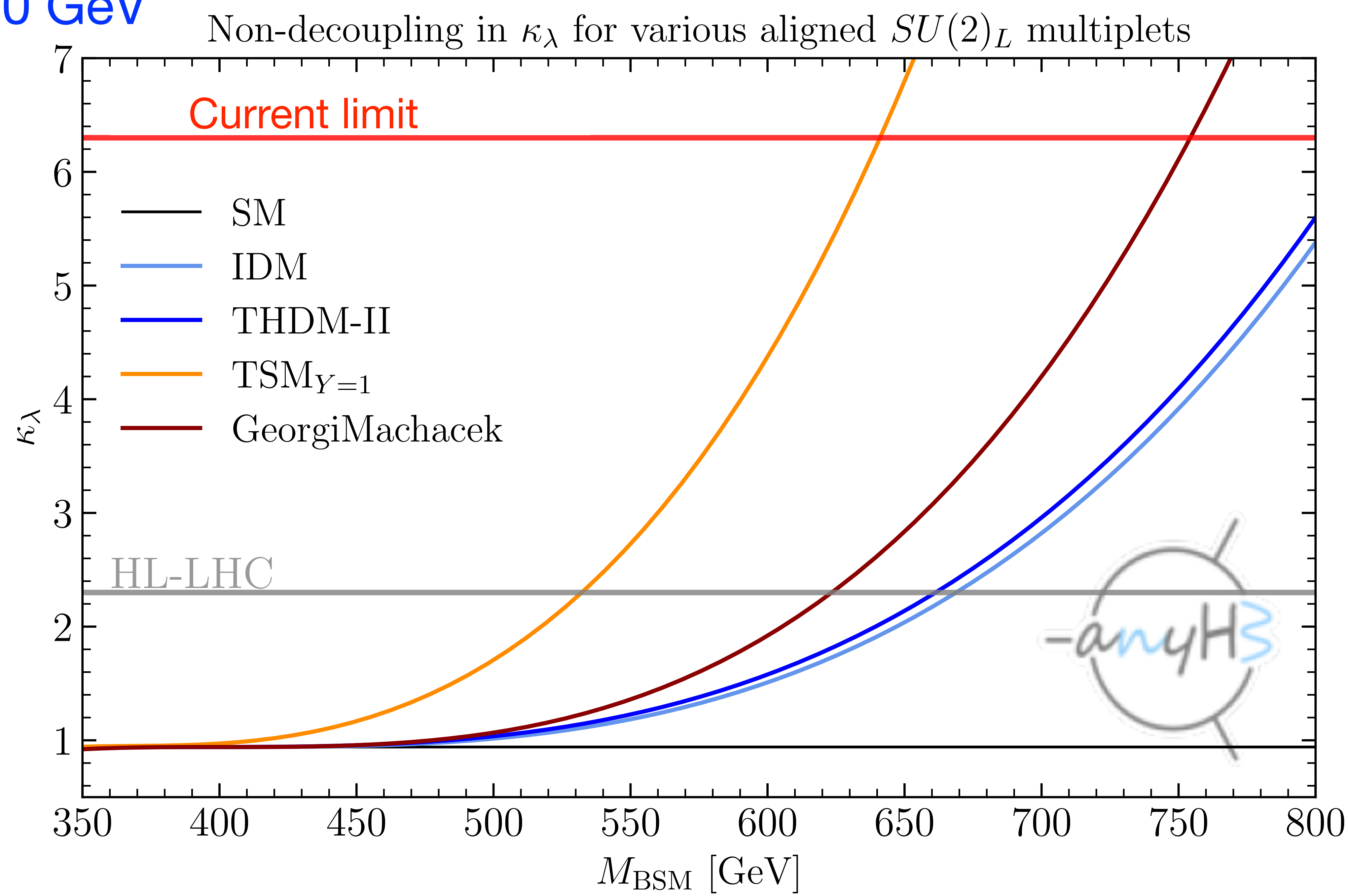
Experimental limit on the trilinear Higgs coupling already has sensitivity to probe extended Higgs sectors!

Trilinear Higgs self-coupling in extended Higgs sectors

Effect of splitting between BSM Higgs bosons: generic feature in extended Higgs sectors (here: one-loop results)

[H. Bahl, J. Braathen, M. Gabelmann, G. W. ’23]

M_L = 400 GeV



How about uncertainties from non-perturbative effects?

[see talks by Giuseppe, …]

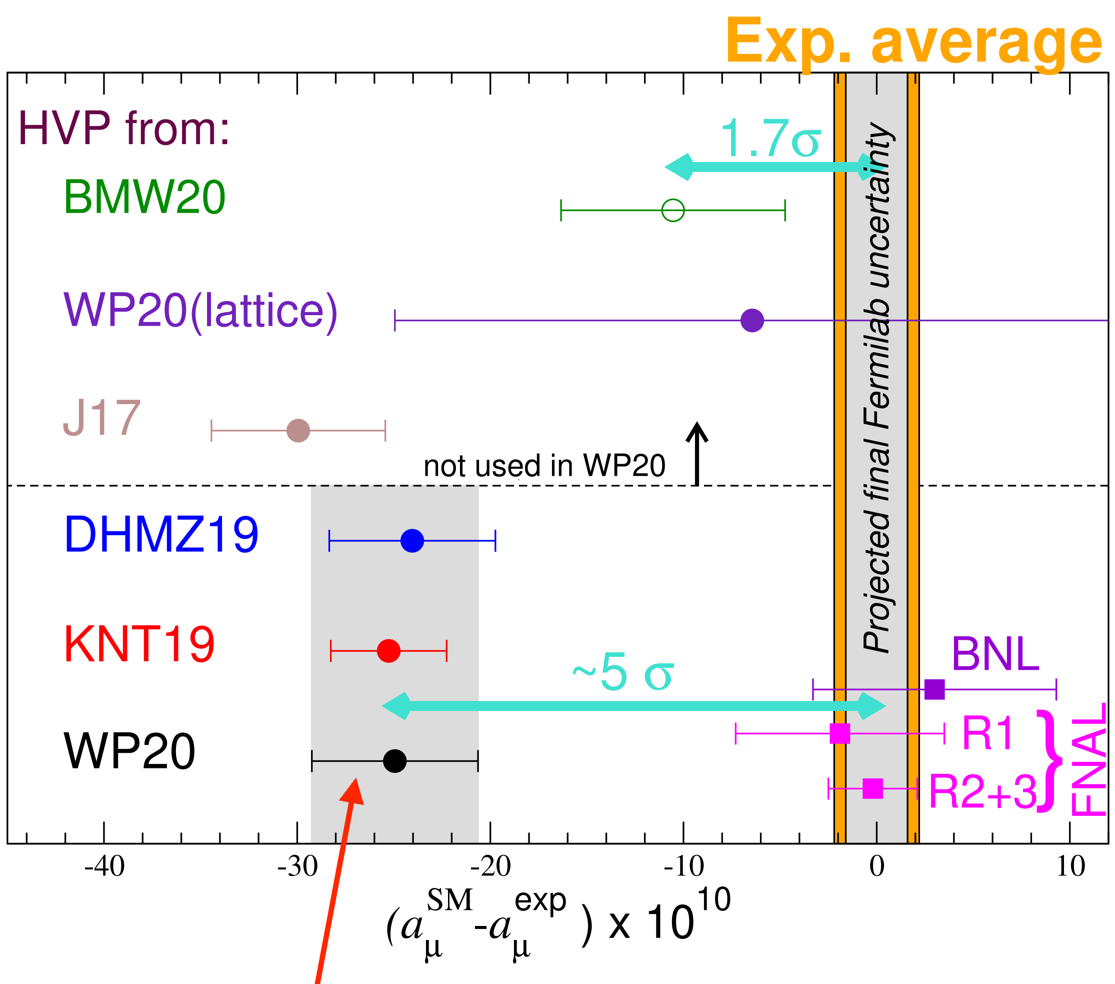
Recent example: g_μ–2, muon anomalous magnetic moment

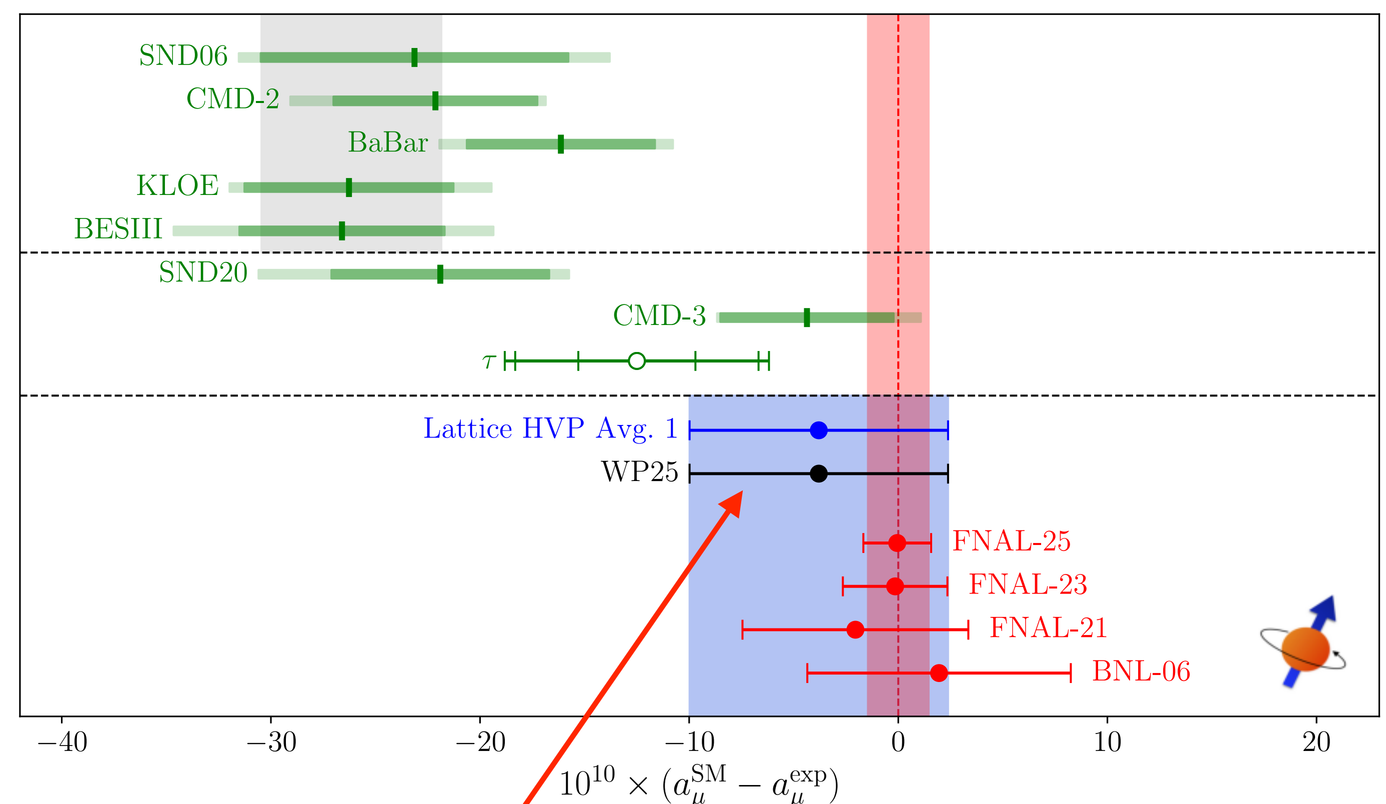
Until six years ago the SM prediction for g_μ–2 seemed very robust and well-established:

[G. Colangelo '25]

After the 2023 Fermilab result

Exp. band=current world average





SM prediction 2020

SM prediction 2025

Uncertainties from non-perturbative effects

[K. Melnikov '25]

Yet, as the precision increases, it becomes more and more difficult to access the credibility of claims that precision can really be controlled at the required level, complicating the whole approach.

Because of high stakes associated with potential outcomes of such precision physics studies, the observables and the theoretical tools that are needed to describe them should be as transparent and simple as possible.

1) Perturbation theory is the best possible tool that we have, but the perturbative expansion alone is almost never sufficient.

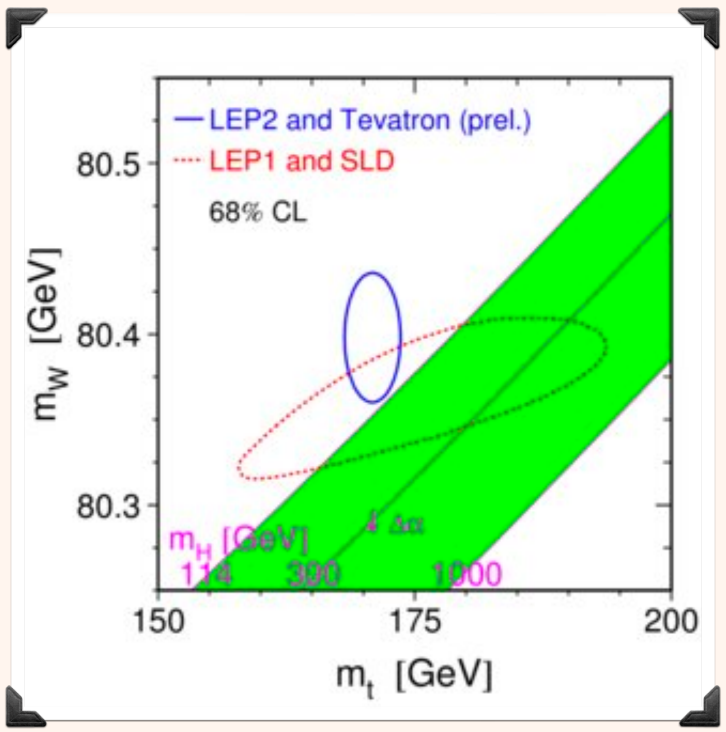
2) Standard Model precision physics and perturbative Standard Model physics are certainly not the same thing, even at the energy frontier. It is only the question of the requested precision, that the non-perturbative physics starts playing a role.

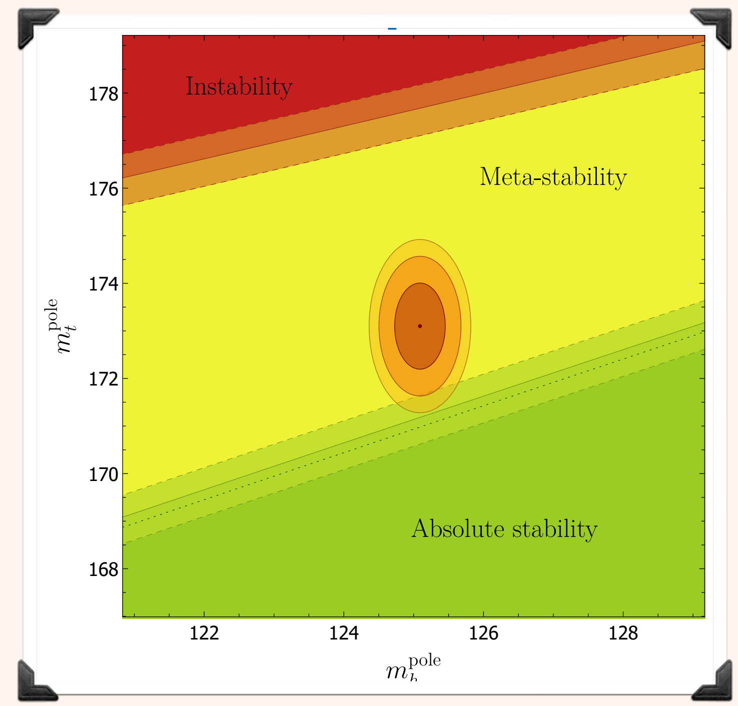
3) The question is one of balance. If parton showers or lattice methods, or other ways to estimate non-perturbative effects are critical for claiming discrepancy (or agreement) with the Standard Model, it is inevitable that the reliability of such complex theoretical tools will be scrutinized.

Non-perturbative effects w.r.t. top-quark pole mass and its relation to the measured Monte Carlo mass

[K. Melnikov '25]

Another quantity where non-perturbative effects play a role and may obscure the outcome, is the top quark mass. According to PDG, top quarks have different masses (pole, MS, MC..) which are all quoted there.





1) The MC top quark mass is a complex issue. Probably it is a combination of a technical issue (hard cut-off in the event generators) and the physics question of how energy of heavy quark jets is calibrated.

2) The pole mass of a top quark cannot be measured from a $t\bar{t}$ production cross section with the accuracy that is better than $\mathcal{O}(\Lambda_{\rm QCD})$.

How to treat theoretical uncertainties together with other types of uncertainties?

The discussed ``intrinsic” theoretical uncertainties do not have a statistical interpretation

Linear / quadratic combination with other sources of uncertainties?

Express theoretical uncertainties in terms of nuisance parameters?

…

Theoretical uncertainties in BSM predictions

In BSM calculations the remaining theoretical uncertainties are in general parameter-space dependent (simplest example: if all BSM particles are very heavy, the theoretical uncertainties should be the same as in the SM; if some BSM particles are relatively light there will be additional uncertainties arising from the BSM part of the calculation)

BSM codes should therefore provide parameter-space dependent estimates of the remaining uncertainties (example: Higgs sector observables in *FeynHiggs*)

Theoretical uncertainties in BSM predictions

For various observables the comparison between experiment and theory is sensitive to higher-order corrections that are only known for the SM but not for BSM models

$\Rightarrow$ BSM effects need to be tested relative to the SM predictions

Examples:

- Electroweak precision observables
- Gluon fusion Higgs production: need to incorporate state-of-the art SM predictions for the SM-like Higgs boson
- BR predictions in models where the Higgs mass is predicted (e.g. SUSY): need to rescale the mass prediction to the measured value and use state-of-the art SM predictions for $h \to 4$ fermions

BSM predictions for EWPOs

Experimental accuracy for M_W and $\sin^2\theta_{eff}$ provides high sensitivity to loop contributions at the two-loop level and beyond!

While within the SM the predictions for M_W and $\sin^2\theta_{eff}$ are known at the level of full two-loop and leading higher-order contributions, no full two-loop predictions exist in any BSM model

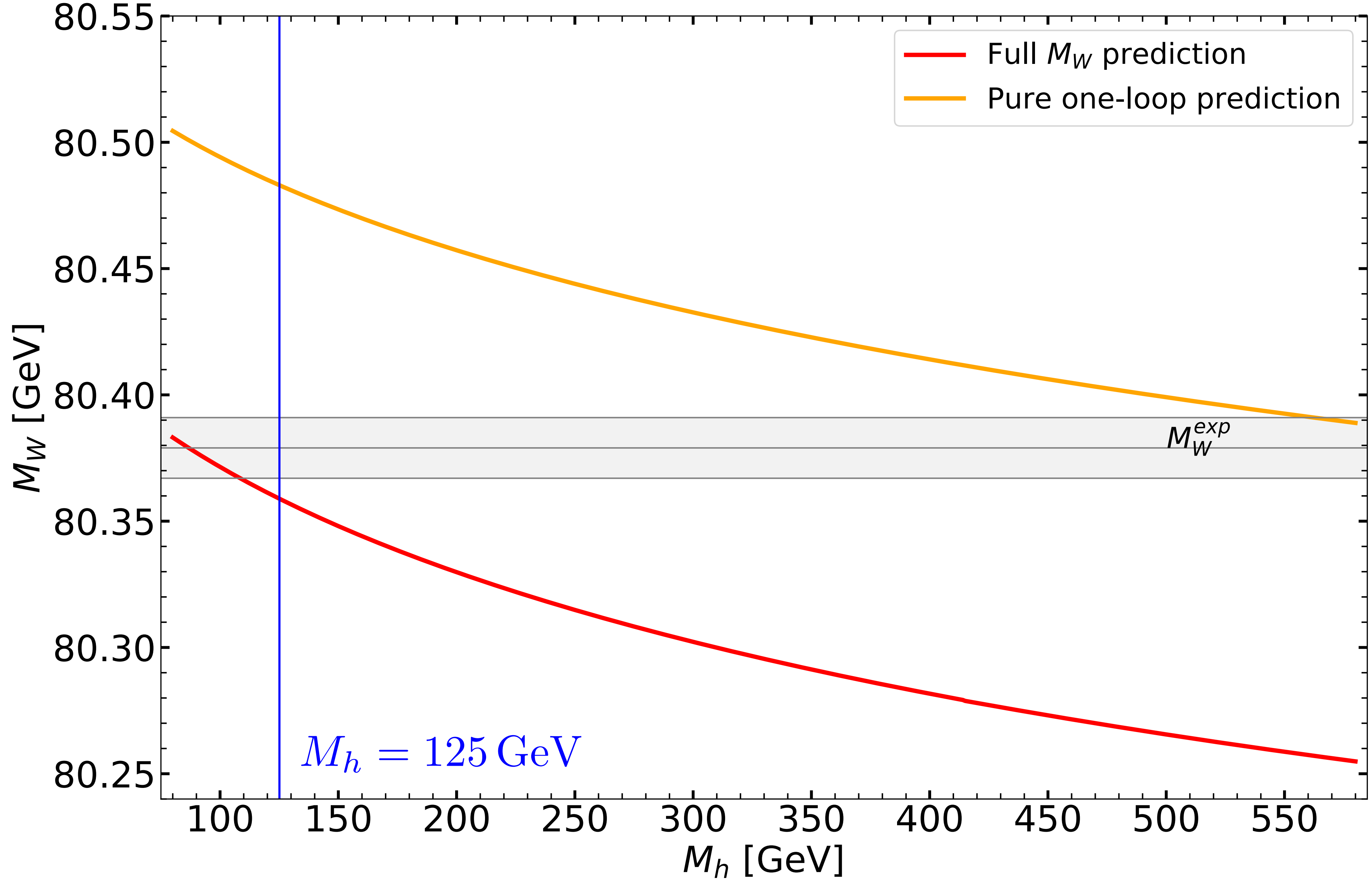
However, restricting the BSM predictions for the EWPOs to the 1-loop level would result in a prediction that is completely off because of the missing SM-like higher-order contributions!

$\Rightarrow$ BSM predictions for (at least) M_W and $\sin^2\theta_{eff}$ need to take into account all known SM-like contributions + the prediction for (BSM – SM) at the level of accuracy for which the BSM prediction is known!

W-mass prediction within the SM: one-loop result vs. state-of-the-art prediction

(up to a few weeks ago)

[M. Berger, S. Heinemeyer, G. Moortgat-Pick, G. W. '22]



⇒ Pure one-loop result would imply preference for heavy Higgs, M_h > 500 GeV

Corrections beyond one-loop order are crucial for reliable prediction of M_W

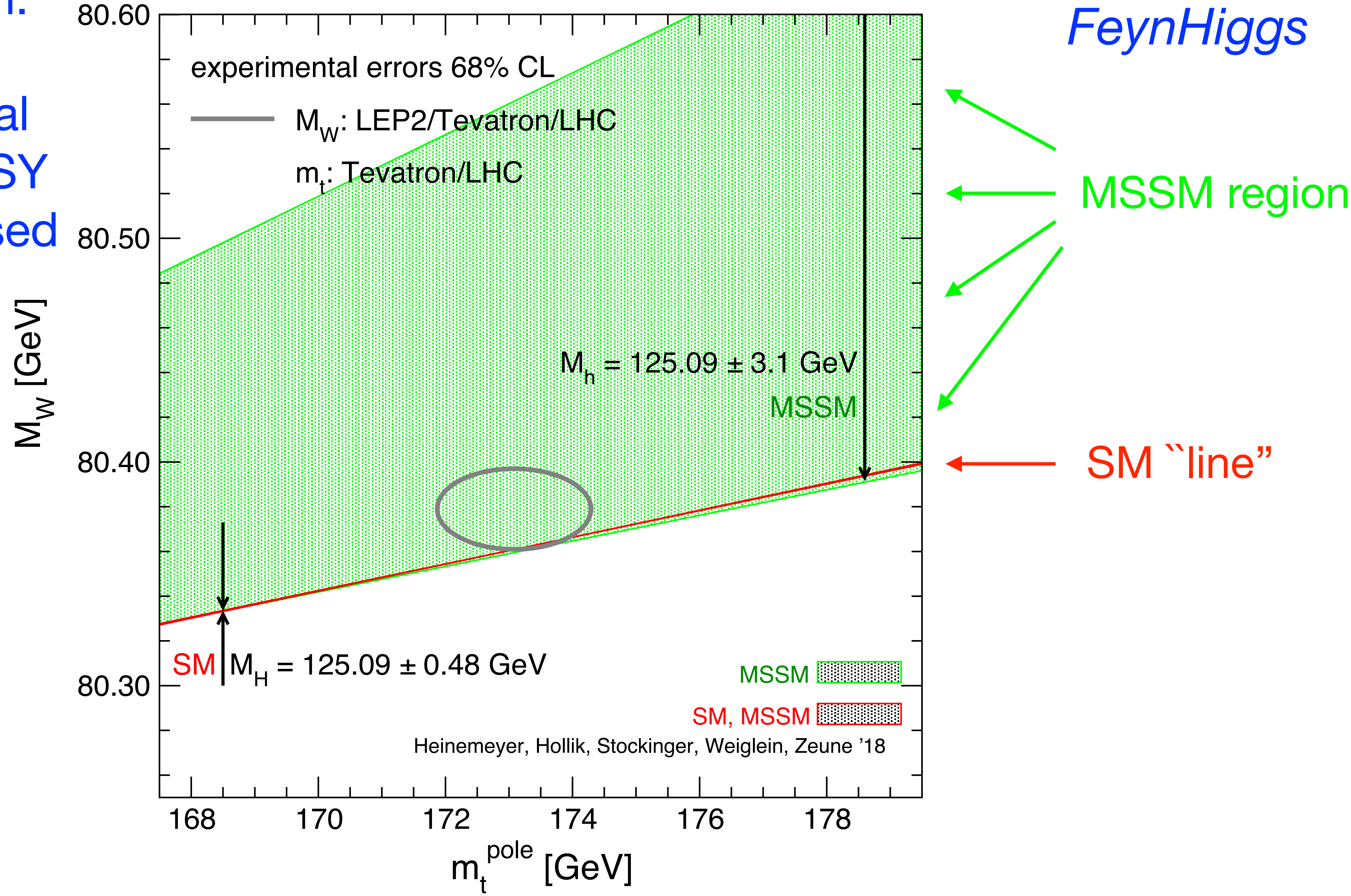
Prediction for M_W in the SM and the MSSM vs. experimental results for M_W and m_t

[S. Heinemeyer, W. Hollik, G. W., L. Zeune ’18]

Parameter scan:

No experimental bounds on SUSY particles imposed

FeynHiggs



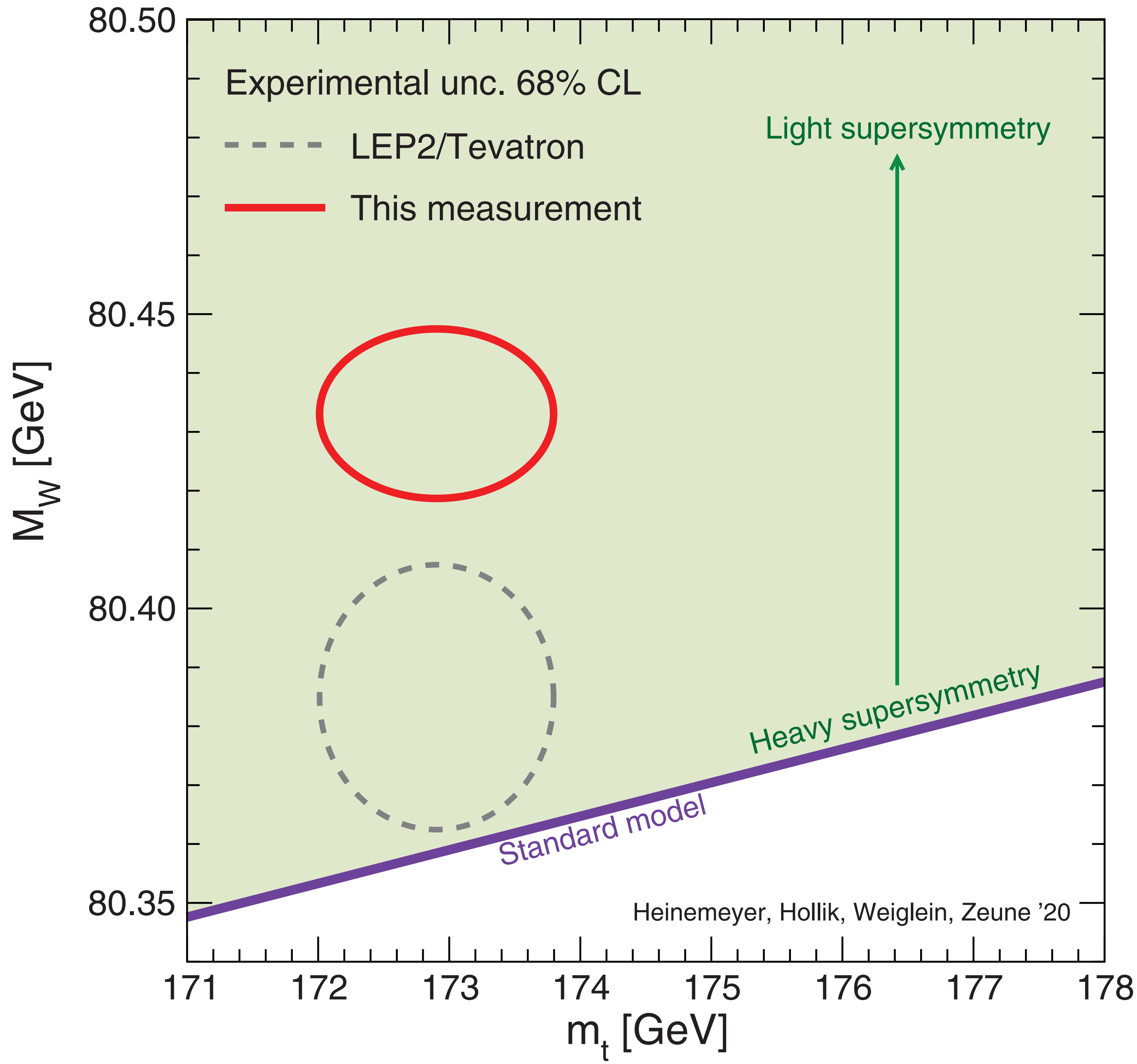
⇒ Large upward shift in M_W possible, large sensitivity to BSM effects

From the CDF paper on their M_W measurement

[CDF Collaboration ’22]   [S. Heinemeyer, W. Hollik, G. W., L. Zeune ’20]

SUSY: in principle large upward shifts in M_W are possible (main source: $\Delta\rho$)

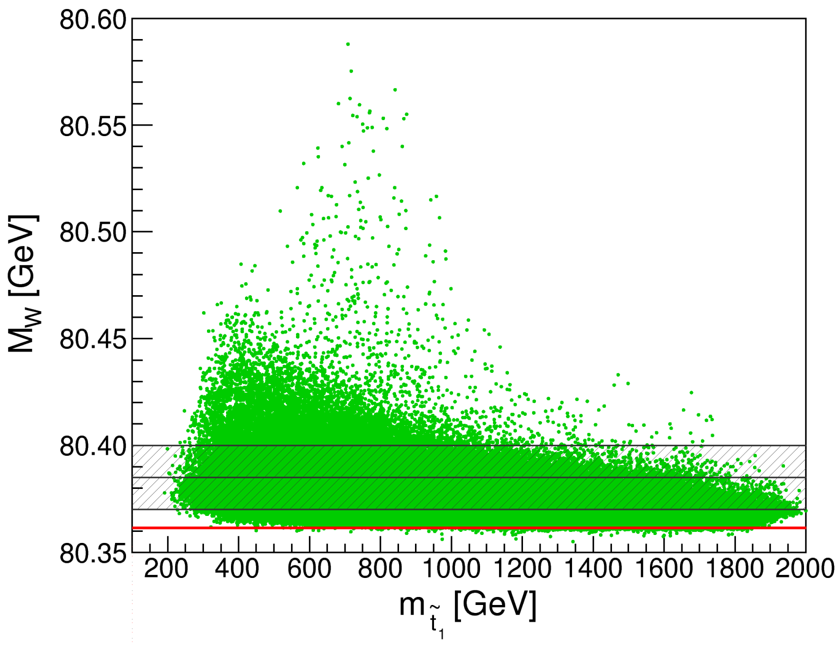
But: no experimental bounds on SUSY particles imposed here!

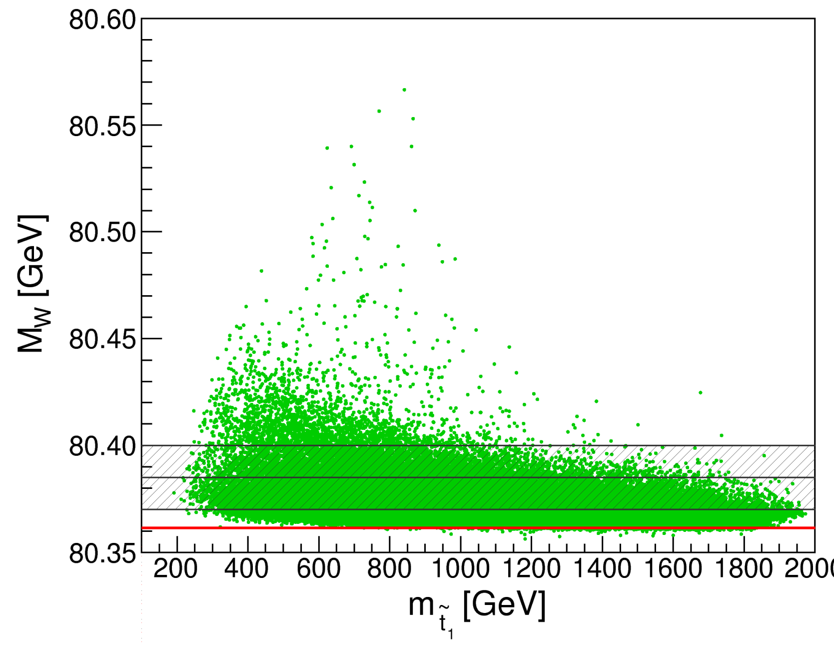


Prediction for M_W in the MSSM depending on the lighter stop mass (parameter scan) *FeynHiggs*

[S. Heinemeyer, W. Hollik, G. W., L. Zeune ’13]

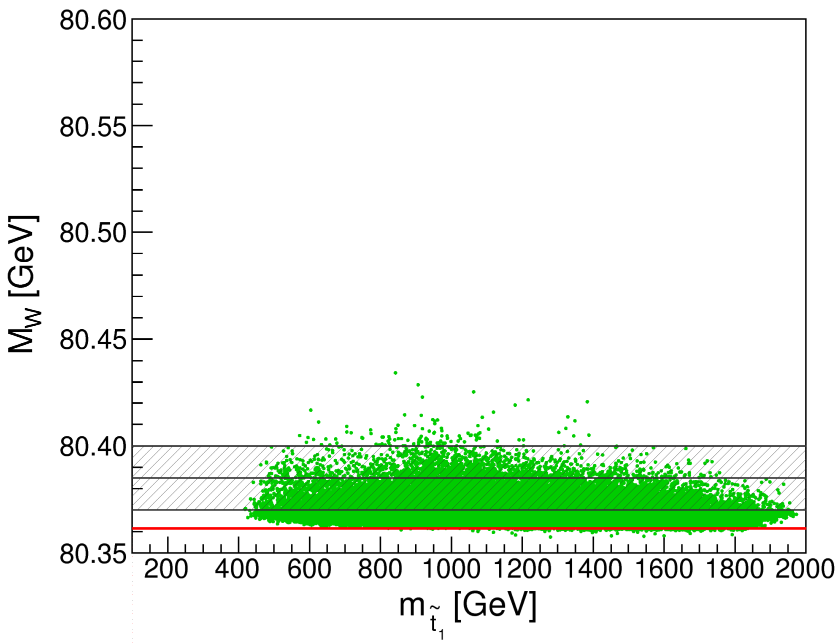
All particles allowed to be light

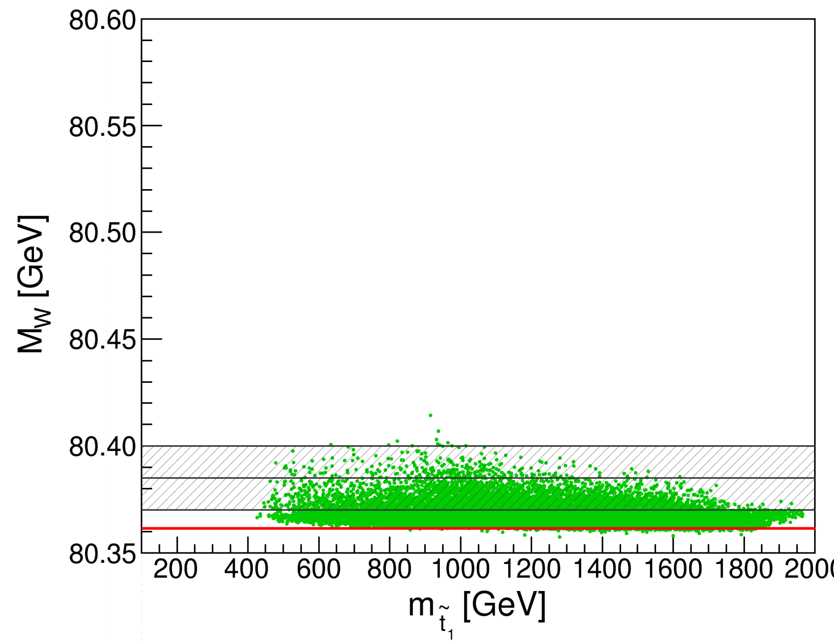




Heavy gluino, heavy first and second generation squarks

+ heavy sbottoms



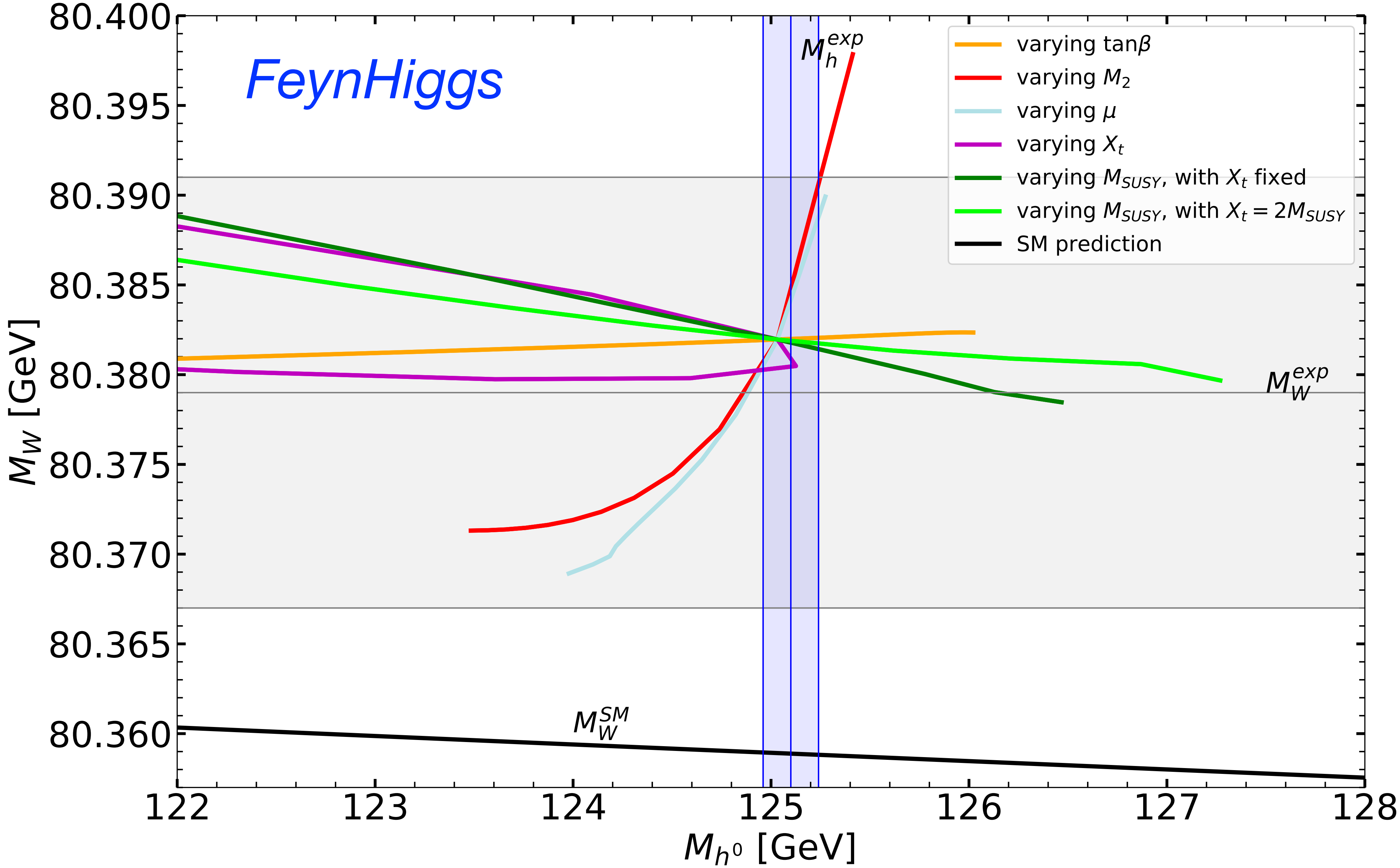


+ heavy sleptons and charginos

⇒Sizeable enhancements possible even for relatively heavy SUSY
Important further constraint: prediction for M_h has to agree with exp. value

Experimental result for M_W vs. prediction in the SM and the MSSM (different parameters varied)

[M. Berger, S. Heinemeyer, G. Moortgat-Pick, G. W. '22]



⇒ Good agreement in the MSSM, comparison can be used to obtain indirect constraints on the SUSY parameters

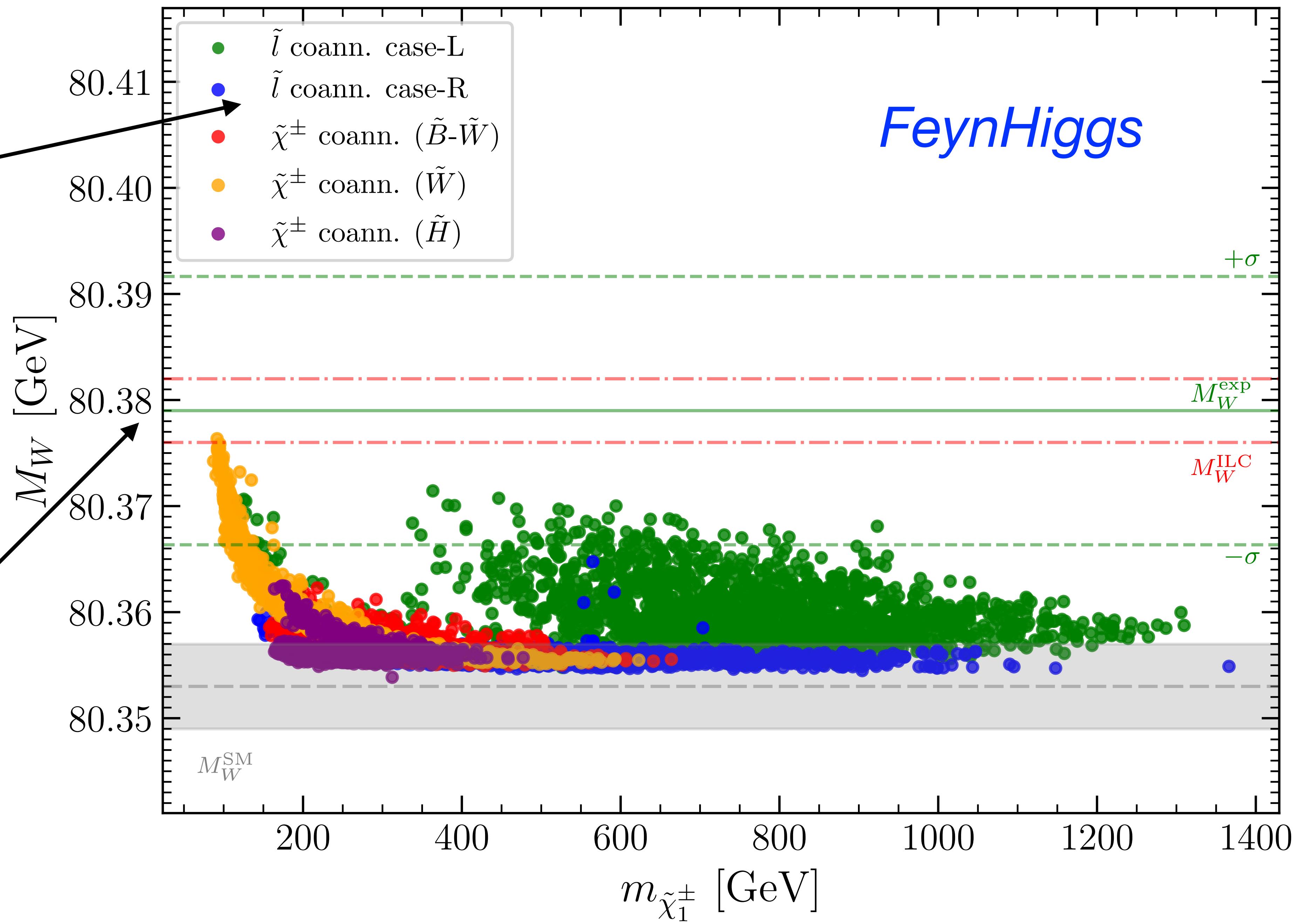
M_W prediction vs. the mass of the lightest chargino

[E. Bagnaschi, M. Chakraborti, S. Heinemeyer, I. Saha, G. W. '22]

Impact of light electroweak SUSY particles (squarks assumed very heavy!):

Different mechanisms for obtaining the right amount of dark matter

2022 world average without the CDF value



⇒ Upward shift w.r.t. SM prediction for light electroweak SUSY particles
Additional shift possible if stops, sbottoms are close to the exp. bounds

Conclusions

Estimate of theoretical uncertainties from unknown higher-order contributions (perturbative / non-perturbative): try to quantify something that we do not know

No algorithm available that will for sure produce reliable results BSM predictions: need parameter-space dependent estimates

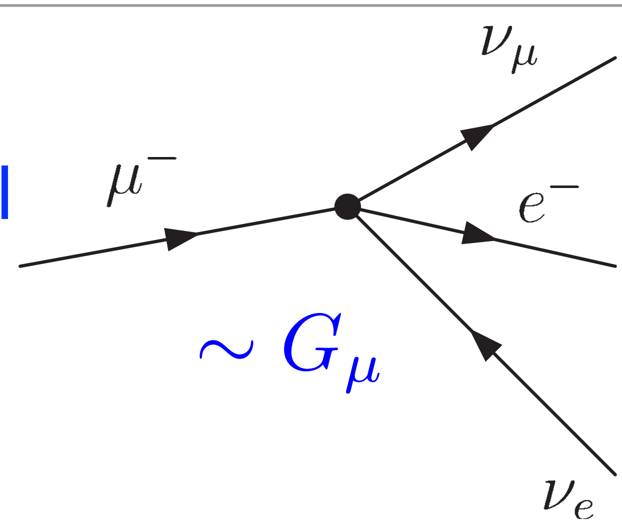
Main ideas: compare different versions of the predictions that are in principle equally valid but differ by higher-order contributions (different ren. schemes, options, …), identify appropriate coupling and enhancement factors

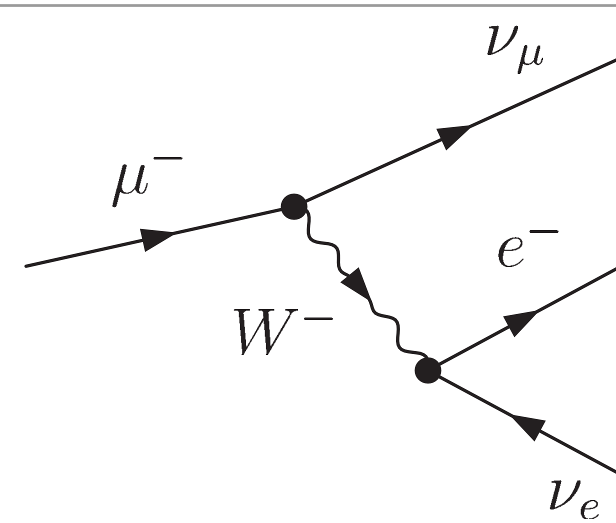
Most of our work goes into trying to reduce the remaining uncertainties (by doing better calculations), but we should be careful to keep a realistic view on the possible size of the unknown contributions

Backup

Theoretical prediction for the W-boson mass from muon decay: relation between M_W, M_Z, α, $G_μ$

Fermi model





SM

$M_{\rm W}$: Comparison of prediction for muon decay with experiment (Fermi constant G_μ); QED corrections in Fermi model incl. in def. of $G_μ$

$$\Rightarrow \quad M_{\rm W}^2\left(1-\frac{M_{\rm W}^2}{M_{\rm Z}^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1+\Delta r\right),$$

$\Updownarrow$

$$\Rightarrow M_W^2 = M_Z^2\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\alpha\pi}{\sqrt{2}G_\mu M_Z^2}\left(1+\Delta r\right)}\right)$$

loop corrections
(except QED corrections in the Fermi model)

$\Rightarrow$ Theo. prediction for $M_{\rm W}$ in terms of $M_{\rm Z}$, α, G_μ, $\Delta r(m_{\rm t}, m_{\tilde{\rm t}}, \ldots)$

Tree-level prediction: $M_W^{\rm tree}$ = 80.939 GeV, $M_W^{\rm exp}$ = 80.369 ± 0.013 GeV
⇒ off by many σ (accuracy of 1.6 x 10^{-4})

W-mass prediction in the Standard Model (SM)

One-loop contribution:

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\Delta\rho + \Delta r_\mathrm{rem}(M_H, \ldots)$$

$\approx 6\%$ $\quad\approx -3\%\quad$ $< 1\%$

$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}$$

contribution from isospin splitting: $\sim \left(m_t^2 - m_b^2\right) \approx m_t^2$

custodial symmetry: $\rho = 1$ at lowest order

M_W prediction in the Standard Model

Contributions beyond one-loop order:

$$\Delta r^{\mathrm{SM(h.o.)}} = \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(\alpha^2)}_{\mathrm{ferm}} + \Delta r^{(\alpha^2)}_{\mathrm{bos}} + \Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)} + \Delta r^{(G_\mu m_t^2 \alpha_s^3)}$$

Chetyrkin, Kuhn, Steinhauser, Djouadi, Verzegnassi, Awramik, Czakon, Freitas, Weiglein, Faisst, Seidensticker, Veretin, Boughezal, Kniehl, Sirlin, Halzen, Strong, ...

Impact of different contributions to Δr (x 10^4) for fixed M_W = 80.385 GeV and M_H^{SM} = 125.09 GeV:

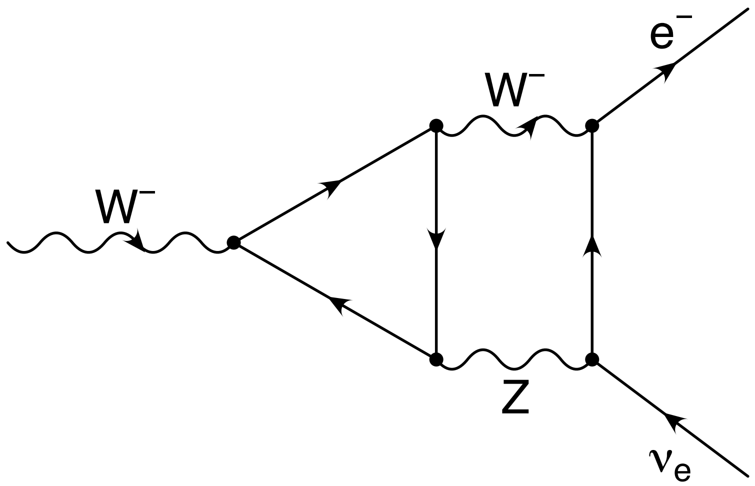
[O. Stål, G. W., L. Zeune '15]

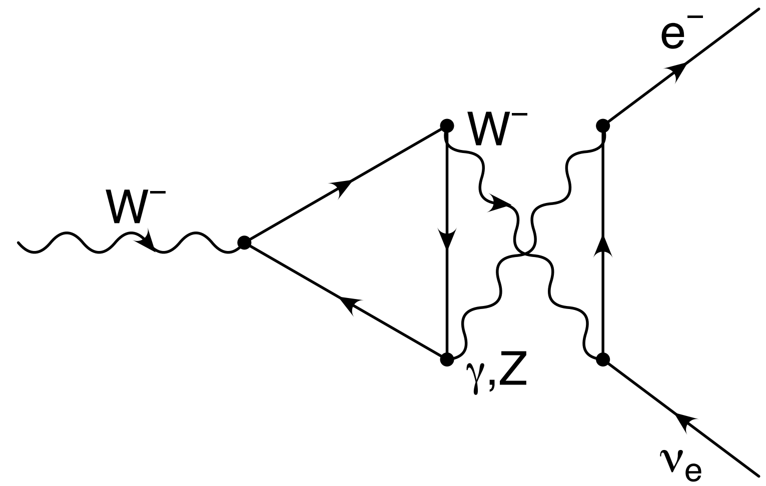
$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha\alpha_s)}$	$\Delta r^{(\alpha\alpha_s^2)}$	$\Delta r^{(\alpha^2)}_{\mathrm{ferm}} + \Delta r^{(\alpha^2)}_{\mathrm{bos}}$	$\Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)}$	$\Delta r^{(G_\mu m_t^2 \alpha_s^3)}$
297.17	36.28	7.03	29.14	-1.60	1.23

Shift of 1 x 10^{-4} in Δr roughly corresponds to –1.5 MeV shift in M_W

Needed for obtaining the full electroweak 2-loop res.

``Anomaly-type'' contributions, careful treatment of γ_5 needed

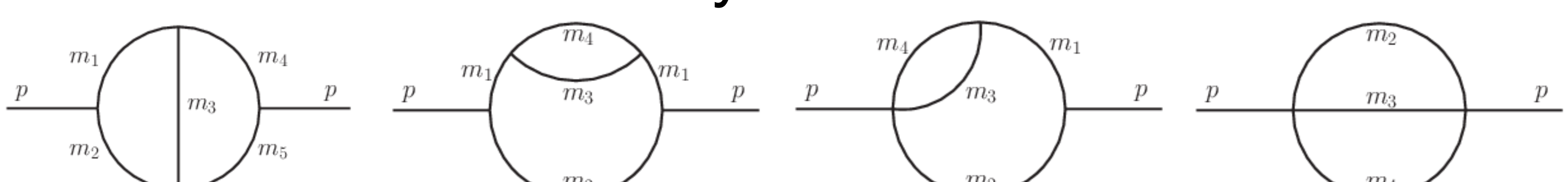




Consistent treatment of W and Z as unstable particles up to the 2-loop level, determination of the physical mass from expansion around the complex pole

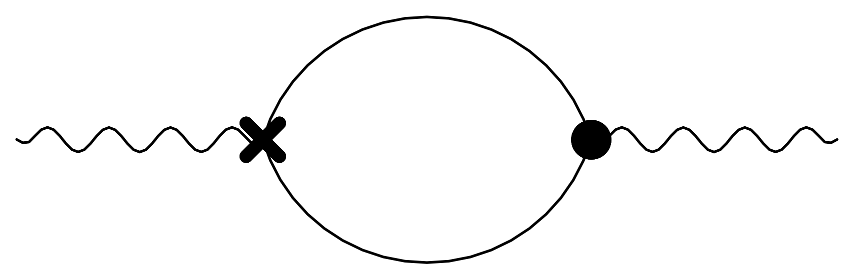
$$M^2 - iM\Gamma - m^2 + \underbrace{\hat{\Sigma}(M^2 - iM\Gamma)}_{\cong \hat{\Sigma}(M^2) - iM\Gamma\hat{\Sigma}'(M^2) + \ldots} = 0$$

2-loop 2-point integrals with non-vanishing external momentum and different internal masses: only numerical results available



Pure fermion-loop contributions at n-loop order

Contribution of n fermion loops at n-loop order:
no irreducible n-loop diagram, but e.g. 1-loop diagram with n–2 counterterm insertions
2-loop example:



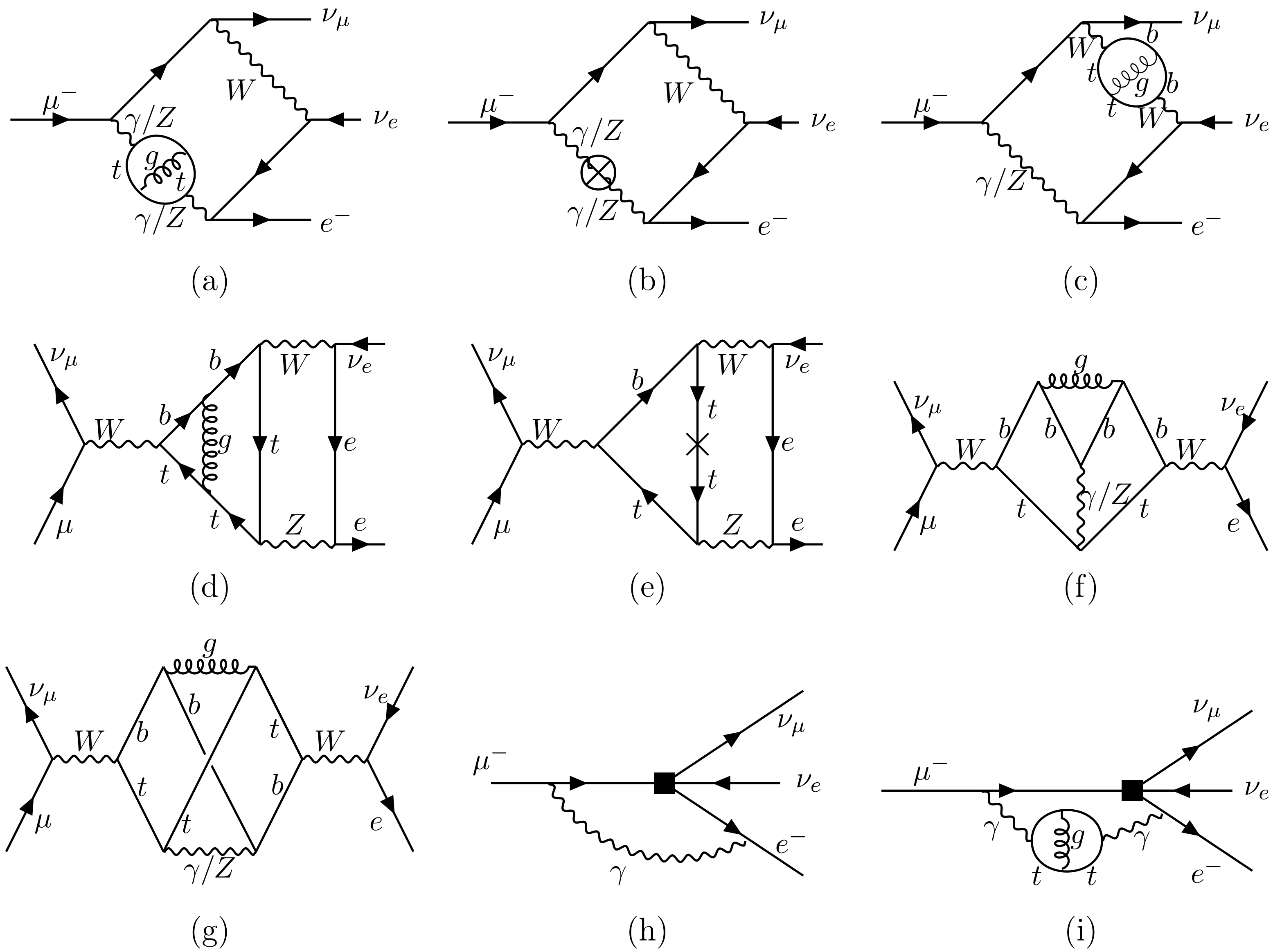
Result up to n-loop order (n = 2, 3, 4, …) for running width definition of M_W and M_Z *[A. Stremplat '98]* *[G. W. '98]*

3-loop result for fixed width definition of M_W and M_Z *[L. Chen, A. Freitas '20]*

3-loop result for 2-loop QCD contribution + closed fermion loop
[L. Chen, A. Freitas '20]

New 3-loop result of O($N_f \alpha^2 \alpha_s$)

3-loop contributions with one fermion loop, mixed electroweak + QCD



[I. Dubovyk, A. Freitas, J. Gluza, J. Usovitsch '26]

New contribution to Δr of $-2.0\ 10^{-4}$ ⇔ ≈ +3 MeV in M_W

⇒ Needs to be implemented into BSM predictions

BSM prediction for M_W, example: MSSM, NMSSM

Δr in the MSSM and the NMSSM, treatment of higher-order contributions:
full one-loop + higher orders (SM) + higher orders (SUSY)

$$\Delta r^{\rm (N)MSSM} = \Delta r^{\rm (N)MSSM(\alpha)} + \Delta r^{\rm (N)MSSM(h.o.)}$$

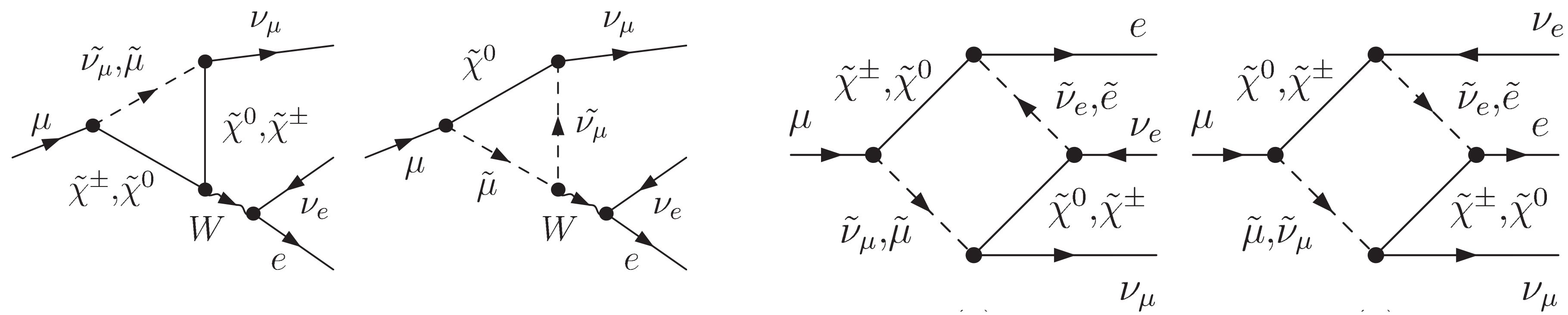
$$\Delta r^{\rm (N)MSSM(h.o.)} = \Delta r^{\rm SM(h.o.)} + \Delta r^{\rm SUSY(h.o.)}$$

⇒ State-of-the art SM prediction recovered in decoupling limit, all available higher-order corrections of SUSY-type included

For relatively light SUSY particles: additional theoretical uncertainty from higher-order SUSY-loop corrections

SUSY higher-order contributions

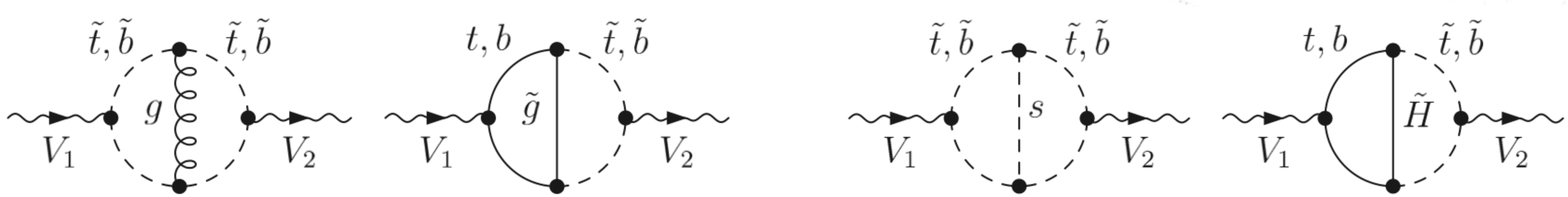
One-loop: complete result known in the MSSM and NMSSM; leading contributions from the scalar superpartners of the top and bottom quarks via Δϱ: additional source of isospin splitting



Two-loop:

leading reducible 2-loop corrections, gluon/gluino 2-loop corrections, higgsino 2-loop corrections

$$\Delta r^{\mathrm{SUSY(h.o.)}} = \Delta r_{\mathrm{red}}^{\mathrm{SUSY}(\alpha^2)} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{SUSY},(\alpha\alpha_s)} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{SUSY},(\alpha_t^2,\alpha_t\alpha_b,\alpha_b^2)}$$



MRSSM prediction for M_W

Extended Higgs sectors consisting of doublets and singlets:
custodial symmetry ⇒ $\varrho = 1$ at lowest order

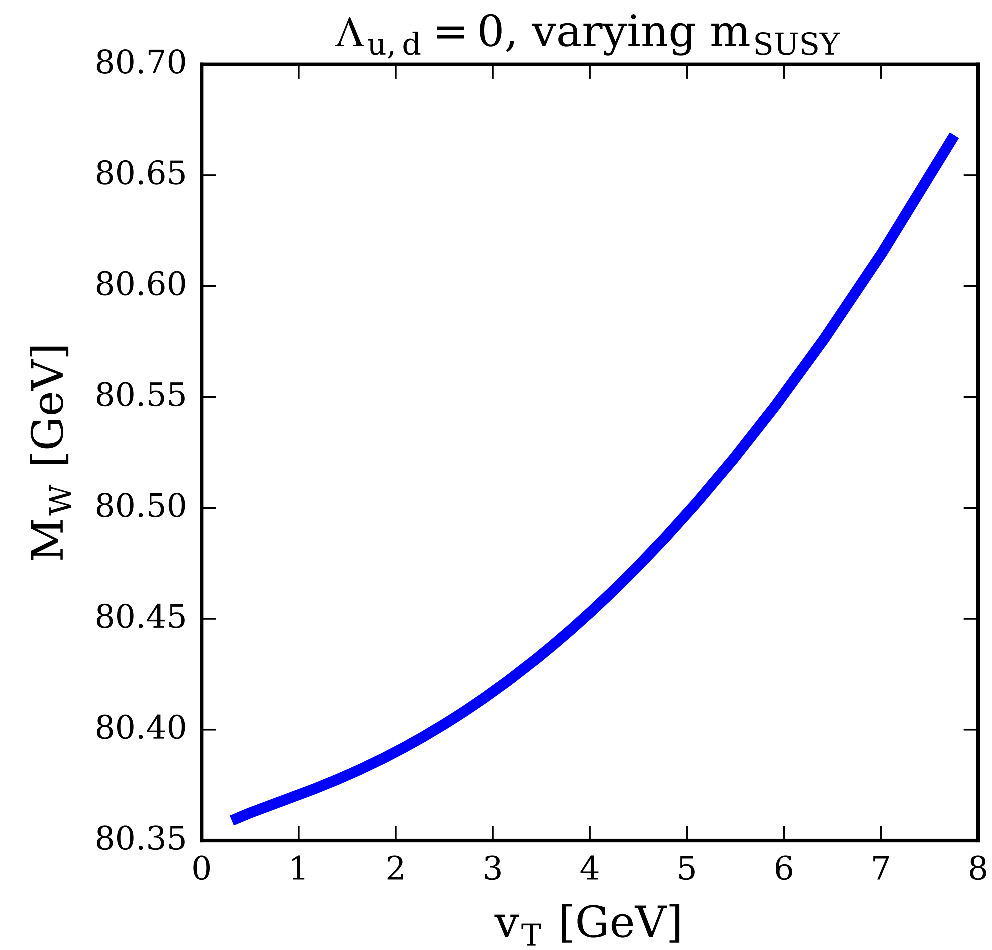
Lowest-order charged Higgs exchange contribution: ~ $(m_\mu m_e)/M_W^2$

⇒Main BSM contributions enter at 1-loop level: $\Delta r(m_i^{SM}, m_j^{BSM}, \ldots)$

Extended Higgs sectors involving triplets:
tree-level contribution from triplet v.e.v. v_T:
$M_W^2 = 1/4\, g_2^2\, v^2 + g_2^2\, v_T^2$

Example: MRSSM [P. Diessner, G. W. '19]

⇒Triplet v.e.v. v_T is constrained to be small



How to deal with tadpole scheme dependencies?

Some proposals for curing the unphysical tadpole scheme dependencies just address the unphysical gauge-parameter dependence. The actual benefit of this may be questioned. Not having a visible gauge-parameter dependence does not automatically make a quantity physical!

Ideally one should focus on relations between physical observables, even if this requires predictions for additional process-specific observables, etc.

I suggest to compare, at the very least, with on-shell type renormalisations for mixing angles, $\tan\beta$, etc.

Specific comment on use of the ``pinch technique”

People making statements like ``we have used the pinch technique to make our result gauge-parameter independent” either disregard or have not understood results that have been obtained more than 30 years ago!

It has been shown that the pinch technique corresponds to the special case of the background-field method (BFM) where the quantum gauge parameter is set to $\xi_Q = 1$. One cannot get rid of the dependence on the quantum gauge parameter in this way; other choices of the quantum gauge parameter are equally well motivated as $\xi_Q = 1$!

[A. Denner, S. Dittmaier, G. W. ’94]

Simplified BSM predictions for the W-boson mass

S, T, U parameters: only BSM contributions taken into account that enter via gauge-boson self-energies (only one-loop contributions), external momentum neglected

$$M_W^2 = M_W^2\big|_{\rm SM}\left(1 + \frac{s_w^2}{c_w^2 - s_w^2}\Delta r'\right)$$

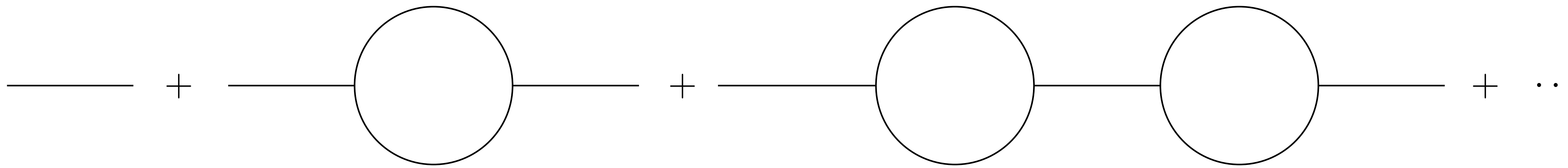
$$\Delta r' = \frac{\alpha}{s_w^2}\left(-\frac{1}{2}S + c_w^2 T + \frac{c_w^2 - s_w^2}{4 s_w^2}U\right)$$

SM prediction for the experimental values of $M_{\rm H}$, $m_{\rm t}$, …

Global fits to electroweak precision observables:
SM, SM + S, T, U parameters: *GFitter*, …
BSM models (SUSY, …): *MasterCode*, *Gambit*, …
EFT fits

What is meant by the mass of an unstable particle?

Mass of a physical particle: pole of the propagator



$$\frac{i}{p^2-m^2} \qquad \frac{i}{p^2-m^2}\; i\Sigma(p^2)\; \frac{i}{p^2-m^2} \qquad \frac{i}{p^2-m^2}\; i\Sigma(p^2)\; \frac{i}{p^2-m^2}\; i\Sigma(p^2)\; \frac{i}{p^2-m^2}$$

$$= \frac{i}{p^2 - m^2 + \Sigma(p^2)}$$

$\Rightarrow$ Pole of the propagator: $\mathcal{M}^2$

Renormalised self-energy (denoted below by $\hat{\Sigma}$)

$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0$$

For a stable particle: $\Sigma(\mathcal{M}^2)$ is real

If $\Sigma(\mathcal{M}^2) \neq 0 \Rightarrow$ Pole shifted by higher-order contributions

Mass of an unstable (elementary) particle

For an unstable particle:

$\Sigma(\mathcal{M}^2)$ is complex $\Rightarrow$ Pole in the complex plane

$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0, \quad \mathcal{M}^2 = M^2 - iM\Gamma$$

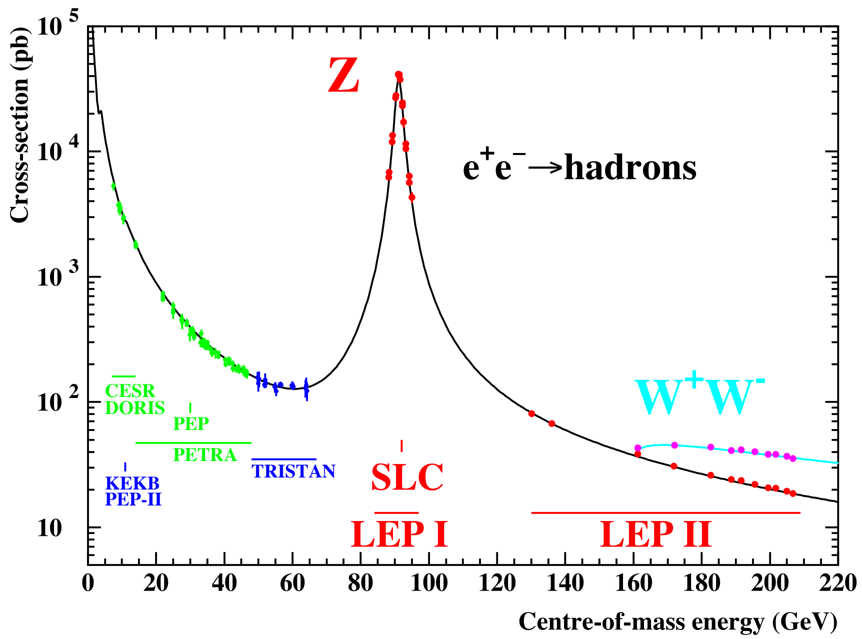
M: physical mass, Γ: decay width of the unstable particle

$\Rightarrow$ The mass of an unstable (elementary) particle is defined according to the real part of the complex pole

Example:

resonant production

of the Z boson and its decay



Which parameter is actually measured?

On the experimental side masses of unstable particles are not directly physical observables (can only measure cross sections, branching ratios, kinematical distributions, …): masses are ``pseudo-observables" whose determination involves a deconvolution procedure (unfolding)

Different parameterisations of the resonance: Breit-Wigner shape with running or constant width (difference: 27 MeV!)

The experimental mass parameter is obtained from the comparison data — Monte Carlo prediction

⇒ The experimental mass parameters M_W, M_Z, m_t, … are not strictly model-independent (example: exp. value for M_Z depends on M_H!)
Note: relation between Monte Carlo mass and well-defined Lagrangian par. is most difficult for m_t (coloured state, renormalon ambiguities, …), but much higher accuracy needed for M_W!

Physical mass of unstable particles: real part of complex pole

$\Rightarrow$ Only the complex pole is gauge-invariant

Expansion around the complex pole leads to a Breit–Wigner shape with constant width

For historical reasons, the experimental values of M_{Z}, M_{W} are defined according to a Breit–Wigner shape with running width

$\Rightarrow$ Need to correct for the difference in definition when comparing theory with experiment

M_W: measurements and theoretical predictions

Measurements at LEP, the Tevatron and the LHC:

- LEP: $e^+e^- \to W^+W^-$ in the continuum and at threshold (small amount of data); impact of fully hadronic final state suffered from uncertainties due to BE correlations, colour reconnections

- Tevatron, LHC: transverse mass distribution

In the SM: M_W is a free input parameter

From muon decay (yields Fermi constant G_μ): relation between the four extremely precisely measured quantities G_μ, α, M_W, M_Z

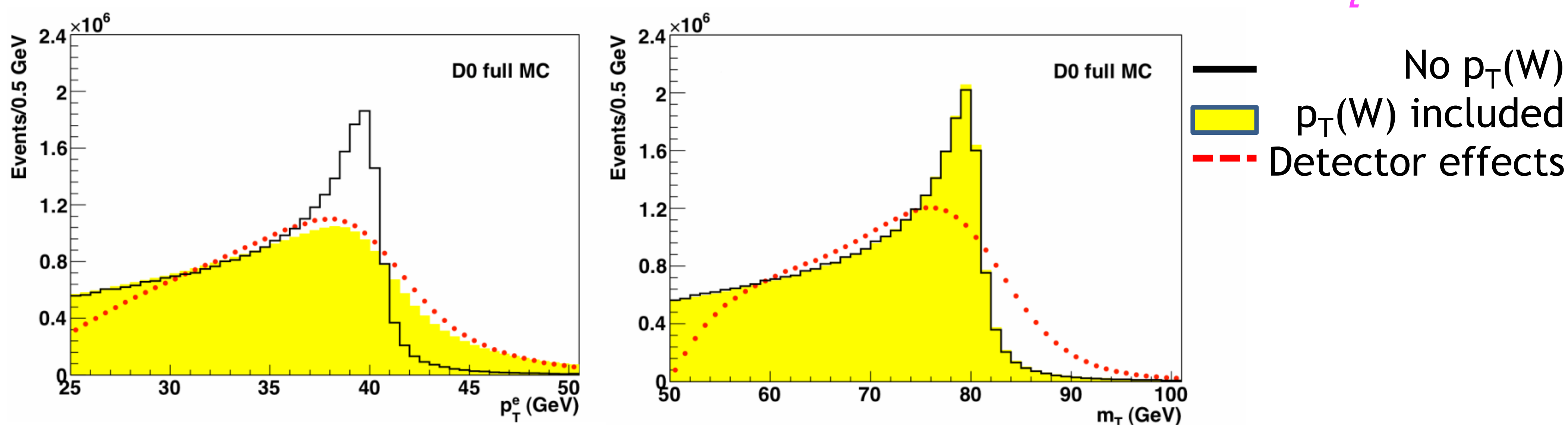
$\Rightarrow$ Use as prediction for M_W in different models, compare with experimental value

Important: precise theoretical predictions are needed both for its predictions in the SM and beyond and for the extraction of M_W from the data (source of systematic uncertainty)!

Extraction of M_W from the data at hadron colliders

problems are due to · the smearing of the distributions due to difficult neutrino reconstruction
· strong sensitivity to the modelling of initial state QCD effects

[A. Vicini '22]



Templates for different M_W values:

The templates are perturbative predictions.
Their residual theoretical uncertainties will propagate as theoretical systematic errors on the determination of (G_μ, m_W, m_Z)

Given the very high precision goal $\delta m_W / m_W \sim 1 \cdot 10^{-4}$, $\delta \sin^2\theta_{eff} / \sin^2\theta_{eff} \sim 1 \cdot 10^{-3}$
control on the shape of the distributions at the sub-percent level is needed, at a hadron collider…

- very large impact of initial-state QCD radiation on the ptlep distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large interplay of QCD and QED corrections redefining the precise shape of the jacobian peak

Expansion around the complex pole for a single resonance

$$p^2 - m^2 + \hat{\Sigma}(p^2) = \underbrace{(p^2 - \mathcal{M}^2)} \underbrace{\left\{ 1 + \frac{d\,\hat{\Sigma}}{d\,p^2} \right\}\bigg|_{p^2 = \mathcal{M}^2}} + \ldots$$

$\longrightarrow$ Breit-Wigner factor with fixed width

$\longrightarrow$ Field renormalisation and wave function normalisation factor of unstable particle

Note:

Wave-function normalisation factor needs to be evaluated at the complex pole

One-loop field renormalisation:
Complex quantity, no restriction to Re(…)

$$\delta Z^{(1)} = - \frac{\partial \Sigma(p^2)}{\partial p^2}\bigg|_{p^2 = m^2}$$

Expansion around the complex pole (example: *M*$_Z$)

Expansion of amplitude around complex pole:

$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_\mathrm{Z}^2} + S + (s - \mathcal{M}_\mathrm{Z}^2)\, S' + \cdots$$

$$\mathcal{M}_\mathrm{Z}^2 = \overline{M}_\mathrm{Z}^2 - i\overline{M}_\mathrm{Z}\,\overline{\Gamma}_\mathrm{Z}$$

Expanding up to $\mathcal{O}(\alpha^2)$ using $\mathcal{O}(\overline{\Gamma}_\mathrm{Z}/\overline{M}_\mathrm{Z}) = \mathcal{O}(\alpha)$

From 2-loop order on:

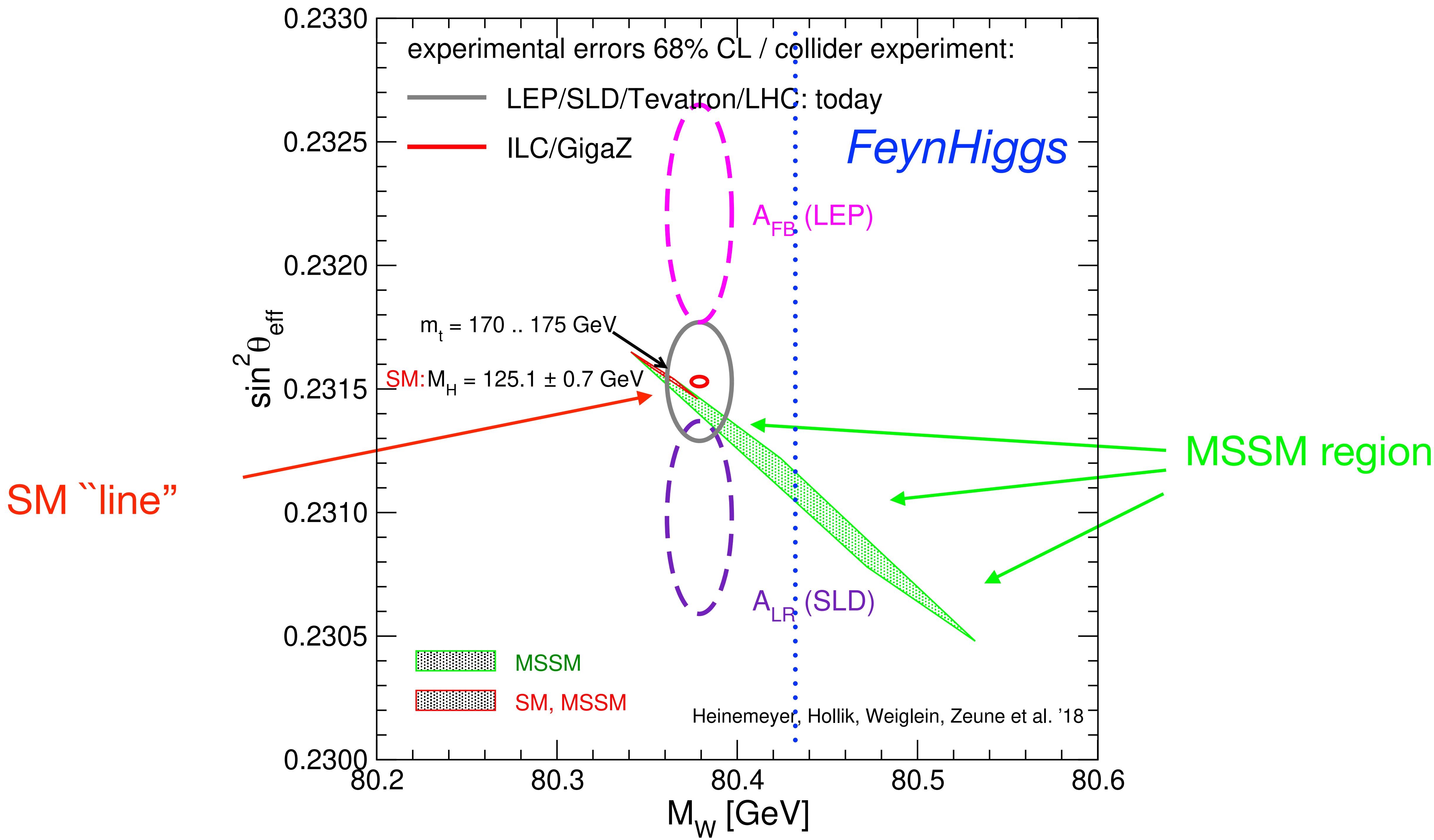
real part of complex pole, $\overline{M}_\mathrm{Z} \neq$ pole of real part, $\widetilde{M}_\mathrm{Z}^2$

$$\delta\overline{M}^2_{(2)} = \delta\widetilde{M}^2_{(2)} + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\}\ \mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}$$

gauge-parameter dependent!

Prediction for M_W and $\sin^2\theta_{eff}$ in the SM and MSSM vs. experimental accuracies

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



⇒ M_W and $\sin^2\theta_{eff}$ have high sensitivity for model discrimination