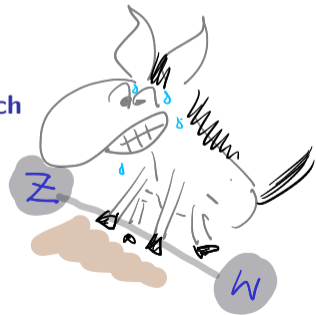


MITP Workshop: Electroweak corrections at current and future accelerators

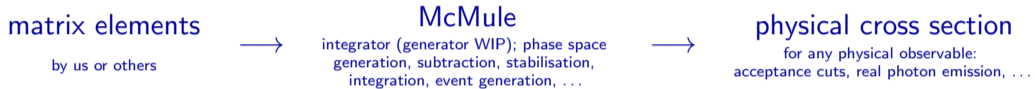
# McMule feels weak – EW corrections for MOLLER (& P2)

Sophie Kollatzsch & David Radic

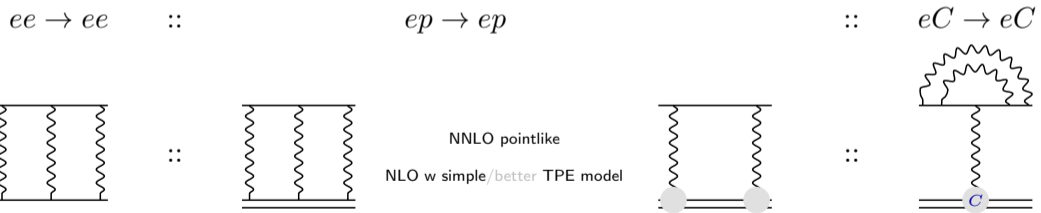
Paul Scherrer Institute / University of Zurich



McMule :: Fixed-order Monte Carlo framework for lepton processes at low energies



we provide (and WIP) NNLO QED for



and EW NNLO+NLL for logs  $\log \left( \frac{m_e^2, s, t, \dots}{M_Z^2, M_W^2, \dots} \right)$  in  $ee \rightarrow ee$  in LEFT

challenges to overcome

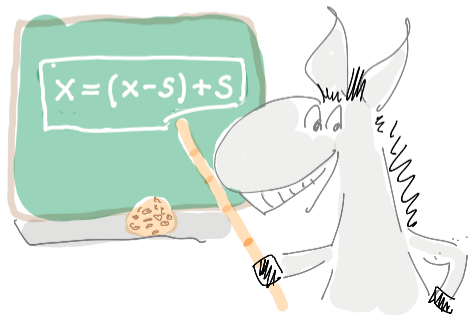
$$\begin{aligned}
 \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \right|^2 \\
 &+ \int d\Phi_3 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \right|^2 \\
 &+ \int d\Phi_4 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \right|^2 \\
 &+ \dots
 \end{aligned}$$

challenges to overcome

$$\sigma = \int d\Phi_2 \left[ \text{tree} + \text{loop} \right]^2 + \int d\Phi_3 \left[ \text{loop} \right]^2$$

- divergent phase space (PS) int  $\Rightarrow$  FKS<sup>ℓ</sup> subtraction

$$\int d\Phi_2 \left( \text{loop} + \int d\Phi_\gamma \text{CT} \right) + \int d\Phi_2 \int d\Phi_\gamma \left( \text{loop} - \text{CT} \right)$$



### challenges to overcome

$$\begin{aligned} \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{green wavy} \\ \text{blue wavy} \\ \text{red wavy} \\ \dots \end{array} \right|^2 \\ &+ \int d\Phi_3 \left| \begin{array}{c} \text{blue wavy} \\ \text{red wavy} \\ \dots \end{array} \right|^2 \\ &+ \int d\Phi_4 \left| \begin{array}{c} \text{red wavy} \\ \dots \end{array} \right|^2 \\ &+ \dots \end{aligned}$$



- divergent phase space (PS) int  
 $\implies$  FKS<sup>ℓ</sup> subtraction
- numerical instabilities in PS int  
 $\implies$  next-to-soft stabilisation  
 + OpenLoops [Buccioni et al 17,19]

$$\begin{aligned} &\frac{\text{red wavy}}{\text{red wavy}} \xrightarrow{E_\gamma \rightarrow 0} \frac{1}{E_\gamma^2} \frac{\text{blue wavy}}{\text{blue wavy}} \\ &+ \frac{1}{E_\gamma} (\mathcal{D} + \mathcal{S}) \frac{\text{blue wavy}}{\text{blue wavy}} + \mathcal{O}(E_\gamma^0) \\ &+ \text{polarisation terms} \end{aligned}$$

[Low 58; Burnett, Kroll 67; Balsach 23;...]

$$\begin{aligned} \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \dots \right|^2 \\ &+ \int d\Phi_3 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \dots \right|^2 \\ &+ \int d\Phi_4 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \dots \right|^2 \\ &+ \dots \end{aligned}$$



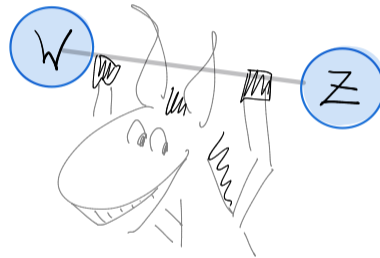
## challenges to overcome

- divergent phase space (PS) int  
 $\implies$  FKS<sup>ℓ</sup> subtraction
- numerical instabilities in PS int  
 $\implies$  next-to-soft stabilisation  
 + OpenLoops [Buccioni et al 17,19]
- loop amplitudes with  $m \neq 0$   
 $\implies$  massification  
 [Penin 05; Mitov et al 06; Becher et al 07; ...]

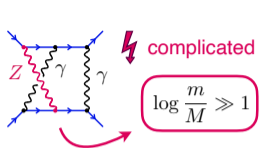
$$\mathcal{M}(m) = \mathcal{M}(0) \times (\sqrt{Z(m)})^{\# \text{ legs}} + \mathcal{O}(m)$$

iff  $m^2 \ll$  all other scales

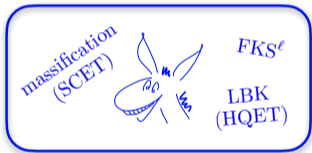
... cannot recover helicities  $\sim m$



EFT framework



### McMule framework



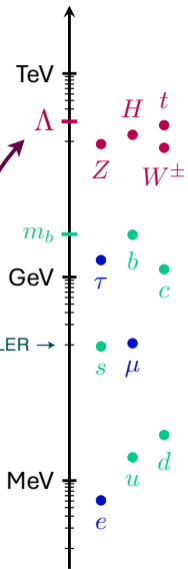
$m \ll M$

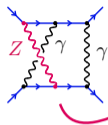


flexible & fully differential!



include EW effects via ...



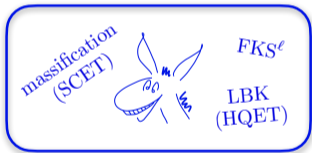


⚡ complicated

$$\log \frac{m}{M} \gg 1$$



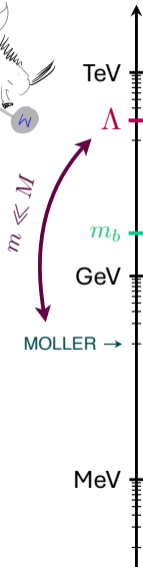
### McMule framework



flexible & fully differential!



include EW effects via low-energy effective field theory (LEFT)



### matching + RG running

$\mathcal{L}_{SM}$

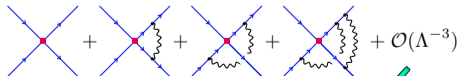
$$\mathcal{L}_{LEFT} = \mathcal{L}_4 + \sum L_i \mathcal{O}_i$$

QED + QCD

$L_i$

$\mathcal{L}_{LEFT, \beta}$  (w/o b quark)

	LL	NLL	N <sup>2</sup> LL
LO	1		
NLO	$\alpha \log$	$\alpha$	
N <sup>2</sup> LO	$\alpha^2 \log^2$	$\alpha^2 \log$	$\alpha^2$
⋮	⋮	⋮	⋮



we use the `San Diego` conventions

→ basis definition, evanescent operators, ...

[Jenkins, Manohar, Stoffer 18]

[Dekens, Stoffer 19]

### power counting ...

$$\mathcal{L}_{\text{LEFT}} \subset L_{pr}^{e\gamma} (\bar{e}_{Lp} \sigma^{\mu\nu} e_{Rr}) F_{\mu\nu} + \text{h.c.} + L_{prst}^{V,LL} (\bar{e}_{Lp} \gamma^\mu e_{Lr})(\bar{e}_{Ls} \gamma_\mu e_{Lt})$$

dipole (dim. 5)                      4-fermion (dim. 6)

↖ loop-induced & of dim. 6 when derived from the SM!

$\mathcal{O}(\Lambda^{-2}) \Rightarrow$  only **single-insertions** of higher-dim. operators (naively counted as  $\sim \alpha^2$ )

we use the 'San Diego' conventions

→ basis definition, evanescent operators, ...

[Jenkins, Manohar, Stoffer 18]

[Dekens, Stoffer 19]

### power counting ...

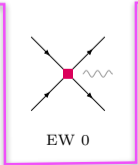
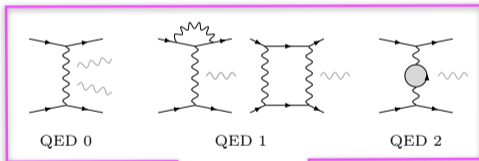
$$\mathcal{L}_{\text{LEFT}} \subset L_{pr}^{e\gamma} (\bar{e}_{Lp} \sigma^{\mu\nu} e_{Rr}) F_{\mu\nu} + \text{h.c.} + L_{prst}^{V,LL} (\bar{e}_{Lp} \gamma^\mu e_{Lr}) (\bar{e}_{Ls} \gamma_\mu e_{Lt})$$

dipole (dim. 5)                      4-fermion (dim. 6)

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LO



we use the 'San Diego' conventions

→ basis definition, evanescent operators, ...

[Jenkins, Manohar, Stoffer 18]

[Dekens, Stoffer 19]

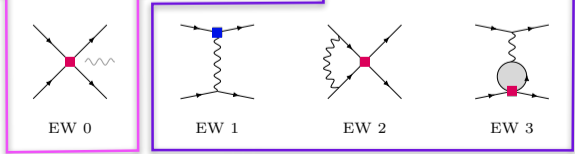
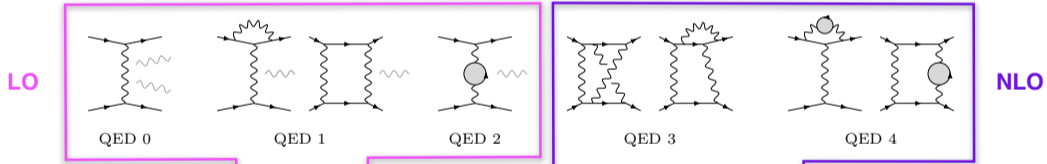
power counting ...

$$\mathcal{L}_{\text{LEFT}} \subset L_{pr}^{e\gamma} (\bar{e}_{Lp} \sigma^{\mu\nu} e_{Rr}) F_{\mu\nu} + \text{h.c.} + L_{prst}^{V,LL} (\bar{e}_{Lp} \gamma^\mu e_{Lr}) (\bar{e}_{Ls} \gamma_\mu e_{Lt})$$

dipole (dim. 5)
4-fermion (dim. 6)

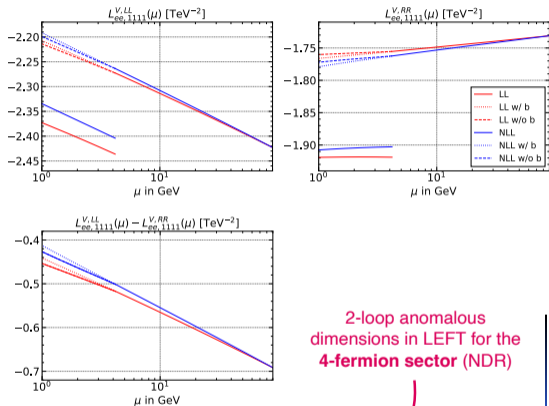
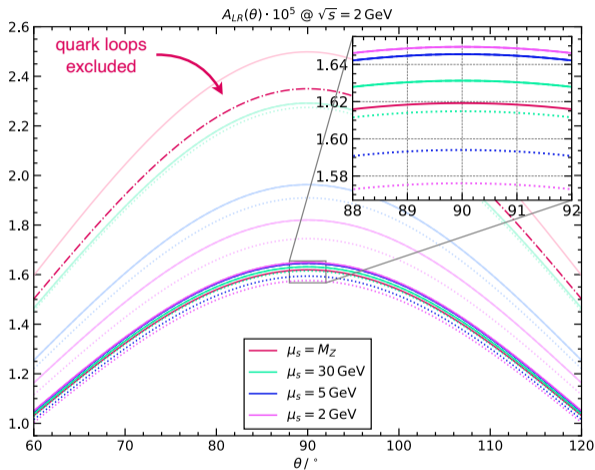
loop-induced & of dim. 6 when derived from the SM!

$\mathcal{O}(\Lambda^{-2}) \Rightarrow$  only **single-insertions** of higher-dim. operators (naively counted as  $\sim \alpha^2$ )





[SK, Moreno, DR, Signer 25]



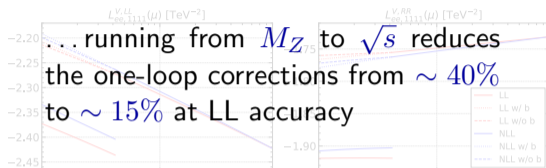
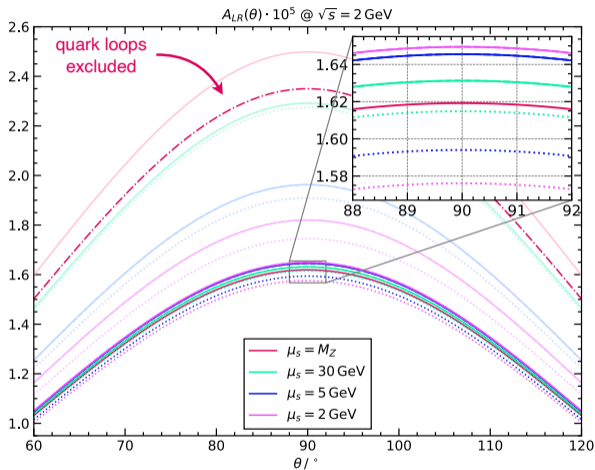
NLL results generated with DsixTools

[Celis, Fuentes-Martin, Vicente, Virto 17]

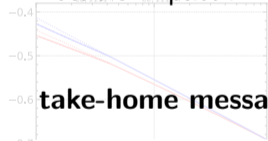
[Fuentes-Martin, Ruiz-Femenia, Vicente, Virto 21]

tree-level (light colors) and one-loop (dark colors) in RG improved perturbation theory at LL (solid lines) and NLL (dotted lines) accuracy

[SK, Moreno, DR, Signer 25]



... switching from LL to NLL has a notable impact



# 1 use LEFT  
resummation is important

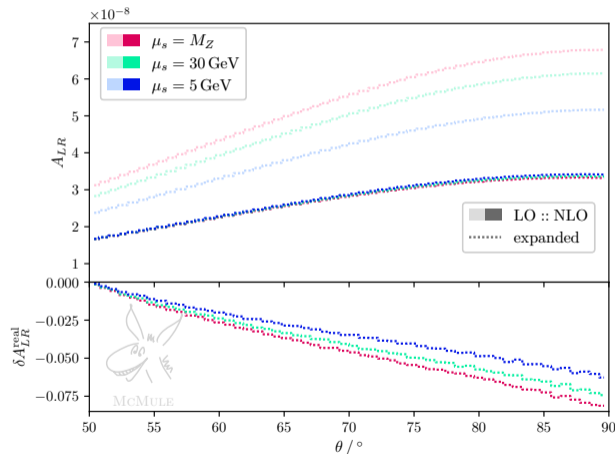
[Celis, Fuentes-Martin, Vicente, Virto 17]

[Fuentes-Martin, Ruiz-Femenia, Vicente, Virto 21]



some examples for MOLLER  
based on [SK, Moreno, DR, Signer 25]

$\sqrt{s} = 106 \text{ MeV}$ ,  $\theta = \frac{1}{2} (\pi - |\theta_3 - \theta_4|)$ ,  $50^\circ \leq \theta_\ell \leq 130^\circ$ , 90% long. pol.



$A_{LR}$  expanded in  $\alpha$  &  $\lambda$

accidental cancellation of IR divergences

$$\delta A_{LR}^{\text{real}} = \frac{\text{real+virtual} - \text{virtual}}{\text{virtual}}$$

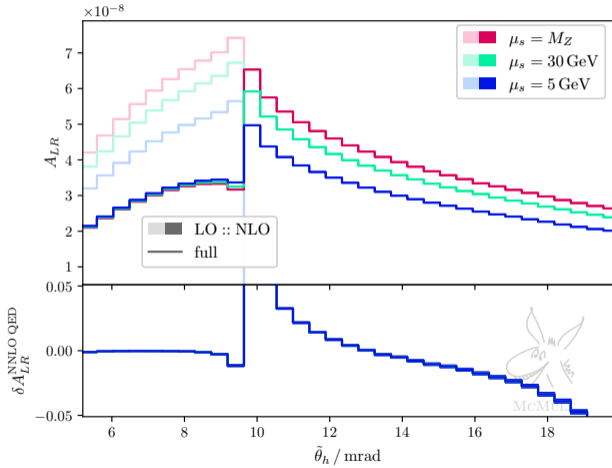
... can be  $\sim 7.5\%$  (or more)

# 2 use McMule

real corrections are important and cannot be neglected

scattering angle of more-energetic electron  $\tilde{\theta}_h$

$$A_{LR} = \frac{\text{best}}{\text{best}} \text{ incl NNLO QED}$$



# 3 use McMule

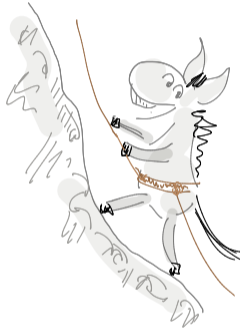
fully differential implementation is required

$$\delta A_{LR}^{\text{NNLO QED}} = \frac{w \text{ NNLO} - w/o \text{ NNLO}}{w/o \text{ NNLO}}$$

... can be ~ 5%

# 4 use McMule

NNLO QED can be important



higher orders within LEFT

**NNLO+NLL in LEFT**

**+**

**matching onto the SM(EFT) at two loops**

**NNLO+NLL in LEFT**

+

matching onto the SM(EFT) at two loops

significant progress on **2-loop anomalous dimensions** in the literature ✓

- **NDR** vs. **HV**? scheme dependencies?

**4-fermion sector (NDR)** → DsixTools  
 [Aebischer, Morell, Pesut, Virto 25]



**full LEFT (HV)**

[Naterop, Stoffer 25 (dim.-five)]  
 [Naterop, Stoffer 25 (dim.-six BNV)]  
 [Naterop, Stoffer 26 (dim.-six)]

compute all relevant **matrix elements in LEFT** up to dimension 6

- automation via in-house **Mathematica routines** interfacing various software packages
- analytic results for 2-loop master integrals known with full mass dependence  
 ... or use expanded results from *AsyInt* [Zhang 24]

⇒ **no showstoppers!**

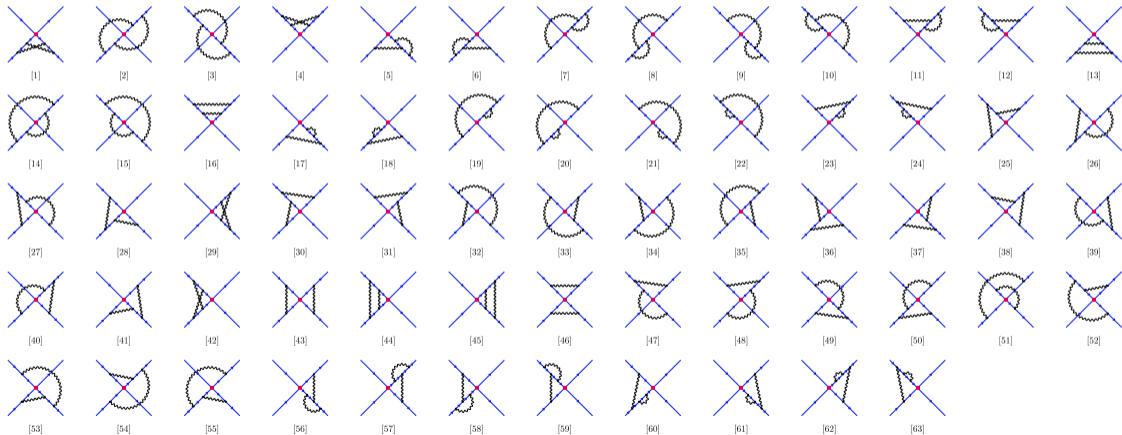


```
In[4]:= InitModel["LEFT"]
```

Initialize model: LEFT

workflow :: ... QGRAF → Mathematica → FORM → Kira → Reduze 2 ...

```
In[5]:= Diagrams[{"e", "e"}, {"e", "e"}, 2, Dim -> {6}, Filter -> {"Single4Fermion", "nf0"}] // Paint[#, Size -> {5, 13}] &
```



NNLO+NLL in LEFT

+

matching onto the SM(EFT) at two loops

WIP

... scheme choices?

renormalization beyond one loop?

$\gamma_5$  treatment?

separate evanescent sector from physical sector

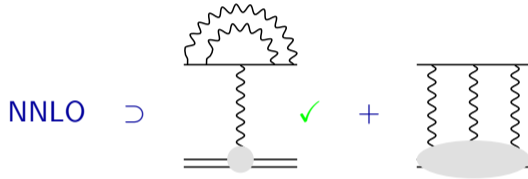


•  
•  
•

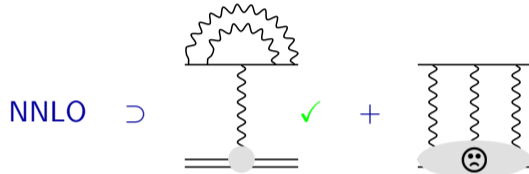


status of  $ep \rightarrow ep$

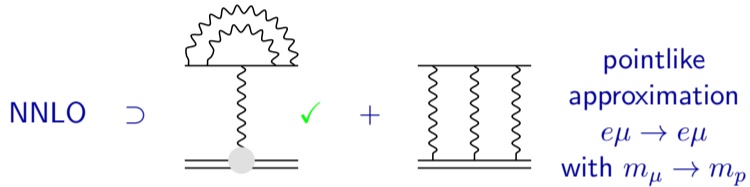
how we deal with protons [Engel, Hagelstein, Rocco, Sharkovska, Signer, Ulrich 23]



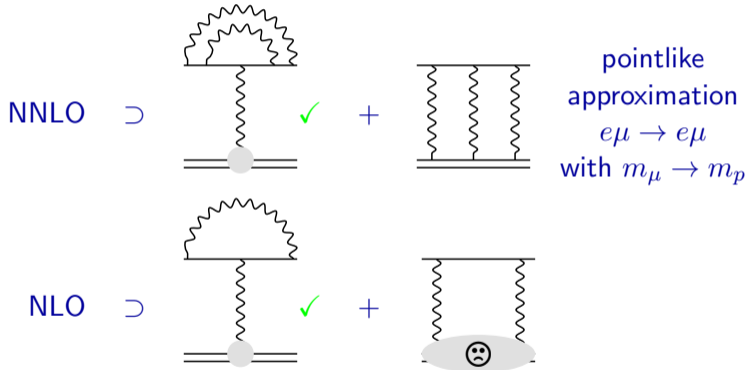
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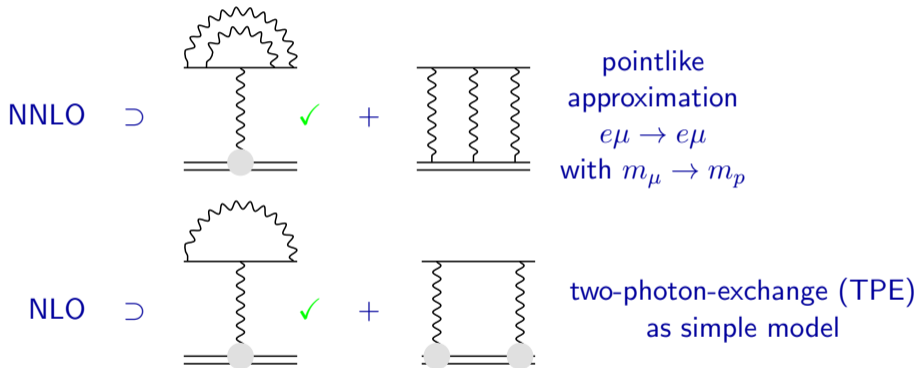
how we deal with protons [Engel, Hagedorn, Rocco, Sharkovska, Signer, Ulrich 23]



how we deal with protons [Engel, Hagedorn, Rocco, Sharkovska, Signer, Ulrich 23]



how we deal with protons [Engel, Hagestein, Rocco, Sharkovska, Signer, Ulrich 23]



ongoing :: implement state-of-the-art TPE, including inelastic TPE

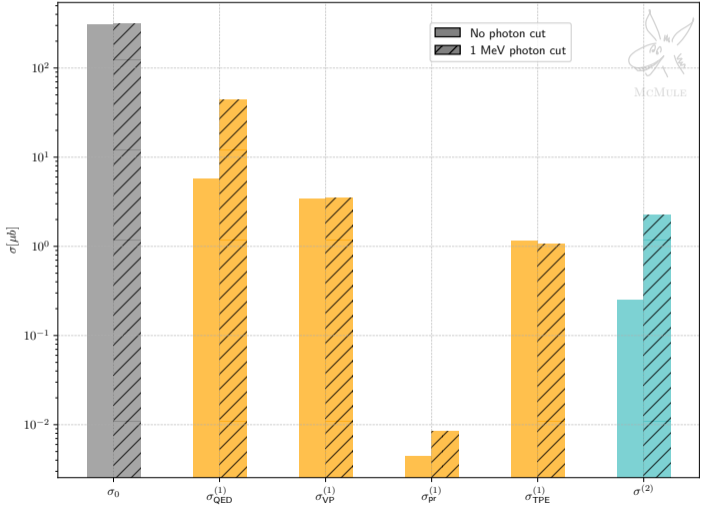
MAGIX example ::  $ep \rightarrow ep$

$E_{in} = 105 \text{ MeV}, 15^\circ \leq \theta_e \leq 150^\circ, 73.5 \text{ MeV} \leq p_e \leq 105 \text{ MeV}, E_\gamma \leq 1 \text{ MeV}$

$e^-p / \mu\text{b } E_{beam} = 105 \text{ MeV}$

NNLO effects can be larger than TPE !

# 5 use McMule  
similar studies needed for  
EW effects for P2





outlook

focus on EW for  $ee \rightarrow ee$

- NLO+LL  $\rightarrow$  NNLO+NLL
- heavy BSM  $\subset$  LEFT

&

repeat QED approach for EW  $ep \rightarrow ep$

- simple TPE model at NLO  
we have methods to deal with dispersive input  
[Fang, SK, Rocco, Signer, Ulrich, Zoller 25]
- pointlike NNLO via  $ee \rightarrow \mu\mu$

... and make McMule even better

- integrator  $\rightarrow$  generator ongoing, see talk by [Y. Ulrich]
- combine with parton shower / soft photon resummation
- make McMule ready for high energies  $\rightarrow$  FCC-ee, ...

to take home

- # 1 use LEFT  $\Rightarrow$  resummation & systematic higher orders, heavy BSM for free
- # 2 use McMule  $\Rightarrow$  real corrections, cuts, NNLO effects





use McMule!  
Interest? Get in touch!



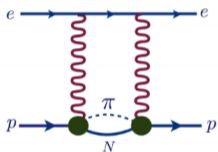
McMULE

[mule-tools.gitlab.io](https://mule-tools.gitlab.io)

f.l.t.r.: A.Signer (Zurich & PSI), F.Hagelstein (Mainz), M.Ronchi (Mainz), M.Rocco (Turin)  
S.Kollatzsch (Zurich & PSI), Y.Ulrich (Liverpool), S.Gündogdu (Zurich & PSI),  
D.Radic (Zurich & PSI), J.Wilson (Liverpool), S.Zanoli (Oxford), Y.Fang (Zurich & PSI)  
not shown: P.Banerjee (Chennai), A.Coutinho (IFIC) V.Sharkovska (Zurich & PSI),  
G.Billis (PSI), T.Oruç (ETHZ)

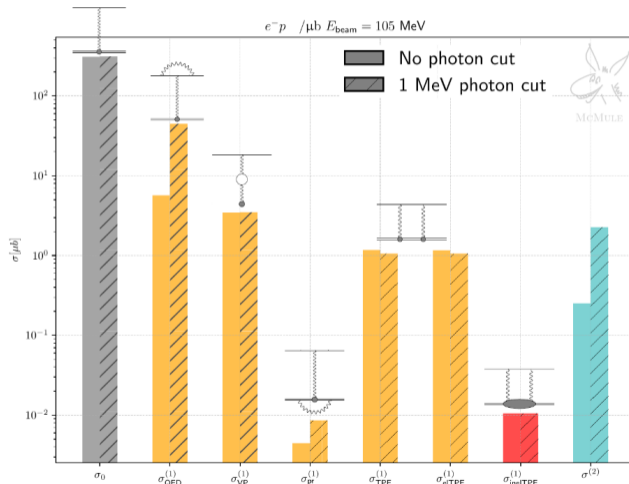
see talk by [A. Signer]

plan: include state-of-the-art TPE models



[Tomalak, Pasquini, Vanderhaeghen, 1708.03303]

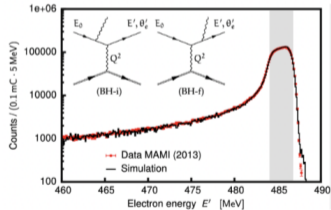
so far 'only' virtual part  
 Oops, recall message # 3  
 ⇒ work on real counterpart



see talk by [A. Signer]

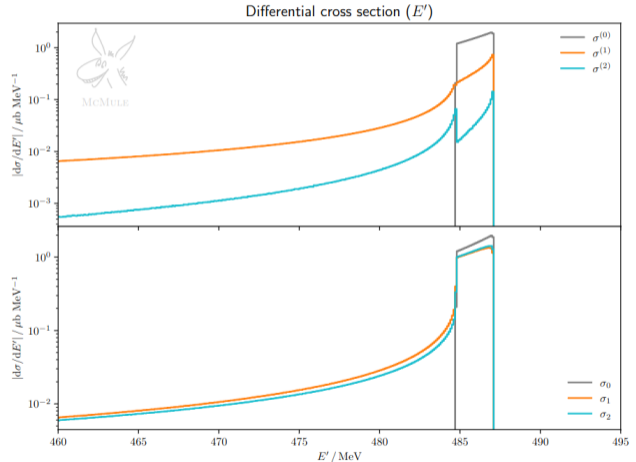
full NLO for  $e^- p \rightarrow e^- p \gamma$   
 ( $\supset$  NNLO for  $e^- p \rightarrow e^- p$ )

$E = 495$  MeV



[Mihovilovic et al. 1905.11182]

10% correction in the tail



$$A_{LR}^{\text{LO full}} = \frac{\lambda d\sigma_{L-R}^{(0,1)}}{2(\alpha d\sigma^{(0)} + \alpha^2 d\sigma^{(1)}) + \lambda d\sigma_{L+R}^{(0,1)}}$$

$$d\sigma_{L\pm R}^{(j,1)} \equiv d\sigma_L^{(j,1)} \pm d\sigma_R^{(j,1)}$$

$$A_{LR}^{\text{NLO full}} = \frac{\lambda d\sigma_{L-R}^{(0,1)} + \alpha \lambda d\sigma_{L-R}^{(1,1)}}{2(\alpha d\sigma^{(0)} + \alpha^2 d\sigma^{(1)} + \alpha^3 d\sigma^{(2)}) + \lambda d\sigma_{L+R}^{(0,1)} + \alpha \lambda d\sigma_{L+R}^{(1,1)}}$$

$$A_{LR}^{\text{LO exp}} = \frac{\lambda}{\alpha} \frac{d\sigma_{L-R}^{(0,1)}}{2 d\sigma^{(0)}}$$

$$A_{LR}^{\text{NLO exp}} = \frac{\lambda}{\alpha} \frac{d\sigma_{L-R}^{(0,1)}}{2 d\sigma^{(0)}} + \lambda \frac{d\sigma^{(0)} d\sigma_{L-R}^{(1,1)} - d\sigma^{(1)} d\sigma_{L-R}^{(0,1)}}{2 (d\sigma^{(0)})^2}$$



4-fermion (fermion-number-violating)

$\Delta L = 4 + \text{h.c.}$

$$\mathcal{O}_{\nu\nu}^{S,LL} \left| (\nu_{Lp}^T C \nu_{Lr}) (\nu_{Ls}^T C \nu_{Lt}) \right| \text{"SLL \ [Nu] \ [Nu] "}$$

$\Delta B = -\Delta L = 1 + \text{h.c.}$

$\mathcal{O}_{ddd}^{S,LL}$	$\varepsilon_{abc} (d_{Lp}^T C d_{Lr}^b) (\bar{e}_{Rs} \bar{d}_{Lt}^c)$	"SLLddd"
$\mathcal{O}_{udd}^{S,LL}$	$\varepsilon_{abc} (u_{Lp}^T C d_{Lr}^b) (\bar{\nu}_{Ls} \bar{d}_{Rt}^c)$	"SLRudd"
$\mathcal{O}_{ddu}^{S,LR}$	$\varepsilon_{abc} (d_{Lp}^T C d_{Lr}^b) (\bar{\nu}_{Ls} \bar{u}_{Rt}^c)$	"SLRddu"
$\mathcal{O}_{ddd}^{S,LR}$	$\varepsilon_{abc} (d_{Lp}^T C d_{Lr}^b) (\bar{e}_{Ls} \bar{d}_{Rt}^c)$	"SLRddd"
$\mathcal{O}_{ddd}^{S,RL}$	$\varepsilon_{abc} (d_{Rp}^T C d_{Rr}^b) (\bar{e}_{Ls} \bar{d}_{Lt}^c)$	"SRLddd"
$\mathcal{O}_{udd}^{S,RR}$	$\varepsilon_{abc} (u_{Rp}^T C d_{Rr}^b) (\bar{\nu}_{Ls} \bar{d}_{Rt}^c)$	"SRRudd"
$\mathcal{O}_{ddd}^{S,RR}$	$\varepsilon_{abc} (d_{Rp}^T C d_{Rr}^b) (\bar{e}_{Ls} \bar{d}_{Rt}^c)$	"SRRddd"

$\Delta L = 2 + \text{h.c.}$

$\mathcal{O}_{\nu e}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr}) (\bar{e}_{Rs} e_{Lt})$	"SLL \ [Nu] e"
$\mathcal{O}_{\nu e}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr}) (\bar{e}_{Rs} \sigma_{\mu\nu} e_{Lt})$	"TLL \ [Nu] e"
$\mathcal{O}_{\nu e}^{S,LR}$	$(\nu_{Lp}^T C \nu_{Lr}) (\bar{e}_{Ls} e_{Rt})$	"SLR \ [Nu] e"
$\mathcal{O}_{\nu u}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr}) (\bar{u}_{Rs} u_{Lt})$	"SLL \ [Nu] u"
$\mathcal{O}_{\nu u}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr}) (\bar{u}_{Rs} \sigma_{\mu\nu} u_{Lt})$	"TLL \ [Nu] u"
$\mathcal{O}_{\nu u}^{S,LR}$	$(\nu_{Lp}^T C \nu_{Lr}) (\bar{u}_{Ls} u_{Rt})$	"SLR \ [Nu] u"
$\mathcal{O}_{\nu d}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr}) (\bar{d}_{Rs} d_{Lt})$	"SLL \ [Nu] d"
$\mathcal{O}_{\nu d}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr}) (\bar{d}_{Rs} \sigma_{\mu\nu} d_{Lt})$	"TLL \ [Nu] d"
$\mathcal{O}_{\nu d}^{S,LR}$	$(\nu_{Lp}^T C \nu_{Lr}) (\bar{d}_{Ls} d_{Rt})$	"SLR \ [Nu] d"
$\mathcal{O}_{\nu e}^{S,LL}$	$(\nu_{Lp}^T C e_{Lr}) (\bar{d}_{Rs} u_{Lt})$	"SLL \ [Nu] edu"
$\mathcal{O}_{\nu e}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} e_{Lr}) (\bar{d}_{Rs} \sigma_{\mu\nu} u_{Lt})$	"TLL \ [Nu] edu"
$\mathcal{O}_{\nu e}^{S,LR}$	$(\nu_{Lp}^T C e_{Lr}) (\bar{d}_{Ls} u_{Rt})$	"SLR \ [Nu] edu"
$\mathcal{O}_{\nu e}^{V,RR}$	$(\nu_{Lp}^T C \gamma^\mu e_{Rr}) (\bar{d}_{Ls} \gamma_\mu u_{Lt})$	"VRL \ [Nu] edu"
$\mathcal{O}_{\nu e}^{V,RR}$	$(\nu_{Lp}^T C \gamma^\mu e_{Rr}) (\bar{d}_{Rs} \gamma_\mu u_{Rt})$	"VRR \ [Nu] edu"

$\Delta B = \Delta L = 1 + \text{h.c.}$

$\mathcal{O}_{udd}^{S,LL}$	$\varepsilon_{abc} (u_{Lp}^T C d_{Lr}^b) (d_{Ls}^T C \nu_{Lt})$	"SLLudd"
$\mathcal{O}_{duu}^{S,LL}$	$\varepsilon_{abc} (d_{Lp}^T C u_{Lr}^b) (u_{Ls}^T C e_{Lt})$	"SLLduu"
$\mathcal{O}_{uud}^{S,LR}$	$\varepsilon_{abc} (u_{Lp}^T C u_{Lr}^b) (d_{Rt}^T C e_{Rt})$	"SLRuud"
$\mathcal{O}_{duu}^{S,LR}$	$\varepsilon_{abc} (d_{Lp}^T C u_{Lr}^b) (u_{Rt}^T C e_{Rt})$	"SLRduu"
$\mathcal{O}_{duu}^{S,RL}$	$\varepsilon_{abc} (u_{Rp}^T C u_{Rr}^b) (d_{Ls}^T C e_{Ls})$	"SRLduu"
$\mathcal{O}_{duu}^{S,RL}$	$\varepsilon_{abc} (d_{Rp}^T C u_{Rr}^b) (u_{Ls}^T C e_{Ls})$	"SRLduu"
$\mathcal{O}_{duu}^{S,RL}$	$\varepsilon_{abc} (d_{Rp}^T C u_{Rr}^b) (d_{Ls}^T C \nu_{Lt})$	"SRLduu"
$\mathcal{O}_{duu}^{S,RL}$	$\varepsilon_{abc} (d_{Rp}^T C d_{Rr}^b) (u_{Ls}^T C \nu_{Lt})$	"SRLduu"
$\mathcal{O}_{duu}^{S,RR}$	$\varepsilon_{abc} (d_{Rp}^T C u_{Rr}^b) (u_{Rt}^T C e_{Rt})$	"SRRduu"

on-shell redundant

$(\nu\nu)D^2 + \text{h.c.}$

$\mathcal{O}_{\nu D}$	$\nu_{Lp}^T C (i\mathbb{D})^2 \nu_{Lr}$	"[Nu]D"
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$(\bar{L}R)D^2 + \text{h.c.}$

$\mathcal{O}_{cD}$	$\bar{e}_{Lp} (i\mathbb{D})^2 e_{Rr}$	"eD"
$\mathcal{O}_{uD}$	$\bar{u}_{Lp} (i\mathbb{D})^2 u_{Rr}$	"uD"
$\mathcal{O}_{dD}$	$\bar{d}_{Lp} (i\mathbb{D})^2 d_{Rr}$	"dD"

$(\bar{L}L)D^3$

$\mathcal{O}_{\nu D}^L$	$\bar{\nu}_{Lp} (i\mathbb{D})^3 \nu_{Lr}$	"L[Nu]D"
$\mathcal{O}_{cD}^L$	$\bar{e}_{Lp} (i\mathbb{D})^3 e_{Lr}$	"LeD"
$\mathcal{O}_{uD}^L$	$\bar{u}_{Lp} (i\mathbb{D})^3 u_{Lr}$	"LuD"
$\mathcal{O}_{dD}^L$	$\bar{d}_{Lp} (i\mathbb{D})^3 d_{Lr}$	"LdD"

$(\bar{R}R)D^3$

$\mathcal{O}_{cD}^R$	$\bar{e}_{Rp} (i\mathbb{D})^3 e_{Rr}$	"ReD"
$\mathcal{O}_{uD}^R$	$\bar{u}_{Rp} (i\mathbb{D})^3 u_{Rr}$	"RuD"
$\mathcal{O}_{dD}^R$	$\bar{d}_{Rp} (i\mathbb{D})^3 d_{Rr}$	"RdD"

$(\bar{L}L)XD$

$\mathcal{O}_{\nu D\gamma}^L$	$(\bar{\nu}_{Lp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} \nu_{Lr}) F_{\mu\nu}$	"LD\ [Nu] \ [Gamma]"
$\mathcal{O}_{\nu D\gamma}^L$	$(\bar{\nu}_{Lp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} \nu_{Lr}) F_{\mu\nu}$	"L\ [Nu]D \ [Gamma]"
$\mathcal{O}_{\nu\gamma D}^L$	$(\bar{\nu}_{Lp} \gamma^\mu \nu_{Lr})(\partial^\nu F_{\mu\nu})$	"L\ [Nu] \ [Gamma]D"
$\mathcal{O}_{De\gamma}^L$	$(\bar{e}_{Lp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} e_{Lr}) F_{\mu\nu}$	"LDe \ [Gamma]"
$\mathcal{O}_{cD\gamma}^L$	$(\bar{e}_{Lp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} e_{Lr}) F_{\mu\nu}$	"LeD \ [Gamma]"
$\mathcal{O}_{e\gamma D}^L$	$(\bar{e}_{Lp} \gamma^\mu e_{Lr})(\partial^\nu F_{\mu\nu})$	"Le \ [Gamma]D"
$\mathcal{O}_{Du\gamma}^L$	$(\bar{u}_{Lp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} u_{Lr}) F_{\mu\nu}$	"LDu \ [Gamma]"
$\mathcal{O}_{uD\gamma}^L$	$(\bar{u}_{Lp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} u_{Lr}) F_{\mu\nu}$	"LuD \ [Gamma]"
$\mathcal{O}_{u\gamma D}^L$	$(\bar{u}_{Lp} \gamma^\mu u_{Lr})(\partial^\nu F_{\mu\nu})$	"Lu \ [Gamma]D"
$\mathcal{O}_{Dd\gamma}^L$	$(\bar{d}_{Lp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} d_{Lr}) F_{\mu\nu}$	"LDd \ [Gamma]"
$\mathcal{O}_{dD\gamma}^L$	$(\bar{d}_{Lp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} d_{Lr}) F_{\mu\nu}$	"LdD \ [Gamma]"
$\mathcal{O}_{d\gamma D}^L$	$(\bar{d}_{Lp} \gamma^\mu d_{Lr})(\partial^\nu F_{\mu\nu})$	"Ld \ [Gamma]D"
$\mathcal{O}_{DuG}^L$	$(\bar{u}_{Lp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} t^\alpha u_{Lr}) G_{\mu\nu}^\alpha$	"LDuG"
$\mathcal{O}_{uDG}^L$	$(\bar{u}_{Lp} \sigma^{\mu\nu} t^\alpha i\overleftrightarrow{\mathbb{D}} u_{Lr}) G_{\mu\nu}^\alpha$	"LuDG"
$\mathcal{O}_{uGD}^L$	$(\bar{u}_{Lp} \gamma^\mu t^\alpha u_{Lr})(D^\nu G_{\mu\nu}^\alpha)$	"LuGD"
$\mathcal{O}_{DdG}^L$	$(\bar{d}_{Lp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} t^\alpha d_{Lr}) G_{\mu\nu}^\alpha$	"LDdG"
$\mathcal{O}_{dDG}^L$	$(\bar{d}_{Lp} \sigma^{\mu\nu} t^\alpha i\overleftrightarrow{\mathbb{D}} d_{Lr}) G_{\mu\nu}^\alpha$	"LdDG"
$\mathcal{O}_{dGD}^L$	$(\bar{d}_{Lp} \gamma^\mu t^\alpha d_{Lr})(D^\nu G_{\mu\nu}^\alpha)$	"LdGD"

$(\bar{R}R)XD$

$\mathcal{O}_{De\gamma}^R$	$(\bar{e}_{Rp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} e_{Rr}) F_{\mu\nu}$	"RDe \ [Gamma]"
$\mathcal{O}_{eD\gamma}^R$	$(\bar{e}_{Rp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} e_{Rr}) F_{\mu\nu}$	"ReD \ [Gamma]"
$\mathcal{O}_{e\gamma D}^R$	$(\bar{e}_{Rp} \gamma^\mu e_{Rr})(\partial^\nu F_{\mu\nu})$	"Re \ [Gamma]D"
$\mathcal{O}_{Du\gamma}^R$	$(\bar{u}_{Rp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} u_{Rr}) F_{\mu\nu}$	"RDu \ [Gamma]"
$\mathcal{O}_{uD\gamma}^R$	$(\bar{u}_{Rp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} u_{Rr}) F_{\mu\nu}$	"RuD \ [Gamma]"
$\mathcal{O}_{u\gamma D}^R$	$(\bar{u}_{Rp} \gamma^\mu u_{Rr})(\partial^\nu F_{\mu\nu})$	"Ru \ [Gamma]D"
$\mathcal{O}_{Dd\gamma}^R$	$(\bar{d}_{Rp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} d_{Rr}) F_{\mu\nu}$	"RDd \ [Gamma]"
$\mathcal{O}_{dD\gamma}^R$	$(\bar{d}_{Rp} \sigma^{\mu\nu} i\overleftrightarrow{\mathbb{D}} d_{Rr}) F_{\mu\nu}$	"RdD \ [Gamma]"
$\mathcal{O}_{d\gamma D}^R$	$(\bar{d}_{Rp} \gamma^\mu d_{Rr})(\partial^\nu F_{\mu\nu})$	"Rd \ [Gamma]D"
$\mathcal{O}_{DuG}^R$	$(\bar{u}_{Rp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} t^\alpha u_{Rr}) G_{\mu\nu}^\alpha$	"RDuG"
$\mathcal{O}_{uDG}^R$	$(\bar{u}_{Rp} \sigma^{\mu\nu} t^\alpha i\overleftrightarrow{\mathbb{D}} u_{Rr}) G_{\mu\nu}^\alpha$	"RuDG"
$\mathcal{O}_{uGD}^R$	$(\bar{u}_{Rp} \gamma^\mu t^\alpha u_{Rr})(D^\nu G_{\mu\nu}^\alpha)$	"RuGD"
$\mathcal{O}_{DdG}^R$	$(\bar{d}_{Rp} i\overleftrightarrow{\mathbb{D}} \sigma^{\mu\nu} t^\alpha d_{Rr}) G_{\mu\nu}^\alpha$	"RDdG"
$\mathcal{O}_{dDG}^R$	$(\bar{d}_{Rp} \sigma^{\mu\nu} t^\alpha i\overleftrightarrow{\mathbb{D}} d_{Rr}) G_{\mu\nu}^\alpha$	"RdDG"
$\mathcal{O}_{dGD}^R$	$(\bar{d}_{Rp} \gamma^\mu t^\alpha d_{Rr})(D^\nu G_{\mu\nu}^\alpha)$	"RdGD"

$X^2 D^2$

$\mathcal{O}_{\gamma D}$	$(\partial_\mu F^{\mu\nu})(\partial^\rho F_{\rho\nu})$	" \ [Gamma]D"
$\mathcal{O}_{GD}$	$(D_\mu G^{\alpha,\mu\nu})(D^\rho G_{\rho\nu}^\alpha)$	"GD"