Higher-order corrections to Higgs observables ($\Gamma_{H\to b\bar{b}}$)

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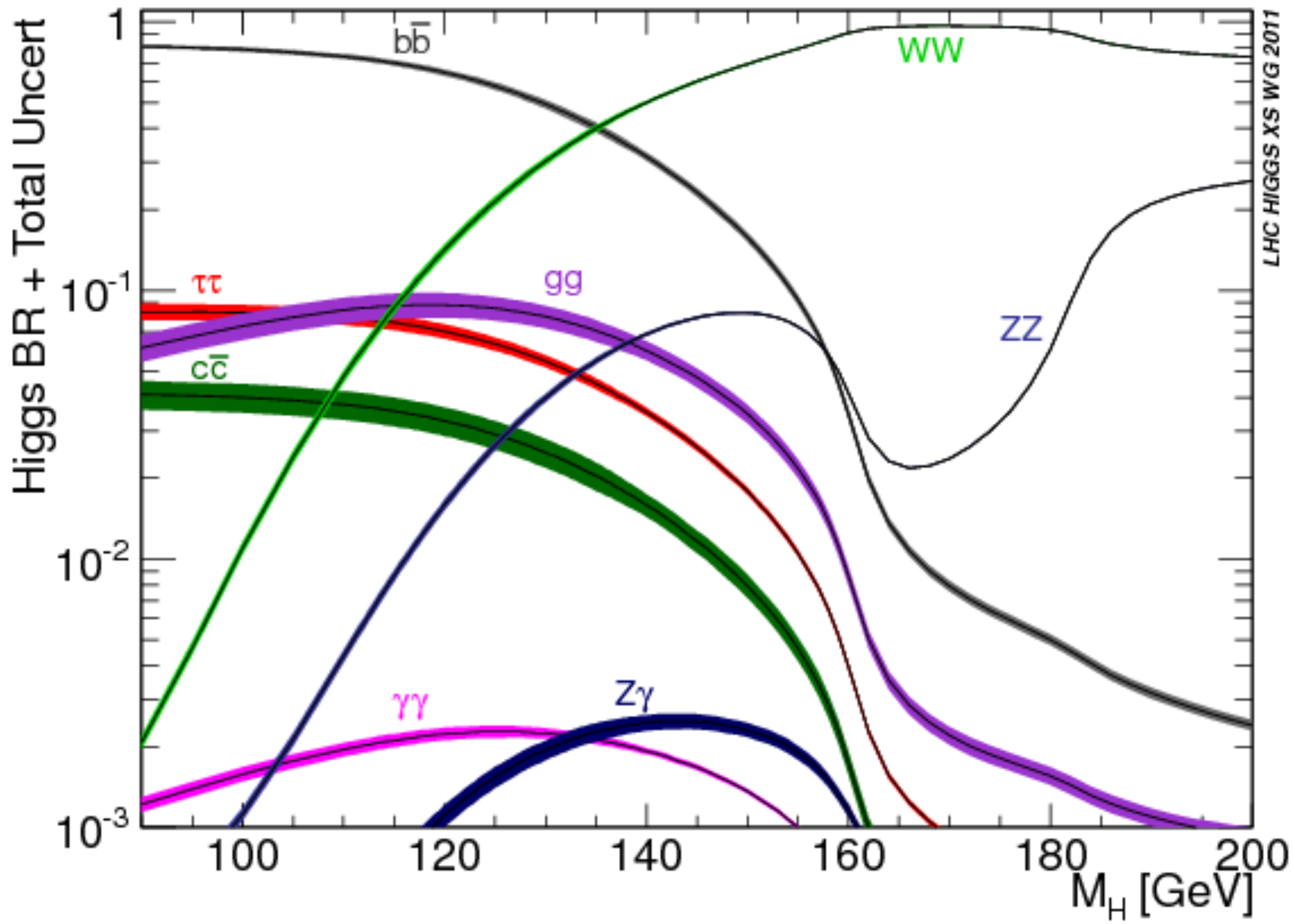
Electroweak Corrections at Current and Future accelerators

Mainz, May 6, 2026

Higgs decays



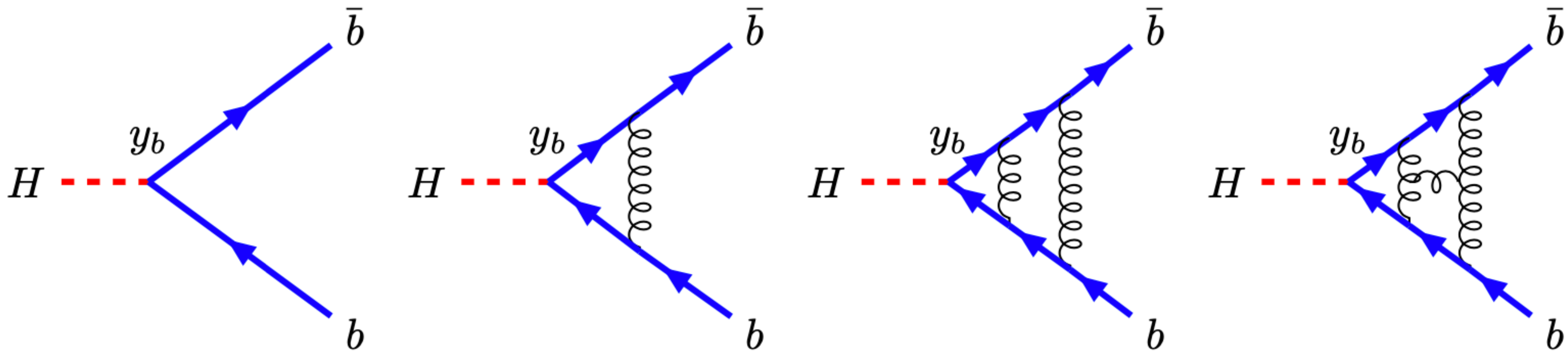
Higgs boson decay to bottom quarks is dominant.

$\sqrt{s}$	240 GeV		365 GeV	
channel	**ZH**	**WW → H**	**ZH**	**WW → H**
ZH → any	±0.31		±0.52	
γH → any	±150			
H → bb	±0.21	±1.9	±0.38	±0.66
H → cc	±1.6	±19	±2.9	±3.4
H → ss	±120	±990	±350	±280
H → gg	±0.80	±5.5	±2.1	±2.6
H → $\tau\tau$	±0.58		±1.2	±5.6 (*)
H → $\mu\mu$	±11		±25	
H → WW*	±0.80		±1.8 (*)	±2.1 (*)
H → ZZ*	±2.5		±8.3 (*)	±4.6 (*)
H → $\gamma\gamma$	±3.6		±13	±15
H → Zγ	±11.8		±22	±23
H → $\nu\nu\nu\nu$	±25		±77	
H → inv.	$< 5.5 \times 10^{-4}$		$< 1.6 \times 10^{-3}$	
H → dd	$< 1.2 \times 10^{-3}$			
H → uu	$< 1.2 \times 10^{-3}$			
H → bs	$< 3.1 \times 10^{-4}$			
H → bu	$< 2.2 \times 10^{-4}$			
H → sd	$< 2.0 \times 10^{-4}$			
H → cu	$< 6.5 \times 10^{-4}$			

HEWT summary

(*) analyses ongoing, results scaled from FCC CDR

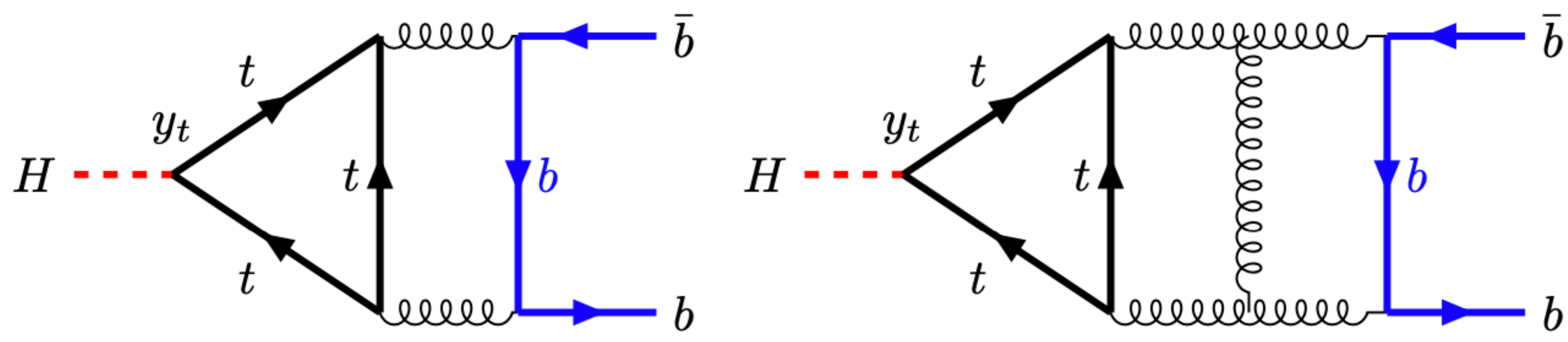
Status of theoretical predictions



- Inclusive decay width up to $\mathcal{O}(y_b^2\alpha_s^2)$ with massive b-quarks [Chetyrkin, Kwiatkowski 1995, Harlander, Steinhauser, 1997, JW, Wang, Zhang, 2024]
- Inclusive decay width up to $\mathcal{O}(y_b^2\alpha_s^4)$ with massless b-quarks [Gorishnii, Kataev, Larin, Surguladze 1990, Chetyrkin 1997, Baikov, Chetyrkin, Kuhn 2006, Herzog, Ruijl, Ueda, Vermaseren, Vogt, 2017]
- Differential decay width at $\mathcal{O}(y_b^2\alpha_s^2)$ with massive b-quarks [Bernreuther, Chen, Si 2018, Behring, Bizoń 2020, Somogyi, Tramontano 2020]
- Differential decay width at $\mathcal{O}(y_b^2\alpha_s^3)$ with massless b-quarks [Anastasiou, Herzog, Lazopoulos, 2012, Duca, Duhr, Somogyi, Tramontano, Trocsanyi, 2015, Mondini, Schiavi, Williams, 2019, Chen, Jakubcik, Marcoli, Stagnitto, 2023, Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss, 2025]

The higher-order corrections beyond NNLO are less than 0.2%

Status of theoretical predictions



- The decay induced by the t-quark Yukawa coupling was calculated at $\mathcal{O}(\alpha_s^2)$ [Primo, Sasso, Somogyi, Tramontano, 2019]

- The extension to $\mathcal{O}(\alpha_s^3)$ was obtained in the large top mass limit [Modini, Schubert, Williams, 2020, JW, Wang, Wang, 2025]

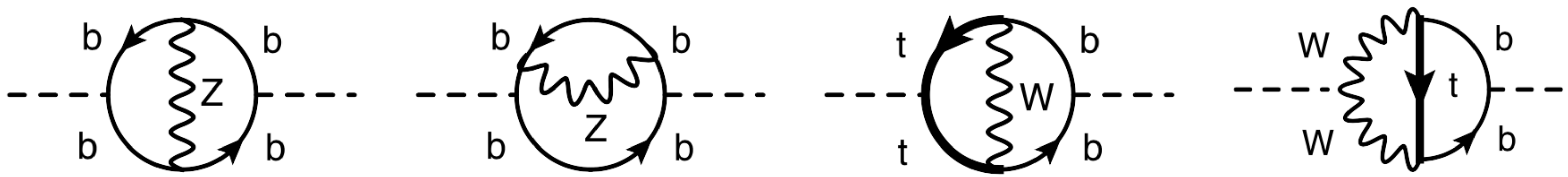
$$\mathcal{L}_{\text{eff}} = -\frac{H}{v}\left(C_1 \mathcal{O}_1^R + C_2 \mathcal{O}_2^R\right) + \mathcal{L}_{\text{QCD}}\,,$$

$$\mathcal{O}_1 = (G^0_{a,\mu\nu})^2, \quad \mathcal{O}_2 = m_b^0 \bar{b}^0 b^0.$$

$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}.$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right]$$

Status of theoretical predictions



- The electroweak correction was calculated at $\mathcal{O}(\alpha)$ [Dabelstein, Hollik, 1992, Kniehl, 1992]

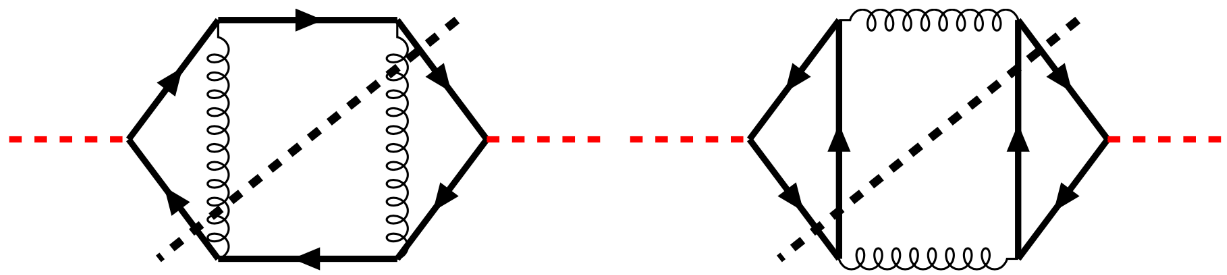
- The mixed $QCD \times EW$ correction was computed at $\mathcal{O}(\alpha\alpha_s)$ [Kataev,1997, Mihaila, Schmidt, Steinhauser, 2015]

	$\Delta^{(\alpha_s)}$	$\Delta^{(\alpha_s^2)}$	$\Delta^{(\alpha_s^3)}$	$\Delta^{(\alpha_s^4)}$
QCD	0.2040	0.0378	0.0020	−0.0014
	$\Delta^{(\mathrm{QED})}$	$\Delta^{(\mathrm{QED},\alpha_s)}$		
QED/QCD	0.0011	0.0001		
	$\Delta^{(\mathrm{weak})}$	$\Delta^{(\mathrm{weak},\alpha_s)}$	$\Delta^{(\mathrm{weak},Z)}$	$\Delta^{(\mathrm{weak},\alpha_s,Z)}$
Weak/QCD	−0.0100	−0.0029	−0.0097	−0.0020

Evaluate the loop diagrams in the limit $m_H^2 \ll M_Z^2$ and apply a Pade approximation to construct a result for the physical mass.

The contribution from $H \to Zb\bar{b}$ was neglected.

Analytical QCD NNLO result



$$\Gamma^{y_b y_b}_{Hb\bar b} \equiv \tilde\Gamma^{y_b y_b}_{Hb\bar b} + \Gamma^{y_b y_b}_{Hb\bar b b\bar b}$$

$$\Gamma(H\to b\bar b) = \Gamma_{\rm LO}\left[1+\frac{\alpha_s(\mu)}{\pi}X_1^{y_b y_b} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 (X_2^{y_b y_b} + X_2^{y_b y_t})\right]$$

- $X^{y_b y_b}_{2,b\bar b}$ is written in terms of MPLs while $X^{y_b y_b}_{2,4b}$ is expressed as complete elliptic integrals or their derivatives/integrals.

$$X^{y_b y_b}_{2,b\bar b} \approx -\frac{\ln^3 z}{27} + \cdots + \frac{1}{z}\left(-\frac{5\ln^4 z}{36} + \frac{\ln^3 z}{18} + \cdots\right) + \cdots$$

$$X^{y_b y_b}_{2,4b} \approx \frac{\ln^3 z}{27} + \cdots + \frac{1}{z}\left(\frac{\ln^4 z}{18} - \frac{\ln^3 z}{18} + \cdots\right) + \cdots$$

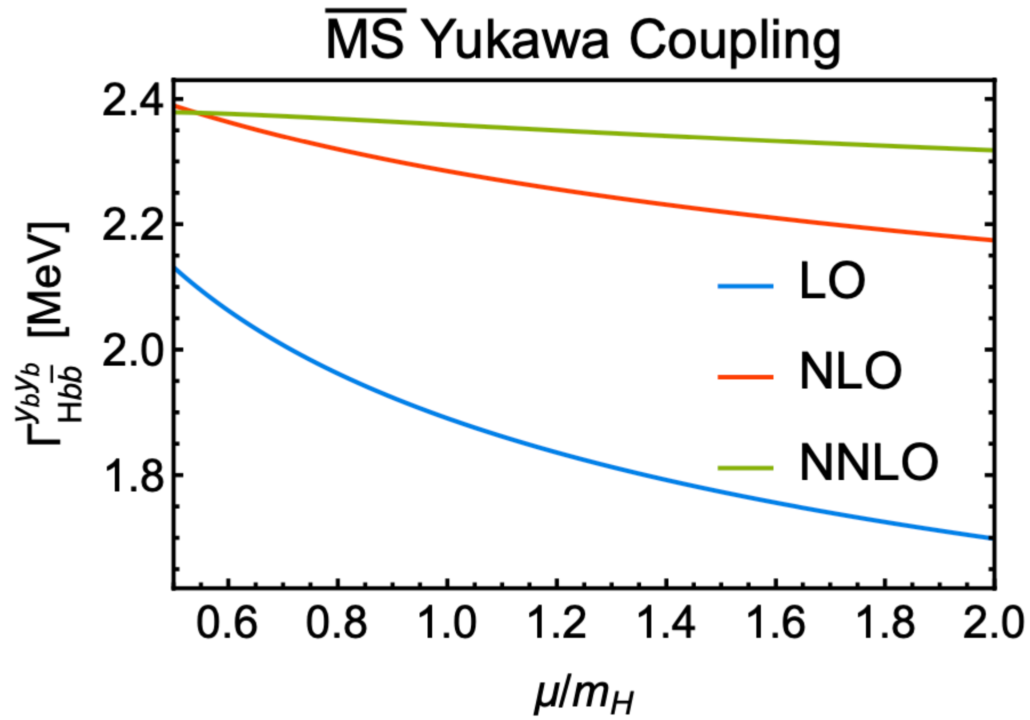
$$z = m_H^2/m_b^2$$

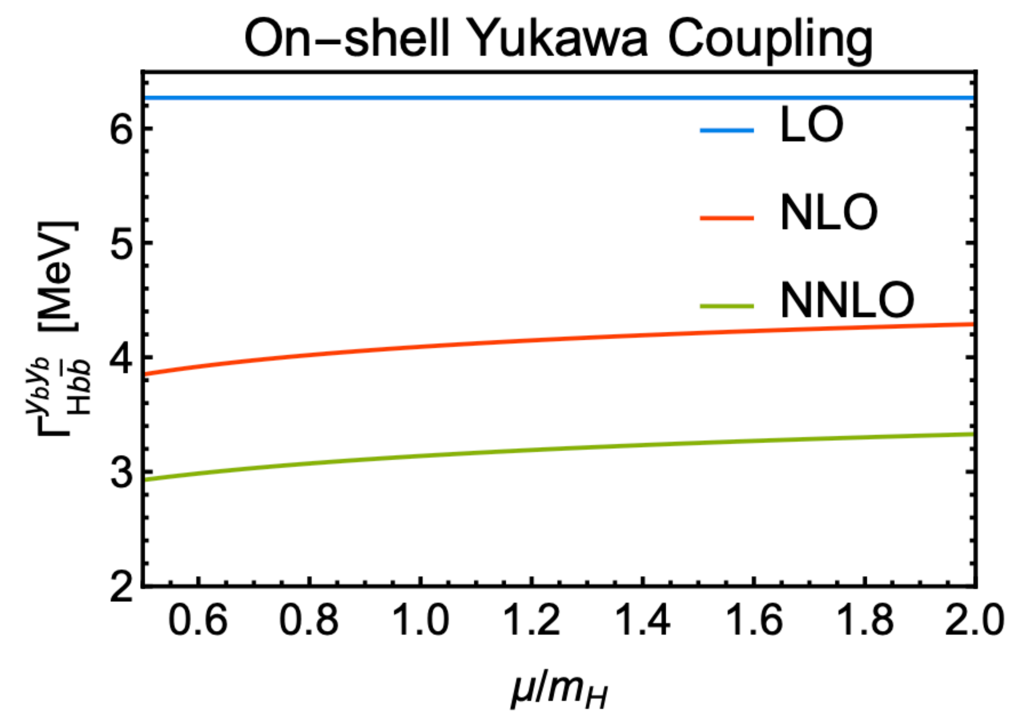
Large logs cancel at $\mathcal{O}(z^0)$, but still survive at $\mathcal{O}(z^{-1})$.

- $X^{y_b y_t}_{2,b\bar b}$ is calculated numerically or as a series of $1/m_t^2$.

[JW, Wang, Zhang, 2023]

Analytical QCD NNLO result





- In the $\overline{\text{MS}}$ scheme, the scale uncertainty is 23%, which is reduced to 9% at NLO and to 3% at NNLO. The NNLO correction is small at $\mu = m_H/2$ and reaches 7% at $\mu = 2m_H$.

- In the onshell scheme, the NLO correction is -35% and the NNLO reduces the width further by 23%. The scale uncertainty is slightly reduced.

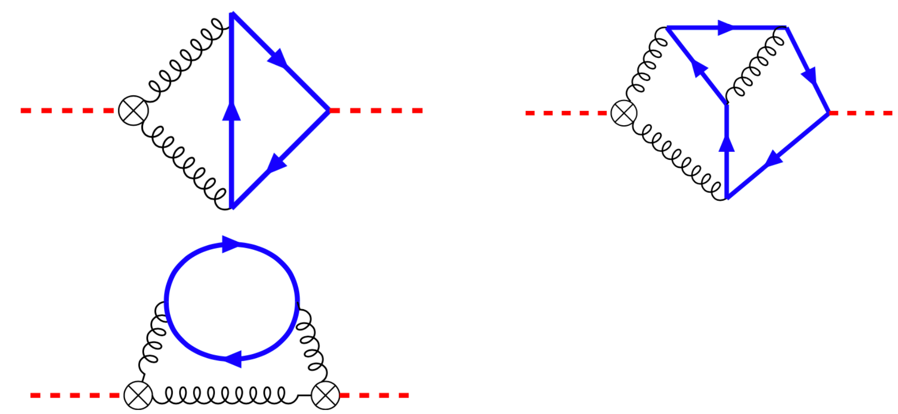
width	LO	NLO	NNLO(y_b^2)	NNLO($y_b y_t$)	N^{3}LO(y_b^2)	N^{4}LO(y_b^2)
$\overline{\text{MS}}$	$1.891^{+0.241}_{-0.192}$	$2.285^{+0.105}_{-0.110}$	$2.359^{+0.020}_{-0.041}$	$2.376^{+0.026}_{-0.046}$	$2.379^{+0.005}_{-0.015}$	$2.377^{+0.006}_{-0.006}$
on-shell	6.269	$4.092^{+0.197}_{-0.242}$	$3.138^{+0.191}_{-0.210}$	$3.193^{+0.181}_{-0.197}$	$2.804^{+0.112}_{-0.096}$	$2.649^{+0.065}_{-0.049}$

[JW, Wang, Zhang, 2023]

Analytical QCD NNNLO result

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right]$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$



The results are expressed in terms of MPLs and elliptic integrals.

$$F_1^{4b}(z) = (z-16)f(z) = \frac{16\pi(z-16)}{z}\left[\mathrm{K}(1-k_-)\mathrm{K}(k_+) - \mathrm{K}(k_-)\mathrm{K}(1-k_+)\right]$$

$$F_4^{4b}(z) = \frac{\beta s F_1^{4b}(z)}{s-4} - \int_{16}^{z} dz_1 \frac{4\beta_1(z_1+2)F_1^{4b}(z_1)}{(z_1-16)(z_1-4)^2}$$

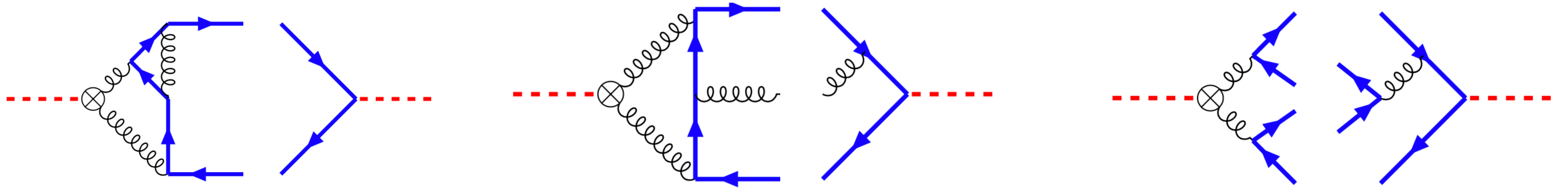
$$\Delta^{C_1C_2}_{1,b\bar{b}}|_{z\to\infty} = \frac{m_H m_b \overline{m_b}(\mu)}{\pi v^2} C_A C_F\left[-\frac{1}{8}\log^2(z) - \frac{3}{4}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{\pi^2}{8} - \frac{19}{8}\right.$$
$$\left. + \frac{1}{2}\frac{\log^2(z)}{z} + 2\frac{\log(z)}{z} + \frac{9}{2z}\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{\pi^2}{2z} + \frac{15}{2z}\right] + \mathcal{O}(z^{-2}),$$

$$\Delta^{C_1C_1}_{1,b\bar{b}}|_{z\to\infty} = \frac{m_H^3}{\pi v^2} C_A C_F\left[\frac{1}{6}\log(z) - \frac{7}{12} + \frac{3}{z}\right] + \mathcal{O}(z^{-2}).$$

- The C_1C_1 channel is power (and log) enhanced.
- The C_1C_2 channel contains double logs, which is induced by soft massive quarks.

[JW, Wang, Wang, 2024]

Analytical QCD NNNLO result



$$-\frac{m_H m_b \overline{m_b}(\mu)}{8\pi v^2} C_A C_F \log^2(z) \times \frac{1}{24} (C_A - C_F) \log^2(z)$$

- Both 2b and 4b final states contribute double logarithmic enhancement. The special color structure is a typical feature of subleading power resummation, which appears in the quark-gluon splitting function [Vogt, 2010, Beneke, Garny, Jaskiewicz, Szafron, Vernazza, JW, 2020], $Hb\bar{b}/Hgg$ form factors [Liu, Penin, 2017, Liu, Neubert, 2020], and off-diagonal "gluon" thrust [Moult, Stewart, Vita, Zhu, 2019, Beneke, Garny, Jaskiewicz, Strohm, Szafron, Vernazza, JW, 2022].

- The subleading logs and all logs at higher powers have not been fully explored.

[JW, Wang, Wang, 2024]

Analytical QCD NNNLO result

$$\Gamma_{H\to b\bar{b}} = \frac{3m_H\overline{m_b}^2}{8v^2\pi}\Bigg\{1+\left(\frac{\alpha_s}{\pi}\right)\frac{17}{3}+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{1}{9}\log^2(\bar{z})-\frac{2}{3}\log(x)-\frac{97\zeta(3)}{6}-\frac{17\pi^2}{12}+\frac{9235}{144}\right]$$

$$+\left(\frac{\alpha_s}{\pi}\right)^3\left[\frac{5}{648}\log^4(\bar{z})+\frac{59}{324}\log^3(\bar{z})-\frac{31\pi^2}{324}\log^2(\bar{z})+\frac{989}{648}\log^2(\bar{z})+\frac{32\zeta(3)}{27}\log(\bar{z})\right.$$

$$-\frac{41\pi^2}{324}\log(\bar{z})+\frac{137}{216}\log(\bar{z})-\frac{23}{18}\log^2(x)-\frac{49}{6}\log(x)+\frac{1945\zeta(5)}{36}-\frac{13\pi^4}{3240}-\frac{81239\zeta(3)}{216}$$

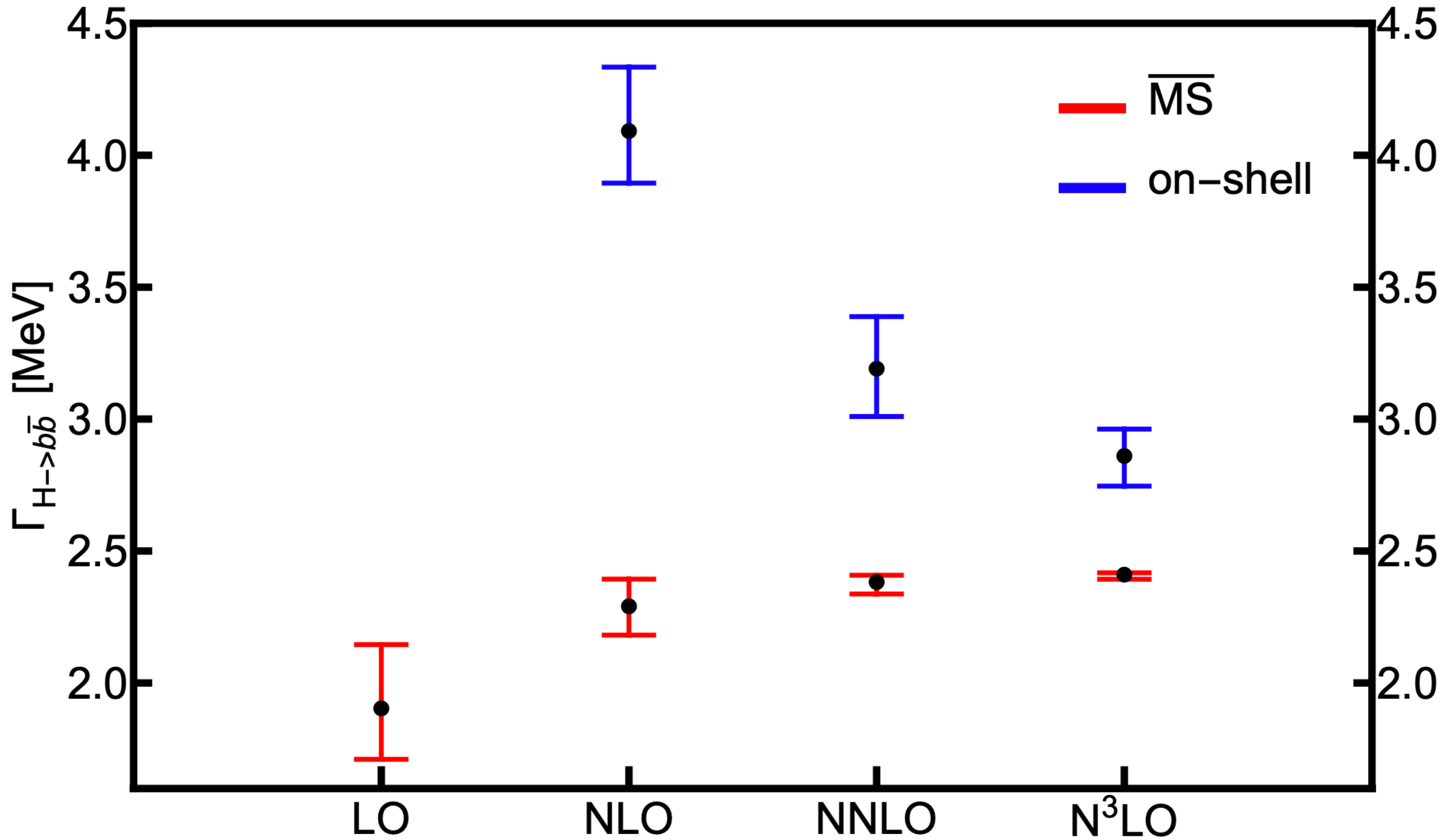
$$\left.-\frac{22291\pi^2}{648}+\frac{37434709}{46656}\right]\Bigg\}+\frac{m_H^3}{v^2\pi}\left(\frac{\alpha_s}{\pi}\right)^3\left[\frac{\log(\bar{z})}{216}-\frac{7}{432}\right]+\mathcal{O}(\bar{z}^{-1})+\mathcal{O}(x)+\mathcal{O}(\alpha_s^4)$$

$$x = m_H^2/m_t^2$$

- The logarithmic terms provide significant corrections.
- The power enhanced term does not play a significant role.
- The large logarithms cancel in the decay to all hadronic states, which agrees with results in [Chetyrkin, Steinhauser 1997, Davies, Steinhauser, Wellmann 2017].

[JW, Wang, Wang, 2024]

Analytical QCD NNNLO result

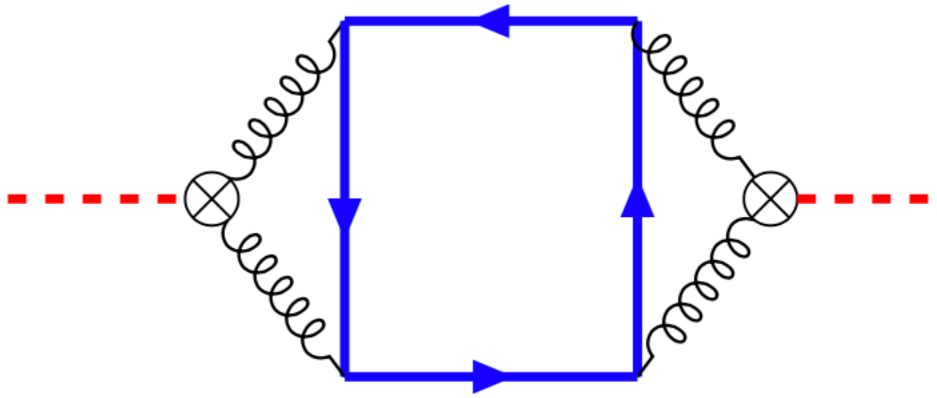


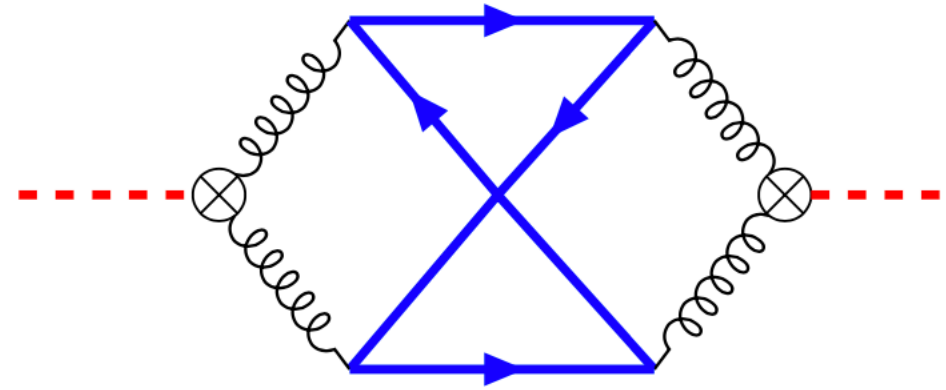
- The $\mathcal{O}(\alpha_s^3)$ correction increases the decay rate by 1%, much larger than naive α_s power-counting expectation.

$$\Gamma_{H\to b\bar{b}}^{\mathrm{N^3LO\ \ QCD}}\left(\overline{\mathrm{MS}}\right) = 2.410^{+0.007}_{-0.017}\ \mathrm{MeV}$$

[JW, Wang, Wang, 2024]

Partial QCD N4LO result





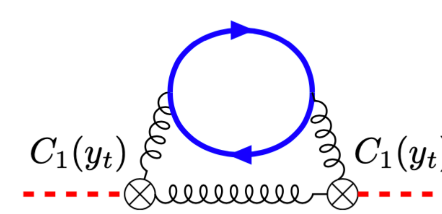
$$F_{37,b\bar{b}}(z) = \int_4^z \frac{\left(K\left(-\frac{x}{16}\right)K\left(\frac{z+16}{16}\right) - K\left(\frac{x+16}{16}\right)K\left(-\frac{z}{16}\right)\right)(x+16)\sqrt{z}}{\pi\sqrt{x}} R_{b\bar{b}}(x)dx,$$

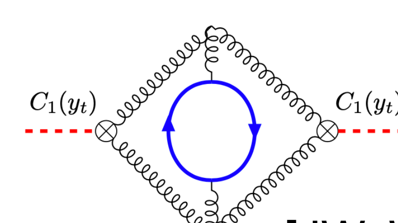
$$F_{38,b\bar{b}}(z) = \int_4^z \frac{2\left(K\left(\frac{x+16}{16}\right)E\left(-\frac{z}{16}\right) + K\left(-\frac{x}{16}\right)\left(E\left(\frac{z+16}{16}\right) - K\left(\frac{z+16}{16}\right)\right)\right)(x+16)\sqrt{z}}{\pi(z+16)\sqrt{x}} R_{b\bar{b}}(x)dx,$$

- $R_{b\bar{b}}$ is a linear combination of logarithmic and Li2 functions.
- Similar result for the 4b final state with R_{4b} being one-fold integrals

$$\Delta^{C_1C_1}_{1,b\bar{b}}|_{m_b\to 0} = -\frac{m_H^3}{6\pi v^2} C_A C_F \log\left(\frac{m_b^2}{m_H^2}\right) + \cdots$$

$$\Delta^{C_1C_1}_{2,b\bar{b}}|_{m_b\to 0} = -\frac{m_H^3}{72\pi v^2} C_A^2 C_F \log^3\left(\frac{m_b^2}{m_H^2}\right) + \cdots$$





[JW, Wang, Wang, 2026]

Partial QCD N4LO result

- The ratio of leading logarithms of $\Delta_{2,b\bar{b}}^{C_1C_1}$ over that of $\Delta_{1,b\bar{b}}^{C_1C_1}$ at $\mathcal{O}(m_b^2)$ is $-\frac{1}{288}C_F\ln^4 z$, which arises from the diagram with exchanges of two soft b-quarks.

$\mu = m_H$	[MeV]	$\Gamma_{Hb\bar{b}}^{C_2C_2}$	$\Gamma_{Hb\bar{b}}^{C_1C_2}$	$\Gamma_{Hb\bar{b}}^{C_1C_1}$	
$\Gamma_{H\to b\bar{b}}\,(\overline{\mathrm{MS}})$	$\mathcal{O}(\alpha_s^0)$	1.9076	-	-	
	$\mathcal{O}(\alpha_s^1)$	0.3873	-	-	
	$\mathcal{O}(\alpha_s^2)$	0.0735	0.0183	-	1/4
	$\mathcal{O}(\alpha_s^3)$	0.0048	0.0142	0.0090	5x
	$\mathcal{O}(\alpha_s^4)$	-0.0025	*	0.0087	+0.4%

Slow convergence

$$\Gamma_{H\to b\bar{b}}^{\mathrm{N^4LO\ QCD}}\,(\overline{\mathrm{MS}}) = 2.421^{+0.008}_{-0.010}(\mathrm{scl.})^{+0.005}_{-0.005}(\alpha_s)\ \mathrm{MeV}$$

[JW, Wang, Wang, 2026]

Mixed QCD×EW correction

- Previous calculation was performed with approximation and omitted the contribution from $H \to Zb\bar{b}$. Dependence on the EW renormalization schemes is not discussed.
- Keep all the masses at their physical values and expand in m_b^2.

On-shell scheme

[MeV]	LO	+NLO EW	+NLO QCD	+QCD *EW
$\alpha(0)$	5.33	5.45 (9%)	3.57	3.54 (3%)
$\alpha(M_Z)$	5.66	5.45 (5%)	3.44	3.54 (1%)
G_μ	5.53	5.48 (18%)	3.52	3.54 (6%)
diff.	6%	0.5%	3%	<0.1%

-0.8%

+3%

+0.6%

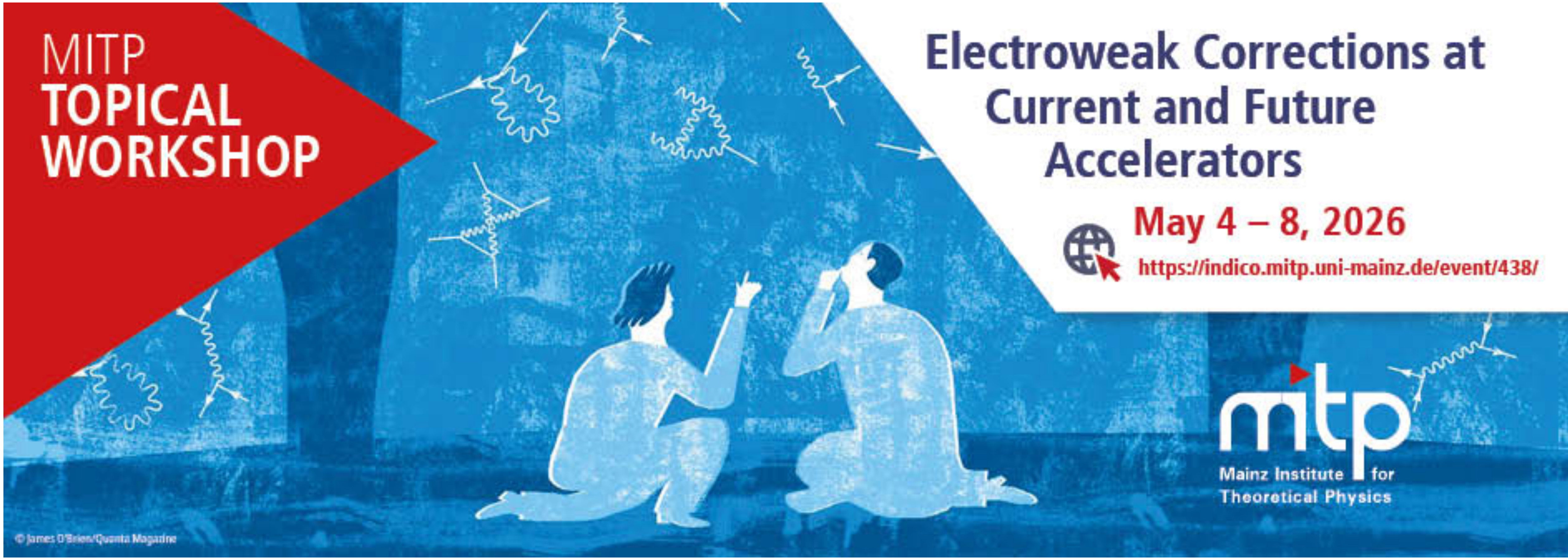
Preliminary

[Chen, Sang, JW, Wang, 2026]

Conclusion

Decay channels	$b\bar{b}$	$c\bar{c}$	gg	WW^*	ZZ^*
BR	57.7%	2.9%	8.6%	21.5%	2.6%
Rel. Stat. Un.	0.3%	2.2%	1.3%	1.1%	7.6%
Rel. Syst. Un.	0.1%	3.6%	1.8%	0.4%	4.3%
Rel. Total Un.	0.3%	4.2%	2.2%	1.2%	8.7%

- Ultimate precision is required at future lepton colliders.
- The QCD corrections to $\Gamma_{H\to b\bar{b}}$ have been (will be) obtained at $\mathcal{O}(\alpha_s^4)$, while the EW corrections are computed at $\mathcal{O}(\alpha_s\alpha)$.
- Higher-order corrections to the other decay channels are also crucial.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Thanks!