Hadronic running of the electromagnetic coupling from first principles

Alessandro Conigli

AC, Djukanovic, von Hippel, Kuberski, Meyer, Miura, Ottnad, Risch, Wittig

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Electroweak Corrections at Current and Future Accelerators

May 7th, 2026

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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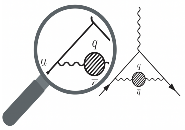
Precision observables and the hadronic bottleneck



- Precision observables test the accuracy of the Standard Model
- Precision depends on controlling **low-energy hadronic contributions**

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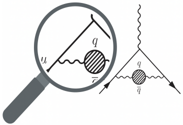


The muon as a precision probe

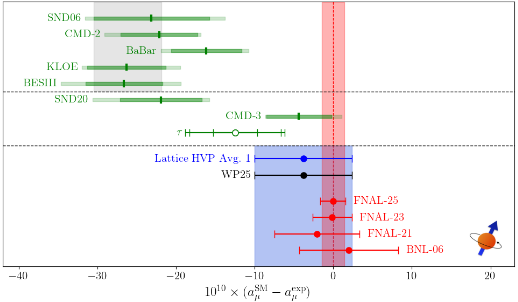
- Anomalous magnetic moment of the muon

$$a_\mu = \frac{g_\mu - 2}{2} = \frac{\alpha}{2\pi} + O(\alpha^2)$$

The White Paper 2025 [2505.21476]

- Sub-percent lattice determinations of a_μ^{hvp}
- WP25 average based on lattice results

[Mainz/CLS-24 2411.07969] [RBC/UKQCD-24+18 2410.20590]
[BMW/DMZ-24 2407.10913] [BMW-20 2002.12347]



- Tension originated **entirely** from dispersive evaluations

Another key hadronic quantity: $\Delta\alpha_{\rm had}$

- The **electromagnetic gauge coupling** is a fundamental parameter in the Standard Model

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \qquad \Delta\alpha(q^2) = \Delta\alpha_{\rm lep}(q^2) + \Delta\alpha^{(5)}_{\rm had}(q^2) + \Delta\alpha_{\rm top}(q^2)$$

- Uncertainty on $\alpha(M_Z^2)$ dominated by non-perturbative QCD effects in $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

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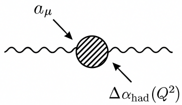
- Uncertainty on $\alpha(M_Z^2)$ dominated by non-perturbative QCD effects in $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

- $\alpha(M_Z^2)$ is a **key input quantity** of EW precision program

- Future colliders such as FCC-ee require $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ at much higher precision

- Both a_μ and $\Delta\alpha_{\rm had}(q^2)$ determined by the same HVP function



$g-2$ success motivates a first-principles determination of $\Delta\alpha_{\rm had}$ **beyond experimental input**

Lattice strategy for $\Delta\alpha_{\mathrm{had}}$

- $\Delta\alpha_{\mathrm{had}}$ is computed from the subtracted HVP $\bar{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

Time Momentum Representation (TMR) [Bernecker, Meyer 2011; Francis *et al.* 2013]

$$\Delta\alpha_{\mathrm{had}}(q^2) = 4\pi\alpha\bar{\Pi}^{\gamma\gamma}(q^2), \qquad \bar{\Pi}^{\gamma\gamma}(q^2) = \int_0^\infty \mathrm{d}t G^{(\gamma,\gamma)}(t) K(t, q^2)$$

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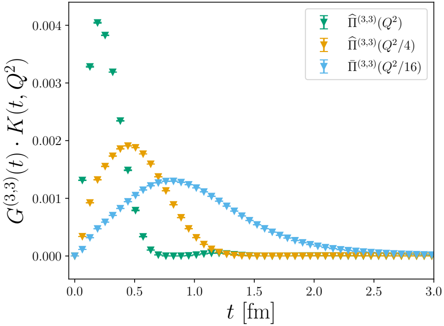
- **Telescopic decomposition:** tailored kernels for better control of systematics
- Better constrained $a \to 0$ and $m_\pi \to m_\pi^{\mathrm{phys}}$
- State-of-the-art techniques **improve precision in the long-distance region**

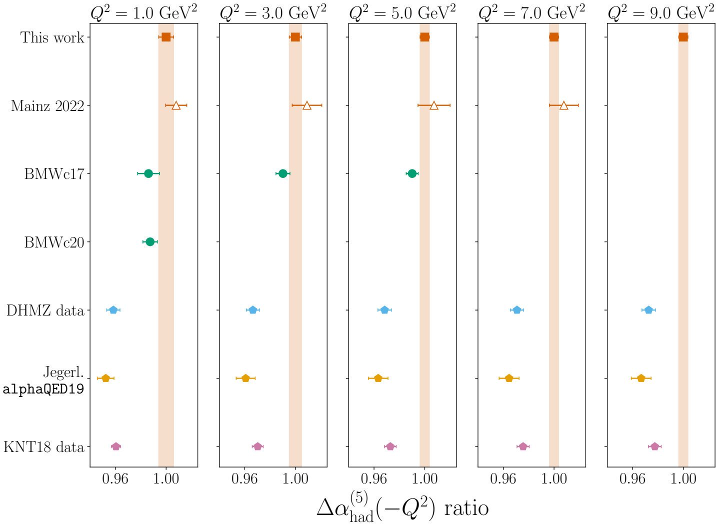


Ensemble E250: $m_\pi \sim m_\pi^{\mathrm{phys}}$, $a = 0.064$ fm

Comparison with other determinations at space-like momenta

Ratio of $\Delta\alpha_{\rm had}$ divided by our results for different Q^2

- Agreement with Mainz 2022 [M. Cè *et al.*, 2021]
- Slight tension between this work and [Borsanyi *et al.*, 2018; Borsanyi *et al.*, 2021]
- 3-7 σ tension between this work and R-ratio based data driven estimates [Keshavarzi, Nomura, and Teubner 2020; Davier *et al.* 2020; Jegerlehner 2020]



Hadronic running at the Z-pole

- Convert lattice results for $\Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$ to an estimate of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$
- Euclidean Split Technique: the Adler function approach [Jegerlehner, hep-ph/9901386, 0807.4206]

$$\Delta\alpha_{\rm had}^{(5)}(M_z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2) + \left[\Delta\alpha_{\rm had}^{(5)}(-M_z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)\right] + \left[\Delta\alpha_{\rm had}^{(5)}(M_z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_z^2)\right]$$

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- $\Delta\alpha^{(5)}_{\rm had}(-Q_0^2)$: input from Lattice QCD
- From the Adler function $D(Q^2)$, known in massive pQCD at three-loops

$$\left[\Delta\alpha^{(5)}_{\rm had}(-M_z^2) - \Delta\alpha^{(5)}_{\rm had}(-Q_0^2)\right]_{\rm pQCD/Adler} = \frac{\alpha}{3\pi}\int_{Q_0^2}^{M_Z^2} \frac{{\rm d}Q^2}{Q^2} D(Q^2)$$

Determination of $D(Q^2)$

- **AdlerPy** [R. F. Hernández, 2311.04849; J. Erler *et al.*, 2308.05740]
- Jegerlehner's software package **pQCDAdler**

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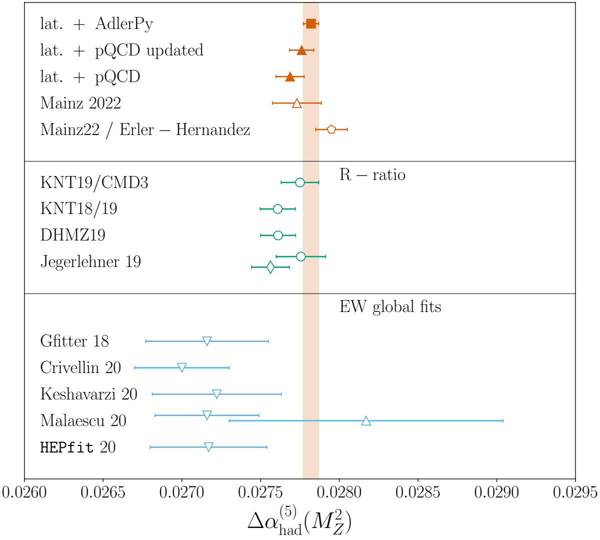
- Perturbation theory: $\left[\Delta\alpha_{\rm had}^{(5)}(M_z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_z^2)\right] = 0.000\ 045(2)$ [Jegerlehner, CERN Yellow Report, 2020]

Comparison with phenomenology and electroweak fits

- We obtain at the Z-pole

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027\,821(34)_{\mathrm{lat}}(35)_{\mathrm{pQCD}}$$

- Large tension observed in space-like region is substantially reduced at the Z-pole
- Tension at the level of $1-2\sigma$ with dispersive evaluations
- Global electroweak fits compatible within 2σ



Future prospects and FCC-ee targets

Are we on track?

Mainz 2022:

$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(15)$ [0.54%]

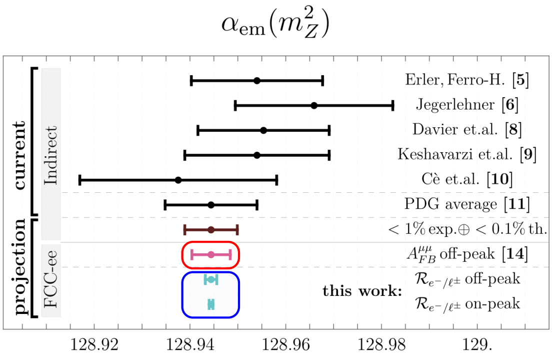
Mainz 2025:

$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,821(49)$ [0.17%]

Future prospects and FCC-ee targets

- The electroweak precision program at FCC-ee requires $\delta\Delta\alpha^{(5)}_{\rm had} \sim 3 \times 10^{-5}$, based on FB asymmetry muon production measurements
  [P. Janot, JHEP 02 (2016) 053]

- A novel method based on Z-pole measurements could reach a statistical sensitivity of $\delta\Delta\alpha^{(5)}_{\rm had} \sim 1 \times 10^{-5}$
  [M. Riembau, 2501.05508]



[M. Riembau, 2501.05508]

How do we get there, given the current precision of $\delta\Delta\alpha_{\rm had} = 4.9 \times 10^{-5}$?

Parametric error modelling

- Hadronic running on the lattice up to Q_0^2, then evolution to the Z-pole using pQCD
- Increasing the matching scale Q_0^2
  - $\rightarrow$ decreases the weight of perturbative running, and thus its uncertainty
  - $\rightarrow$ requires higher spacelike momenta on the lattice, where discretization effects become more relevant

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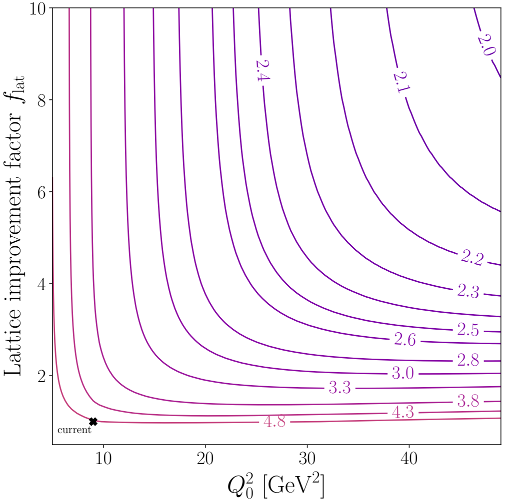
- Increasing the matching scale Q_0^2
  - $\rightarrow$ decreases the weight of perturbative running, and thus its uncertainty
  - $\rightarrow$ requires higher spacelike momenta on the lattice, where discretization effects become more relevant
- We construct a predictive model of the total uncertainty as a function of three ingredient:
  - $\rightarrow$ the precision of the non-perturbative inputs entering pQCD
  - $\rightarrow$ the precision of the lattice determination at the matching point
  - $\rightarrow$ the choice of the matching scale itself

$$\sigma^2_{\mathrm{tot}}(Q_0^2; \{f_i\}, f_{\mathrm{lat}}) = \sigma^2_{\mathrm{pQCD}}(Q_0^2; \{f_i\}) + \sigma^2_{\mathrm{lat}}(Q_0^2; f_{\mathrm{lat}})$$

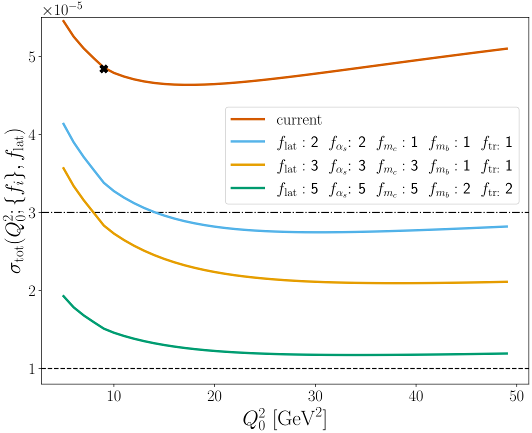
Exploring improvement scenarios

- Projected total uncertainty σ_{tot} in the $(Q_0^2, f_{\mathrm{lat}})$ plane. Shown are contour lines of the total error in units of 10^{-5}
- The current uncertainty is denoted by a black cross
- No improvement factor assumed in pQCD input parameters
- Pushing to higher Q_0^2 alone does not meet the desired precision level. Combined with moderate lattice improvements, it delivers substantial boost in precision



Pathways to ultimate precision

- Total uncertainty σ_{tot} as a function of the matching scale Q_0^2 for different improvement scenarios
- Moderate improvement scenarios are capable of reaching the required accuracy of $\delta\Delta\alpha_{\text{had}}^{(5)} \sim 3 \times 10^{-5}$ foreseen by [P. Janot, JHEP 02 (2016) 053]
- Reaching the target precision of $\delta\Delta\alpha_{\text{had}}^{(5)} \sim 1 \times 10^{-5}$ anticipated in [M. Riembau, 2501.05508] demands substantial advances in both lattice and pQCD inputs. Further suppression of the truncation uncertainty is required



- Lattice QCD can help improve the pQCD inputs [ALPHA 25, 2501.06633]

Conclusions

Summary

- Updated determination of the HVP contribution to the running of the electroweak gauge couplings
- Significant tension at space-like momenta with data-driven estimates
- Our result at the Z-pole is a factor of two more precise than recent phenomenological evaluations

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = 0.027\,821(34)_{\mathrm{lat}}(35)_{\mathrm{pQCD}}[49]\ [0.17\ \%]$$

Outlook: path to FCC-ee precision

- The FCC-ee EW program requires $\delta\Delta\alpha_{\mathrm{had}} \sim 3\times10^{-5}$, and possibly $\delta\Delta\alpha_{\mathrm{had}} \sim 1\times10^{-5}$
- Increasing the matching scale Q_0^2 alone is insufficient, but combining this with moderate lattice improvements ($\times2 - \times4$) yields substantial gains
- Significant progress in pQCD inputs needed for the 1×10^{-5} regime, and LQCD can play a crucial role

Thank You!

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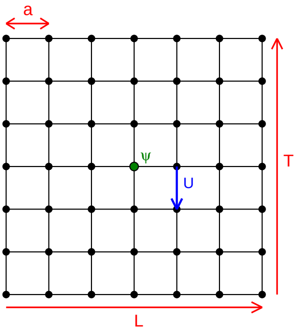
How to Lattice QCD

- First-principles numerical approach for non-perturbative regularization
- Fields on a discrete space-time
  $L^{-1} \ll E_i \ll a^{-1}$ IR and UV cutoff
- Mathematically solid definition of QCD
- Tune physical parameters $m_q^{\mathrm{phys}}, \ \Lambda_{\mathrm{QCD}}^{\mathrm{phys}}$ and renormalise
- Take $a \to 0, \ L, T \to \infty$ to recover continuum theory

$U_\mu(x) = e^{iaA_\mu(x)}$



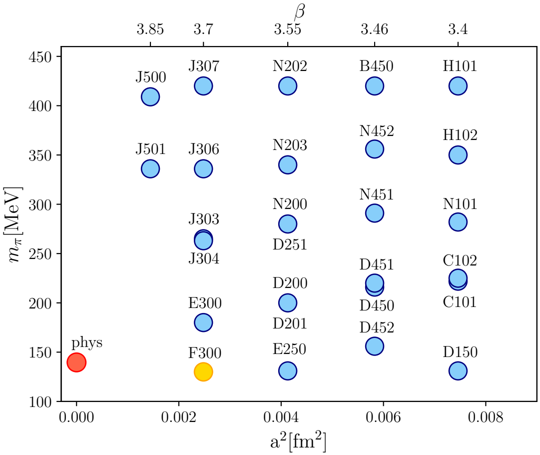
- Euclidean path integral formalism

$$\begin{aligned} \langle \mathcal{O} \rangle &= \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] e^{-S_G - S_F} \mathcal{O}[\psi, \bar{\psi}, U] \\ &= \int \mathcal{D}[U] e^{-S_G} \det(D_W) \mathcal{O}[U] \end{aligned}$$

- Evaluate path integral numerically through Monte Carlo sampling
- Massive conceptual and numerical challenge ($\sim 10^{11}$ d.o.f.)

Lattice setup - CLS ensembles

- $N_f = 2+1$ non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions [Lüscher and Schaefer, JHEP 1107 036 - JHEP 1502 043 - 1712.04884 - 2003.13359]
- Five values of $a \in [0.039, 0.087]$ fm
- Full range of pion masses $m_\pi \in [130, 420]$ MeV
- New ensembles / significantly improved statistics since [M. Cè *et al.*, 2203.0867]
- Newly added F300 with 256×128^3 at 0.05 fm and $m_\pi \sim m_\pi^{\text{phys}}$



Computational strategy

- Two discretisations of the vector current, the local (L) and point-split (C)

Set 1: Improvement coefficients from large-volume [1811.08209]

Set 2: Improvement coefficient from SF setup [1805.07401, 2010.09539]

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- Isovector contribution [S. Kuberski *et al.* 2024]

$$\bar{\Pi}^{(3,3)}(Q^2) = \bar{\Pi}^{(3,3)}_{\mathrm{sub}}(Q^2) + b^{(3,3)}(Q^2, Q_m^2)$$

where

$$\widehat{K}(x_0, Q^2, Q_m^2)_{\mathrm{sub}} = \frac{16}{Q^2}\sin^4\left(\frac{Qx_0}{4}\right) - \frac{Q^2}{Q_m^4}\sin^4\left(\frac{Q_m x_0}{2}\right), \qquad b^{(3,3)}(Q^2, Q_m^2) = \frac{Q^2}{4Q_m^2}\left(\Pi^{(3,3)}(4Q_m^2) - \Pi^{(3,3)}(Q_m^2)\right)$$

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- Isoscalar contribution

$$\text{HV}: \widehat{\Pi}^{(8,8)} = \widehat{\Pi}^{(3,3)} + \widehat{\Delta}_{\text{ls}}(Q^2), \qquad \text{MV}, \ \text{LV}: \ \bar{\Pi}^{(8,8)} = \int_0^\infty \mathrm{d}x_0 G^{(8,8)} K(x_0, Q^2)$$

where

$$\Delta_{\text{ls}}(Q^2) = G^{(8,8)} - G^{(3,3)} \propto \alpha_s (m_s^2 - m_l^2), \qquad \widehat{K}(x_0, Q^2) = \frac{16}{Q^2} \sin^4\left(\frac{Qx_0}{4}\right), \qquad K(x_0, Q^2) = x_0^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qx_0}{2}\right)$$

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- Charm connected contribution

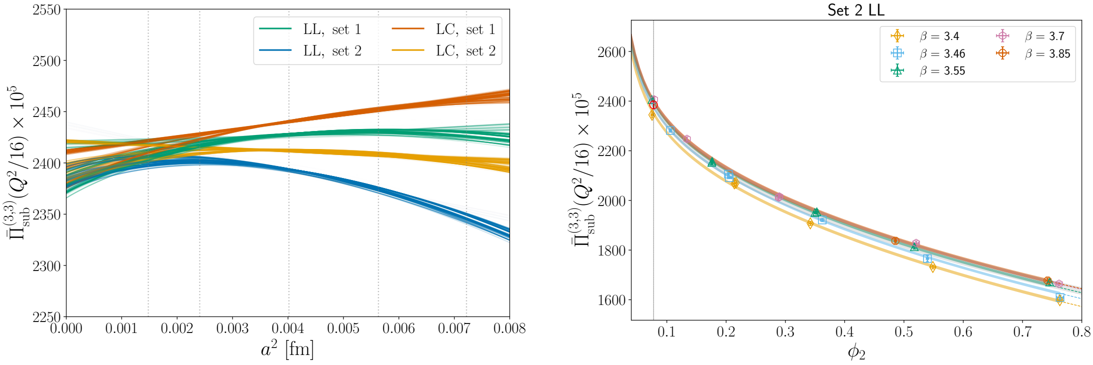
$$\bar{\Pi}^{(c,c)} = \bar{\Pi}^{(c,c)}_{\text{sub}}(Q^2) + b^{(c,c)}(Q^2)$$

where

$$b^{(c,c)}(Q^2) = 2b^{(3,3)}(Q^2, Q_m^2) + \Delta_{lc} b(Q^2, Q_m^2), \qquad \Delta_{lc} b(Q^2, Q_m^2) = \frac{Q^2}{4Q_m^2}\Big(\Pi(4Q_m^2) - \Pi(Q_m^2)\Big)$$

A glimpse into chiral-continuum extrapolations

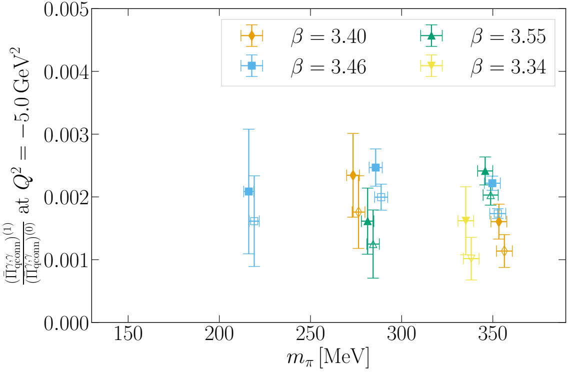
- Approaching the physical point $\mathcal{O}(0, m_\pi^{\text{phys}}, m_K^{\text{phys}}) = \lim_{a\to 0} \lim_{m_\pi \to m_\pi^{\text{ph}}} \lim_{m_K \to m_K^{\text{ph}}} \mathcal{O}(a, m_\pi, m_K)$



- Dependence of $\bar{\Pi}^{(3,3)}$ on a^2 (*left*) and $\phi_2 = 8t_0 m_\pi^2$ (*right*) at $Q^2 \sim 0.5$ GeV^2
- Each line is a fit ansatz we explore: four set of data differ by $O(a^2)$

Isospin-breaking effects

- **Lattice calculation:**
  - → Down to $m_\pi \sim 200$ MeV
  - → Only connected diagrams



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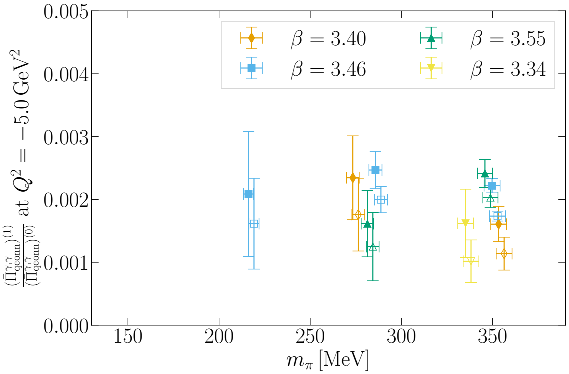
  - b Pseudoscalar meson exchanges $(\pi^0,\ \eta,\ \eta',\ \eta_c)$ [V. Biloshytskyi *et al.* 2209.02149]
  - c Strong IB, based on $\omega - \rho$ mixing [D. Erb *et al.* 2505.24344]



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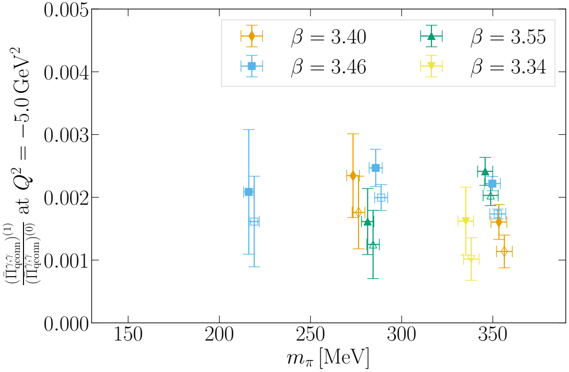

$$\Delta_{\rm IB}\bar\Pi(9\ \mathrm{GeV}^2) = 8.2(14) \times 10^{-5}$$

FVC and noise reduction strategies

- Finite-volume corrections via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] pion form factor
  - → Correct to $(m_\pi L)_{\mathrm{ref}} = 4.29$ for $a \neq 0$ [Borsanyi et al., 2002.12347] [Djukanovic et al., 2411.07696]
  - → Correct to $L = \infty$ in the continuum at the physical point

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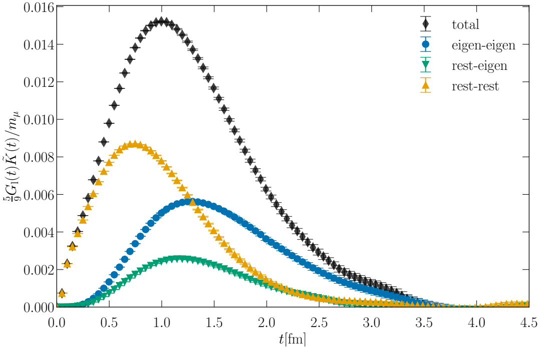
- Low-Mode Averaging (LMA) [L. Giusti *et al.*, hep-lat/0402002, T.A. DeGrand *et al.*, hep-lat/0401011]
  - → Improved estimator for the light-connected correlation function
  - → LMA is used for all ensembles with $m_\pi < 280\ \mathrm{MeV}$



Ensemble F300
$m_\pi = 130\ \mathrm{MeV}, \quad a = 0.049\ \mathrm{fm}$
[Plot by S. Kuberski]

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- Low-Mode Averaging (LMA) [L. Giusti *et al.*, hep-lat/0402002, T.A. DeGrand *et al.*, hep-lat/0401011]
- Bounding Method in isovector and isoscalar channels

$$0 \leq G^{(\alpha,\gamma)}(t_c)e^{-E_{\mathrm{eff}}(t-t_c)} \leq G^{(\alpha,\gamma)}(t) \leq G^{(\alpha,\gamma)}(t_c)e^{-E_0(t-t_c)}$$

  where

  $\rightarrow$ E_0 is the ground-state energy level contributing to the vector correlation function

  $\rightarrow$ E_{eff} is the logarithmic derivative of the vector correlation function

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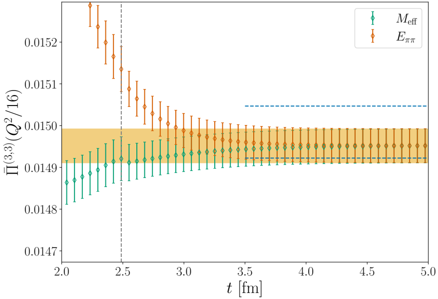
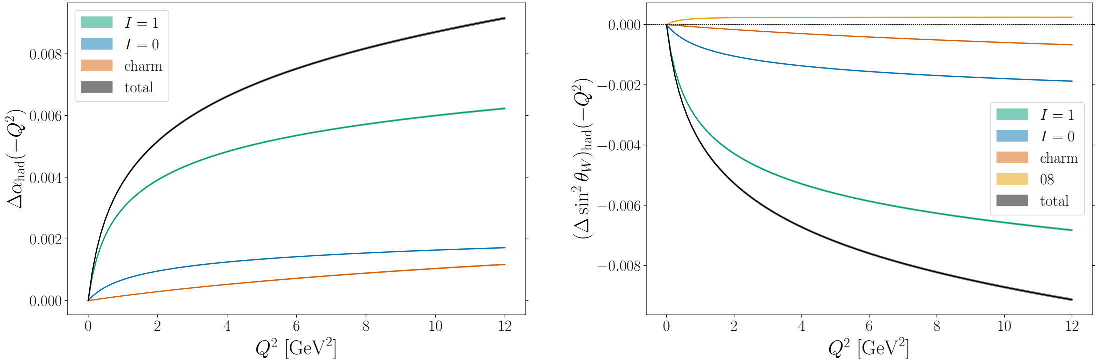
- Spectral reconstruction of the $\pi\pi$ contribution in isovector channel [Djukanovic et al., 2411.07696][Nolan Miller @ Lattice24]

- $N_{\pi\pi} = 4$ states used to reconstruct the correlator
- Solves signal-to-noise problem beyond $t = 2.5$ fm
- Increase the precision by a factor of 2 on physical point ensemble E250: 0.2% for $\bar{\Pi}^{(3,3)}$



Running of $\Delta\alpha_{\rm had}$ and $(\sin^2\theta_W)_{\rm had}$



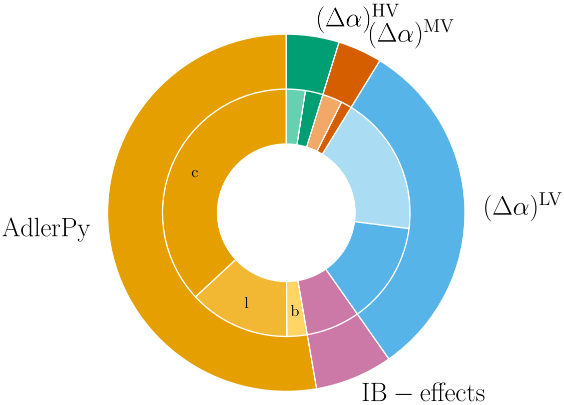
- Total HVP contribution to $\Delta\alpha_{\rm had}$ and $(\Delta\sin^2\theta_W)_{\rm had}$
- Results in the range $0 \leq Q^2 \leq 12$ GeV^2
- Rational approximation of the running through a Padé Ansatz [AC *et al.*, 2511.01623]

Main contribution to the variance of $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$

- **Threshold energy:** $Q_0^2 = 9\ \mathrm{GeV}^2$
- Good balance between lattice and pQCD

Error budget dominated by:

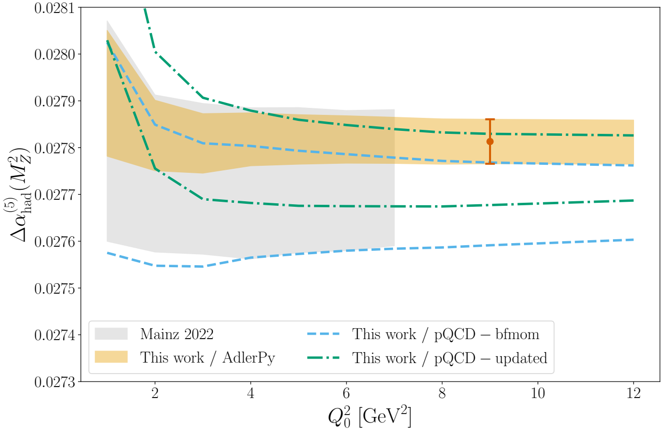
- **AdlerPy**: charm contribution to $D(Q^2)$
- **Lattice**: low-energy regime
- **IB-effects:** by far subleading



Evaluation of $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

- **Lattice input:**
  $\Delta\alpha_{\text{had}}^{(5)}(-Q_0^2)$ for $1\ \text{GeV}^2 \leq Q_0^2 \leq 12\ \text{GeV}^2$
- **Perturbative running:**
  Evaluation of the Adler function with
  1. AdlerPy
  2. pQCD-updated

  Input values from [FLAG24]
- **Threshold energy:** $Q_0^2 = 9\ \text{GeV}^2$
- Excellent stability of the results for $Q_0^2 \geq 3\ \text{GeV}^2$



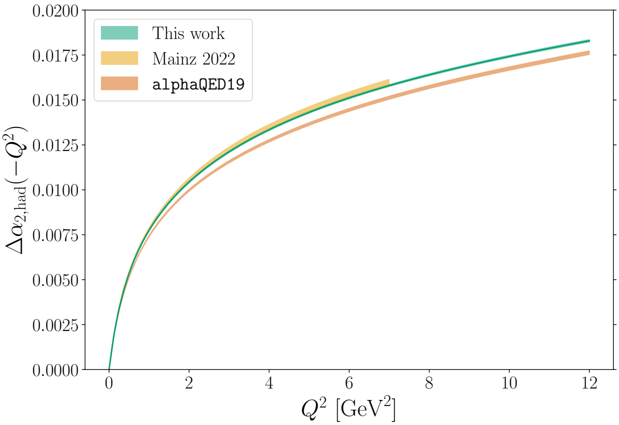
Electroweak mixing angle

- Running of $\alpha_2 = g^2/(4\pi^2)$

$$\alpha_2(q^2) = \frac{\alpha_2}{1 - \Delta\alpha_2(q^2)}$$

- Phenomenological estimates systematically below our determinations, by about 3.5% at $Q^2 = 12\ \mathrm{GeV}^2$

- Factor of two improvement in precision for the running of the electroweak mixing angle compared to [J. Erler *et al.*, 2406.16691]



- New measurements expected from P2 [D. Becker *et al.*, 1802.04759] and MOLLER [MOLLER Experiment, 1411.4088] collaborations

Parametric error modelling

- We quantify how much each perturbative input α_s, $m_c(m_c)$, $m_b(m_b)$ shifts $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

$$g_i = \frac{\partial \Delta\alpha_{\text{had}}^{(5)}(M_Z^2)}{\partial p_i}, \qquad p_i \in \{\alpha_s,\, m_c(m_c),\, m_b(m_b)\}$$

- This allows to parametrize the total uncertainty

$$\sigma^2_{\text{pQCD}}(Q_0^2) = \sum_{i,j} g_i(Q_0^2) C_{ij} g_j(Q_0^2) + \sigma^2_{\text{trunc}} \quad \Longrightarrow \quad \sigma^2_{\text{tot}}(Q_0^2) = \sigma^2_{\text{pQCD}}(Q_0^2) + \sigma^2_{\text{lat}}(Q_0^2)$$

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- Introduce improvement factors that rescale the corresponding uncertainties

$$f_i = \frac{\sigma_{p_i}}{\sigma^{\rm curr}_{p_i}}, \qquad p_i \in \{\alpha_s,\, m_c(m_c),\, m_b(m_b)\}, \qquad f_{\rm lat} = \frac{\sigma_{\rm lat}}{\sigma^{\rm curr}_{\rm lat}}$$

$$\sigma^2_{\rm tot}(Q_0^2; \{f_i\}, f_{\rm lat}) = \sigma^2_{\rm pQCD}(Q_0^2; \{f_i\}) + \sigma^2_{\rm lat}(Q_0^2; f_{\rm lat})$$