Theory needs for extracting Precision Observables from e^+e^-data

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MITP Workshop
"Electroweak Radiative Corrections at Current and Future Accelerators"

Relevant thresholds and processes

- revisit LEP physics with unprecedented statistics
  - at Z pole (~ 0.1% at LEP1) $e^+e^- \to f\bar{f}$
  - at WW threshold (~ 1% at LEP2) $e^+e^- \to 4f$
- explore for the first time at a leptonic collider
  - ZH threshold $e^+e^- \to 4f$
  - $t\bar{t}$ threshold and beyond $e^+e^- \to 6f$

General aspects: MC generators

- MC generators are the link between data and the extraction of observables
- The high precision target of future e^+e^- accelerators requires corresponding developments in MC event generators (and more in general simulation codes)
- Exclusive simulation of all phases of the e^+e^- collisions (schematically)
  - Initial State QED radiation
  - Hard Scattering
  - Final State Radiation

Loosly and schematically

- **ISR**
  - perturbative series for a ew-mediated cross section $2 \to n$

$$\sigma \sim \alpha^n \sum_{m=0}^{\infty} \alpha^m \sum_{i=0}^{m} \sum_{j=0}^{m} c_m^{\{i,j\}} L^i \ell^j$$

$$L = \ln\left(\frac{Q^2}{m_e^2}\right) \qquad \text{collinear log}$$

$$\ell = \ln\left(\frac{Q^2}{(\Delta E_\gamma)^2}\right) \qquad \text{IR log}$$

- orders of magnitude for $\sqrt{Q^2}$ of the order of 100 GeV
  - for an annihilation process $L \sim 25$
  - for a (small angle) scattering $\ell \sim 15$
  - with strong dependence on the event selection and underlying physics (e.g. at the Z pole Γ_Z acts as a natural cutoff for hard photon radiation)
- $\Longrightarrow$ for precise predictions, matching between fixed order and resummation is needed (already for $\mathcal{O}(0.1\%)$ accuracy)

Two approaches for ISR to match fixed order and resummation

- **YFS algorithm, which resums IR logs**

$$d\sigma^{\infty} \sim \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_n \prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i)\Theta(k_i,\Omega) \left\{ \tilde{\beta}_0(\Phi_n) + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(\Phi_{n+1})}{\tilde{S}(k_j)} + \dots \right\}$$

D.R. Yennie, S.C. Frautschi and H. Suura, Annals of Physics 13 (1961) 379;

S. Jadach and B.F.L. Ward, Comput. Phys. Commun. 56 (1990) 351

- **SFs/PDFs approach, which resums collinear logs**

$$d\sigma^{\infty} \sim \int dx_1 dx_2 D(x_1,Q^2) D(x_2,Q^2) d\hat{\sigma}(s') \delta(s' - x_1 x_2 s)$$

E. Kuraev, V. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466;
G. Altarelli, G. Martinelli, Physics at LEP, Vol. 1 (1986) 47;
O. Nicrosini, L. Trentadue, Phys. Lett. B 196 (1987) 551;

F.A. Berends, G. Burgers, W.L. van Neerven, Phys. Lett. B185 (1987) 395; Nucl. Phys. B297 (1988) 429

- both approaches developed and used at LEP/flavour factories
- matching leading log resummation with NLO matrix elements $\Longrightarrow \mathcal{O}(0.1\%)$ accuracy

see e.g. S. Actis et al., Eur. Phys. J. C66 (2010) 585

  - **at least an additional order of magnitude in precision is required**

Recent results

see talk by A. Price tomorrow

- on YFS resummation

A. Price, F. Krauss, “Toward a fully automated differential $\mathrm{NNLO}_{\mathrm{EW}}$ generator for lepton colliders”, PRD113 (2026) 7

- on collinear resummation with NLL accuracy

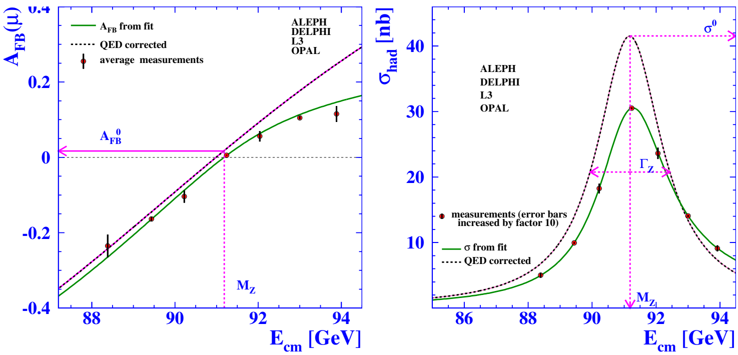
V. Bertone, M. Cacciari, S. Frixione, G. Stagnitto, “The partonic structure of the electron at the next-to-leading accuracy in QED”, JHEP 03 (2020) 135;

“Improving methods and predictions at high-energy e^+e^- colliders within collinear factorisation”, JHEP 10 (2022) 089

hard scattering cross sections: $e^+e^- \to f\bar{f}$ at the Z

- accuracy of $\mathcal{O}(0.1\%)$, around the Z-pole at LEP1
  - with full NLO matrix elements for $e^+e^- \to f\bar{f}$ processes, including higher-order $(\Delta\alpha)^n$, $(\frac{m_t}{M_W})^n$ and Δr^n enhanced contributions
  - in association with resummation of leading log QED radiation plus all IR contributions and pure collinear terms of $\mathcal{O}(\alpha^n L^n)$ and partial $\mathcal{O}(\alpha^2 L)$
  - (α, G_μ, M_Z) input parameter scheme, in order to minimize the parametric uncertainties

Effect of QED ISR



LEP EWWG, SLD WG, ALEPH, DELPHI, L3, OPAL, Phys. Rept. 427 (2006) 257

Deconvolution performed at LEP by means of “semianalytical”

- TOPAZ0

  G. Montagna, O. Nicrosini, G. Passarino, F.P., R. Pittau, 1993, 1996, 1999

- ZFITTER

  D. Bardin et al., 1989, 1991, 1992, 1994, 2001

$$d\sigma = \int dx_1 dx_2 D(x_1, Q^2) D(x_2, Q^2) d\hat{\sigma}(s') \delta(s' - x_1 x_2 s)$$

- by analytical integration over one x dimension, we get the convolution with the radiatior/flux function $H(z, Q^2)$
  - at the prize of being not fully exclusive on both leptons, being able to treat only simplified event selections (e.g. $s' > s_0$)
  - ~ 30 years ago one analytical integration allowed to avoid CPU time problems

$$\sigma_{\rm T}(s) = \int_{z_0}^{1} dz H(z; s) \hat{\sigma}_{\rm T}(zs) \qquad A_{FB}(s) = \frac{\pi \alpha^2 Q_e^2 Q_f^2}{\sigma_{\rm tot}} \int_{z_0}^{1} dz \frac{1}{(1+z)^2} H_{\rm FB}(z; s) \hat{\sigma}_{\rm FB}(zs)$$

- H functions known at $\mathcal{O}(\alpha^3)$ for cross sections and $\mathcal{O}(\alpha^2)$ for A_{FB}

ansatz for the kernel cross section

- model-independent parameterization of $\hat{\sigma}(e^+e^- \to f\bar{f})$

$$A_{\rm SM} = A_\gamma + A_Z + \text{non-factorizable}$$

  - aim: write the Z-line shape in a model independent way, i.e. describe the exchange of a spin-1 relativistic resonance

Borrelli, Consoli, Maiani, Sisto, NPB333 (1990) 357

$$\sigma^Z_{f\bar{f}} = \sigma^{\rm peak}_{f\bar{f}} \frac{s\Gamma_Z^2}{(s-M_Z)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\sigma^{\rm peak}_{f\bar{f}} = \frac{\sigma^0_{f\bar{f}}}{R_{\rm QED}}; \qquad \sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

- partial widths (or, even better, ratios) can be fitted from data and calculated within SM (and/or possible extension of it) to the desired accuracy
- calculating decay widths is easier w.r.t. a complete cross section
- also easier data combination between different experiments

Uncertainty on the ISR deconvolution

- obtained through the comparison of additive and factorized form of the radiator function

	LEP 1 energy in GeV				
	$M_Z - 3$	$M_Z - 1$	M_Z	$M_Z + 1$	$M_Z + 3$
	$10^4 \times$ (fact/add-1)				
σ_μ	0.44 0.88	0.63 0.63	0.61 0.68	0.72 0.72	0.49 0.49
$\sigma_{\rm had}$	0.58 0.61	0.58 0.62	0.64 0.67	0.73 0.76	0.59 0.62
	fact-add [pb]				
σ_μ	0.01 0.02	0.03 0.03	0.09 0.10	0.05 0.05	0.02 0.02
$\sigma_{\rm had}$	0.26 0.27	0.56 0.60	1.95 2.04	1.04 1.08	0.48 0.51
	$10^5 \times$ (fact-add)				
A^μ_{FB}	1.00 -4.00	1.00 -2.00	0.00 0.00	0.00 1.00	-1.00 1.00

- The level of agreement between TOPAZ0 and ZFITTER around the Z peak is below the 0.01% level $\longrightarrow$ analysis at the 0.1% level on the derived observables are robust

example: interference $\gamma - Z$

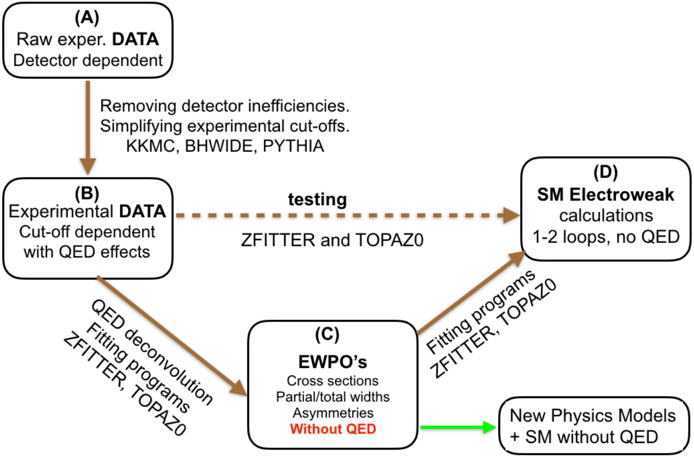
	Centre-of-mass energy in GeV				
	$M_Z - 3$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3$
$\delta^{\rm int}\sigma_\mu$	$-0.209\,\%$	$-0.136\,\%$	$-0.028\,\%$	$+0.072\,\%$	$+0.132\,\%$
$\delta^{\rm int}\sigma_{\rm had}$	$-0.492\,\%$	$-0.301\,\%$	$-0.029\,\%$	$+0.226\,\%$	$+0.384\,\%$
$\Delta^{\rm int}\mu$	-0.27703	-0.16611	0.00145	0.15923	0.25441

$$\delta^{\rm int}\sigma = \frac{\sigma_{\rm T}^{\rm DD}}{\sigma_{Z+\gamma}^{\rm DD}} - 1 \quad \text{in percent}$$

$$\Delta^{\rm int} A_{FB}^{\mu} = A_{FB}^{\mu,{\rm DD}} - \left(A_{FB}^{\mu,{\rm DD}}\right)_{Z+\gamma} .$$

m_H [GeV]	Centre-of-mass energy in GeV				
	$M_Z - 3$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3$
			$\delta^{\rm int}\sigma_\mu$		
10	$-0.229\,\%$	$-0.150\,\%$	$-0.030\,\%$	$+0.082\,\%$	$+0.149\,\%$
100	$-0.209\,\%$	$-0.136\,\%$	$-0.028\,\%$	$+0.072\,\%$	$+0.132\,\%$
1000	$-0.181\,\%$	$-0.119\,\%$	$-0.026\,\%$	$+0.060\,\%$	$+0.111\,\%$
			$\delta^{\rm int}\sigma_{\rm had}$		
10	$-0.518\,\%$	$-0.317\,\%$	$-0.029\,\%$	$+0.240\,\%$	$+0.407\,\%$
100	$-0.492\,\%$	$-0.301\,\%$	$-0.029\,\%$	$+0.226\,\%$	$+0.384\,\%$
1000	$-0.457\,\%$	$-0.281\,\%$	$-0.028\,\%$	$+0.207\,\%$	$+0.353\,\%$

D.Y. Bardin, M. Grünewald, G. Passarino, hep-ph/9902452



A. Freitas, J. Gluza, S. Jadach, arXiv:1809.01830

- for $\sim 0.1\%$ precision, it was thoroughly checked that any uncertainty in the procedure was not larger than 0.01% level

From Z physics at LEP to future accelerators

- NNLO corrections to the hard scattering

- together with higher precision on ISR

- QED IFI resummation in the presence of a resonance, which gives a suppression Γ_Z/M_Z

M. Greco, G. Pancheri, Y. Srivastava, Nucl.Phys. B101 (1975) 234; NPB171 (1980) 118

  - however it is important for
    - total cross sections out of peak
    - asymmetry around the peak
  - implemented in KKMC

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- can be still applied a deconvolution reliable with 10^{-5} precision?
- alternatively one should compare directly data/theory predictions through a Monte Carlo generator implementing any New Physics model or SMEFT

Key observable common to all energies: luminosity

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

Several key measurements at an e^+e^- machine depend on L, e.g.

- σ_Z^0, the Z peak cross section
- light neutrino species from radiative return $(e^+e^- \rightarrow \nu\bar{\nu}\gamma)$
  - alternatively, from $(e^+e^- \rightarrow \nu\bar{\nu}\gamma)/(e^+e^- \rightarrow \mu^+\mu^-\gamma)$
- Γ_Z from the line-shape of $e^+e^- \rightarrow f\bar{f}$
- M_W and Γ_W from line-shape of $e^+e^- \rightarrow W^+W^-$ close to threshold
- total cross section for $e^+e^- \rightarrow HZ \Longrightarrow HZZ$ coupling and total Γ_H

Absolute luminosity precision level

- **Uncertainty target for future e^+e^- machines**
  - at Z pole 10^{-4} or better for the overall luminosity calibration
  - $\mathcal{O}(10^{-4})$ at $\sqrt{s} \sim 2M_W$ to get $\Delta M_W \simeq 1$ MeV
  - 10^{-3} at higher energies

- **Reference processes**
  - ⋆ Small-angle Bhabha scattering
    - used at LEP $\sim 0.05\%$
    - $d\sigma/d\theta \sim 1/\theta^3$ limiting factor
  - ⋆ $e^+e^- \to \gamma\gamma$
    - $d\sigma/d\cos\theta \sim 1/\sin^2\theta$
    - lower statistics w.r.t Bhabha scattering

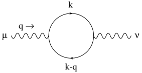
Sources of theory systematics

- **QED corrections**

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Blue: Leading-Log PS, Leading-Log YFS, SF

- **hadronic contribution to photon vacuum polarization**



  - $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\qquad \Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\text{top}}(q^2) + \Delta\alpha^{(5)}_{\text{had}}(q^2)$
  - low energy physics of muon $g-2$ is triggering new (present and future) data analysis (BaBar, BESIII, KLOE, VEPP2M, Belle2, MUonE) and new results on the Lattice for $\Delta\alpha_{\text{had}}$

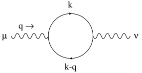
Sources of theory systematics

- **QED corrections**

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Red: matched PS, YFS, SF + NLO

- **hadronic contribution to photon vacuum polarization**



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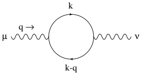
Sources of theory systematics

- **QED corrections**

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

- **hadronic contribution to photon vacuum polarization**



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Past and recent updates

- **theoretical error in SABS at LEP1 by the end of operation**

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

- **experimental systematics: 0.034%**

G. Abbiendi et al., (OPAL), Eur. Phys. J. C14 (2000) 373

- **recent reanalysis**
  - "The path to 0.01% theoretical luminosity precision for the FCC-ee"

S. Jadach, W. Placzek, M. Skrzypek, B.F.L. Ward and S.A. Yost, Phys Lett B790 (2019) 314

  - "Improved Bhabha cross section at LEP and the number of light neutrino species"

P. Janot and S. Jadach, Phys. Lett. B803 (2020) 135319

Updates and future collider projections

Type of correction / Error	Update 2018	FCC-ee forecast
(a) Photonic $[O(L_e\alpha^2)]$ $O(L_e^2\alpha^3)$	0.027%	0.1×10^{-4}
(b) Photonic $[O(L_e^3\alpha^3)]$ $O(L_e^4\alpha^4)$	0.015%	0.6×10^{-5}
(c) Vacuum polariz.	0.014% [26]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [28]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

S. Jadach, W. Placzek, M. Skrzypek, B.F.L. Ward and S.A. Yost, Phys Lett B790 (2019) 314

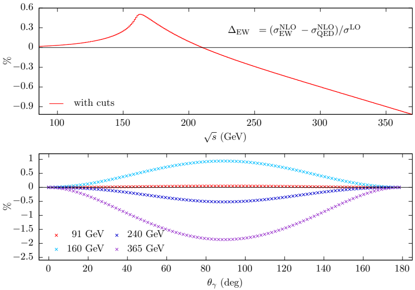
Forecast study for FCCee$_{M_Z}$		
Type of correction / Error	Published [2]	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_e^0)$		0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.70×10^{-4}

Forecast			
Type of correction / Error	ILC$_{500}$	ILC$_{1000}$	CLIC$_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0\times10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$<0.1\times10^{-4}$	$<0.1\times10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

B.F.L. Ward, S. Jadach, W. Placzek, M. Skrzypek, S.A. Yost, arXiv:2410.09095

Interesting additional channel: $e^+e^- \to \gamma\gamma$

- pure QED process, no vacuum polarization at LO
- NLO weak corrections "small" in a wide range of energies

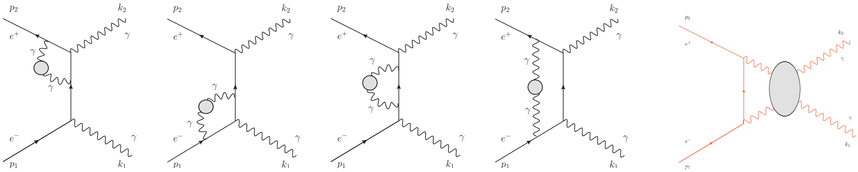


C.M. Carloni Calame, M. Chiesa, G. Montagna, O. Nicrosini, FP, Phys. Lett. B798 (2019) 134976

- exclusive event generation available with NLOPS accuracy
- recent NNLO calculation in QED for low c.m. energies

T. Engel, M. Rocco, A. Signer, Y. Ulrich, Phys. Lett. B874 (2026) 140236

Rough estimate of (NNLO) VP hadronic corrections



$$\sigma^{\text{NNLO}}_{\Delta\alpha_{had}} \pm \delta\sigma \overset{\text{very naive!}}{\approx} \left(\sigma^{\text{NLO}}_{\text{QED}} - \sigma^{\text{LO}}\right) \times \left[\Delta\alpha_{had}(s) \pm \delta\Delta\alpha_{had}\right]$$

$\sqrt{s}$ (GeV)	$\Delta\alpha_{had}(s)^*$	$\delta\sigma/\sigma_{LO}$ [1]	$\delta\sigma/\sigma_{LO}$ [2]
91	$(276.7 \pm 1.2) \cdot 10^{-4}$	$2.8 \cdot 10^{-5}$	$3.7 \cdot 10^{-6}$
160	$(309.1 \pm 1.2) \cdot 10^{-4}$	$3.0 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$
240	$(333.2 \pm 1.2) \cdot 10^{-4}$	$3.1 \cdot 10^{-5}$	$3.9 \cdot 10^{-6}$
365	$(358.5 \pm 1.2) \cdot 10^{-4}$	$3.4 \cdot 10^{-5}$	$4.0 \cdot 10^{-6}$

- LbL contribution, with its uncertainty, should be quantified

*from F. Jegerlehner `hadr5n16.f`

What about possible New Physics contamination?

- A study on small angle Bhabha scattering

  M., Chiesa, C.L. Del Pio, G. Montagna, O. Nicrosini, , F.P., F.P. Ucci, arXiv:2501.05256

- preliminary study for $e^+e^- \to \gamma\gamma$

  J.A. Maestre, arXiv:2206.07564

- **given the present and projected bounds on BSM physics, can we expect any uncertainty above the target precision?**

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- **Collider scenarios**

Exp.	$[\theta_{\text{min}}, \theta_{\text{max}}]$	$\sqrt{s}$ [GeV]	$\Delta L/L$
FCC	$[3.7^\circ, 4.9^\circ]$	91 160 240 365	$< 10^{-4}$ 10^{-4}
ILC	$[1.7^\circ, 4.4^\circ]$	250 500	$< 10^{-3}$
CLIC	$[2.2^\circ, 7.7^\circ]$	1500 3000	$< 10^{-2}$

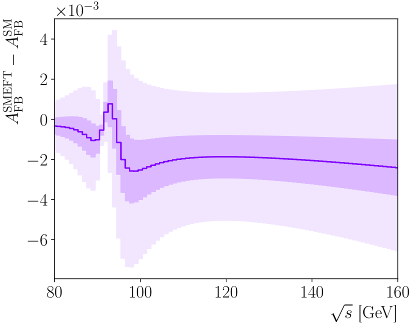
Results

- $\Longrightarrow \delta\sigma_{\text{SABS}}$ **from** $\Delta g^{Ze}_{L/R}$ **negligible**
- $4f$ **dim-6 operators could give a contamination**

Exp.	$[\theta_{\text{min}}, \theta_{\text{max}}]$	$\sqrt{s}$ [GeV]	$(\delta \pm \Delta\delta)_{\text{SMEFT}}$	$\Delta L/L$
FCC	$[3.7°, 4.9°]$	91	$(-4.2 \pm 1.7) \times 10^{-5}$	$< 10^{-4}$
		160	$(-1.3 \pm 0.5) \times 10^{-4}$	10^{-4}
		240	$(-2.9 \pm 1.2) \times 10^{-4}$	
		365	$(-6.7 \pm 2.7) \times 10^{-4}$	
ILC	$[1.7°, 4.4°]$	250	$(-2.5 \pm 0.9) \times 10^{-4}$	$< 10^{-3}$
		500	$(-4.9 \pm 1.9) \times 10^{-4}$	
CLIC	$[2.2°, 7.7°]$	1500	$(-9.7 \pm 3.9) \times 10^{-3}$	$< 10^{-2}$
		3000	$(-4.2 \pm 1.7) \times 10^{-2}$	

Final luminosity determination?

- possible strategies to remove the uncertainty using asymmetry data



- $\sqrt{s} = 89, 93, 98$ GeV
- one year of running at each $\sqrt{s_i}$ with $\mathcal{L} = 1.4 \times 10^{36}$cm $^{-2}$s^{-1}

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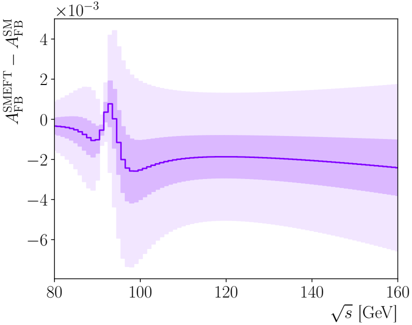

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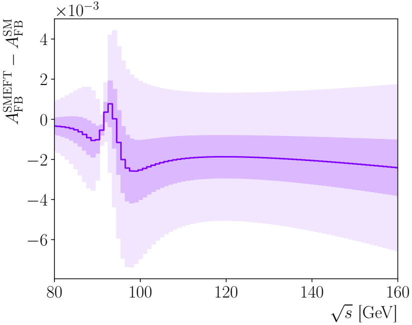

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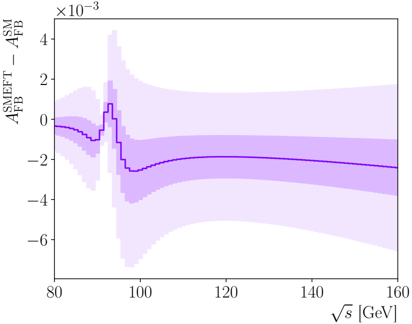

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hard scattering cross sections: $e^+e^- \to 4f$ for M_W

see talk by J. Reuter

- at LEP2, at and above the WW threshold, NLO matrix elements for $e^+e^- \to 4f$ (first complete calculation)

  A. Denner, S. Dittmaier, M. Roth, L.H. Wieders, Nucl. Phys. B724 (2005) 247

  together with partial h.o. effects in EFT expansion $(\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta_W^2)$ at WW threshold $\Longrightarrow$ uncertainty on $M_W \sim 3$ MeV

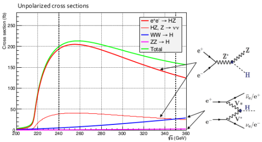
  M. Beneke, P. Falgari, C. Schwinn, A. Signer, G. Zanderighi, Nucl. Phys. B792 (2008) 89

  S. Actis, M. Beneke, P. Falgari, C. Schwinn, Nucl. Phys. B807 (2009) 1

- target exp. precision of $\mathcal{O}(0.5\,\mathrm{MeV})$ requires $\Delta\sigma \sim 0.01\%$
  - NNLO corrections to $e^+e^- \to W^+W^-$ and to $W \to f\bar{f}'$
  - to reach the 0.01% level accuracy, the next relevant corrections are the Coulomb-enhanced N3LO, A. Freitas et al., arXiv:1906.05379
  - the $4q$ final state would require also $\mathcal{O}(\alpha_s^2)$, in particular the leading resonant contribution of non-factorizable QCD exchanges between

hard scattering cross sections: $e^+e^- \to 4f$ for ZH

- present automatic Monte Carlo generators can simulate the whole $2 \to 4$ process at LO and NLO



- $e^+e^- \to ZH$ dominant $(\sim 80\%)$ at $\sqrt{s} \sim 240$ GeV

- $\Gamma_H/M_H \ll 1(\sim 3 \cdot 10^{-5}) \Longrightarrow$ Higgs NWA reliable
- beyond NLO accuracy
  - mixed $\mathcal{O}(\alpha\alpha_s)$ $(\sim 1.3\%)$

    Y. Gong, Z. Li, X. Xu, L.L. Yang, X. Zhao, Phys. Rev. D95 (2017) 093003

    Q.F. Sun, F. Feng, Y. Jia, W.L. Sang, Phys. Rev. D96 (2017) 051301(R)
    - calculated also for $e^+e^- \to \mu^+\mu^- H$ $(\sim 1.5\%)$

      W. Chen, F. Feng, Y. Jia, W.L. Sang, Chin. Phys. C43 (2019) 013108
  - NNLO corr. with closed fermion loops to $e^+e^- \to ZH$ $(< 1\%)$ on σ at $\sqrt{s} = 240$ GeV

    A. Freitas and Q. Song, Phys. Rev. Lett. 130 (2023) 031801; Phys. Rev. D108 (2023) 053006
  - NNLO corrections to $e^+e^- \to ZH$

    X. Chen et al., arXiv:2209.14953