Analytic techniques for electroweak corrections for current and future colliders

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CERN

MITP Workshop — Mainz 2026

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MARIE CURIE ACTIONS

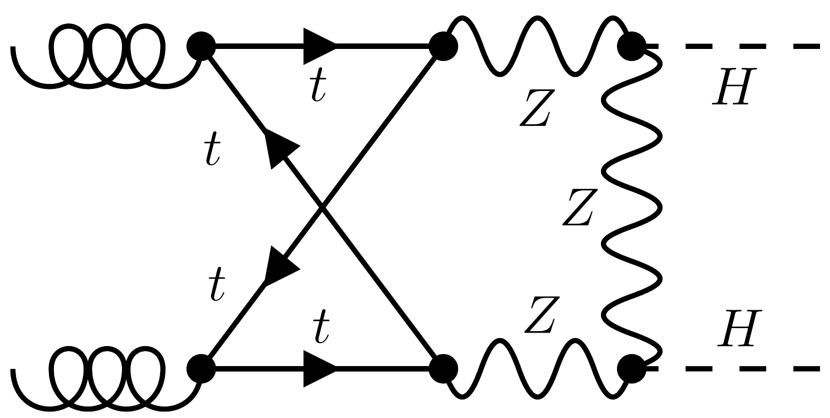
Can we compute two-loop EW corrections analytically?

Yes, e.g. Higgs pair production

Large-mass expansion

Davies, Schönwald, Steinhauser, Zhang

JHEP 10 (2023) 033



High-energy expansion

Davies, Schönwald, Steinhauser, Zhang

JHEP 08 (2022) 259

JHEP 04 (2025) 193

2603.08789

&

Zhang, JHEP 09 (2024) 069

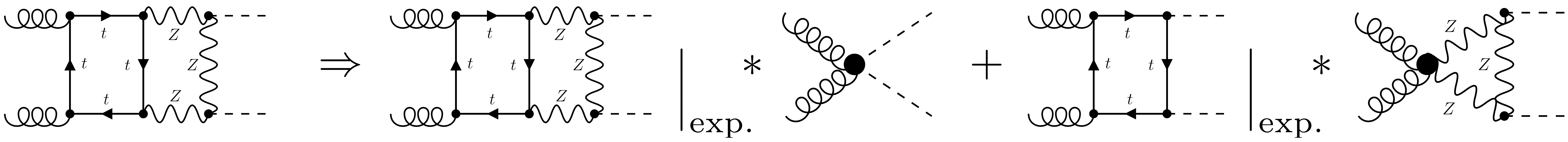
Applicable to low-energy experiments and BSM physics @ FCC-ee

For SM @ LHC

Large-m_t expansion and EW renormalisation

[Davies, Schönwald, Steinhauser, **Zhang**, JHEP 10 (2023) 033]

- Expansion hierarchy: $m_t^2 \gg s, t, m_H^2, m_W^2, m_Z^2$
- Expand and calculate in **general R_ξ gauge** with large gauge fixing parameters $\xi_Z, \xi_W \gg 1$

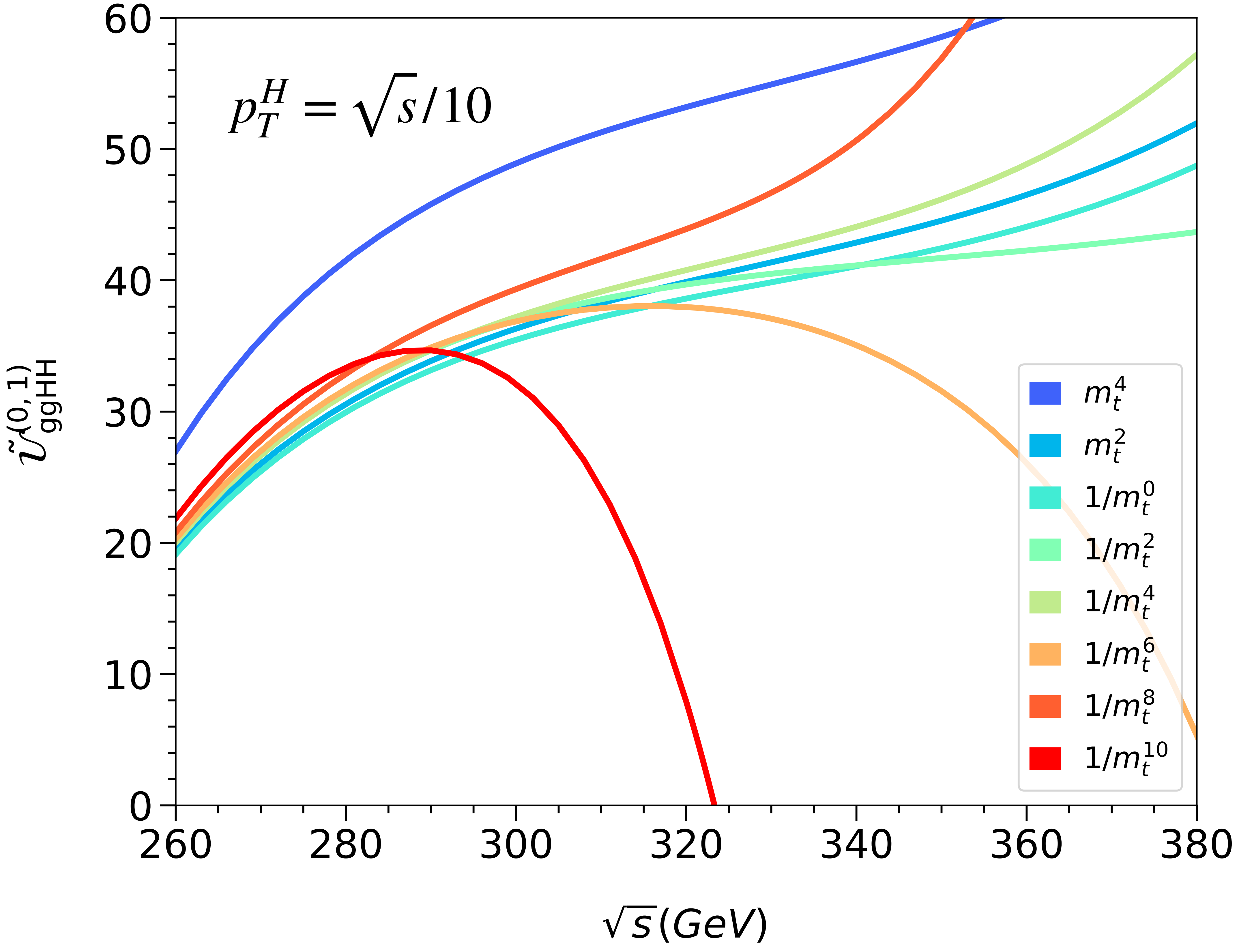


- **On-Shell renormalise** input parameters $\{e, m_W, m_Z, m_t, m_H\}$ in G_μ scheme
- ξ_W, ξ_Z **cancel** after external Higgs **fields OS renormalisation**

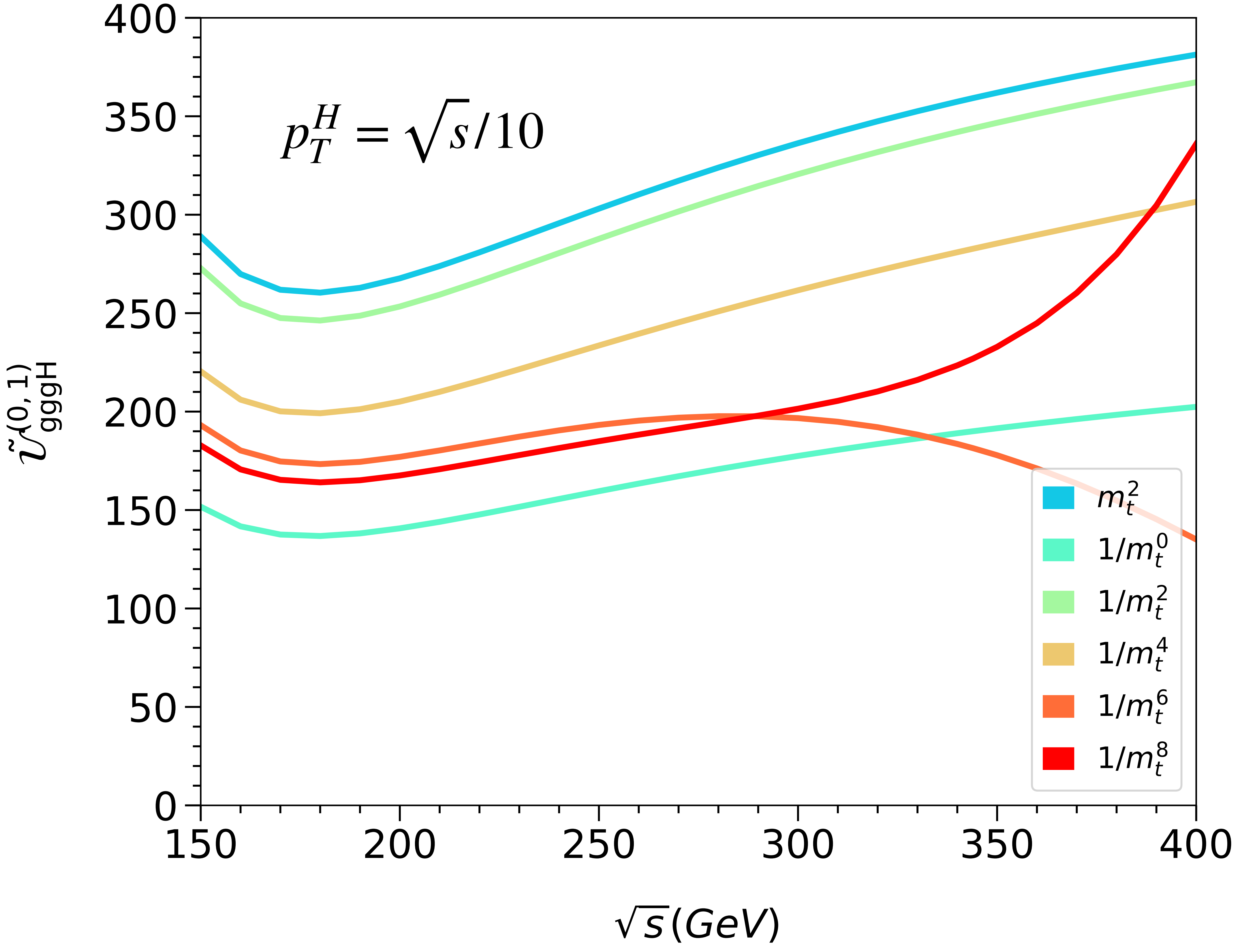
Large-m_t expansion to $gg \to HH$ and $gg \to Hg$

[Davies, Schönwald, Steinhauser, **Zhang**, JHEP 10 (2023) 033]

$$\mathscr{M} = X_0 \sum_{\text{col}} \sum_{\text{pol}} |\mathscr{A}|^2 \propto \tilde{\mathscr{U}}^{(0)}_{\text{HH/H+j}} + \frac{\alpha}{\pi} \tilde{\mathscr{U}}^{(0,1)}_{\text{HH/H+j}}$$



$\tilde{\mathscr{U}}^{(0,1)}_{\text{HH}}$ plot up @ NLO EW



$\tilde{\mathscr{U}}^{(0,1)}_{\text{H+j}}$ plot @ NLO EW

Large-mass expansion prospects @ FCC-ee

Large-m_t expansion works perfectly for NNLO mixed QCD-EW corrections for $e^+e^- \to ZH$ for $\sqrt{s} < 300 GeV$

[Gong, Li, Xu, Yang, Zhao, Phys.Rev.D 95 (2017) 9, 093003]
[Wang, Xu, Yang, Phys.Rev.D 100 (2019) 7, 071502]

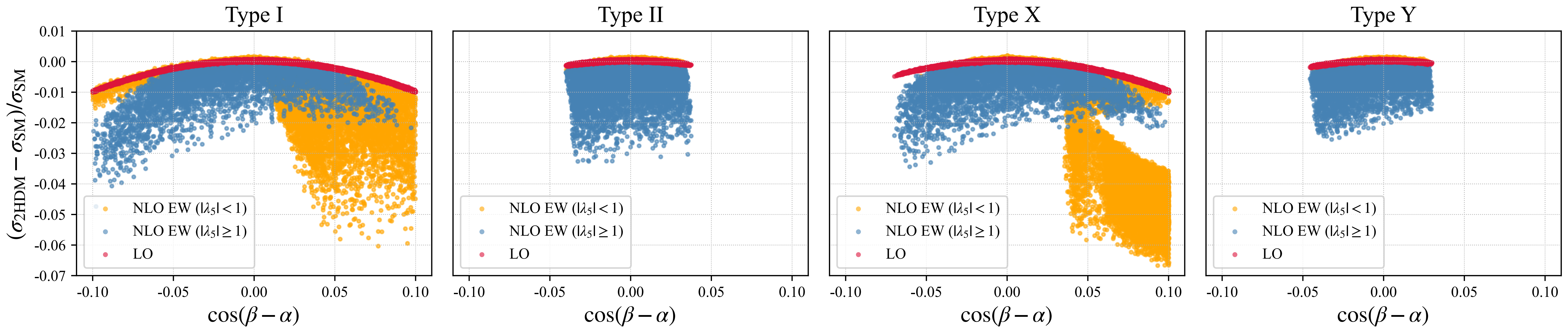
For BSM scale $\Lambda > \sqrt{s}$

Large-Λ expansion for FCC-ee processes at NNLO (two loops)

BSM at NLO EW for FCC-ee processes is important and necessary

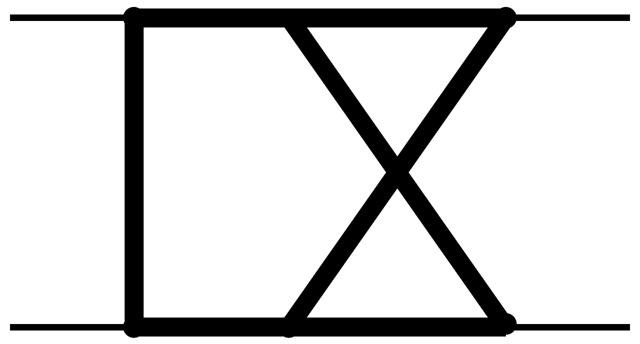
[Bredt, Banno, Höfer, Iguro, Kilian, Ma, Reuter, **Zhang**, Phys.Rev.Lett. 136 (2026) 8, 081801]

Relative differences between **2HDM** and **SM** predictions for $e^+e^- \to H\nu\bar{\nu}$ at $\sqrt{s} = 365$ GeV



LO (red) and NLO EW (orange for $|\lambda_5| < 1$, blue for $|\lambda_5| > 1$)

Roadmap for analytic EW at high energies



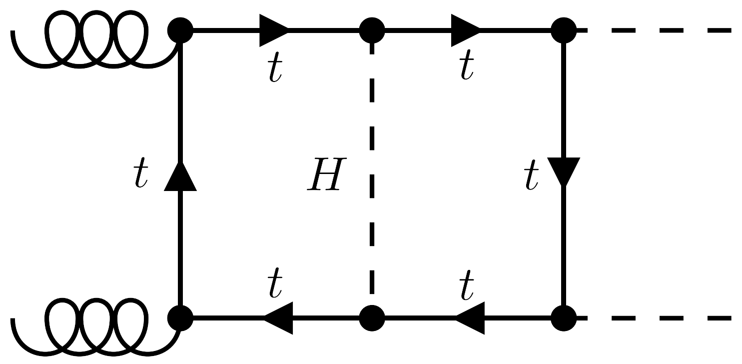
AsyInt toolbox for
master integrals (MIs) in high-energy limit

[**Zhang**, JHEP 09 (2024) 069]

Download at: https://gitlab.com/asyint/asyint-public

[Davies, Mishima Schönwald, Steinhauser, **Zhang**,
JHEP 08 (2022) 259]

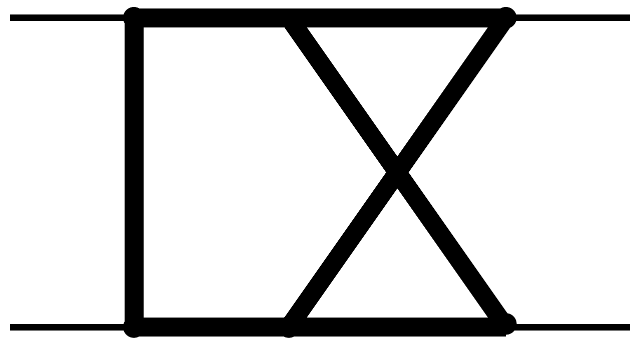
First attempt on leading
Yukawa corrections



Recent (semi)-analytic EW calculations

- Two-loop logarithmic (Sudakov) EW corrections in **OpenLoops** for V+jets [Lindert, Mai, 2604.14320]
- Mixed QCD-EW corrections to Drell-Yan, e.g. [Bonciani et al. 2106.11953, Buccioni et al., 2203.11237]

Roadmap for analytic EW at high energies



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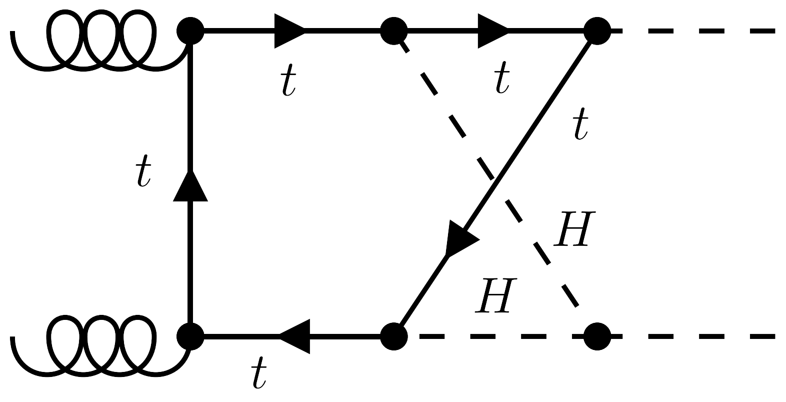

Full set of MIs in deep high-energy expansion
&
Yukawa and Higgs self-coupling corrections

[Davies, Schönwald, Steinhauser, **Zhang**,
JHEP 04 (2025) 193]

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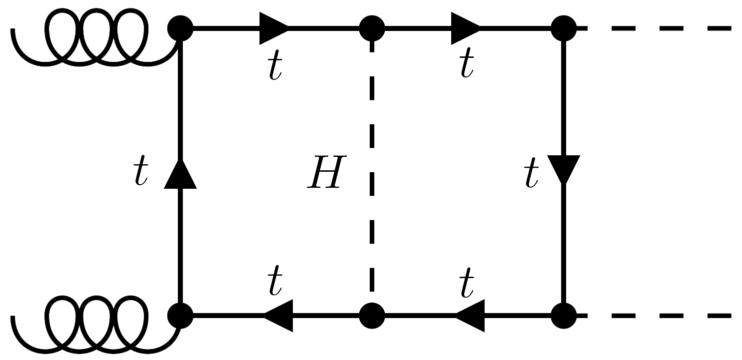

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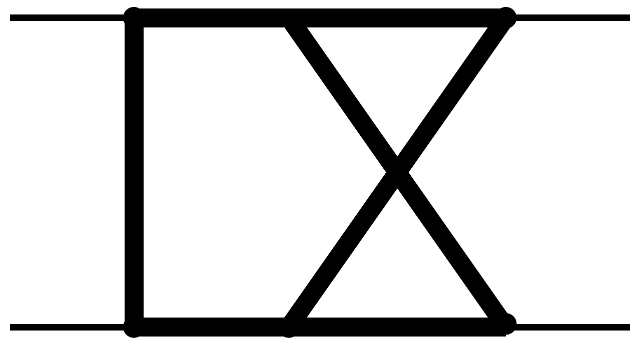

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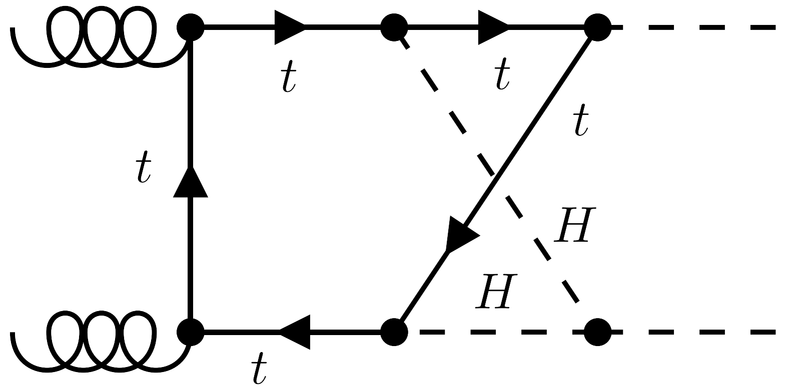

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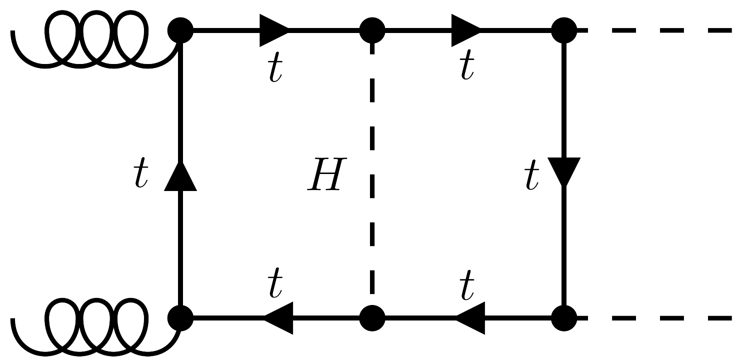
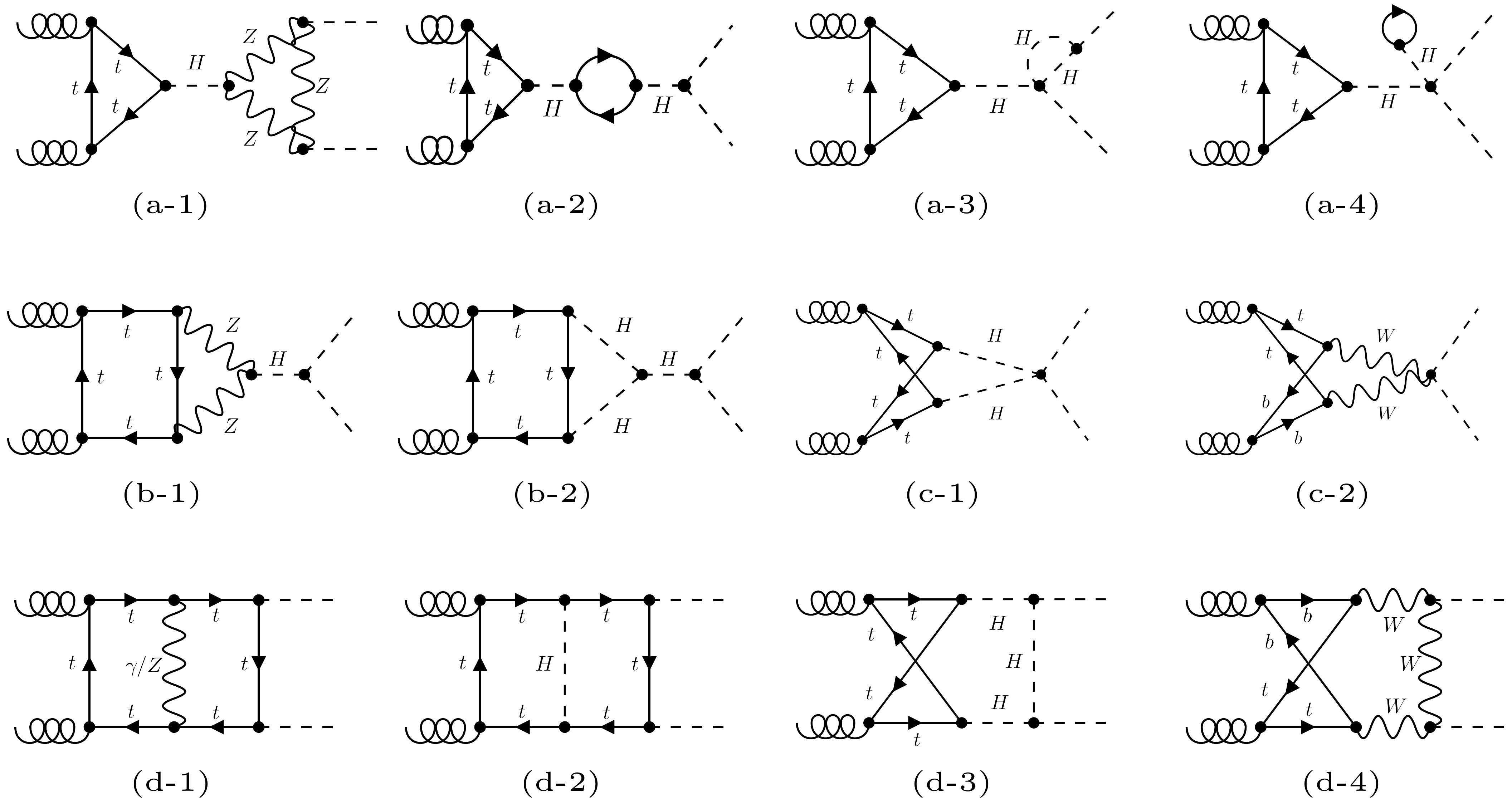


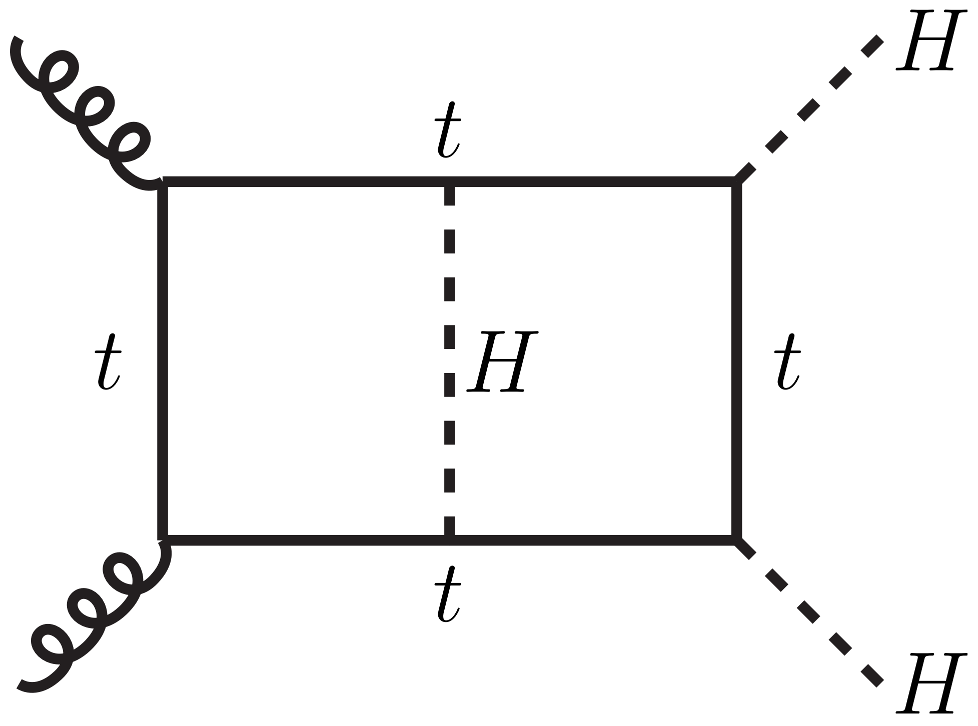


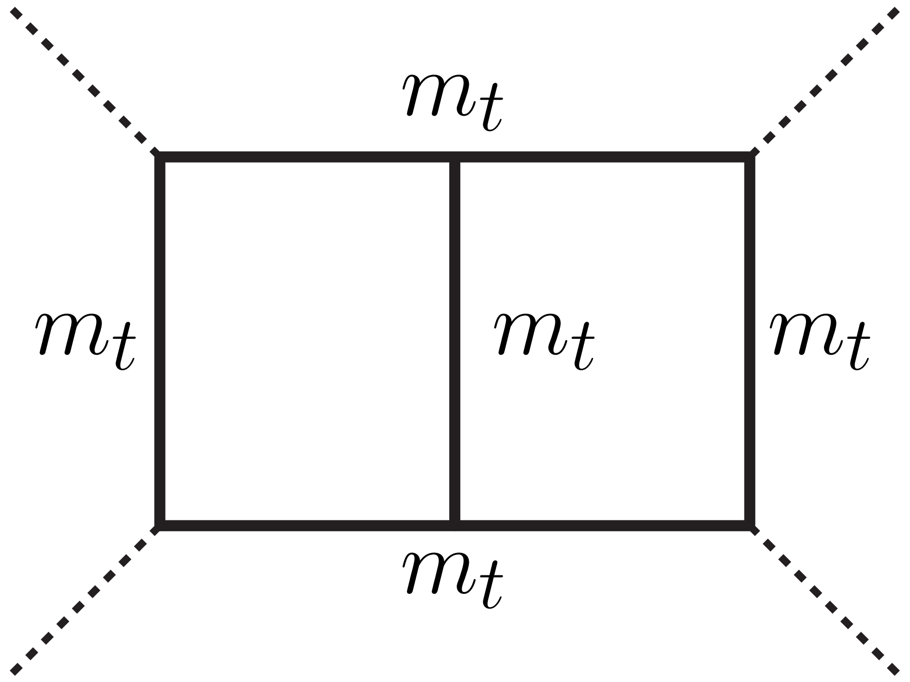
[Davies, Schönwald, Steinhauser, **Zhang**,
2603.08789]

Full top-quark contribution at NLO EW

Expansion strategies at high energies

- At high energies, SM masses are of a similar order: $m_t^2 \approx m_W^2, m_Z^2, m_H^2 \ll s, |t|$
- Two **Taylor expansions**: equal-internal-mass and small-external-mass expansions





$s, t \gg m_t^2$

High-energy expansion

$$\delta_X = 1 - \frac{m_X}{m_t}, \;\; X = H, W, Z$$

High energy expansion of master integrals

1. **Asymptotic expansion:** $s, t \gg m_t^2$

2. **System of differential equations for Master Integrals** from IBP reduction [**Kira, FIRE**]

$$\frac{\partial}{\partial\left(m_t^2\right)}\mathbf{I} = M(s,t,m_t^2,\epsilon)\,\mathbf{I} \quad \text{with} \quad \mathbf{I} = \left(\mathcal{I}_1, \ldots, \mathcal{I}_N\right)^T$$

3. Plug in **power-log ansatz** for each master integral

$$\mathcal{I}_n = \sum C_{(n)}^{ijk}(s,t)\,\epsilon^i\left[m_t^2\right]^j\left[\log(m_t^2)\right]^k$$

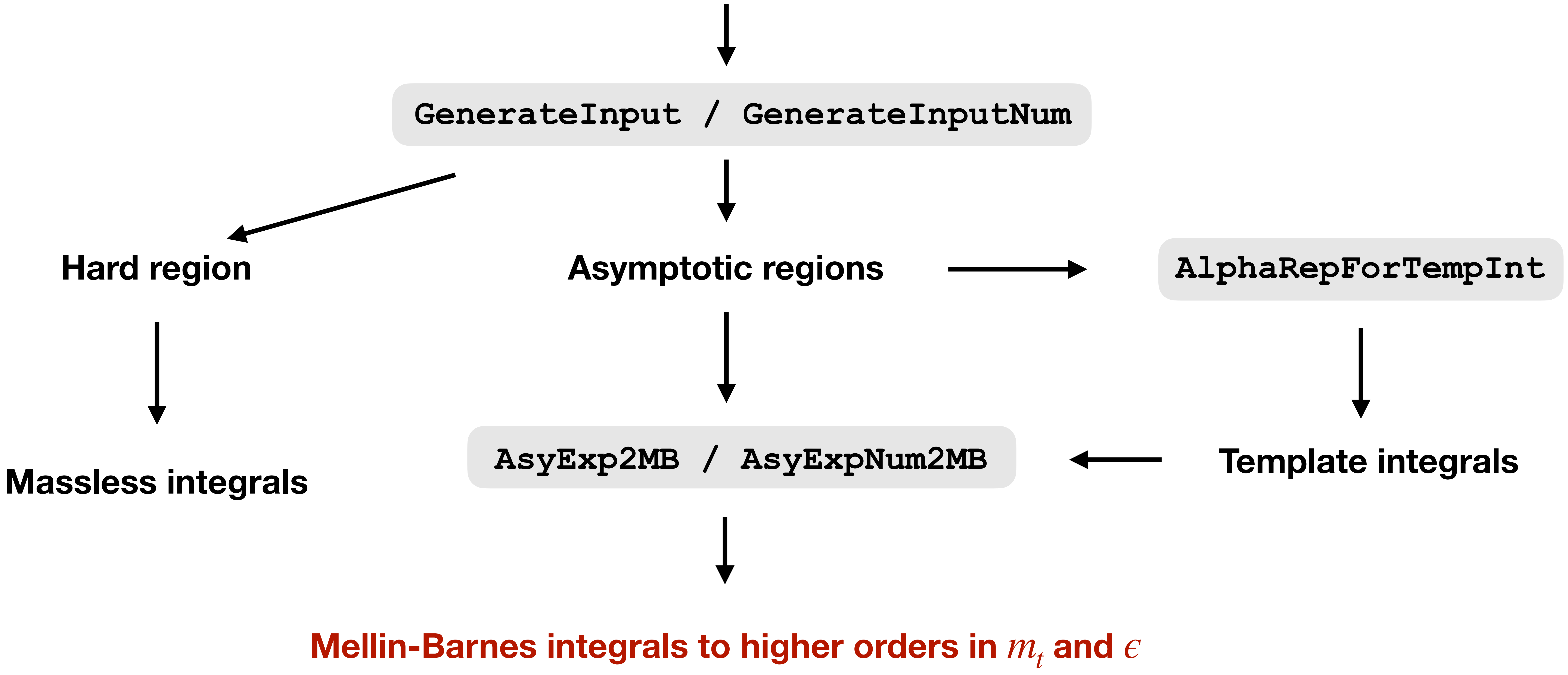
4. Solve **boundary master integrals** in $m_t^2/s \to 0$ to higher orders in m_t^2 and ϵ using **AsyInt**

AsyInt toolkit I: generate MB-integral representations

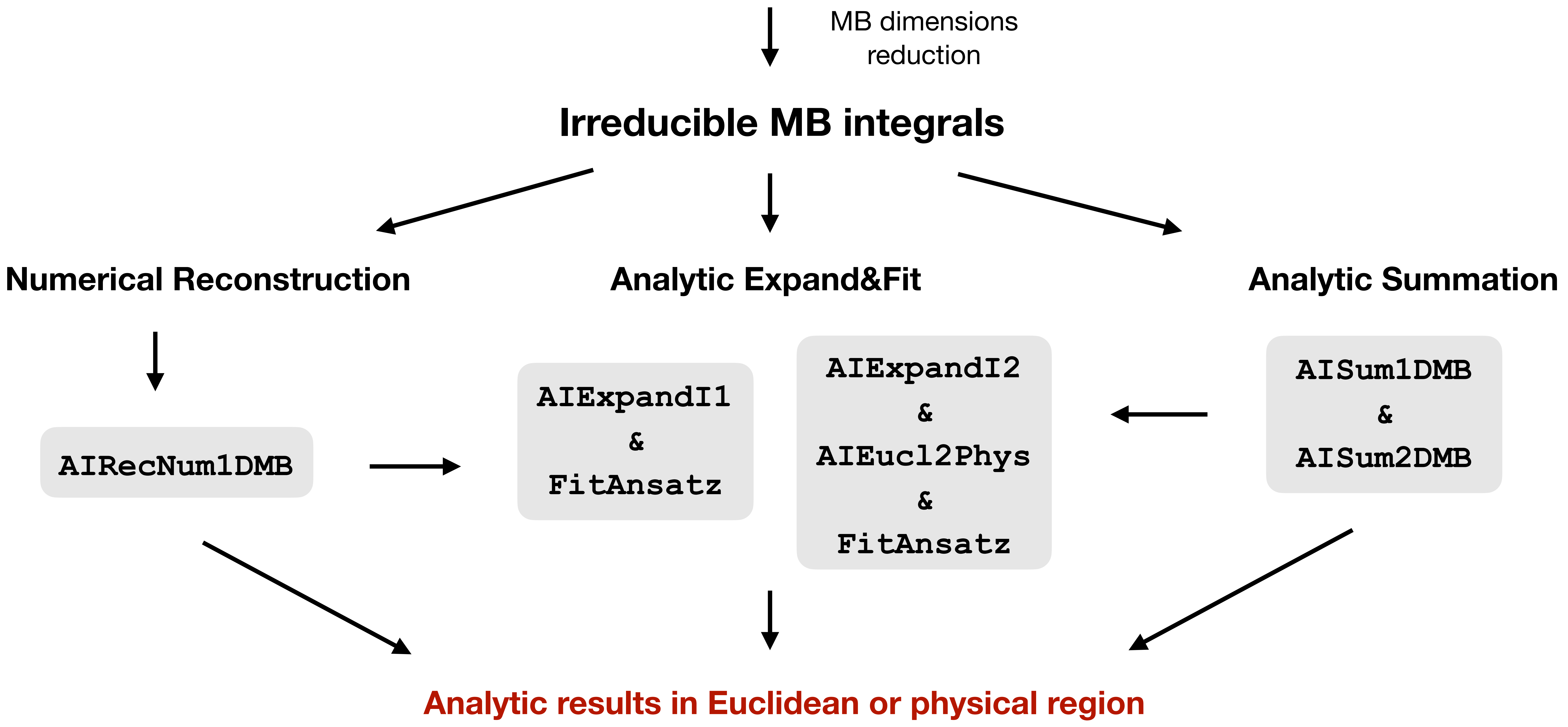
- Two-loop Feynman integral with n propagators and k numerators

$$\mathcal{I}_{n,k} = \int \prod_{j=1}^{2} \mathrm{d}l_j \, \frac{N_1^{\lambda_1} \cdots N_k^{\lambda_k}}{D_1^{1+\delta_1} \cdots D_n^{1+\delta_n}}$$

δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)



AsyInt toolkit II: solve MB integrals



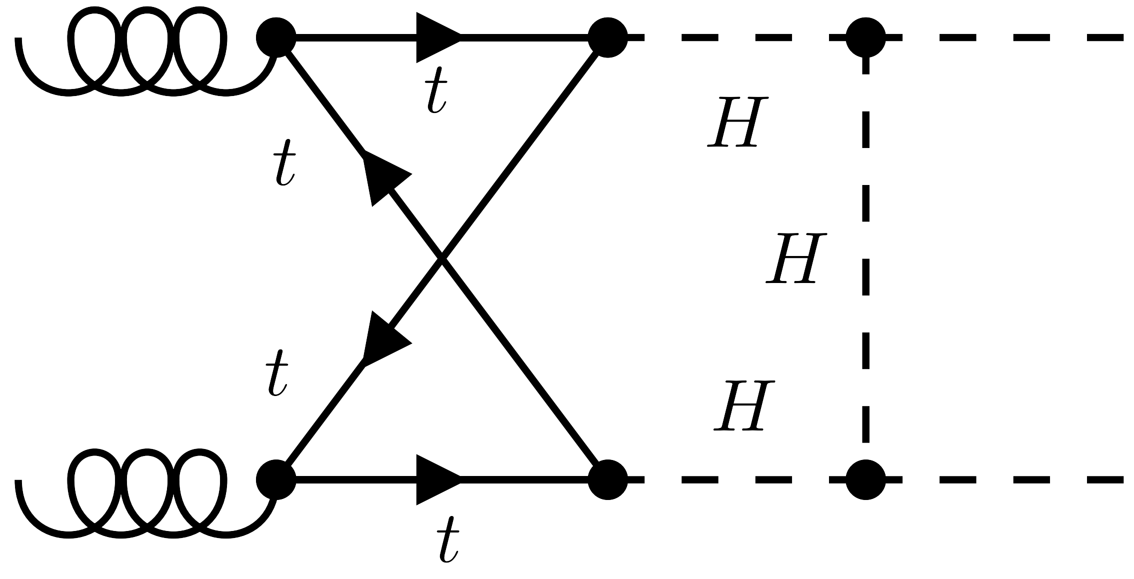
Expand&Fit for complicated irreducible MB integrals

(type-1): 2-dim 1-scale MB integrals with non-vanishing arc contributions

(type-2): 2-dim 2-scale MB integrals

Comparison of Yukawa and Higgs self-coupling corrections

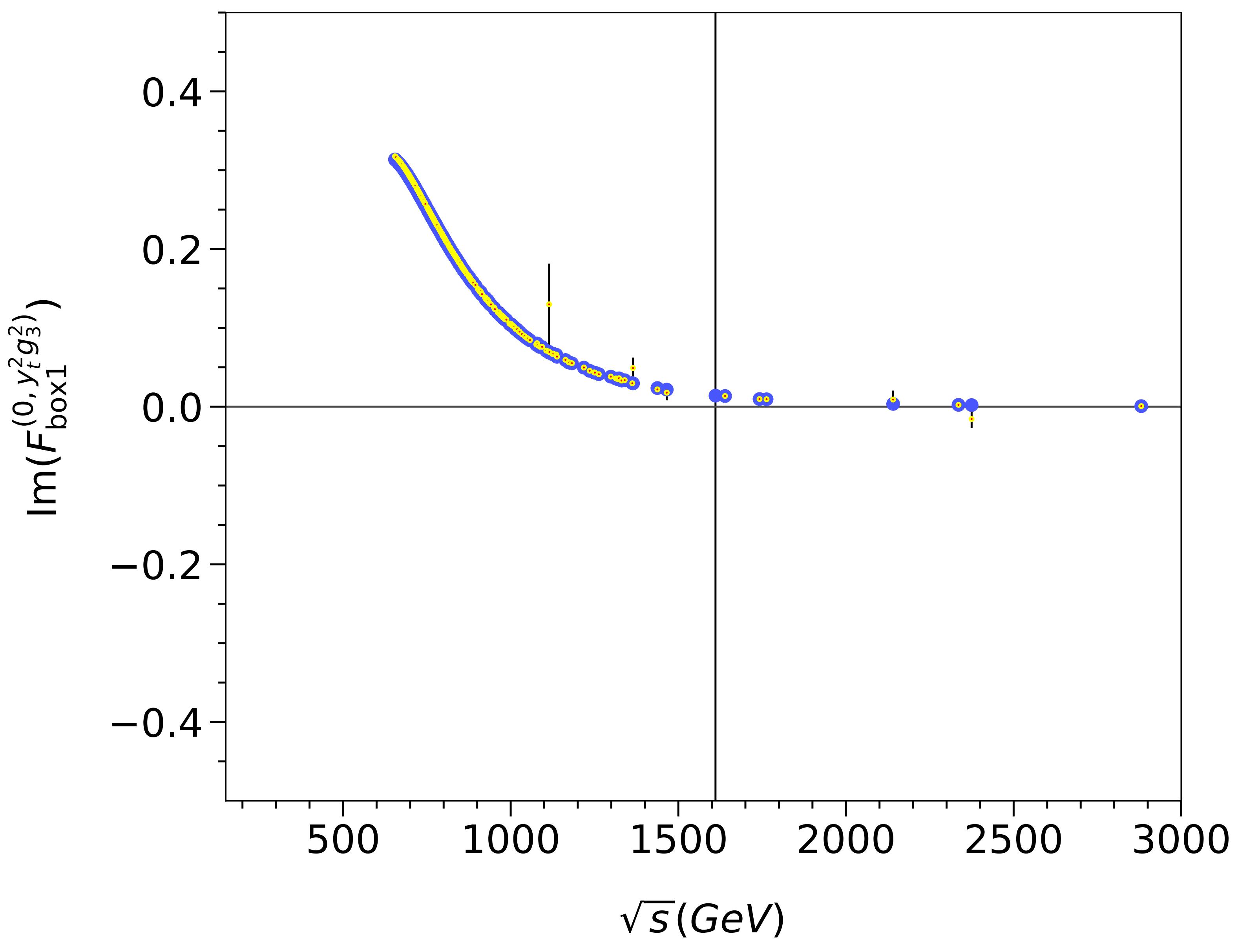
[Davies, Schönwald, Steinhauser, **Zhang**, JHEP 04 (2025) 193]



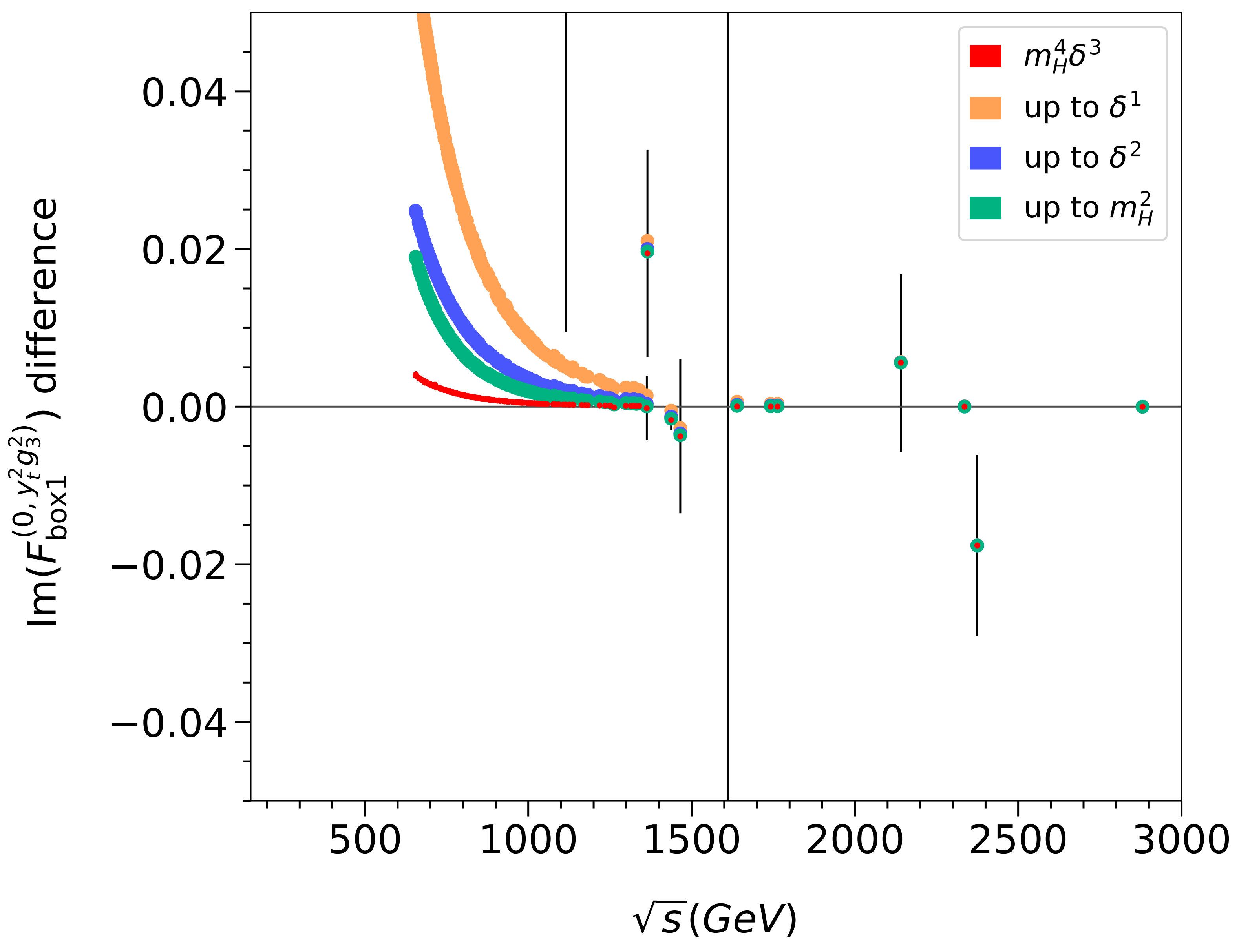
Comparison to numerical results from SecDec group - $y_t^2 \lambda^2$ (bare) form factor

[Heinrich, Jones, Kerne, Stone, Vestner, JHEP 11 (2024) 040]

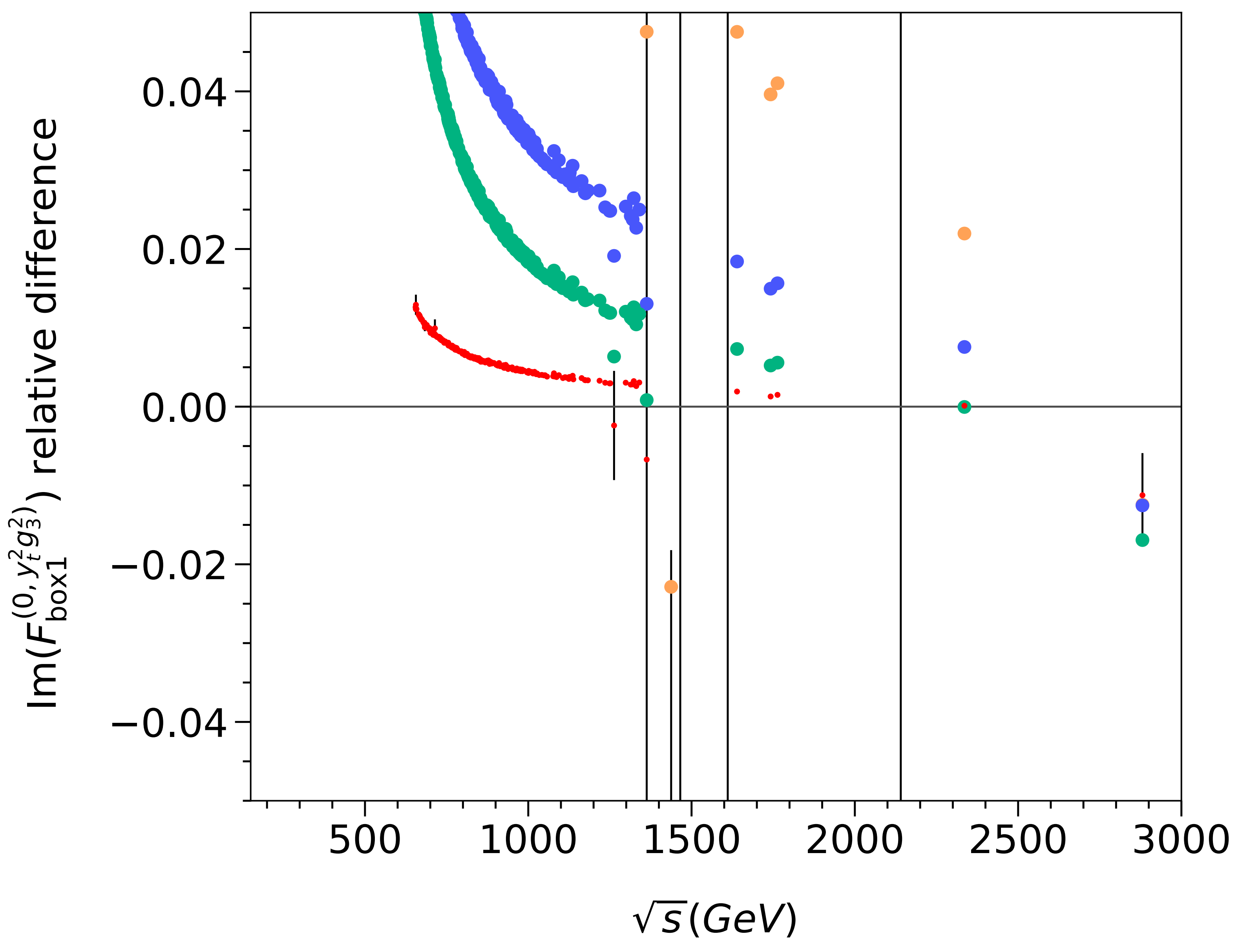
Note the expansion is only up to δ^3 order ($\delta = 1 - m_H/m_t$) for this comparison



$F_{\mathrm{box}}^{(y_t^2 \lambda^2)}$ form factor



Absolute difference



Relative difference

Analytic EW results - form factors

[Davies, Schönwald, Steinhauser, **Zhang**, 2603.08789]

Perturbative QCD and EW expansion: $F = F^{(0)} + \frac{\alpha_s(\mu)}{\pi} F^{(1,0)} + \frac{\alpha}{\pi} F^{(0,1)} + \cdots$

Central EW mass scale $m_{EW} := m_t$

Deep high-energy expansion: $F = \sum_{n=-4}^{108} \sum_{i=0}^{2} \sum_{k_W=0}^{4} \sum_{k_Z=0}^{4} \sum_{k_H=0}^{4} c_{ni}^{k_W k_Z k_H} \, m_t^n \, (m_H^{\text{ext}})^{2i} \, \delta_W^{k_W} \, \delta_Z^{k_Z} \, \delta_H^{k_H}$, with $\delta_X = 1 - \frac{m_X}{m_t}$

- **In the $m_t \to 0$ and $m_H = 0$ limits**, one-loop box form factors are

$$F^{(0)}_{\text{box1}} = \frac{8\,m_t^2}{s} + \mathcal{O}\left(\frac{m_t^4}{s^2}, \frac{m_H^2}{s}\right)$$

$$F^{(0)}_{\text{box2}} = \frac{2m_t^2}{st(s+t)}\left[-l_{1ts}^2 (s+t)^2 - l_{ts}^2 t^2 - \pi^2 (s^2 + 2st + 2t^2)\right] + \mathcal{O}\left(\frac{m_t^4}{s^2}, \frac{m_H^2}{s}\right)$$

$l_{ms} = \log\left(\frac{m_t^2}{s}\right) + i\pi$, $l_{ts} = \log\left(-\frac{t}{s}\right) + i\pi$, $l_{1ts} = \log\left(1 + \frac{t}{s}\right) + i\pi$, $c_w = \frac{m_W}{m_Z}$

$gg \to HH$ at NLO EW contains several coupling structures
Logarithmic / power corrections in different limits
can be obtained from the ancillary file
https://www.ttp.kit.edu/preprints/2026/ttp26-006

- Two-loop box form factors are

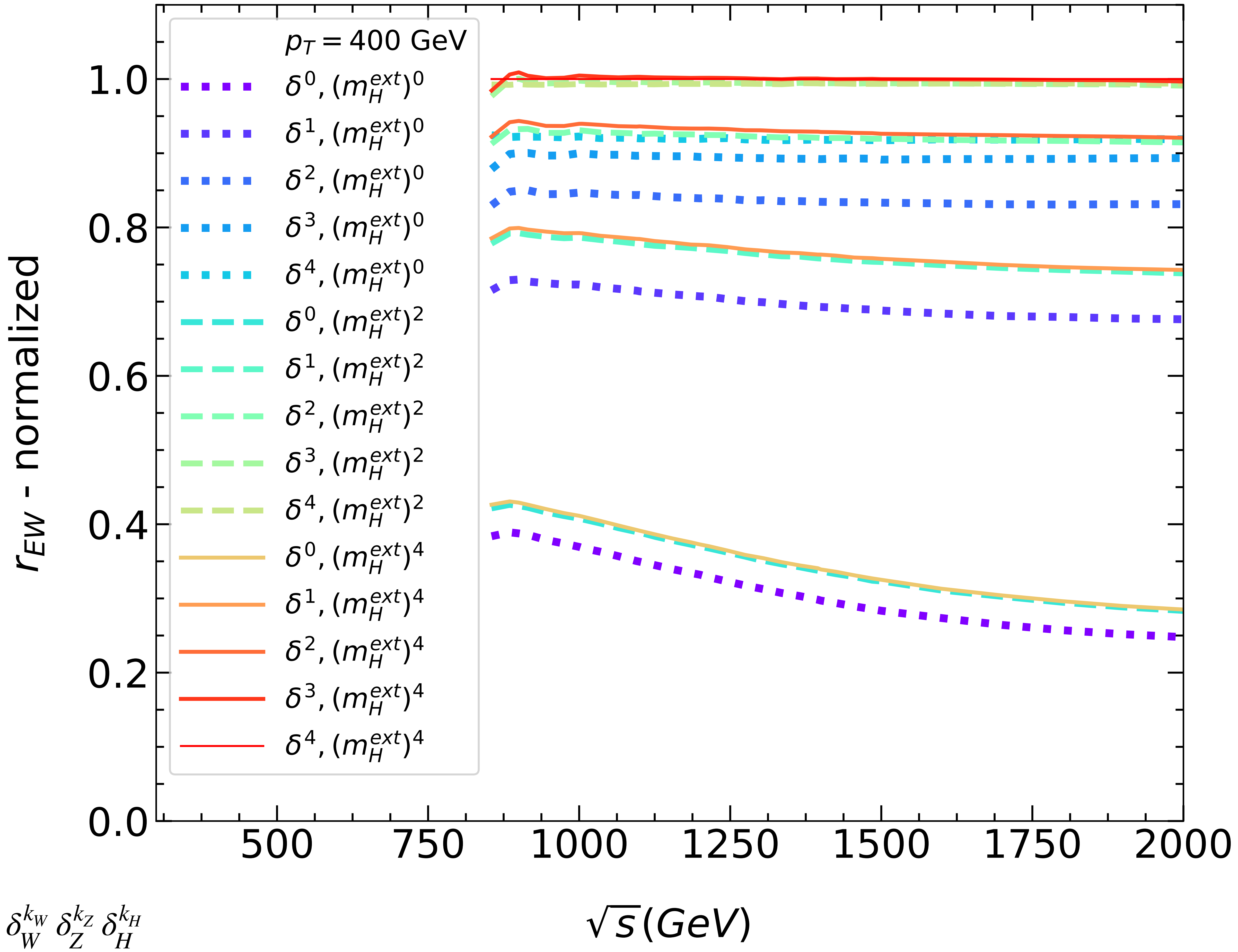
$$F^{(0,1)}_{\text{box1}} = \frac{m_W^4}{s^2 c_w^6 s_w^2}\left[l_{ms}^2 \frac{36c_w^6 + 32c_w^4 - 40c_w^2 + 17}{36}\left(l_{1ts}^2 + l_{ts}^2 - 2l_{1ts}l_{ts} + \pi^2 - 4\right)\right] + \cdots$$

$$F^{(0,1)}_{\text{box2}} = \frac{m_W^4}{s^2 c_w^6 s_w^2}\left[l_{ms}^3 \frac{36c_w^6 + 32c_w^4 - 40c_w^2 + 17}{27} + l_{ms}^2 \left(-\frac{2}{9}\,\delta_Z \left(32c_w^4 - 40c_w^2 + 17\right) - 8\,\delta_W\, c_w^6 + \cdots\right)\right] + \cdots$$

Analytic EW results - convergence

[Davies, Schönwald, Steinhauser, **Zhang**, 2603.08789]

$$r_{\rm EW} = \frac{\alpha}{\pi}\frac{\mathscr{U}^{(0,1)}}{\mathscr{U}^{(0)}}$$ in perturbative expansion of squared matrix element $|\mathscr{M}|^2 = \bar{X}_0\left(\mathscr{U}^{(0)} + \frac{\alpha_s}{\pi}\mathscr{U}^{(1,0)} + \frac{\alpha}{\pi}\mathscr{U}^{(0,1)} + \cdots\right)$



We estimate a conservative $1\,\%$ uncertainty due to truncation of higher-order δ, m_H expansion

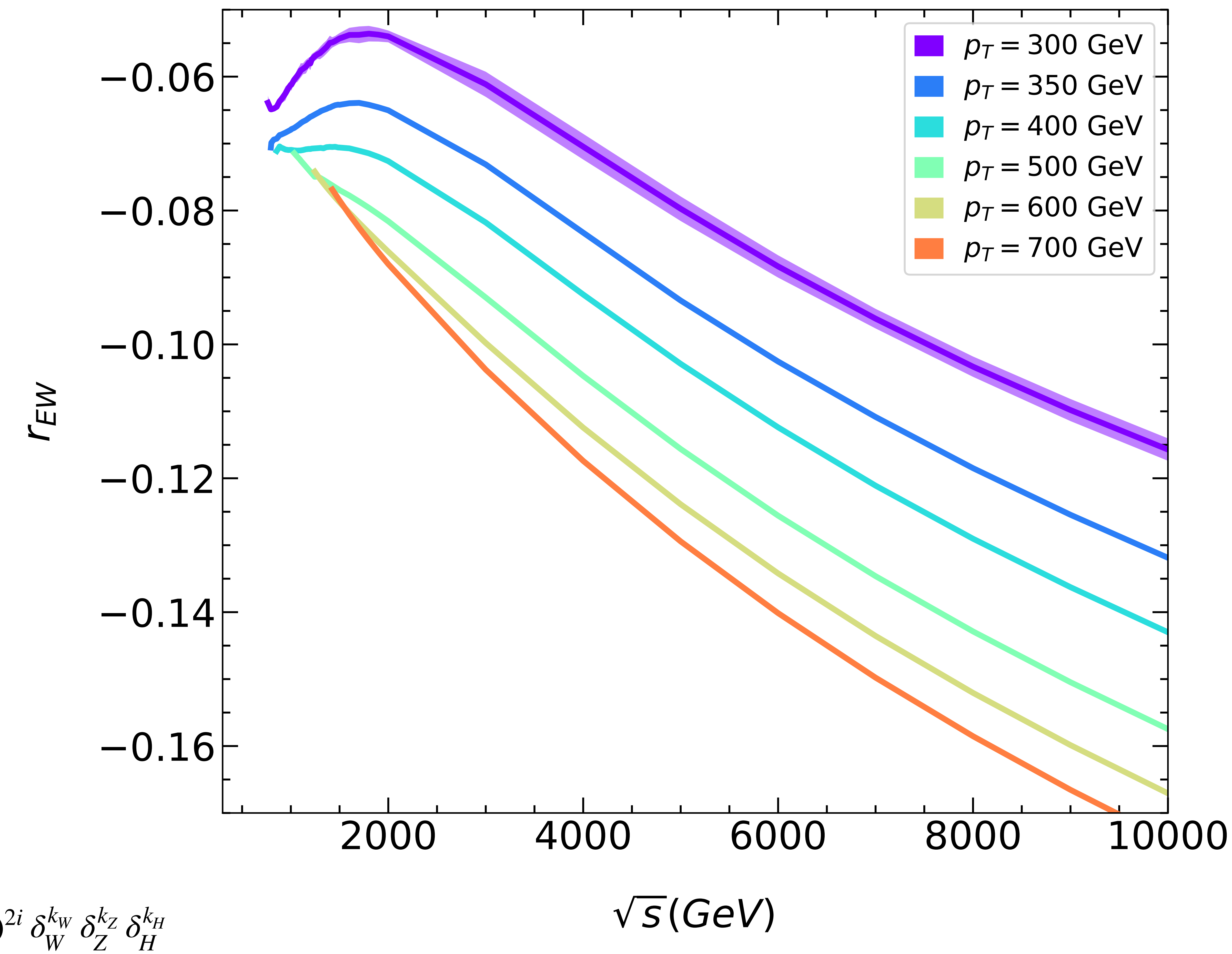
Deep high-energy expansion:

$$F = \sum_{n=-4}^{108}\sum_{i=0}^{2}\sum_{k_W=0}^{4}\sum_{k_Z=0}^{4}\sum_{k_H=0}^{4} c_{ni}^{k_W k_Z k_H}\, m_t^n\, (m_H^{\rm ext})^{2i}\, \delta_W^{k_W}\, \delta_Z^{k_Z}\, \delta_H^{k_H}$$

Analytic EW results - partonic K factor

[Davies, Schönwald, Steinhauser, **Zhang**, 2603.08789]

$$r_{\mathrm{EW}} = \frac{\alpha}{\pi}\frac{\mathcal{U}^{(0,1)}}{\mathcal{U}^{(0)}}$$ in perturbative expansion of squared matrix element $|\mathcal{M}|^2 = \bar{X}_0\left(\mathcal{U}^{(0)} + \frac{\alpha_s}{\pi}\mathcal{U}^{(1,0)} + \frac{\alpha}{\pi}\mathcal{U}^{(0,1)} + \cdots\right)$



NLO EW corrections from top-quark contribution is of **the order of $-10\,\%$ at high energies**

Padé approximation uncertainty band only visible at $p_T = 300$ GeV (Padé used for $m_t/\sqrt{s}$ expansion)

Deep high-energy expansion:

$$F = \sum_{n=-4}^{108}\sum_{i=0}^{2}\sum_{k_W=0}^{4}\sum_{k_Z=0}^{4}\sum_{k_H=0}^{4} c_{ni}^{k_W k_Z k_H}\, m_t^n\,(m_H^{\mathrm{ext}})^{2i}\,\delta_W^{k_W}\,\delta_Z^{k_Z}\,\delta_H^{k_H}$$

Summary

- **Analytic two-loop EW machinery for $2 \to 2$ processes at high & low energies**
  - A technical roadmap developed for MIs and amplitudes calculations at high energies [JHEP 08 (2022) 259, JHEP 04 (2025) 193]
    - **AsyInt** toolbox for MIs at high energies public available [JHEP 09 (2024) 069]
  - Large-mass expansion demonstrated for $gg \to HH$ and $gg \to Hg$ [JHEP 10 (2023) 033]
    - Applicable to precision NNLO BSM physics @ FCC-ee
    - Importance of BSM effects at NLO EW demonstrated @ FCC-ee [Phys.Rev.Lett. 136 (2026) 8, 081801]
- **NLO EW corrections to Higgs pair production at high energies** [2603.08789]
  - Rich structures of logarithmic / power corrections, given multiple EW coupling structures
  - Good convergence observed for multiple expansions in a vast phase space, from high-energy limit down to $p_T \approx 350$ GeV
  - Show top-quark induced EW corrections are of the order of $-10\,\%$

Backup Slides

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, JHEP 08 (2022) 259]

- Close the contour to left or right semi-circle, and sum up residues, e.g.

$$\int_{-i\infty}^{+i\infty} \frac{dz_1}{2\pi i} f(z_1) = -\sum_{k=0}^{\infty} \mathrm{Res}_{z_1=k}\left[f(z_1)\right] - \int_{\mathrm{arc}} \frac{dz_1}{2\pi i} f(z_1) \quad \text{with} \quad f(z_1) = \frac{z_1^8 \Gamma(-z_1)^2 \Gamma(z_1)^2}{(z_1+1)^3 (z_1+2)^3}$$

$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

arc integral non-zero

solve arc contribution by adding auxiliary scale:

$$\int_{\mathrm{arc}} \frac{dz_1}{2\pi i} \xi^{z_1} f(z_1) = -\sum_{k=0}^{\infty} \frac{k^6}{(1+k)^3(2+k)^3} \xi^k \log(\xi) \overset{\xi\to 1}{=} -1$$

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, JHEP 08 (2022) 259]

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Expand&Fit method

[**Zhang**, JHEP 09 (2024) 069]

- for 2-dim 1-scale MB integral with **nested non-vanishing arc contributions**

$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{-t}{-s}\right)^{z_1} f\left(\Gamma, \psi^{(i)}; z_1, z_2\right) \Rightarrow \text{HPLs}$$

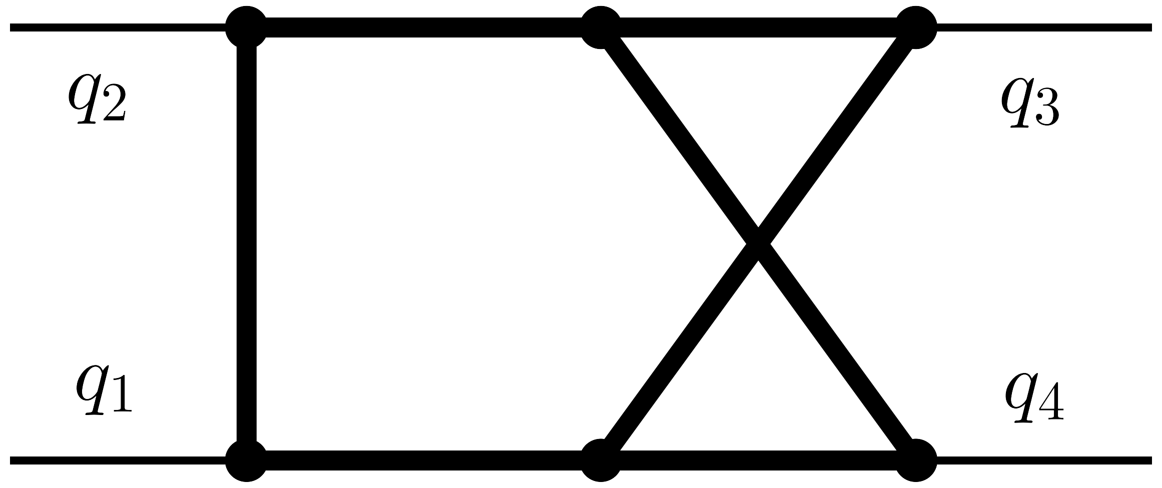
- for 2-dim 2-scale MB integral in non-planar diagrams

$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{-t}{-s}\right)^{z_1} \left(\frac{-u}{-s}\right)^{z_2} f\left(\Gamma, \psi^{(i)}; z_1, z_2\right) \Rightarrow \text{HPLs}$$

(1). **Expand in Regge limit** ($t \ll s$) for more than a hundred terms
(2). **Solve Regge limit analytically in Euclidean region**
(3). **Reconstruct full analytic results with ansatz** in Euclidean region (for planar integrals) or in physical region (with analytic continuation for non-planar integrals)

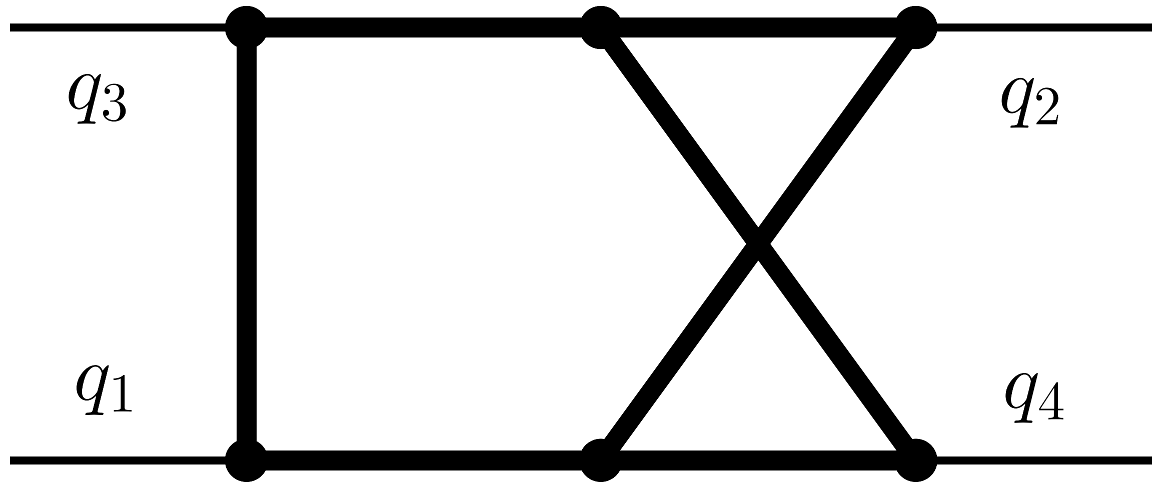
Non-planar fully massive (EW) integrals

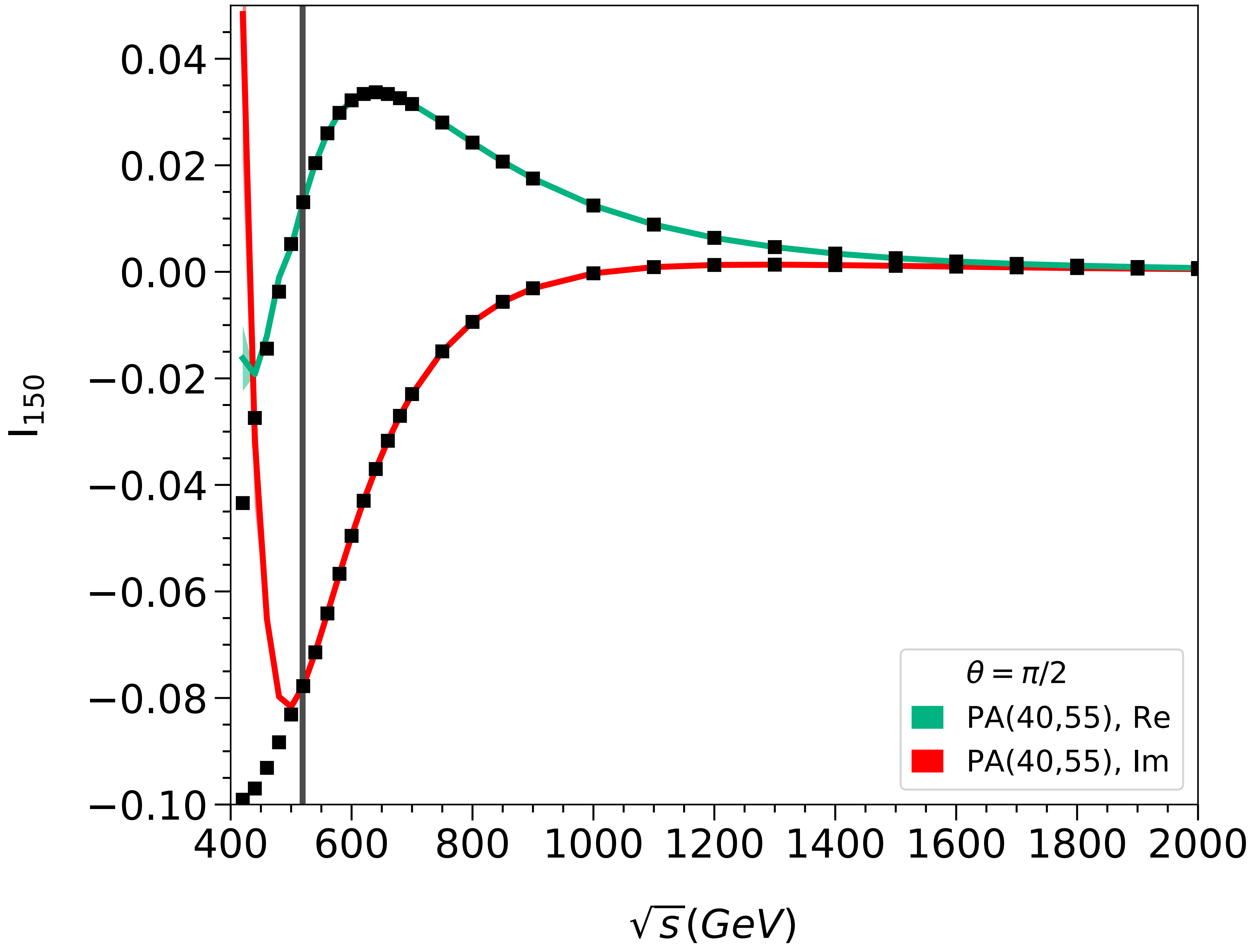
Analytic solution in terms of Harmonic PolyLogarithms (HPLs) [Davies, Schönwald, Steinhauser, **Zhang**, JHEP 04 (2025) 193]

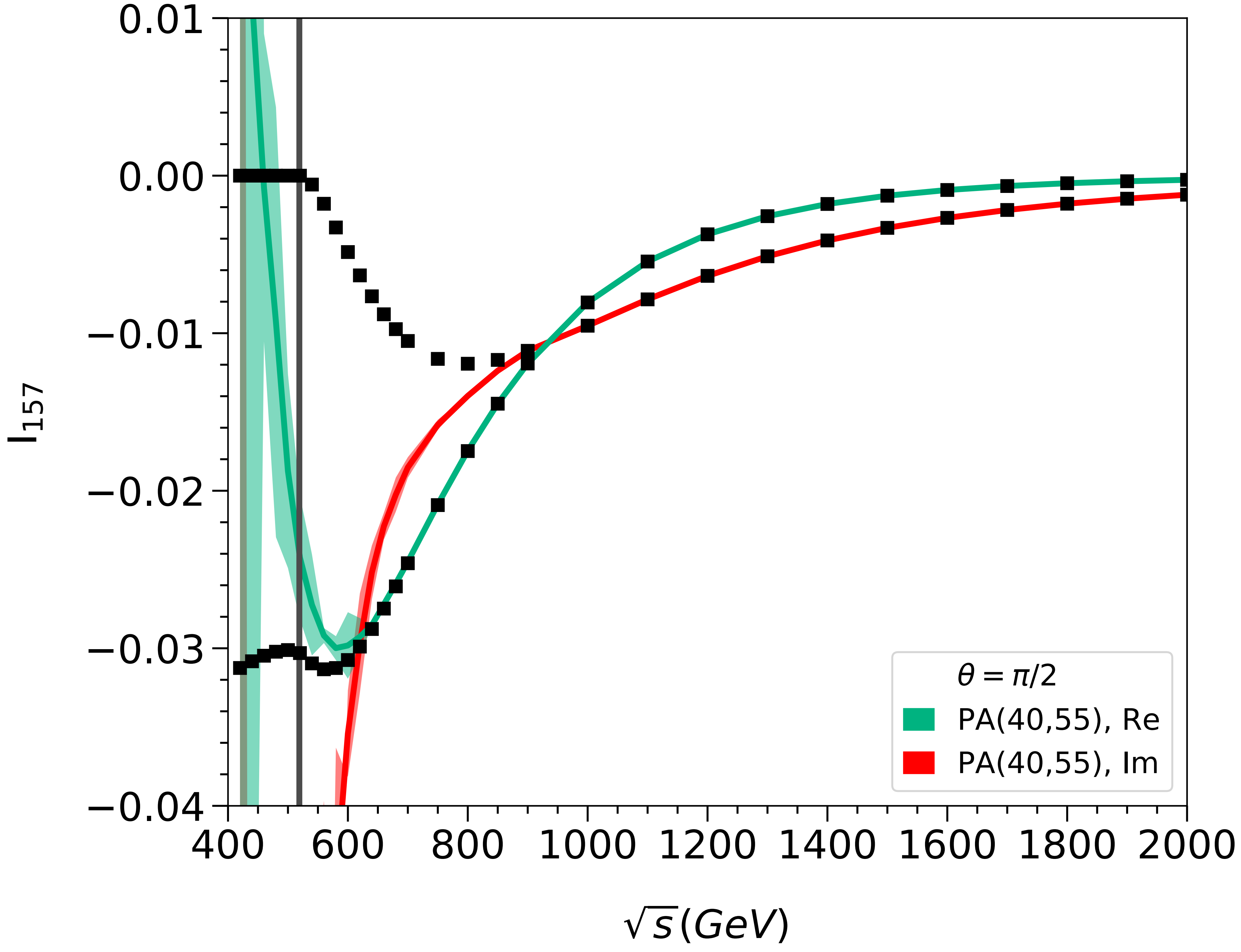


> 100 expansion terms in $\frac{m_t}{\sqrt{s}}$

Fixed scattering angle plots







Red Green curves (analytic), Black (numerics from AMFlow)