Status of Predictions for Pseudo Observables in the SM

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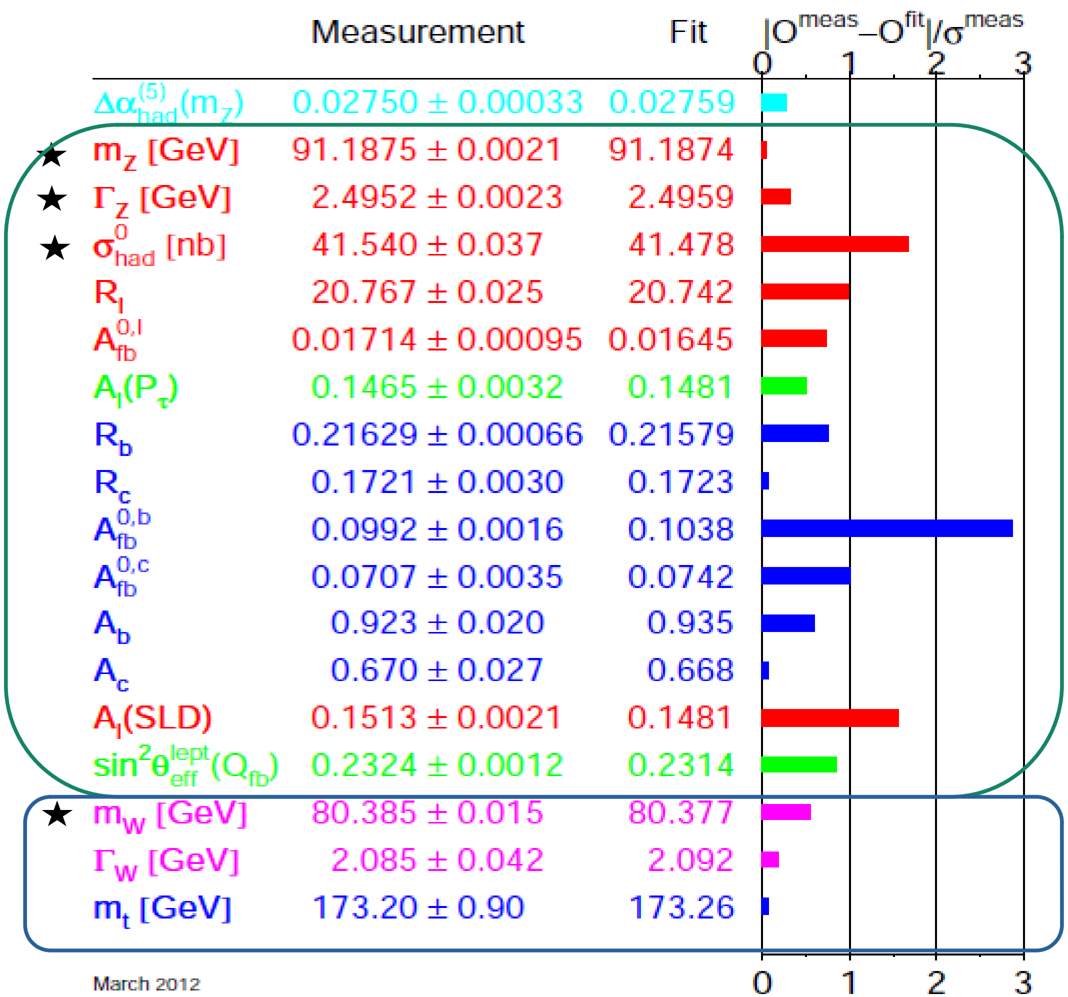
Outline

- The Lep and Tevatron legacy vs today
- Pseudo Orbservables: precise calculations
- Pseudo Observables: precise inputs
- Conclusions

Electroweak Precision Observable: The LEP/SLC and Tevatron legacy

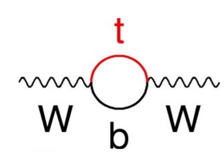


Reevaluation of Bhabha cross section

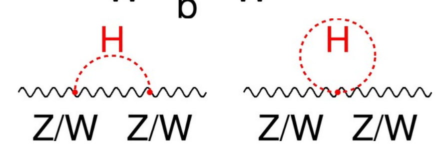
$$
\begin{aligned}
m_Z\ [\text{GeV}] &= 91.1875 \pm 0.0021 \\
\Gamma_Z\ [\text{GeV}] &= 2.4995 \pm 0.0023 \\
\sigma^0_{had}\ [\text{nb}] &= 41.4802 \pm 0.0325
\end{aligned}
$$

Two-loop massive b-quark corrections

$$A_{fb}^{0,b} = 0.0996 \pm 0.0016$$



$\sim \alpha \frac{M_t^2}{m_W^2}$



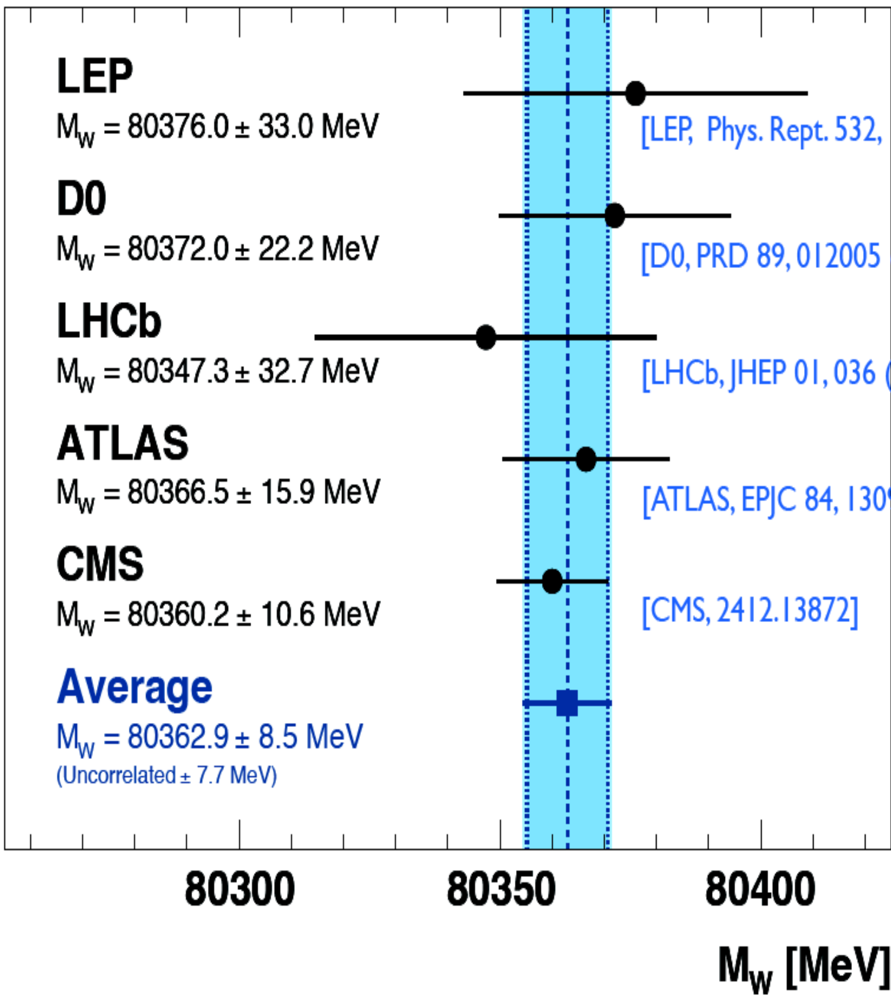
$\sim \alpha \log \frac{m_H^2}{m_Z^2}$

precision $\sim 5 - 1 \times 10^{-3}$

★ better than 10^{-3}

Sensitivity to quantum effects

Electroweak Precision Pseudo Observable today: W mass



- Combination following LHC-TeV MW Working Group [MW WG, EPJC 84, 451 (2024)]
- Values adjusted to CT18
- Correlations accounted for
- $\chi 2/\mathrm{ndf} = 0.7 / 5$

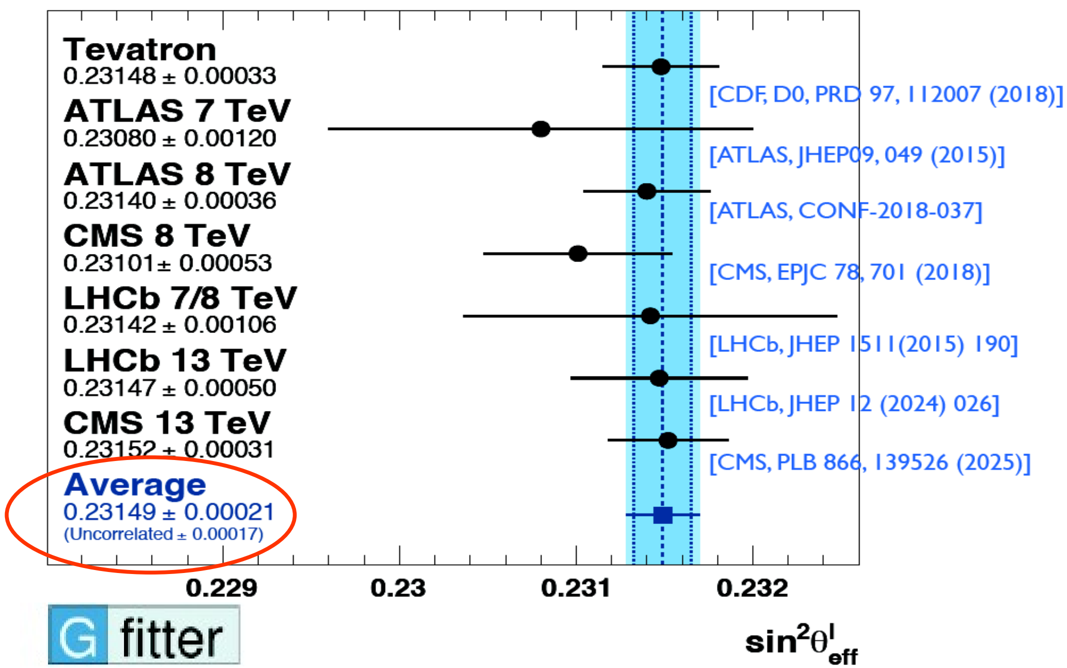
Precision on m_W:
15 MeV (2015)
8.5 MeV (today)

(about 10 years ago: envisioned 8 MeV for LHC Run 3)

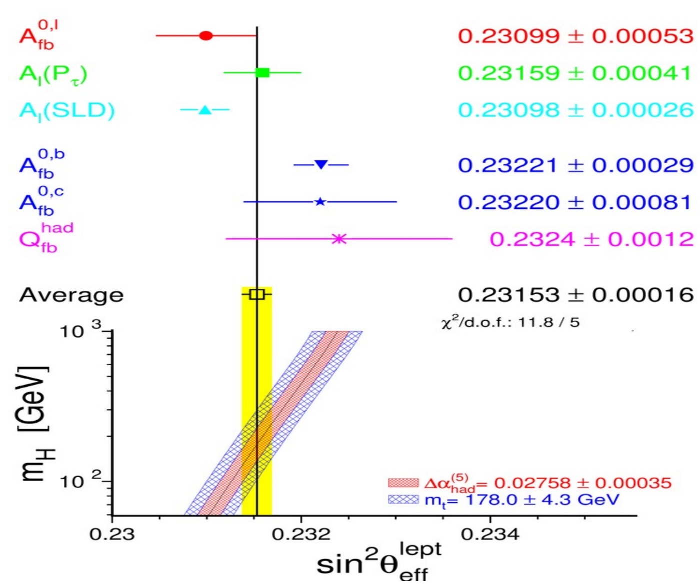
Electroweak Precision Pseudo Observable today: $\sin^2\theta_{eff}^{lept}$

- LEP/SLD determination from fermion asymmetries
- Hadron determination via A_{fb} in Drell-Yan for $e^+ e^-$, $\mu^+ \mu^-$ final state

hadronic



leptonic



After corrections of $A_{fb}^{0,b}$

$\sin^2\theta_{eff}^{lept} = 0.23151 \pm 0.0016$

hadronic more consistent than leptonic

hadronic and leptonic in very good agreement

Electroweak Precision Pseudo Observable: SM results

SM does not predict any mass: $m_W,\, m_Z,\, M_t,\, m_H \cdots \leftrightarrow g,\, g',\, v,\, Y_t,\, \lambda \ldots$
masses (weak angle) are correlated to measurable quantities

m_W obtained from α, G_μ and m_Z via muon decay

$$m_W^2 = \frac{m_Z^2}{2}\left\{1+\left[1-\frac{4\pi\alpha}{\sqrt{2}\,G_\mu\, m_Z^2}(1+\Delta r)\right]^{1/2}\right\}$$

$$\Delta r = \Delta r(M_t,\, \Delta\alpha^{(5)}_{\rm had}(m_Z^2),\, m_H,\, \alpha_s, \ldots)$$

$\sin^2\theta^{\rm lept}_{eff}$ extracted from asymmetries as an effective coupling in the $Z\bar{f}f$ vertex

$$\rho^f\gamma^\mu\left[\frac{1}{4}I_3^f(1-\gamma_5) - Q^f\sin^2\theta^{\rm lept}_{eff}\right] = \gamma^\mu(g_V^f - g_A^f\gamma_5)$$

$$\Rightarrow \sin^2\theta^{\rm lept}_{eff} = \frac{1}{4}\left[1-{\rm Re}\left(\frac{g_V^l}{g_A^l}\right)\right] \equiv {\rm Re}(1+\Delta\kappa^l)\sin^2\theta_W$$

$$\Delta\kappa^l = \Delta\kappa^l(M_t,\, \alpha_s, m_H, \ldots)$$

Accurate predictions:

- Need precise calculations (higher order corrections)
- Need precise inputs (parametric uncertainties)

Precise calculations

The renormalization scheme (RS) defines the counterterms

"All RS are equal but some RS are more equal than others"

adapted from the book "Animal Farm" by G. Orwell

Popular schemes:

On-shell scheme: $\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$ $\quad (s_W \equiv \sin\theta_W,\ c_W \equiv \cos\theta_W)$ Sirlin (1980),

$$\frac{\delta\alpha}{\alpha} = \Pi_{\gamma\gamma}(0) \longrightarrow$$ define α (Thompson)

$$\delta m_X = Re A_{XX}(m_X) \longrightarrow$$ define pole masses

$$\frac{\delta s_W^2}{s_W^2} = \frac{c_W^2}{s_W^2}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right)$$

Pure $\overline{\text{MS}}$-scheme: UV divergent parts only

(Hybrid) $\overline{\text{MS}}$-scheme: distinguish between couplings (g, g', $\overline{\text{MS}}$) and masses (g v, m)
m_W, m_Z kept as a pole mass to absorb non-decoupling contributions of heavy particles

Precise calculations: large radiative contributions

On-shell:

✔ Δr: large contributions associated to the running of alpha and to $\frac{\delta s_W^2}{s_W^2} \sim -\frac{c_W^2}{s_W^2}\Delta\rho$

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\rm rem}$$

~6.5 x 10^{-2} ~ -3.2 x 10^{-2}

$$\Delta\alpha = \Pi_{\gamma\gamma}^{\rm ferm}(m_Z^2) - \Pi_{\gamma\gamma}^{\rm ferm}(0) \quad \sim \alpha \log\frac{m_f^2}{m_Z^2}$$

$$\Delta\rho = \frac{A_{ZZ}(0)}{m_Z^2} - \frac{A_{WW}(0)}{m_W^2} \quad \sim \alpha\frac{M_t^2}{m_W^2}$$

✔ Δκ: large contributions associated to $\frac{\delta s_W^2}{s_W^2}$ but indirect sensitivity to Δα via W mass

Resummation of large reducible contributions:

$$1+\Delta r \to \frac{1}{1-\Delta r}$$

Includes correctly only $\mathcal{O}((\alpha \ln m_Z^2/m_f^2)^n)$

Sirlin (1984)

$$1+\Delta r \to \frac{1}{(1+\Delta\alpha)(1+\frac{c_W^2}{s_W^2}\Delta\rho)+\ldots}$$

Includes also $\mathcal{O}((\alpha M_t^2/m_W^2)^2)$

Consoli, Hollik, Jegerlehner (1989)

Precise calculations: large radiative contributions

$\overline{\text{MS}}$-scheme:

No large contribution associated to the counterterms but absorbed in the definition of the parameters.

✔ m_W -m_Z interdependence written as (but same “ingredients” as in Δr):

$$\begin{aligned}
\frac{G_\mu}{\sqrt{2}} &= \frac{\pi\hat{\alpha}(m_z)}{2m_w^2\sin^2\hat{\theta}_W(m_z)}[1+\Delta\hat{r}_W] \\
\hat{\alpha}(M_z) &= \frac{\alpha}{1-\Delta\hat{\alpha}(m_z)} \\
\hat{\rho} &\equiv \frac{m_W^2}{m_Z^2\,\cos^2\hat{\theta}_W(m_z)} = \frac{1}{1-\Delta\hat{\rho}} \\
\Delta\hat{\rho} &= \mathrm{Re}\left[\frac{A_{WW}(m_W^2)}{m_W^2} - \hat{c}^2\frac{A_{ZZ}(m_Z^2)}{m_W^2}\right]_{\overline{MS}}
\end{aligned}$$

$$\begin{aligned}
m_W^2 &= \frac{\hat{\rho}\,m_Z^2}{2}\left\{1+\left[1-\frac{4\pi\hat{\alpha}(m_z)}{\sqrt{2}G_\mu m_Z^2\hat{\rho}}(1+\Delta\hat{r}_W)\right]^{1/2}\right\} \\
\sin^2\hat{\theta}_W(m_z) &= \frac{1}{2}\left\{1-\left[1-\frac{4\pi\hat{\alpha}(m_z)}{\sqrt{2}G_\mu m_Z^2\hat{\rho}}(1+\Delta\hat{r}_W)\right]^{1/2}\right\}
\end{aligned}$$

exact resummed expressions

✔ $\sin^2\hat{\theta}_W(m_z)$ is very close to $\sin^2\theta_{eff}^{\mathrm{lept}}$, $\Delta\hat{\kappa}^l(m_z)\sim 0.001$

$$\begin{aligned}
\Delta\alpha &\sim 6.5\times 10^{-2} \\
X &\equiv \frac{c_W^2}{s_W^2}\mathrm{Re}\left[\frac{A_{WW}(m_W^2)}{m_W^2}-\frac{A_{ZZ}(m_Z^2)}{m_Z^2}\right]\sim -3.3\times 10^{-2}
\end{aligned}$$

Resummation is not completely neglibible:

Rule of thumb: $\delta m_w \sim -15\,\delta(\Delta r)$

$1\times 10^{-4}\ (\Delta r)\to -1.5\,\mathrm{MeV}\ (m_W)$

3-loop

	$(\Delta\alpha)^3$	$(\Delta\alpha)^2X$	$\Delta\alpha X^2$	$\Delta\alpha X^3$	$\Delta\alpha X$	X^2
$O(\alpha^3)\times 10^4$	~ 2.7	~ -1.4	~ 0.7	~ -0.4		
$O(\alpha^2\alpha_s)\times 10^4$					~ 2.4	~ -1.2

Few MeV

Precision calculation the full two-loop level: m_W

✔ Δr known at the complete two-loop level in the OS scheme:

- $O(\alpha\alpha_s)$ $\delta m_W \sim -55$ MeV — Djouadi, Verzegnassi (87); Djouadi (90); Kniehl (90); Djouadi, Gambino(94)
- $O(\alpha^2)$ $\delta m_W \sim -45$ -60 MeV — Freitas, Hollik, Walter, Weiglein (02); Awramik, Czakon (02); Onishchenko, Veretin (02), Awramik, Czakon, Freitas, Weiglein (04);

Inclusion of "large" higher order contributions

- $O(\alpha\alpha_s^2)$ $\delta m_W \sim -10$ MeV — Chetyrkin, Kuehn, Steinhauser (95, 96)
- $O(\alpha^2 \alpha_s M_t^4)$ $\delta m_W \sim 1.5$ MeV — Faisst et al. (03);
- $O(\alpha^3 M_t^6)$ $\delta m_W < 1$ MeV — Faisst et al. (03);

$\Big\}$ "leading" contributions

- $O(\alpha\, \alpha_s^3 M_t^2)$ $\delta m_W \sim 1.5$ MeV — Boughezal, Czakon (06), Chetyrkin et al. (06)

✔ m_W compute at the two-loop level in $\overline{\text{MS}}$ scheme

two-loop evaluation of: $\hat{\alpha}(m_Z),\ \Delta\hat{r}_W,\ \hat{\rho}$ — Degrassi, Gambino, Giardino (2014)

Precision calculations at the full two-loop level: m_W

The OS and $\overline{\text{MS}}$ calculations are very different also in the treatment of the W, Z masses.
W and Z are unstable particle. Different possible definitions of the masses.

$\overline{\text{MS}}$ calculation uses masses, *m,* identified as the experimental value $m_{\rm exp}$ as extracted using a Breit-Wigner function with an energy-dependent width

$$\sigma(s)_{\rm exp} \sim \frac{1}{(s-m^2_{\rm exp})^2+s^2\frac{\Gamma^2_{\rm exp}}{m^2_{\rm exp}}}$$

OS calculation uses a mass define in term of the real part of the complex pole $s_p = M^2 - iM\Gamma$
Evaluation of σ(s) in terms of s_p gives a constant width dependence

$$\sigma(s)_{\rm th} \sim \frac{1}{(s-M^2)^2+M^2\Gamma^2}$$

$$m^2_{\rm exp} = M^2+\Gamma^2, \quad m_{\rm exp}/\Gamma_{\rm exp} = M/\Gamma$$

$$M = K\, m_{\rm exp}, \qquad \Gamma = K\,\Gamma_{\rm exp}, \quad K = \left(1+\Gamma^2_{\rm exp}/m^2_{\rm exp}\right)^{-1/2}$$

$M_Z - m_Z \approx$ -34 MeV, $M_W - m_W \approx$ -27 MeV

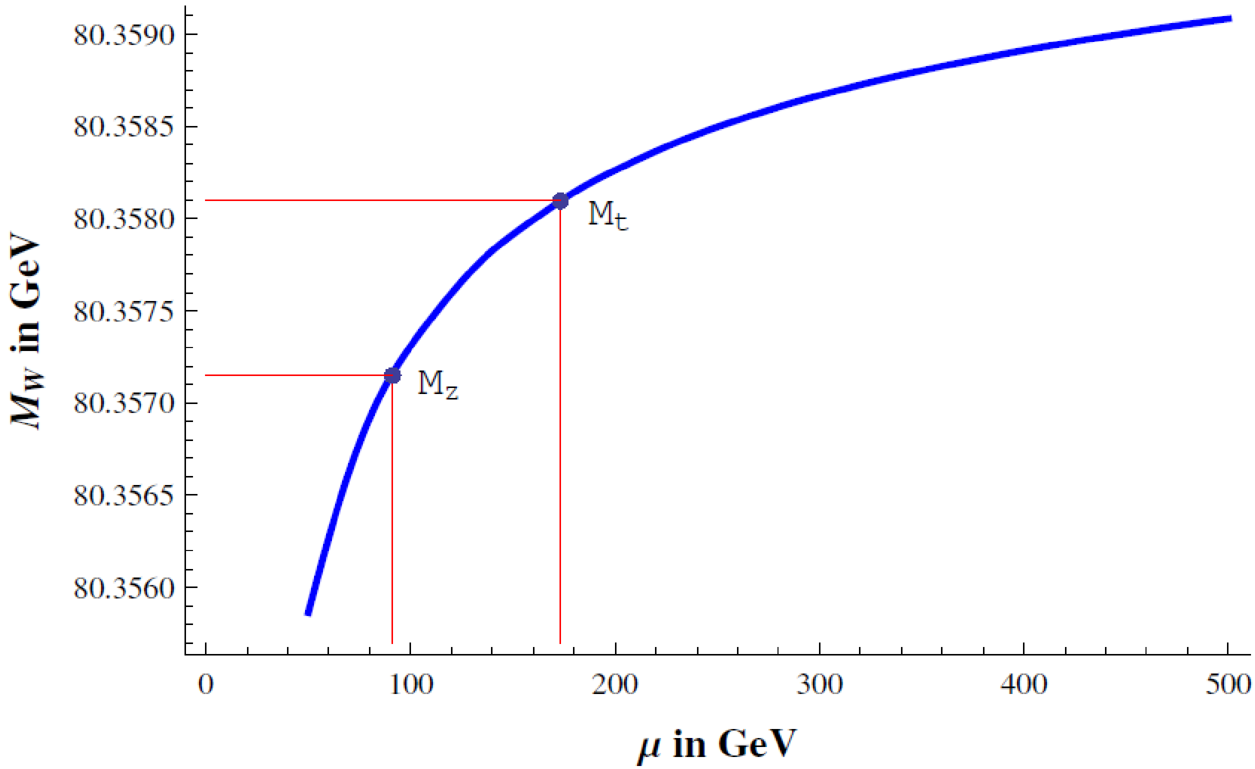
However, using the same inputs:

OS: m_W = 80.353 GeV Bagnaschi et al. (2023)

Very good agreement!

MS: m_W = 80.351 GeV

μ dependence in the $\overline{MS}$ result



$100 \leq \mu \leq 200$ GeV, $\delta m_w \sim 1$ MeV

$50 \leq \mu \leq 500$ GeV, $\delta m_w \sim 3$ MeV

Precision calculations: going beyond two-loop

$O(\alpha\alpha_s^2)$

$\Delta\rho$:

3-loop integrals with a single mass scale (M_t). Analytic expression

Analytic result: a result expressed in terms of " functions" that can be computed with a (public) code in a reasonable (very short) amount of time (ex. Log → HPL, GHPL …)

Δr:

Top contribution: integrals with two mass scales (M_t, m_V) but we can compute via a series expansion in $(m_V/M_t)^2$

1996 Chetyrkin, Kuehn, Steinhauser 4 terms
2026 Pati, Rana, Vicini 15 terms

Light quarks contribution: integrals with a single mass scale (m_V)

		$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$
δm_W (MeV)	tb	-59.62	-11.61
	lq	-1.98	-0.23
$10^5 \times \delta \sin^2 \theta^f_{eff}$	tb	-67.1	-14.3
	lq	-3.4	-0.5

Estimate of $\mathcal{O}(\alpha\alpha_s^3)$

$$\delta m_W \sim \left(\frac{11.84}{61.60}\right)(-11.84) \sim 2\,\mathrm{MeV}$$

Precision calculations: going beyond two-loop in m_W

$O(\alpha^2\alpha_s)$

New result for Δr:
3-loop tadpole integrals with several masses
3-loop bubble integrals at non-zero momentum

Dubovyk, Freitas, Gluza, Usovitsch, 2026

$\delta m_W \sim$ 4 MeV quite large, similar to what is usually assumed as theory error

Previous estimate of this contribution in Δr from the "leading" contribution $O(\alpha^2 \alpha_s M_t^4)$ in $\Delta\rho$. But the M_t^4 term is not a good approximation for the true value of the top mass.

$$\frac{\Delta\rho^{(\alpha^2\alpha_s)}}{\Delta\rho^{(\alpha^2\alpha_s)}_{M_t^4}} \sim 1.81$$

Precision calculation at the full two-loop level: $\sin^2\theta_{eff}^{\text{lept}}$

$$\sin^2\theta_{eff}^{\text{lept}} = \frac{1}{4}\left[1 - \mathrm{Re}\left(\frac{g_V^l}{g_A^l}\right)\right] = \mathrm{Re}(1+\Delta\kappa^l)\sin^2\theta_W$$

two-loop vertex at $q^2 \neq 0$

from G_μ

$\Delta\kappa^l$ known at the complete two-loop level in the OS scheme:

- $O(\alpha\alpha_s)$ adapted from m_w calculation
- $O(\alpha^2)$

Awramik, Czakon, Freitas, Weiglein (04-06); Hollik, Meier, Uccirati (05-06)
Dubovyk et al. (19)

Inclusion of “large” higher order contributions adapted from m_w calculation

In the OS scheme there are large cancellations between $\Delta\kappa^l$ and the calculation of $\sin^2\theta_w$

Precise inputs
Parametric uncertainties

What we do not care:

✓ α(Thomson)

$$\begin{aligned} \alpha^{-1}(a_e) &= 137.035999166(15)\ [11\,ppt] \\ \alpha^{-1}(Cs) &= 137.035999046(27)\ [20\,ppt] \\ \alpha^{-1}(Rb) &= 137.035999206(11)\ [81\,ppt] \end{aligned}$$

Most precise α from g-2 of the electron: pure QED process, hadronic and weak contributions at the level of the error

$$a_e(exp) = 1\,159\,652\,180.59\,(13) \times 10^{-12} \quad [0.13\,ppt]$$

✓ G_μ Fermi constant

✓ M_H Higgs mass

Parametric uncertainties: G_μ

G_μ is a number extract from the muon lifetime using the below formula

$$\frac{1}{\tau} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F[x] \left(1 + \frac{\alpha(m_\mu)}{\pi} H_1[x] + \frac{\alpha(m_\mu)^2}{\pi^2} H_2[x] + \frac{\alpha(m_\mu)^3}{\pi^3} H_3[x]\right) \qquad x \equiv m_e^2/m_\mu^2, \quad \alpha(m_\mu)^{-1} = 135.901$$

$$F[x] = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x = 0.99981295$$

$$H_1[x] = \frac{25}{8} - \frac{\pi^2}{2} + \mathcal{O}(x) = -1.80793, \qquad H_2[x] = ... = 6.64, \qquad H_3[x] = ... = -15.3(2021)$$

Kinoshita, Sirlin (1959)

Van Ritbergen, Stuart (2000)

Fael, Schoenwal, Steinhauser (2021)
Czakon, Czarnecki, Dowling (2021)

40 yr

20 yr

$$G_\mu = 1.1663788(6) \times 10^{-5} \ \ \mathrm{GeV}^{-2} \qquad \delta G_\mu / G_\mu = 5 \times 10^{-7}$$

Parametric uncertainties: G_μ

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$$\frac{1}{\tau} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F[x] \left(1 + \frac{\alpha(m_\mu)}{\pi} H_1[x] + \frac{\alpha(m_\mu)^2}{\pi^2} H_2[x] + \frac{\alpha(m_\mu)^3}{\pi^3} H_3[x]\right)\left(1 + \frac{3m_\mu^2}{5m_W^2}\right) \quad x \equiv m_e^2/m_\mu^2,\ \alpha(m_\mu)^{-1} = 135.901$$

$$F[x] = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x = 0.99981295$$

$$H_1[x] = \frac{25}{8} - \frac{\pi^2}{2} + \mathcal{O}(x) = -1.80793, \qquad H_2[x] = ... = 6.64, \qquad H_3[x] = ... = -15.3(2021)$$

Kinoshita, Sirlin (1959)

Van Ritbergen, Stuart (2000)

Fael, Schoenwal, Steinhauser (2021)
Czakon, Czarnecki, Dowling (2021)

40 yr

20 yr

$$G_\mu = 1.1663788(6) \times 10^{-5} \longrightarrow G_\mu = 1.1663782(6) \times 10^{-5} \quad \mathrm{GeV}^{-2} \qquad \delta G_\mu / G_\mu = 5 \times 10^{-7}$$

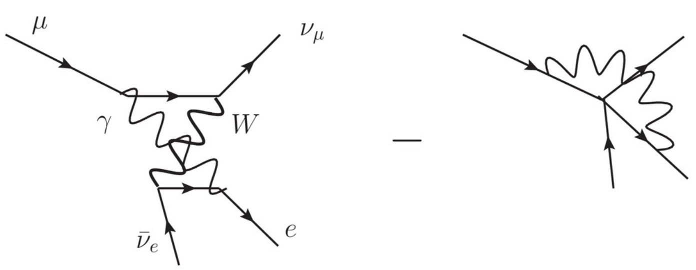
Difference at the level of the experimental error but irrelevant for prediction of PO

Technical detail: separation of the photonic corrections

Historical way:

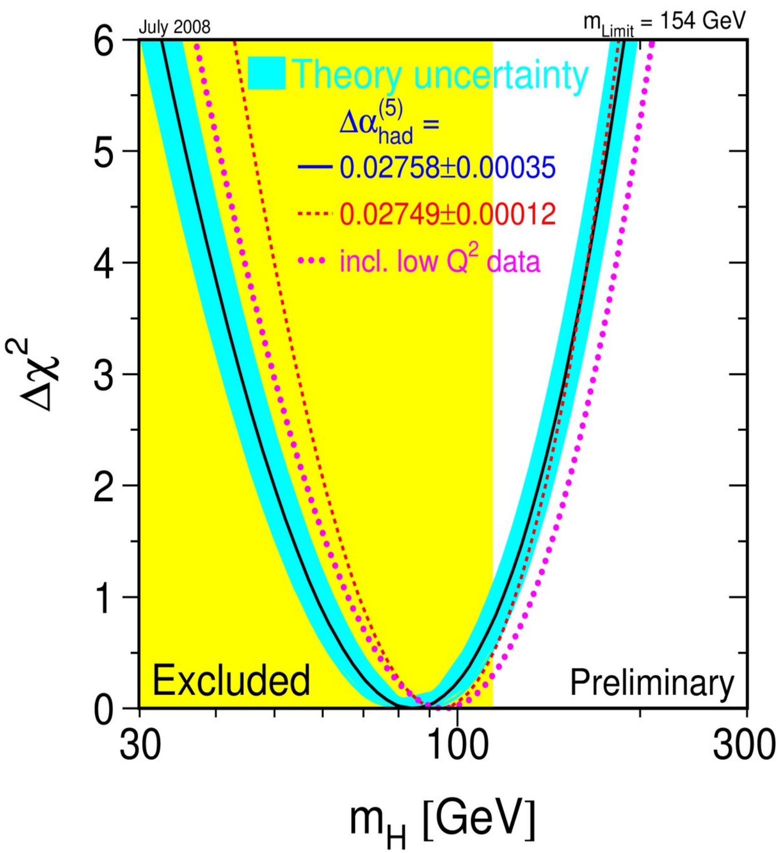
Pauli-Villars cutoff factor

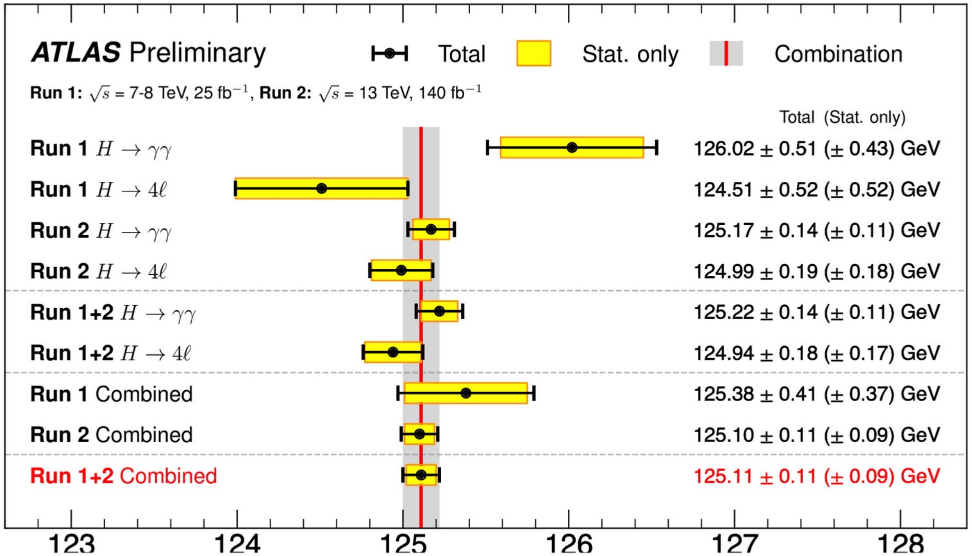
$$\frac{1}{k^2} = \underbrace{-\frac{m_W^2}{k^2 - m_W^2}\frac{1}{k^2}}_{\text{Fermi}} + \underbrace{\frac{1}{k^2 - m_W^2}}_{\Delta r}$$

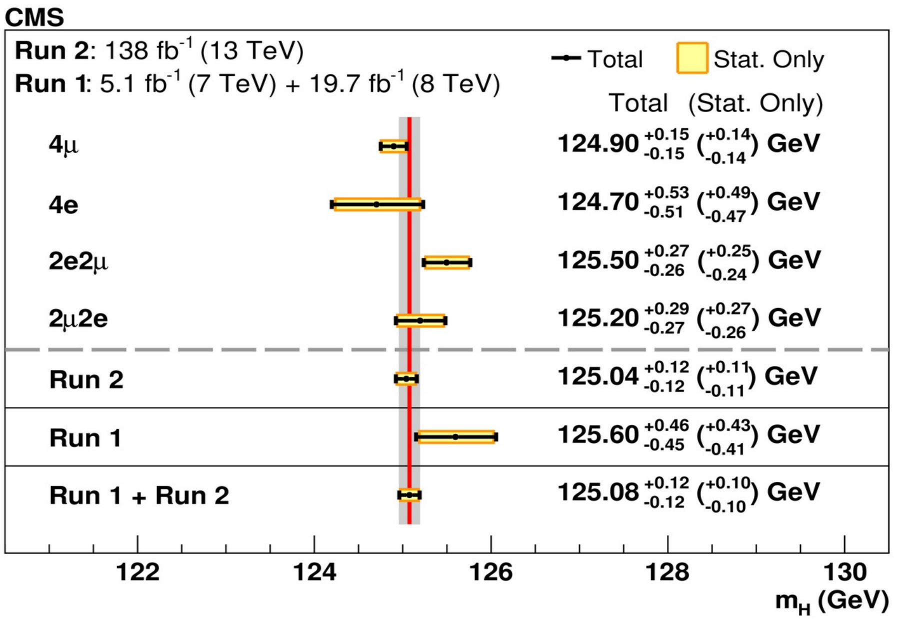


Modern way: Matching off-shell at zero external momenta

Parametric uncertainties: Higgs







PDG:

$$m_H = 125.20 \pm 0.11 \quad \text{GeV}$$

$$\frac{\delta m_H}{m_H} \sim 9 \cdot 10^{-4}$$

Precise inputs
Parametric uncertainties

What we do care:

Estimate of “Theory uncertainties”

	current	future (conservative)	future (aggressive)
σ_{had} [pb]	6	1.6	0.3
R_ℓ [10^{-3}]	6	1.2	0.2
R_c [10^{-5}]	5	1	0.2
R_b [10^{-5}]	10	2	0.35
Γ_Z [MeV]	0.4	0.08	0.016
$\sin^2\theta_{eff,lept}$ [10^{-5}]	4.5	0.7	0.06
m_W [MeV]	4	1	0.1

Bagnaschi, Freitas Giardino
ESPP, Venice 2025

✓ Z boson mass: $\Delta m_Z/m_Z = 2\cdot 10^{-5}$ $\rightarrow$

$$\delta m_W \sim 2.5\ \text{MeV}$$
$$\delta \sin^2\theta_{\text{eff}}^{\text{lept}} \sim 1.3\times 10^{-5}$$

✓ Top mass

✓ Hadronic contribution to the running of α

Precise inputs: M_t

t

$$I(J^P) = 0(\tfrac{1}{2}^+)$$

Charge $= \frac{2}{3}\, e$ Top $= +1$

Mass (direct measurements) $m = 172.56 \pm 0.31$ GeV [a,b]   (S = 1.6)
Mass (from cross-section measurements) $m = 162.5^{+2.1}_{-1.5}$ GeV [a]
Mass (Pole from cross-section measurements) $m = 172.4 \pm 0.7$ GeV

Direct measurements: rely upon observables that are strongly sensitive to the mass of the decay products and are less affected by production/decay dynamics.
Monte Carlo are used to reconstruct the top mass M_t^{MC} from its decays products. Modeling.
Indirect measurements: Performed fitting kinematic distribution unfolded at the parton level. Same kind of modeling error that affect direct measurements.

M_t^{MC} is a pole mass? Pole mass is ambiguous by an amount $\mathcal{O}(\Lambda_{QCD})$ (Δ) due to its IR sensitivity in the top self-energy (renormalon).

$$M_t = M_t^{MC} + \Delta, \qquad \delta M_t^{MC} = \pm 0.31 \text{ GeV}, \qquad \delta M_t^{MC}/M_t^{MC} \sim 1.8 \times 10^{-3}$$

The agreement between the central values of
Mass (direct measurements) and Mass (Pole from x-section measurements)
to me indicates that M_t^{MC} “smells like a pole mass”

How can estimate Δ?

Direct measurements

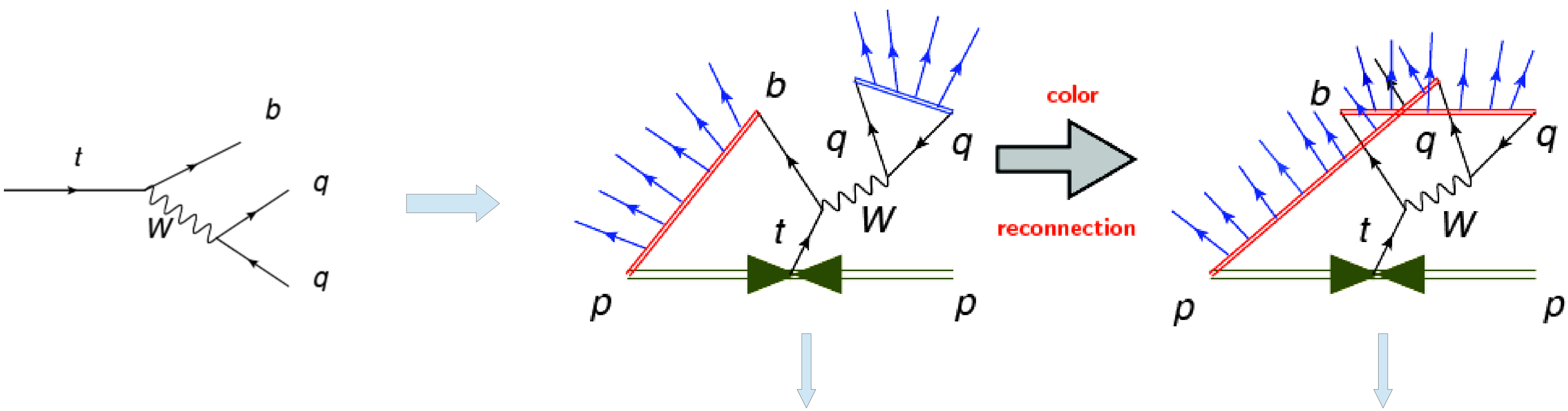
It is often argued that direct measurement cannot be interpreted in a precise renormalization scheme because they rely upon MC's...
This is wrong!

- They attempt to measure the mass of the system of decay products; this is the Pole Mass. Neglecting finite width effects, there are no perturbative errors affecting this identification.
- Uncertainties on the relation between the MC parameter and the distribution that you are fitting are modelling uncertainties. They can be assessed by varying MC parameters, by comparing the reference MC to more accurate ones, etc.
- As you increase precision, you unavoidably clash with corrections that cannot be computed reliably, but only modelled. These again can be treated as modelling errors (by changing MC cutoff scales, varying parameters in the hadronization model, considering colour reconnection models, using alternative shower models, etc.).

Example: Color Reconnection

pp event description:
Hard subprocess → Parton Shower (W decay product are color connected)
→ colorless combination of partons (strings) → hadrons

CR affects the reconstruction of the top system



$$M_t^2 = (P_b + P_{j1} + P_{j2})^2 \qquad M_t^2 \neq (P_b + P_{j1} + P_{j2})^2$$

This can be only estimate by different models in our Monte Carlo $\delta M_t \sim 300$ MeV

In general: the Pole mass ambiguity is not the only mechanism that can generate linear power corrections to the top pole mass.

Precise input: definition of M_t

Ambiguity in the top pole mass

$$
\begin{aligned}
M_t &= M_t^{\overline{MS}} \times \left[1 + c_1 \frac{\alpha_s}{\pi} + c_2 \left(\frac{\alpha_s}{\pi}\right)^2 + c_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \left(\frac{\alpha_s}{\pi}\right)^4 + \dots \right] \\
&= M_t^{\overline{MS}} \times [1 + 0.046 + 0.010 + 0.003 + 0.001 + \dots]
\end{aligned}
$$

Marquad, A. Smirnov, V. Smirnov, Steinhauser (15)

$$
\delta M_t \sim \mathcal{O}(0.001 \times M_t^{\overline{MS}}) \approx \mathcal{O}(200\,\mathrm{MeV})
$$

From truncation of the series

M_t^{MC} is interpreted as M_t within the intrinsic ambiguity in the definition of M_t
$\Delta \sim O(\Lambda_{QCD})$ ~300 MeV

To go below O(300) MeV in δM_t we need a better theoretically defined mass:
a short-distance mass: any mass not affected by the renormalon ambiguity.

$M_t^{MS}(M_t)$ is a SD mass but not a threshold mass $\longrightarrow$ $$\frac{1}{\not{p} - M_t^{\overline{MS}} - \Sigma^{\rm fin}}$$
very different from M_t^{MC}

In general: you can define a SD mass but you have to give the same relation to M_t^{MC} for any observables

Alternative: use observables thet do not require a threshold mass:
ex. total production cross section $\sigma(t\bar{t} + X)$

$$M_t^{\overline{MS}}(M_t) = 162.5^{+2.1}_{-1.5}\,\mathrm{GeV} \to M_t = 172.5^{+2.1}_{-1.5} + \ldots\ \mathrm{GeV}$$

$$\begin{aligned} \delta m_W &\sim 12\,\mathrm{MeV} \\ \delta \sin^2\theta_{\rm eff}^{\rm lept} &\sim 1\times 10^{-4} \end{aligned}$$

Bottom line:
an error on the top pole mass ~ 500 MeV is probably optimistic
an error on the top pole mass ~ 1 GeV is probably pessimistic
but really quantify precisely the error is very hard

$$\begin{aligned} \delta m_W &\sim 3-6\,\mathrm{MeV} \\ \delta \sin^2\theta_{\rm eff}^{\rm lept} &\sim (1.5-3)\times 10^{-5} \end{aligned}$$

Precise inputs: $\Delta\alpha$

Running of α at the scale m_Z : $\alpha(m_Z^2) = \dfrac{\alpha(0)}{1-\Delta\alpha(m_Z^2)}$,

$$\begin{aligned}\Delta\alpha(m_Z^2) &\equiv Re\,\Pi_{\gamma\gamma}(m_Z^2) - \Pi_{\gamma\gamma}(0)\\ &= \Delta\alpha_{\rm lept}(m_Z^2) + \Delta\alpha^{(5)}_{\rm had}(m_Z^2)\\ &\sim \mathcal{O}\left(\alpha\log\frac{m_f}{m_Z}\right) \sim 6.5\times 10^{-2}\end{aligned}$$

$\Delta\alpha_{\rm lept}(m_Z^2) = 0.031498$ (4-loops)

Steinhauser (1998), Sturm (2013)

"Classical" data driven evaluation of $\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$ via subtracted dispersion relation

$$\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = -\frac{\alpha\, m_Z^2}{3\pi} Re\int_{4m_\pi^2}^{s_0} ds\frac{R(s)}{s(s-m_Z^2-i\epsilon)} - \frac{\alpha\, m_Z^2}{3\pi} Re\int_{s_0}^{\infty} ds\frac{R(s)}{s(s-m_Z^2-i\epsilon)}$$

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma_{tot}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

Exp. data　　　　Perturbative QCD

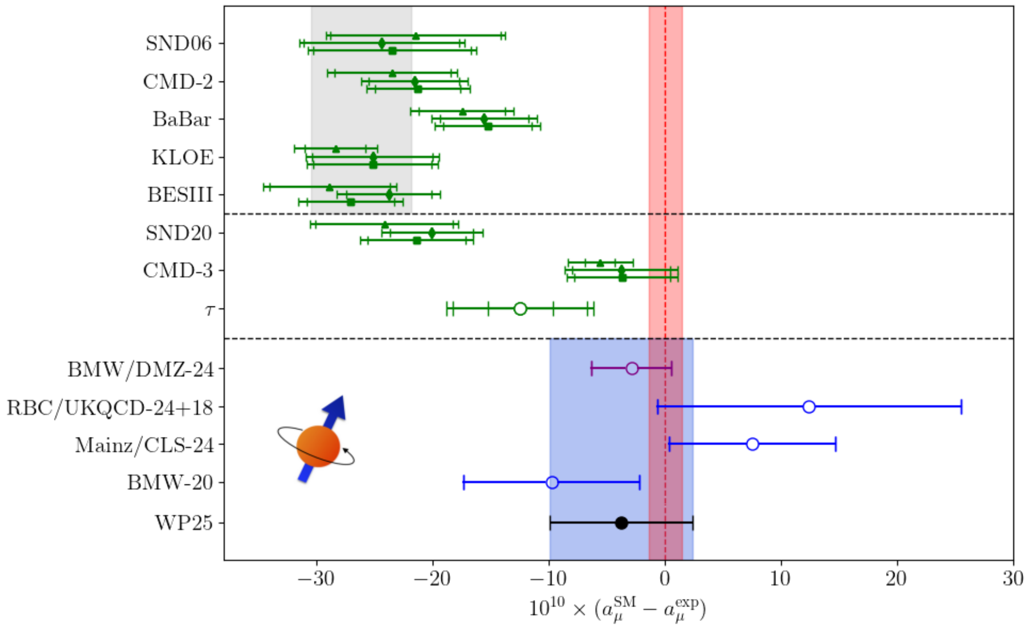
$\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$

From PDG

Reference	Result	Comment
Krasnikov, Rodenberg (1997) [157]	0.02737 ± 0.00039	pQCD for $\sqrt{s} > 2.3$ GeV
Kühn & Steinhauser (1998) [158]	0.02778 ± 0.00016	full $\mathcal{O}(\alpha_s^2)$ for $\sqrt{s} > 1.8$ GeV
Groote *et al.* (1998) [156]	0.02787 ± 0.00032	use of QCD sum rules
Martin *et al.* (2000) [159]	0.02741 ± 0.00019	incl. new BES data
de Troconiz, Yndurain (2001) [160]	0.02754 ± 0.00010	pQCD for $s > 2$ GeV^2
Burkhardt, Pietrzyk (2011) [161]	0.02750 ± 0.00033	pQCD for $\sqrt{s} > 12$ GeV
Erler, Ferro-Hernández (2017) [162]	0.02761 ± 0.00010	conv. from $\overline{\rm MS}$ scheme
Jegerlehner (2019) [163]	0.02755 ± 0.00013	Euclidean split technique
Davier *et al.* (2019) [164]	0.02761 ± 0.00010	pQCD for $\sqrt{s} = 1.8$–3.7 & > 5 GeV
Keshavarzi *et al.* (2019) [165]	0.02761 ± 0.00011	pQCD for $\sqrt{s} > 11.2$ GeV

Does The g-2 "revolution" imply a $\Delta\alpha^{(5)}_{\rm had}(m_z^2)$ "revolution"?

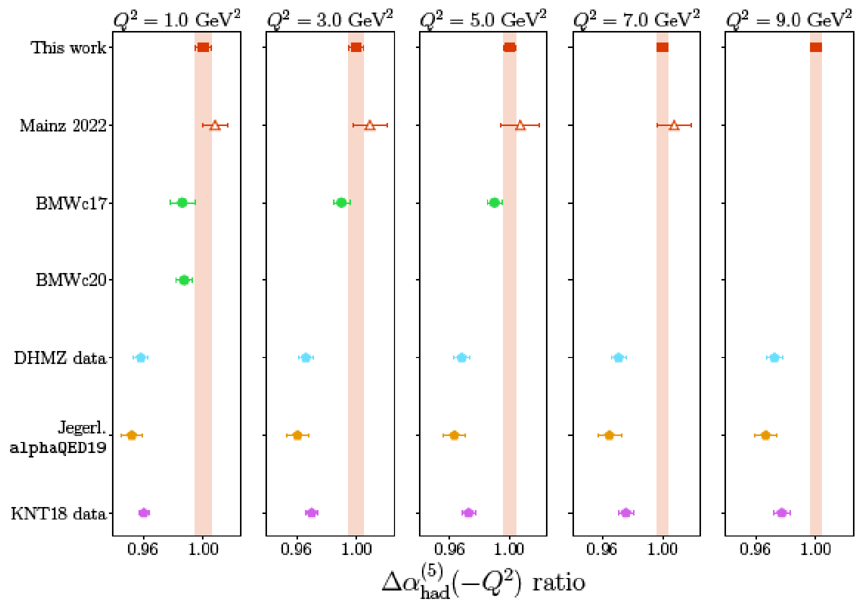
The "classical" evaluation of the hadronic vacuum polarization contributions to the muon anomalous magnetic moment is based on a dispersion relation (with a different kernel) that employs the same R data. Then, it came the lattice...



We moved for a ~4 sigma discrepancy with the experimental result to agreement

Does the g-2 “revolution” imply a $\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$ “revolution”?

Lattice determination of $\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$



Same kind of discrepancy with data driven results as in the hadronic vacuum polarization contributions to the muon anomalous magnetic moment

Conigli et al. 2025

Lattice results:

$$\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = 0.02773 \pm 0.00015$$ Ce' et al. 2022

$$\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = 0.027813(33)_{\rm lat}(35)_{\rm pQCD}$$ Conigli et al. 2025

Data driven:

$$\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = 0.02760 \pm 0.00010$$

From data driven to lattice:

$$\delta m_W \sim -4\,\mathrm{MeV}$$
$$\delta \sin^2\theta^{\rm lept}_{\rm eff} \sim 6.5\times 10^{-5}$$

Not a revolution but not completely negligible

Conclusions

- The SM works fine

- The present level of precision in the theoretically determination of m_W is comparable to the present experimental error. But
  m_W(exp.) = 80.363
  m_W(th.) = 80.353 + 0.004 (data driven)
  m_W(th.) = 80.348 + 0.004 (lattice)
  Lattice is on the low side

- A solid prediction of m_W with a precision below 10 MeV requires a better understanding of the top pole mass (and an improvement in $\Delta\alpha_{\mathrm{had}}^{(5)}(m_z^2)$)

- A prediction of m_W with a precision 1 MeV seems not feasible in the near future