W^+W^- in e^+e^- annihilation at threshold and beyond

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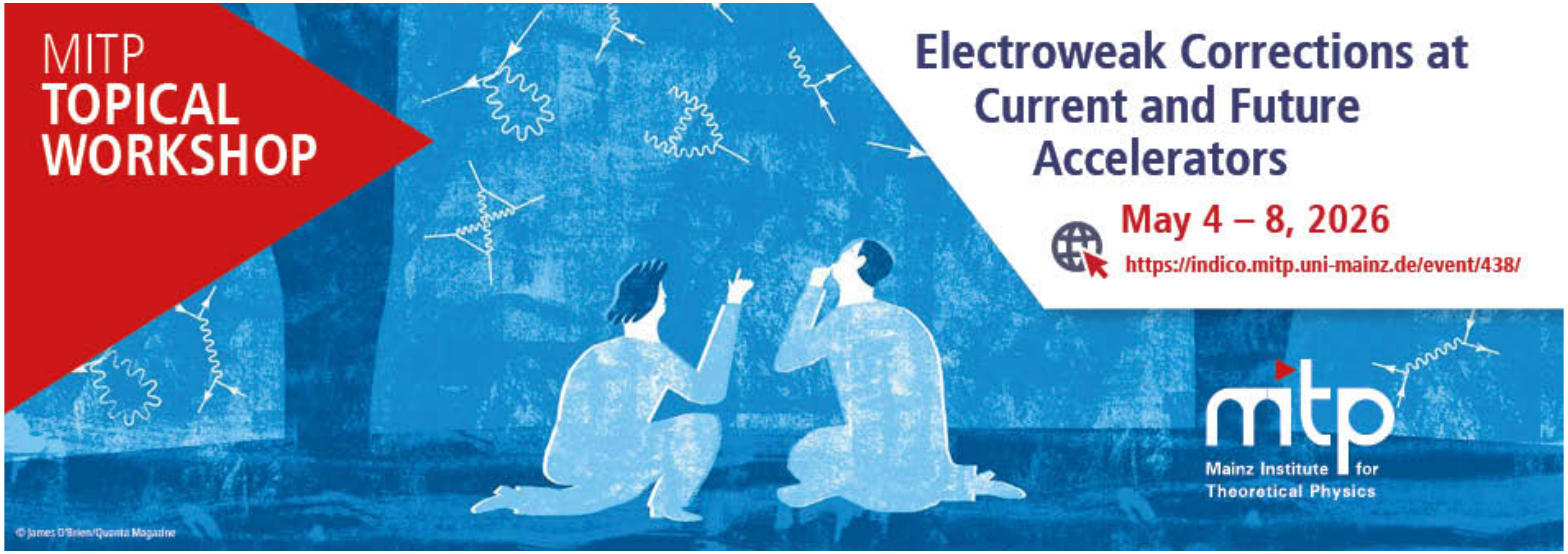
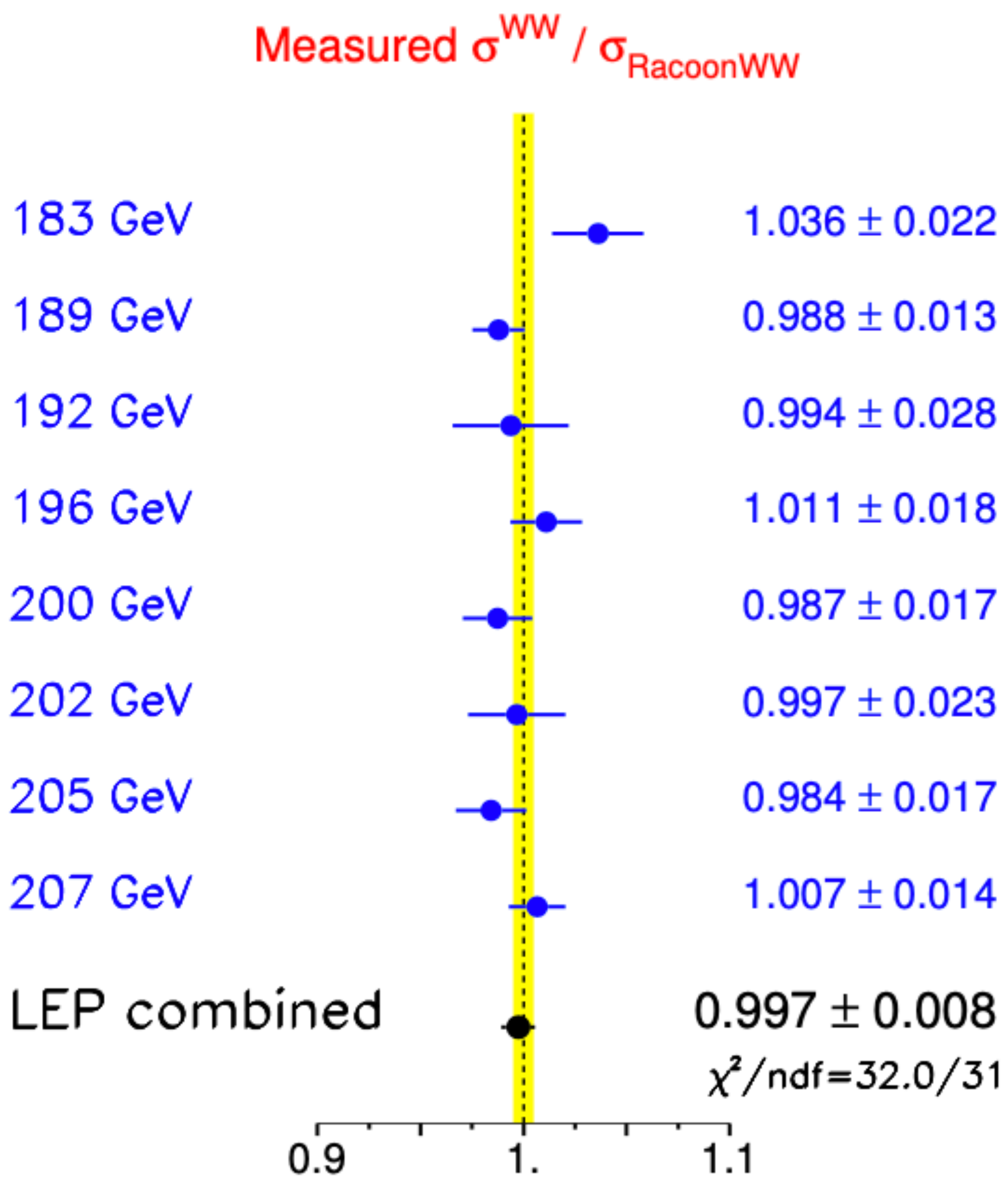
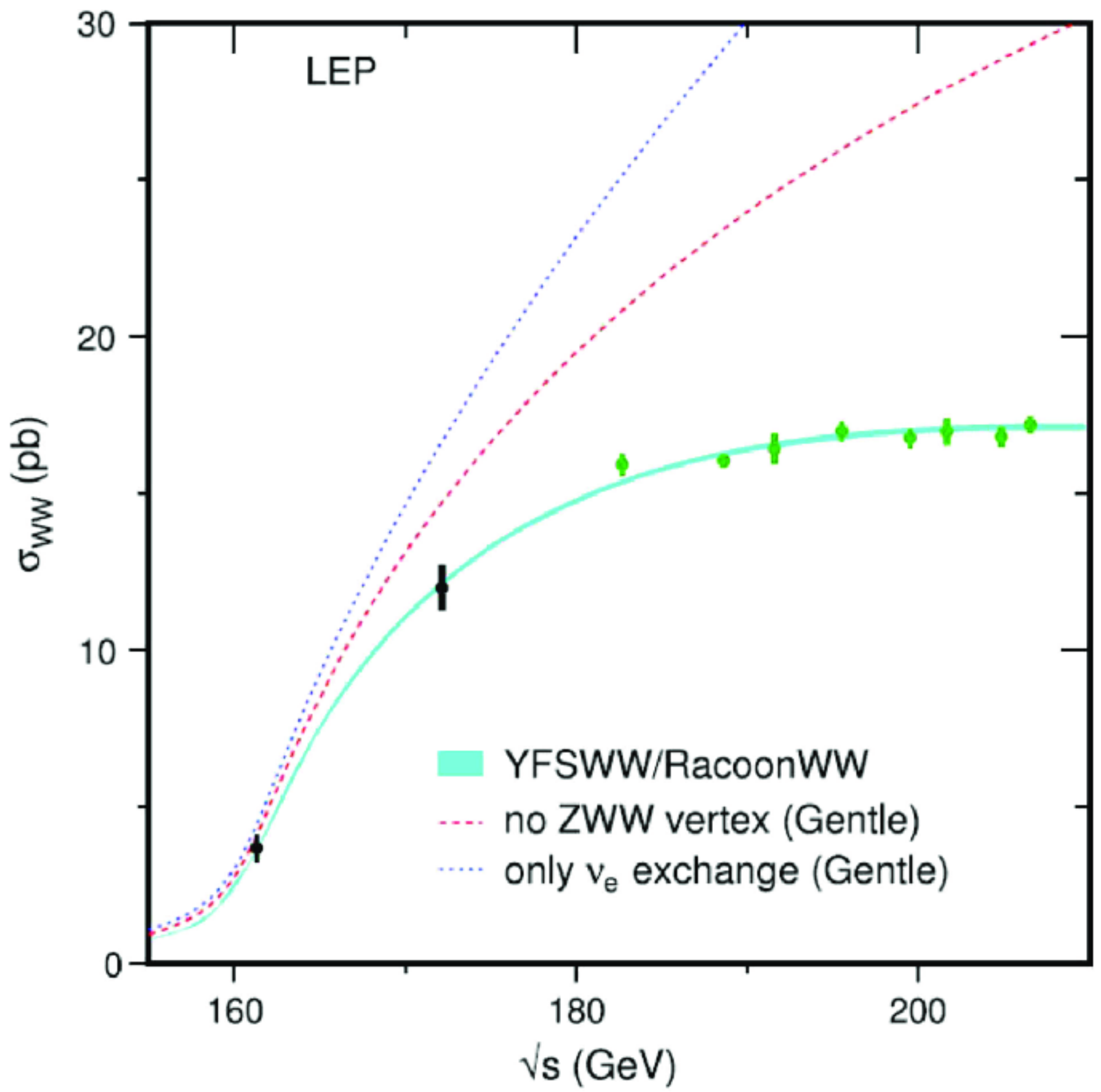


Jürgen R. Reuter

WW threshold — LEP legacy

- σ_{WW}: 1% agreement with NLO EW
- Test EW sector at quantum level (after the *Z* pole)
- Lepton universality (BR measurements)
- 1st test of non-Abelian EW structure
- Bounds on anom. TGC ("SMEFT" ops)
- *W* mass measurement by ...





1. Kinematic Reconstruction
2. Threshold scan/fit (σ_{WW})

$$\Delta m_{\rm W}({\rm stat}) = \left(\frac{d\sigma_{\rm WW}}{dm_{\rm W}}\right)^{-1} \frac{\sqrt{\sigma_{\rm WW}}}{\sqrt{\mathcal{L}}} \frac{1}{\sqrt{\epsilon p}} = \left(\frac{d\sigma_{\rm WW}}{dm_{\rm W}}\right)^{-1} \frac{\sqrt{\sigma_{\rm WW}}}{\sqrt{\epsilon \mathcal{L}}} \sqrt{1 + \frac{\sigma_B}{\epsilon \sigma_{\rm WW}}}$$

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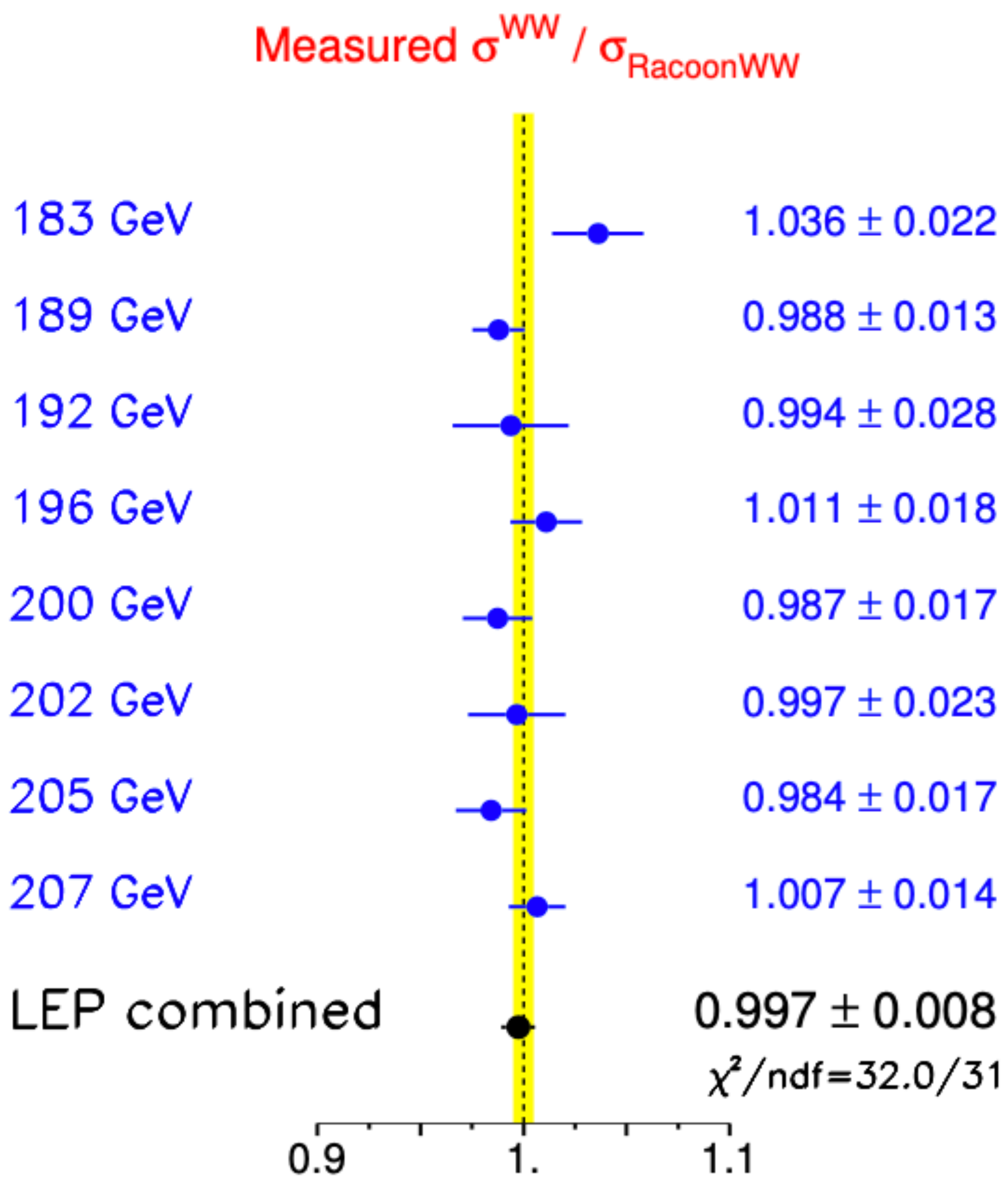

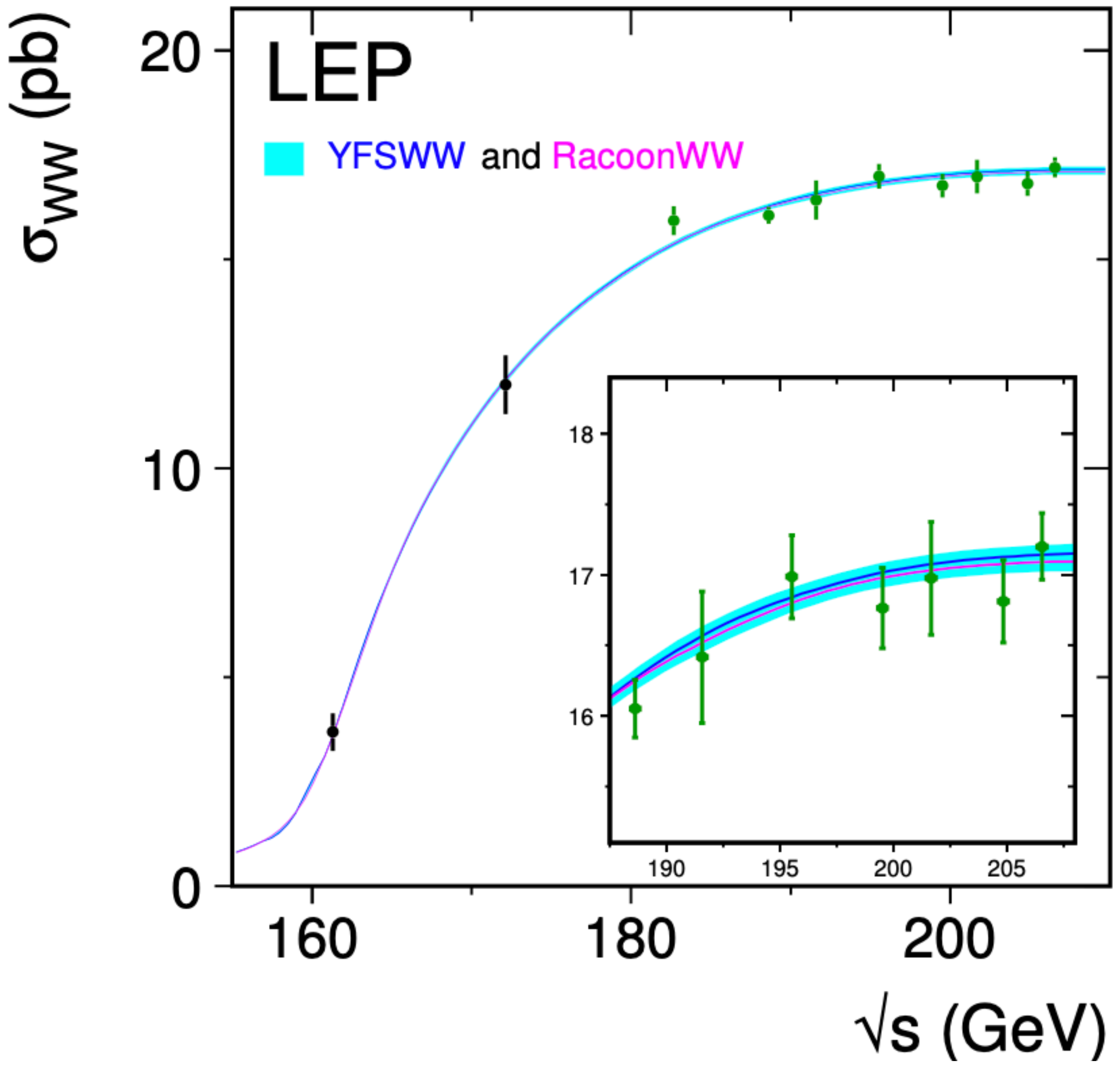

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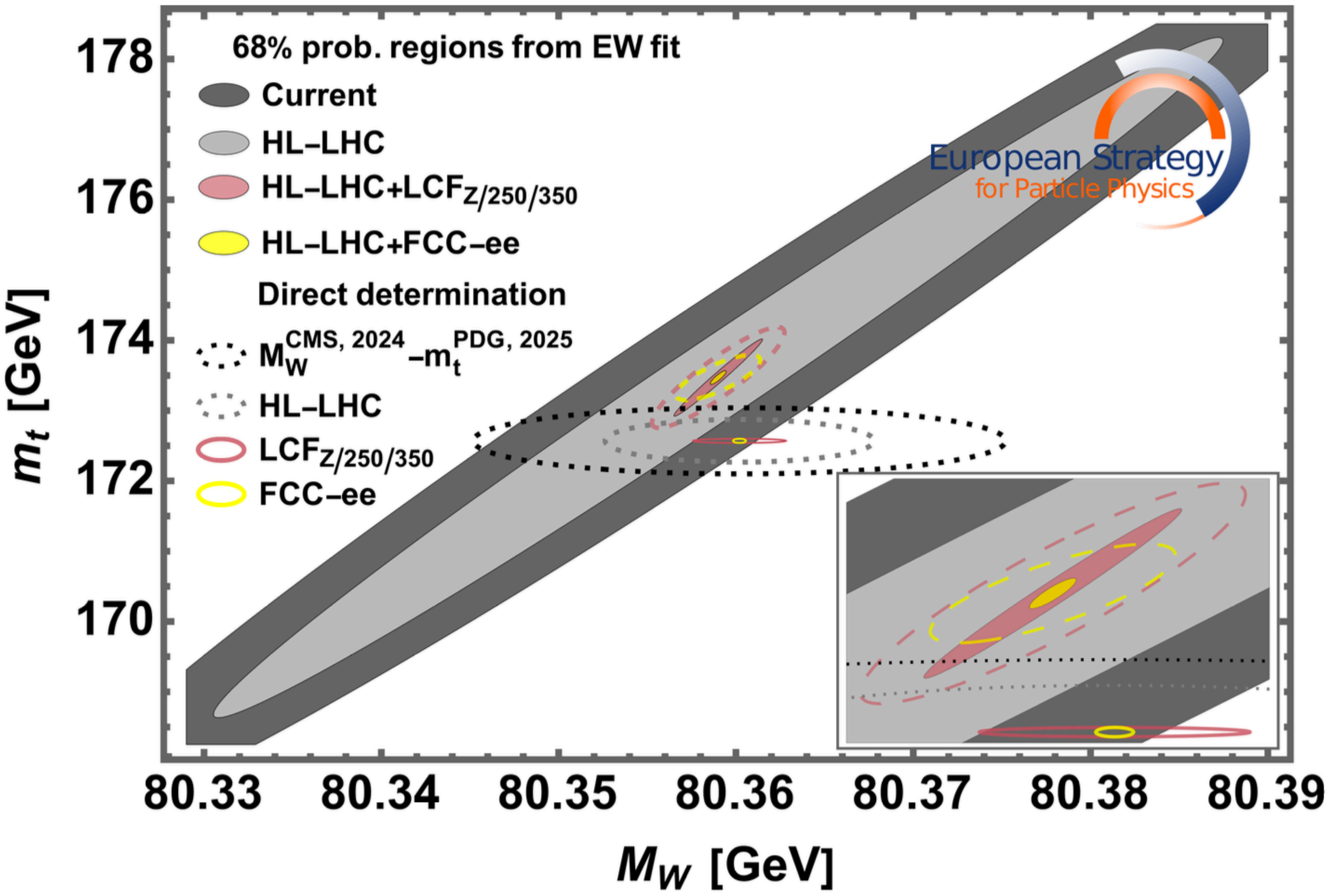

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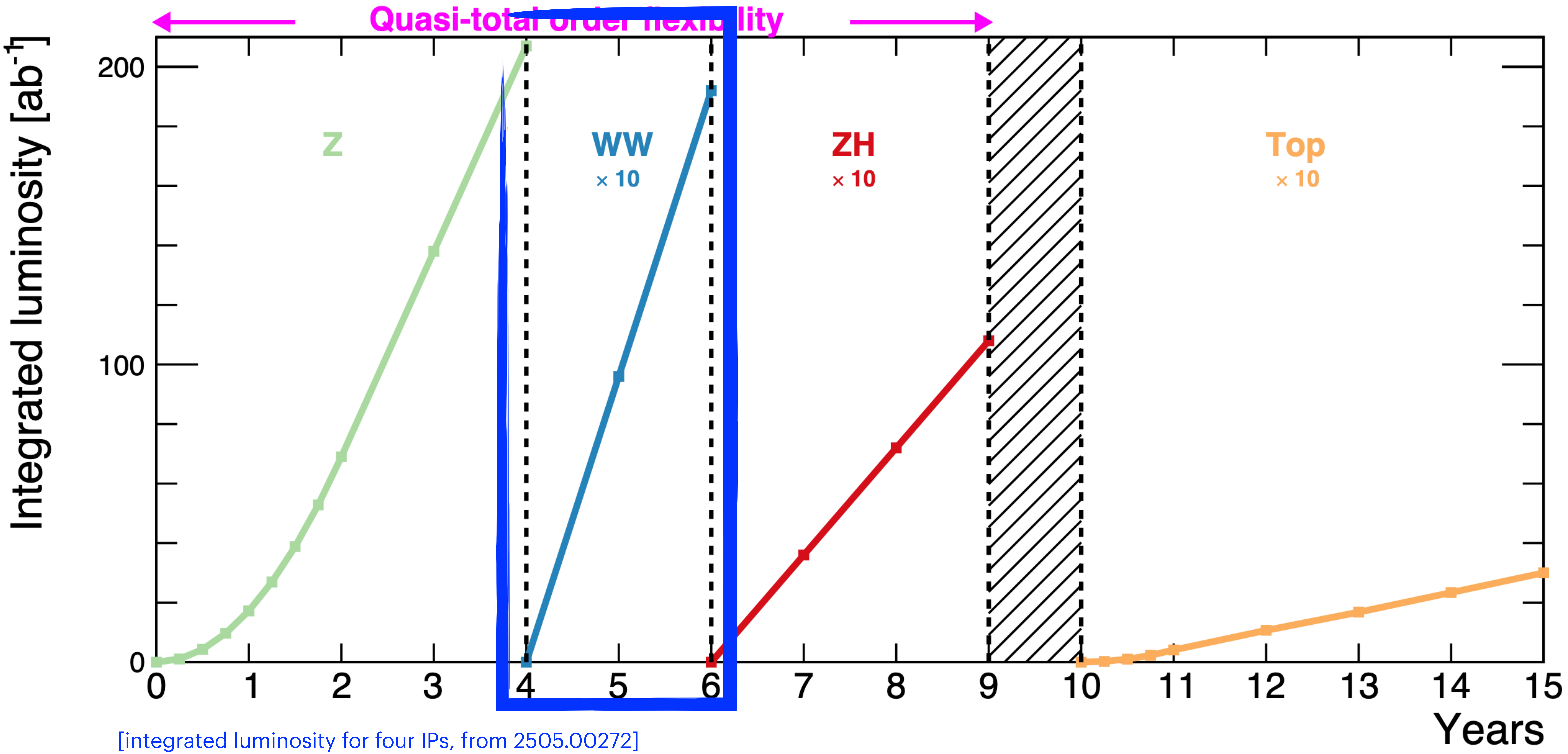
(EW) Precision Program at FCC-ee

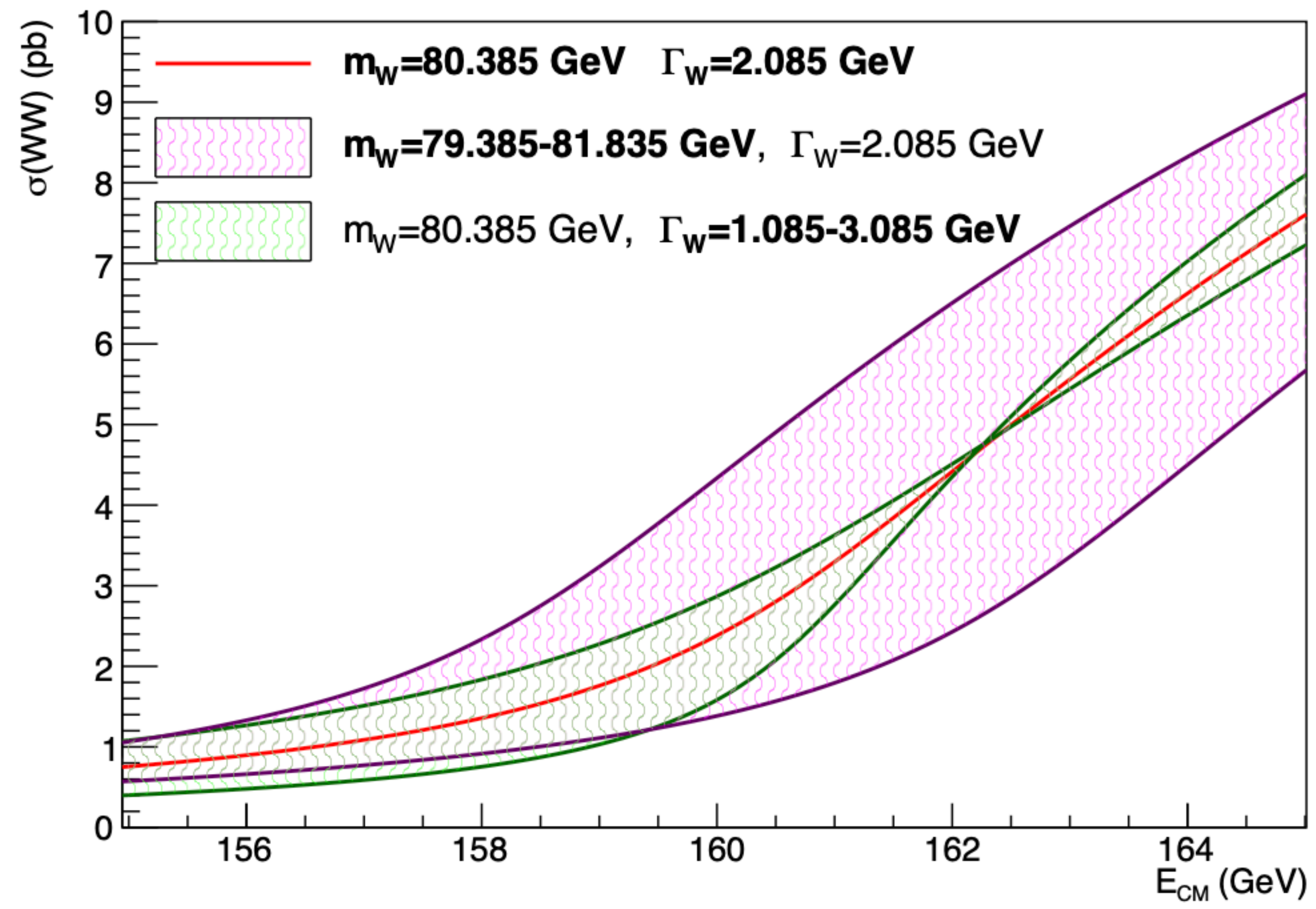


[integrated luminosity for four IPs, from 2505.00272]

WW threshold — FCC-ee

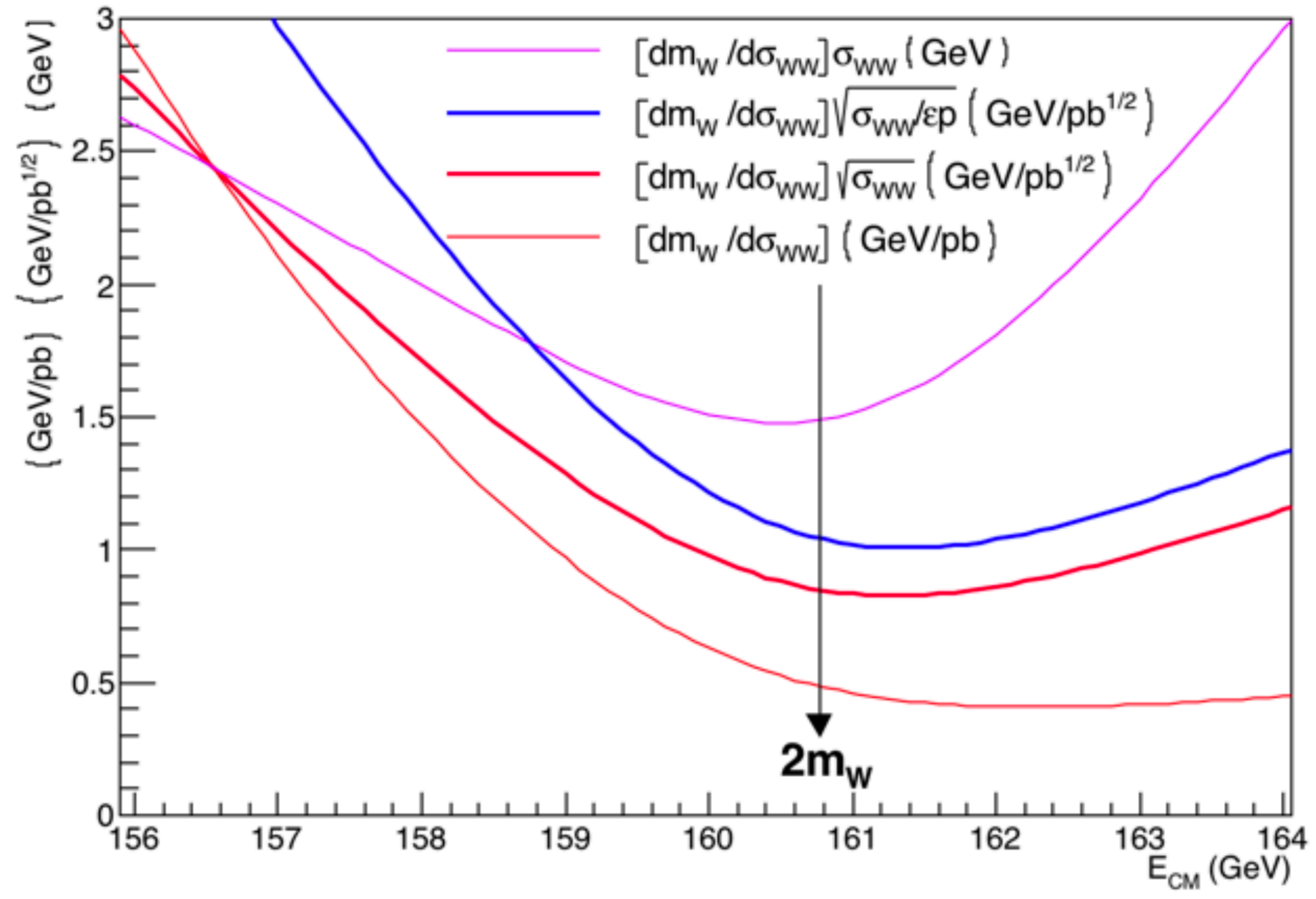
FCC study: 1703.01626

Observable	present value	±	uncertainty	FCC-ee Stat.	FCC-ee Syst.	Comment and leading uncertainty
m_W (MeV)	80 360.2	±	9.9	**0.18**	0.16	From WW threshold scan Beam energy calibration
Γ_W (MeV)	2 085	±	42	**0.27**	0.2	From WW threshold scan Beam energy calibration
$\alpha_S(m_W^2)$ ($\times 10^4$)	1 010	±	270	**2**	2	Combined R_ℓ^W, Γ_{tot}^W fit



Tiny statistical uncertainties, systematic uncertainties often severe

True bottleneck: theory uncertainties => disruptive advances needed



References and reviews:

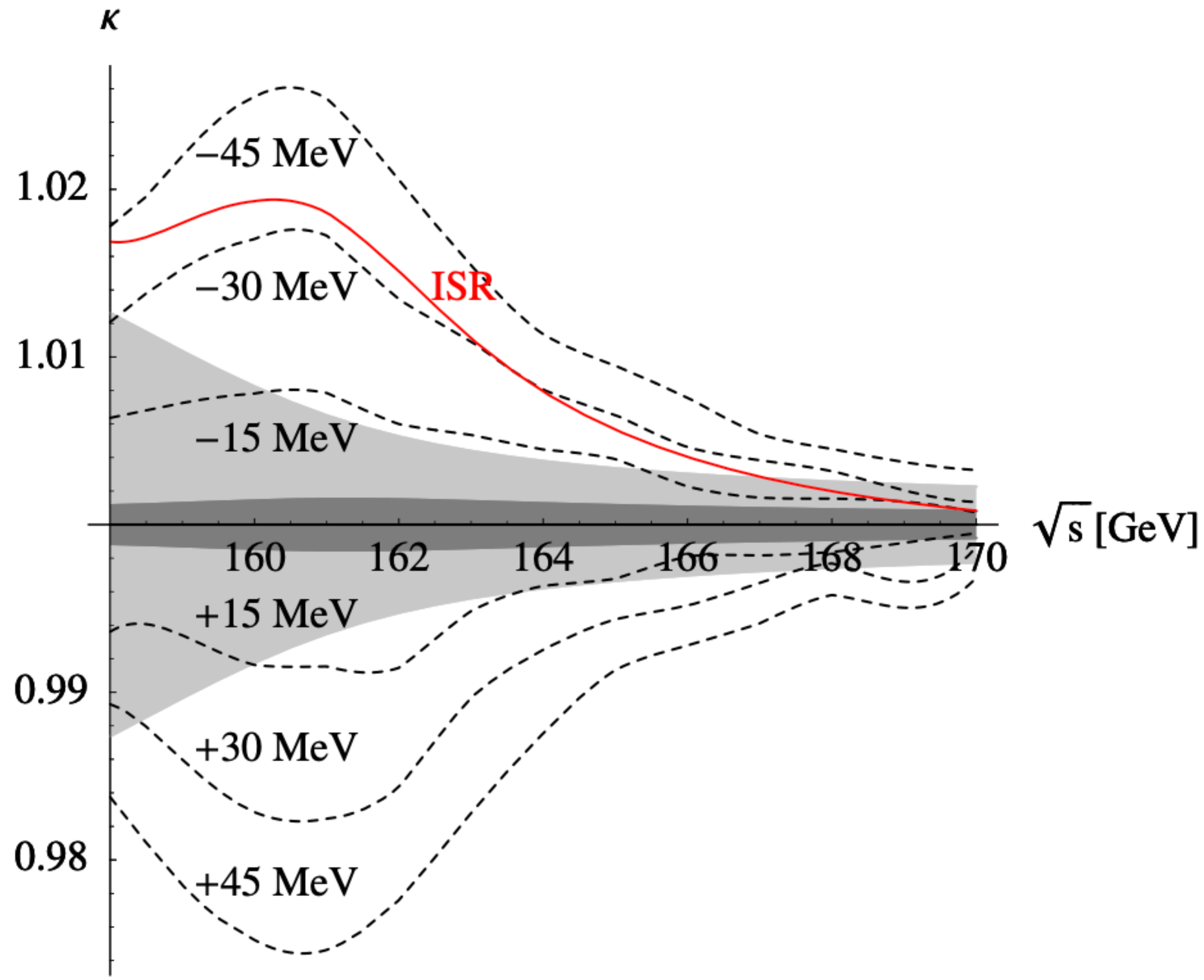
- ECFA Higgs-Top-EW Factory Report, 2506.15390
- FCC-ee Feasibility (FFS) Study, Vol. 1, Physics etc., 2505.00272
- LC Vision (LCF) Report, 2503.19983
- PPG Physics Briefing Book, 2511.03883
- PPG Physics Working Group Theory Uncertainty Spreadsheet (link)
- Snowmass 2021 EW Working Group Report, 2209.08078
- Theory requirements for SM Higgs and EW precision physics at FCC-ee, S. Heinemeyer, S. Jadach, JRR, 2106.11802
- Talk by Stefan Dittmaier, 2nd HTE Factory Workshop Paestum (link)
- Talk. C. Schwinn CEPC Workshop 2019 ... and many more ...

WW threshold — *W* mass

M_W sensitivity of σ_{WW}:

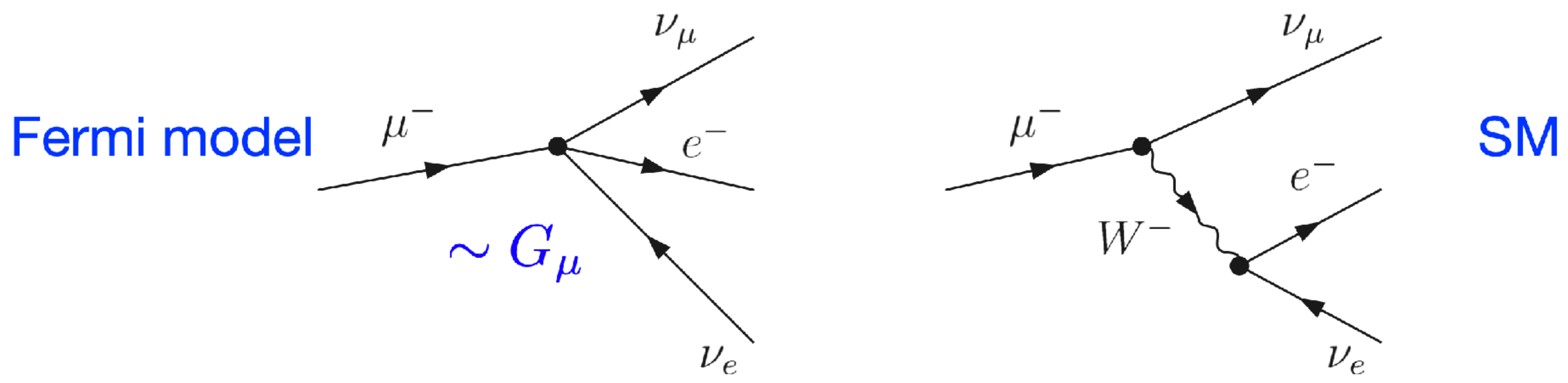
$$\kappa = \frac{\sigma(s, M_W + \delta M_W)}{\sigma(s, M_W)}$$

	Experimental accuracy				Theory uncertainty				
		σ_{WW} @ threshold				intrinsic		parametric	
	current	LEP2	LCF	FCC-ee	current	source	prospect	prospect	source
ΔM_W [MeV]	13.3	200	1.3-2.0	0.3	3	$\alpha^3, \alpha^2\alpha_s$	1	1(0.6)	$\Delta\alpha_{\text{had}}$



Beneke/Falgari/Schwinn/Signer/Zanderighi, 0707.0773

$\sqrt{s} = 161$ GeV : $\Delta\kappa = 0.1\,\%\,(0.01\%) \iff \delta M_W = 1.5\,(0.15)$ MeV



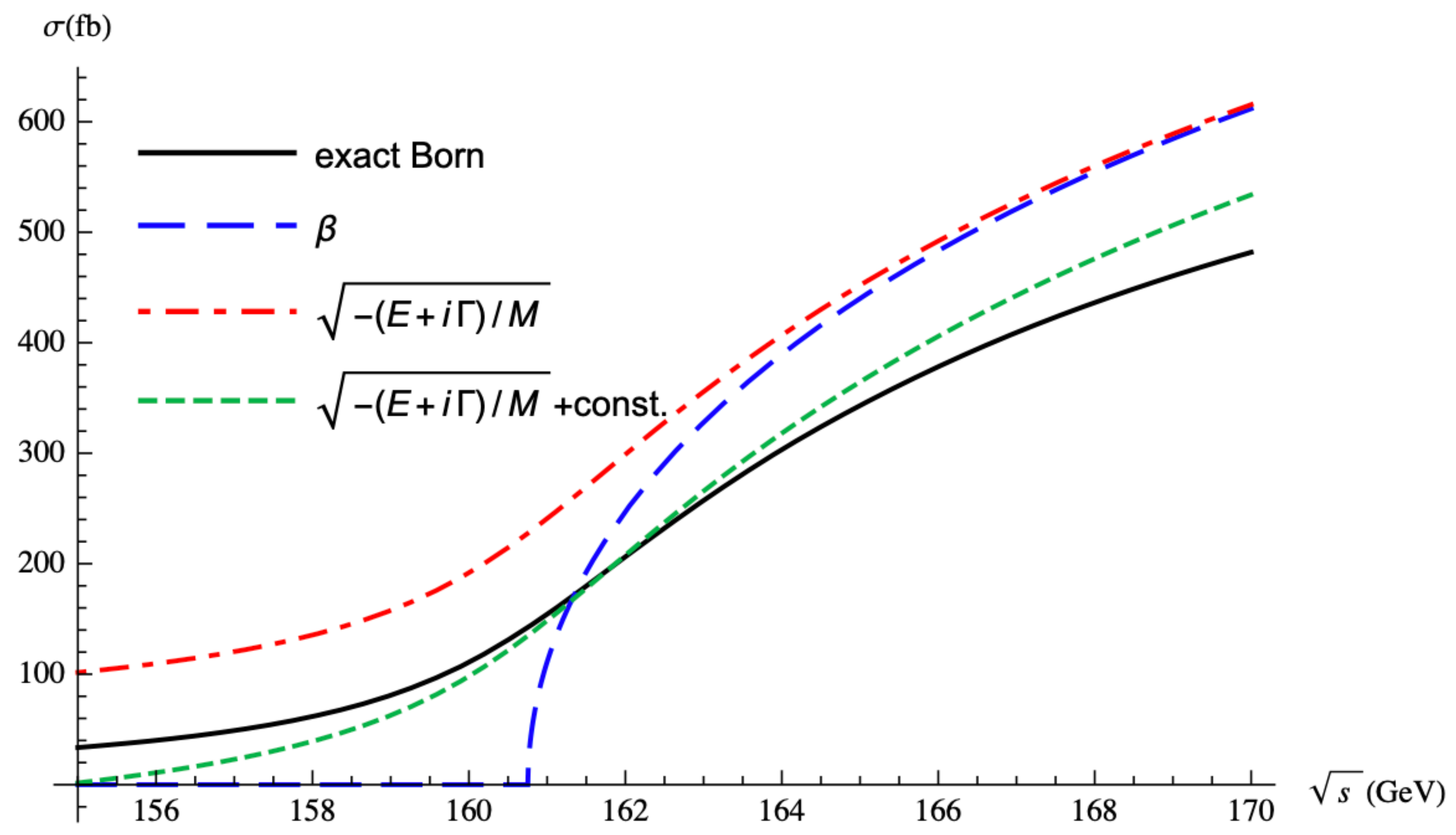
↪ cf. Talk by Giuseppe Degrassi

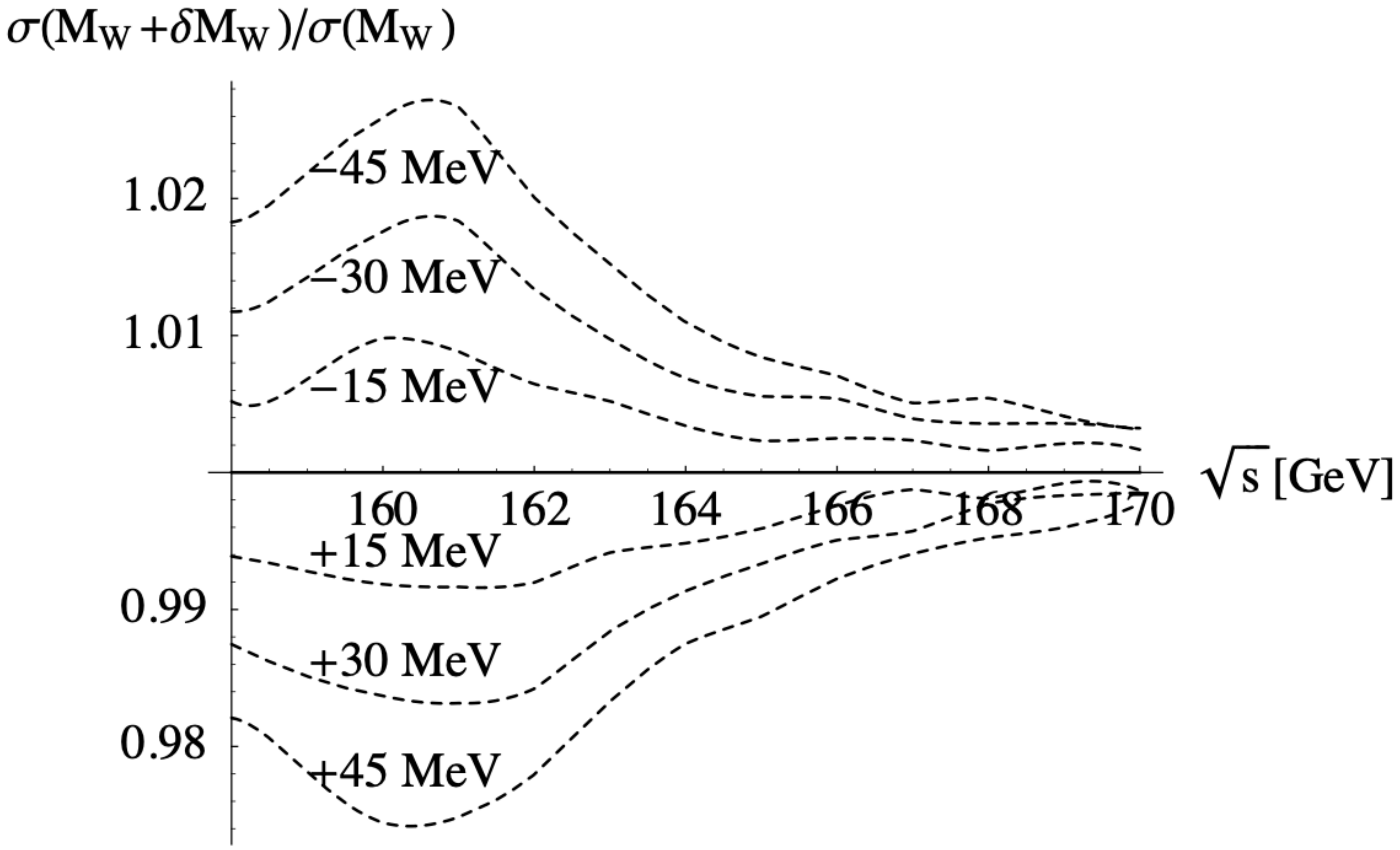
Kinematics at threshold

- Threshold dominated by *t* channel
- On-shell *WW* at threshold (or NWA): $\sigma_{WW} \sim \beta = \sqrt{1 - \frac{4M_W^2}{s}}$
- Considering the finite decay width $(E = \sqrt{s} - 2M_W)$, so eff.

$$\beta = \frac{1}{\sqrt{2}} \sqrt{\frac{E}{M_W} + \sqrt{\frac{E^2}{M_W^2} + \frac{\Gamma^2}{M_W^2}}}$$

- Higher order terms ($\sim \beta^3$) irrelevant at threshold

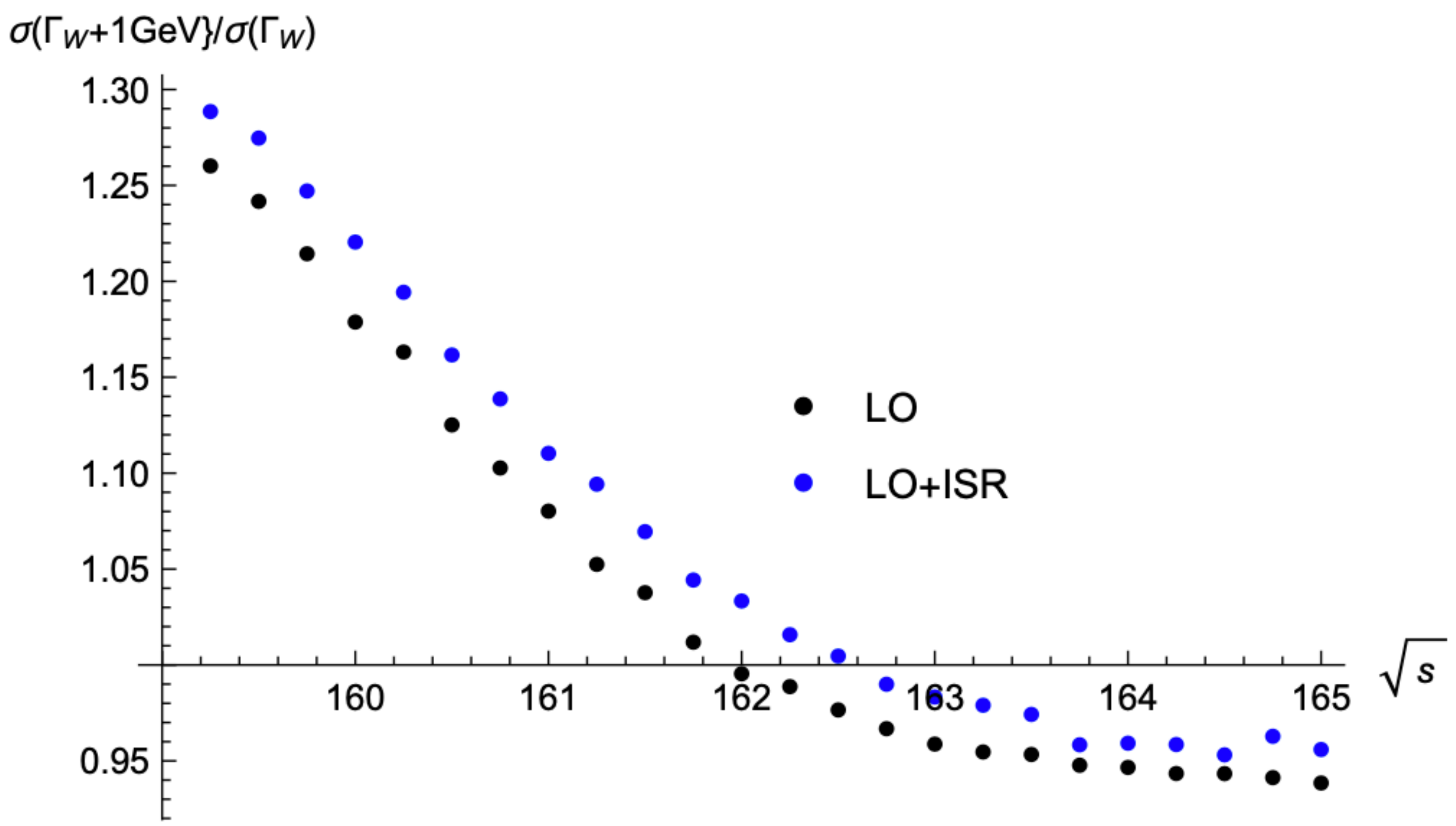




- Phase-space constraints and non-resonant contrib.: constant shift
- Direct sensitivity to M_W: $\Delta\sigma \sim 1\,\% \Leftrightarrow \Delta M_W \sim 15$ MeV
- Maximal dependence at $\approx \sqrt{s} = 161$ GeV

Off-Shell Process & W width

- Resonant part: $\sigma(e^+e^- \to 4f) \sim \Gamma^{\text{SM}}_{W\to f_1\bar{f}_2}\Gamma^{\text{SM}}_{W\to f_3\bar{f}_4}/\Gamma^2_W$
- BSM contributions: usually rescale by $(\Gamma_W/\Gamma_W^{\text{SM}})^2$ to keep BR$(W \to f\bar{f})$ const.
- Γ_W sensitivity below threshold: $\Delta\Gamma_W = 10\,\text{MeV} \Leftrightarrow \Delta\sigma \sim 4\,‰$
- $d\sigma/d\Gamma_W = 0$ due to interplay of β dependent and constant terms
- ISR effect to width dependence: 500 MeV (!)



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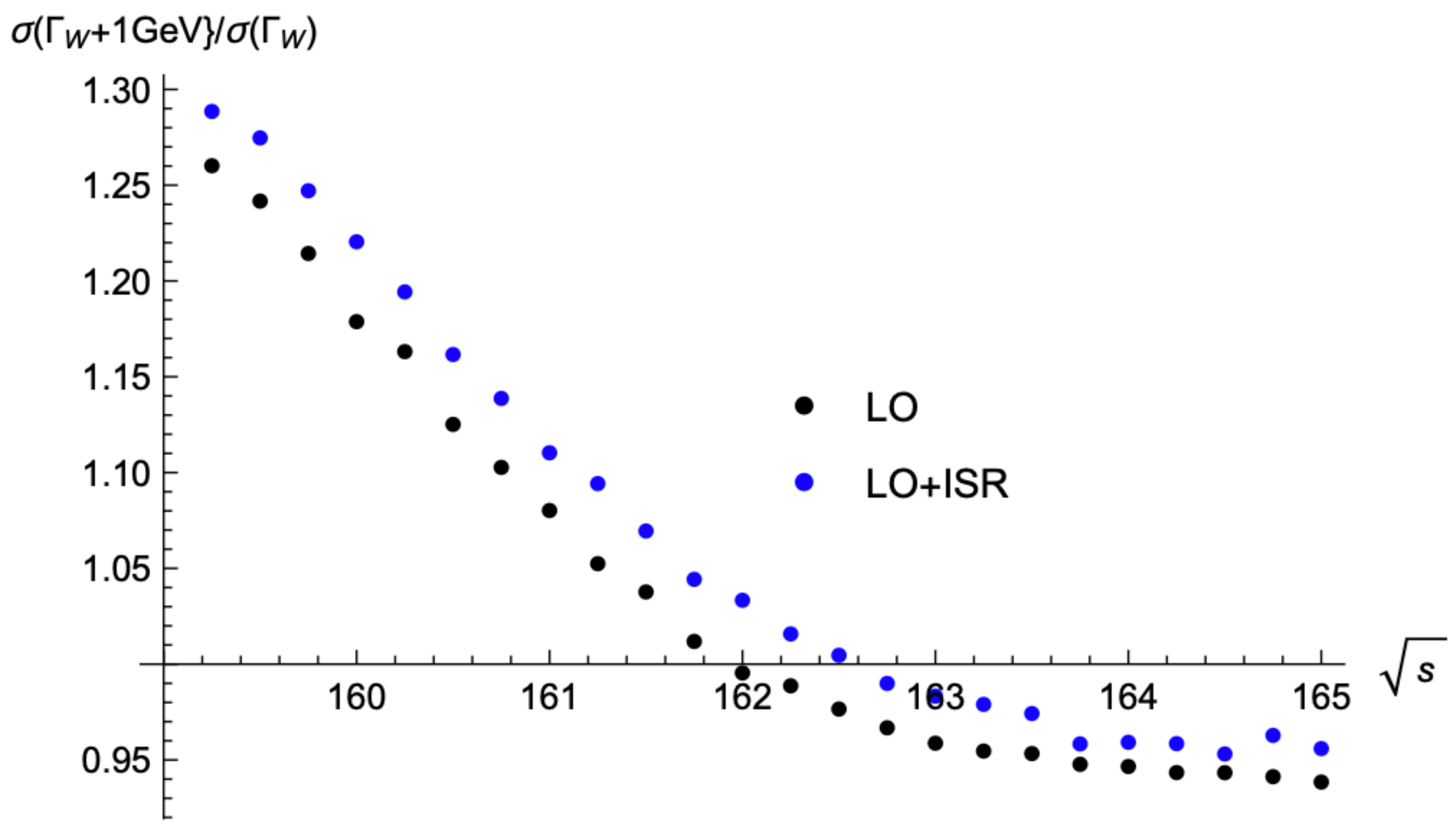
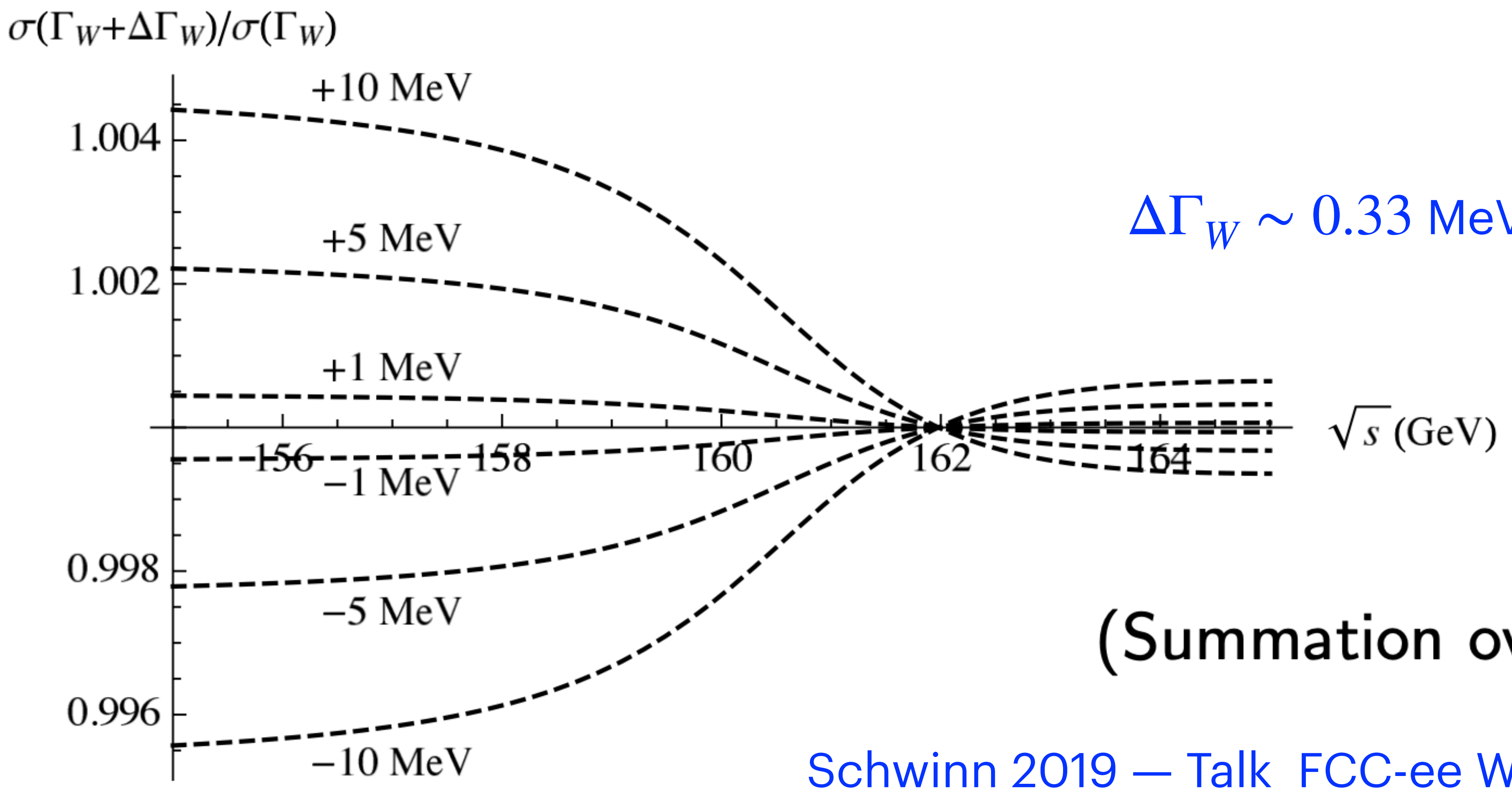




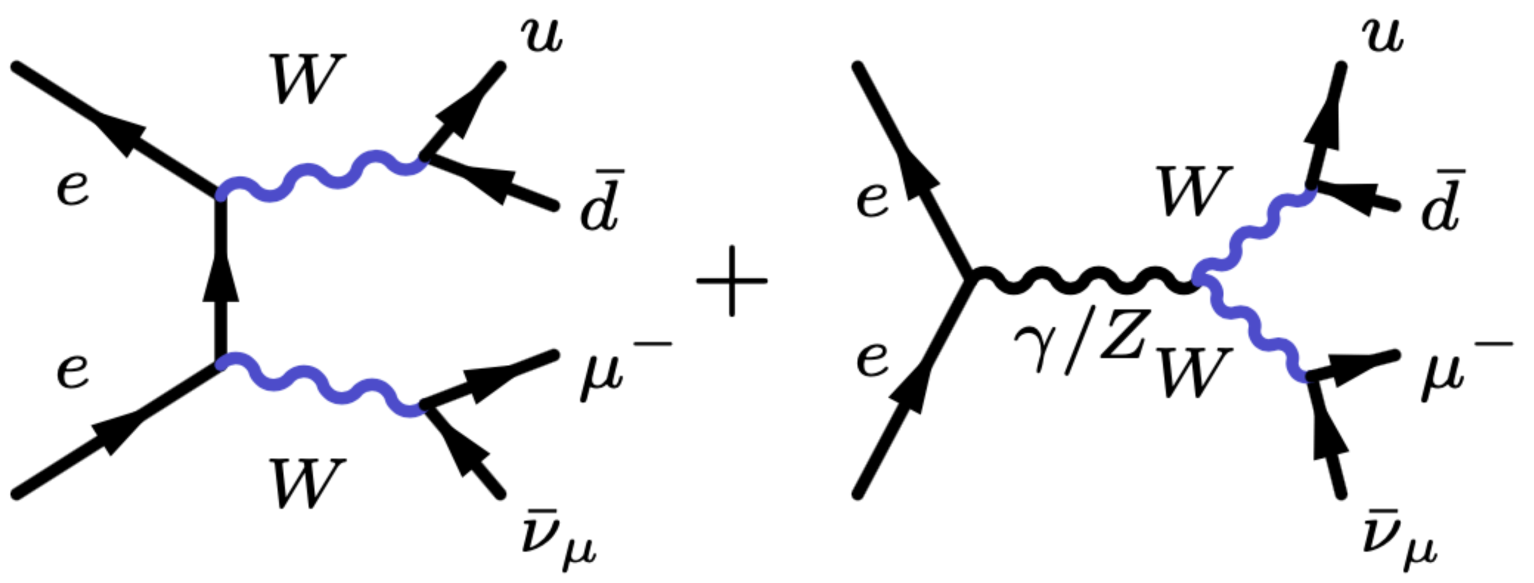
$\Delta\Gamma_W \sim 0.33$ MeV @ FCC-ee stat. + sys.

(Summation over all $4f$ states)

Schwinn 2019 — Talk FCC-ee Workshop CERN

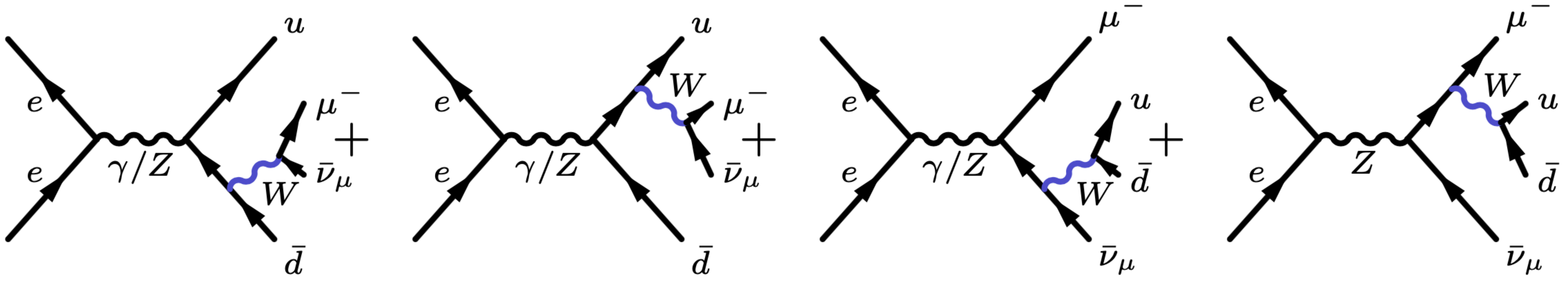
Off-Shell Process & NLO corrections

- Need to consider full 4-fermion process: $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}, \qquad f_1\bar{f}_2 f_3\bar{f}_4$



- Double-resonant ("signal") process: "CC3"
- Only full process ("CC10" @ tree-level) gauge-invariant:

CC3 +



- Fully hadronic [leptonic] gets contribution from $e^+e^- \to \gamma\gamma, \gamma Z, ZZ, ZH$ and QCD backgrounds
- Consistent EW scheme for W width needed [cf. Beenakker et al., 1996ff]

$\sqrt{s}$	200 GeV	500 GeV	1 TeV	5 TeV
Running width	672.96	225.45	62.17	123.76
Constant width	673.08	224.05	56.90	2.212

Off-Shell Process & NLO corrections

cf. also Denner/Dittmaier, 1912.06823

- Improved Born Approximation (IBA): Leading-Log ISR + universal EW corrections $\Delta \sim 2\,\%$ (e.g. in GENTLE)

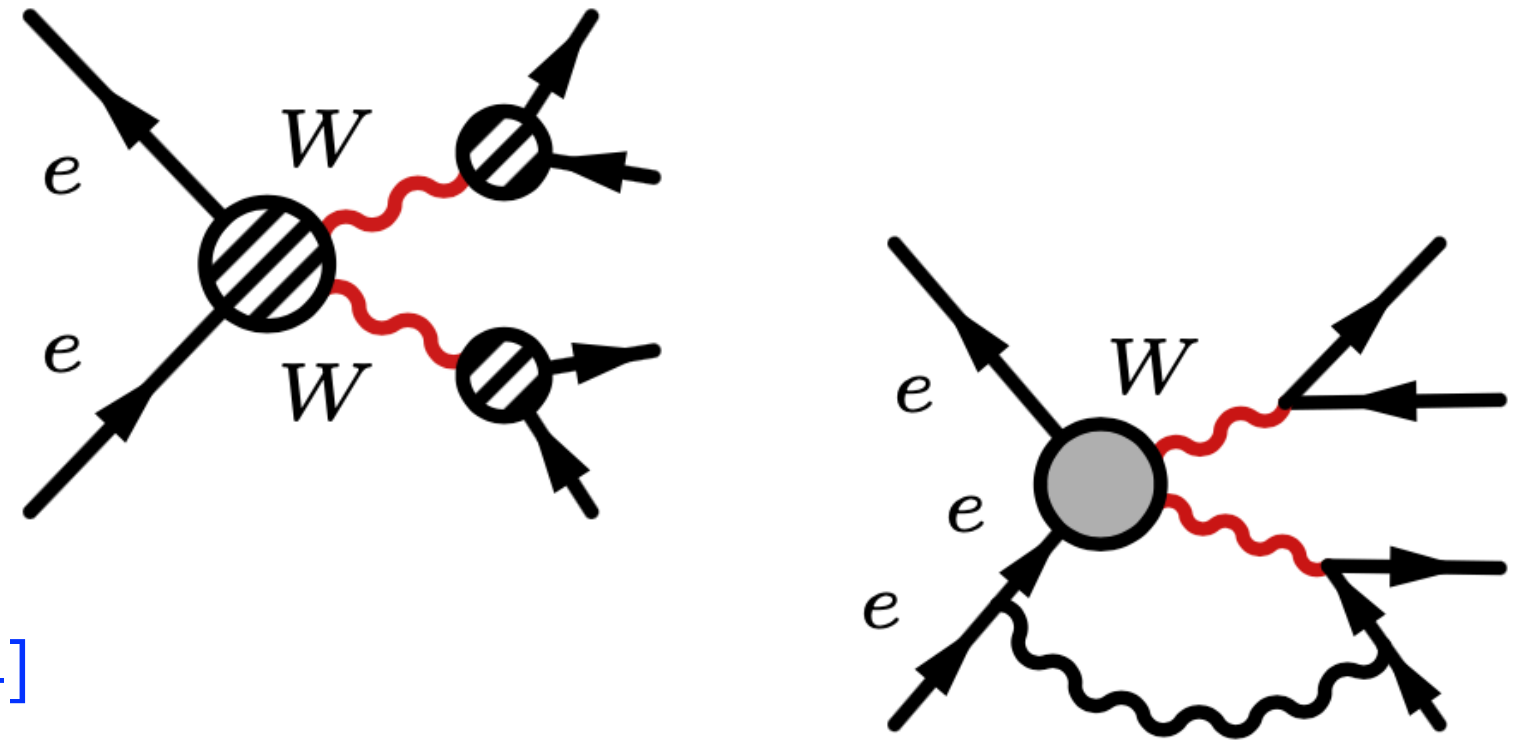
- Double Pole Approx. (DPA) (`RacoonWW/YFSWW`) resonance expansion leading term: $\Delta \sim 0.5\,\%$ above threshold (✗ threshold)

  - Factorizable EW corrections, decay of on-shell *W*s

    FCC-ee energies: $\Delta\sigma_{\text{DPA}} \propto \frac{\Gamma_W}{M_W} \sim 0.3 - 0.5\,\% \longrightarrow \frac{\Gamma_W}{\sqrt{s} - 2M_W} \sim 2 - 3\,\%$

  - Nonfactorizable soft photon corrections [Berends et al., 1998; Denner et al., 1999]

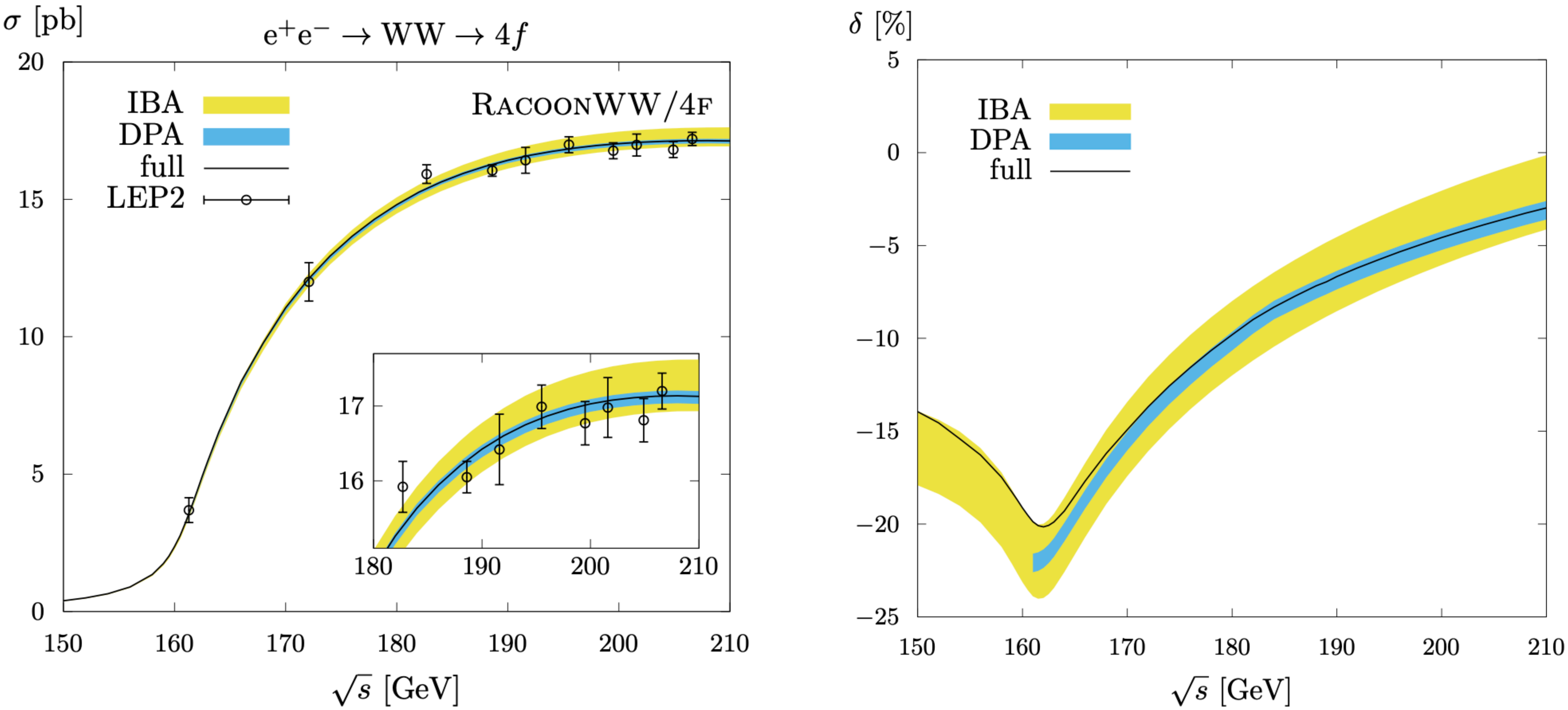
    [vanish for total cross section: [Fadin/Khoze/Martin; Melnikov/Yakovlev, 1994]



- "Full" NLO calculation $e^+e^- \to 4f$ (charged current) NLO + ISR LL [Denner/Dittmaier/Roth/Wieders, 2005]

  - $\Delta \sim 0.5\,\%$ full kinematic range, 1000 1-loop diagrams:
  - Pioneering for pentagons/hexagons, today (fully) automated `GoSam/MadLoop/Openloops/Recola`...
  - Development of complex mass scheme (CMS)

Off-Shell Process & NLO corrections



EFT and WW threshold improvement

- At threshold EW EFT expansion of σ_{WW} in 3 parameters with power counting: $\sqrt{\alpha} \sim \sqrt{\Gamma_W/M_W} \sim \beta_W = \sqrt{1 - 4M_W^2/s}$

- Schematic structure of total cross section:

$\mathrm{NLO}^{\mathrm{EFT}}$:	v^2	α	α^2/v^2		
$\mathrm{N}^{3/2}\mathrm{LO}^{\mathrm{EFT}}$:	αv	α^2/v	α^3/v^3		
$\mathrm{NNLO}^{\mathrm{EFT}}$:	v^4	αv^2	α^2	α^3/v^2	α^4/v^4

Beneke/Falgari/Schwinn/Signer/Zanderighi, 0707.0773

$$\sigma_{WW} = C\alpha^2\beta\Big[\,1 + c^{(0)}\beta \qquad \text{LO}$$
$$+\,\alpha\left(\frac{c_1^{(1)}}{\beta} + c_2^{(1)}\ln\beta L_e + c_3^{(1)} L_e + c_4^{(1)} + c_5^{(1)}\beta\right) \qquad \text{NLO}$$
$$+\,\alpha^2\underbrace{\left(\frac{c_1^{(2)}}{\beta^2} + \frac{c_2^{(2)}}{\beta} + c_3^{(2)}\ln^2\beta L_e^2 + c_4^{(2)}\ln\beta L_e^2 + \ldots\right)}_{\text{leading NNLO parts known}} + \ldots\Big] \qquad \text{NNLO}$$

NNLO ↓ required for FCC-ee

- Systematically possible to improve cross section precision with EFT, but limited to near-threshold region

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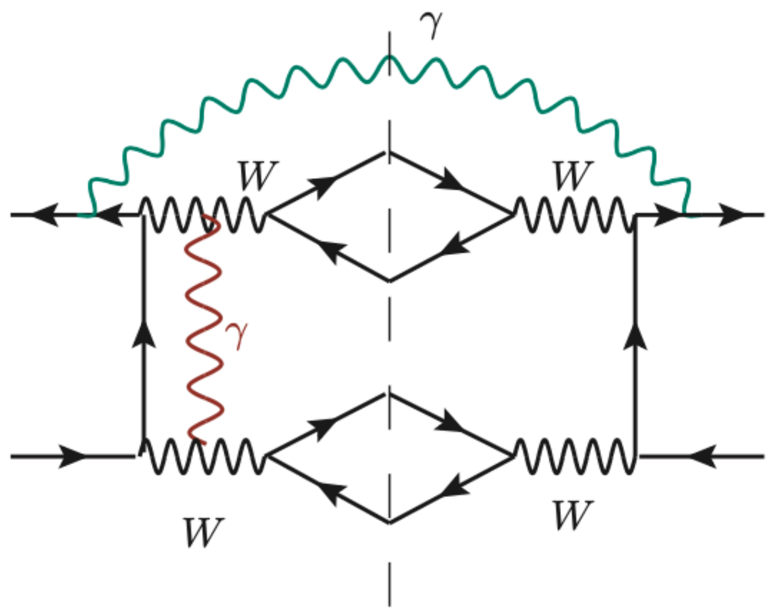
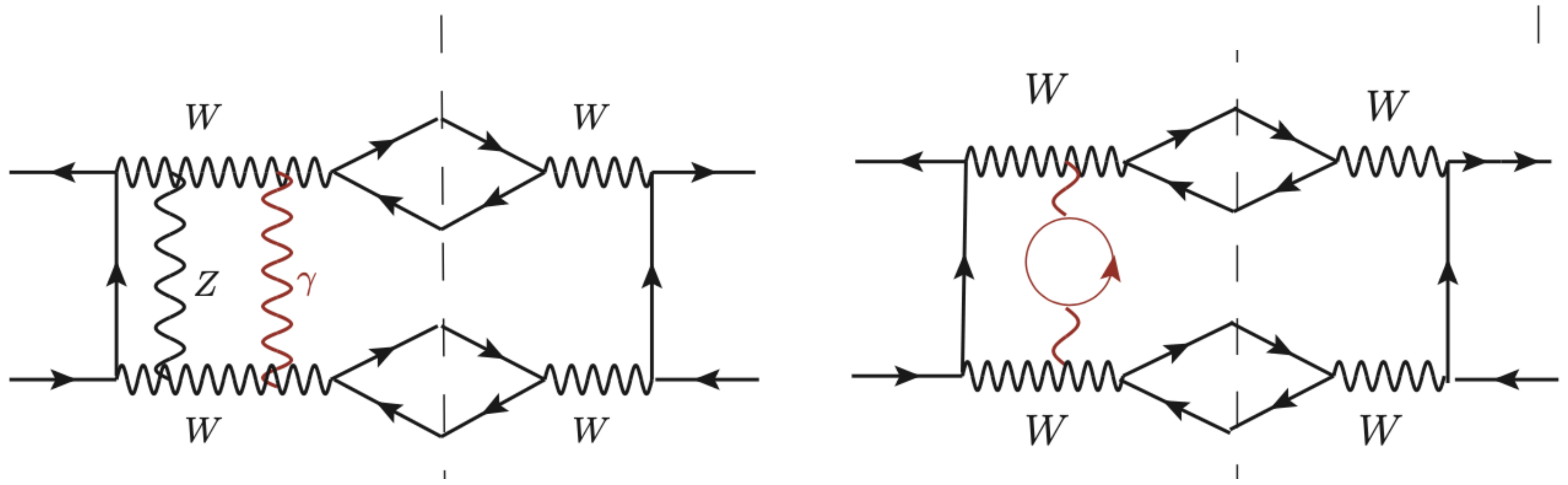
- Mass logarithms: $L_e = \log\left(m_e/M_W\right)$ resummed in "ISR structure function" (lepton PDF) [cf. talk Alan Price]
- Coulomb corrections α/β: enhanced, resummation [maybe] not necessary (screened by finite width): [Fadin et al., 1995]
  tower of $\alpha^n (M_W/\Gamma_W)^{n/2} \sim \alpha^{n/2}$ [method for all-order resummation known from $t\bar{t}$]
- Soft $\log\beta$ corrections $\sim \alpha\log\alpha \sim 0.04$: resummation (likely) not necessary

WW threshold predictions



- 2nd Coulomb correction: $\alpha^2/\beta^2 \sim \alpha$ [Fadin et al., 1995]
- Coulomb-enhanced: $\alpha^2/\beta \sim \alpha^{3/2}$ [Actis et al., 2008]



- Numerical effect: $\Delta\sigma_{WW} \sim 5$ ‰; [$\Delta M_W \lesssim 3$ MeV]

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)			
$\sqrt{s}$ [GeV]	NLO_{EFT}	NLO_{ee4f} [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7‰)	0.15 (1.3‰)
170	397.8	404.5	0.25 (0.6‰)	1.6 (3.9‰)

Beneke/Falgari/Schwinn/Signer/Zanderighi, 0707.0773
Actis/Beneke/Falgari/Schwinn, 0807.0102

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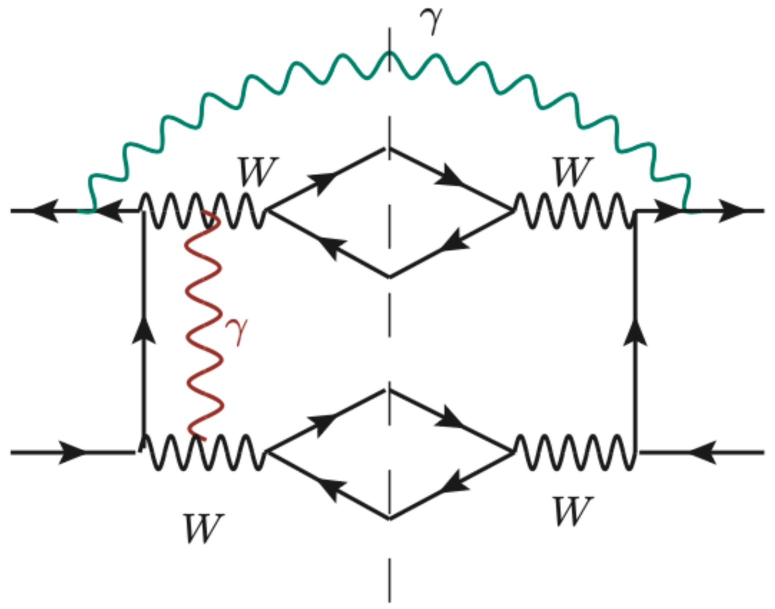

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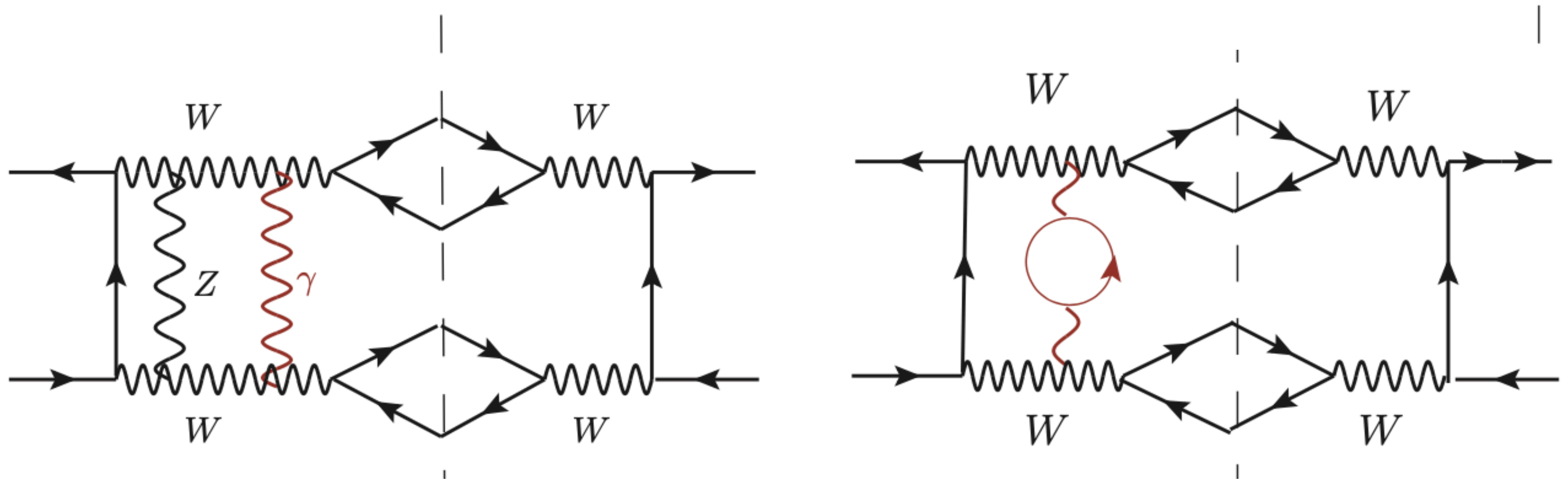

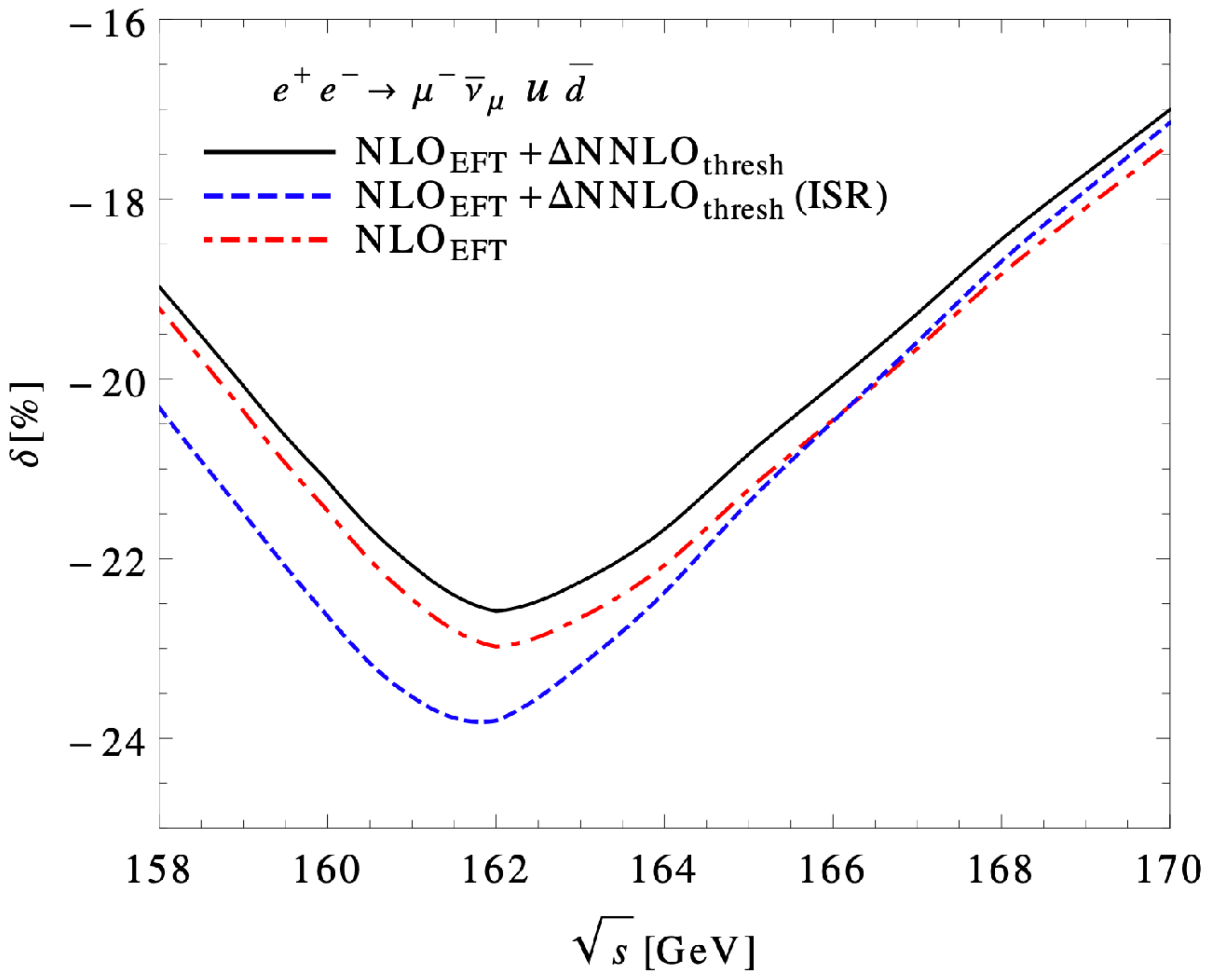

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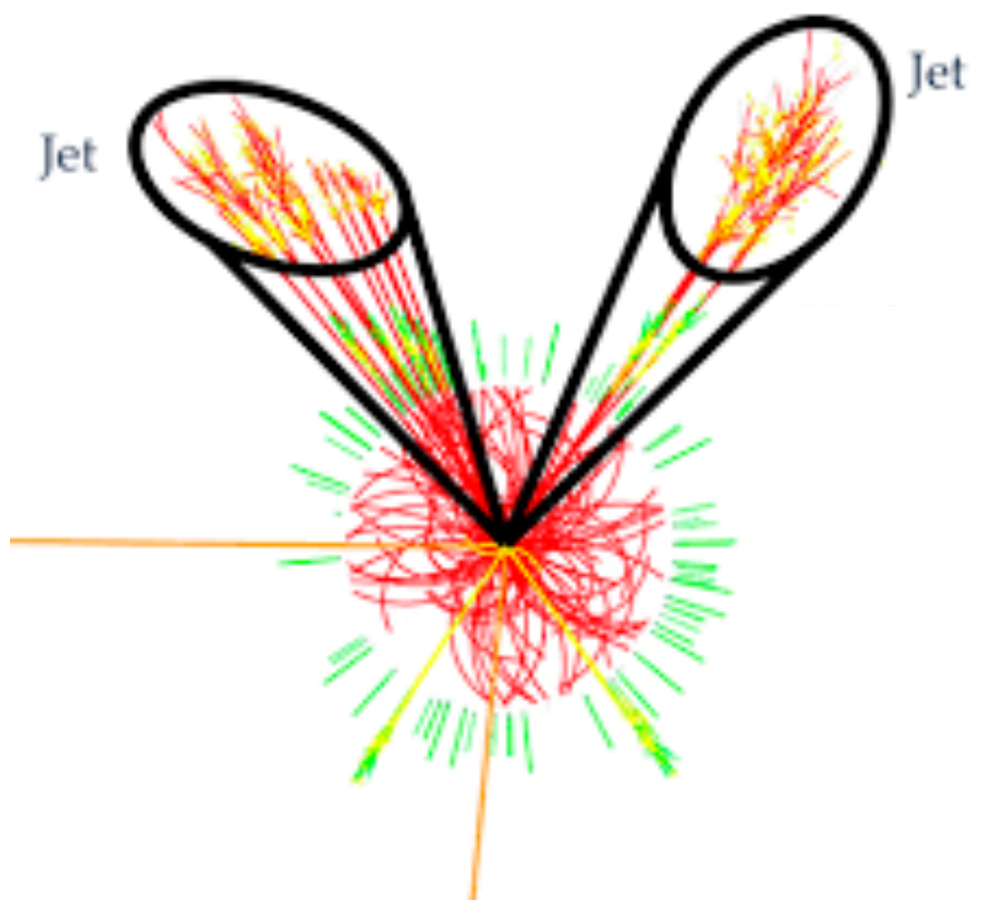
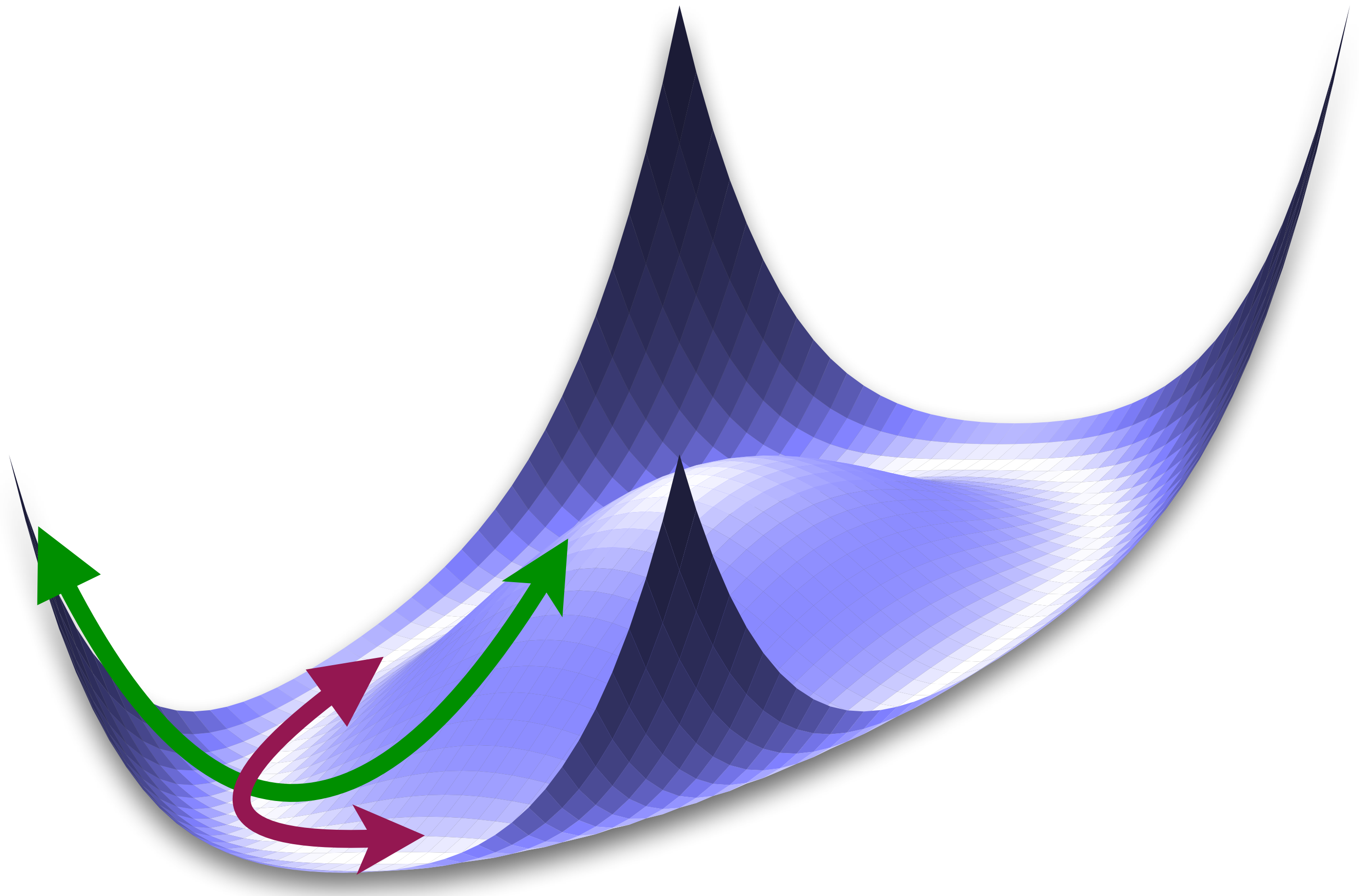
ISR logarithm: $L = \ln(m_e/M_W)$

Need for resummation of
LL $(\alpha L)^n$ and NLL $\alpha\,(\alpha L)^{n-1}$

The mystery of electroweak interactions

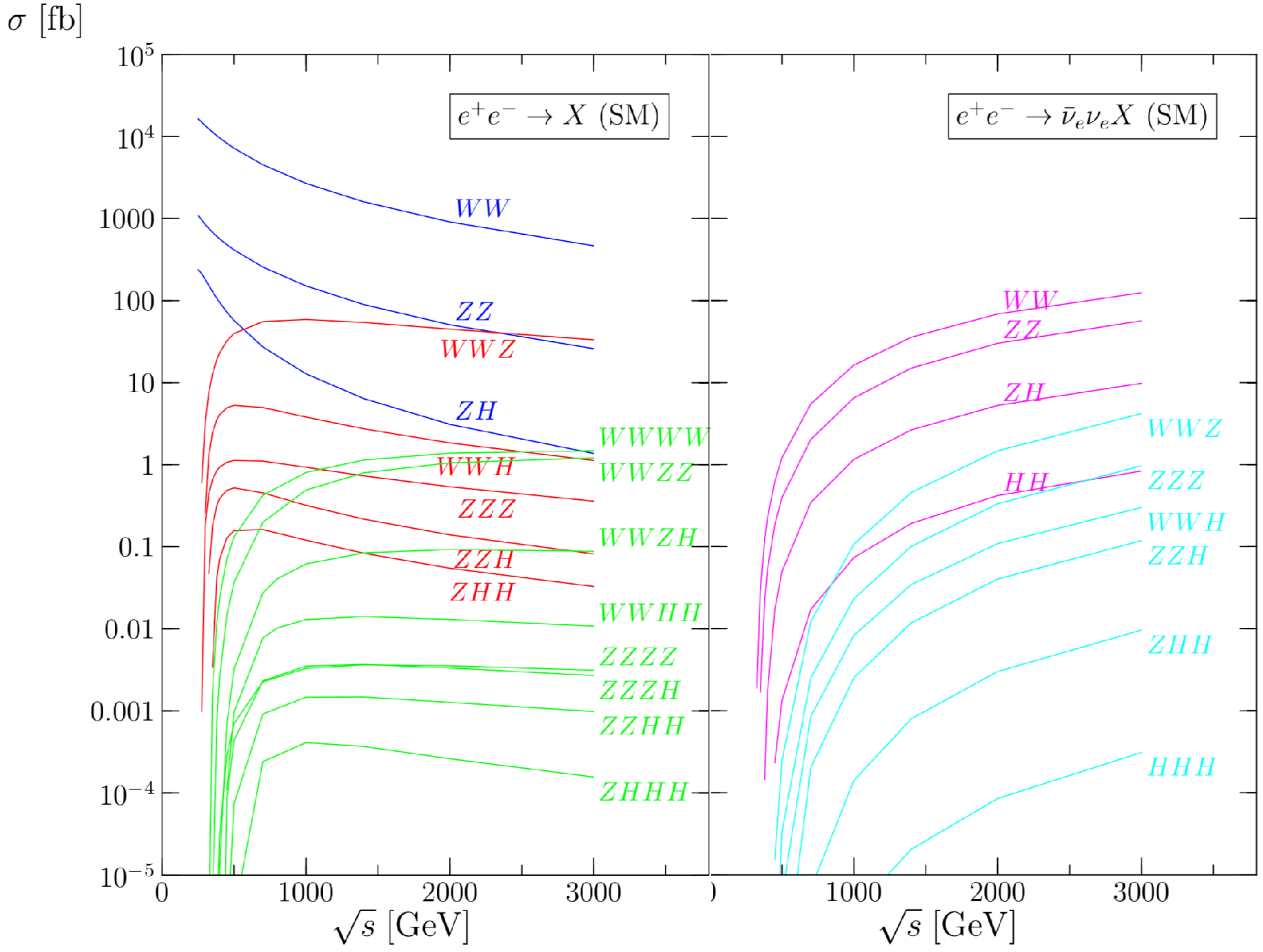


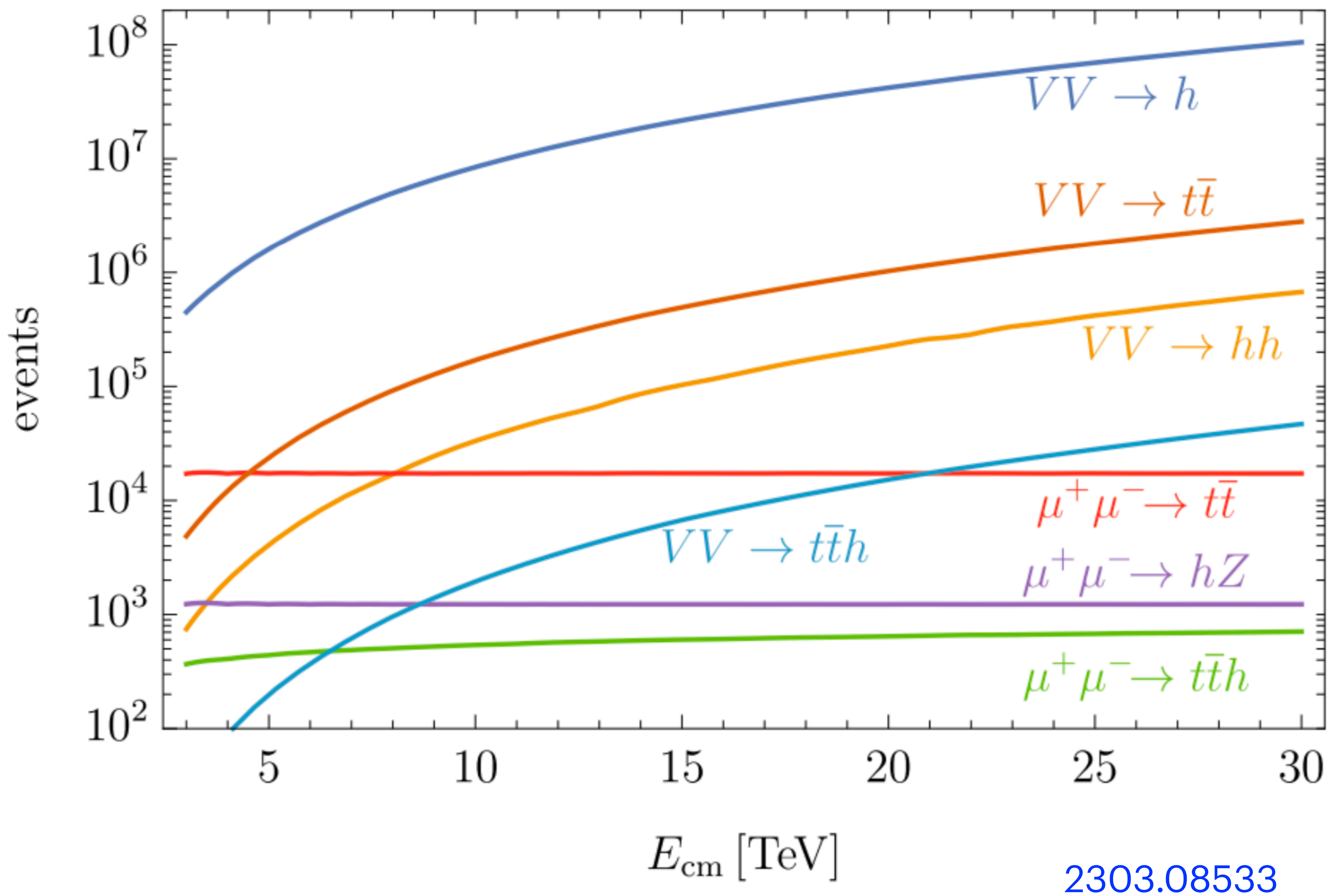


- Microscopic origin of EWSB not yet understood
- EW interactions in unbroken phase (almost) untested: EW jets, EW Sudakov logs, EW splittings
- Mostly tested at the level of “pion scattering”   (1.0-1.5 TeV EW ≙ ca. 1-1.5 GeV in QCD)
- EW $SU(2)_L$ conceptually different than QCD $SU(3)_c$:   (broken vs. unbroken) asymptotically free
- EW precision measurements do not only reside in the (*Z* and *WW*) pole regions
- EW “jet” regime starts becoming relevant at a 10 TeV parton-level CM energy collider

Vector-boson fusion (VBF) vs. prompt production

VBS beats multi-boson at high energies, e.g.
from 1812.02093 & Brass/Kilian/JRR/Shim



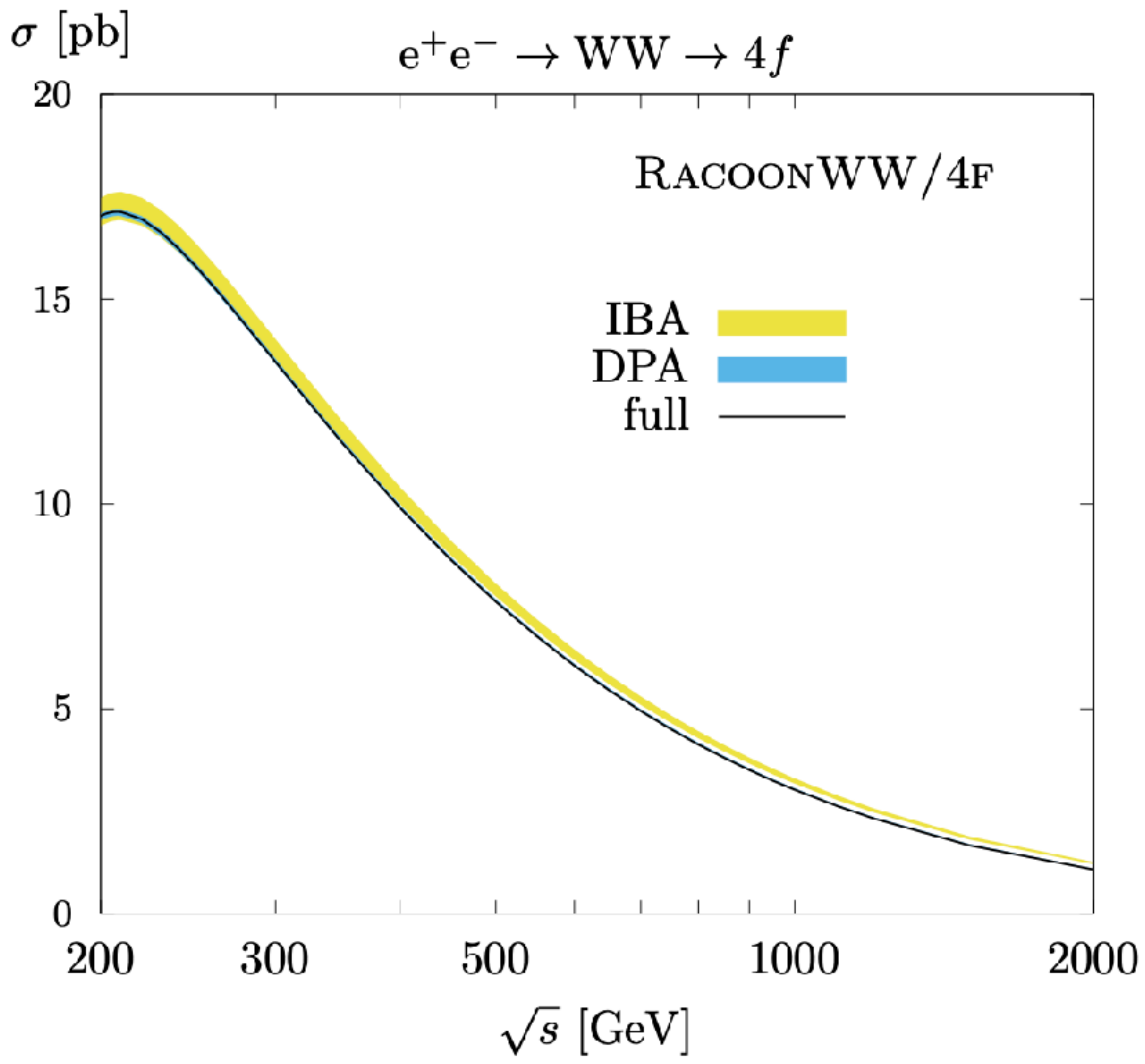


2303.08533

$$\frac{\sigma^{\rm SM}_{\rm VBF}}{\sigma^{\rm SM}_{\rm ann}} \propto \alpha_W^2 \frac{s}{m_V^2} \log^3 \frac{s}{m_V^2}$$

2005.10289

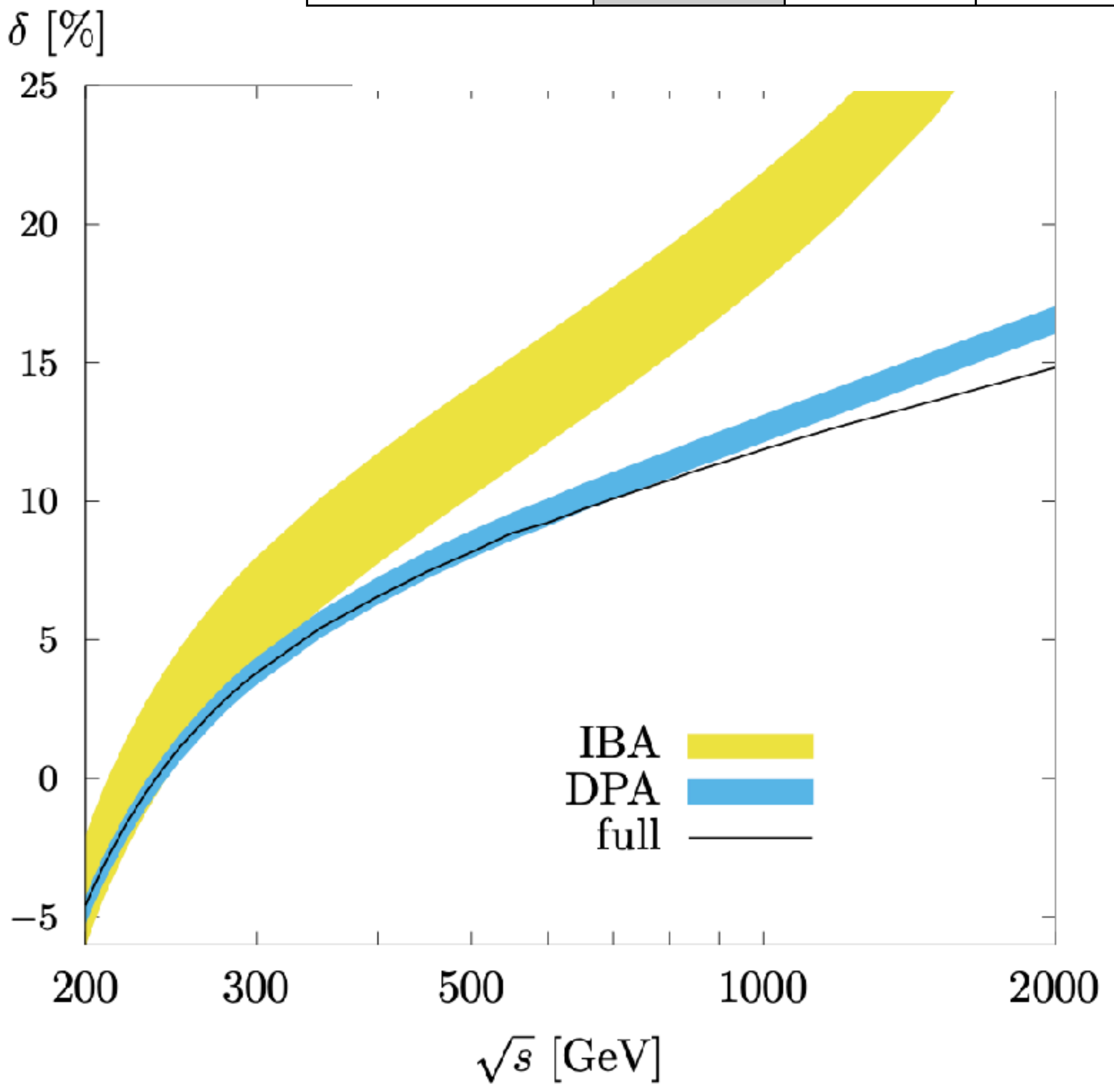
EW corrections at high energies



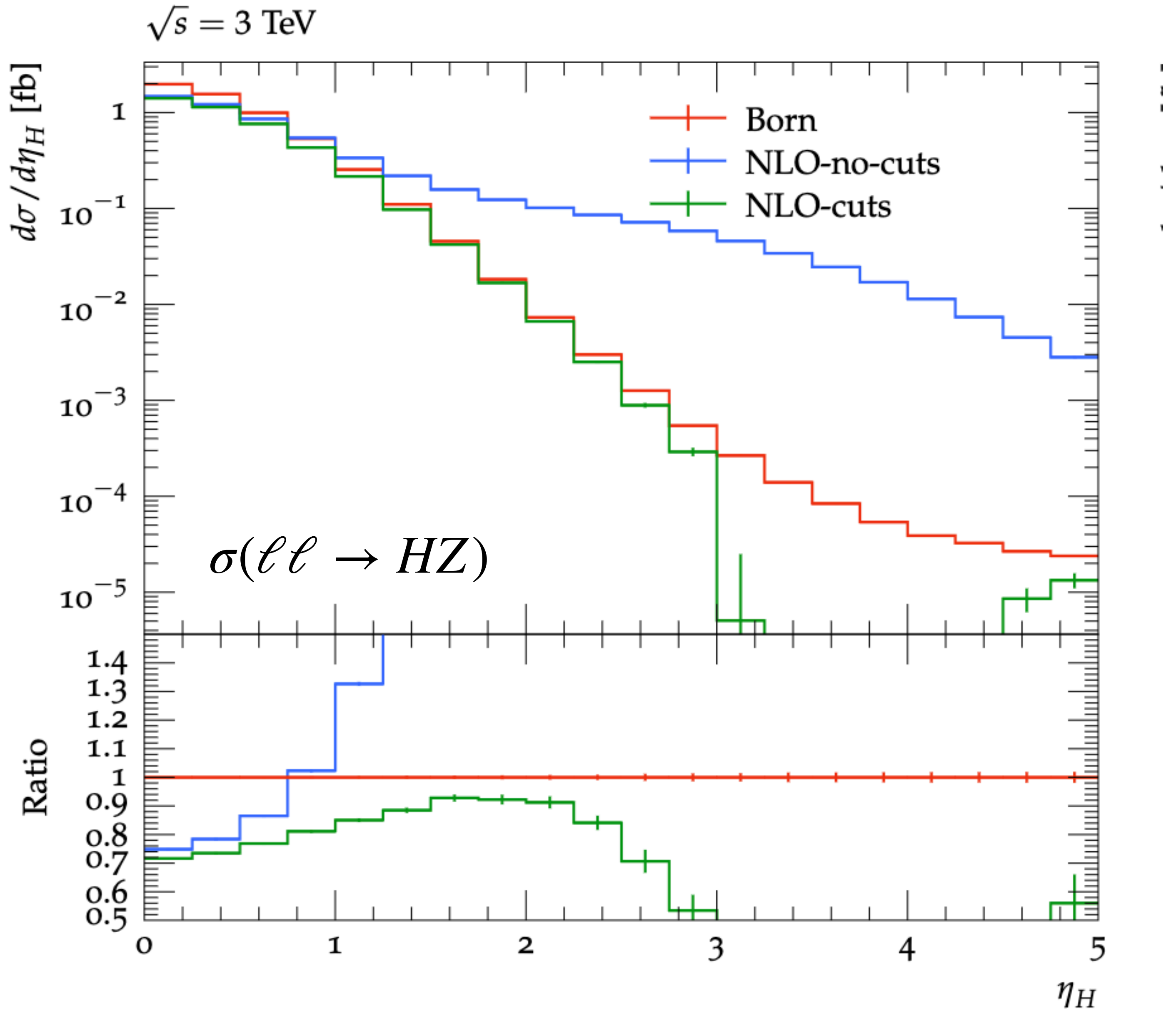
Denner/Dittmaier, 1912.06823

$\sqrt{s}$ [GeV]	$\sigma^{\rm Born}$	$\sigma^{\rm Born}_-$	$\sigma^{\rm Born}_+$	$\sigma^{\rm Born}_T$	$\sigma^{\rm Born}_L$	$\sigma^{\rm Born}_M$
165	10.66	21.27	0.05	5.19	1.34	4.13
176	16.47	32.74	0.21	8.84	1.79	5.84
190	18.12	35.92	0.32	10.76	1.65	5.70
250	15.40	30.52	0.27	11.77	0.75	2.87
500	6.71	13.37	0.05	6.34	0.11	0.25
1000	2.51	5.00	0.01	2.46	0.03	0.02

$\sigma(e^+e^-_\sigma \to W^+_\kappa W^-_\lambda)$



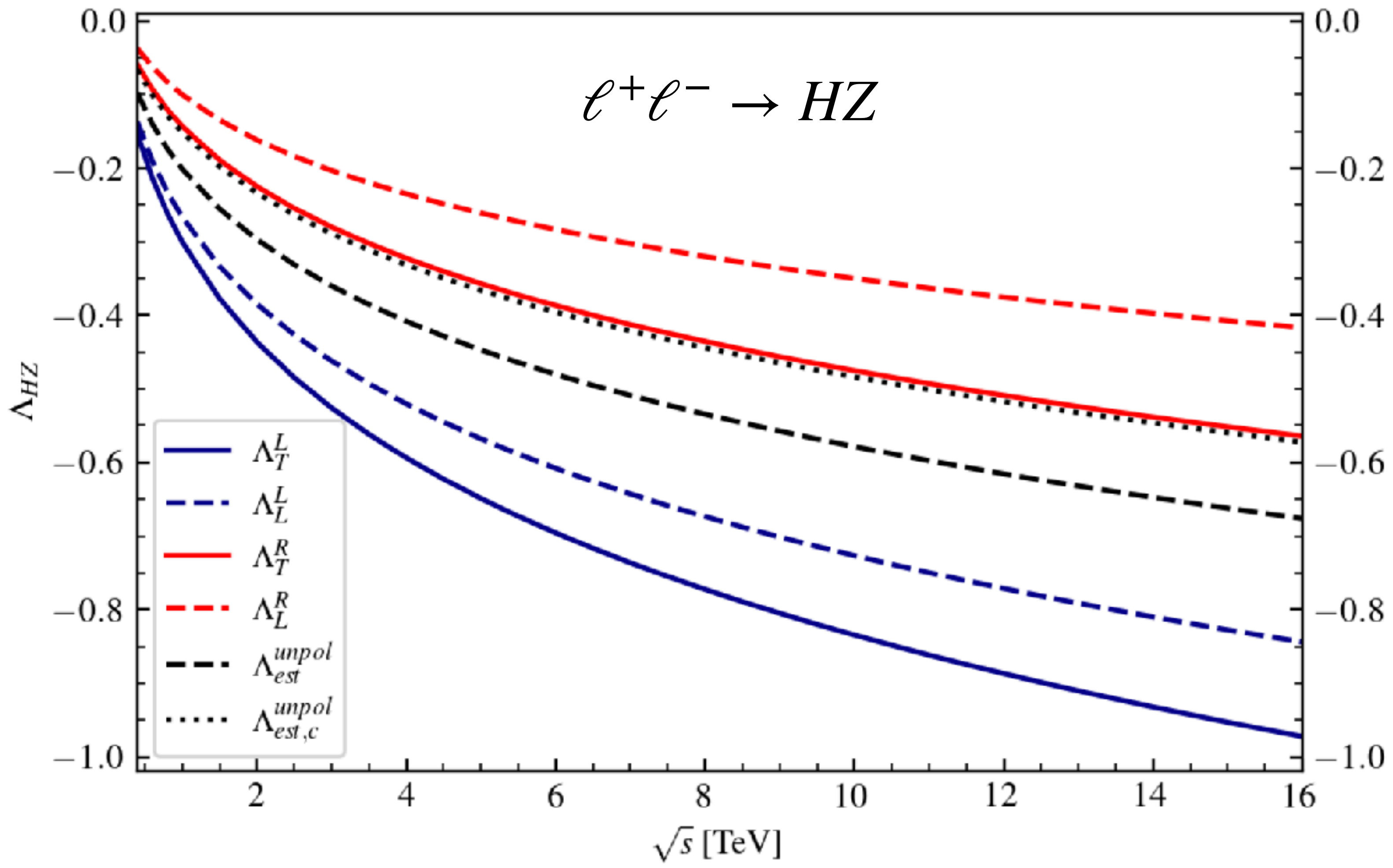
EW corrections at high energies



Exclusive vs. inclusive corrections

Bredt/Kilian/JRR/Stienemeier, 2208.09438

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$4.6591(2) \cdot 10^2$	$4.847(7) \cdot 10^2$	$+4.0(2)$
ZZ	$2.5988(1) \cdot 10^1$	$2.656(2) \cdot 10^1$	$+2.19(6)$
HZ	$1.3719(1) \cdot 10^0$	$1.3512(5) \cdot 10^0$	$-1.51(4)$
HH	$1.60216(7) \cdot 10^{-7}$	$5.66(1) \cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2) \cdot 10^1$	$2.568(8) \cdot 10^1$	$-22.9(2)$
W^+W^-H	$1.1253(5) \cdot 10^0$	$0.895(2) \cdot 10^0$	$-20.5(2)$
ZZZ	$3.598(2) \cdot 10^{-1}$	$2.68(1) \cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4) \cdot 10^{-2}$	$6.60(3) \cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1) \cdot 10^{-2}$	$2.451(5) \cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6) \cdot 10^{-8}$	$0.86(7) \cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$0.993(6) \cdot 10^0$	$-33.1(4)$
W^+W^-ZZ	$1.209(1) \cdot 10^0$	$0.699(7) \cdot 10^0$	$-42.2(6)$
W^+W^-HZ	$8.754(8) \cdot 10^{-2}$	$6.05(4) \cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1) \cdot 10^{-2}$	$0.655(5) \cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$1.799(7) \cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$1.766(6) \cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	$-25.7(2)$



Large EW Sudakov logarithms

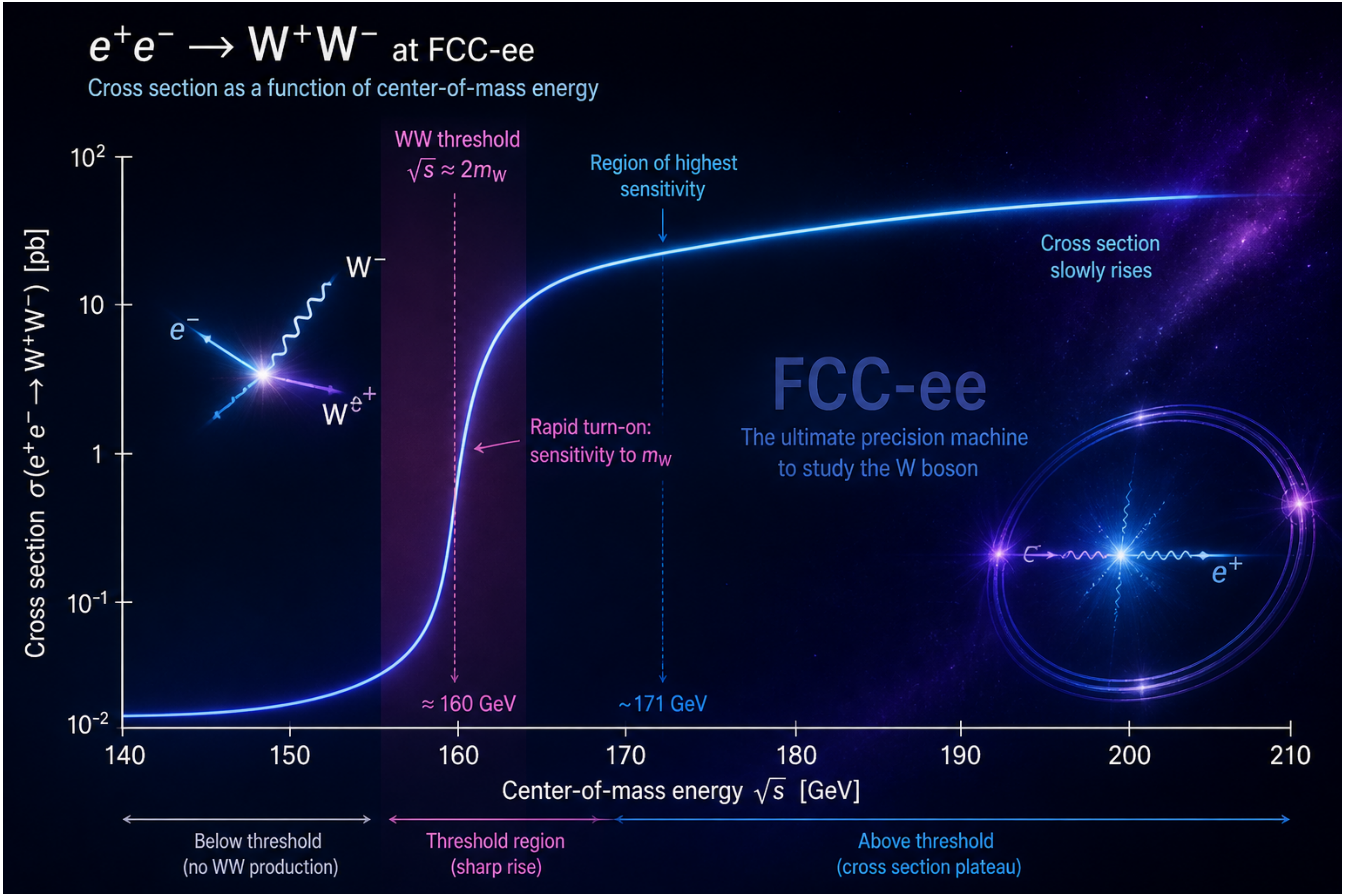
Automation of EW Sudakov logarithms:

Bothmann / Napoletano, 2006.14635;
Pagani/Zaro, 2110.03714

Conclusions & Outlook

- Huge experimental leap in electroweak sector at FCC-ee (or alternative Higgs factory)
- Electroweak (pseudo-) observables will improve by factor of few up to three orders of magnitude!
- Ultimate stress test of the SM with the aim to go beyond [ἐντελέχεια, not ἀταραξία]
- $e^+e^- \to W^+W^-$: need to go at $e^+e^- \to 4f$ level (double-pole, CMS, EW/width scheme)
- *WW* threshold: ultimate precision of *W* mass and width, full NNLO EFT and leading NNNLO needed
- Theoretical problems: EFT calculation, matching to SM, NLL ISR, soft resummation, color reconnection, BEC etc.
- Comparison to *W* mass/G_F calculation from μ decay: needed at full 3-loop and leading 4-loop level
- High energies: Sudakov regime, EW symmetry restoration, EW evolution
- Major challenges: Life will be interesting for theorists

  [uncovered: matching to shower/hadronization, polarized particles, spin correlations, ...]



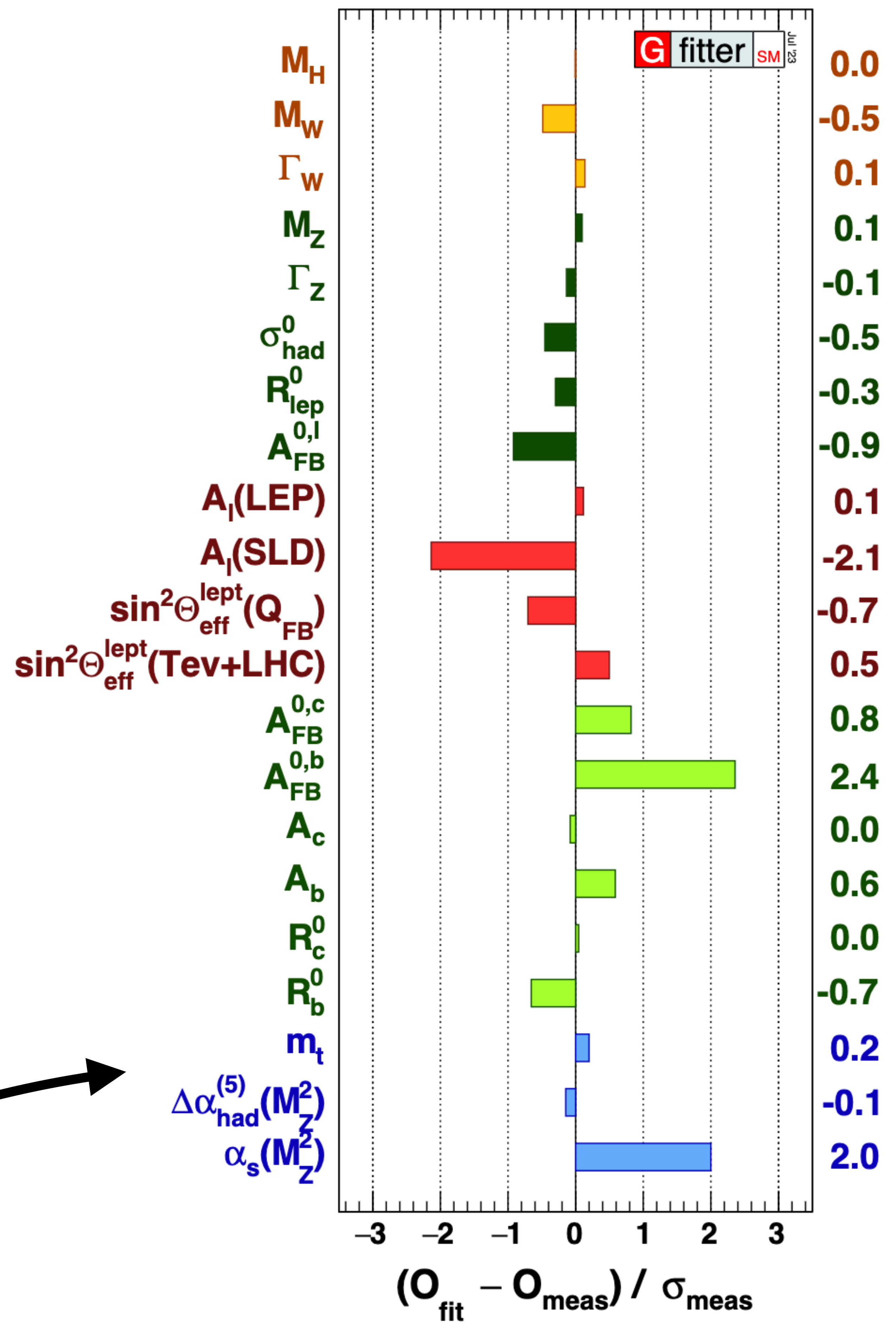
Find the error ...

chatgpt.com

BACKUP

Electroweak Pseudo-Observables

- Experimental observables: cross sections, differential distributions, branching ratios
- Relate this to theory predictions (“pseudo observables”) via deconvolution/unfolding
- Simplest pseudo-observables analytically accessible
- Otherwise: Comparison of Monte Carlo simulation with data
- Different schemes for resonances (Breit-Wigner running vs. constant width etc.)
- Different EW parameter schemes (connected to parametric / scheme uncertainties)
- So far: assumption of (EW) precision program at FCC-ee using EWPO
- Different paradigms: More LHC-like, cf. e.g. S. Frixione, FCC workshop Annecy 2024



EWPO include state-of-the-art theory calculations (cf. later):

Y. Fischer, 2023

Theoretical Uncertainties

- Comparison of EWPOs / HPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs / HPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ backgrounds (in full SM), QED/QCD, MC tools

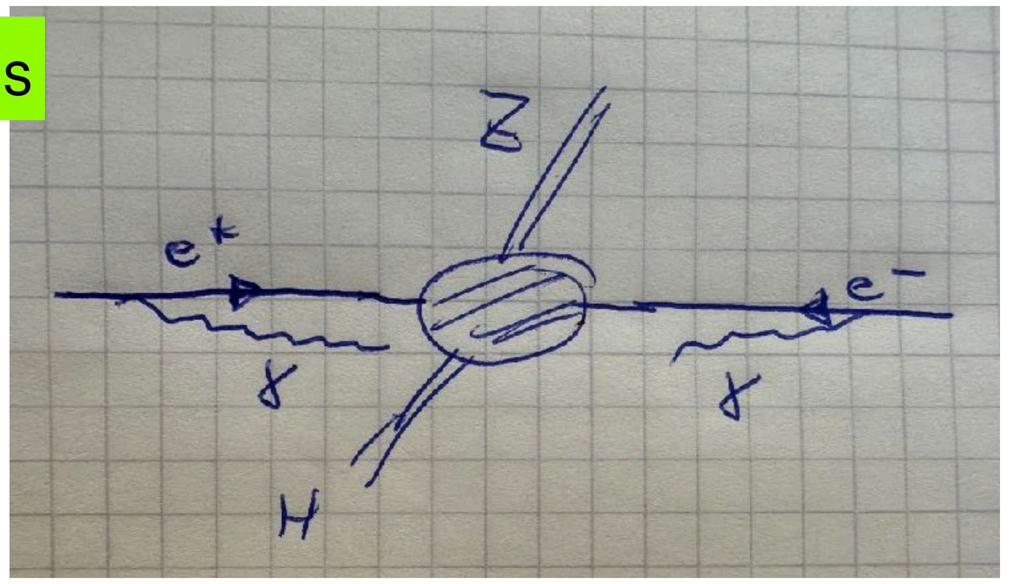
- "Other" eletroweak parameters (**"input" parameters**)
  $\longrightarrow$ m_t, α_s, etc. extracted from other processes

- Scaling of coupling factors [Strip loop amps. of group theory/mass ratios/multiplicities/coupl. → $\mathcal{O}(1)$]
- Extrapolate to higher orders from geometric series, e.g. $\Delta_{\text{NNLO}} \sim \delta^2_{\text{NLO}}$ (beware of renormalons)
- Scale dependence for missing higher order (MHO) corrections (QCD, $\overline{\text{MS}}$, not useful for EW)
- Compare differences in renormalisation schemes (e.g. On-Shell vs. $\overline{\text{MS}}$)
- Different approaches for resummation and matching

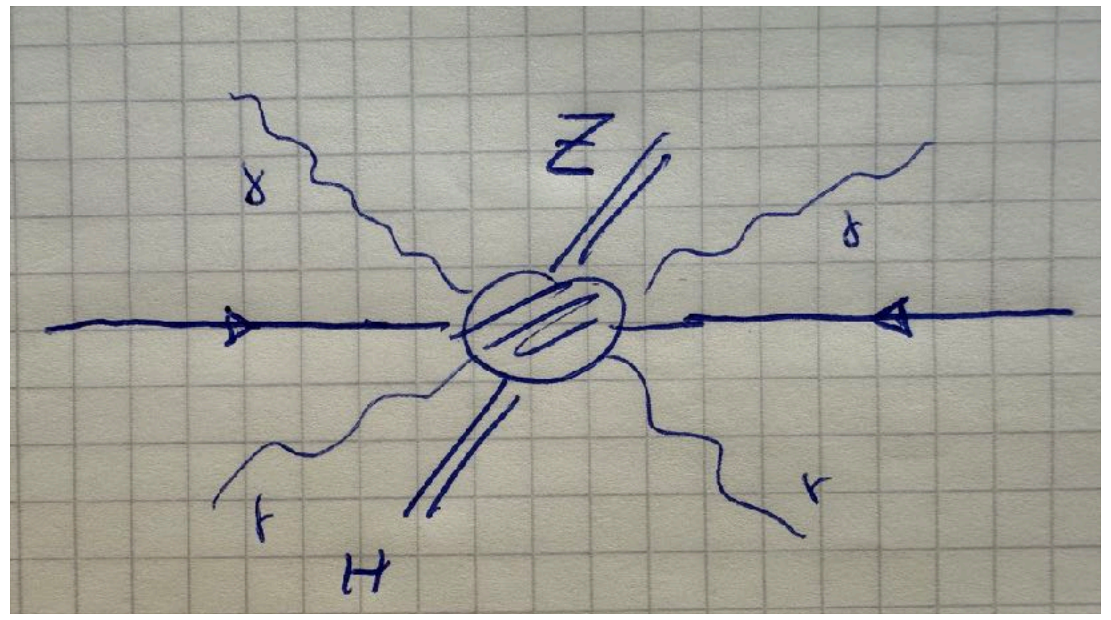
QED corrections / Photon radiation

Collinear logarithms

$$L = \log \frac{Q^2}{m^2}$$



$$\sigma = \alpha^b \sum_{n=0}^{\infty} \alpha^n \sum_{i=0}^{n} \sum_{j=0}^{n} \varsigma_{n,i,j} L^i \ell^j$$



Soft logarithms

$$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}$$

- Collinear resummation LO/LL Gribov/Lipatov, 1972; Kuraev/Fadin, 1985; Skrzypek/Jadach, 1992; Cacciari/Deandrea/Montagna/Nicrosini, 1992
- NLO QED PDFs, collinear evolution @ NLL Frixione, 1909.0388; Bertone/Cacciari/Frixione/Stagnitto, 1911.12040 + 2207.03265
- Crucial: numerical stability at kinematically peaked limit $z \to 1$
- YFS automated Kraus/Price, 2203.10948; 2512.04959
- Both approaches can be systematically improved

$$d\sigma_{kl}(p_k, p_l) = \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \Gamma_{i/k}(z_+, \mu^2, m^2) \Gamma_{j/l}(z_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \mathcal{O}\left(\left(\frac{m^2}{s}\right)^p\right)$$

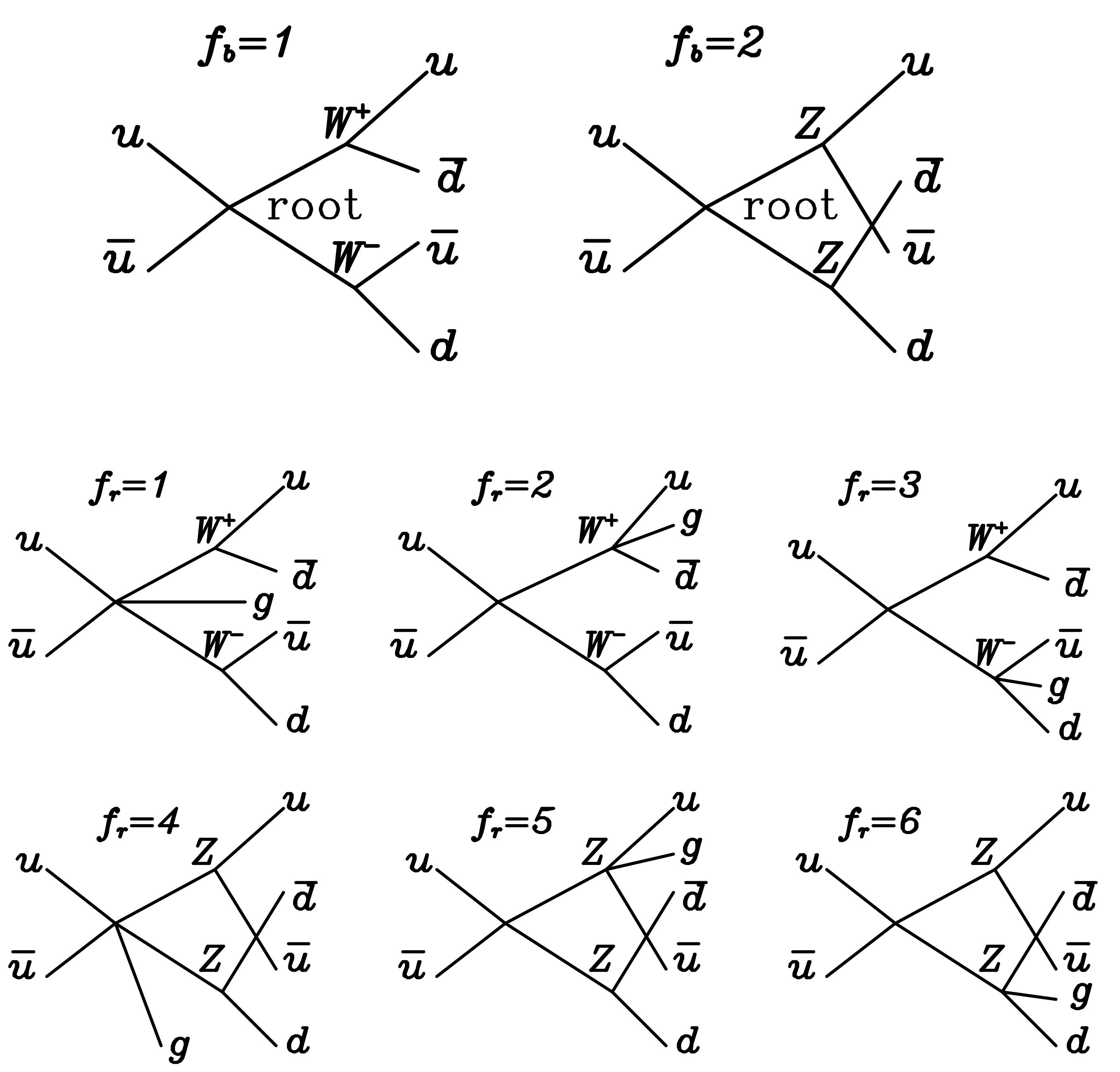
$$\begin{aligned} d\sigma(L, \ell) &= \mathcal{K}_{soft}(\ell; L) \beta(L) d\mu \\ &= e^{Y(p_1, p_2, p_X)} \sum_{n=0}^{\infty} \beta_n \left(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}p_X; \{k_i\}_{i=0}^n\right) d\mu_{X+n\gamma} \end{aligned}$$

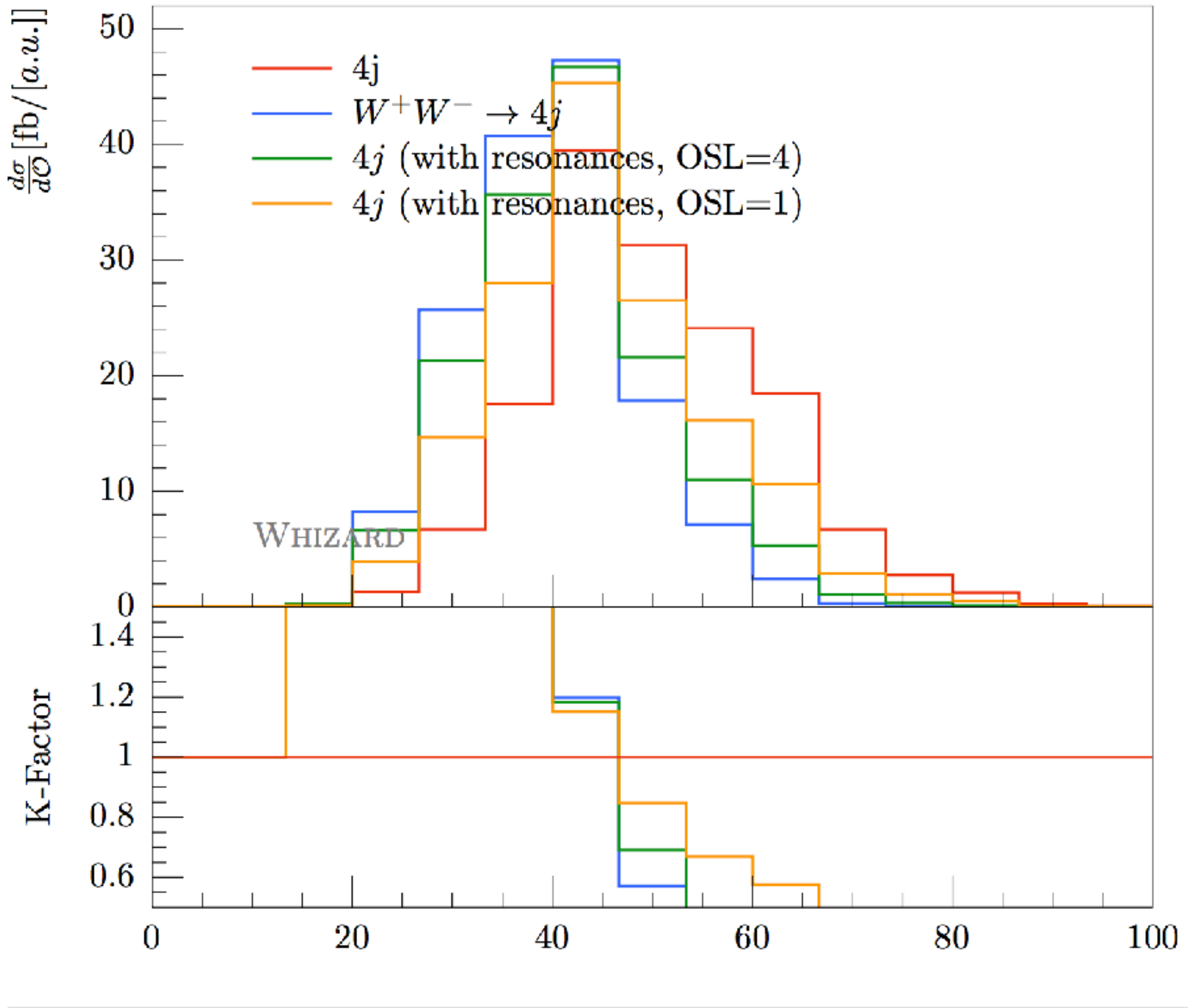
Recent efforts in $e^+e^- \to f\bar{f}$ (2-loop, logarithmic corrections, radiator functions)
Blümlein/de Freitas/Raab/Schönwald, 1901.08018, 1910.05759, 2003.14283, 2004.04287 etc.

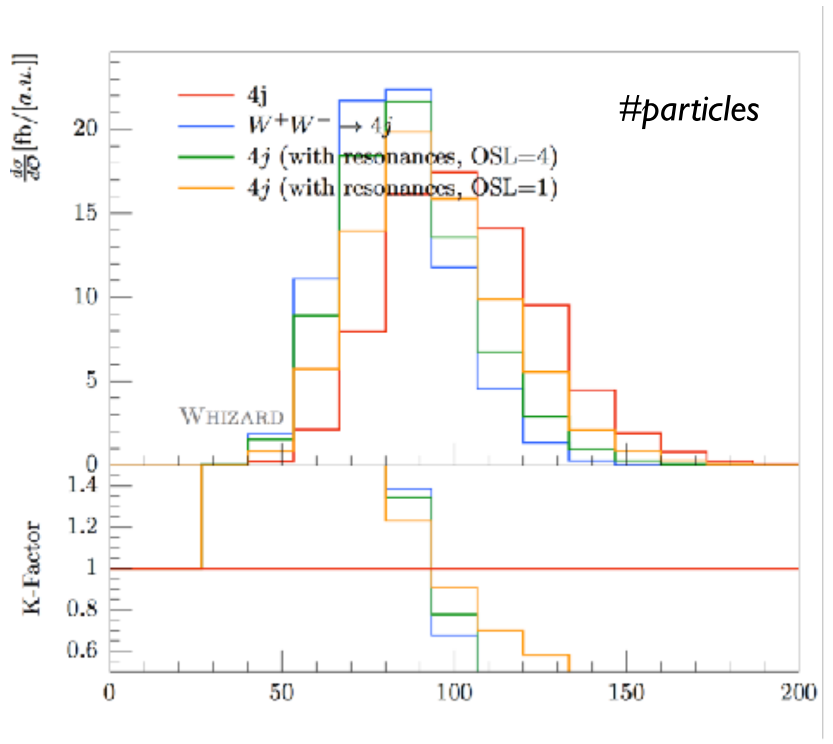
(Resonance) Matching to shower / hadronization

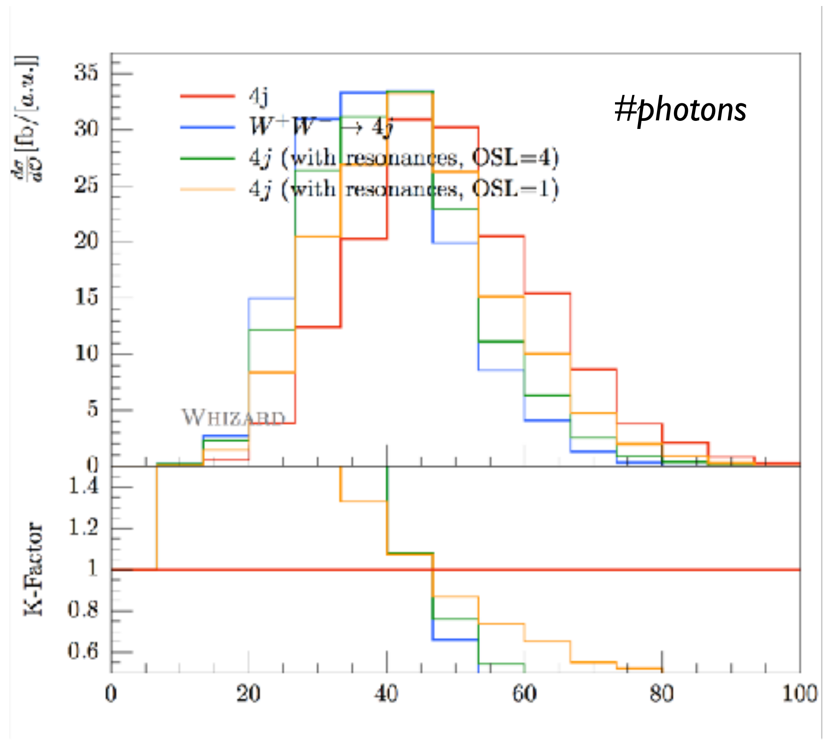

```
?resonance_history = true
resonance_on_shell_limit = 4
resonance_on_shell_turnoff = 1
resonance_background_factor = 1e-10
```

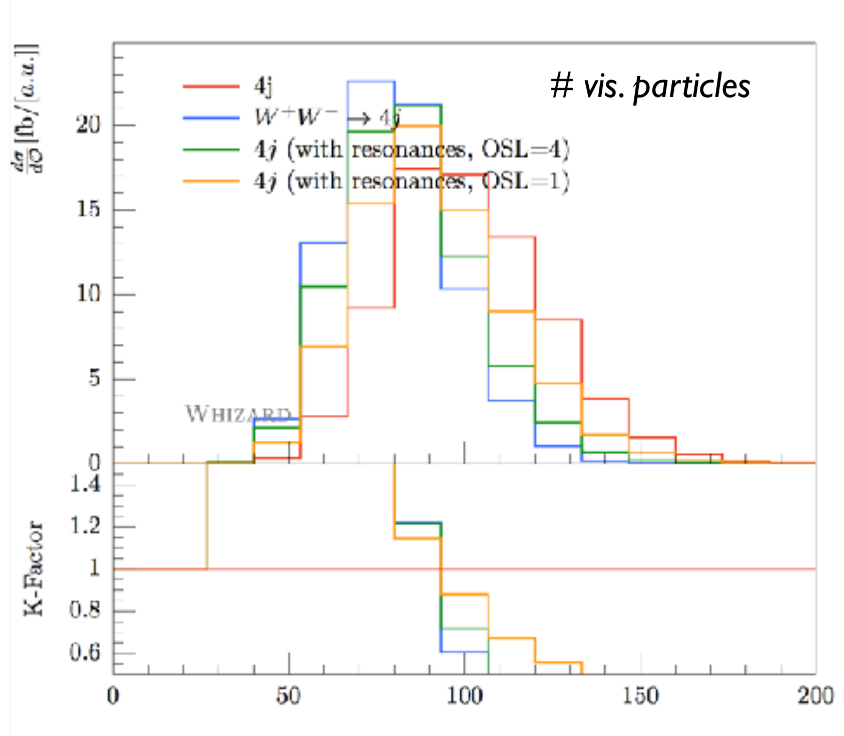
- Problem: $e^+e^- \to jjjj$ not dominated by highest α_s power, but by resonances $e^+e^- \to WW/ZZ \to (jj)(jj)$
- Solution I: proper merging w/ resonant subprocesses by resonance histories

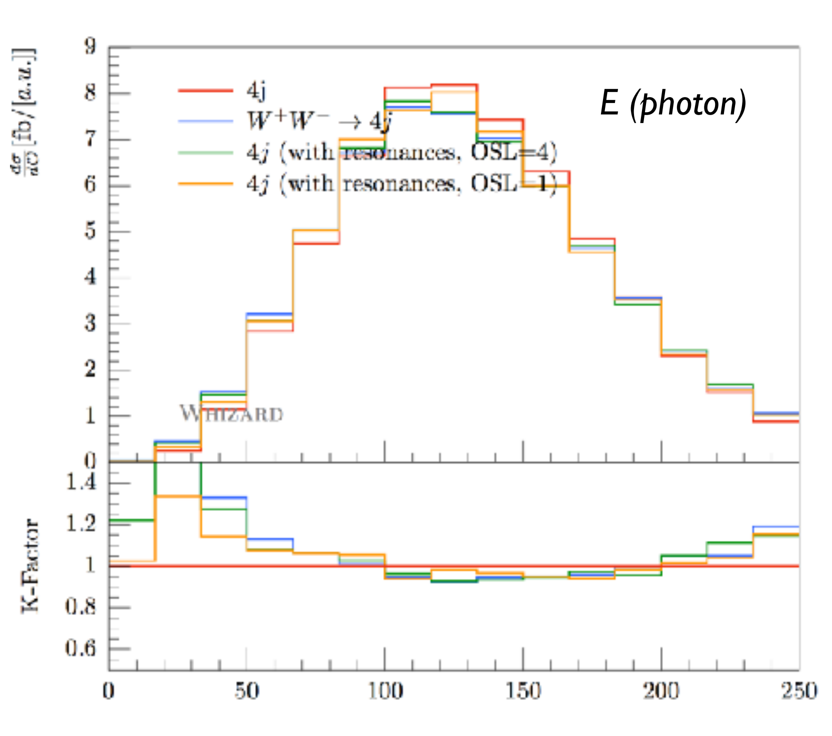












- Solution II: resonance-aware parton showers Höche/Reichelt, 2604.13978

WW threshold: theory issues

Experimental extraction of σ_{WW}:

- $e^{\pm}$ final states: separate or include single-W (compromise theory precision/exp. accuracy)
- Hadronic final states: separation of multi-jet events (2j, 3j, 4j, ...), color reconnections

Required theory corrections for σ_{WW}:

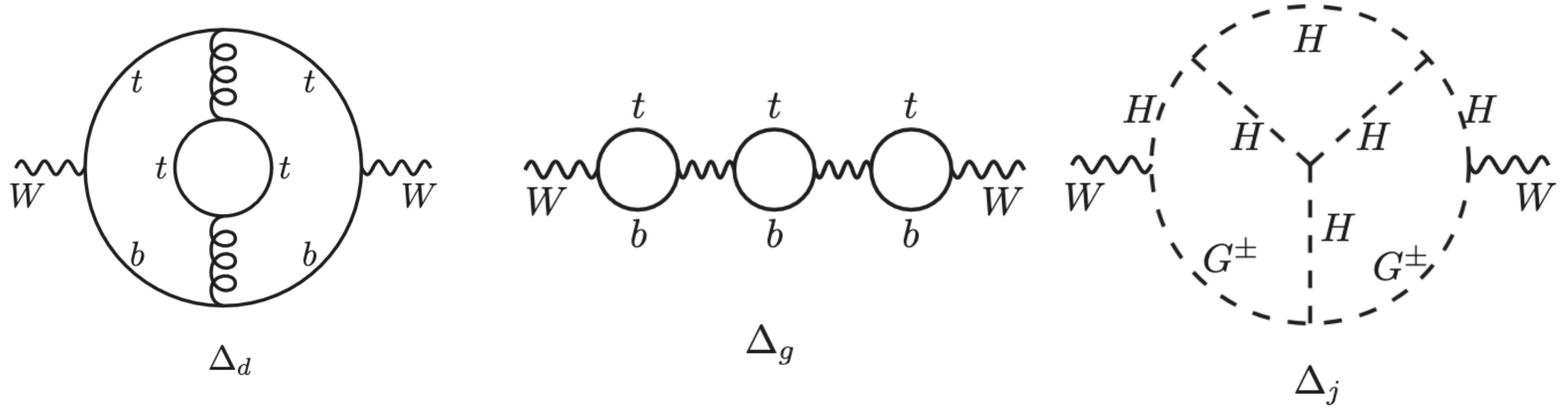
- Full NLO $e^+e^- \to 4f$ (incl. interference of W, Z, forward $e^{\pm}$) for *all* fermion species
- Full NNLO EFT calculation [only leading terms are available]
- Leading 3-loop Coulomb-enhanced EFT corrections
- Matching fixed order $e^+e^- \to 4f$ to threshold EFT
- Convolution of matched cross section with higher order ISR (electron PDF)

Estimate of theory uncertainty at threshold for σ_{WW}: $\Delta \sim 0.01 - 0.04\,\%$ Freitas et al., 1906.05379

Improved prediction for M_W from μ decay:

Massive 3-loop corrections (vacuum graphs + self energies)

e.g. S. Martin, 2507.15946



[conversion from $\overline{\text{MS}}$ to on-shell scheme needed]