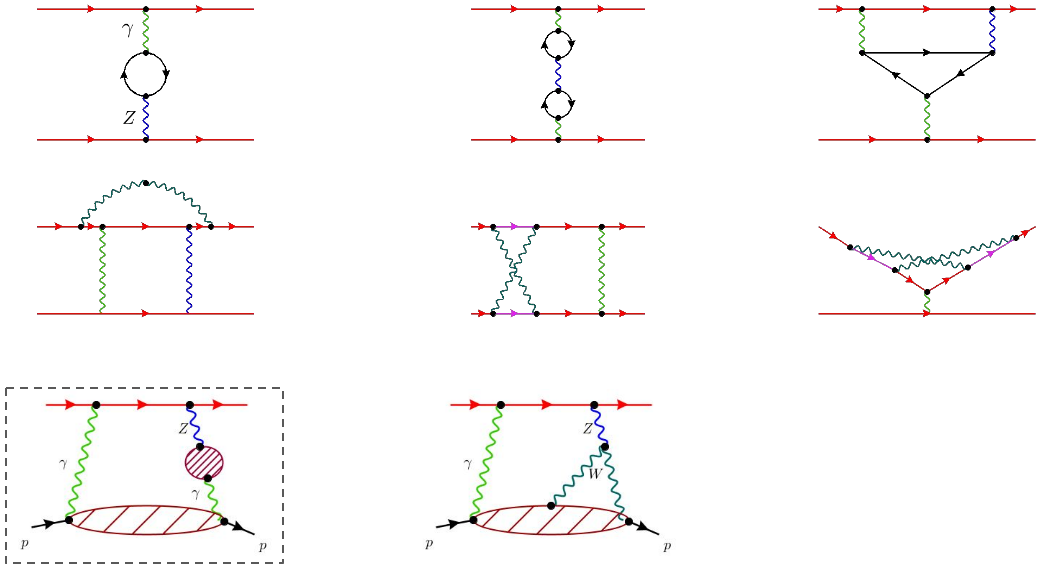
NNLO Electroweak Correction @ MOLLER and P2

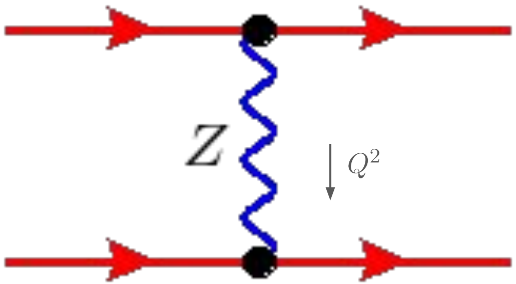
- Physics Motivation
- Current Status
- Outlook

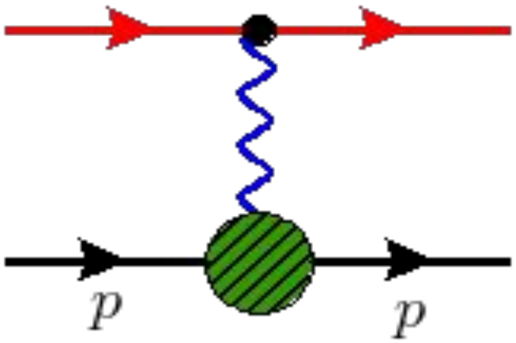


work with Jens Erler, Ayres Freitas, Juhun Kwak

PVES experiments

$$A_{LR} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$





- $e^- - e^-$ Møller scattering
- $Q^2 \sim 0.005\,\mathrm{GeV}^2$
- $\delta Q^e_{\mathrm{W}} \sim 2.3\% \to \delta\sin^2\theta_W \sim 0.1\%$

- $e^- - p$ elastic scattering
- $Q^2 \sim 0.006\,\mathrm{GeV}^2$
- $\delta Q^p_{\mathrm{W}} \sim 1.5\% \to \delta\sin^2\theta_W \sim 0.14\%$

$$A_{LR}^{\mathrm{MOLLER}} = \frac{G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4}(1 - 4\sin^2\theta_W + \Delta Q^e_W)$$

$$y = Q^2/s$$

$$\mathcal{A}_{LR}^{\mathrm{P2}} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}((1 - 4\sin^2\theta_W) + \Delta Q^p_W + F(Q^2, E_i))$$

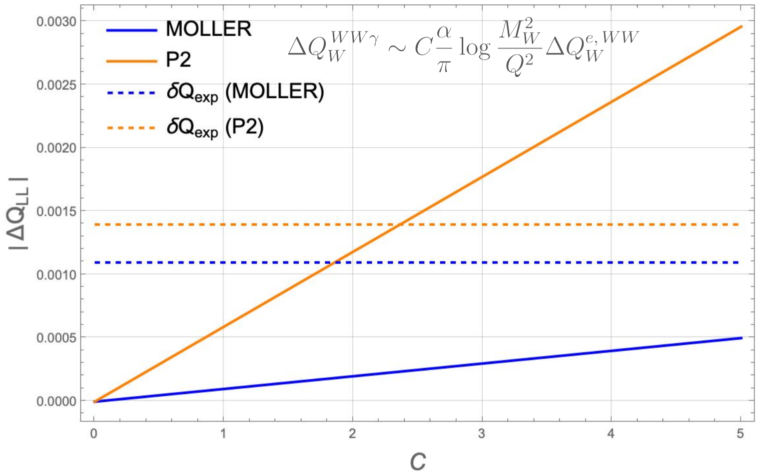
MOLLER: purely leptonic

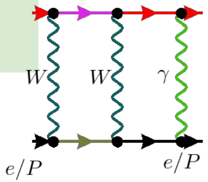
P2: ep, hadronic structure enters

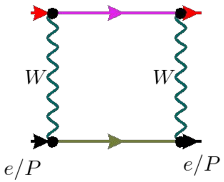
Both need NNLO EW at the required precision.

the Large Logs (LL) @ PVES @ *NNLO*

A naive estimate:







$$\Delta Q^{e,WW\,(1)} = \frac{\alpha}{4\pi \hat{s}^2}$$

$$\Delta Q_W^{p,WW} = \frac{\alpha}{4\pi \hat{s}^2}\left[2+5\left(1-\frac{\alpha_s(M_W^2)}{\pi}\right)\right]$$

See more about the LL of PVES @ NLO in talks by Sophie and David

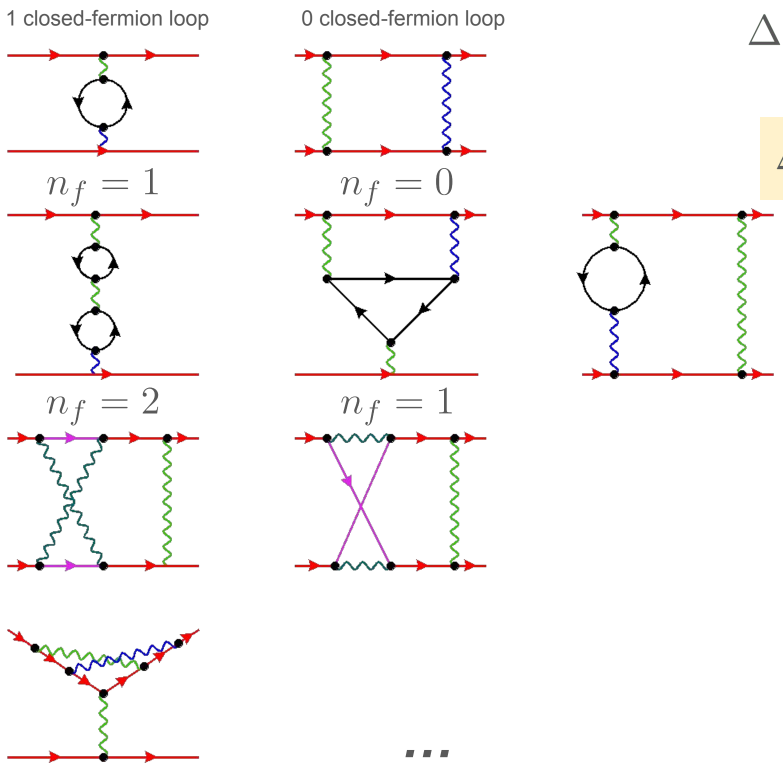
Having ***complete*** NNLO calculation for MOLLER means

- ❑ Testing the weakest estimation in the current theory error budget.
- ❑ Completing one of the very few 2-to-2 full NNLO EW corrections.
- ❑ Providing a bridge for P2 (especially the large-log issue).
- ❑ Protecting the BSM interpretation.

Electroweak Radiative Correction Categorization for MOLLER

1-loop

2-loop



$$\Delta Q_W^{(L,n_f)} = \Delta Q_W^{(1,1)} + \Delta Q_W^{(1,0)} + \Delta Q_W^{(2,2)} + \Delta Q_W^{(2,1)} + \Delta Q_W^{(2,0)}$$

$$\Delta Q_W^{(2,0)} = \Delta Q_W^{(2)}(\log(Q^2/M^2)) + \Delta Q_W^{(2,res)}$$

- ❏ fermion loop corrections are often numerical leading.
- ❏ In boson loop correction, large-logs are also important.
- ❏ Loops that are (1-4sw^2) suppressed are numerically less important.

EW Radiative corrections @ MOLLER

$$A_{\rm LR} = \frac{G_F Q^2}{\sqrt{2}\pi\alpha}\mathcal{G}(y) Q_W^{e,(0)}(1 + N_f \Delta Q_W^{(1,1)} + \Delta Q_W^{(1,0)} + N_f^2 \Delta Q_W^{(2,2)} + N_f \Delta Q_W^{(2,1)} + \Delta Q_W^{(2,0)})$$

-0.39   +0.77   +0.07   -1.15   ~±0.13, est*

Low-energy scheme, (s,y)=(0.011GeV^2, 04),sw(0)-scheme, ***in unit of*** $\mathbf{10^{-3}}$

LO : Derman and Marciano, 79'

NLO : Marciano, Sirlin 80'; Czarnecki and Marciano, 96'; Denner and Pozzorini, 98; Petriello, 03; J.Erler, A.Kurylov, M.J.Ramsey-Musolf 03'; Kolomensky et al, 05'; Zykunov et al, 05'; Zykunov, 09'; Aleksejevs et al 10',11',12', *S. Kollatzsch, D.Moreno, D. Radic, A.Signer '25 (LEFT+LL Res)*

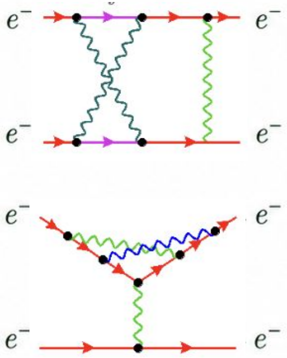
NNLO: Aleksejevs et al '12 '15; Freitas,Yong, M.J.Ramsey-Musolf, H.Patel '19; Freitas, J.Erler, R.Ferro-Hernandez'22

Relevant Integrals: S.Weinzierl, N.Schwanemann, N.Böttcher '23, S.Weinzierl, N.Schwanemann '24

A systematic, complete NNLO calculation, along with the proper SM phenomenology study is yet to be carried out.

Workflow of the Calculation

1



• • •

Generating two-loop bosonic Møller diagrams with FeynArts

2

$$\gamma^\mu \gamma^\alpha \gamma_\mu = (2-d)\gamma^\alpha$$

$$\mathrm{Tr}\left[\gamma^\mu \gamma^v \gamma^\rho \gamma^\sigma\right] = 4\left(g^{\mu v} g^{\rho\sigma} - g^{\mu\rho} g^{v\sigma} + g^{\mu\sigma} g^{v\rho}\right)$$

• • •

Algebraic simplification with FeynCalc

3

Beneke, Smirnov '88

1. scale hierarchy: $m_f,\, s,\, t,\, u \ll M_Z, M_W$
2. hard-hard region: $|q_1|,\, |q_2| \sim M_Z$ or M_W
3. soft-hard region: $|q_1| \sim M_Z$ or M_W, $|q_2| \sim t$

• • •

Expansion-by-Regions with home-cooked code

4

$$0 = \int d^d k\, d^d l\, \frac{\partial}{\partial k^\mu} \left(\frac{v^\mu}{D_1^{a1}\, D_2^{a2} \cdots D_m^{am}} \right)$$

with $D_i = (\ell_i^2 - m_i^2)$

$$\Rightarrow \sum_i c_i\,(a_j, d)\, I_i = 0$$

(IBP relations reduce integrals to a smaller set)

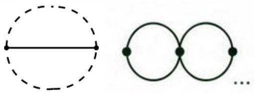
Integral reduction with FIRE, etc.

5

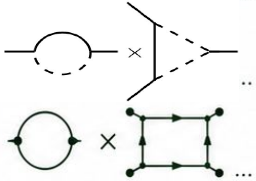
$$\mathcal{A} = \sum_{i=1}^{N_{\mathrm{MI}}} c_i\,(\epsilon)\, I_i$$

MI basis

Two-loop vacuum bubbles



Products of one-loop topologies



Collecting results in terms of MIs

Technical Aspects

- Analytical results can be obtained via expansion by region (EBR).
- Soft and collinear factorization ensure the IR-divergences from virtual corrections cancel out at asymmetry-level. IR-divs are regularized by m_e(coll.) and m_γ (soft).
- γ^5 is handled with diagram-dependent prescription.
- All results are obtained at least twice via independent calculations. Analytical forms, UV- and IR-finiteness provide cross check.

Practical take-away on γ_5 in SM calculations

Assumption: infrared singularities are consistently tracked and under control.

1. In full Standard Model calculations with well-traced IR singularities, naive dimensional regularization is the practical default framework for higher-order calculations.

2. For $2 \to 2$ fermion scattering, γ_5-dependent traces on open fermion chains may be evaluated in four dimensions, provided the coefficient of the parity-odd trace is free of unresolved UV poles.

Core rule: use 4D γ_5 only after the coefficient of the parity-odd trace is pole-free.

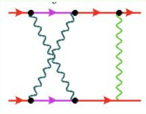
General γ_5 Prescription in SM Calculations

(for calculations with well-traced IR poles)

Start: Identify where γ_5 appears

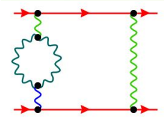
Closed fermion loop?

No → Open fermion chains / external traces

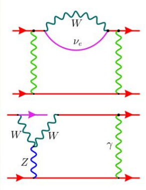


Does the γ_5-dependent sector multiply any unresolved UV/IR pole?

- **No →** Safe: evaluate the remaining γ_5 traces in 4D
- **Yes →** **Is the pole-carrying loop structure factorizable or projectable from the external spinor structure?**
  - **Yes →** Keep the pole-carrying loop reduction in $d = 4 - 2\varepsilon$; perform tensor decomposition / projection of the UV-divergent subloop to scalar coefficients; then evaluate the remaining external γ_5 traces in 4D → Safe after reduction / projection

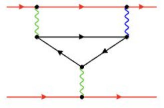


  - **No →** Construct a local UV subtraction or counterterm, or project out the problematic subloop basis; combine / subtract the pole first → Evaluate γ_5 traces in 4D only after subtraction

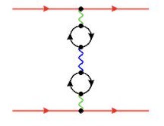


Yes → Closed fermion loop

VVA / AVV-type structure?



- **Yes →** Yes: special nontrivial closed-loop γ_5 case. Use a dedicated anomaly-consistent prescription (dimensionally regularize the loop, treat the odd γ_5 trace with a 4D Levi-Civita tensor, and check Ward identities). → In the SM, sum over the anomaly-free fermion set
- **No →** No: simple closed fermion bubble / non-VVA-AVV case. The γ_5 issue disappears after NDR, so no special γ_5 treatment is needed beyond NDR.



Core rule: Evaluate γ_5 traces in 4D only after projecting that their coefficient is free of unresolved poles, either because the sector is finite, factorized/projected, or explicitly subtracted.

Practical note: for projected self-energy insertions, keep explicit D-dependent prefactors (such as $D-2$) until the UV-sensitive subloop or counterterm sector has been combined; external momenta are treated as 4D.

Renormalization

- All masses except M_W, couplings : on-shell scheme.
- $\sin\theta_W$: $\overline{MS}$ with $\mu = 0$ and/or $\mu = M_Z$ $\hat{s}_W \equiv \sin\theta_W^{\overline{MS}}$
- $\delta M_W^2 = (1 - \hat{s}_W^2)\delta M_Z^{2\,OS} - M_Z^2 \delta \hat{s}_W^2$

J.Erler, Rodolfo F, A.Freitas 22

NNLO fermionic contribution

Contribution $(\times 10^{-3})$	$\hat{s}(m_Z)$–α scheme	$\hat{s}(0)$–α scheme	$\hat{s}(0)$–G_μ scheme
LO: $1 - 4\hat{s}^2$	74.40	45.56	45.56
NLO total: $\Delta Q_W^e(1,1) + \Delta Q_W^e(1,0)$	-25.98	$+1.16$	$+1.27$
NNLO total: $\Delta Q_W^e(2,2) + \Delta Q_W^e(2,1)$	$+1.00$	-1.08	-1.25

Progress and Outlook

- ❏ The NNLO bosonic corrections in full SM is close to completion.
- ❏ Some validation and cross-check are still in progress.
- ❏ It would be interesting to write the large-log structure systematically in the final result and communicate it with the low-energy EFT/LEFT description.

Existing Framework for *e-p* PVES

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[Q_W^p + F^p(Q^2,\theta)\right]$$

Q_W^p : effective EW couplings at $Q^2 = 0$

@ NLO (Marciano & Sirlin, Erler, Kurylov & Ramsey-Musolf,)

$$Q_W^p = [\rho_{NC} + \Delta_e]\left[1 - 4\sin^2\hat{\theta}_W(0) + \Delta_e'\right] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

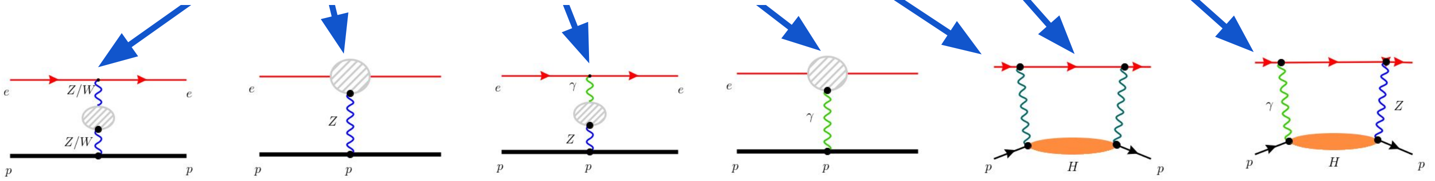


Diagram type	How treated
WW, ZZ boxes	short-distance OPE + free-quark matching + pQCD
γZ box	dispersive treatment; high-Q^2 pQCD plus low-Q^2 hadronic input
self-energies	universal renormalization: ρ_{NC}, running $\sin^2\theta_W$, γZ mixing
electron vertex	explicit Δ_e, Δ_e' corrections
proton/quark vertex	weak charge protected at $Q^2 = 0$; finite-Q^2 structure put into form factors

Don't define proton weak-charge

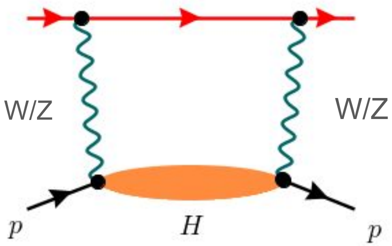
$$\mathcal{L}_{\rm PV} = -\frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma_\mu\gamma_5 e\sum_q C_{1q}\bar{q}\gamma^\mu q + \bar{e}\gamma_\mu e\sum_q C_{2q}\bar{q}\gamma^\mu\gamma_5 q\right]$$

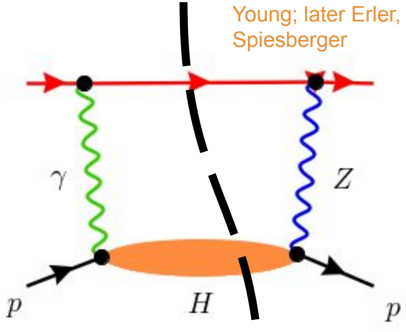
$$Q_W^p = -2(2C_{1u} + C_{1d})$$

NLO Boxes

$$T_{\mu\nu}(k) = \int d^4x\, e^{-ikx} \langle p'|T\{J_\mu(x)J_\nu(0)\}|p\rangle$$

Gorchtein & Horowitz; Sibirtsev, Blunden, Melnitchouk & Thomas; Rislow & Carlson; Gorchtein, Horowitz & Ramsey-Musolf; Blunden, Melnitchouk & Thomas; Hall, Blunden, Melnitchouk, Thomas & Young; later Erler, Gorchtein, Koshchii, Seng & Spiesberger





F_i are data-driven with certain model assumptions (see Misha's talk)

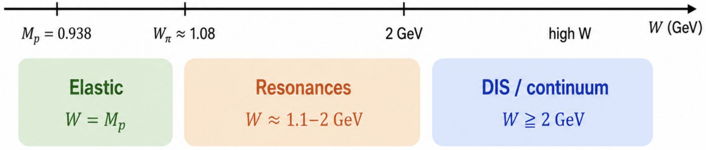
$k \sim M_Z$ down to $k \sim \Lambda_{\rm QCD}$

- ❏ Hard physics dominant.
- ❏ OPE $J_W(x)J_W(0) \to \sum_i C_i O_i(0)$
- ❏ Free-quark matching + pQCD (Erler/Kurylov/Ramsey-Musolf)

$$\mathrm{Im}\,\Box_{\gamma Z} \to \mathrm{Re}\,\Box_{\gamma Z}(E) \quad \Box_{\gamma Z}(E) = \Box^V_{\gamma Z}(E) + \Box^A_{\gamma Z}(E)$$

$$\mathrm{Im}\,\Box_{\gamma Z}(E) = \int dW^2 \int dQ^2 \left[K_1 F_1^{\gamma Z} + K_2 F_2^{\gamma Z} + K_3 F_3^{\gamma Z}\right]$$

$$\Box^V_{\gamma Z} \leftrightarrow F_1^{\gamma Z},\ F_2^{\gamma Z}, \qquad \Box^A_{\gamma Z} \leftrightarrow F_3^{\gamma Z}$$



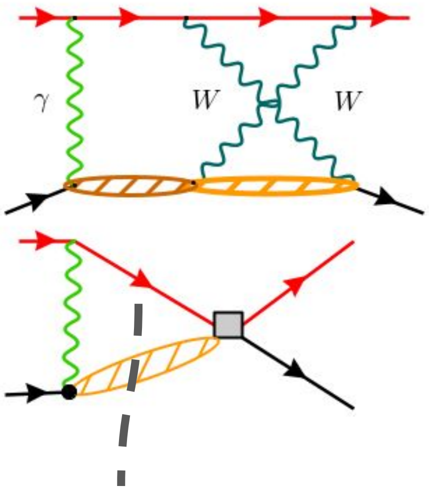
NNLO Box with Large Log

subloop corr. on hadronic-side

subloop corr. On leptonic-side, including SE of photon-Z mixing.

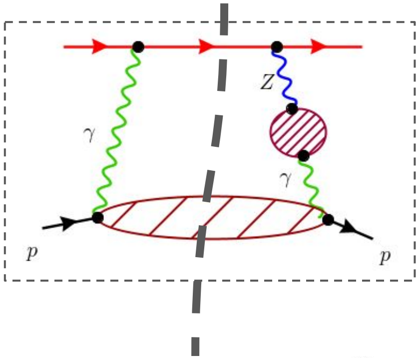
$$\mathrm{Im}\,\Box_{WW\gamma}(E) = \int dW^2\, dQ^2 \sum_{i=1}^{3} K_i^{WW\gamma}(E, W^2, Q^2) F_i^{\gamma WW}(W^2, Q^2)$$

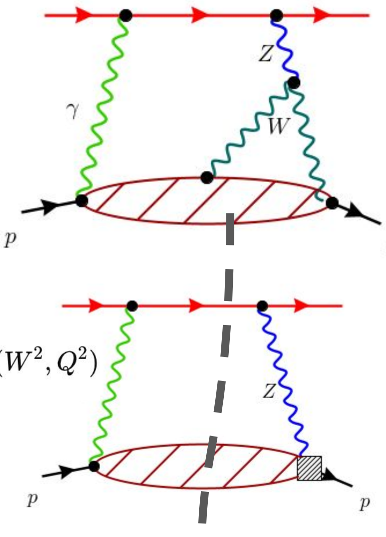
$$K_i^{WW\gamma} = \text{leptonic/photon kernel} \times C_{WW}(M_W, \mu).$$



But question: how good do we know about $F_i^{\gamma WW}(W^2, Q^2)$

$$\mathrm{Im}\,\Box^{(2)}_{\gamma Z} \sim \int dW^2\, dQ^2 \sum_i \left[K_i^{(0)} F_i^{(1)} + K_i^{(1)} F_i^{(0)} \right]$$





$$K_i^{(1)} F_i^{(0)} \longrightarrow K_i^{\gamma Z,e}(E, W^2, Q^2) \frac{\widehat{\Pi}_{Z\gamma}(Q^2)}{Q^2 + M_Z^2} F_i^{\gamma\gamma}(W^2, Q^2)$$

$$\delta F_i^{\gamma Z,\mathrm{EW}} \approx \delta_{\mathrm{EW}}\, F_i^{\gamma Z}$$

Outlook for P2 @ NNLO

- ❑ P2 reuses many building blocks from NNLO @ MOLLER.
- ❑ The only conceptually new part left is the box containing photon.
- ❑ Dispersion relation seems to be the most consistent way of bookkeeping EW effects from elastic regime to DIS.

Open questions:

- ❑ How proton weak-charge Q^p should be defined beyond NLO?
- ❑ How to organize the large log structure in the dispersion relation?